**Taking the risk out of systemic risk measurement**

**by**

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ABSTRACT

Conditional value at risk (CoVaR) and marginal expected shortfall (MES) have been proposed as measures of systemic risk. Some argue these statistics should be used to impose a "systemic risk tax" on financial institutions. These recommendations are premature because CoVaR and MES are ad hoc measures that: (1) eschew statistical inference; (2) confound systemic and systematic risk; and, (3) poorly measure asymptotic tail dependence in stock returns. We introduce a null hypothesis to separate systemic from systematic risk and construct hypothesis tests. These tests are applied to daily stock returns data for over 3500 firms during 2006-2007. CoVaR (MES) tests identify almost 500 (1000) firms as systemically important. Both tests identify many more real-side firms than financial firms, and they often disagree about which firms are systemic. Analysis of hypotheses tests' performance for nested alternative distributions finds: (1) skewness in returns can cause false test rejections; (2) even when asymptotic tail dependence is very strong, CoVaR and MES may not detect systemic risk. Our overall conclusion is that CoVaR and MES statistics are unreliable measures of systemic risk.

**Key Words:** systemic risk, conditional value at risk, CoVaR, marginal expected shortfall, MES, systemically important financial institutions, SIFIs

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**Taking the risk out of systemic risk measurement**

## I. Introduction

A number of economists have proposed using specific measures of stock return tail dependence as indicators for the "systemic risk" created by large financial institutions.[2] In this paper, we focus on two proposed systemic risk measures: conditional value at risk (CoVaR) and marginal expected shortfall (MES). Proponents of these measures have argued that CoVaR and MES statistics should be used as a basis to tax large complex financial institutions to penalize them for the systemic risk that they create [Acharya, Engle and Richardson (2012), Acharya, Pedersen, Philippon and Richardson (2010)] or to indirectly tax these institutions by requiring enhanced regulatory capital and liquidity requirements calibrated using these measures [Adrian and Brunnermeier (2011)].

The argument for using CoVaR and MES to measure of systemic risk is simple enough. Should a systemically important financial institution teeter on the brink of default, its elevated risk of failure will negatively impact the lower tail of the stock return distributions of many other firms in the economy. The loss-tail of firms' return distributions will experience a negative shift because the institution's failure will spread losses throughout the financial sector and choke off credit intermediation to the real economy. Formally, stock returns will have asymptotic left-tail dependence with the stock returns of a systemically important financial institution. Intuitively, CoVaR and MES are empirical measures of the shift in the loss-tail of stock return distributions brought on by a specific conditioning event, so it is reasonable to think they might be useful for identifying and measuring the systemic risk potential of an institution.

The CoVaR measure is the difference between two 1 percent value-at-risk (VaR)[3] measures. First, the 1 percent VaR of a portfolio is calculated using returns conditioned on the event that a single large financial institution experiences a return equal to the 1 percent quantile of its unconditional return distribution. The second step is to subtract the VaR of the same portfolio conditioned on the event that the large financial institution in question experiences a median

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[2] These papers include: Acharya, Engle and Richardson (2012), Acharya, Pedersen, Philippon and Richardson (2010), Adrian and Brunnermeier (2011) and Brownlees and Engle (2012). See Flood, Lo and Valavanis (2012) for a recent survey of this literature. Kupiec (2012) or Benoit, Colletaz, Hurlin and Perignon (2012) provide a critical assessment.

[3] In this literature, a 1 percent VaR measure is defined as the 1 percent quantile of the underlying return distribution.

return. CoVaR is typically estimated using quantile regression on the grounds that such estimates are non-parametric and free from biases that may be introduced by inappropriately restrictive parametric distributional assumptions.

MES is the expected shortfall calculated from a conditional return distribution for an individual financial institution where the conditioning event is a large negative market return realization. Two proposed measures of systemic risk, Expected Systemic Shortfall (SES) and the Systemic Risk Index (SRISK) are simple transformations of the MES that generate an approximation of the extra capital the financial institution will need to survive a virtual market meltdown. The primary input into SES and SRISK measures is the financial institution's MES which is estimated as the institution's average stock return on days when the market portfolio experiences a return realization that is in its 5 percent unconditional lower tail. This measure is non-parametric as it requires no maintained hypothesis about the probability density that generates observed stock return data.

The existing literature argues that CoVaR or MES estimates that are large relative to other financial firm estimates are evidence that a financial institution is a source of systemic risk. The validity of these systemic risk measures is established by showing that virtually all of the large financial institutions that required government assistance (or failed) during the recent financial crisis exhibited large CoVaR or MES measures immediately prior to the crisis. Moreover, the nonparametric nature of CoVaR and the MES estimators has been portrayed as a strength since they avoid biases that may be introduced by inappropriate parametric distributional assumptions.

We identify a number of important weaknesses in the CoVaR and MES literature. One important flaw is that it eschews formal statistical hypothesis tests to identify systemic risk. A second weakness is that the CoVaR and MES measures are contaminated by systematic risk.[4] Firms that have large systematic risk are more likely to produce large (negative) CoVaR and MES statistics even when there is no systemic risk in stock returns.

To address these two short comings, we introduce the null hypothesis that returns have a multivariate Gaussian distribution. The Gaussian distribution is asymptotically tail independent which rules out systemic risk. Using the Gaussian null, we construct classical hypothesis test

[4] See, for example Kupiec (2012) or Benoit, Colletaz, Hurlin and Perignon (2012).

statistics that use alternative CoVaR and MES estimators to detect asymptotic tail dependence. We use Monte Carlo simulations to construct the critical values of the small sample distributions of our two test statistics.

We use daily return data on 3518 firms and the CRSP equally-weighted market portfolio over the sample period 2006-2007 to construct CoVaR and MES hypothesis tests and identify systemically important firms. The CoVaR test identifies roughly 500 firms as sources of systemic risk; the MES test identifies nearly 1000 firms. Both tests identify many more real-side than financial firms, and the tests often disagree about which firms are a source of systemic risk.

To better understand our hypothesis test results, we perform simulations to evaluate the performance of our hypothesis tests for nested alternative return distributions, some with asymptotic tail dependence, and others asymptotically tail independent. Our analysis shows that our hypothesis tests may reject the null hypothesis in the presence of return skewness patterns that are common in the data, even when the return distributions are asymptotically tail independent.[5] Thus a rejection of the null need not be an indication of systemic risk. This finding highlights the potential benefit of choosing a more general null hypothesis to construct the test statistics, but other findings suggest that the search for a better null is probably moot.

The final and fatal weakness we identify is a lack of power: CoVaR and MES measures are unable to reliably detect asymptotic tail dependence, even when asymptotic tail dependence is exceptionally strong. We show that distributions with weak but non-zero asymptotic tail dependence produce CoVaR and MES sampling distributions that significantly overlap the sampling distributions of CoVaR and MES estimators from nested distributions that are asymptotically tail independent. Unless asymptotic tail dependence is very strong, the CoVaR and MES test statistics will have low power meaning that they are unable to reliably distinguish returns with asymptotic tail dependence from returns that come from an asymptotically tail-independent return distribution. Even when asymptotic tail dependence is strong, the CoVaR and MES test statistics have poor power characteristics because, as asymptotic tail dependence increases, the variance of the CoVaR and MES estimates grow significantly. Paradoxically, the

[5] A distribution's skewness can cause the hypothesis tests to under- or over-reject, depending on the pattern of skewness in the data. When the market return distribution is strongly negatively skewed, and individual returns are weakly positively skewed, the tests over-reject the null hypothesis. This pattern is particularly common in our sample of stock returns.

CoVaR and MES systemic risk proxies are noisier and less efficient when there is more systemic risk.

The remainder of the paper is organized as follows. Section II provides a brief overview of the existing literature. Section III applies the CoVaR and MES measures to the daily returns of 3518 individual stocks over the sample period 2006-2007 and highlights many features that are counterintuitive if CoVaR and MES are reliable measuring systemic risk. Section IV derives CoVaR and MES measures when returns are multivariate Gaussian, and it constructs classical hypothesis test statistics and calculates the critical values of their sampling distribution under the Gaussian null. Section V applies our hypothesis tests to the pre-crisis sample of individual sock returns data. Section VI investigates the properties of our hypothesis test statistics under nested alternative return distribution assumptions that both include and exclude asymptotic tail dependence. Section VII provides our summary and conclusions.

## II. Literature Review

### *1. Conditional Value at Risk (CoVaR)*

Let $\tilde{R}_P$ represent the return on a reference portfolio of stocks, $\tilde{R}_j$represent the return on an individual stock, $\tilde{R}_P|R_j = r$ be the return on the reference portfolio conditional on a specific realized value for $\tilde{R}_j$, $R_j = r$, and $VaR(\tilde{R}_j$,p) represent the p-percent value at risk for stock j, or the critical value of the return distribution for $\tilde{R}_j$ below which, there is at most p-percent probability of a smaller return realization.

Adrian and Brunnermeier (2012) define the q-percent CoVaR measure for firm j to be the q-percent quantile of a reference portfolio's conditional return distribution, where the reference portfolio's distribution is conditioned on a return realization for stock j equal to its q-percent VaR value. Using our notation, firm j's q-percent CoVaR is formally defined as:

$$CoVaR\left(\tilde{R}_{P|j}, q\ \right) = VaR[\tilde{R}_P|R_j = VaR\left(\tilde{R}_{j,}\text{q}\right), \text{q}] \qquad (1)$$

Adrian and Brunnermeier define ΔCoVaR is the difference between the stock j's q-percent CoVaR and the stock's median CoVaR defined as stock j's CoVaR calculated conditional on a median market return.

$$\Delta CoVaR(\tilde{R}_{P|j}, q) = CoVaR(\tilde{R}_{P|j}, q) - CoVaR(\tilde{R}_{P|j}, 50\%) \quad (2)$$

Adrian and Brunnermeier (2011) set "q" equal to 1-percent and estimate ΔCoVaR using quantile regressions. They also show that GARCH-based ΔCoVaR estimates yield values similar to quantile regression estimators.  They argue that ΔCoVaR measures how an institution contributes to the systemic risk of the overall financial system.[6]

*2. Marginal Expected Shortfall (MES)*

Acharya, Pedersen, Phillipon, and Richardson (2010) define MES as the marginal contribution of firm j to the expected shortfall of the financial system. Formally, MES for firm j is the expected value of the stock return $\tilde{R}_j$ conditional on the market portfolio return $\tilde{R}_M$ being at or below the sample q-percent quantile.

$$MES(\tilde{R}_j, q) = E(\tilde{R}_j | R_M < VaR(\tilde{R}_M, q)) \quad (3)$$

Higher levels of MES imply that firm j is more likely to be undercapitalized in the bad states of the economy and thus contribute more to the aggregate risk of the financial system.

Acharya, Pedersen, Phillipon, and Richardson (2010) use a 5 percent left-tail market return threshold and estimate MES by taking a selected-sample average. They argue that that MES calculated over the 2006-2007 period can predict stock returns during the crisis.  Brownlees and Engle (2012) and Acharya, Engle and Richardson (2012) refine the MES measure and use dynamic volatility and correlation models to estimate MES from firm and market returns.

***Systemic Expected Shortfall (SES)***

Acharya, Pedersen, Phillipon, and Richardson (2010) define Systemic Expected Shortfall (SES) as the expected undercapitalization of bank *i* when the aggregate banking system as a whole is undercapitalized.  Acharya, Engle, and Richardson (2012) rename SES as SRISK and define SRISK for firm *j*:

$$SRISK_j = \ max\ (\ 0\ ,\ Equity_j\ \ [\ K\ Leverage_j - (1 - K)\ exp(-18\ MES_j)\ ]\ ) \quad (4)$$

[6] Adrian and Brunnermeier (2011) also estimate a measure they call "forward-ΔCoVaR" by projecting estimates of ΔCoVaR on bank-level characteristics.

where $Equity_j$ denotes market capitalization of firm $j$, $K$ is the minimum capital requirement for banks, and leverage is the book value of bank j's debt divided by equity. The authors appeal to extreme value theory to argue that the transformation t $exp\left(-18\ MES_j\right)$ converts MES, which is estimated on moderately bad days into a proxy for MES measured during the extreme event of a systemic banking crisis.[7]

MES only depends on stock return moments while SRISK/SES incorporates additional information about firm size and leverage. Since the underlying intuition in these systemic risk measures is that systemic risk potential can be measured using information compounded in observed stock return distributions, we focus our interest on the MES component of the SES risk measure.

## III. Do CoVaR and MES Statistics Measure Systemic Risk?

The existing CoVaR and MES literature focuses on the stock returns of large financial instructions just prior to the crisis. It argues that CoVaR and MES are measures of systemic risk because there is a high correspondence between institutions that had large MES and CoVaR measures immediately prior to the crisis and institutions that subsequently required extensive government capital injections or failed. While this justification can seem convincing in the context of a selected sample of financial firms, the argument becomes less convincing when it is viewed in a broader context. CoVaR and MES statistics can be calculated for all firms with stock return data and it is informative to see the magnitude of CoVaR and MES statistics for firms outside of the financial services industry.

In this section we calculate the MES and ΔCoVaR measures using the nonparametric methods recommended in the literature for all CRSP stocks with close to 500 days of daily return data in the sample period 2006-2007. We start with all US firms identified in CRSP database between 2006 and 2007. We calculate daily log returns on days where reported closing prices are based on transactions. We exclude security issues other than common stock such as ADRs and REITs.

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[7] MES is estimated on days when the market portfolio realization is in the lower 5 percent tail. This within-sample condition is likely less severe than the returns associated with a financial crisis. Scaled MES is an approximation for the expected losses on financial crisis days. It is based on an extreme value approximation that links expected losses under more extreme events to sample MES estimates based on less restrictive conditions.

We eliminate firms if: the market capitalization is less than $100 million. These filters result in 3518 return series.

Our reference portfolio is the CRSP equally-weighted index. For each firm in the sample, we estimate 1 percent ΔCoVaR statistics using quantile regression.[8] We measure MES for each stock as the average stock returns on days when the equally-weighted market portfolio experiences a return in the 5 percent lower tail of its sampling distribution. The reference portfolio is the CRSP equally-weighted market portfolio.

Table 1 describes the sample. It lists the number of firm in the sample by industry and the average ΔCoVaR and MES estimates for each industry. The ΔCoVaR estimates indicate that, on average, the construction industry has the largest (negative) ΔCoVaR estimates suggesting that, on average, this industry has the most systemic risk. Following the construction industry, according to the ΔCoVaR measure, on average, broker-dealers and mining firms are the next most important industry sources of systemic risk. According to the MES statistic, on average broker-dealers are the largest contributors to systemic risk, followed by mining companies and then the construction industry.

Table 2 lists the fifty companies that exhibit the largest ΔCoVaR measures in descending order of "systemic risk" importance as indicated by the magnitude of their 1-percent ΔCoVaR statistics. Most of the firms listed in Table 1 are part of the "real-side" of the economy and have nothing to do with the financial services sector. Among the firms listed in Table 2 are 7 financial firms. It is very unlikely that anyone would view any of these 7 firms as "systemically important." The depository institution with the largest ΔCoVaR statistic, Citizens First Bancorp, has less than half the indicated systemic risk of Proquest, the company with the largest ΔCoVaR systemic risk measure among traded firms.

Table 3 lists, in descending order, the fifty companies with the largest (negative) MES systemic risk measures. While the top-fifty MES firms are still dominated by the real sector, the MES statistic does generate a list of "high risk" financial firms that did subsequently flounder during the crisis. CompuCredit, E-trade, Countrywide Financial, IndyMac, BankUnited Financial, Net

[8] ΔCoVaR is the difference between the two quantile regression estimates: the 1 percent quantile regression of the market portfolio on an individual firm's return less the estimate of the 50 percent quantile regression of the market return on the individual firm's return.

Bank, Accredited Home Lenders and Fremont General Corporation all experienced serious distress or failed subsequent to the onset of the financial crisis. Still, none of these firms are exceptionally large and none was considered to be systemically important or "too-big-to-fail" during the financial crisis. The firm with the largest MES, China Precision Steel, is not a financial firm. It is not particularly important for the U.S. domestic economy as more than 60 percent of its operations are in China.

The results in Table 1 through 3 demonstrate that when ΔCoVaR and MES statistics are calculated for all traded stocks in the years immediately preceding the crisis (2006-2007), the largest financial institutions that are alleged to be the primary sources of systemic risk for the economy do not even appear among the list of firms with the fifty most negative ΔCoVaR or MES estimates. The omission of the largest financial institutions from either list calls into question previous claims that ΔCoVaR and MES provide accurate measures of a firm's potential to cause systemic risk.

Another troubling feature of MES and ΔCoVaR estimates is that there is a systematic relationship between ΔCoVaR and MES statistics and common market factor beta risk. A significant share of the cross sectional variation in ΔCoVaR and MES statistics can be attributed to variation in firms' systematic risk or the firm's return correlation with the returns from the equally-weighted CRSP portfolio.[9] Figure 1 shows the fit of a regression of the sample individual stock MES estimates on their market model beta coefficients estimates. The simple market model beta coefficient (systematic market risk) explains nearly three-quarters of the cross-section variation in MES estimates. Figure 2 shows the fit of a cross-sectional regression of ΔCoVaR estimates on individual stocks' sample correlation estimates with the equally-weighted market portfolio return. This regression explains nearly 30 percent of the observed cross-sectional variation in the sample ΔCoVaR estimates.

The data clearly show that, the greater a firm's systematic risk, the greater the potential that it produces a large value ΔCoVaR or MES statistic. Hence, to construct a ΔCoVaR or MES-based test for systemic risk, it is first necessary to remove the effects of a firm's systematic risk. In the next section, we construct ΔCoVaR and MES-based test statistics that remove the effects of

[9] Our estimates for ΔCoVaR or MES are similar when we use S&P 500 or value-weighted market portfolio.

systematic risk and common factor correlation that are compounded in the "raw" ΔCoVaR and MES measures.

## IV. Systematic Risk Hypothesis Test based on ΔCoVaR and MES Statistics

To construct classical hypothesis tests to detect systemic from ΔCoVaR and MES statistics, we must adopt a null hypothesis for stock returns that excludes the possibility of systemic risk. Under the null hypothesis, we develop test statistics using ΔCoVaR and MES estimates that allow us to assess the probability that the MES and ΔCoVaR estimates would be observed if the null hypothesis is true. When we observe ΔCoVaR and MES test statistics that are so large that they are highly unlucky to be generated under the null hypothesis return distribution, we reject the null hypothesis in favor of alternative stock return generating process that includes the possibility of systemic risk.

Admissible candidates for the distribution under the null hypothesis must exhibit asymptotic left-tail independence since we formally define systemic risk as asymptotic dependence in the left tail of the distribution. The bivariate Gaussian distribution satisfies the independence condition, is analytically tractable, and given the long history of using the Gaussian distribution to model stock returns, it is a logical distribution to use initially. In the penultimate section of this paper, we discuss the performance of our test statistic under alternative hypotheses that deviate from the Gaussian null hypothesis including alternatives that are asymptotically tail independent and thereby lack systemic risk.

When $(\tilde{R}_j, \tilde{R}_P)$ have a bivariate normal distribution, $\Phi\left[\begin{pmatrix}\mu_j \\ \mu_P\end{pmatrix}, \begin{pmatrix}\sigma_j^2 & \rho\sigma_j\sigma_P \\ \rho\sigma_j\sigma_P & \sigma_P^2\end{pmatrix}\right]$, where $\Phi(a, b)$ represent the Gaussian distribution function with mean "a" and variance "b". The conditional distributions are also normal random variables,

$$\tilde{R}_j|\left(\tilde{R}_P = r_{Pi}\right) \sim \Phi\left[\mu_j + \rho\frac{\sigma_j}{\sigma_P}(r_{Pi} - \mu_P), (1-\rho^2)\sigma_j^2\right] \tag{5a}$$

$$\tilde{R}_P|\left(\tilde{R}_j = r_{ji}\right) \sim \Phi\left[\mu_P + \rho\frac{\sigma_P}{\sigma_j}\left(r_{ji} - \mu_j\right), (1-\rho^2)\sigma_P^2\right] \tag{5b}$$

where $\mu_j$ and $\mu_P$ represent the individual (univariate) return means, $\sigma_j^2$ and $\sigma_P^2$ represent the individual return variances and $\rho$ represents the correlation between the returns.

*Parametric CoVaR for Gaussian Returns*

CoVaR can be measured in two ways. One CoVaR measures the conditional VaR of a reference portfolio conditional on an individual stock experiencing an extreme left-tail return event. A second possible CoVaR calculation (called the Exposure CoVaR) measures the conditional value at risk of an individual stock conditional on the reference portfolio experiencing an extreme left-tail return event. While we derive closed form expressions for both measures, we focus our analysis on the former measure during the rest of the paper.

First we derive the CoVaR measure for the reference portfolio conditioned on the extreme negative return of an individual stock. The conditional distribution function for $\tilde{R}_P$ conditional on $\tilde{R}_j$ equal to its 1 percent value at risk is,

$$\tilde{R}_P | \left(\tilde{R}_j = \Phi^{-1}(.01,\ \tilde{R}_j)\right) \sim \Phi\left[\mu_P + \rho \frac{\sigma_P}{\sigma_j}\left(\Phi^{-1}(.01,\ \tilde{R}_j) - \mu_j\right), (1-\rho^2)\sigma_P^2\right], \tag{6}$$

and observing $\Phi^{-1}(.01,\ \tilde{R}_j) = \mu_j - 2.32635\ \sigma_j$, the 1 percent CoVaR for the portfolio conditional on $\tilde{R}_j$ equal to its 1 percent VaR is,

$$CoVaR\left(\tilde{R}_P | \left(\tilde{R}_j = \Phi^{-1}(.01,\ \tilde{R}_j)\right)\right) = \mu_P - \rho \frac{\sigma_P}{\sigma_j}\left(2.32635\ \sigma_j\right) - 2.32635\ \sigma_P\sqrt{1-\rho^2}\ , \tag{7}$$

The conditional return distribution for the portfolio, conditional on $\tilde{R}_j$ equal to its median is,

$$\tilde{R}_P | \left(\tilde{R}_j = \Phi^{-1}(.50,\ \tilde{R}_j)\right) \sim N[\mu_P, (1-\rho^2)\sigma_P^2]. \tag{8}$$

Consequently, the CoVaR for the portfolio with $\tilde{R}_j$ evaluated at its median return is,

$$CoVaR\left(\tilde{R}_P | \left(\tilde{R}_j = \Phi^{-1}(.50,\ \tilde{R}_j)\right)\right) = \mu_P - 2.32635\sigma_P\sqrt{1-\rho^2}\ , \tag{9}$$

Subtracting (8) from (7) and defining $\beta_{jP} = \frac{Cov(\tilde{R}_j, \tilde{R}_P)}{\sigma_P^2}$, the contribution CoVaR measure is,

$$\Delta CoVaR\left(\tilde{R}_P | \left(\tilde{R}_j = \Phi^{-1}(.01,\ \tilde{R}_j)\right)\right) = -\ \beta_{jP} \cdot 2.32635\ \frac{\sigma_P^2}{\sigma_j} \tag{10a}$$

$$= -\rho \cdot 2.32635\sigma_P \tag{10b}$$

Reversing the order of the conditioning variable (i.e., the CoVaR for $\tilde{R}_j$ conditional on $\tilde{R}_P$ equal to its 1 percent VaR ), it is straight-forward to show that the so-called exposure CoVaR measure is,

$$\Delta CoVaR\left(\tilde{R}_i | \left(\tilde{R}_P = \Phi^{-1}(.01,\ \tilde{R}_P)\right)\right) = -\beta_{jP} \cdot 2.32635\, \sigma_P, \tag{11a}$$

$$= -\rho \cdot 2.32635 \sigma_j \tag{11b}$$

Regardless of which return is used to do the conditioning, both ΔCoVaR measures are negatively related to the correlation between the stock and the reference portfolio return.

*1. Parametric MES for Gaussian Returns*

The marginal expected shortfall measure is the expected shortfall calculated from a conditional return distribution. In Acharya, Pedersen, Philippon and Richardson (2010), the conditioning event is when the return on a reference portfolio, $\tilde{R}_P$, is less than or equal to its 5 percent VaR value. The reference portfolio could be a well-diversified portfolio representing the entire stock market, or a portfolio of bank stocks.

Under the assumption of bivariate normality, the conditional stock return is normally distributed, and consequently,

$$E\left(\tilde{R}_j | \tilde{R}_P = r_P\right) = \mu_j - \rho \frac{\sigma_j}{\sigma_P}\, \mu_P + \rho \frac{\sigma_j}{\sigma_P} r_P. \tag{12}$$

Now, if $\tilde{R}_P$ is normally distributed with mean $\mu_P$ and standard deviation $\sigma_P$, then the expected value of the market return truncated above the value "b" is,

$$E\left(\tilde{R}_P | \tilde{R}_P < b\right) = \mu_P - \sigma_P \left[\frac{\phi\left(\frac{b-\mu_P}{\sigma_P}\right)}{\Phi\left(\frac{b-\mu_P}{\sigma_P}\right)}\right], \tag{13}$$

If b is the lower 5 percent tail value, $b = \mu_P - 1.645\sigma_P$, and the expected shortfall measure is,

$$E\left(\tilde{R}_j | \tilde{R}_P < VaR(\tilde{R}_P, 95\%)\right) = \mu_j - \rho\sigma_j \left[\frac{\phi(-1.645)}{\Phi(-1.645)}\right] \tag{14a}$$

$$= \mu_j - 2.062839\, \sigma_M\, \beta_{jP} \tag{14b}$$

where the constant (2.062839) is a consequence of the 5 percent tail conditioning on the market return, i.e., $\frac{\phi(-1.645)}{\Phi(-1.645)} = 2.062839$.

*2. Systemic Risk Test Statistics when Returns are Gaussian*

Our hypothesis test statistic is constructed as difference between non-parametric and parametric (Gaussian) ΔCoVaR (or MES) estimates, scaled to remove dependence on a volatility parameter. Under the null hypothesis of Gaussian returns, the parametric MES and ΔCoVaR estimators are unbiased and efficient since they are maximum likelihood estimates. Similarly, under the null hypothesis the alternative non-parametric ΔCoVaR and MES estimators are unbiased, but they are not efficient as they do not use any information on the parametric form of the stock return distribution. If the alternative hypothesis is true, nonparametric ΔCoVaR and MES estimators should have expected values that differ from their parametric Gaussian counterparts. Under the alternative hypothesis, the magnitude of the nonparametric estimators reflect tail-dependence in the sample data while their parametric Gaussian estimates do not. Thus, if the alternative hypothesis that stock returns are in part driven by systemic risk is correct, the nonparametric estimators should produce larger (more negative) ΔCoVaR and MES statistics.

Under the null hypothesis, the difference between the two estimators (nonparametric and parametric) has an expected value of 0, but has a sampling error in any given sample. An important issue is whether the variance of this sampling error is independent of the characteristics of the stock and portfolio returns that are being analyzed. It turns out that the difference between the non-parametric and the Gaussian parametric estimators has a sampling error that depends on both the return correlation and a volatility parameter. We can control for volatility dependence by normalizing the differences between the nonparametric and parametric measures by an estimate of the relevant volatility parameter, but we are still left with the returns correlation as a "nuisance" parameter that must be controlled for when we construct our small sample Monte Carlo test statistic simulations.

Let $\Delta\widetilde{CoVaR}$ represent the non-parametric quantile regression estimator for the contribution ΔCoVaR measure. Let $\Delta\widehat{CoVaR}$ represent the sample parametric Gaussian estimator for ΔCoVaR, and let $\widehat{\sigma_P}$ represent the sample standard deviation of the returns on the reference portfolio. We define the CoVaR test statistic as,

$$\kappa_{CoVaR} = -\frac{\Delta\widetilde{CoVaR} - \Delta\widehat{CoVaR}}{\widehat{\sigma_P}}. \quad (15)$$

Under the Gaussian null hypothesis, it can be easily demonstrated the sampling distribution of $\kappa_{CoVaR}$ depends only on the correlation parameter between the stock returns and the returns on the reference portfolio.

Under the alternative hypothesis of systemic risk, $\Delta\widetilde{CoVaR} - \Delta\widehat{CoVaR}$ is expected to be negative, and since $\hat{\sigma}_P$ is positive, systemic risk is evident when the test statistic produces a large positive value. Statistical significance is determined by comparing the test value for $\kappa_{CoVaR}$ with its sampling distribution under the null. When the test value of $\kappa_{CoVaR}$ is in the far right-hand tail of its sampling distribution, we can reject the null hypothesis of no systemic risk. The critical value used to establish statistical significance determines the type 1 error rate for the test. For example, rejecting the null hypothesis for test values at or above the 95 percent quantile of the sampling distribution for $\kappa_{CoVaR}$ is consistent with a 5 percent type 1 error meaning there is at most 5 percent chance of rejecting a true null hypothesis.

Let $\widetilde{MES}$ represent the non-parametric estimator for MES. The literature defines it as the average individual stock return on days when the reference portfolio has a return realization in its lower tail. We condition on reference portfolio returns in the 5 percent tail. Let $\widehat{MES}$ represent the sample parametric Gaussian estimator for MES and $\hat{\sigma}_i$ represent the sample standard deviation of the individual stock return in the calculation. We define the MES test statistic as,

$$\kappa_{MES} = -\frac{\widetilde{MES} - \widehat{MES}}{\hat{\sigma}_i} \quad (17)$$

Under the alternative hypothesis of systemic risk, $\widetilde{MES}$ is expected to produce a larger negative number compared to $\widehat{MES}$, and since $\hat{\sigma}_i$ is positive, systemic risk is evident when the test statistic produces a large positive value. As in the ΔCoVaR case, this test statistic still depends on the correlation between the stock and the reference portfolio, so correlation is a "nuisance" parameter that enters into the sampling distribution critical value calculations.

In Table 4 we report the small sample distribution 1, 5 and 10 percent critical value estimates for the $\kappa_{CoVaR}$ and $\kappa_{MES}$ for 12 different portfolio-stock return correlation assumptions between -0.2

and 0.9.[10] The critical values are calculated using Monte Carlo Simulation[11] for a sample size of 500 observations, the equivalent of about two years of daily data. We focus on a two-year estimation window because the characteristics of institutions, especially large financial institutions, change very quickly over time through mergers and acquisitions. The critical value statistics we report are based on 50,000 Monte Carlo simulations.

## V. Systemic Risk Test Application to 2006-2007 Stock Returns Data

*1. Estimation Methodology*

We use daily CRSP stock return data from the period 2006-2007 to calculate $\kappa_{CoVaR}$ and $\kappa_{MES}$. The data set construction is discussed in Section III.

### Nonparametric CoVaR

We estimate the nonparametric ΔCoVaR statistic, $\Delta\widetilde{CoVaR}$, in three steps:

- We run a 1-percent quantile regression of the CRSP equally weighted market return, $R_M$ on $R_j$ and estimate $\widehat{\beta_q}$, the stock return coefficient in the quantile regression.
- Estimate the 1-percent sample quantile and the median of the firm's stock return, $R_j$, $\widetilde{VaR}(R_j, q)$ and $\widetilde{VaR}(R_j, 0.50)$.
- Nonparametric ΔCoVaR estimator is defined as:

$$\Delta\widetilde{CoVaR}\left(R_M \,|R_j = VaR(R_j, q)\right) = \widehat{\beta_q}\left(\widetilde{VaR}(R_j, q) - \widetilde{VaR}(R_j, 0.50)\right) \tag{18}$$

We estimate our parametric ΔCoVaR statistic, $\Delta\widehat{CoVaR}$, using equation (11) and the sample moments of individual stock returns and the equally-weighted market portfolio returns.

[10]Under the null hypothesis, we use Monte Carlo simulations to construct the sampling distribution for the hypothesis test statistics. The 90th, 95th, and 99th percentiles of the estimated sampling distribution determine, respectively, the 10 percent, 5 percent and 1 percent critical values of the test statistic. If the null hypothesis true, and we reject the null hypothesis if the sample statistic exceeds the critical threshold value, the type I error is 10 percent, 5 percent or 1 percent respectively because there is less than a 10 percent, 5 percent or 1 percent probability of observing a larger value than the threshold in any random sample of 500 observations if the null hypothesis is true.

[11] We use the quantile regression package QUANTREG in R written by Roger Koenker.

**Nonparametric MES**

We estimate nonparametric MES, $\widetilde{MES}$, as the average of individual stock returns on sample subset of days that correspond with the 5 percent worst days of the equally-weighted broad stock market index.

$$\widetilde{MES}\left(R_j, 5\%\right) = \frac{\sum R_j \, I\left(R_m < VaR(R_m, 5\%)\right)}{\sum I\left(R_m < VaR(R_m, 5\%)\right)} = \frac{1}{N} \sum_{R_m < VaR_{5\%}} R_j \tag{19}$$

where I(.) is the indicator function and N is the number of 5 percent worst days for the market.

We measure parametric MES, $\widehat{MES}$, using expression (14) and sample moments for individual stock returns and the returns on the equally-weighted market portfolio.

*2. Hypothesis Test Results*

Table 5 summarizes $\kappa_{CoVaR}$ and $\kappa_{MES}$ hypothesis test results. Evaluated at the 5 percent level of the test, the $\kappa_{CoVaR}$ test identifies 496 of 3518 firms as systemically important, or 14 percent of all sample firms. Ranked by industry, the mining industry has the largest share of firms identified as systemically important (36.4 percent), followed by "other financial" (25.7 percent) and wholesale trade (22.1 percent). Among the remaining financial services industries, broker dealers have the largest share of firms identified as systemically important (14.5 percent), followed by insurance (13 percent) and depository institutions (9.4 percent).

The $\kappa_{MES}$ test identifies a much larger number of firms as systemically important. At the 5 percent level, the MES test identifies 979 firms as potential sources of systemic risk, or 28 percent of the firms in the sample. Among specific industries, public administration has the largest share of firms with systemic risk potential (54.5 percent), followed by broker dealers (49.1 percent), and transportation, communications and utilities (46.9 percent). Among the remaining financial services industries, insurance had the largest share of firms identified as systemically risky (43.5 percent), followed by depository institutions (37.7 percent) and other financial (28.4 percent).

These formal hypothesis tests identify many more firms as sources of systemic compared to previous papers that examined only a small subset of financial institutions. And, unlike prior papers, because we examine all firms with actively traded equity, we identify many real-side firms as systemically important. If systemic risk is manifest as asymptotic tail dependence in stock returns, and if our MES and CoVaR tests reliably identify tail dependence, then a lot firms will require systemic risk taxes or heightened prudential standards as prophylactic measures to keep the financial system safe. Clearly, the issue of the reliability of our test statistics is central. The Gaussian null hypothesis is very restrictive and it is important to understand whether commonly observed non-Gaussian stock return characteristics can lead to a test rejection even when stock returns are asymptotically tail independent.

## VI. Do $\kappa_{CoVaR}$ and $\kappa_{MES}$ Rejections Detect Asymptotic Tail Dependence?

Let $(\widetilde{R_1}, \widetilde{R_2})$ represent a bivariate random vector with individual univariate marginal distributions defined by $F_1(r) = Pr(\tilde{R}_1 \leq r)$, and $F_2(r) = Pr(\tilde{R}_2 \leq r)$. Let $L(u)$ represent the conditional probability,

$$L(u) = Pr(F_1(R_1) < u |\, F_2(R_2) < u) \tag{20}$$

The asymptotic left tail dependence between $\widetilde{R_1}$ and $\widetilde{R_2}$ is defined as, $L = \lim_{u \to 0} L(u)$. If this limit is 0, the random variables are asymptotically independent in their left tails. If the limit is positive, then the random variables have asymptotic tail dependence in the left tail. The larger is the value of the limit, the stronger is the left-tail dependence.

Many distributions can be used to model stock returns, and among these, any distribution that exhibits asymptotic left-tail independence is a candidate for use as the null hypothesis in the construction of our CoVaR and MES test statistics. In this section, we consider the properties of the Gaussian null hypothesis against a family of alternative distributions nested within the bivariate skew-t distribution. Depending on parameter values, the bivariate skew-t-distribution nests the bivariate skewed Gaussian distribution and the bivariate symmetric student-t and Gaussian distributions.

Let $d$ be the dimension of the multivariate distribution. Let $y$ be a $(d \times 1)$ random vector, and $\beta$ and $\alpha$ $(d \times 1)$ vectors of constants. $\Omega$ is a $(d \times d)$ positive definite matrix. $\nu \in (0, \infty)$ is the scalar that represents the degrees of freedom parameter. The multivariate skew-t density $y \sim f_T(y, \beta, \Omega, \alpha, \nu)$ is defined in Azzalini (2005):

$$f_T(y, \beta, \Omega, \alpha, \nu) = 2t_d(y; \beta, \Omega, \nu)T_1\left(\alpha^T\omega^{-1}(y-\beta)\left(\frac{\nu+d}{Q_y+\nu}\right)^{0.5}; \nu+d\right) \quad (21)$$

where, $$t_d(y; \beta, \Omega, \nu) = \frac{\Gamma\left(0.5(\nu+d)\right)}{|\Omega|^{0.5}(\pi\nu)^{d/2}\Gamma(0.5\nu)\left(1+Q_y/\nu\right)^{(\nu+d)/2}}$$

is the density function of a d-dimensional student t variate with $\nu$ degrees of freedom, and $T_1(x;\nu+d)$ denotes the scalar t distribution with $\nu+d$ degrees of freedom. $\beta$ is the location parameter which controls the distribution means, $\alpha$ is the parameter which controls the skewness of the distribution,[12] and Ω is a generalized covariance matrix.[13] The remaining parameters are, $\omega = diag(\Omega)^{0.5}$ and $Q_y = (y-\beta)^T\Omega^{-1}(y-\beta)$.

The skew-t distribution nests the symmetric t-distribution, $(\alpha = 0, 0 < \nu < \infty)$, the skewed Gaussian distribution, $(\alpha \neq 0, \nu = \infty)$, and the symmetric Gaussian distribution, $(\alpha = 0, \nu = \infty)$. The flexibility of the skew-t distribution facilities the process of evaluating the behavior of the ΔCoVaR and MES hypothesis test statistics under a number of plausible alternative hypothesis.

It is well-known that the Gaussian distribution is asymptotically tail independent. The skewed-Gaussian distribution is also asymptotically tail independent [Bortot (2010)]. Both the skewed and symmetric t-distributions have asymptotic tail dependence where the tail dependence depends on the degrees of freedom parameter, the generalized correlation parameter, and the skewness parameters.

The expression for the asymptotic left tail dependence of a bivariate symmetric t distribution is given by,

$$L_{sym\,t} = 2T_1\left(-\frac{\sqrt{(\nu+1)(1-\rho)}}{\sqrt{1+\rho}}, \nu+1\right) \quad (22)$$

where $T_1(x, \nu)$ is the distribution function for a univariate t distribution with $\nu$ degrees of freedom. Figure 3 illustrates the relationship between the asymptotic tail dependence in the symmetric bivariate t distribution and the distribution's correlation and degrees of freedom

[12] α is not equal to but monotonically related to skewness. α=0 implies a symmetric non-skewed distribution, whereas $\alpha > 0$ ($\alpha < 0$) implies positive (negative) skewness. It is also called the "shape" or the "slant" parameter.
[13] Ω is equal to the covariance matrix of y only when the skewness and kurtosis are 0 (α=0, ν=∞).

parameters. The higher the bivariate correlation and smaller the degrees of freedom, the stronger the distribution's asymptotic tail dependence.

In most cases, there is no closed-form expression for the asymptotic tail dependence of a bivariate skew-t distribution. In the special case where the skew is identical across the individual random variables, $\alpha_1 = \alpha_2 = \alpha$ , the distribution's asymptotic tail dependence is given by

$$L_{skew-t} = K(\alpha, \nu, \rho)\ \ 2T_1\left(-\frac{\sqrt{(\nu+1)(1-\rho)}}{\sqrt{1+\rho}}, \upsilon + 1\right), \tag{23}$$

where, $$K(\alpha, \nu, \rho) = \frac{T_1\left(2\alpha\sqrt{\frac{(\nu+2)(1+\rho)}{2}}; \nu+2\right)}{T_1\left(\frac{\alpha(1+\rho)\sqrt{\nu+1}}{\sqrt{1+\alpha^2(1-\rho^2)}}; \nu+1\right)}$$

Figure 4 illustrates the relationship between strength of the asymptotic tail dependence for the skew-t distribution and the degrees of freedom parameter, $\nu$, correlation parameter $\rho$, and skewness parameters $\alpha_1 = \alpha_2 = \alpha$. We observe that negative skewness increases tail dependence, but only when the degree of freedom is low, that is when tails of the distribution are fat.

We estimated the parameters for the bivariate skew-t distribution using maximum likelihood for each of the 3518 stocks in our sample paired with the CRSP equally-weighted market return.[14] The subscript 1 (2) indicates a parameter estimate that is associated with the individual stock return (equally-weighted CRSP portfolio return). The distribution of resulting parameter estimates is reported in Table 6. Key parameters of interest in Table 6 are $\alpha_1, \alpha_2,\ \upsilon$, and $\rho$.

Table 6 shows that, for most of the stocks in the sample over the 2006-2007 sample period, $\hat{\alpha}_1 > 0$, indicating that most individual stock's return distributions are positively skewed. This finding is consistent with the literature [e.g., Singleton and Wingender (2006), or Carr et al, (2002)].

$\alpha_2$ is the parameter that determines the skewness of the market return distribution. While its magnitude varies among the 3518 bivariate maximum likelihood estimations, in all cases, $\hat{\alpha}_2 < 0$, indicating that the market return distribution is negatively skewed.[15] Again, a negatively

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[14] We used the "sn" package in "R" written by Adelchi Azzalini

[15] The market skewness parameter is estimated simultaneously with the individual stock's skewness parameter and the single degrees of freedom parameter for the bivariate distribution. Ideally we would prefer to estimate the entire

skewed market return distribution is consistent with the literature [e.g., Fama (1965), Duffee (1995), Carr et al, (2002)), or Adrian and Rosenberg (2008)].

The degrees of freedom parameter, $\upsilon$, is the most important determinant of the asymptotic tail dependence of the bivariate distribution. The results in Table 6 show that in more than 75 percent of the sample, $\upsilon < 4.5$.

Our test statistics are derived under the null hypothesis that stock returns are bivariate normal. The results in Table 6 suggest that the Gaussian null hypothesis is incorrect in a large number of cases as indicated by non-zero skewness parameter estimates and a predominance of small values for the degrees of freedom estimate. It is important to understand how alternative return distributions affect the sampling distribution of our test statistics. For example, will return distributions with asymptotic tail dependence reliably cause our test statistic to reject the Gaussian null? Will any degree of asymptotic tail dependence cause a rejection of the null hypothesis or must the asymptotic tail dependence reach a critical strength before it is detected? Will skewness trigger a rejection of null hypothesis even when returns have no asymptotic tail dependence? The bivariate skew-t distribution provides an ideal model of stock returns that can be used to investigate all of these questions.

In the remainder of this section we generate simulated stock return data from alternative parameterizations of the skew-t distributions and estimate the sampling distribution for our hypothesis test statistics using Monte Carlo simulation and kernel density estimation. For alternative parameterizations, we calculate our hypothesis test statistics for 10,000 samples of 500 observations. We consider alternative parameterizations of the skew-t distribution that use parameter values that more realistically represent the underlying distribution of the actual stock return series.

We first investigate whether $\kappa_{CoVaR}$ and $\kappa_{MES}$ have statistical power against alternative hypothesis that include negative asymptotic tail dependence. Figure 5 plots the sampling distribution for the $\kappa_{CoVaR}$ test statistic under the Gaussian null hypothesis distribution and two

multivariate (3518 dimensions) skew-t distribution simultaneously and thereby get one set of parameters for the market portfolio. However, with only 500 daily observations, the joint return system not be identified unless additional structural simplification are imposed to reduce the number of independent parameters that must be estimated.

symmetric t distributions with degrees of freedom that are characteristic of the estimates produced in our cross section of stock returns.

Recall that the asymptotic tail dependence of the t distribution is largest when the correlation is strong and the degrees of freedom parameter is small. Across the panels in Figure 5, as $\rho$ increases, we plot the sampling distributions for $\kappa_{CoVaR}$ test statistics as the underlying distributions have stronger and stronger asymptotic tail dependence. For most of the distributions, the sampling distribution for $\kappa_{CoVaR}$ under the null hypothesis substantially overlaps the sampling distributions for $\kappa_{CoVaR}$ under the alternative hypothesis, and the alternative hypothesis sampling distributions have relatively little cumulative probability to the right of the 5 percent critical value under the null hypothesis. This relationship indicates that is highly probable that, should the alternative hypothesis be true, the calculated test statistics may not reject the null. In other words, the $\kappa_{CoVaR}$ test has lower power for detecting asymptotic tail dependence. This is especially true for $\rho = 0$ and $\rho = .2$ panels when the asymptotic tail dependence of the alternative t distributions is under 30 percent.

Figure 6 repeats the symmetric t distribution power simulations for the $\kappa_{MES}$ test statistic and the results are similar to those for the $\kappa_{CoVaR}$ statistic. While the $\kappa_{MES}$ test has slightly better power characteristics compared to $\kappa_{CoVaR}$, it still has poor power unless the returns have very strong asymptotic tail dependence. For example, the $\rho$=.7 panel shows that, even when asymptotic tail dependence is 0.489, the power of the $\kappa_{MES}$ test is still only about 50 percent meaning that if the alternative hypothesis of tail dependence is true, the test will not reject the null hypothesis about half the time.

What is more, the figures also show that as, asymptotic tail dependency increases, the variance of $\kappa_{CoVaR}$ and $\kappa_{MES}$ test statistics grow significantly. Hence, almost paradoxically, these test statistics become noisier and less efficient when the distribution has greater systemic risk. This tradeoff between the magnitude of systemic risk to be detected and the efficiency of the tests statistic estimators may be small sample issue. Relatively short time series (about 500 daily observations) are too small to effectively detect tail dependence. However, the use of longer time series is impractical because the characteristics of the individual firms, especially the largest financial institutions, can change dramatically within a period of only a few years.

Taken together Figures 5 and 6 make a strong case against using ΔCoVaR and MES as measures of systemic risk. Even after correcting for market risk, and developing a formal hypothesis test, the ΔCoVaR and MES are not going to be able to accurately distinguish returns with asymptotic tail dependence from returns that are independent in the asymptotic tails. While the lack of statistical power calls into question the value of further research on refining our ΔCoVaR and MES hypothesis test statistics, it is still valuable to better understand the consequences of adopting the Gaussian null hypothesis.

Figures 7 and 8 analyze the behavior of the $\kappa_{CoVaR}$ and $\kappa_{MES}$ test statistics under alternative return distributions in which market returns are symmetric Gaussian, but individual firm returns are positively skewed Gaussian. Under these alternative distributions, returns are asymptotically independent and the null hypothesis should not be rejected. Figures 7 and 8 show that the sampling distributions for $\kappa_{CoVaR}$ and $\kappa_{MES}$ test statistics under the alternative hypothesis are very similar to the sampling distribution under the null, and there is little risk that positively skewed Gaussian individual returns (with symmetric Gaussian market returns) will generate many false rejections of the null.

Figures 9 and 10 analyze the behavior of the $\kappa_{CoVaR}$ and $\kappa_{MES}$ test statistics under alternative return distributions in which market returns are negatively skewed Gaussian, but individual firm returns are symmetric Gaussian. Returns are asymptotically independent and the null hypothesis should not be rejected. Figures 9 and 10 clearly show that negatively skewed market returns can cause false rejection of the null hypothesis. This is especially true when the individual stocks and the market are highly positively correlated. The $\kappa_{CoVaR}$ seems to be more susceptible to this type of error, but for both statistics, the Gaussian null hypothesis critical values will cause false rejections that could be misinterpreted as evidence of tail dependence and systemic risk.

Figures 11 and 12 analyze the behavior of the $\kappa_{CoVaR}$ and $\kappa_{MES}$ test statistics under alternative return distributions in which market returns are negatively skewed Gaussian, and individual firm returns are positively-skewed Gaussian. Negative market skewness with positive individual stock skewness is the most prevalent pattern observed in the data. Here again, returns are asymptotically independent and the null hypothesis should not be rejected. The pattern in Figures 11 and 12 is similar. Mild positive individual stock skewness with relatively strong negative market skewness and strong positive return correlation is a pattern that is likely to generate false rejections of the null hypothesis. This particular skewness and correlation pattern is predominant in the 2006-2007 sample data suggesting that the large number of rejections in our sample may owe in part to an overly restrictive null hypothesis. Generalizing the test statistic to incorporate a skewed Gaussian distribution under the null would be a step in the right direction, but again the poor ΔCoVaR and MES power characteristics show there will be limited benefits for further refinements.

## VII. Summary and Conclusions

In this paper, we develop a new methodology to control for systematic risk biases inherent in CoVaR and MES risk measures and construct classical hypothesis tests for the presence of systemic risk. The methodology and test statistics are based on the Gaussian model of stock returns. We use Monte Carlo simulation to estimate the critical values of the sampling distributions of our proposed test statistics and use these critical values to test for evidence of systemic risk in a wide cross section of stocks using daily return data over the period 2006-2007.

Our methodology introduces formal hypothesis tests to detect systemic risk, tests which heretofore have been absent from the literature. However, our hypothesis tests are composite tests, and as a consequence they do not provide an ideal solution to the CoVaR and MES measurement issues we identify. Our hypothesis tests for MES and ΔCoVaR require a maintained hypothesis of a specific return distribution under the null hypothesis. The composite nature of the null hypothesis is problematic because it can lead to false indications of systemic risk. For example, we show that, depending on the return generating process, our tests reject may the null hypothesis of no systemic risk even when returns are generated by a tail-independent distributions. Thus, the choice of a specific return distribution to characterize returns under the null hypothesis is a crucial aspect of systemic risk test design that has garnered little attention in the literature. While this finding suggests a research agenda focused on clarifying the nature of stock returns under null hypothesis, our findings on the power of ΔCoVaR and MES tests suggest that these efforts would be wasted.

In particular, our simulation results suggest that ΔCoVaR and MES statistics are unlikely to detect asymptotic tail dependence unless the tail dependence is very strong. And even in cases with strong tail dependence, the power of these tests is limited. The power limitations are a consequence of sampling variance of ΔCoVaR and MES statistics. In small samples (e.g. 500

observations), as tail dependence strengthens, the variance of the MES and CoVaR estimators increases. If systemic risk is truly manifest as asymptotic left-tail dependence in stock returns, ΔCoVaR and MES seem incapable of providing a reliable measure of a firm's systemic risk potential.

## References


Acharya, V., Engle, R., Richardson, M., 2012. "Capital Shortfall: A New Approach to Ranking and Regulating Systemic Risks", *The American Economic Review* 102, 59-64.

Acharya, V. V., Pedersen, L., Philippon, T., and Richardson, M., 2010. "Measuring Systemic Risk", Technical report, Department of Finance, Working paper, NYU Stern School of Business.

Adrian, T. and Brunnermeier, M. K., 2011. "CoVaR", FRB of New York. Staff Report No. 348.

Adrian, Tobias, and Joshua Rosenberg, 2008. "Stock Returns and Volatility: Pricing the Short-Run and Long-Run Components of Market Risk, *Journal of Finance*, Vol. 63, No. 6, pp. 2997-3030.

Azzalini, A., (2005). Skew-normal distribution and related multivariate families. *Scandinavian Journal of Statistics*, Vol 32, pp. 159-188.

Azzalini, A. and A. Capitanio, (2003). Distributions generated by perturbation of symmetry with emphasis on a multivariate skew t distribution. *Journal of the Royal Statistical Society*, B, Vol 65, pp. 367-389.

Benoit, S. G. Colletaz, C. Hurlin, and C. Perignon, 2012. "A Theoretical and Empirical Comparison of Systemic Risk Measures." Working Paper, University of Orleans, France.

Bisias, D., Flood, M., Lo, A., and Valavanis, S., 2012. A survey of systemic risk analytics. U.S. Department of Treasury, Office of Financial Research Working Paper.

Bortot, Paola, 2010, Tail dependence in bivariate skew-Normal and skew-t distributions. Department of Statistical Sciences, Working paper, University of Bologna

Brownlees, C. and Engle, R., 2012. "Volatility, Correlation and Tails for Systemic Risk Measurement", Working Paper, New York University.

Carr, P., H. Geman, D. B. Madan, and M. Yor, (2002). The fine structure of asset returns: An empirical investigation. The Journal of Business, 75(2), 305-333.

Duffee, G. R., 1995. "Stock Returns and Volatility: A Firm-Level Analysis. *Journal of Financial Economics,* Vol. 37, pp. 399-420.

Fama, E. F., 1965. "The Behavior of Stock Market Prices," *Journal of Business,* Vol. 38, pp. 34-105.

Kupiec, Paul, 2012. "Discussion of Acharya, V.; Engle, R.; Richardson, M., 'Capital Shortfall: A New Approach to Ranking and Regulating Systemic Risks,' American Economic Association Meetings, Chicago, Ill., January 2012.

Kupiec, Paul, 1998. "Stress Testing in a Value at Risk Framework," *The Journal of Derivatives,* Vol. 6, No. 1.

Singleton, Clay, and J. Wingender, 1986. "Skewness Persistence in Common Stock Returns," *The Journal of Financial and Quantitative Analysis,* Vol. 21, No. 3, pp. 335-341.

**Table 1: Sample Characteristics by Industrial Classification**

ΔCoVaR is estimated as the difference between two linear quantile regressions estimates of the market portfolio return on individual stock returns: the 1 percent quantile estimate less the 50 percent quantile estimate. The reference portfolio is the CRSP equally-weighted market index. Quantile regressions are estimated using the R package Quantreg. MES is estimated as the average of individual stock returns on sample subset of days that correspond with the 5 percent worst days of the equally-weighted CRSP stock market index.

| Industry | Number of Firms | Mean ΔCoVaR | Mean MES |
|---|---|---|---|
| Broker Dealers | 55 | -0.0321 | -0.0102 |
| Construction | 41 | -0.0345 | -0.0095 |
| Depository Institutions | 426 | -0.0196 | -0.0067 |
| Insurance | 138 | -0.0206 | -0.0075 |
| Manufacturing | 1336 | -0.0231 | -0.0076 |
| Mining | 151 | -0.0272 | -0.0100 |
| Other Financial | 74 | -0.0267 | -0.0087 |
| Public Administration | 11 | -0.0085 | -0.0033 |
| Retail Trade | 227 | -0.0231 | -0.0075 |
| Services | 626 | -0.0213 | -0.0070 |
| Transportation, Communications | 320 | -0.0211 | -0.0086 |
| Wholesale Trade | 113 | -0.0234 | -0.0084 |

## Table 2. Fifty Firms with the Largest ΔCoVaR Estimates over 2006-2007

ΔCoVaR is estimated as the difference between two linear quantile regressions estimates of the market portfolio return on individual stock returns: the 1 percent quantile estimate less the 50 percent quantile estimate. The reference portfolio is the CRSP equally-weighted market index. Quantile regressions are estimated using the R package Quantreg. The median market capitalization is the median value of the closing share price times the number of shares outstanding.

| Rank | Company Name | Median Market Capitalization ($thous) | Industry | CoVaR |
|---|---|---|---|---|
| 1 | PROQUEST CO | 370,338 | Manufacturing | -0.0400 |
| 2 | TETRA TECHNOLOGIES INC | 1,739,641 | Manufacturing | -0.0246 |
| 3 | NEWSTAR FINANCIAL INC | 516,271 | Other Financial | -0.0245 |
| 4 | IBASIS INC | 324,017 | Transportation, Communications | -0.0226 |
| 5 | ISILON SYSTEMS INC | 852,653 | Manufacturing | -0.0215 |
| 6 | NEOWARE SYSTEMS INC | 265,074 | Manufacturing | -0.0207 |
| 7 | MATTSON TECHNOLOGY INC | 493,509 | Manufacturing | -0.0207 |
| 8 | TITAN INTERNATIONAL INC ILL | 397,960 | Manufacturing | -0.0203 |
| 9 | VALERO ENERGY CORP NEW | 37,153,498 | Manufacturing | -0.0200 |
| 10 | ROBERT HALF INTERNATIONAL INC | 6,167,046 | Services | -0.0200 |
| 11 | DOVER CORP | 9,885,112 | Manufacturing | -0.0196 |
| 12 | MURPHY OIL CORP | 9,977,096 | Manufacturing | -0.0194 |
| 13 | VENOCO INC | 814,278 | Mining | -0.0192 |
| 14 | CONSTELLATION BRANDS INC | 4,953,574 | Manufacturing | -0.0188 |
| 15 | CITIZENS FIRST BANCORP INC | 203,931 | Depository Institutions | -0.0187 |
| 16 | M S C INDUSTRIAL DIRECT INC | 2,149,047 | Manufacturing | -0.0186 |
| 17 | SMURFIT STONE CONTAINER CORP | 2,928,624 | Manufacturing | -0.0184 |
| 18 | SOUTHERN COPPER CORP | 16,450,064 | Mining | -0.0180 |
| 19 | STILLWATER MINING CO | 1,095,439 | Mining | -0.0180 |
| 20 | A M R CORP DEL | 5,753,965 | Transportation, Communications | -0.0179 |
| 21 | AMERIPRISE FINANCIAL INC | 13,095,030 | Broker Dealers | -0.0178 |
| 22 | NASDAQ STOCK MARKET INC | 3,617,941 | Broker Dealers | -0.0178 |
| 23 | U A L CORP | 4,264,734 | Transportation, Communications | -0.0177 |
| 24 | PARKER HANNIFIN CORP | 9,950,826 | Manufacturing | -0.0176 |
| 25 | TECH DATA CORP | 2,065,805 | Wholesale Trade | -0.0176 |
| 26 | ORMAT TECHNOLOGIES INC | 1,368,098 | Transportation, Communications | -0.0175 |
| 27 | WESTERN REFINING INC | 1,830,752 | Manufacturing | -0.0175 |
| 28 | AMERICA SERVICE GROUP INC | 152,270 | Services | -0.0174 |
| 29 | CIRCOR INTERNATIONAL INC | 568,005 | Manufacturing | -0.0174 |
| 30 | CATERPILLAR INC | 46,739,598 | Manufacturing | -0.0174 |
| 31 | RADIO ONE INC | 595,811 | Transportation, Communications | -0.0174 |
| 32 | CIMAREX ENERGY CO | 3,213,883 | Mining | -0.0173 |
| 33 | EMPIRE RESOURCES INC DEL | 104,041 | Wholesale Trade | -0.0173 |
| 34 | CLEVELAND CLIFFS INC | 2,149,713 | Mining | -0.0173 |
| 35 | R P C INC | 1,463,770 | Mining | -0.0173 |
| 36 | EVERCORE PARTNERS INC | 175,262 | Other Financial | -0.0172 |
| 37 | PROCENTURY CORP | 194,361 | Insurance | -0.0172 |
| 38 | KOMAG INC | 1,111,125 | Manufacturing | -0.0172 |
| 39 | LEGG MASON INC | 12,749,954 | Broker Dealers | -0.0171 |
| 40 | PULTE HOMES INC | 7,395,542 | Construction | -0.0171 |
| 41 | DYNAMEX INC | 244,848 | Transportation, Communications | -0.0170 |
| 42 | GATEHOUSE MEDIA INC | 745,417 | Manufacturing | -0.0170 |
| 43 | FREEPORT MCMORAN COPPER & GOLD | 11,781,130 | Mining | -0.0170 |
| 44 | ROWAN COMPANIES INC | 4,092,873 | Mining | -0.0170 |
| 45 | MOSAIC COMPANY | 9,378,757 | Manufacturing | -0.0170 |
| 46 | HOME DIAGNOSTICS INC | 198,165 | Manufacturing | -0.0169 |
| 47 | OIL STATES INTERNATIONAL INC | 1,680,397 | Manufacturing | -0.0169 |
| 48 | COOPER CAMERON CORP | 6,046,972 | Manufacturing | -0.0168 |
| 49 | HANMI FINANCIAL CORP | 910,360 | Depository Institutions | -0.0168 |
| 50 | MARINE PRODUCTS CORP | 358,240 | Manufacturing | -0.0168 |

## Table 3. Fifty Firms with the Largest MES Estimates over 2006-2007

MES is estimated as the average of individual stock returns on sample subset of days that correspond with the 5 percent worst days of the equally-weighted CRSP stock market index. The median market capitalization is the median value of the closing share price times the number of shares outstanding.

| Rank | Company Name | Median Market Capitalization ($thous) | Industry | MES |
|---|---|---|---|---|
| 1 | CHINA PRECISION STEEL INC | 191,188 | Manufacturing | -0.0780 |
| 2 | FREMONT GENERAL CORP | 1,083,450 | Depository Institutions | -0.0771 |
| 3 | ESCALA GROUP INC | 177,063 | Services | -0.0742 |
| 4 | ACCREDITED HOME LENDERS HLDG CO | 697,385 | Other Financial | -0.0697 |
| 5 | URANERZ ENERGY CORP | 133,479 | Mining | -0.0688 |
| 6 | CRAWFORD & CO | 160,407 | Insurance | -0.0629 |
| 7 | ORBCOMM INC | 360,348 | Transportation, Communications | -0.0591 |
| 8 | TRIAD GUARANTY INC | 674,462 | Insurance | -0.0590 |
| 9 | STANDARD PACIFIC CORP NEW | 1,525,954 | Construction | -0.0589 |
| 10 | DELTA FINANCIAL CORP | 211,543 | Other Financial | -0.0587 |
| 11 | W C I COMMUNITIES INC | 792,183 | Construction | -0.0583 |
| 12 | COMPUCREDIT CORP | 1,786,092 | Other Financial | -0.0582 |
| 13 | BEAZER HOMES USA INC | 1,601,873 | Construction | -0.0573 |
| 14 | EMPIRE RESOURCES INC DEL | 104,041 | Wholesale Trade | -0.0570 |
| 15 | HOUSTON AMERICAN ENERGY CORP | 120,614 | Mining | -0.0566 |
| 16 | RADIAN GROUP INC | 4,673,175 | Insurance | -0.0557 |
| 17 | I C O GLOBAL COMMS HLDGS LTD DE | 590,548 | Transportation, Communi | -0.0555 |
| 18 | E TRADE FINANCIAL CORP | 9,718,507 | Broker Dealers | -0.0552 |
| 19 | BUCKEYE TECHNOLOGIES INC | 451,391 | Manufacturing | -0.0548 |
| 20 | BADGER METER INC | 383,436 | Manufacturing | -0.0545 |
| 21 | CHESAPEAKE CORP VA | 280,014 | Manufacturing | -0.0543 |
| 22 | AVANEX CORP | 376,476 | Manufacturing | -0.0543 |
| 23 | BANKUNITED FINANCIAL CORP | 945,137 | Depository Institutions | -0.0538 |
| 24 | NASTECH PHARMACEUTICAL CO INC | 328,747 | Services | -0.0533 |
| 25 | IDAHO GENERAL MINES INC | 254,642 | Mining | -0.0533 |
| 26 | GEORGIA GULF CORP | 694,153 | Manufacturing | -0.0524 |
| 27 | WINN DIXIE STORES INC | 1,001,750 | Retail Trade | -0.0523 |
| 28 | PANACOS PHARMACEUTICALS INC | 241,728 | Manufacturing | -0.0520 |
| 29 | CROCS INC | 1,864,861 | Manufacturing | -0.0517 |
| 30 | GRUBB & ELLIS CO | 266,193 | Other Financial | -0.0514 |
| 31 | K B W INC | 877,814 | Broker Dealers | -0.0513 |
| 32 | FRANKLIN BANK CORP | 425,906 | Depository Institutions | -0.0511 |
| 33 | ASIAINFO HOLDINGS INC | 289,401 | Services | -0.0508 |
| 34 | FIRST AVENUE NETWORKS INC | 613,862 | Transportation, Communications | -0.0508 |
| 35 | WHEELING PITTSBURGH CORP | 293,215 | Manufacturing | -0.0505 |
| 36 | STILLWATER MINING CO | 1,095,439 | Mining | -0.0504 |
| 37 | TOREADOR RESOURCES CORP | 353,326 | Mining | -0.0500 |
| 38 | COUNTRYWIDE FINANCIAL CORP | 21,663,474 | Depository Institutions | -0.0497 |
| 39 | CHAMPION ENTERPRISES INC | 781,824 | Manufacturing | -0.0495 |
| 40 | REVLON INC | 554,749 | Manufacturing | -0.0494 |
| 41 | TIENS BIOTECH GROUP USA INC | 285,336 | Services | -0.0493 |
| 42 | AIRSPAN NETWORKS INC | 142,206 | Transportation, Communications | -0.0492 |
| 43 | FREEPORT MCMORAN COPPER & GOLD | 11,781,130 | Mining | -0.0490 |
| 44 | MERITAGE HOMES CORP | 1,069,891 | Construction | -0.0486 |
| 45 | EDDIE BAUER HOLDINGS INC | 273,345 | Retail Trade | -0.0485 |
| 46 | A Z Z INC | 235,577 | Manufacturing | -0.0483 |
| 47 | GRAFTECH INTERNATIONAL LTD | 722,492 | Manufacturing | -0.0483 |
| 48 | P M I GROUP INC | 3,817,586 | Insurance | -0.0483 |
| 49 | ASYST TECHNOLOGIES INC | 341,391 | Manufacturing | -0.0480 |
| 50 | NEUROGEN CORP | 213,849 | Manufacturing | -0.0480 |

**Table 4. Critical Values for ΔCoVaR and MES Test Statistics**

For each correlation value we run 50,000 simulations of the bivariate normal distribution with 500 pairs of returns of zero mean and unit variance. In each simulation we estimate the ΔCoVaR test statistic, $K_{CoVaR}$ and the MES test statistic, $K_{MES}$. 10% , 5% , and 1% Type I errors correspond to the 90th, 95th and 99th percentile values of the sample distibutions of the test statistics.

| | Critical Value of ΔCoVaR Test Statistic $K_{CoVaR}$ | | | Critical Value of MES Test Statistic, $K_{MES}$ | | |
|---|---|---|---|---|---|---|
| Correlation | Type I error=10% | Type I error=5% | Type I error=1% | Type I error=10% | Type I error=5% | Type I error=1% |
| -0.2 | 47.6 | 60.7 | 87.0 | 21.7 | 27.8 | 39.6 |
| -0.1 | 47.8 | 62.3 | 88.5 | 21.9 | 28.1 | 39.8 |
| 0.0 | 47.4 | 61.3 | 88.4 | 21.9 | 28.2 | 39.9 |
| 0.1 | 47.1 | 61.5 | 88.9 | 21.7 | 28.0 | 39.7 |
| 0.2 | 46.7 | 60.7 | 87.2 | 21.3 | 27.6 | 39.2 |
| 0.3 | 44.6 | 58.1 | 83.8 | 20.8 | 26.9 | 38.4 |
| 0.4 | 43.0 | 56.2 | 81.9 | 20.1 | 25.9 | 37.1 |
| 0.5 | 40.8 | 53.4 | 79.8 | 19.1 | 24.8 | 35.3 |
| 0.6 | 38.1 | 50.2 | 74.0 | 18.0 | 23.3 | 33.1 |
| 0.7 | 34.4 | 45.7 | 68.2 | 16.4 | 21.4 | 30.4 |
| 0.8 | 30.0 | 40.3 | 59.5 | 14.7 | 19.0 | 27.0 |
| 0.9 | 23.7 | 32.1 | 48.6 | 12.3 | 16.0 | 22.9 |

**Table 5. Summary of ΔCoVaR and MES Hypothesis Test Results**

The ΔCoVaR estimate is statistically significant at 1% (5%) level if the sample estimate of the $\kappa_{CoVaR}$ test statistic exceeds its critical value for a given correlation level and Type I error of 1% (5%) provided in Table 4. Similarly MES estimate is statististically significant if the sample estimate of the $\kappa_{MES}$ test statistic exceeds its critical value provided in Table 4 for a given correlation and Type I error level .

| Industry | Number of Firms | ΔCoVaR Statistically Significant at 1% level | ΔCoVaR Statistically Significant at 5% level | Percentage of Firms Sig. at 5% Level under ΔCoVaR Test | MES Statistically Significant at 1% level | MES Statistically Significant at 5% level | Percentage of Firms Sig. at 5% Level under MES Test |
|---|---|---|---|---|---|---|---|
| Broker Dealers | 55 | 3 | 8 | 14.5% | 22 | 27 | 49.1% |
| Construction | 41 | 1 | 4 | 9.8% | 4 | 11 | 26.8% |
| Depository Institutions | 426 | 16 | 40 | 9.4% | 88 | 158 | 37.1% |
| Insurance | 138 | 5 | 18 | 13.0% | 36 | 60 | 43.5% |
| Manufacturing | 1336 | 75 | 159 | 11.9% | 132 | 326 | 24.4% |
| Mining | 151 | 22 | 55 | 36.4% | 11 | 28 | 18.5% |
| Other Financial | 74 | 9 | 19 | 25.7% | 13 | 21 | 28.4% |
| Public Administration | 11 | 1 | 2 | 18.2% | 0 | 6 | 54.5% |
| Retail Trade | 227 | 10 | 27 | 11.9% | 27 | 63 | 27.8% |
| Services | 626 | 38 | 80 | 12.8% | 50 | 129 | 20.6% |
| Transportation, Communications | 320 | 21 | 59 | 18.4% | 80 | 150 | 46.9% |
| Wholesale Trade | 113 | 9 | 25 | 22.1% | 8 | 24 | 21.2% |

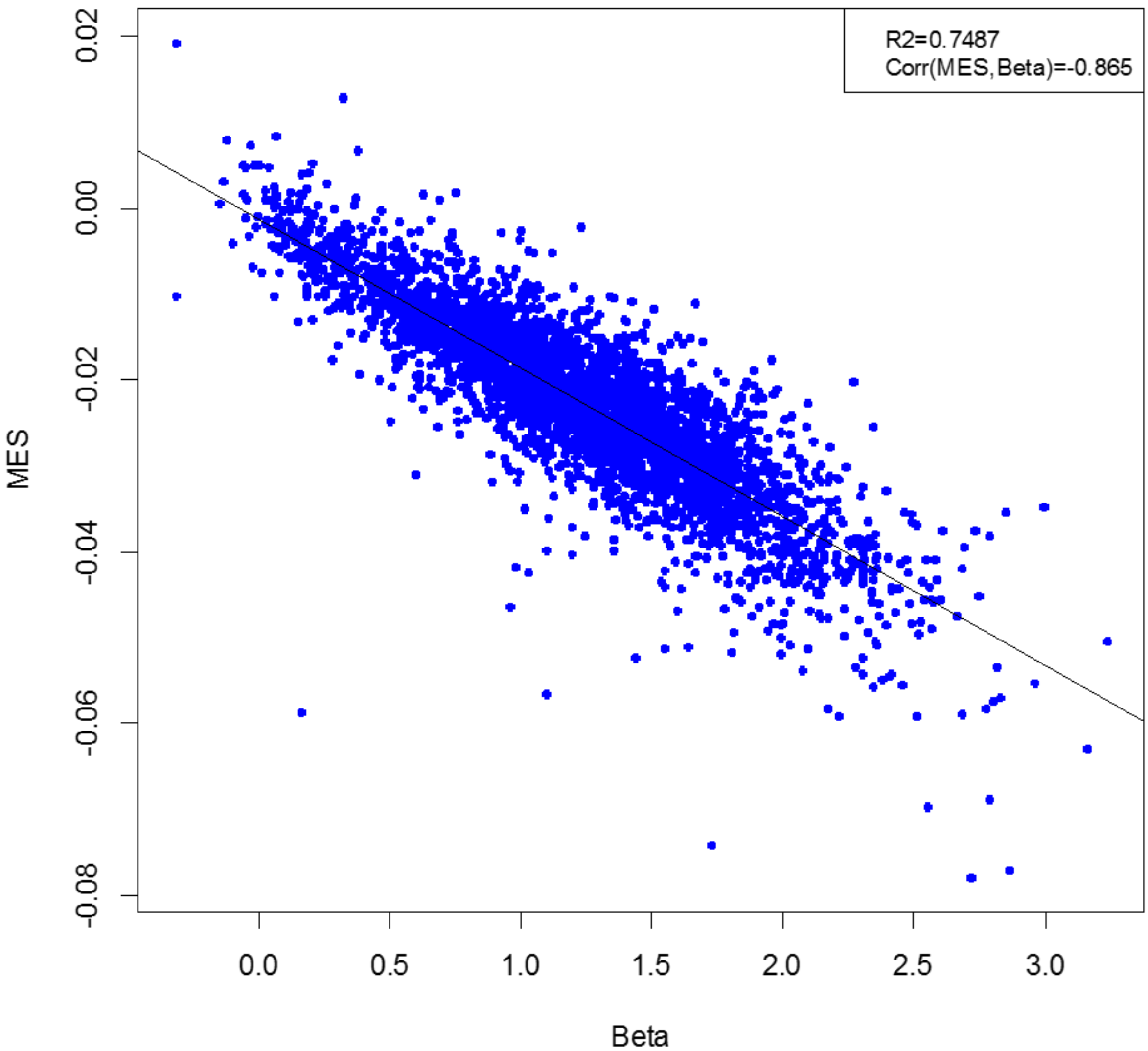
**Table 6. Maximum Likelihood Estimates of Bivariate Skew-t Distribution for 3518 firms**

This table summarizes the distribution quantiles of the bivariate skew-t parameter estimates for 3518 daily stock and market return pairs between January 2006 and December 2007. The parameters are estimated for each firm by maximum likelihood using the Adelchi Azzalini's "SN" package in "R". The subscript 1 (2) indicates a parameter estimate that is associated with the individual stock return (equally weighted CRSP portfolio return). Parameter ρ is not estimated but calculated as $\rho = \sigma_{12}(\sigma_{11}\sigma_{22})^{-0.5}$ . Estimates are for 100 times daily log returns.

| Distribution Quantiles | $\beta_1$ | $\beta_2$ | $\Omega_{11}$ | $\Omega_{12}$ | $\Omega_{22}$ | $\alpha_1$ | $\alpha_2$ | $\nu$ | $\rho$ |
|---|---|---|---|---|---|---|---|---|---|
| 1% | -1.050 | 0.239 | 0.400 | 0.002 | 0.243 | -0.304 | -1.611 | 2.306 | 0.002 |
| 5% | -0.408 | 0.332 | 0.705 | 0.079 | 0.311 | -0.165 | -1.301 | 2.805 | 0.100 |
| 25% | 0.010 | 0.398 | 1.501 | 0.293 | 0.387 | 0.062 | -1.056 | 3.426 | 0.313 |
| 50% | 0.197 | 0.435 | 2.608 | 0.445 | 0.426 | 0.213 | -0.925 | 3.875 | 0.454 |
| 75% | 0.389 | 0.469 | 4.234 | 0.601 | 0.470 | 0.382 | -0.815 | 4.480 | 0.549 |
| 95% | 0.776 | 0.522 | 7.767 | 0.908 | 0.552 | 0.698 | -0.629 | 5.737 | 0.651 |
| 99% | 1.189 | 0.574 | 11.369 | 1.137 | 0.663 | 0.987 | -0.468 | 7.617 | 0.713 |

**Figure 1: Relationship between 5 % MES Estimates and Market Beta**

Figure 1 plots the linear regression estimate of 5 percent MES statistics for 3518 individual stocks estimated over the sample period 2006-2007 on a constant and the individual stocks' market model beta coefficient estimates. MES statistics are calculated as the average individual stock returns on days in which the market portfolio experiences a return realization that is in its 5 percent (left) tail.



### Figure 2: Relationship between 1% ΔCoVaR Estimate and Market Return Correlation

Figure 2 plots the linear regression estimate of the relationship between 1-percent ΔCoVaR sample estimates for 3518 individual stocks over the sample 2006-2007 on a constant and the sample correlation estimate between the returns on the individual stocks and the equally-weighted CRSP market portfolio. The 1 percent ΔCoVaR is measured as the difference between the 1 percent conditional Value at Risk estimate for the market portfolio where the conditioning event is a 1 percentile realization of the individual stock, and the "median" conditional value at risk for the market portfolio where the conditioning event is a median return realization from the individual stock. The individual CoVaRs are estimated using quantile regressions on daily log returns.

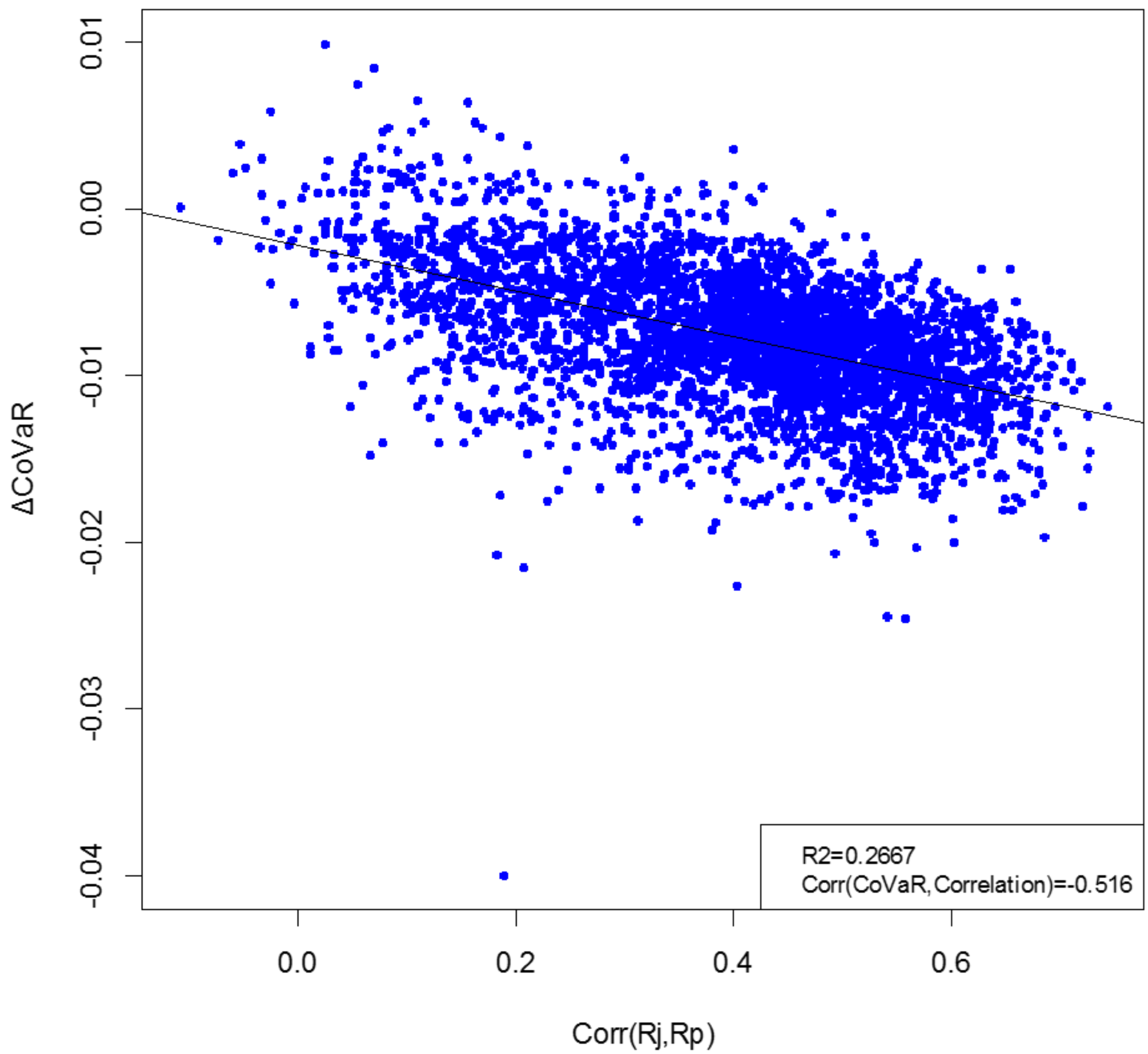
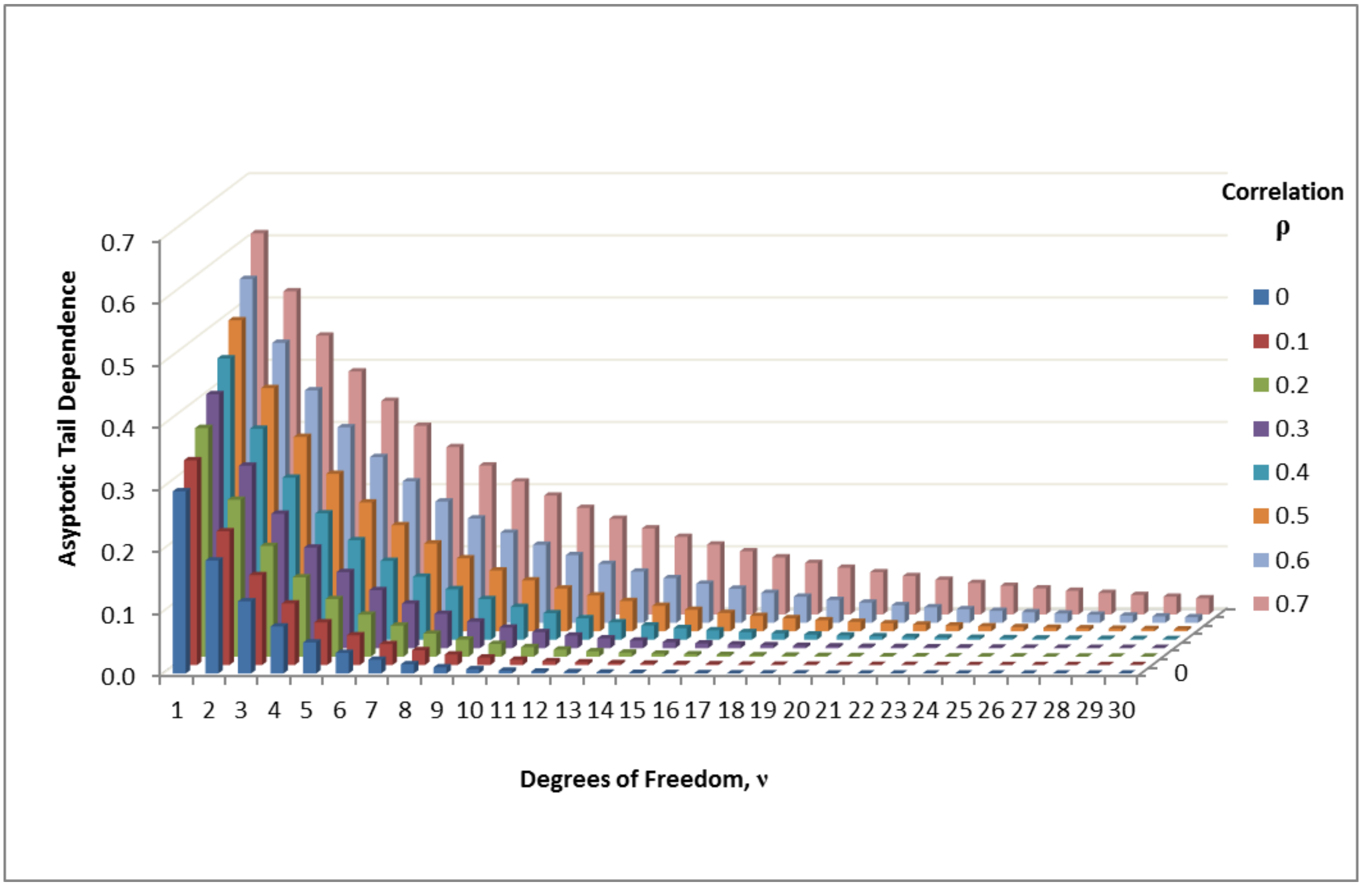


**Figure 3: Symmetric T-Distribution Asymptotic Tail Dependence as a Function of Correlation and Degrees of Freedom**

The figure plots the asymptotic tail dependence for various levels of degrees of freedom and correlation for the symmetric bivariate t distribution. Asymptotic tail dependence is given by:

$$L_{sym\,t} = 2T_1\left(-\frac{\sqrt{(\nu+1)(1-\rho)}}{\sqrt{1+\rho}}, \upsilon + 1\right)$$

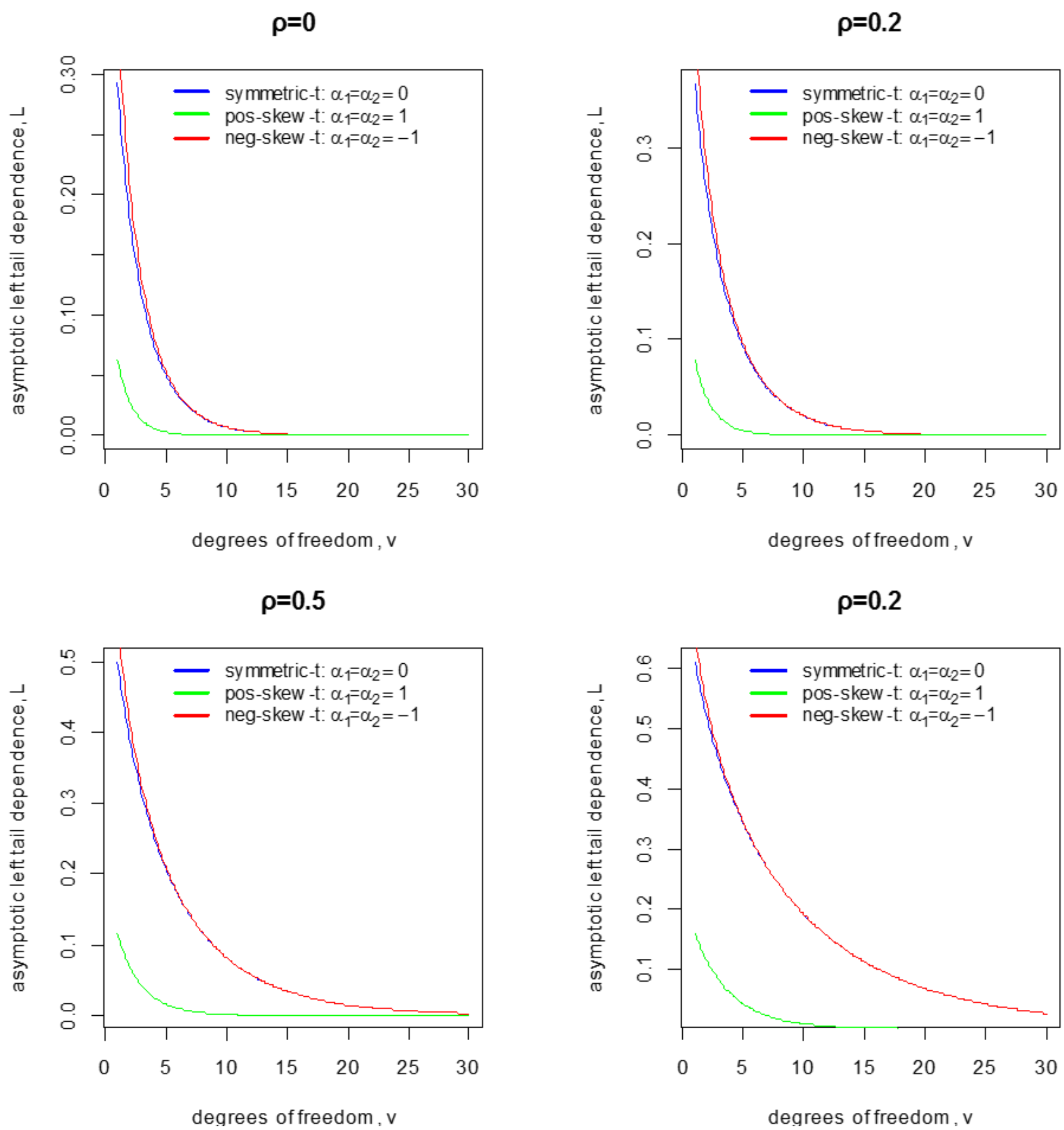


## Figure 4: Magnitude of the Asymptotic Left Tail Dependence in a Skew-t Distribution

The figure illustrates the relationship between strength of the asymptotic left tail dependence for the skew-t distribution and the degrees of freedom parameter, $\nu$, correlation parameter, $\rho$ and skewness parameters, $\alpha_1 = \alpha_2 = \alpha$. The distribution's asymptotic tail dependence, $L_{skew-t}$ is given by:

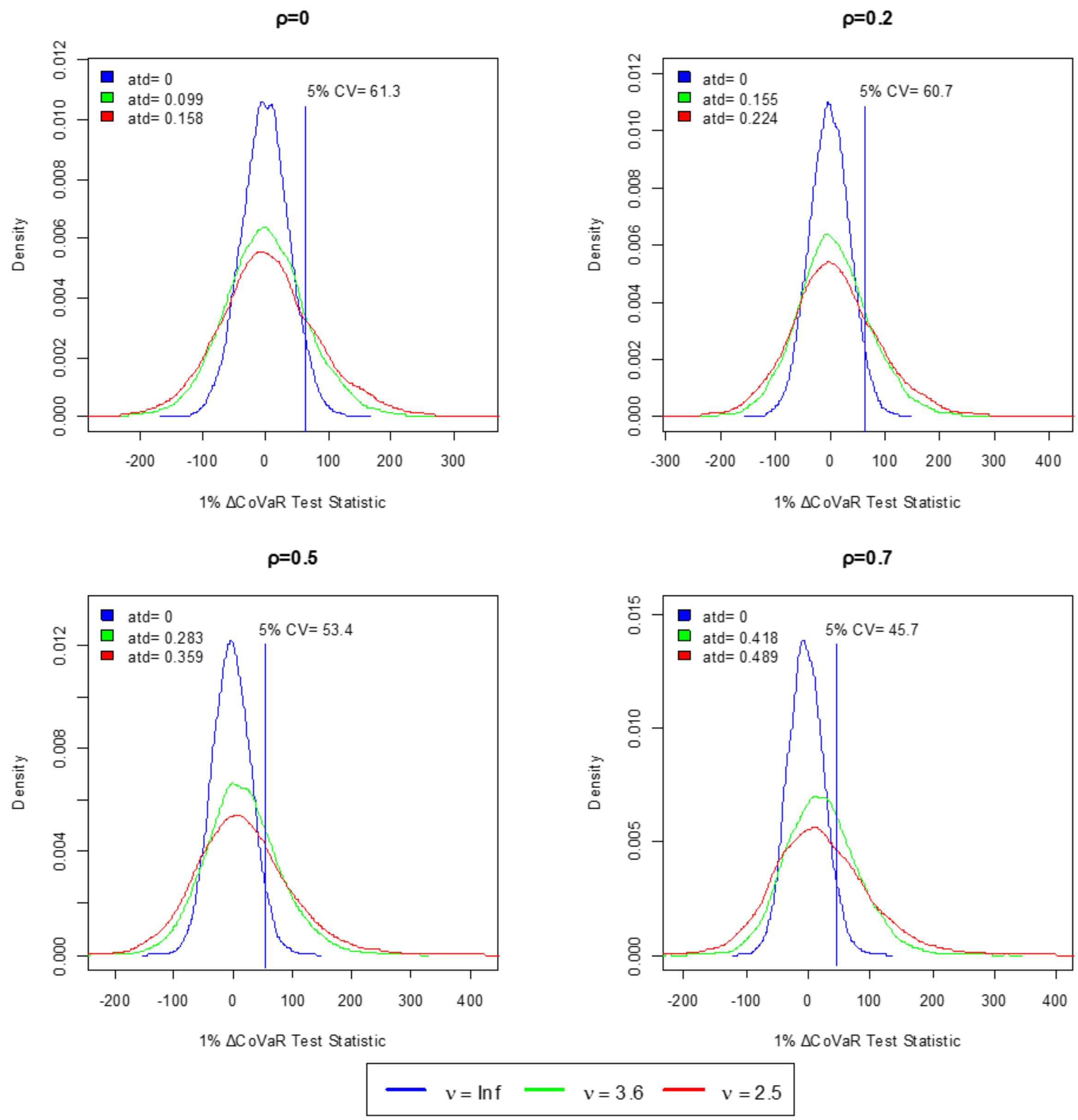
$$L_{skew-t} = \frac{T_1\left(-2\alpha\sqrt{\frac{(\nu+2)(1+\rho)}{2}}; \nu+2\right) * 2T_1\left(-\frac{\sqrt{(\nu+1)(1-\rho)}}{\sqrt{1+\rho}}, \upsilon+1\right)}{T_1\left(\frac{-\alpha(1+\rho)\sqrt{\nu+1}}{\sqrt{1+\alpha^2(1-\rho^2)}}; \nu+1\right)}$$

where $T_1$ represents the cumulative distribution function of univariate student t.



## Figure 5: $\kappa_{CoVaR}$ Power against Symmetric T-Distribution Alternatives

The figure shows the simulated sampling distributions $\kappa_{CoVaR}$ based on a sample size of 500 and 10 thousand Monte Carlo replications, and a Gaussian kernel density estimator. The blue distributions are test statistics calculated from the bivariate Gaussian distribution $(\alpha_1 = \alpha_2 = 0, \nu = \infty)$ using the median parameter values in Table 6 for $\beta_1, \beta_2, \Omega_{11}, \Omega_{22}$. When $\rho$ varies $\Omega_{12}$ varies, $\Omega_{12} = \rho * \sqrt{\Omega_{11}\Omega_{22}}$. The blue vertical lines are the 5 percent critical values of the $\kappa_{CoVaR}$ test statistic under the null hypothesis. The red and light green distributions are symmetric t distributions $(\alpha_1 = \alpha_2 = 0)$ with degrees of freedom $\nu = 3.6$ $(light\ green)$, $\nu = 2.5$ $(red)$ and median parameter values from Table 6 for the remaining parameters. "atd" is an abbreviation for the asymptotic tail dependence of the t-distribution.



## Figure 6: $\kappa_{MES}$ Power against Symmetric T-Distribution Alternatives

The figure shows the simulated sampling distributions for $\kappa_{MES}$ based on a sample size of 500 with 10,000 Monte Carlo replications, and a Gaussian kernel density estimator. The blue distributions are test statistics calculated from the bivariate Gaussian distribution ($\alpha_1 = \alpha_2 = 0, \nu = \infty$) using the median parameter values in Table 6 for $\beta_1, \beta_2, \Omega_{11}, \Omega_{22}$. When $\rho$ varies $\Omega_{12}$ varies, $\Omega_{12} = \rho * \sqrt{\Omega_{11}\Omega_{22}}$. The blue vertical lines are the 5 percent critical values of the $\kappa_{MES}$ test statistic under the null hypothesis. The red and light green distributions are symmetric t distributions ($\alpha_1 = \alpha_2 = 0$) with degrees of freedom $\nu = 3.6$ ($light\ green$), $\nu = 2.5$ ($red$) and median parameter values from Table 6 for the remaining parameters. "atd" is an abbreviation for the asymptotic tail dependence of the t-distribution.

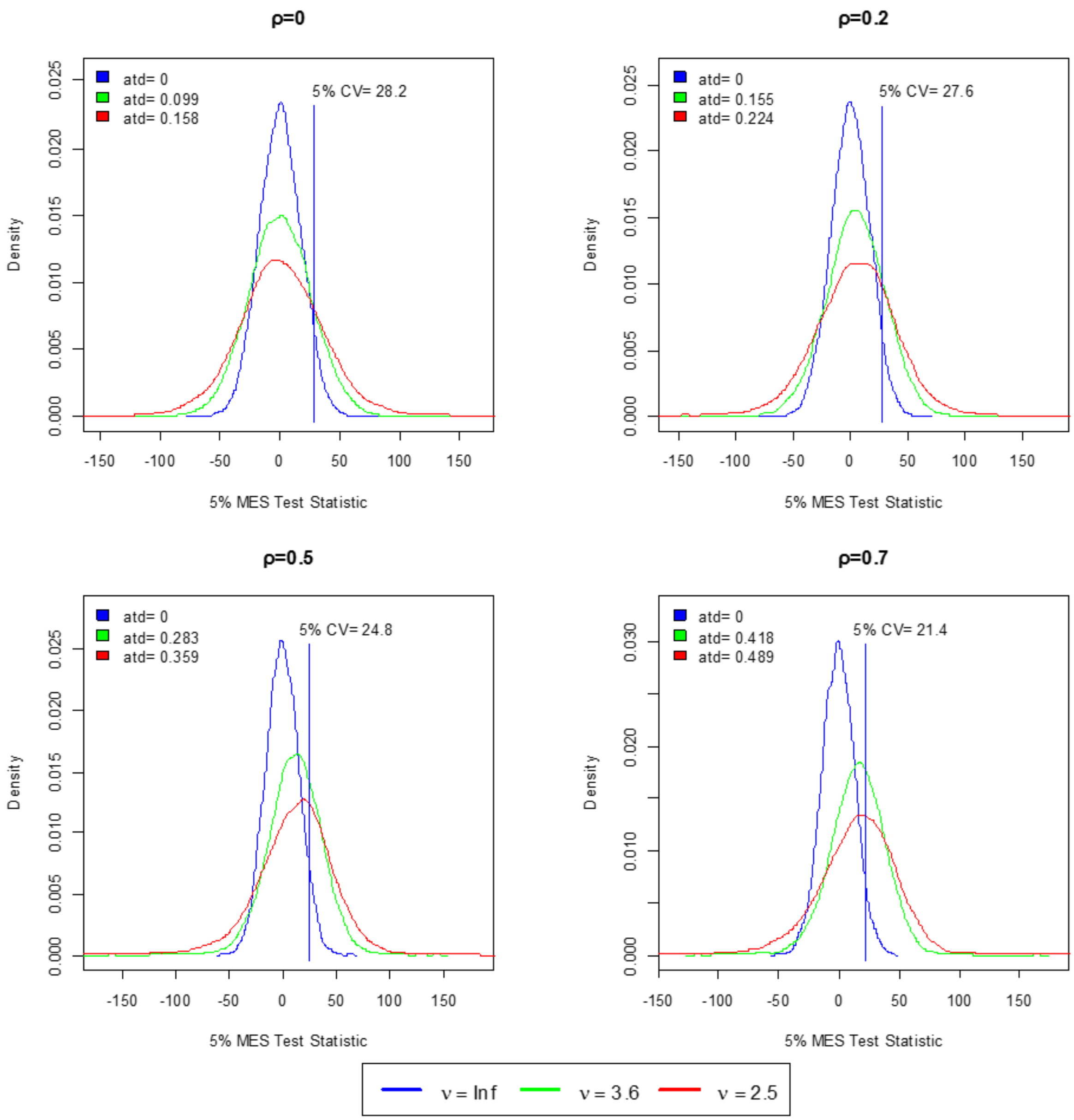
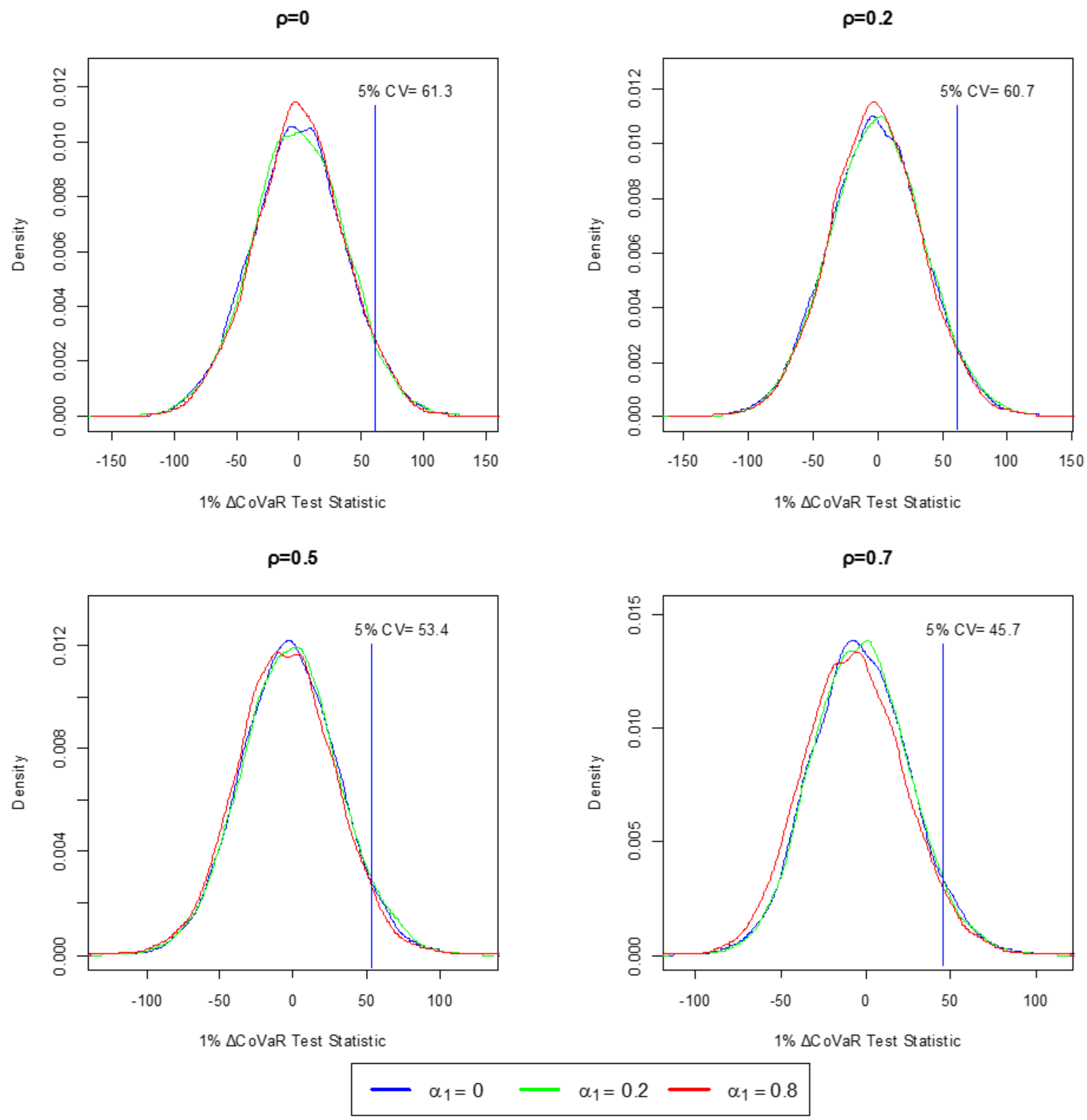


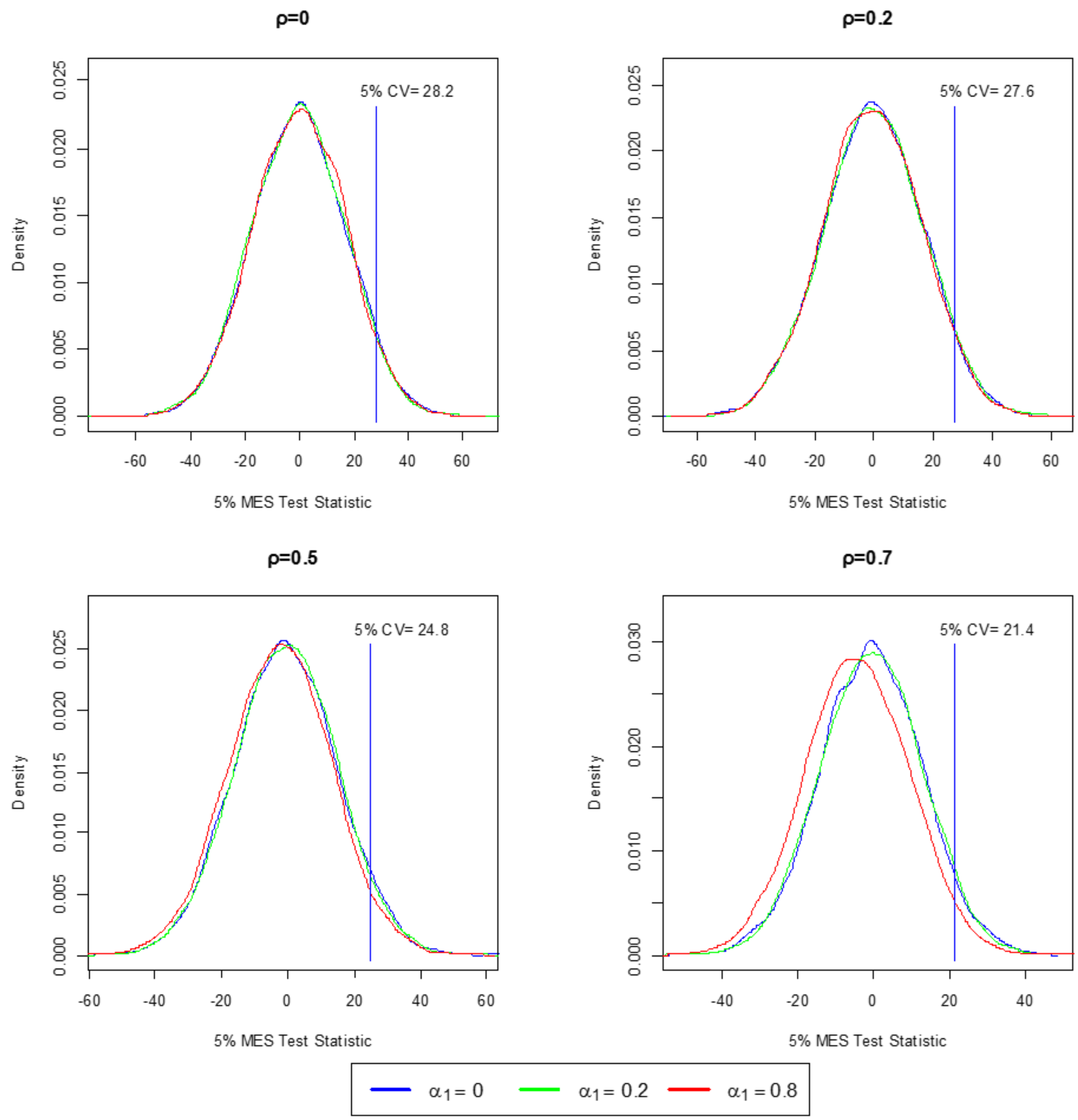
**Figure 7: $\kappa_{CoVar}$ Tests Statistics when Individual Stocks are Positive-Skewed Gaussian**

The figure shows the simulated sampling distributions for $\kappa_{CoVaR}$ based on a sample size of 500, with 10,000 Monte Carlo replications, and a Gaussian kernel density estimator. The blue distributions are test statistics calculated from the bivariate Gaussian distribution $(\alpha_1 = \alpha_2 = 0, \nu = \infty)$ using the median parameter values in Table 6 for $\beta_1, \beta_2, \Omega_{11}, \Omega_{22}$. When $\rho$ varies $\Omega_{12}$ varies, $\Omega_{12} = \rho * \sqrt{\Omega_{11}\Omega_{22}}$. The blue vertical lines are the 5 percent critical values of the $\kappa_{CoVaR}$ test statistic under the null hypothesis. The red and light green distributions are Gaussian distributions where the market return is symmetric and the individual stock return is positively skewed. The light green distribution has mild positive skewness $(\alpha_1 = .2, \alpha_2 = 0, \nu = \infty)$ while the red distribution has a stronger positive skew $(\alpha_1 = .8, \alpha_2 = 0, \nu = \infty)$. Remaining parameters are set to median parameter values from Table 6. All of the distributions are asymptotically tail independent.



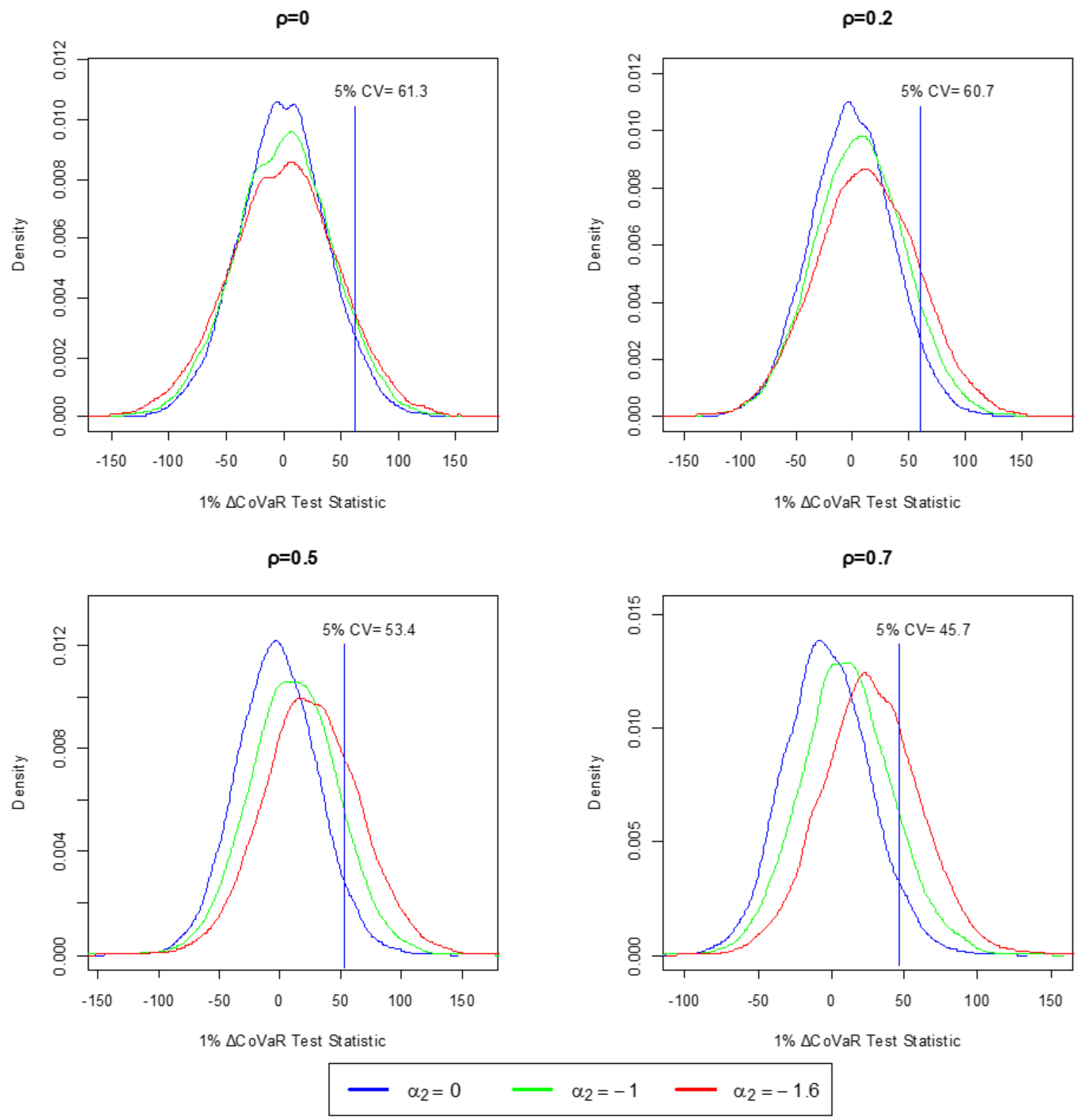
## Figure 8: $\kappa_{MES}$ Tests Statistics when Individual Stocks are Positive-Skewed Gaussian

The figure shows the simulated sampling distributions for $\kappa_{MES}$ based on a sample size of 500 with 10,000 Monte Carlo replications, and a Gaussian kernel density estimator. The blue distributions are test statistics calculated from the bivariate Gaussian distribution ($\alpha_1 = \alpha_2 = 0, \nu = \infty$) using the median parameter values in Table 6 for $\beta_1, \beta_2, \Omega_{11}, \Omega_{22}$. When $\rho$ varies $\Omega_{12}$ varies, $\Omega_{12} = \rho * \sqrt{\Omega_{11}\Omega_{22}}$. The blue vertical lines are the 5 percent critical values of the $\kappa_{MES}$ test statistic under the null hypothesis. The red and light green distributions are Gaussian distributions where the market return is symmetric and the individual stock return is positively skewed. The light green distribution has mild positive skewness ($\alpha_1 = .2, \alpha_2 = 0, \nu = \infty$) while the red distribution has a stronger positive skew ($\alpha_1 = .8, \alpha_2 = 0, \nu = \infty$). Remaining parameters are set to median parameter values from Table 6. All of the distributions are asymptotically tail independent.



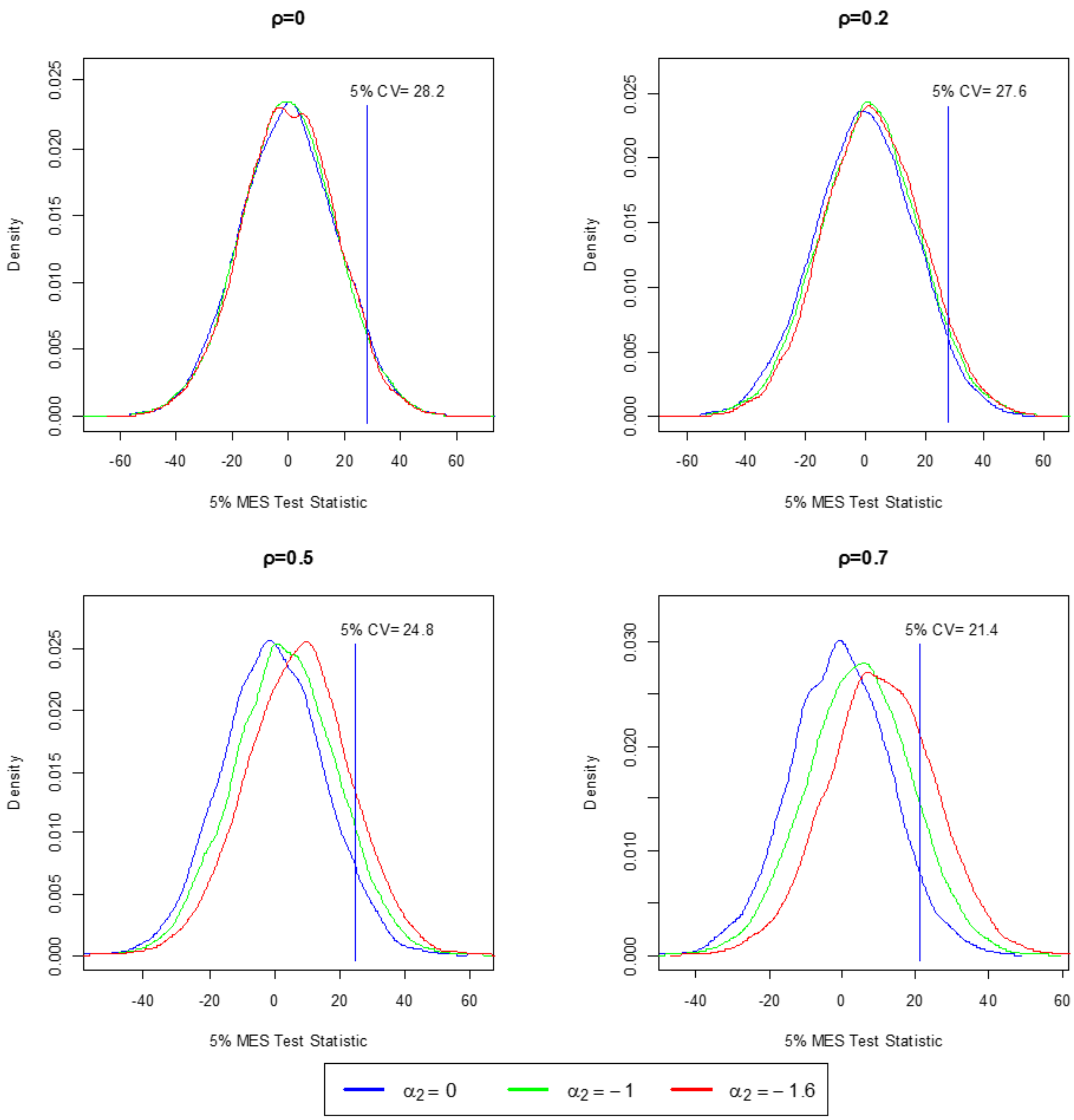
## Figure 9: $\kappa_{CoVar}$ Tests Statistics when Market Returns are Negative-Skewed Gaussian

The figure shows the simulated sampling distributions for $\kappa_{CoVaR}$ based on a sample size of 500 with 10,000 Monte Carlo replications, and a Gaussian kernel density estimator. The blue distributions are test statistics calculated from the bivariate Gaussian distribution ($\alpha_1 = \alpha_2 = 0, \nu = \infty$) using the median parameter values in Table 6 for $\beta_1, \beta_2, \Omega_{11}, \Omega_{22}$. When $\rho$ varies $\Omega_{12}$ varies, $\Omega_{12} = \rho * \sqrt{\Omega_{11}\Omega_{22}}$. The blue vertical lines are the 5 percent critical values of the $\kappa_{CoVaR}$ test statistic under the null hypothesis. The red and light green distributions are Gaussian distributions where the individual stock return is symmetric and the market return is negatively skewed. The light green distribution has mild negative skewness ($\alpha_1 = 0, \alpha_2 = -1, \nu = \infty$) while the red distribution has a stronger negative skew ($\alpha_1 = 0, \alpha_2 = -1.6, \nu = \infty$). Remaining parameters are set to median parameter values from Table 6. All of the distributions are asymptotically tail independent.



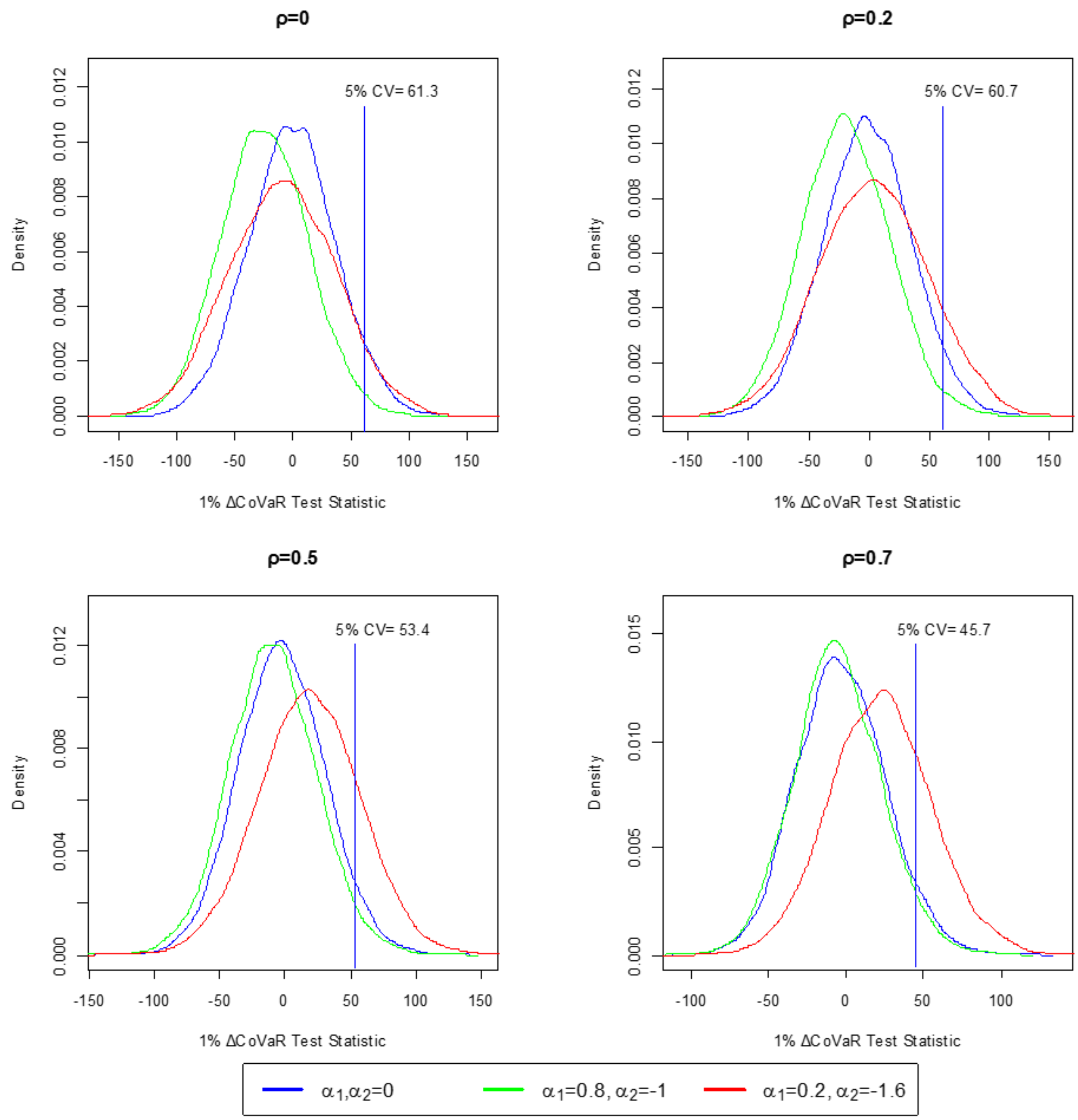
## Figure 10: $\kappa_{MES}$ Tests Statistics when Market Returns are Negative-Skewed Gaussian

The figure shows the simulated sampling distributions for $\kappa_{MES}$ based on a sample size of 500, 10,000 Monte Carlo replications, and a Gaussian kernel density estimator. The blue distributions are test statistics calculated from the bivariate Gaussian distribution ($\alpha_1 = \alpha_2 = 0, \nu = \infty$) using the median parameter values in Table 6 for $\beta_1, \beta_2, \Omega_{11}, \Omega_{22}$. When $\rho$ varies $\Omega_{12}$ varies, $\Omega_{12} = \rho * \sqrt{\Omega_{11}\Omega_{22}}$. The blue vertical lines are the 5 percent critical values of the $\kappa_{MES}$ test statistic under the null hypothesis. The red and light green distributions are Gaussian distributions where the market return is symmetric and the individual stock return is positively skewed. The light green distribution has mild negative skewness ($\alpha_1 = 0\,, \alpha_2 = -1, \nu = \infty$) while the red distribution has a stronger negative skew ($\alpha_1 = 0\,, \alpha_2 = -1.6, \nu = \infty$). Remaining parameters are set to median parameter values from Table 6. All of the distributions are asymptotically tail independent.



## Figure 11: $\kappa_{CoVar}$ Tests Statistics when Individual Stocks are Positive-Skewed and Market Returns are Negative-Skewed Gaussian

The figure shows the simulated sampling distributions for $\kappa_{CoVaR}$ based on a sample size of 500, 10,000 Monte Carlo replications, and a Gaussian kernel density estimator. The blue distributions are test statistics calculated from the bivariate Gaussian distribution ($\alpha_1 = \alpha_2 = 0, \nu = \infty$) using the median parameter values in Table 6 for $\beta_1, \beta_2, \Omega_{11}, \Omega_{22}$. When $\rho$ varies $\Omega_{12}$ varies, $\Omega_{12} = \rho * \sqrt{\Omega_{11}\Omega_{22}}$. The blue vertical lines are the 5 percent critical values of the $\kappa_{CoVaR}$ test statistic under the null hypothesis. The red and light green distributions are Gaussian distributions where the market return is negatively skewed and the individual stock return is positively skewed. The light green distribution has strong positive skewness ($\alpha_1 = .8\,, \alpha_2 = -1, \nu = \infty$) while the red distribution has a lower positive skew ($\alpha_1 = .2\,, \alpha_2 = -1.6, \nu = \infty$). Remaining parameters are set to median parameter values from Table 6. All of the distributions are asymptotically tail independent.



## Figure 12: $\kappa_{MES}$ Tests Statistics when Individual Stocks are Positive-Skewed and Market Returns are Negative-Skewed Gaussian

This figure plots the simulated sampling distributions for $\kappa_{MES}$ based on a sample size of 500, 10,000 Monte Carlo replications, and a Gaussian kernel density estimator. The blue distributions are test statistics calculated from the bivariate Gaussian distribution $(\alpha_1 = \alpha_2 = 0, \nu = \infty)$ using the median parameter values in Table 6 for $\beta_1, \beta_2, \Omega_{11}, \Omega_{22}$. When $\rho$ varies $\Omega_{12}$ varies, $\Omega_{12} = \rho * \sqrt{\Omega_{11}\Omega_{22}}$. The blue vertical lines are the 5 percent critical values of the $\kappa_{MES}$ test statistic under the null hypothesis. The red and light green distributions are Gaussian distributions where the market return is symmetric and the individual stock return is positively skewed. The light green distribution has strong positive skewness $(\alpha_1 = .8, \alpha_2 = -1, \nu = \infty)$ while the red distribution has lower positive skew $(\alpha_1 = .2, \alpha_2 = -1.6, \nu = \infty)$. Remaining parameters are set to median parameter values from Table 6. All of the distributions are asymptotically tail independent.