

The Rise of China's Shadow Banking System*

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Abstract

Shadow banking in China has grown very rapidly during the past decade. This paper studies the causes and impending consequences. We begin by documenting important differences in the cross-section of Chinese banks to isolate the regulatory triggers for shadow banking. We then build a model that rationalizes the facts and use it to conduct policy experiments. We find that asymmetric competition between banks is both a short-run stabilizer and a long-run risk, with new regulations potentially exacerbating the tipping point.

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1 Introduction

The 2007-2009 financial crisis has led to acute concerns about shadow banking, broadly defined by the Financial Stability Board as:

“credit intermediation [that] takes place in an environment where prudential regulatory standards ... are applied to a materially lesser or different degree than is the case for regular banks engaged in similar activities” (FSB, 2011)

The *scale* of shadow banking is still highest in the U.S. and Western Europe but the *growth* of shadow banking is now fastest in China (Figure 1). Lending by China’s shadow sector has increased by more than 30% annually since 2009 and is becoming an increasingly important source of external financing. It is also fuelling a rapid rise in China’s debt-to-GDP ratio, with the latter hitting 1.9 in 2013 (Figure 2). Despite growing concerns about the fragility of China’s financial system and the potential spillovers to the world economy, China’s shadow sector remains a mystery to academia. Who are the shadow banks in China? Why are they emerging now? Do they pose the same financial stability risks that played out in the 2007-2009 crisis? Our paper tackles these questions.

We argue that the regulatory trigger for shadow banking in China was stricter enforcement of a 75% cap on bank loan-to-deposit ratios in an environment with deposit rate ceilings. The enforcement action was complemented by a large increase in reserve requirements, making the 75% cap akin to a liquidity standard. Upon enforcement, several banks – in particular, small and medium-sized banks – found themselves constrained by the cap and unable to comply with it by increasing deposit rates to attract more deposits. As a result, they moved activities off balance sheet.

Issuing “wealth management products” (henceforth WMPs) is the core off balance sheet activity in China. We will provide a detailed description of WMPs below. In short, they are best described as asset-backed term deposits.¹ There are no regulations on the returns to

¹In other words, a term deposit which derives its cash flows from a specific pool of investments.

WMPs and most WMPs are not consolidated into bank balance sheets. By issuing WMPs with returns in excess of deposit rates, banks can attract household savings then funnel these savings into trust companies which are not subject to loan-to-deposit rules.

Importantly though, not all banks are constrained by the 75% cap. In particular, the four biggest banks (henceforth the Big Four) have much lower loan-to-deposit ratios than the average small or medium-sized bank. The Big Four do issue WMPs but keep a disproportionately larger fraction on balance sheet. We argue that the Big Four are using WMPs to defend their market share. Overall savings in China are not yet sufficiently elastic to WMP returns so high-return WMPs by cap-constrained banks poach deposits from the Big Four. The latter respond by issuing WMPs with competitive returns and are content to keep at least some on balance sheet since their goal is not to evade regulators. However, once the Big Four enter the fray, cap-constrained banks must be more aggressive and offer even higher returns in order to attract enough WMPs to skirt loan-to-deposit rules.

While most WMPs are short-term, with a maturity of three months or less, they are often used to fund long-term projects. This maturity mismatch has led to increasingly active interbank markets. The transaction volume on the repo market, for instance, more than tripled between 2008 and 2013. The Big Four turn out to be the main liquidity provider on this market. In 2012, they provided net lending of RMB 55 trillion, roughly 40% of the total transaction volume. The rise of the shadow sector has also coincided with much higher interest rates. In the repo market, the average interest rate increased from 2.8% in 2008 to 4.4% in 2013. Similar patterns are also visible in the uncollateralized money market.

Higher interbank rates, *ceteris paribus*, discourage cap-constrained banks from expanding their off balance sheet activities to evade liquidity standards. This, together with the Big Four's dominant role in providing interbank liquidity, implies that the Big Four can also defend their market share by manipulating the interbank market. More precisely, when shadow banking by cap-constrained banks begins poaching deposits from the Big Four, the

latter can issue their own WMPs and/or respond strategically by reducing liquidity supply to these other banks. Strategic reductions in liquidity supply increase interbank rates and compel the other banks to scale back their WMP issuance.

Having identified that the Big Four have two ways to respond to the shadow banking activities of cap-constrained banks, we build a simple model to understand what the Big Four ultimately do. Our model has three key ingredients: maturity transformation in the spirit of Diamond and Dybvig (1983), an interbank market for reserves, and heterogeneity in market power. We start by showing that big banks are typically not constrained by the loan-to-deposit cap because they internalize the effect of their reserve holdings on the interbank market. We then show that a tighter cap has two effects. First, it pushes cap-constrained banks off balance sheet and fuels a credit expansion. Second, it leads to more aggressive on balance sheet lending by big banks as the latter try to fend off the cap-constrained banks by reducing interbank liquidity. The second effect curtails some of the initial credit expansion but also contributes directly to credit growth. We show that the net effect is an increase in overall credit and an increase in the equilibrium interbank rate.

Our paper thus sheds light on a few puzzling facts. As noted above, Chinese regulators have increased liquidity standards and cracked down on loan-to-deposit ratios yet debt-to-GDP has only grown faster. Our model provides an explanation for this seemingly counterintuitive outcome. More generally, it also warns against assuming standard policy implications in an environment with non-standard transmission mechanisms. Another puzzle is convergence in the loan-to-deposit ratios of different banks. While falling ratios among small and medium-sized banks are easily explained by the regulatory tightening, rising ratios among the Big Four are more subtle. Our model provides a novel explanation: the Big Four are putting strategic pressure on interbank markets to protect their deposit base from off balance sheet competition. At least in the short-term, this has helped regulators by curtailing some of the shadow banking that would have otherwise been pursued by cap-constrained

banks. However, in order to manipulate the interbank market, the Big Four are approaching their loan-to-deposit constraint. If this constraint becomes binding on them, then China's financial system will suddenly get a lot more fragile.

The rest of the paper proceeds as follows. Section 2 describes the basic features of China's banking system. Section 3 then presents our empirical evidence using both bank-level and product-level data. There is a large and compelling literature that uses disaggregated data to understand financial crises post-mortem.² Our goal is to provide an ante-mortem analysis. The remaining sections build our theory. In particular, Section 4 lays out a simple framework, Section 5 augments to our full model, and Section 6 summarizes policy experiments. Section 7 concludes. All proofs and derivations are collected in the appendix.

2 Institutional Background

There are two main features of China's regulatory environment: a ceiling on bank deposit rates and a cap on bank loan-to-deposit ratios. China has a long history of regulating deposit rates. Prior to 2004, deposit rates were simply set by the People's Bank of China (China's central bank). The central bank introduced some flexibility in 2004 by allowing deposit rates to fall below the benchmark rate but all banks just stayed at the benchmark. In other words, the benchmark rate turned out to be a binding ceiling.³

The 75% loan-to-deposit cap is formally written into China's Law on Commercial Banks. The law was enacted in 1995 but enforcement of the cap was initially loose. Things changed around 2008 when the China Banking Regulatory Commission (CBRC) began stricter enforcement in response to rising loan-to-deposit ratios among small and medium-sized banks (henceforth SMBs). CBRC then established China's "CARPALS" regulatory system in 2011,

²Examples for the 2007-2009 crisis include Brunnermeier (2009), Gorton and Metrick (2012), Acharya et al (2013), Covitz et al (2013), Kacperczyk and Schnabl (2013), and Krishnamurthy et al (2014).

³The central bank relaxed this ceiling slightly in 2012, allowing banks to set deposit rates 10% above the benchmark rate. Almost all banks responded by increasing their deposit rates 10%.

where the loan-to-deposit ratio is one of thirteen supervised indicators. Regulation was further strengthened by monitoring the average daily ratio rather than just the end-of-year ratio. The tightening of loan-to-deposit rules has also been echoed by a very rapid increase in reserve requirements (Figure 3). Official requirements went from 9% in early 2007 to 15.5% in February 2010. They were then increased twelve times to reach 21.5% by December 2011.

The loan-to-deposit ratio across all commercial banks averaged 67% between 2007 and 2013 so the 75% cap does not appear to bind at the aggregate level. However, this largely reflects the Big Four. Many SMBs have actually been constrained since the late 2000s. For example, the ten national banks (excluding the Big Four), had an average loan-to-deposit ratio of 74% between 2007 and 2013. Section 3 will establish that SMBs moved most forcefully into off balance sheet activities following stricter enforcement of the 75% cap.

Figure 4 illustrates the nature of off balance sheet activities in China. At the heart of the figure are “wealth management products” or WMPs for short. A WMP is best described as an asset-backed term deposit. However, unlike traditional term deposits, WMPs may or may not be principal-guaranteed by the issuing bank. Without a guarantee, the WMP and the assets it invests in are not consolidated into the bank’s balance sheet and thus not subject to loan-to-deposit rules. According to CBRC, non-guaranteed products accounted for 70% of total WMP issuance in 2012. We will show that this percentage is driven primarily by SMB activity. We will also show that these activities are inherently fragile in the sense of Diamond and Dybvig (1983): most wealth management products involve a maturity mismatch and are thus susceptible to runs.

Disclosure about the exact assets backing WMPs is sparse but an important component appears to be trust company assets. Trust companies raise most of their funds as paid-in capital. Roughly 70% of the capital is money already pooled together by other institutions rather than money pooled independently by the trust. This is consistent with Figure 5 which shows that rapid growth in WMPs outstanding has been matched by rapid growth in the

trust industry’s assets under management. In response, the CBRC has also clamped down on how much bank-issued WMPs can invest in trusts. A formal limit of 35% was announced in March 2013 but government warnings reportedly began around 2011.

The result is a second wave of shadow banking which operates as per Figure 6. In short, trust companies offer up beneficiary rights which make their way to banks via “offline” interbank repos.⁴ This arrangement still channels money from WMPs to trusts but we argue that it is a separate, potentially riskier, wave of shadow banking.

To see why, notice that the reverse repo in Figure 6 does not count against Bank A’s loan-to-deposit ratio. The need to record WMPs off balance sheet is thus mitigated and, indeed, the fraction of non-guaranteed WMPs falls from 70% in 2012 to 65% in 2013. But therein lies a dark side: now that banks can use reverse repos to hide their trust exposure *on* balance sheet, they can also channel *traditional deposits* into trust companies. This is a problem if China goes through with deposit insurance as recently proposed by the State Council.⁵ In theory, CBRC could ban offline repos so that trusts – which are less transparent than banks – do not benefit from a government safety net. However, the natural response would be more intricate arrangements to hide trust lending. This could then lengthen the intermediation chain and increase fragility given maturity mismatch.

3 Empirical Evidence

This section presents the key facts that motivate our paper. We first document the rise of shadow banking within a particular bank then turn to the main differences across banks. Our primary dataset is the Wind Financial Terminal which contains information about individual wealth management products. In cases where Wind is insufficient, we collect data from bank annual reports, regulatory agencies, and financial association websites.

⁴Offline transactions are ones which do not go through the China Foreign Exchange Trade System (the platform for normal repo or money market trades).

⁵See http://english.gov.cn/policies/latest_releases/2014/11/30/content_281475017400928.htm

3.1 The Case of China Merchants Bank

Among small and medium-sized banks, China Merchants Bank (CMB) is an important issuer of wealth management products. In 2012, it accounted for only 3% of total banking assets but 5.2% of WMPs outstanding at year-end and 17.7% of all WMPs issued during the year.⁶ It will thus be informative to chart the rise of CMB's shadow banking activities.

Figure 7(a) illustrates the bank's loan-to-deposit ratio. The blue line is the ratio of gross loans to deposits, both measured at the end of the year. The red line is the same ratio but using daily averages over the course of the year. The blue line tends to be well below the red one, suggesting that some window-dressing occurs at year-end. This is particularly true around 2008 when regulators began requiring a loan-to-deposit ratio of at most 75%. The solid gray line in Figure 7(a) plots the official ratio used by regulators. Relative to the blue line, it excludes certain agricultural and micro loans from the numerator.⁷ On the surface, CMB's 2008 ratio is visibly below the 75% cap. However, as the dashed gray line reveals, the cap binds when the regulator drills down to only RMB-denominated activities.

Figure 8(a) illustrates the subsequent growth in CMB's wealth management products. Annual issuance increased from RMB 0.1 trillion in 2007 to RMB 0.7 trillion in 2008 before reaching almost RMB 5 trillion in 2013. From the buyer's perspective, a WMP resembles a term deposit in that withdrawals are usually not permitted before maturity. CMB, like other WMP issuers, offers different products with different maturity dates. Products also differ based on where they invest the funds and whether or not the buyer's principal is guaranteed. As explained above, the key is in the principal guarantee. A non-guaranteed product (and the investments made by that product) can be booked off balance sheet where the loan-to-deposit cap does not apply. At the end of both 2012 and 2013, CMB had about 83% of its outstanding WMPs booked off balance sheet. Based on notes to the financial statements, figures for earlier years were likely higher.

⁶Based on data from KPMG, CBRC, and China Merchants Bank.

⁷Prior to 2007, bills were also not part of CMB's official calculation.

Another feature is that off balance sheet WMPs can be set to mature just prior to a regulatory exam. Upon maturity, the principal and interest are automatically deposited into the buyer's formal bank account. As long as the buyer does not immediately withdraw the funds or roll over the WMP, the bank will record an increase in deposits. To avoid violating capital rules, it will also need an increase in assets. These assets should not be recorded as loans otherwise the bank will hit its loan-to-deposit cap. One approach is to keep the automatic deposits as reserves. Another is to bring loans back on balance sheet through the repo market. The idea is similar to Figure 6 but with very short-term repos (i.e., only around exam dates). Figure 7(b) suggests CMB just kept reserves between 2009 and 2011. Naturally, the regulatory response to either approach is more frequent exams. Figure 9 shows that this prompted a temporary drop in the maturity of CMB's wealth management products. Reducing the amount of time until WMPs are automatically deposited helps lower average loan-to-deposit ratios during the year. This is consistent with the declining red line in Figure 7, raising concerns that banks exacerbated maturity mismatch to evade regulators.

So far though, non-guaranteed WMPs have proven fairly safe. Defaults on principal are virtually unheard of and the expected interest usually materializes. Whether this is sustainable going forward ultimately depends on the underlying investments. Disclosure at the bank level is patchy but Figure 10 provides an aggregate perspective. Loans were the key assets backing WMPs in 2008 and 2009, consistent with our view that loan-to-deposit caps matter. The subsequent decline of WMPs backed by loans coincides with new regulation. In August 2010, CBRC announced that WMPs could invest at most 30% in trust loans. This was not a restriction on the flow from banks to trusts per se but rather a restriction on how exactly trust companies used the funds. In March 2013, CBRC went even further and officially ruled that WMPs could invest a maximum of 35% in non-standard debt assets. This was a more direct crimp on bank-trust cooperation since most trust activity falls within the definition of non-standard debt.

CMB’s 2013 annual report references the March rule, stating that the bank “actively took ... measures to conduct its wealth management business in compliance with those requirements.” However, the offline repos discussed in Section 2 are visible on CMB’s balance sheet. As Figure 8(b) shows, CMB recorded a huge increase in trust beneficiary rights held either as investment or for resale in 2013. WMPs deriving their returns from these rights would be advertised as WMPs backed by interest rate products. In other words, CMB data echoes the recent rise of interest-related WMPs in Figure 10. Figure 8(b) also shows that CMB recorded a big jump in deposits by banks and other financial institutions, alternatively called placements from counterparts. Along with using beneficiary rights to generate returns for its own wealth management products, CMB can use rights to generate returns for other banks. The latter practice was innovated by Industrial Bank (another SMB) well before 2013 but it did not become widespread until CBRC cracked down on bank-trust cooperation.

3.2 Differences Between Small and Big Banks

The analysis of China Merchants Bank raises two questions. First, how similar are other small and medium-sized banks? Second, how similar are the four biggest banks?⁸

Figure 11 plots the evolution of raw loan-to-deposit ratios by bank size. As a group, SMBs resemble Merchants: constrained by the 75% cap and taking measures to comply with it. Big banks, on the other hand, do not appear similarly constrained. Even unadjusted, the Big Four’s loan-to-deposit ratio has not exceeded 65% for at least a decade. A popular explanation is that the government uses individual loan quotas to impose even stricter limits on the big banks. However, many of the banking insiders we spoke with conceded that quotas are open to negotiation and that the Big Four have sufficient sway to loosen any quotas imposed on them. In this sense, big banks are less constrained than the SMBs.

Heterogeneity in the bindingness of the 75% cap suggests a natural test: if enforcement

⁸The Big Four are ICBC, Construction Bank of China, Agricultural Bank of China, and Bank of China.

of the cap did indeed trigger shadow banking, then we should see small and medium-sized banks moving much more heavily into WMPs (and in particular off balance sheet WMPs) than the Big Four. We should also see much higher holdings of trust beneficiary rights by SMBs once CBRC restricts bank-trust cooperation. Figures 12 to 15 confirm this.

Figure 12 shows that big banks issue at most 30% of new WMP batches in any given year. This number is based on product counts since Wind does not yet have complete data on the total funds raised by each product. However, using data from CBRC and the annual reports of the Big Four, we estimate that big banks accounted for 40.5% of WMP balances outstanding at the end of 2012 and 39.6% at the end of 2013. The count data for these two years is within the same ballpark and, like the balance data, exhibits no obvious trend. Figure 13 then shows that WMP issuance by SMBs causes (in the Granger sense) WMP issuance by big banks. The reverse is not true at any reasonable level of significance, suggesting that the impetus for WMPs is indeed coming from small and medium-sized banks.

Turning next to non-guaranteed WMPs, we find that roughly 56% of WMP batches were issued without a principal guarantee. Figure 14 decomposes this percentage into big banks versus SMBs. For any minimum investment amount, we find that SMBs are the main provider of non-guaranteed batches.⁹ Figure 15 then shows a dramatic rise in “other investments” by SMBs as CBRC begins cracking down on bank-trust cooperation. Other investments include purchases of trust beneficiary rights or holdings of such rights through reverse repos. Figure 15 also shows that SMBs recorded a big jump in placements from counterparts, consistent with our discussion of China Merchants Bank. In contrast, there is no rise in other investments or placements from counterparts at the Big Four.

That SMBs are the driving force behind shadow banking in China stands in sharp contrast

⁹We are working on a decomposition by RMB value. CBRC data indicates that 65% of WMP balances outstanding at the end of 2013 were not guaranteed. This works out to RMB 6.64 trillion. Based on annual report data, *at most* RMB 2.82 trillion came from the Big Four. We say *at most* because the entire WMP balance reported by Bank of China is described as an unconsolidated balance yet the micro data captures several guaranteed batches for this bank.

to other regions. In the U.S. and Europe, for example, big banks are generally seen as the main drivers.¹⁰ Prior to the 2007-2009 crisis, large U.S. and European banks used off balance sheet vehicles to issue asset-backed commercial paper (ABCP). Both ABCP and WMPs are short-term debts supported by longer-term investments. In other words, both involve a maturity mismatch. Figure 16 summarizes the maturity distribution of WMPs. The median maturity has been around 3 months since 2008, with roughly 25% of WMPs having a maturity of 1 month or less. The presence of a mismatch can then be gleaned from Figure 17. First, the majority of trust capital is money already pooled together by other institutions (e.g., either directly or indirectly coming from bank WMPs). Second, trust assets are mainly held as loans and long-term investments. Consistent with the fairly long horizon of trust assets, trust companies issued products with an average maturity of 1.7 years when attempting to pool money on their own during the first half of 2013.¹¹

3.3 Discussion

This section laid out some key facts about shadow banking in China. We argued that stricter enforcement of loan limits has triggered a sharp rise in off balance sheet activities by small and medium-sized banks. What tradeoffs do banks face when deciding between on and off balance sheet activities and why do general equilibrium forces permit a rise in the debt-to-GDP ratio? These questions are best answered through the lens of a model. First, we want to prove that tightening the loan-to-deposit cap does indeed trigger shadow banking and increase overall lending. Second, we want to better understand the heterogeneity between SMBs and the four biggest banks.

The second point goes back to Figure 11 which reveals an interesting reversal among big banks: their loan-to-deposit ratio was falling prior to 2008 but has been rising ever since. Why did big banks become more aggressive at the exact moment that regulators

¹⁰See, for example, Cetorelli and Peristiani (2012) and Acharya et al (2013).

¹¹Annual Report of the Trust Industry in China (2013).

began enforcing loan-to-deposit caps? A common explanation is the two-year RMB 4 trillion stimulus package announced by the State Council in late 2008. The central government funded 30% of this package directly, with the remaining RMB 2.8 trillion to be borrowed by local governments. However, balance sheet data shows that gross loans at the Big Four jumped by RMB 4.8 trillion between 2008 and 2009 then by RMB 3.5 trillion between 2009 and 2010. Therefore, even if big banks were pressured to finance the full RMB 2.8 trillion for local governments (an assumption not supported by our discussions with CBRC), there is still a sizeable jump in big bank lending left to be explained.

We argue that the Big Four have strategically become less liquid in order to tighten interbank money market conditions and pressure SMBs to pare down off balance sheet activities. The goal is essentially to stop SMBs from severely impinging on the Big Four's deposit base. This strategy by the big banks is consistent with higher and more volatile interbank money market rates (Figure 18) despite liquidity injections by the People's Bank of China.¹² The next two sections build our model and show that competition between big and small banks is key to explaining the universe of facts presented here.

4 Simple Model

Our paper proposes a banking model with three main ingredients: (i) maturity transformation, (ii) an interbank market for reserves, and (iii) heterogeneity in market power. The third ingredient is not found in most banking models so, to understand its contribution, we begin without (iii) and consider a perfectly competitive representative agent model. Such a model clearly cannot explain heterogeneity between big and small banks. This section will show that it also cannot explain a rise in total lending or an increase in interbank interest rates after regulatory tightening. Heterogeneity in market power will thus be important for matching features of the Chinese economy.

¹²For data on the PBOC's liquidity injections, see "Continual PBOC Injections Forestall Banking System Pain" by Andrew Polk, Conference Board, November 2013.

4.1 Environment

There are three periods, $t \in \{0, 1, 2\}$, and a continuum of banks, $j \in [0, 1]$. All banks are identical at $t = 0$ and perform maturity transformation in the spirit of Diamond and Dybvig (1983). More precisely, a dollar of household savings deposited in a bank at $t = 0$ becomes $1 + i_B$ if withdrawn at $t = 1$ and $(1 + i_B)^2$ if withdrawn at $t = 2$. The bank creates these returns by pooling savings and investing at least some of the pool in an asset that pays $(1 + i_A)^2$ at $t = 2$, where $i_A > i_B$. We refer to the bank's investment as a loan. For simplicity, liquidating a loan at $t = 1$ yields nothing and i_A and i_B are fixed.¹³ Anything the bank does not lend at $t = 0$, it holds as reserves. We will first elaborate on household savings then explain the market for reserves.

Households are endowed with one unit of savings which can be split between traditional deposits and wealth management products (WMPs). Compared to the deposits described above, WMPs pay an additional return ξ_j . To ease the exposition, suppose ξ_j only accrues if the WMP is held until $t = 2$. Denote by $W_j(\xi_j)$ the demand for bank j 's WMPs, where $W_j(0) = 0$, $W_j'(\cdot) > 0$, and $W_j''(\cdot) \leq 0$. In words, WMPs are only purchased if they pay more interest than regular deposits. The demand for WMPs then increases with the amount of interest paid but the increase is bounded. Moreover, WMP demand is a continuous function: deposits have an (unmodelled) convenience value which stops households from switching entirely to wealth management products once $\xi_j > 0$.

We use $D_j(\xi_j)$ to denote the demand for bank j 's deposits. In lieu of specific functional forms, we simply write the total amount of household savings attracted by bank j as:

$$D_j(\cdot) + W_j(\cdot) \equiv \rho_0 + \rho_1 W_j(\cdot)$$

where $\rho_0 > 0$ and $\rho_1 \in [0, 1]$. If $\rho_1 = 0$, then each bank attracts a fixed amount of household

¹³The last part is a stand-in for competition pushing loan rates down to the loan rate floor and deposit rates up to the deposit rate ceiling.

savings. Therefore, any WMPs issued by bank j will cut one-for-one into its own deposit base. At the other extreme, $\rho_1 = 1$, each bank views itself as attracting a fixed amount of deposits: its WMPs will just cut into the savings available for other banks. Recall that total household savings sum to one so, given all other parameters, the value of ρ_0 must be consistent with an optimal ξ_j^* such that $D_j(\xi_j^*) + W_j(\xi_j^*) = 1$ in a symmetric equilibrium.

The only risk in our model (for now) is an idiosyncratic liquidity risk. In particular, each bank can be in one of two states at $t = 1$. The first is a low withdrawal state where fraction θ_ℓ of households cash out their deposits and WMPs. The second is a high withdrawal state where the fraction is $\theta_h > \theta_\ell$. The low withdrawal state occurs with probability $\pi \in (0, 1)$ and the high withdrawal state occurs with probability $1 - \pi$. The expected withdrawal rate can thus be written as $\bar{\theta} \equiv \pi\theta_\ell + (1 - \pi)\theta_h$.

To cover withdrawals, the banking system must have some reserves. Denote bank j 's reserve holdings by R_j . More precisely, bank j attracts household savings $D_j(\cdot) + W_j(\cdot)$ at $t = 0$, holds R_j as reserves, and lends the rest. If R_j proves insufficient to cover bank j 's withdrawals at $t = 1$, then j borrows from an interbank market at interest rate i_L . Interbank lenders are banks with surplus reserves. We also allow for a supply of external funds $\Psi(i_L) = \psi(i_L - i_B)$, where $\psi \geq 0$. These funds can be interpreted as liquidity injections by the central bank. We will conduct some policy experiments with them in the full model so introduce them here for completeness.

In a symmetric equilibrium, interbank market clearing requires:

$$R_j^* + \Psi(i_L^*) = \bar{\theta}(1 + i_B) \tag{1}$$

Total liquidity available at $t = 1$ is the sum of bank reserves, R_j , and external liquidity, $\Psi(i_L)$. Total liquidity required is the sum of household withdrawals. By symmetry, each bank attracts a unit of household savings at $t = 0$ and is thus liable for $1 + i_B$ at $t = 1$.

With an average of $\bar{\theta}$ households withdrawing at $t = 1$, the banking system needs liquidity $\bar{\theta}(1 + i_B)$. The i_L that solves equation (1) is the equilibrium interbank rate. This rate clearly enters (1) through $\Psi(\cdot)$ but it can also enter indirectly through the optimal choice of R_j .

The last modeling element before moving to bank optimization is a regulatory standard. Suppose the government imposes a loan limit on each bank. This limit can also be viewed as a liquidity rule which says that the ratio of reserves to on-balance-sheet liabilities must be at least $\alpha \in (0, 1)$. Given the structure of our model, reserves are only needed for use in $t = 1$ so enforcement of the liquidity rule is confined to $t = 0$. The relevant liabilities are deposits and WMPs. Whereas deposits must be booked on balance sheet, banks can choose where to manage WMPs and the loans financed by those WMPs. If fraction $\tau_j \in [0, 1]$ is managed in an off balance sheet vehicle, then bank j 's reserve holdings only need to satisfy:

$$\lambda_j \equiv \frac{R_j}{D_j(\cdot) + (1 - \tau_j)W_j(\cdot)} \geq \alpha \quad (2)$$

Use of off balance sheet vehicles is “regulatory arbitrage” as defined in Adrian et al (2013).¹⁴ Note that $1 - \lambda_j$ is our model’s counterpart to the loan-to-deposit ratio in Section 3.

4.2 Results

The representative bank chooses the attractiveness of its WMPs ξ_j , the extent of its off balance sheet activities τ_j , and its reserve holdings R_j to maximize expected profit at $t = 0$ subject to the liquidity rule set out in (2). Mathematically:

$$\max_{\xi_j, \tau_j, R_j} \left\{ \begin{array}{l} (1 + i_A)^2 [D_j(\xi_j) + W_j(\xi_j) - R_j] \\ + (1 + i_L) [R_j - \bar{\theta}(1 + i_B) [D_j(\xi_j) + W_j(\xi_j)]] \\ - (1 - \bar{\theta}) [(1 + i_B)^2 [D_j(\xi_j) + W_j(\xi_j)] + \xi_j W_j(\xi_j)] \end{array} \right\} \quad (3)$$

¹⁴Their definition is “a change in structure of activity which does not change the risk profile of that activity, but increases the net cash flows to the sponsor by reducing the costs of regulation.”

subject to

$$R_j \geq \alpha [D_j(\xi_j) + (1 - \tau_j) W_j(\xi_j)]$$

$$\tau_j \in [0, 1]$$

Let μ_j denote the Lagrange multiplier on the liquidity rule. It can be interpreted as the shadow cost of reserves. Also let η_j^0 and η_j^1 denote the multipliers on $\tau_j \geq 0$ and $\tau_j \leq 1$ respectively. Finally, define the interest rate spread:

$$\Delta_j \equiv \frac{[1 - \bar{\theta}(1 + i_B)](1 + i_A)^2}{1 - \bar{\theta}} - (1 + i_B)^2$$

We assume $\Delta_j > 0$ so that the bank's problem is not trivial. The first order conditions with respect to R_j , τ_j , and ξ_j are:

$$\mu_j = (1 + i_A)^2 - (1 + i_L) \quad (4)$$

$$\eta_j^1 = \eta_j^0 + \alpha \mu_j W_j(\xi_j) \quad (5)$$

$$\xi_j + \frac{W_j(\xi_j)}{W_j'(\xi_j)} = \rho_1 \Delta_j - \frac{\rho_1 [\alpha - \bar{\theta}(1 + i_B)]^{-\alpha \tau_j}}{1 - \bar{\theta}} \mu_j \quad (6)$$

If $1 + i_L = (1 + i_A)^2$, then $\mu_j = 0$. The liquidity rule is not binding and equation (6) reveals that the value of ξ_j^* hinges on ρ_1 . The interpretation of $\rho_1 = 0$ is that bank j 's WMP issuance cuts one-for-one into its own deposit base. Deposits are a cheaper liability than WMPs so the result is $\xi_j^* = 0$ and $W_j(\xi_j^*) = 0$. With $\rho_1 > 0$, the cut into bank j 's deposits is only partial: the rest comes from the market share of other banks. This prompts $\xi_j^* > 0$ and $W_j(\xi_j^*) > 0$ but with $\frac{\partial \xi_j^*}{\partial \alpha} = 0$. Moreover, $\eta_j^1 = \eta_j^0$ from equation (5) so the bank is indifferent between any $\tau_j^* \in [0, 1]$. WMP issuance thus stems from competition for a larger share of household savings, not from a desire to evade liquidity requirements.

Things are different if $1 + i_L < (1 + i_A)^2$, in which case $\mu_j > 0$ and the liquidity rule binds. Return to $\rho_1 = 0$ which previously resulted in no WMPs. Now we have a solution with

positive WMP issuance. To see this, suppose the bank chooses $\tau_j^* = 1$. Equation (6) then returns $\xi_j^* > 0$ which, when substituted into (5), implies $\eta_j^1 > 0$ and confirms the choice of $\tau_j^* = 1$.¹⁵ Notice that the incentive to issue WMPs no longer comes from competition: with $\rho_1 = 0$, the bank is simply substituting within its own liabilities. Instead, WMPs are issued because they can be booked off-balance sheet, away from the binding liquidity rule. It now remains to check whether the equilibrium value of i_L does indeed satisfy $1 + i_L < (1 + i_A)^2$. The results are summarized next:

Proposition 1 *Suppose $\rho_1 = 0$ so regulatory arbitrage is the only motive for issuing WMPs. If $W_j(\cdot)$ is sufficiently concave, then there exists a scalar $\bar{\alpha} \in [0, 1]$ such that:*

1. $1 + i_L^* = (1 + i_A)^2$ and $\xi_j^* = 0$ with $\lambda_j^* = \bar{\alpha}$ for any $\alpha \leq \bar{\alpha}$
2. $1 + i_L^* < (1 + i_A)^2$ and $\xi_j^* > 0$ with $\tau_j^* = 1$, $\frac{di_L^*}{d\alpha} < 0$, $\frac{d\xi_j^*}{d\alpha} > 0$, and $\lambda_j^* = \alpha$ for any $\alpha > \bar{\alpha}$

In words, sufficiently stricter regulation (i.e., increasing α from below $\bar{\alpha}$ to above $\bar{\alpha}$) triggers the issuance of off balance sheet WMPs and leads to a lower interbank rate. Our simple model thus accounts for the rise of shadow banking but it cannot account for tighter interbank conditions. As shown below, this shortcoming is not an artifact of $\rho_1 = 0$:

Proposition 2 *If $\rho_1 \in (0, 1]$, then ψ below some positive upperbound ensures $D_j^* > 0$. Moreover, there is a scalar $\bar{\bar{\alpha}} \in [0, 1]$ such that $1 + i_L^* = (1 + i_A)^2$ for any $\alpha \leq \bar{\bar{\alpha}}$ and $1 + i_L^* < (1 + i_A)^2$ with $\frac{di_L^*}{d\alpha} < 0$ otherwise.*

It is easy to see from equation (1) that a lower interbank rate implies a decrease in aggregate lending. Household savings are normalized to one and banks hold R_j in reserves so the total amount lent at $t = 0$ is $1 - R_j$. With lower i_L , less external liquidity is available to satisfy the same withdrawals. Banks must therefore hold more of their own reserves, prompting a fall in total lending. If we eliminate external liquidity altogether, then total

¹⁵In principle, also have a solution with $\xi_j = 0$: using $\tau_j = 0$ in (6) gives $\xi_j = 0$ which, when subbed into (5), is consistent with any $\tau_j \in [0, 1]$. But can't go from any $\tau_j \in [0, 1]$ to $\tau_j = 0$ so eliminate by refinement.

lending is constant at $1 - \bar{\theta}(1 + i_B)$ for any α . Either way, the simple model outlined here cannot generate more lending in the midst of tightening liquidity rules. By virtue of focusing on a representative bank, it is also silent on differences between big and small banks: all we can glean from Proposition 1 is that small banks become constrained by the new liquidity rules and, to comply with these rules, their loan-to-deposit ratio falls.

5 Asymmetric Competition

Based on the discussion above, we now introduce a big bank. To generate the differences in loan-to-deposit ratios established in Section 3, we cannot have this bank take the interbank rate as given. Stated more formally, we need a different shadow cost of reserves to get differences in the bindingness of the liquidity rule.¹⁶ Another approach could be to make the big bank a price-taker and just assume different preferences than the small banks. However, the general view among regulators is that the Big Four care about profitability and are driven by market forces so assuming differences in preferences would be ad hoc. We find it much more fruitful to keep banks as similar as possible except that the big – by definition of being big – recognize their decisions are economically significant.

5.1 Extending the Simple Model

We keep the continuum of small banks, $j \in [0, 1]$, and introduce one big bank indexed by k . The big bank is subject to the same liquidity risk described in the simple model. However, since the big bank is a price-setter on the interbank market, the interbank rate will depend

¹⁶Suppose everyone is a price-taker. Then, unless α is large, need $1 + E(i_L) = (1 + i_A)^2$ otherwise reserves are insufficient which cannot be an equilibrium. With $1 + E(i_L) = (1 + i_A)^2$, everyone is indifferent between holding reserves and lending. This is consistent with the argument in Farhi et al (2009). Given indifference, it is then possible that the small bank loan-to-deposit ratio is exactly $1 - \alpha$ while the big bank ratio is below $1 - \alpha$ but this is only one of many possibilities. It is also possible that an increase in α leads to convergence in these ratios but, again, there are many other possibilities. In short, an objective equilibrium selection criterion is missing: the model with everyone being a price-taker does not provide clear microfoundations for converging loan-to-deposit ratios. Such a model, by virtue of being stuck at $1 + E(i_L) = (1 + i_A)^2$, would also not generate a change in the interbank rate after an increase in α .

on the big bank's withdrawal fraction. We can thus interpret the big bank's individual state as an aggregate state. In particular, suppose the big bank is hit by withdrawal fraction θ_s where $s \in \{\ell, h\}$. Then market clearing in state s requires:

$$R_j + R_k + \Psi(i_L^s) = \bar{\theta}(1 + i_B)(D_j + W_j) + \theta_s(1 + i_B)(D_k + W_k)$$

There are two important points here. First, all choices are made ex ante so the market can only clear in one state unless $\Psi(\cdot)$ is very potent. Let's keep $\Psi(\cdot)$ in the background for now. If the market clears at i_L^h , then i_L^ℓ is associated with an excess supply of reserves. If the market instead clears at i_L^ℓ , then i_L^h is associated with excess demand for reserves. We assume that the market clears at i_L^h and set $i_L^\ell = i_B$ to reduce notation.¹⁷ The second important point is that the amount of liquidity needed at $t = 1$ is endogenous. In particular, it depends on the split between $D_j + W_j$ and $D_k + W_k$ which will itself depend on ξ_j and ξ_k . This differs from the simple model where the right-hand side of equation (1) was constant.

Aggregate savings are again normalized to one. In addition, we use the following functional forms for WMP demand:

$$\begin{aligned} W_j &= \beta \xi_j (\xi_j + \xi_k)^{\gamma-1} \\ W_k &= \beta \xi_k (\xi_j + \xi_k)^{\gamma-1} \end{aligned}$$

where $\beta > 0$ and $\gamma \in [0, 1]$. If $\gamma = 1$, then the demand for big bank WMPs only responds to ξ_k . If $\gamma = 0$, then it only responds to the relative value ξ_k/ξ_j . The demand for small bank WMPs depends on γ in the same way. The balance of aggregate savings is then divided between banks in the form of traditional deposits. Mathematically, $\delta \in (0, 1)$ with:

$$\begin{aligned} D_j &= \delta [1 - \beta (\xi_j + \xi_k)^\gamma] \\ D_k &= (1 - \delta) [1 - \beta (\xi_j + \xi_k)^\gamma] \end{aligned}$$

¹⁷One can interpret $i_L^\ell > 0$ as interest on reserves at the end of $t = 1$.

Small banks still solve (3) but using the functional forms here along with the expected interbank rate, $1 + \pi i_B + (1 - \pi) i_L^h$, in place of $1 + i_L$. The big bank solves:

$$\max_{\xi_k, \tau_k, R_k} \left\{ \begin{array}{l} (1 + i_A)^2 (D_k + W_k - R_k) \\ + [1 + \pi i_B + (1 - \pi) i_L^h] [R_k - \bar{\theta} (1 + i_B) (D_k + W_k)] \\ - (1 - \bar{\theta}) [(1 + i_B)^2 (D_k + W_k) + \xi_k W_k] \\ - \pi (1 - \pi) (\theta_h - \theta_\ell) (1 + i_B) (i_L^h - i_B) (D_k + W_k) \end{array} \right\}$$

subject to

$$R_k \geq \alpha [D_k + (1 - \tau_k) W_k]$$

$$\tau_k \in [0, 1]$$

$$\xi_j = \xi_j(\xi_k, \tau_k, R_k) \text{ and } i_L^h = i_L^h(\xi_k, \tau_k, R_k)$$

There are two differences relative to the small bank problem. First is the extra term $\pi (1 - \pi) (\theta_h - \theta_\ell) (1 + i_B) (i_L^h - i_B) (D_k + W_k)$ subtracted from the objective function. This arises because the big bank's state is the aggregate state: if the big bank gets hit by a high withdrawal shock, then it has to pay the higher interest rate on a higher amount of interbank borrowing. Second, and most importantly, is that the big bank internalizes how ξ_j and i_L^h depend on its own choices. The expression for i_L^h comes from interbank clearing while ξ_j comes from the first order conditions of the small banks. Therefore, in our model, big banks can be unconstrained by the loan-to-deposit cap because they internalize how their liquidity affects the market. If they lend too much and are then hit by a high withdrawal shock, they will have to borrow a lot from the interbank market. This increases the demand for liquidity and thus increases borrowing costs.

5.2 Equilibrium Conditions

Given the stylized facts in Section 3, we seek parameters such that small banks are constrained by the loan limit while big banks are not. We first derive the equations that define

this equilibrium then find the parameter conditions to support it. Defining some terms upfront will help streamline the exposition. First, define the following constants:

$$\Phi \equiv \alpha - \bar{\theta}(1 + i_B)$$

$$\Theta \equiv \frac{\pi(\theta_h - \theta_\ell)(1 + i_B)[(1 + i_A)^2 - (1 + i_B)]}{1 - \bar{\theta}}$$

Next, define the ratio $z \equiv \frac{\xi_k}{\xi_j}$ and the following functions of z :

$$m(z) \equiv 2\gamma(\gamma + z)[1 - \delta(1 + z)] - (1 + \gamma + 2z)$$

$$n(z) \equiv \gamma(1 + \gamma) + 4\gamma z + (1 + \gamma^2)z^2$$

$$q(z) \equiv (\gamma + z)[1 + \gamma + 2z - (1 - \gamma)z^2] + z[(1 + \gamma)(1 + 2z) + 2z^2]$$

Finally, define the following function of i_L^h :

$$g(i_L^h) \equiv \frac{(1 + i_A)^2 - (1 + i_B)^2}{1 - \bar{\theta}} - \frac{\bar{\theta}(1 - \pi)(1 + i_B)}{1 - \bar{\theta}}(i_L^h - i_B)$$

Equilibrium is characterized by the first order conditions from the small bank problem, the big bank problem, and interbank market clearing. It is easy to show $\tau_j = 1$ and $\tau_k = 0$. In other words, small banks are the ones who go off balance sheet. Going through the algebra (details in appendix), we then need a triple (z, ξ_j, i_L^h) that solves:

$$i_L^h = i_B + \frac{1}{\bar{\theta}(1 - \pi)(1 + i_B)} \left[(1 + i_A)^2 - (1 + i_B)^2 - \frac{(1 - \bar{\theta}) \left[\frac{\alpha\gamma\delta\Delta_j}{\bar{\theta}(1 + i_B)} + (1 + \frac{\gamma+z}{1+z})\xi_j \right]}{\frac{\gamma+z}{1+z} + \frac{\gamma\delta\Phi}{\bar{\theta}(1 + i_B)}} \right]$$

$$i_L^h = i_B + \frac{(1 + i_A)^2 - (1 + i_B)}{2(1 - \pi)} - \frac{\delta\Phi}{2\psi} + \frac{\beta\bar{\theta}(1 + i_B)\xi_j^\gamma(1 + z)^\gamma}{2\psi(1 - \gamma)} \left[1 + \frac{\delta\Phi}{\bar{\theta}(1 + i_B)} - \frac{[\Theta + 2(z + \frac{\gamma+z}{1-\gamma})\xi_j] \left[\frac{\gamma+z}{1+z} + \frac{\gamma\delta\Phi}{\bar{\theta}(1 + i_B)} \right]^2}{\frac{\alpha\gamma\delta\Delta_j}{\bar{\theta}(1 + i_B)} + \left[\frac{1+z}{\gamma+z} + \frac{\gamma+z}{1+z} + \frac{2\gamma\delta\Phi}{\bar{\theta}(1 + i_B)} \right] \frac{(\gamma+z)\xi_j}{1-\gamma}} \right]$$

$$2q(z)\xi_j^2 - [\Theta m(z) + n(z)g(i_L^h)]\xi_j - (1-\gamma)[\gamma\Theta[1-\delta(1+z)] + zg(i_L^h)]g(i_L^h) = 0$$

To confirm the initial supposition that only small banks are constrained by the loan limit, we now need to check:

$$i_L^h \stackrel{?}{<} i_B + \frac{(1+i_A)^2 - (1+i_B)}{1-\pi} \quad (7)$$

$$\lambda_k \equiv \frac{R_k}{D_k + W_k} = \pi(\theta_h - \theta_\ell)(1+i_B) + \frac{\bar{\theta}(1+i_B) - \psi(i_L^h - i_B) - \alpha\delta[1 - \beta\xi_j^\gamma(1+z)^\gamma]}{1 - \delta - \beta\xi_j^\gamma(1+z)^{\gamma-1}(1-\delta-\delta z)} \stackrel{?}{>} \alpha \quad (8)$$

Figure 19 shows that (7) and (8) require α not too high and ψ between some positive bounds. Recall that $1 - \alpha$ is the loan limit while ψ is the responsiveness of external liquidity to the interbank rate. If ψ is very low, then the interbank rate is so high that small banks choose to hold additional liquidity and are thus not constrained by the loan limit. If ψ is very high, then the big bank has insufficient influence on the interbank market, prompting it to behave like a small bank and hit the loan limit. The restriction on α is also quite intuitive: if regulation is overly strict, then everyone is constrained.

5.3 Comparative Statics

We now want to show that our model is capable of generating the key facts in Section 3. The red area in Figure 19 does this. In particular, for α and ψ within consistent bounds, a tighter loan limit (i.e., an increase in α) leads to: (i) a decrease in the small bank loan-to-deposit ratio; (ii) an increase in the big bank loan-to-deposit ratio; (iii) an increase in the interbank interest rate; (iv) an increase in total lending; and (v) an increase in the fraction of total lending done off balance sheet.

The intuition is as follows. Small banks move (more heavily) into off balance sheet wealth management products after liquidity rules tighten. Once there, they can also offer higher interest rates than the rates allowed on traditional deposits. Mathematically, they

can offer $(1 + i_B)^2 + \xi_j$ in $t = 2$ instead of just $(1 + i_B)^2$. All else constant, this poaches household savings from the big bank. One way for the big bank to respond is by offering its own products with high interest rates. Naturally, this is costly because of the high rates. Another, more indirect, approach is for the big bank to use its influence on the interbank market. Small banks have less incentive to skirt liquidity rules if the price of liquidity is high enough. Therefore, the big bank manipulates the interbank market to make small banks scale back their issuance of wealth management products. This manipulation requires the big bank to become less liquid, consistent with a rise in its loan-to-deposit ratio.

In support of our intuition, we find that the big bank’s loan-to-deposit ratio is much less responsive to changes in α when its market share, $D_k + W_k$, is held constant. We also find that our qualitative results are largely unchanged if k ’s choices are derived as a Nash equilibrium between two big players.

One issue is that the full model does not go from $\xi_j = 0$ to $\xi_j > 0$ when α goes from a low value to a high value. This is true even if small banks are not constrained by the loan limit (i.e., even if $\mu_j = 0$). This is reminiscent of the simple model with $\rho_1 > 0$: regardless of α , there is a motive for WMP issuance that stems from competition for a larger share of household savings. Adding an extra cost of WMP issuance would neutralize some of the competitive motive and “normalize” the model so that the competition benefit of WMPs is offset by the extra cost when liquidity regulations are mild. In short, we would just need to shift the first order condition for ξ_j down so that it starts close to zero for α low.

6 Policy Experiments

With the full model of Section 5 in hand, we can now conduct some policy experiments.

Central Bank Liquidity Injections Suppose the central bank becomes more responsive to interbank market conditions. This maps into our model as an increase in ψ and undermines

the big bank's use of i_L^h to defend its market share. The result (holding all other parameters including α constant) is a higher loan-to-deposit ratio for the big bank, a lower interbank rate, higher values of ξ_j and ξ_k such that ξ_k/ξ_j also increases, an increase in total lending, and an increase in off balance sheet loans. In other words, liquidity injections to stabilize the interbank market could trigger a huge expansion of shadow banking going forward.

Deposit Rate Liberalization The last major barrier to fully liberalized interest rates in China is the cap currently placed on deposit rates. Some have argued that the central government tolerates wealth management products because the interest rates that prevail on these products might reveal something about the deposit rates that will prevail if/when China fully liberalizes. We can use our model to evaluate whether extrapolation is indeed possible here. One way is to shut down off balance sheet WMPs and just let ξ be part of the deposit rate. For high α , this leads to lower choices of ξ_j and ξ_k than the model in Section 5. For low α , it leads to higher choices. Off balance sheet WMPs are thus not a good guide for liberalized deposit rates. The reason is that the rates offered on off balance sheet WMPs reflect both flexible interest rates and different liquidity conditions. If α is high (i.e., if loan limits are strict), then small banks have an incentive to evade liquidity rules. However, evasion requires off balance sheet vehicles: if such vehicles are shut down, then any additional savings attracted by $\xi_j > 0$ are still subject to the high α . This lowers the incentive to attract additional savings and thus lowers the equilibrium choice of ξ_j .

Systemic Risk As described in Section 2, CBRC has recently clamped down on bank-trust cooperation. This has resulted in a new wave of shadow banking which operates as per Figure 6. Banks are essentially becoming more interconnected in order to funnel money from WMPs into trust companies without violating new rules. We are currently extending our model in this direction to formally show that regulation in China is increasing systemic risk. A run-like event on one bank can now trigger a serious chain reaction.

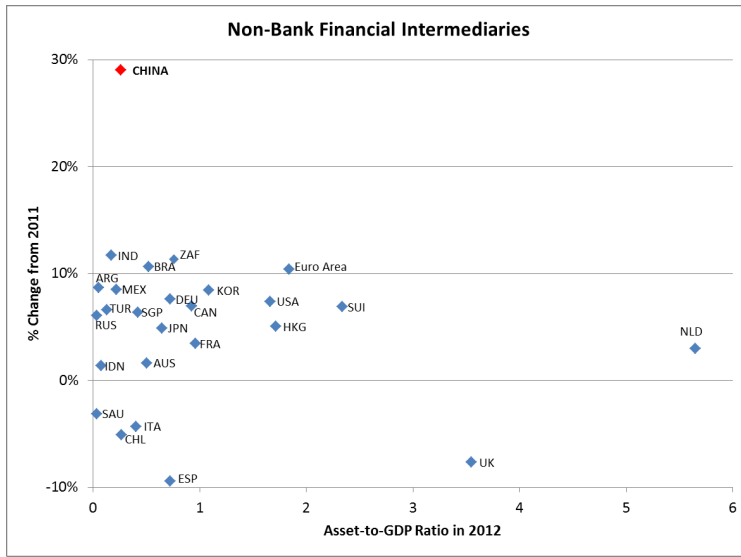
7 Conclusion

This paper has explored the dynamics of China's shadow banking sector. We began by documenting important differences in the cross-section of Chinese banks to isolate the regulatory triggers for shadow banking. There is a large and compelling literature that uses disaggregated data to understand financial crises post-mortem. Our goal was to provide an ante-mortem analysis. We then built a model that rationalizes the facts and used it to conduct policy experiments. We found that asymmetric competition between banks is both a short-run stabilizer and a long-run risk, with new regulations potentially exacerbating the tipping point. We are currently extending the model to explore additional dimensions of systemic risk.

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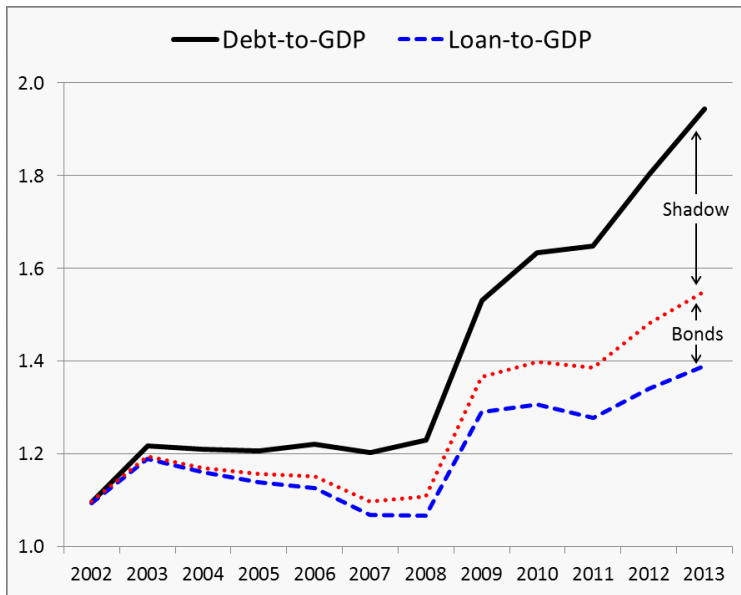
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Figure 1



Source: FSB Global Shadow Banking Monitoring Report 2013

Figure 2



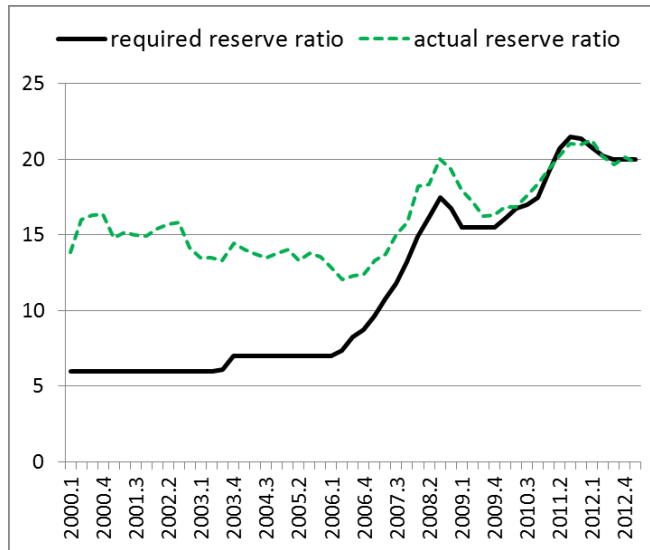
Source: National Bureau of Statistics China

Debt is total social financing excluding domestic equity financing of non-financial enterprises

Loans include RMB loans and converted foreign currency loans

Shadow finance includes trust loans, entrusted loans, and undiscounted bankers' acceptances

Figure 3



Source: People's Bank of China

Figure 4

Anatomy of a Wealth Management Product

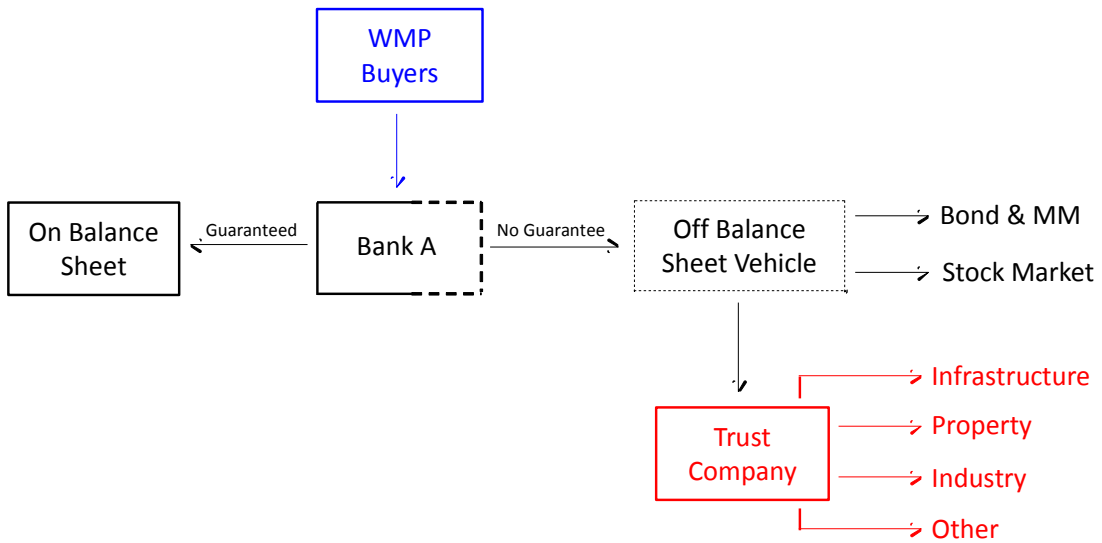
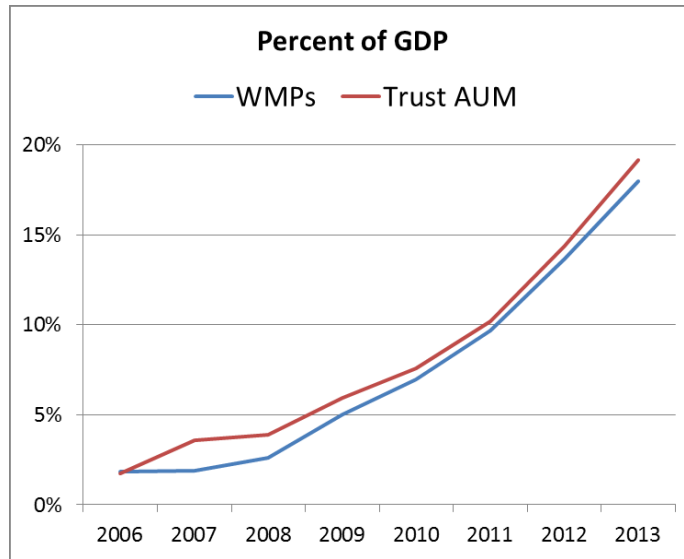


Figure 5



Source: PBOC, CBRC, IMF, China Trustee Association, KPMG China Trust Surveys

Figure 6

Business with Counterparts

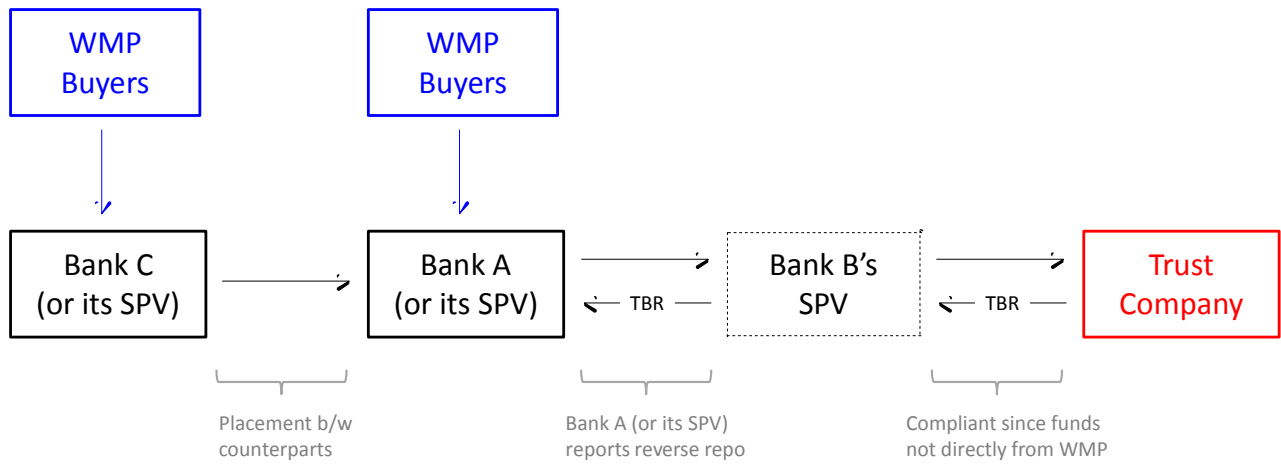
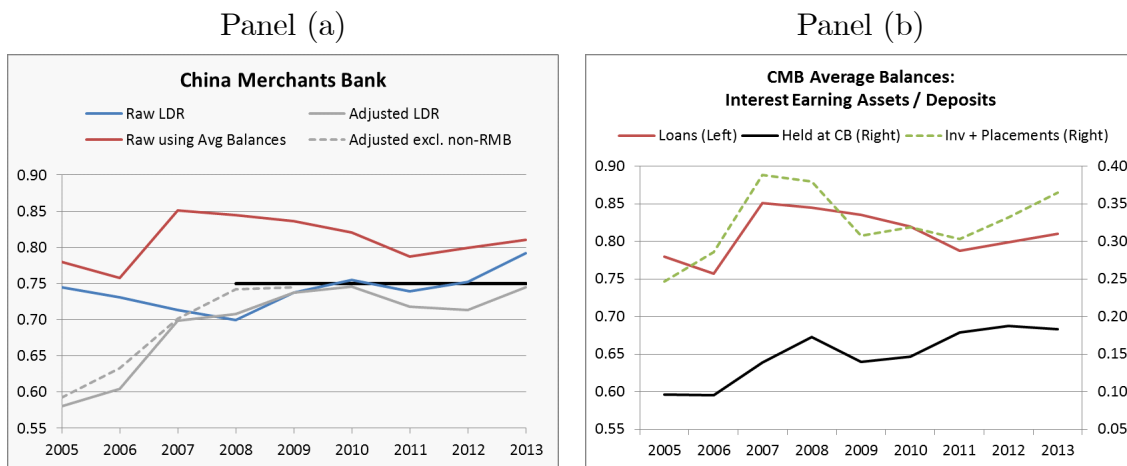
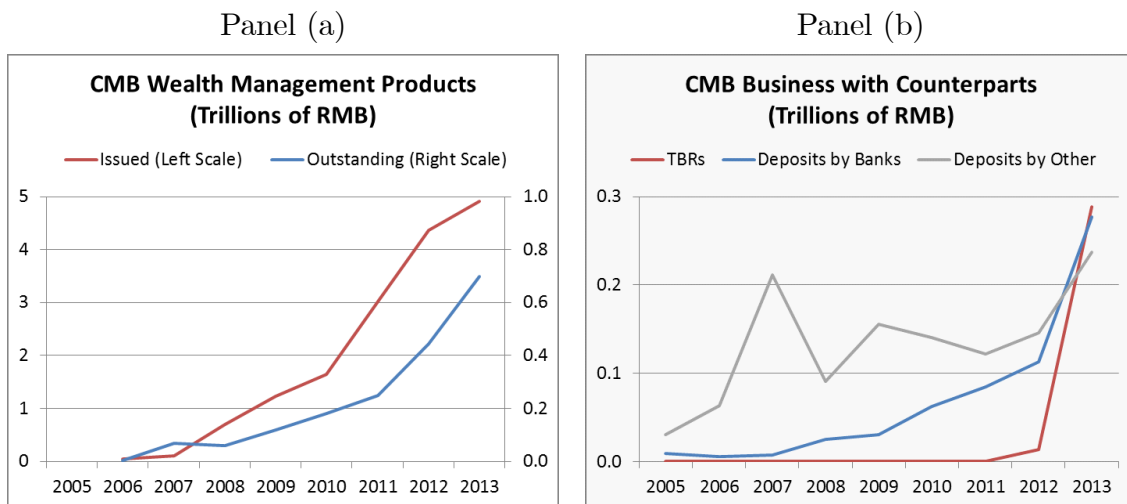


Figure 7



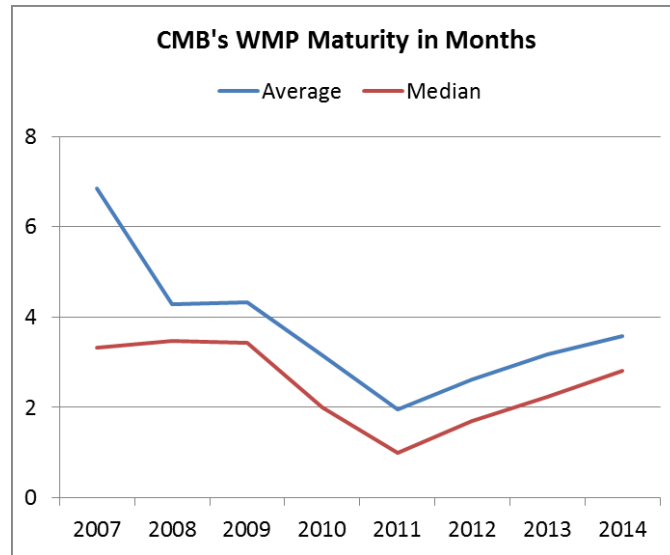
Source: China Merchants Bank Annual Reports

Figure 8



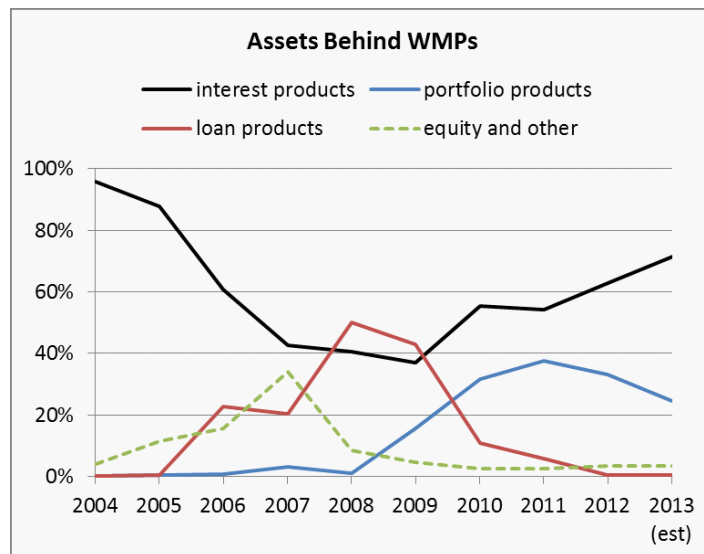
Source: China Merchants Bank Annual Reports

Figure 9



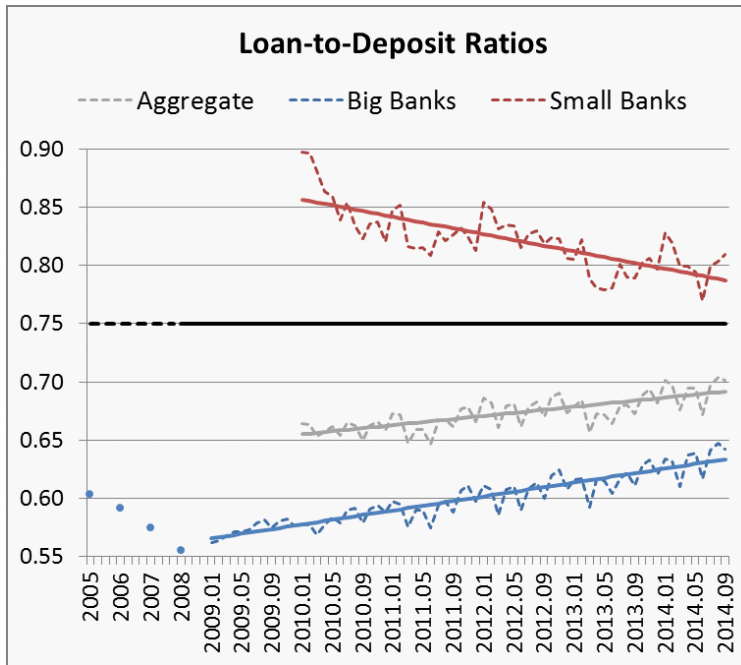
Source: Wind Financial Terminal

Figure 10



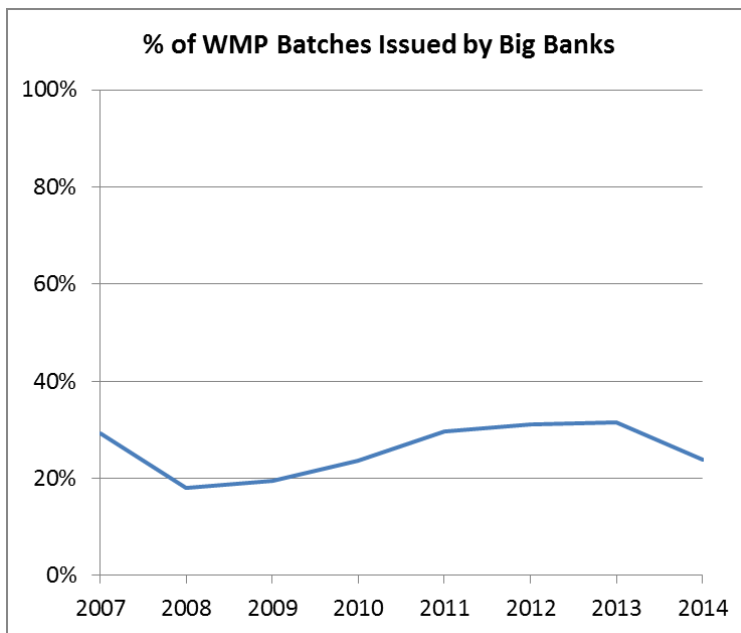
Source: Chinese Academy of Social Sciences as reported in “Reform and Stricter Regulation of Bank Wealth Management Products in China” by Eiichi Sekine, Nomura Research Institute, Autumn 2013.

Figure 11



Source: PBOC (Financial Institutions Statistics) and Bank Annual Reports

Figure 12



Source: Wind Financial Terminal

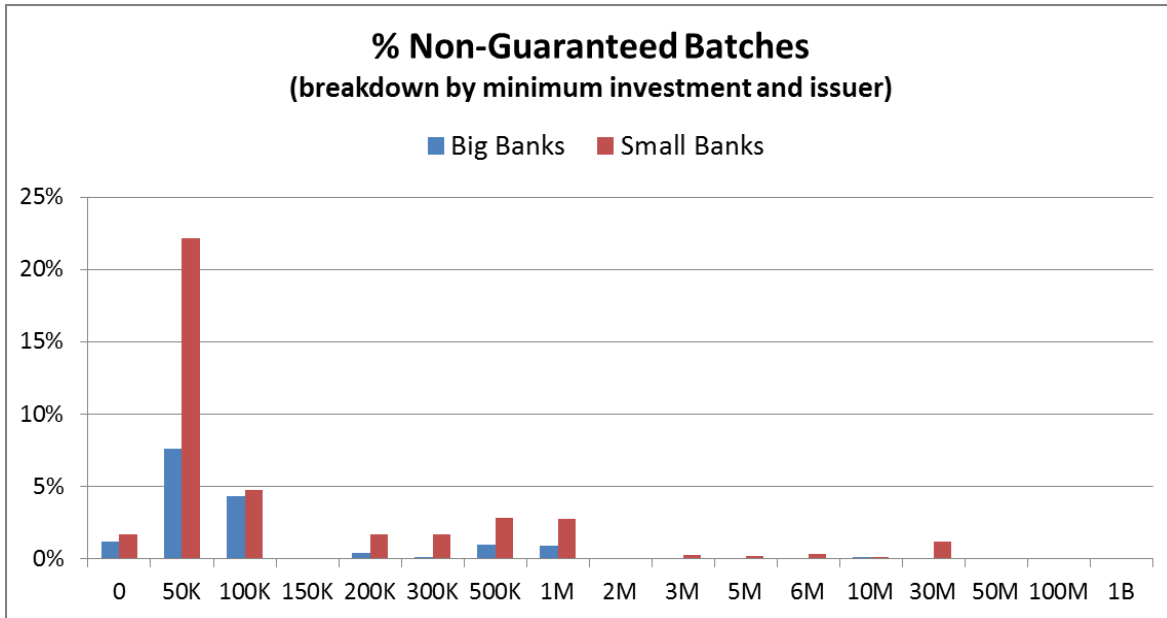
Figure 13

Granger Causality Wald Tests

	Chi2	Prob > Chi2
H0: # of WMP issued by the small banks does not Granger-cause # of WMP issued by the big four	21.104	0.002
H0: # of WMP issued by the big four does not Granger-cause # of WMP issued by the small banks	5.5264	0.478

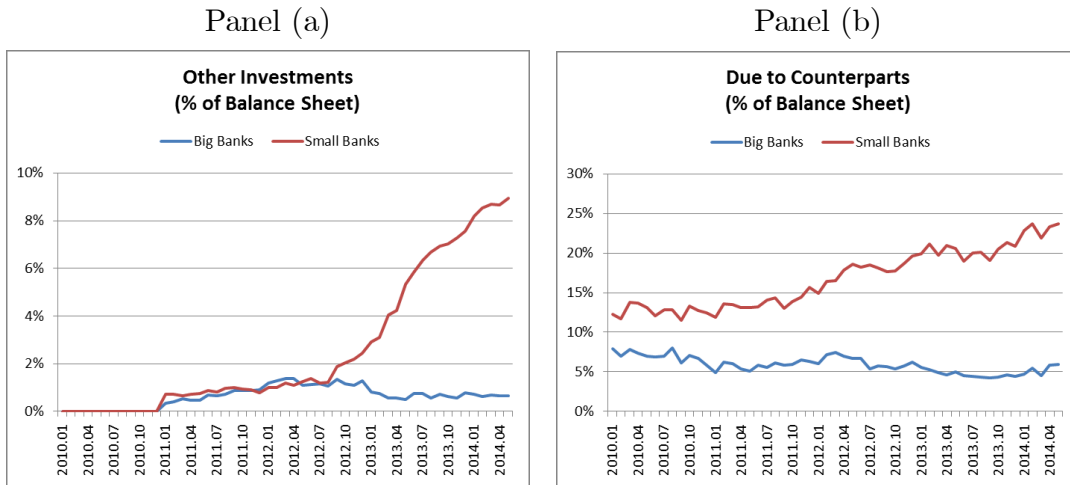
Note: We use detrended monthly data from Wind and estimate VARs with six lags

Figure 14



Source: China Banking Financial Network

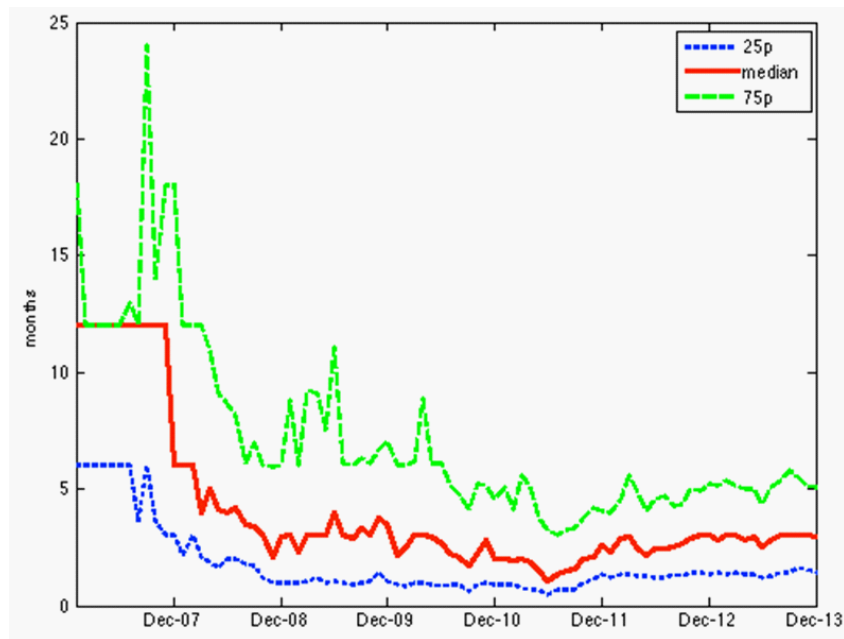
Figure 15



Source: PBOC (Financial Institutions Statistics)

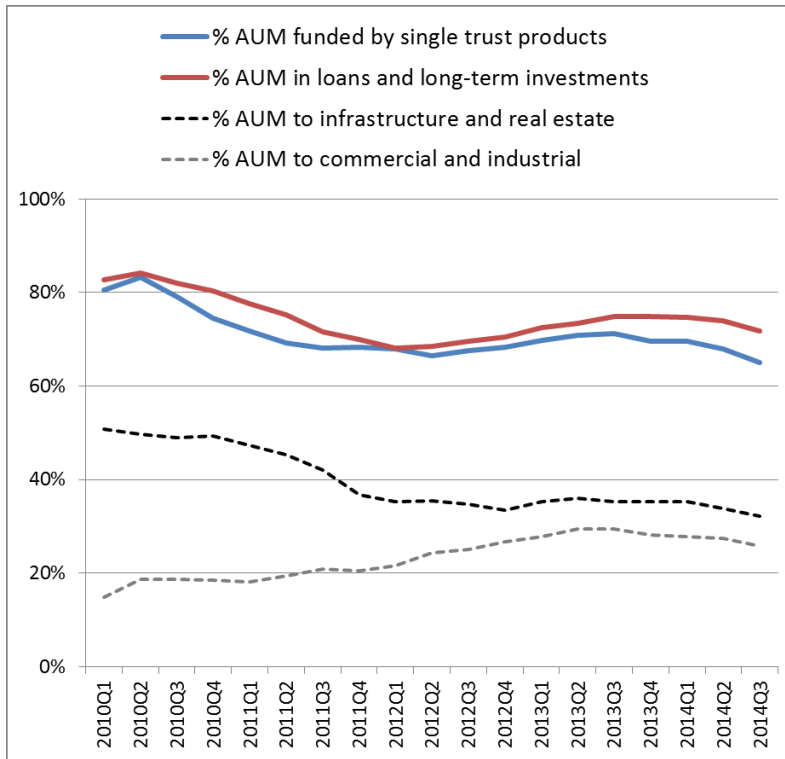
Figure 16

WMP Maturity in Full Sample



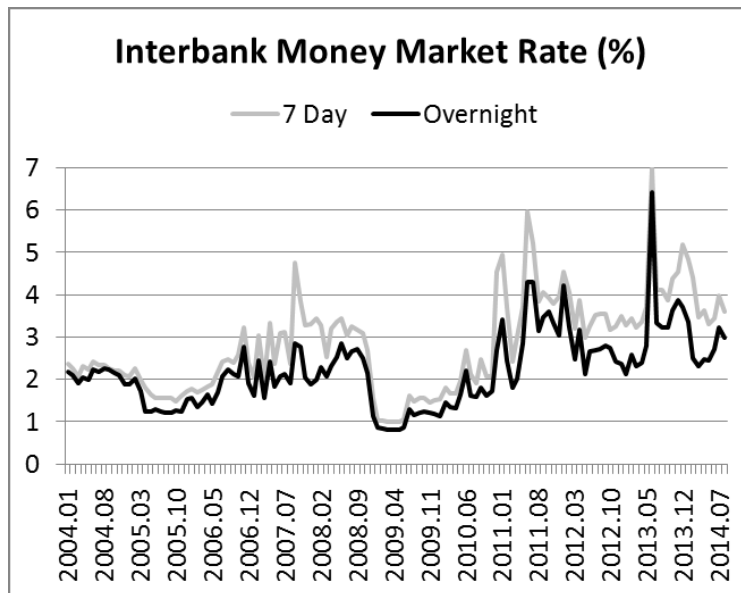
Source: Wind Financial Terminal

Figure 17



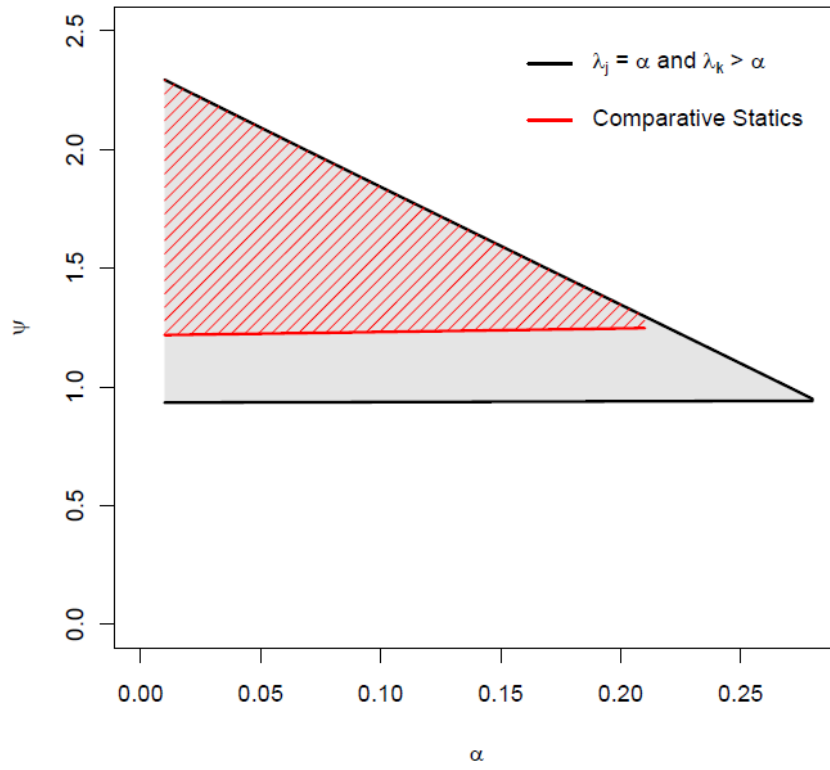
Source: China Trustee Association

Figure 18



Source: People's Bank of China

Figure 19
Parameter Space



Appendix

Proof of Proposition 1

Begin with $1 + i_L = (1 + i_A)^2$. The main text already established $\xi_j = 0$ and $W_j = 0$. In equilibrium, $D_j + W_j = 1$ so $D_j = 1$ and the bank's liquidity rule is $R_j \geq \alpha$. Substituting $1 + i_L = (1 + i_A)^2$ into equation (1) pins down $R_j = \alpha_0 \equiv (\psi + \bar{\theta})(1 + i_B) - \psi(1 + i_A)^2$. For this to satisfy $R_j \geq \alpha$, we need $\alpha \leq \alpha_0$.

Consider now $1 + i_L < (1 + i_A)^2$. We already know $\tau_j = 1$ so equations (4) and (6) give:

$$\xi_j + \frac{W_j(\xi_j)}{W_j'(\xi_j)} = \frac{\alpha[(1+i_A)^2 - (1+i_L)]}{1-\bar{\theta}} \quad (9)$$

We also know that the liquidity rule binds so use $D_j + W_j = 1$ to write $R_j = \alpha(1 - W_j)$.

We can now substitute into equation (1) to get another relationship between ξ_j and i_L :

$$i_L = i_B + \frac{\bar{\theta}(1+i_B) - \alpha[1 - W_j(\xi_j)]}{\psi} \quad (10)$$

Totally differentiate equations (9) and (10) then combine to find:

$$\frac{di_L}{d\alpha} \stackrel{\text{sign}}{=} W_j(\xi_j) + \xi_j W_j'(\xi_j) - [1 - W_j(\xi_j)] \left[2 - \frac{W_j(\xi_j)}{W_j'(\xi_j)} \frac{W_j''(\xi_j)}{W_j'(\xi_j)} \right]$$

The sign of $\frac{di_L}{d\alpha}$ thus depends on the curvature of $W_j(\cdot)$. In particular, if $W_j(\cdot)$ is sufficiently concave, then $\frac{di_L}{d\alpha} < 0$. Notice that $\frac{d\xi_j}{d\alpha} > 0$ follows from equation (9). We now need to confirm $1 + i_L < (1 + i_A)^2$. Using equation (10), this requires $x(\alpha) \equiv \alpha [1 - W_j(\xi_j(\alpha))] > \alpha_0$. The same condition that yields $\frac{di_L}{d\alpha} < 0$ also yields $x'(\alpha) > 0$. Moreover, $x(\alpha_0) = \alpha_0$ in the non-trivial case of $\alpha_0 > 0$. Therefore, $x(\alpha) > \alpha_0$ for any $\alpha > \alpha_0$, confirming $1 + i_L < (1 + i_A)^2$.

It now follows that low α yields the $1 + i_L = (1 + i_A)^2$ equilibrium while high α yields the $1 + i_L < (1 + i_A)^2$ one. Defining $\bar{\alpha} \equiv \min\{\max\{\alpha_0, 0\}, 1\}$ completes the proof. ■

Proof of Proposition 2

If $\rho_1 > 0$, then $W_j = \frac{1-\rho_0}{\rho_1}$ and $D_j = 1 - \frac{1-\rho_0}{\rho_1}$. Begin with $1 + i_L = (1 + i_A)^2$. Substitute i_L into the market clearing equation to get $R_j = \alpha_0$. The bank's liquidity rule is $R_j \geq \alpha \left[1 - \frac{(1-\rho_0)\tau_j}{\rho_1} \right]$ with $\tau_j \in [0, 1]$ so we again need α below some threshold. Turn now to $1 + i_L < (1 + i_A)^2$. The liquidity rule binds with $\tau_j = 1$, implying $R_j = \alpha \left[1 - \frac{1-\rho_0}{\rho_1} \right]$. Substituting into market clearing yields:

$$i_L = i_B + \frac{\bar{\theta}(1+i_B)}{\psi} - \frac{\alpha}{\psi} \left[1 - \frac{1-\rho_0}{\rho_1} \right] \quad (11)$$

Confirming $1 + i_L < (1 + i_A)^2$ thus requires $\left[1 - \frac{1-\rho_0}{\rho_1} \right] \alpha > \alpha_0$. Moreover:

$$\frac{di_L}{d\alpha} = -\frac{1}{\psi} \left[1 - \frac{1-\rho_0}{\rho_1} \right]$$

This has to be negative otherwise $D_j \leq 0$ which we can rule out as follows. Notice that $D_j > 0$ requires $\rho_0 > 1 - \rho_1$. Also notice that equation (6) with $\tau_j = 1$ and i_L as per (11) must yield ξ_j consistent with $W_j(\xi_j) = \frac{1-\rho_0}{\rho_1}$. Such consistency amounts to a particular value of ρ_0 conditional on all other parameters so we just need to show that this particular value satisfies $\rho_0 > 1 - \rho_1$. Using (11), write equation (6) as:

$$\xi_j + \frac{W_j(\xi_j)}{W_j'(\xi_j)} = \rho_1 \Delta_j + \frac{\alpha(1-\rho_1) + \bar{\theta}(1+i_B)\rho_1}{\psi(1-\bar{\theta})} \left[\alpha \left(1 - \frac{1-\rho_0}{\rho_1} \right) - \alpha_0 \right] \quad (12)$$

This expression implies $\frac{d\xi_j}{d\rho_0} > 0$ whereas $W_j(\xi_j) = \frac{1-\rho_0}{\rho_1}$ implies $\frac{d\xi_j}{d\rho_0} < 0$. So, all else constant, there is at most one value of ρ_0 that works. Substitute $\rho_0 = 1 - \rho_1$ into equation (12). The result is clearly impossible if the right-hand side is negative or, equivalently, if:

$$(1 + i_A)^2 - (1 + i_B)^2 + \alpha \left(\frac{1}{\rho_1} - 1 \right) \left[(1 + i_A)^2 - (1 + i_B) - \frac{\bar{\theta}(1+i_B)}{\psi} \right] \leq \frac{\bar{\theta}^2(1+i_B)^2}{\psi}$$

A sufficient condition for the above inequality is:

$$\psi \leq \min \left\{ \frac{\bar{\theta}(1+i_B)}{(1+i_A)^2 - (1+i_B)}, \frac{\bar{\theta}^2(1+i_B)^2}{(1+i_A)^2 - (1+i_B)^2} \right\} = \frac{\bar{\theta}^2(1+i_B)^2}{(1+i_A)^2 - (1+i_B)^2}$$

where the solution to the min operation follows from $\Delta_j > 0$. ■

System of Equations for Subsection 5.2

Define the following constants:

$$\Delta_k \equiv \frac{[1 - \theta_h(1+i_B)](1+i_A)^2 - (1-\theta_h)(1+i_B)^2}{1-\theta}$$

$$Y(\alpha) \equiv \frac{(1+i_A)^2 - (1+i_B)}{1-\pi} + \frac{\delta[\alpha - \bar{\theta}(1+i_B)]}{\psi}$$

$$f(i_L^h) \equiv \frac{(1+i_A)^2 - [1 + \pi i_B + (1-\pi)i_L^h]}{1-\bar{\theta}}$$

Begin with the small bank. The first order conditions with respect to R_j and τ_j are:

$$\mu_j = (1+i_A)^2 - [1 + \pi i_B + (1-\pi)i_L^h]$$

$$\eta_j^1 = \eta_j^0 + \alpha \mu_j W_j$$

As before, $\mu_j > 0$ yields $R_j = \alpha D_j$.¹⁸ The first order condition with respect to ξ_j is:

$$\xi_j + \frac{W_j}{\partial W_j / \partial \xi_j} = \left[\Delta_j - \frac{\alpha - \bar{\theta}(1+i_B)}{1-\bar{\theta}} \mu_j \right] \left[1 + \frac{\partial D_j / \partial \xi_j}{\partial W_j / \partial \xi_j} \right] + \frac{\alpha \tau_j}{1-\bar{\theta}} \mu_j$$

If $\xi_j > 0$, then $\tau_j = 1$ and we can use the functional forms to write the choice of $\xi_j > 0$ as:

$$\left[1 + \frac{\xi_k + \gamma \xi_j}{\xi_j + \xi_k} \right] \xi_j = \alpha \delta \gamma f(i_L^h) + [\Delta_j + \bar{\theta}(1+i_B)] f(i_L^h) \left[\frac{\xi_k + \gamma \xi_j}{\xi_j + \xi_k} - \delta \gamma \right]$$

Based on this expression, the reactions to the big bank's choices are:

¹⁸More precisely, $\mu_j > 0$ yields $R_j = \alpha [D_j + (1-\tau_j)W_j]$ but we can show that $(1-\tau_j)W_j$ is always zero. If $W_j = 0$, then $(1-\tau_j)W_j = 0$ is trivially true. If $W_j > 0$, then $\eta_j^1 > 0$ and thus $\tau_j = 1$ so we again have $(1-\tau_j)W_j = 0$. Of course, the jump from $W_j > 0$ to $\tau_j = 1$ assumes $\alpha > 0$.

$$\frac{d\xi_j}{d\xi_k} = \frac{\xi_j}{\xi_k + \frac{(\xi_j + \xi_k)[(1+\gamma)\xi_j + 2\xi_k]}{(1-\gamma)[\Delta_j + \bar{\theta}(1+i_B)f(i_L^h) - \xi_j]}}$$

$$\frac{d\xi_j}{di_L^h} = -\frac{\frac{1-\pi}{1-\bar{\theta}} \left[\bar{\theta}(1+i_B) + [\alpha - \bar{\theta}(1+i_B)] \frac{\delta\gamma(\xi_j + \xi_k)}{\xi_k + \gamma\xi_j} \right]}{1 + \frac{\xi_j + \xi_k}{\xi_k + \gamma\xi_j} + [\Delta_j + \bar{\theta}(1+i_B)f(i_L^h) - \xi_j] \frac{(1-\gamma)\xi_k}{\xi_j + \xi_k} \frac{1}{\xi_k + \gamma\xi_j}}$$

Turn next to the big bank (without the loan limit constraint). The first order condition with respect to ξ_k is:

$$\Delta_k \left[\frac{\partial(D_k + W_k)}{\partial\xi_k} + \frac{\partial(D_k + W_k)}{\partial\xi_j} \frac{\partial\xi_j}{\partial\xi_k} \right] = W_k + \xi_k \left[\frac{\partial W_k}{\partial\xi_k} + \frac{\partial W_k}{\partial\xi_j} \frac{\partial\xi_j}{\partial\xi_k} \right]$$

$$+ f(i_L^h) \left[\bar{\theta}(1+i_B) \left[\frac{\partial(D_j + W_j)}{\partial\xi_k} + \frac{\partial(D_j + W_j)}{\partial\xi_j} \frac{\partial\xi_j}{\partial\xi_k} \right] - \alpha \left[\frac{\partial D_j}{\partial\xi_k} + \frac{\partial D_j}{\partial\xi_j} \frac{\partial\xi_j}{\partial\xi_k} \right] \right]$$

The first order condition with respect to i_L^h is:

$$\Delta_k \frac{\partial(D_k + W_k)}{\partial\xi_j} \frac{\partial\xi_j}{\partial i_L^h} = \xi_k \frac{\partial W_k}{\partial\xi_j} \frac{\partial\xi_j}{\partial i_L^h} + f(i_L^h) \left[\bar{\theta}(1+i_B) \frac{\partial(D_j + W_j)}{\partial\xi_j} - \alpha \frac{\partial D_j}{\partial\xi_j} \right] \frac{\partial\xi_j}{\partial i_L^h}$$

$$+ f'(i_L^h) \left[\bar{\theta}(1+i_B) (D_j + W_j) - \alpha D_j - \psi(i_L^h - i_B) \right] - \psi f(i_L^h)$$

Using the functional forms, we can write these equations as:

$$\left[\Delta_k + \bar{\theta}(1+i_B) f(i_L^h) \right] \left[[1 - \gamma(1 - \delta)] \xi_j + \gamma\delta\xi_k - [(1 - \gamma\delta)\xi_k + \gamma(1 - \delta)\xi_j] \frac{\partial\xi_j}{\partial\xi_k} \right]$$

$$= \xi_k \left[2\xi_j + (1 + \gamma)\xi_k - (1 - \gamma)\xi_k \frac{\partial\xi_j}{\partial\xi_k} \right] + \alpha\gamma\delta \left[1 + \frac{\partial\xi_j}{\partial\xi_k} \right] (\xi_j + \xi_k) f(i_L^h)$$

$$\left[\Delta_k + \bar{\theta}(1+i_B) f(i_L^h) \right] [(1 - \gamma\delta)\xi_k + \gamma(1 - \delta)\xi_j] \frac{\partial\xi_j}{\partial i_L^h}$$

$$= (1 - \gamma)\xi_k^2 \frac{\partial\xi_j}{\partial i_L^h} - \alpha\gamma\delta \frac{\partial\xi_j}{\partial i_L^h} (\xi_j + \xi_k) f(i_L^h) - f'(i_L^h) \frac{\bar{\theta}(1+i_B)(D_j + W_j) - \alpha D_j}{\beta(\xi_j + \xi_k)^{\gamma-2}} + \frac{\psi[f(i_L^h) + f'(i_L^h)(i_L^h - i_B)]}{\beta(\xi_j + \xi_k)^{\gamma-2}}$$

Defining $z \equiv \frac{\xi_k}{\xi_j}$, we can then rewrite as:

$$\frac{(1-\gamma)z^2}{1+z}\xi_j + \frac{\Delta_k + \bar{\theta}(1+i_B)f(i_L^h) - 2z\xi_j}{1 + \frac{\partial\xi_j}{\partial\xi_k}} = \alpha\gamma\delta f(i_L^h) + [\Delta_k + \bar{\theta}(1+i_B)f(i_L^h)] \left[\frac{\gamma+z}{1+z} - \gamma\delta \right]$$

$$\begin{aligned} & [\alpha\gamma\delta f(i_L^h) + [\Delta_k + \bar{\theta}(1+i_B)f(i_L^h)] \left[\frac{\gamma+z}{1+z} - \gamma\delta \right]] \frac{\partial\xi_j}{\partial i_L^h} \frac{1}{\xi_j(1+z)} \\ &= \frac{(1-\gamma)z^2}{(1+z)^2} \frac{\partial\xi_j}{\partial i_L^h} + \frac{\bar{\theta}(1+i_B)(1-\pi)}{1-\bar{\theta}} \frac{1}{1+z} + \frac{\delta(1-\pi)[\alpha - \bar{\theta}(1+i_B)]}{1-\bar{\theta}} + \frac{\psi \left[2f(i_L^h) - \frac{1-\pi}{1-\bar{\theta}} Y(\alpha) \right]}{\beta\xi_j^\gamma(1+z)^\gamma} \end{aligned}$$

Putting everything together, the equilibrium is summarized by:

$$\left[1 + \frac{\gamma+z}{1+z} \right] \xi_j = \alpha\gamma\delta f(i_L^h) + [\Delta_j + \bar{\theta}(1+i_B)f(i_L^h)] \left[\frac{\gamma+z}{1+z} - \gamma\delta \right] \quad (13)$$

$$\alpha\gamma\delta f(i_L^h) + [\Delta_k + \bar{\theta}(1+i_B)f(i_L^h)] \left[\frac{\gamma+z}{1+z} - \gamma\delta \right] = \frac{(1-\gamma)z^2}{1+z}\xi_j + \frac{\Delta_k + \bar{\theta}(1+i_B)f(i_L^h) - 2z\xi_j}{1 + \frac{\partial\xi_j}{\partial\xi_k}} \quad (14)$$

$$\frac{\Delta_k + \bar{\theta}(1+i_B)f(i_L^h) - 2z\xi_j}{(1+z)\xi_j} \frac{\partial\xi_j}{\partial i_L^h} \frac{1}{1 + \frac{\partial\xi_j}{\partial\xi_k}} = \frac{1-\pi}{1-\bar{\theta}} \left[\delta [\alpha - \bar{\theta}(1+i_B)] + \frac{\bar{\theta}(1+i_B)}{1+z} + \frac{\psi \left[\frac{2(1-\bar{\theta})}{1-\pi} f(i_L^h) - Y(\alpha) \right]}{\beta\xi_j^\gamma(1+z)^\gamma} \right] \quad (15)$$

Subbing in the reaction functions from the small bank's problem and rearranging yields the three equations in the main text.

Derivatives for Subsection 5.3

In addition to i_L^h , we are interested in the big bank's liquidity ratio (λ_k), the total amount of lending (TL), and the fraction of total lending that is done off balance sheet (OBS). We can write these objects as:

$$\lambda_k = \pi(\theta_h - \theta_\ell)(1+i_B) + \frac{\bar{\theta}(1+i_B) - \psi(i_L^h - i_B) - \alpha\delta[1 - \beta\xi_j^\gamma(1+z)^\gamma]}{1 - \delta - \beta\xi_j^\gamma(1+z)^{\gamma-1}[1 - \delta(1+z)]}$$

$$TL = 1 - \bar{\theta}(1+i_B) + \Psi(i_L^h) - \pi(\theta_h - \theta_\ell)(1+i_B) \left[1 - \delta - \beta\xi_j^\gamma(1+z)^{\gamma-1} [1 - \delta(1+z)] \right]$$

$$OBS = \frac{\beta\xi_j^\gamma(1+z)^{\gamma-1}}{TL}$$

To avoid carrying around heavy notation, define $X \equiv \frac{\partial \xi_j}{\partial i_L^h}$ and $Q \equiv 1 + \frac{\partial \xi_j}{\partial \xi_k}$. Also define $h(z) \equiv \frac{\gamma+z}{1+z} - \gamma\delta$ and $U \equiv \Delta_k + \bar{\theta}(1+i_B)f(i_L^h) - 2z\xi_j$. Differentiate equation (13) to get:

$$\underbrace{\left[1 + \frac{\gamma+z}{1+z}\right] \frac{d\xi_j}{d\alpha}}_{\Omega_0} = \gamma\delta f(i_L^h) + \underbrace{\frac{(1-\gamma)[\Delta_j + \bar{\theta}(1+i_B)f(i_L^h) - \xi_j]}{(1+z)^2}}_{\Omega_1} \frac{dz}{d\alpha} - \underbrace{\frac{(1-\pi)[\alpha\gamma\delta + \bar{\theta}(1+i_B)h(z)]}{1-\bar{\theta}}}_{\Omega_2} \frac{di_L^h}{d\alpha}$$

Next, combine (13) and (14) then differentiate to get:

$$\underbrace{\left[2z + \frac{U}{Q} \frac{\partial Q}{\partial \xi_j} + \frac{U + \Theta h(z)Q}{\xi_j}\right] \frac{d\xi_j}{d\alpha}}_{\Omega_3} = \underbrace{\left[\frac{(1-\gamma)Q[\Theta + (1+z)^2\xi_j - 2\xi_j]}{(1+z)^2} - 2\xi_j - \frac{U}{Q} \frac{\partial Q}{\partial z}\right] \frac{dz}{d\alpha}}_{\Omega_4} - \underbrace{\left[\frac{\bar{\theta}(1-\pi)(1+i_B)}{1-\bar{\theta}} + \frac{U}{Q} \frac{\partial Q}{\partial i_L^h}\right] \frac{di_L^h}{d\alpha}}_{\Omega_5}$$

where

$$\begin{aligned} \frac{\partial Q}{\partial \xi_j} &= -\frac{1+z-Qz}{\xi_j} \left[Q - 1 + \frac{1-\gamma}{1+\gamma+2z} \left(1 - \frac{Qz}{1+z}\right)\right] \\ \frac{\partial Q}{\partial z} &= -(Q-1) \left[Q - 1 + \frac{3+4z+\gamma}{1+\gamma+2z} \left(1 - \frac{Qz}{1+z}\right)\right] \\ \frac{\partial Q}{\partial i_L^h} &= -\frac{\bar{\theta}(1-\pi)(1+i_B)}{1-\bar{\theta}} \frac{1+z-Qz}{\xi_j} \left(\frac{1-\gamma}{1+\gamma+2z}\right) \left(1 - \frac{Qz}{1+z}\right) \end{aligned}$$

Finally, differentiate (15) to get:

$$\begin{aligned} &\underbrace{\left[\frac{U}{Q} \frac{\partial X}{\partial \xi_j} + X \left[1 + \frac{\gamma+z}{1+z} - \frac{(1-\gamma)z^2}{1+z}\right] - \frac{\gamma(1-\pi)[\bar{\theta}(1+i_B) + \delta\Phi(1+z)]}{1-\bar{\theta}} - \frac{(1-\gamma)UX}{Q\xi_j}\right] \frac{d\xi_j}{d\alpha}}_{\Omega_6} \\ &= \underbrace{\frac{\delta(1-\pi)}{1-\bar{\theta}} \left[1 - \frac{1}{\beta\xi_j^\gamma(1+z)^\gamma}\right]}_{\Omega_7} (1+z)\xi_j - \underbrace{\frac{U}{Q} \frac{\partial X}{\partial \alpha}}_{\Omega_8} - \underbrace{\left[\frac{U}{Q} \frac{\partial X}{\partial i_L^h} + \frac{2\psi(1-\pi)}{\beta(1-\bar{\theta})} \xi_j^{1-\gamma} (1+z)^{1-\gamma}\right] \frac{di_L^h}{d\alpha}}_{\Omega_8} \\ &\quad - \underbrace{\left[\frac{U}{Q} \frac{\partial X}{\partial z} + \frac{1-\pi}{1-\bar{\theta}} \frac{\bar{\theta}(1+i_B)}{1+z} \xi_j - \frac{(1-\gamma)X[\Theta + (1+z)^2\xi_j - 2\xi_j]}{(1+z)^2} - \frac{\gamma(1-\pi)}{1-\bar{\theta}} \left[\delta\Phi + \frac{\bar{\theta}(1+i_B)}{1+z}\right] \xi_j - \frac{(1-\gamma)UX}{Q(1+z)}\right] \frac{dz}{d\alpha}}_{\Omega_9} \end{aligned}$$

where

$$\frac{\partial X}{\partial \alpha} = \frac{\gamma\delta X}{\alpha\gamma\delta + \bar{\theta}(1+i_B)h(z)}$$

$$\begin{aligned}\frac{\partial X}{\partial \xi_j} &= \frac{X}{\xi_j} \left[1 + \frac{(1-\bar{\theta})X}{1-\pi} \frac{2 - \frac{(1-\gamma)(1+2z)}{(1+z)^2}}{\alpha\gamma\delta + \bar{\theta}(1+i_B)h(z)} \right] \\ \frac{\partial X}{\partial z} &= \frac{X}{1+z} \left[1 - \frac{1}{z} + \frac{\bar{\theta}(1+i_B)(1-\gamma) - \frac{1-\bar{\theta}}{1-\pi} \frac{1+\gamma-2z^2}{z} X}{[\alpha\gamma\delta + \bar{\theta}(1+i_B)h(z)](1+z)} \right] \\ \frac{\partial X}{\partial i_L^h} &= -\frac{X^2}{\alpha\gamma\delta + \bar{\theta}(1+i_B)h(z)} \frac{\bar{\theta}(1+i_B)(1-\gamma)z}{(1+z)^2 \xi_j}\end{aligned}$$

Combine the differentiated expressions to isolate the core derivatives:

$$\frac{di_L^h}{d\alpha} = \frac{\Omega_7 [\Omega_1\Omega_3 - \Omega_0\Omega_4] + \gamma\delta f(i_L^h) [\Omega_3\Omega_9 + \Omega_4\Omega_6]}{\Omega_8 [\Omega_1\Omega_3 - \Omega_0\Omega_4] + \Omega_2 [\Omega_3\Omega_9 + \Omega_4\Omega_6] - \Omega_5 [\Omega_0\Omega_9 + \Omega_1\Omega_6]}$$

$$\frac{dz}{d\alpha} = \frac{\Omega_0\Omega_7 - \gamma\delta f(i_L^h) \Omega_6}{\Omega_0\Omega_9 + \Omega_1\Omega_6} - \frac{\Omega_0\Omega_8 - \Omega_2\Omega_6}{\Omega_0\Omega_9 + \Omega_1\Omega_6} \frac{di_L^h}{d\alpha}$$

$$\frac{d\xi_j}{d\alpha} = \frac{\gamma\delta f(i_L^h)}{\Omega_0} + \frac{\Omega_1}{\Omega_0} \frac{dz}{d\alpha} - \frac{\Omega_2}{\Omega_0} \frac{di_L^h}{d\alpha}$$

We can then write the derivatives for the objects of interest as:

$$\begin{aligned}\frac{d\lambda_k}{d\alpha} &\stackrel{sign}{=} -\psi \frac{di_L^h}{d\alpha} - \delta \left[1 - \beta\xi_j^\gamma (1+z)^\gamma \right] + \alpha\beta\gamma\delta\xi_j^\gamma (1+z)^\gamma \left[\frac{1}{\xi_j} \frac{d\xi_j}{d\alpha} + \frac{1}{1+z} \frac{dz}{d\alpha} \right] \\ &\quad + \frac{\beta[\lambda_k - \pi(\theta_h - \theta_\ell)(1+i_B)]\xi_j^\gamma}{(1+z)^{1-\gamma}} \left[\frac{\gamma[1-\delta(1+z)]}{\xi_j} \frac{d\xi_j}{d\alpha} - \left[\frac{1-\gamma}{1+z} + \gamma\delta \right] \frac{dz}{d\alpha} \right]\end{aligned}$$

$$\frac{dT_L}{d\alpha} = \psi \frac{di_L^h}{d\alpha} + \frac{\beta\pi(\theta_h - \theta_\ell)(1+i_B)\xi_j^\gamma}{(1+z)^{1-\gamma}} \left[\frac{\gamma[1-\delta(1+z)]}{\xi_j} \frac{d\xi_j}{d\alpha} - \left[\frac{1-\gamma}{1+z} + \gamma\delta \right] \frac{dz}{d\alpha} \right]$$

$$\frac{dOBS}{d\alpha} \stackrel{sign}{=} \frac{\gamma}{\xi_j} \frac{d\xi_j}{d\alpha} - \frac{1-\gamma}{1+z} \frac{dz}{d\alpha} - \frac{1}{T_L} \frac{dT_L}{d\alpha}$$