

# *CoVaR*\*

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## **Abstract**

We propose a measure for systemic risk: *CoVaR*, the Value at Risk (*VaR*) conditional on an institution (or the whole financial sector) being under distress. We argue for regulatory requirements that are based on the difference between *CoVaR* and *VaR*, capturing an institution's (marginal) contribution to systemic risk. Countercyclical regulation should take institution's characteristics like maturity mismatch and leverage into account to the extent that they predict systemic risk contributions.

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\*Please apologize typos of this intermediate version.

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# 1 Introduction

During times of financial crisis, losses tend to spread across financial institutions, threatening the financial system as a whole.<sup>1</sup> Measures of systemic risk that capture risk spillovers and tail risk correlations should form the basis of any macro-prudential regulation.

The most common measure of risk used by financial institutions—the Value at Risk (*VaR*)—focuses on the risk of an individual institution in isolation. The  $\pi\%$ -*VaR* is the maximum dollar loss within the  $(1 - \pi\%)$ -confidence interval; see, e.g., Jorion (2006). However, a single institution’s risk measure does not necessarily reflect systemic risk – the risk that the stability of the financial system as a whole is threatened. A measure for systemic risk should achieve two objectives. First, it should indicate which institutions are likely to be in financial difficulties should a systemic risk event occur. Second, it should jointly measure (i) how financial difficulties of one institution spill over to others and (ii) how financial tail risk is correlated among the main financial institutions. Following the classification in Brunnermeier, Crocket, Goodhart, Persaud, and Shin (2009), the first group of institutions are “individually systemic” because they are so massively interconnected and large that they can cause negative risk spillover effects on others. The second group of institutions are “systemic as part of a herd” because they are exposed to a common risk factor. Since it is not essential to distinguish whether an institution is individually systemic or as part of a herd, the risk measure does not need to identify a causal relationship.

In this paper, we propose such a measure for systemic risk that covers both ob-

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<sup>1</sup>Examples include the 1987 equity market crash which started by portfolio hedging of pension funds and led to substantial losses of investment banks; the 1998 crisis started with losses of hedge funds and spilled over to the trading floors of commercial and investment banks; and the 2007/08 crisis spread from SIVs to commercial banks and on to hedge funds and investment banks, see Brady (1988), Rubin, Greenspan, Levitt, and Born (1999), and Brunnermeier (2009).

jectives. We call our risk measure *CoVaR*, where the “Co” stands for *conditional*, *comovement*, *contagion*, or *contributing*. In general terms, it is defined as the *VaR conditional on* either the whole financial sector or the particular institution being in distress. For determining which institutions are likely to experience distress in case of a systemic event, we condition the *VaR* of each institution on the event that the index return of the financial sector is at its *VaR* level. Note that the likelihood of being in distress in case of a systemic event depends to a large extent on the institution’s funding strategy (leverage, maturity mismatch etc.) in addition to its asset holdings. To address the question to what extent a particular institution *contributes* (in a non-causal sense) to the overall systemic risk (by being in distress when there is a system-wide distress), we reverse the conditioning. In this case, an institution’s *CoVaR* is defined as the *VaR* of the whole financial sector conditional on this institution being in distress. The difference between the *CoVaR* and unconditional financial industry *VaR*,  $\Delta CoVaR$ , captures the marginal contribution of a particular institution (in a non-causal sense) to the overall systemic risk.

In practice, we argue for a change of the regulatory framework that emphasizes the institution’s contribution to systemic risk and focuses not only on its individual risk. More specifically, the degree an institution increases the *CoVaR* of the financial sector or of a specific set of financial institutions (such as the institutions with access to the discount window) should determine the macro-prudential regulation of that institution. Capital and liquidity requirements should reflect the potential for risk spillovers and tail correlations. The aim is to internalize externalities and provide the incentive to minimize systemic risk exposure.

Current risk regulation focuses on the risk of an individual institution (in isolation). This leads, in the aggregate, to excessive risk along the systemic risk factors. To see

this more explicitly, consider two institutions,  $A$  and  $B$ , which report the same  $VaR$ , but while institution  $A$ 's  $CoVaR = VaR$ , institution  $B$ 's  $CoVaR$  largely exceeds its  $VaR$ . Based on their  $VaRs$ , both institutions seem to appear to be equally risky. However, the high  $CoVaR$  of institution  $B$  indicates that it is more correlated/exposed to system risk. Since system risk carries a higher risk premium, institution  $B$  will outshine institution  $A$  and competitive pressure will force institution  $A$  to follow suit. Imposing stricter regulatory requirements on institution  $B$  would break this herding tendency. One might argue that regulating institutions'  $VaR$  might be sufficient as long as each institution's  $CoVaR$  goes hand in hand with its  $VaR$ . This is not the case, since (i) it is not desirable that institution  $A$  increases its contribution to systemic risk by following a strategy similar to institution  $B$  and (ii) there is no one-to-one connection between institutions  $CoVaR$  (y-axis) and  $VaR$  (x-axis) as Figure 1 shows. Overall, Figure 1 questions the sole focus on  $VaR$  as the current bank regulation based on Basel II does.

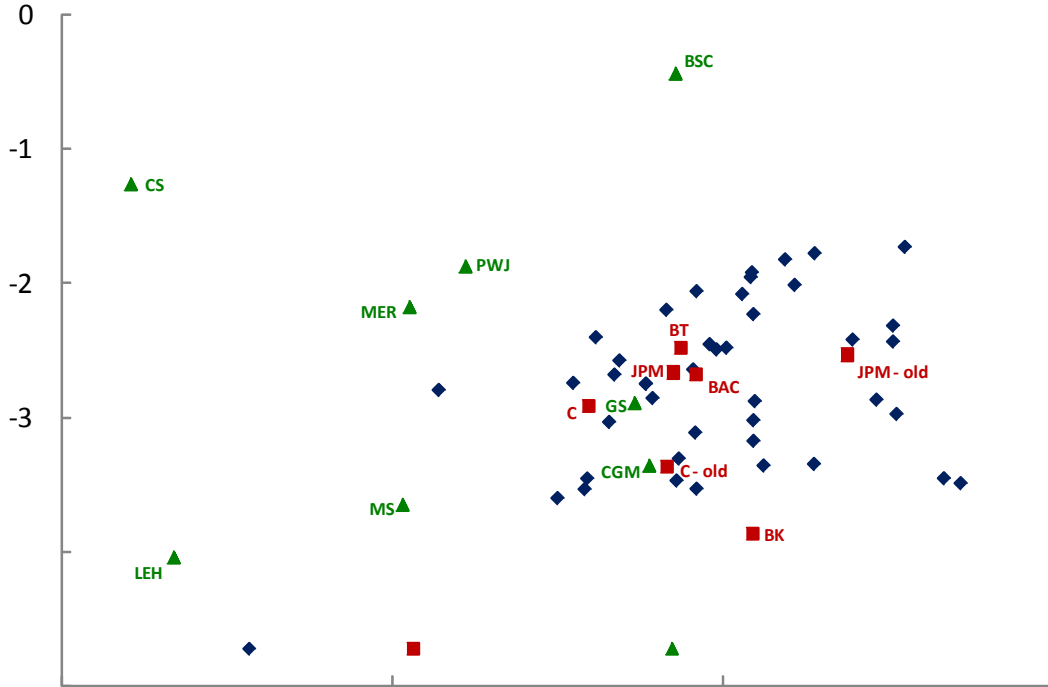


FIGURE 1: The scatter plot shows the weak link between an institution’s risk in isolation, measured by  $VaR^i$  (x-axis), and institution’s contribution to system risk, measured by  $\Delta CoVaR_{add}^i := CoVaR^{index|i} - VaR^{index}$  (y-axis) for each investment bank (green triangles), commercial bank (red squares), and portfolios of financial assets (blue diamonds).

There are many ways to estimate our  $CoVaR$  measure. In this paper we primarily use quantile regressions which are appealing for their simplicity and efficient use of data. Our estimates of  $\Delta CoVaR$  in Figure 1 are based on (weekly) equity returns of traded financial institutions (portfolios). We use equity returns, since we want to capture all

forms of risk, including not only the risk of adverse asset price movements, but—equally importantly—also funding liquidity risk. In other words, focusing exclusively on the quality of an institution’s asset portfolio is insufficient, since it is the funding structure, especially the asset-liability maturity mismatch, that exposes an institution to systemic risk. Ideally, one would like to base the risk measure on exact asset composition and funding structure especially as they can change rapidly over time. For hedge funds’ *CoVaR* measures we rely on reported returns.

One reason why institutions’ *CoVaR* estimates might not line up well with the *VaR* estimates is that their portfolio strategy might have changed over time. For example, a particular institution might have been very levered in the 1990s but may have only a low leverage ratio in the 2000s. Its overall estimated *CoVaR* reflects a mixture of both leverage ratios. We attempt to control for this effect by repeating the analysis for portfolios that are sorted based on leverage, maturity mismatch, volatility, etc.

The second part of the paper addresses the problem that any (empirical) risk measure suffers from the fact that “tail observations” are—by definition—rare. After a string of good news, risk seems tamed, but, when a new tail event occurs, the estimated risk measure may sharply increase. This problem is most pronounced if the data samples are short. Hence, regulatory requirements that are naively based on estimated risk measures would be stringent during a crisis and lax during a boom. This introduces procyclicality – exactly the opposite of the goal of effective regulation. In order to derive a countercyclical risk measure, we derive the  $\Delta CoVaR$  for each institution using the full set of data. We first estimate it conditional on macro variables like slope of yield curve, aggregate credit spread, and implied market volatility from VIX. Using panel regressions we then relate these time-varying  $\Delta CoVaR$  measures to institutions’ maturity mismatch, leverage, and book-to-market. We do so contemporaneously and

in a predictive sense. The regression coefficients indicate how one should weigh the different funding liquidity measures in determining the capital charge or Pigouvian tax imposed on various financial institutions. The predictive regressions allow the regulator to act in advance. Of course, any empirical analysis is limited and has to be complemented with “theorizing”, especially when the banking model changes.

**Related Literature.** Our *co-risk measures* can be interpreted in light of recent economic theories of financial sector amplification. While we do not test any particular theory, *CoVaR* is meaningful in economic settings where financing constraints of financial institutions are linked to risk. As measured risk increases, margin and capital requirements widen, forcing institutions to unwind. This tends to increase market risk, thus leading to further increases of measured risk.

Brunnermeier and Pedersen (2009) propose a theory of margin spirals, where balance sheet constraints lead to risk spillovers among financial institutions. Adrian and Shin (2009) derive a micro foundation for the use of *VaR* by financial institutions and analyze risk spillovers for financial systems of interlocked balance sheets. Kyle and Xiong (2001) provide a model of contagion among financial institutions where the interaction of risk spillovers and wealth effects leads to institutional contagion.

Our paper can also be linked to several other strands of literatures. First, our paper contributes to the growing literature that sheds light on the link between hedge funds and the risk of a systemic crisis. Boyson, Stahel, and Stulz (2006) document contagion across hedge fund styles using logit regressions. Chan, Getmansky, Haas, and Lo (2006) document an increase in correlation across hedge funds, especially prior to the LTCM crisis and after 2003. Adrian (2007) points out that the increase in correlation since 2003 is due to a reduction in volatility – a phenomenon that occurred across many

financial assets – rather than an increase in covariance.

Second, our work relates to the large literature in international finance that focuses on cross-country spillovers. For example, King and Wadhvani (1990) document an increase in correlation across stock markets during the 1987 crash, which in itself – as Forbes and Rigobon (2002) argue – is only evidence for interdependence but not contagion, since estimates of correlation tend to go up when volatility is high. Claessens and Forbes (2001) and the articles therein provide an overview. In contrast to these papers, our analysis focuses on volatility spillovers. The most common method to test for volatility spillover is to estimate GARCH processes, as e.g. Hamao, Masulis, and Ng (1990) do for international stock market returns. While GARCH processes allow for time-variation in conditional volatility, they assume that extreme returns follow the same return distribution as the rest of returns. Hartman, Straetmans, and de Vries (2004) avoid this criticism by developing a contagion measure that focuses on extreme events. Building on extreme value theory, they estimate the expected number of market crashes given that at least one market crashes. However, extreme value theory works best for very low quantiles (see Danielsson and de Vries (2000)). This motivates Engle and Manganelli (2004) to develop *CAViaR* that – like our approach – makes use of quantile regressions as initially proposed by Koenker and Bassett (1978) and Bassett and Koenker (1978). While Engle and Manganelli’s *CAViaR* focuses on the evolution of quantiles over time, we study risk spillover effects across financial institutions as measured by our *CoVaR*. More recently, Rossi and Harvey (2007) estimate time-varying quantiles and expectiles using a state space signal extraction algorithm. The machinery developed by Engle and Manganelli (2004) and Rossi and Harvey (2007) could be used to study the time variation of *CoVaR*.

The remainder of the paper is organized in four sections. In Section 2, we outline



the methodology. We define *CoVaRs*, introduce time-variation and show how one could implement a countercyclical financial regulation. In Section 3, we present estimates of *CoVaRs* for commercial banks, investment banks, and hedge funds and relate them to macro risk factors. In Section 4, we show to what degree *CoVaRs* depend on the financial institutions' characteristics such as leverage, maturity mismatch, and size and whether these variables help to predict future *CoVaRs*. We conclude in Section 5.

## 2 *CoVaR* Methodology

In this section, we first introduce our systemic co-risk measure, *CoVaR*, and then specify two particular forms that are the focus of this paper. Subsequently, we introduce time-varying *CoVaRs* by linking our *CoVaR* estimates to certain macro variables and finally, we outline how one can achieve a countercyclical financial regulation.

### 2.1 Definition of *CoVaR*

Recall that  $VaR_q^i$  is implicitly defined as the  $q$  quantile, i.e.

$$\Pr(R^i \leq VaR_q^i) = q,$$

where  $R^i$  is the return of institution (or portfolio)  $i$ . Note that  $VaR_q^i$  is typically a negative number. Practitioners usually switch the sign, a sign convention we will not follow. It is also noteworthy that all our empirical results are expressed in percentage returns. These can be transformed into dollar amounts by multiplying by total assets.

**Definition 1** We denote the  $CoVaR_q^{i|j}$ , the  $VaR_q^i$  of institution (index)  $i$  conditional on the (unconditional)  $VaR$  of institution (index)  $j$ . That is,  $CoVaR_q^{i|j}$  is implicitly

defined by  $q$ -quantile of the conditional probability distribution

$$\Pr \left( R^i \leq \text{CoVaR}_q^{i|j} | R^j = \text{VaR}_q^j \right) = q.$$

Institutions  $j$ 's contribution to  $\text{CoVaR}_q^{i|j}$  is simply denoted by

$$\Delta \text{CoVaR}_q^{i|j} = \text{CoVaR}_q^{i|j} - \text{VaR}_q^i,$$

The *CoVaR* is typically more negative than the unconditional *VaR* since conditioning on a “bad event” typically shifts the mean downwards and can even increase the variance in an environment with heteroskedasticity. The *CoVaR*, unlike the covariance, reflects both shifts. In addition, *CoVaR* focuses on the tail distribution and, importantly, it is directional. That is, typically  $\text{CoVaR}_q^{i|j} \neq \text{CoVaR}_q^{j|i}$ , since reversing the conditioning matters. Note also, that we condition on the return  $R^j = \text{VaR}_q^j$ . This ensures that the conditioning event is equally likely independently of whether one conditions on the return of a risky or a less risky institution. (In contrast, conditioning on an absolute return level would make the conditioning more extreme for less risky institutions/indexes  $R^j$ .) Another attractive feature of *CoVaR* is that it can be easily adopted for other “corisk-measures”. One of them is the co-expected-shortfall, *Co-ES*. Expected shortfall has a number of advantages relative to *VaR* and can be calculated as a sum of *VaRs*. In the same manner, *Co-ES* can be calculated as an integral of *CoVaRs*.

Finally, the *CoVaR* definition can be applied to analyze the tail dependency across two financial institutions or with respect to indexes. For example, one can calculate the conditional Value at Risk of a particular investment bank conditional on the fact

that hedge funds are in financial difficulty. In this paper we put special emphasis on the following two forms of *CoVaR* measures.

### 2.1.1 Exposure Measure: $CoVaR_{exp}^i$

To investigate which financial institutions are most exposed in the case of a systemic financial crisis, we condition each individual institution's  $VaR^i$  on the event that the portfolio of all financial institutions is in distress, i.e. is at its  $VaR^{index}$  level.

$$\Pr \left( R^i \leq CoVaR_q^{i|index} | R^{index} = VaR_q^{index} \right) = q.$$

To simplify notation we call it  $CoVaR_{exp}^i$ , where the superscript stands for “systemic risk *exposure*” of institution  $i$ .

### 2.1.2 Contribution Measure: $CoVaR_{add}^i$

To investigate which financial institution  $i$ 's marginal contribution to systemic risk is highest, we reverse the conditioning. That is, we calculate the Value of Risk of the whole financial system conditional on institution  $i$  being in distress:

$$\Pr \left( R^{index} \leq CoVaR_q^{index|i} | R^i = VaR_q^i \right) = q.$$

We denote it as  $CoVaR_{add}^i$ , since the difference to the unconditional  $VaR$  of the financial system,  $\Delta CoVaR_{add}^i$ , measures how much this institution “*adds*” to overall systemic risk (in returns). This measure captures externalities that arise because an institution is “too big to fail”, or “too interconnected to fail”, or takes on positions or relies on funding that can lead to crowded traded. Of course, ideally, one would like to have a co-risk measure that satisfies a set of axioms as e.g. the Shapley value

does. Recall that the Shapley value measures the marginal contribution of a player to a grand coalition.

Importantly, the  $CoVaR_{add}^i$  measure does not distinguish whether the contribution is causal or simply driven by a common factor. We view this as a virtue rather than a disadvantage. To see this, suppose a large number of small hedge funds hold similar positions and are funded in a similar way. That is, they are exposed to the same factors. Now, if only one of the small hedge funds falls into distress, this will not necessarily *cause* any systemic crisis. However, if this is due to a common factor all of hedge funds, i.e. all which are “systemic as part of a herd” will be in distress. Hence, each individual’s hedge fund co-risk measure should capture this, even though there is no direct causal link and the  $CoVaR_{add}^i$  measure does so.

### 2.1.3 Endogeneity of Systemic Risk

Note that each institution’s  $CoVaR$  is endogenous and depends on the other institutions’ risk taking. Hence, imposing a regulatory framework that internalizes externalities alters the  $CoVaR$  measures. We view the fact that  $CoVaR$  is an equilibrium measure as a strength, since it adapts to changing environments and provides an incentive for each institution to reduce its exposure to certain risk factors if other institutions load excessively on it.

### 2.1.4 Equity Returns

Our analysis focuses on  $VaR$ -returns rather on absolute dollar amounts since it makes a comparison across institutions of different sizes easier. More importantly, no capital ratios have to be calculated since regulation can impose direct caps on the equity return

$CoVaR_{add}^i$ .<sup>2</sup>

We focus on equity returns since a financial institution's risk is not only driven by the riskiness of its assets but also by the risk of its funding structure. Ideally, one would like to calculate the asset and funding risk across several trading desks separately and relate them to each other. Without detailed P&L data for subdivisions of firms, however, it is best to rely on equity returns. Focusing on asset returns alone and ignoring funding considerations – as the current bank regulatory framework does – is in our view inferior to equity returns.

## 2.2 Time-variation in $CoVaR_t$ and $VaR_t$

Applying our definition directly, we can only estimate a single  $CoVaR$  for each institution that is constant over time. To overcome this limitation, we pursue two modifications. First, to reflect the fact that financial institutions' financing strategy might change over time, we also calculate the  $CoVaRs$  for portfolio sorts. Second, to capture time variation that covaries with certain macro-variables and risk factors, we allow time-variation along these factors.

### 2.2.1 Portfolio Sorts

While we are interested in estimating the evolution of the risk measures  $VaR$  and  $CoVaR$  for individual financial institutions, the nature of any particular institution might have changed drastically over the 1986-2008 sample period. In addition, many of the individual banks merged with other organizations, and some went out of business. One way to control for the changing nature of each individual institution is to form

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<sup>2</sup>Current bank regulation requires that a bank's Value-at-Risk in dollar amounts divided by its capital does not exceed a certain threshold.

portfolios on particularly important balance sheet characteristics. In particular, we form the following sets of quintile portfolios: maturity mismatch, leverage, cash to assets, book to market, and equity volatility. Maturity mismatch is measured as short-term debt - (cash + short-term investments) normalized by dividing it by total assets. Leverage is the ratio of total assets to book equity. Equity volatility is estimated each quarter from the daily equity return data. We form portfolios every quarter.

### 2.2.2 Time-variation linked to Macro Variables

To allow for time-variation we relate the *CoVaR* and the *VaR* to certain macro variables with whom they co-vary. We indicate time-varying  $(Co)VaR_t$  with an additional subscript  $t$ . Taking time-variation into account leads to a panel data set of  $(Co)VaR_t$ s and reduces the problem that tail correlation are overestimated when volatility is high (see e.g. Claessens and Forbes (2001)).

More specifically, we focus on the following “macro” factors to estimate the variation of *VaRs* and *CoVaRs* across institutions and over time. The factors capture certain aspects of risks. They are also liquid and easily tradable. We restrict ourselves to a small set of risk factors to avoid overfitting the data. Our factors are:

(i) *VIX* which captures the implied future volatility in the stock market. This implied volatility index is available on Chicago Board Options Exchange’s website.

(ii) a short term “*liquidity spread*”, defined as the difference between the 3-month repo rate and the 3-month bill rate measures the short-term counterparty liquidity risk. We use the 3-month general collateral repo rate that is available on Bloomberg, and obtain the 3-month Treasury rate, from the Federal Reserve Bank of New York.

(iii) The level of the 3-month term Treasury bill rate.

In addition we consider the following two fixed-income factors that are known to

be indicators in forecasting the business cycle and also predict excess stock returns (Estrella and Hardouvelis (1991), Campbell (1987), and Fama and French (1989)):

(iv) the return to the *slope of the yield curve*, measured by the yield-spread between the 10-year Treasury rate and the 3-months bill rate.

(v) the return to the *credit spread* between BAA rated bonds and the Treasury rate (with same maturity of 10 years).

The last two factors are from the Federal Reserve Board’s H.15 release.<sup>3</sup>

## 2.3 Countercyclical Regulation based on Predictive Characteristics

Instead of relating financial regulation directly to our  $\Delta CoVaR_t^i$  measure, we propose to link them to more frequently observed variables that predict the  $\Delta CoVaR_t^i$  of a financial institution in advance. This ensures that financial regulation is implemented in a proactive and countercyclical way. Like any tail risk measure,  $CoVaR_t$  estimates rely on

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<sup>3</sup>The literature has studied related factors for explaining hedge fund returns. Boyson, Stahel, and Stulz (2006) use the S&P500, Russell 3000, change in VIX, FRB dollar index, Lehman US bond index and the 3-Month Bill return as factors, but – unlike our study – they do not find a link between these factors and contagion. Agarwal and Naik (2004) also focus on tail risk. In addition to out of the money put and call market factors they use the Russell 3000, MSCI excluding US (bonds), MSCI emerging markets, HML, SMB, MOM, Salomon Government and corporate bonds, Salomon world government bonds, Lehman high yield, Federal Reserve trade weighted dollar index, GS commodity index and change in default spread. Factors used in Fung and Hsieh (1997, 2001, 2002, 2003) differ depending on the hedge fund style they analyze. An innovative feature of their factor structure is to incorporate lookback options factors that are intended to capture momentum effects. We opted not to include this factor since restricted ourselves only to highly liquid factors. Fung, Hsieh, Naik, and Ramadorai (2008) try to understand performance of fund of fund managers. They employ the S&P 500 index as factor; a small minus big factor; the excess returns on portfolios of lookback straddle options on currencies, commodities and bonds; the yield spread – our factor (v) – and the credit spread – our factor (vi). Finally, Chan, Getmansky, Haas, and Lo (2006) use the S&P 500 total return, bank equity return index, the first difference in the 6-months LIBOR, the return on the U.S. Dollar spot rate, the return to a gold spot price index, the Dow Jones / Lehman Brothers bond index, Dow-Jones large cap - small cap index, Dow Jones value minus growth index, the KDP high yield minus U.S. 1-year Treasury yield, the 10-year Swap / 6-month Libor spread, and the change in CBOE’s VIX implied volatility index. Bondarenko (2004) introduced the Variance swap contract as a new factor.

relatively few data points. Hence, adverse movements, especially after a “quiet period”, can lead to sizable increases in tail risk measures. Any regulation that naively relies on these estimates would be unnecessarily tight after such adverse events and hence would amplify the initial adverse impact. To overcome this procyclicality, we relate the *CoVaR* measures to characteristics of financial institutions. We focus in particular on institutions’ maturity mismatch, leverage, book to market and relative size. Data limitations restrict our analysis, but regulators can make use of a wider set of institution specific characteristics. We especially emphasize the predictive relationship between *CoVaR* and certain variables since they allow the regulator to act before problems build up. The coefficients for each of these characteristic variables also indicate how much weight one should put on each of them.

### **3 Estimating *CoVaR***

In this section we outline one simple and efficient way to estimate *CoVaR* using quantile regressions, describe the data and then present our main empirical results.

#### **3.1 Estimation Method: Quantile Regression**

The *CoVaR* measure can be computed in various ways. Using quantile regressions is a particularly efficient way to estimate *CoVaR*, but by no means the only one. Alternatively, *CoVaR* can be computed from models with time varying second moments, from measures of extreme events, or by bootstrapping past returns.

To see the attractiveness of quantile regressions, consider the prediction of a quantile



regression of return  $i$  on index return  $j$ :

$$\hat{R}_q^i = \hat{\alpha}_q^{ij} + \hat{\beta}_q^{ij} R^j, \quad (1)$$

where  $\hat{R}_q^i$  denotes the predicted value of excess return of institution  $i$  or portfolio  $j$  (a commercial bank, investment bank, or a hedge fund style index) for quantile  $q$  and  $R^j$  denotes the excess return.<sup>4</sup> In principle, this regression could be extended to allow for nonlinearities by introducing higher order dependence of returns to style  $i$  as a function of returns to index  $j$ . From the definition of Value at Risk, it follows directly that:

$$VaR_q^i | R^j = \hat{R}_q^i. \quad (2)$$

That is, the predicted value from the quantile regression of returns of index  $i$  on return  $j$  gives the Value at Risk conditional on  $R^j$  since the  $VaR$  given  $R^j$  is just the conditional quantile. Using a particular return realization  $R^j = VaR^j$  yields our  $CoVaR^{ij}$  measure.<sup>5</sup> More formally, within the quantile regression framework our  $CoVaR$  measure is simply given by:

$$CoVaR_q^{ij} := VaR_q^i | VaR_q^j = \hat{\alpha}_q^{ij} + \hat{\beta}_q^{ij} VaR_q^j. \quad (3)$$

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<sup>4</sup>Note that a median regression is the special case of a quantile regression where  $q = 50\%$ . We provide a short synopsis of quantile regressions in the context of linear factor models in the Appendix. Koenker (2005) provides a more detailed overview of many econometric issues.

While quantile regressions are regularly used in many applied fields of economics, their applications to financial economics are limited. Notable exceptions are econometric papers like Bassett and Chen (2001), Chernozhukov and Umantsev (2001), and Engle and Manganelli (2004) as well as the working papers by Barnes and Hughes (2002) and Ma and Pohlman (2005).

<sup>5</sup>It differs from the often used conditional VaR (CVaR), mean excess loss, expected/mean shortfall (ES), or tail VaR, which are all defined for a single strategy as  $E[R^i | R^i \leq VaR^i]$ .

## 3.2 Financial Institution Return Data

We focus on three groups of financial institutions in this paper: commercial banks, investment banks and hedge funds. We select the U.S. based primary dealers of the Federal Reserve System as the universe of commercial and investment banks that we consider in the sample. The list of primary dealers can be obtained at <http://www.newyorkfed.org/markets/primarydealers>. We consider equity data since the beginning of 1986, so the list of qualifying institutions comprises a number of banks that have since merged into larger organizations (for example, Salomon Brothers was bought by Citibank, and Citibank in turn merged with Travelers to form Citigroup). We provide a full list of institutions, together with their PERMCO and TICKER in Appendix B. We obtain the daily equity return data from CRSP, and the quarterly balance sheet data from COMPUSTAT. We also use the banking and security broker dealer portfolios from the 49 industry portfolios by Kenneth French available at [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). These portfolios are constructed as value weighted averages from CRSP equity returns according to SIC codes.

In addition to commercial and investment banks, we also include hedge fund returns in our analysis. Hedge funds are private investment partnerships that are largely unregulated. Studying hedge funds is more challenging than the analysis of regulated financial institutions such as mutual funds, banks, or insurance companies, as only limited data on hedge funds is made available through regulatory filings. Consequently, most studies of hedge funds rely on self-reported return data.<sup>6</sup> We follow this approach and use the hedge fund style indices by Credit Suisse/Tremont, which are provided on

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<sup>6</sup>A notable exception is a study by Brunnermeier and Nagel (2004) who use quarterly 13F filings to the SEC and show that hedge funds were riding the tech-bubble rather than acting as price-correcting force.

a monthly basis.<sup>7</sup>

### 3.3 *CoVaR* Estimates

Table 2 provides the estimates of our *CoVaR* measures that we obtain from using quantile regressions. Panel A focuses on 19 commercial banks, Panel B on the 9 investment banks and Panel C provides the summary statistic for monthly hedge fund returns. Estimates are based on weekly equity return data. We opted for a weekly horizon, since we consider daily tail events are too short, while focusing on monthly horizon would reduce the number of data points for our tail estimates. Hedge fund return data are an exception, they are only available at a monthly basis from January 1994 to December 2008.

Table 2 reports institution  $i$ 's individual risk,  $VaR^i$ , the VaR of the whole financial sector conditional on institution  $i$  being in distress, i.e. the  $CoVaR_{add}^i$ , and the  $\Delta CoVaR_{add}^i$  which measures the marginal contribution of institution  $i$  to the overall systemic risk. Recall that  $\Delta CoVaR_{add}^i$  reflects the difference between two value at risk of the portfolio of the “financial universe”. The financial universe contains all traded financial institutions, including real estate financiers (French’s “Portfolio 49”). Finally, we report the exposure  $CoVaR_{exp}^i$  which measures the extent to which institution  $i$  is exposed to a potential systemic event. We report the overall estimates. To obtain the

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<sup>7</sup>There are several papers that compare the self-reported hedge fund returns of different vendors (see e.g. Agarwal and Naik (2005)), and some research compares the return characteristics of hedge fund indices with the returns of individual funds (Malkiel and Saha (2005)). The literature also investigates biases such as survivorship bias (Brown, Goetzmann, and Ibbotson (1999) and Liang (2000)), termination and self-selection bias (Ackermann, McEnally, and Ravenscraft (1999)), backfilling bias, and illiquidity bias (Asness, Krail, and Liew (2001) and Getmansky, Lo, and Makarov (2004)). We take from this literature that hedge fund return indices do not constitute ideal sources of data, but that their study is useful, and the best that is available. In addition, there is some evidence that the Credit Suisse/Tremont indices appear to be the least affected by various biases (Malkiel and Saha (2005)).

between-statistics we take a cross-sectional average across all commercial banks, investment banks or hedge funds, respectively, and then calculate the standard deviation. The within-statistics are obtained by analyzing the time-series averages and focuses on the cross-sectional dispersion.

Our risk measure estimates are surprisingly similar between commercial and investment banks. The estimates for hedge funds are different, especially the  $CoVaR_{exp}^i$  estimates, which is not surprising since they based on a monthly basis. As conjectured the  $\Delta CoVaR^i$  estimates are mostly negative. That is, most financial institutions contribute to the systemic risk ( $\Delta CoVaR_{add}^i < 0$ ) and are exposed to additional risk when the financial system is in distress ( $\Delta CoVaR_{exp}^i < 0$ ). Indeed, a F-test rejects that  $\Delta CoVaR_{add}^i$  is positive with a p-value of X and for  $CoVaR_{exp}^i$  with a p-value of Y. (still needs to be confirmed.) The summary statistic also reveal

**TABLE 1, SUMMARY STATISTICS**

		PANEL A: COMMERCIAL BANKS						PANEL B: INVESTMENT BANKS						PANEL C: HEDGE FUNDS						
	$Var^i$	Mean	Sd	Min	Max	Obs	Mean	Sd	Min	Max	Obs	Mean	Sd	Min	Max	Obs	Mean	Sd	Min	Max
	overall	-10.14	3.95	-60.32	-2.11	N = 15200	-11.81	4.68	-68.95	-3.11	15200	-2.84	2.81	-17.39	2.02	15200	-2.84	2.81	-17.39	2.02
	between		1.93	-15.69	-6.86	n = 19		1.59	-14.89	-10.47	19		2.09	-6.40	-0.37			2.09	-6.40	-0.37
	within		3.42	-60.13	-0.05	T = 800		4.45	-65.86	-1.11	800		1.99	-14.54	4.87			1.99	-14.54	4.87
	overall	-6.91	2.53	-36.65	-2.10	N = 15200	-6.88	2.63	-36.41	-2.01	15200	-7.48	3.87	-25.26	1.66	15200	-7.48	3.87	-25.26	1.66
	between		0.68	-7.75	-5.76	n = 19		0.85	-8.04	-5.96	19		1.67	-9.11	-3.61			1.67	-9.11	-3.61
	within		2.45	-36.11	-1.71	T = 800		2.51	-35.46	-1.36	800		3.52	-24.24	-0.44			3.52	-24.24	-0.44
	overall	-1.84	1.18	-13.23	2.90	N = 15200	-1.62	1.24	-10.27	1.41	15200	-1.27	1.99	-8.71	4.43	15200	-1.27	1.99	-8.71	4.43
	between		0.54	-2.58	-0.57	n = 19		0.61	-2.55	-0.97	19		1.68	-2.92	2.58			1.68	-2.92	2.58
	within		1.08	-13.05	2.28	T = 800		1.11	-9.62	1.45	800		1.17	-7.62	2.27			1.17	-7.62	2.27
	overall	-14.48	5.07	-73.15	-5.05	N = 15200	-16.12	7.11	-91.33	-4.32	15200	-2.97	3.21	-20.04	1.88	15200	-2.97	3.21	-20.04	1.88
	between		2.82	-21.74	-10.98	n = 19		4.05	-24.68	-11.60	19		2.60	-8.24	-0.47			2.60	-8.24	-0.47
	within		4.22	-69.40	-4.46	T = 800		6.02	-82.77	0.96	800		2.03	-14.77	5.30			2.03	-14.77	5.30
	overall	-4.34	2.52	-31.75	4.41	N = 15200	-4.31	3.89	-29.76	7.00	15200	-0.14	0.95	-6.06	2.82	15200	-0.14	0.95	-6.06	2.82
	between		1.55	-7.91	-2.17	n = 19		3.17	-9.79	1.26	19		0.90	-2.55	0.81			0.90	-2.55	0.81
	within		2.11	-32.03	4.90	T = 800		2.58	-24.29	5.92	800		0.40	-3.64	1.88			0.40	-3.64	1.88
Obs	N	15200					6902					1845								
	$\bar{n}$	19					9					11								
	$\bar{T}$	500					767					168								

### 3.4 $CoVaR$ versus $VaR$

Figure 1 in the introduction shows that *across financial institutions* there is only a very loose link between institutions  $VaR^i$  and its contribution to systemic risk measured by  $\Delta CoVaR_{add}^i$ . Hence, imposing financial regulation that is solely based on the individual risk of a institution in isolation is not that useful. Ideally, it has to be complemented by other measures as well. Figure 2 below shows that the same is true for the relationship between  $VaR^i$  and  $CoVaR_{exp}^i$ . Institutions that are perceived to be highly risky, are not necessarily the same ones who are most exposed to systemic risk. Overall, the figure suggests commercial banks and investment banks are roughly equally exposed to systemic crisis even though investment banks appear to have a much higher  $VaR^i$ .

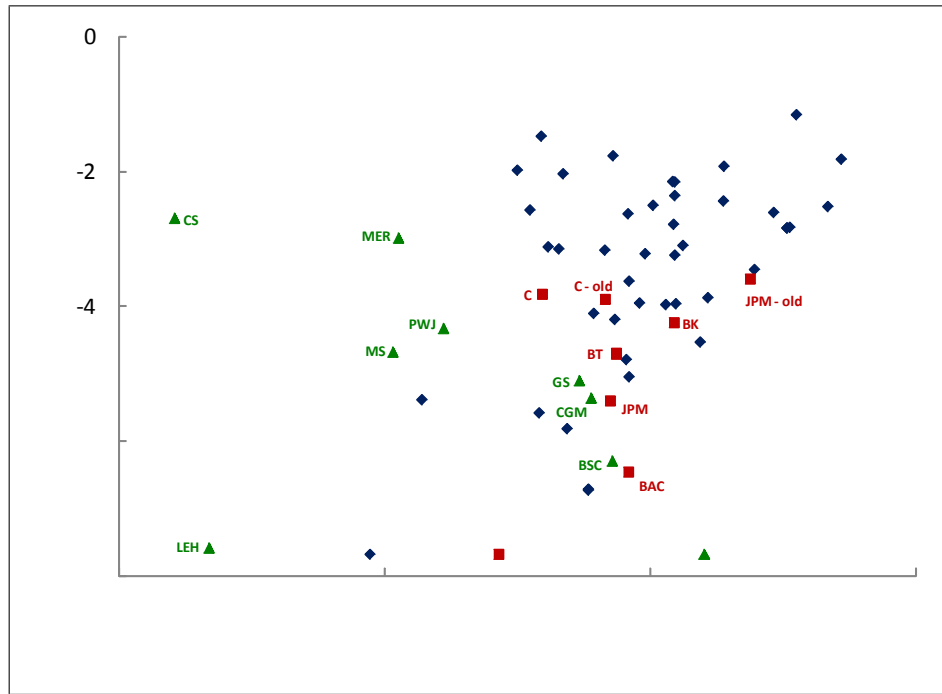


FIGURE 2: Cross-sectional relationship between  $VaR$  (x-axis) and  $CoVaR_{exp}^i$  (y-axis).

The disconnect between  $VaR$  and  $CoVaR$  in the cross-section is in sharp contrast to the close link in the *time series*. Figure 3, Panel A and B show for  $\Delta CoVaR_{add,t}$  and  $CoVaR_{exp,t}$ , respectively, that at times when the institution's risk (in isolation), measured by  $VaR_t$ , is high the co-risk measures is also high.

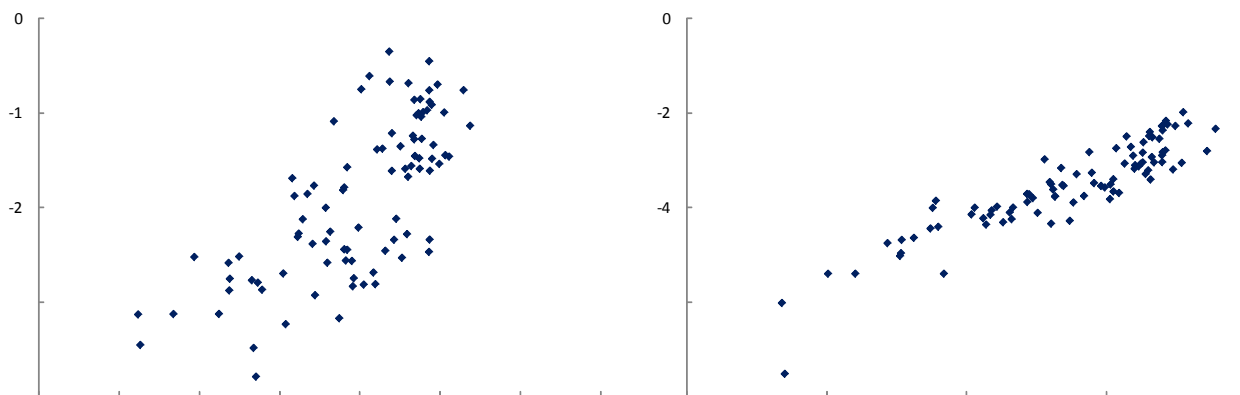


FIGURE 3: Time-series relationship between  $VaR_t$  (x-axis) and  $\Delta CoVaR_{add,t}$  (y-axis) in Panel A and between  $VaR_t$  and  $\Delta CoVaR_{exp,t}$  in Panel B.

### 3.5 Time-varying *CoVaR*

To capture time-variation of risk measures we relate them to macro factors, described in Section 2. More specifically, we quantile regress the weekly returns on on these macro variables. Since institution's investment and funding strategy might have changed over time, we repeat the analysis for portfolios that are sorted as described above. We run this regressions for each institution separately. Table 2 reportst the (equally-weighted) average coefficients across all institutions (in Panel A) and across all portfolios (in Panel B). Recall portfolios are sorted into quintiles according to matuirty mismatch, leverage, cash to assets, book to market, and equity volatility. The number in paratheses reports the average  $t$ -statistic. The second half of the table reports the average coefficients for a different specification. In it, we use the average leverage, maturity mismatch and book-to-market ratio as variable. This alternative specification will serve as a useful robustness check later, when we regress risk measures on institution specific leverage etc.



TABLE 2: AVERAGE RISK FACTOR EXPOSURES

PANEL A:

INSTITUTIONS

PANEL B:

PORTFOLIOS

	$VaR^{\text{index}}$	$VaR^i$	$CoVaR_{add}^i$	$CoVaR_{exp}^i$	$VaR^i$	$CoVaR_{add}^i$	$CoVaR_{exp}^i$
VIX	-0.27 (-11.85)	-0.44 (-9.50)	-0.21 (-15.55)	-0.18 (-9.03)	-0.39 (-3.34)	-0.20 (-6.13)	-0.18 (-2.48)
3 Month	0.65	0.20	0.22	-0.75	0.46	0.23	-0.06
Yield	(3.34)	(-0.10)	(3.12)	(-2.98)	(1.22)	(1.80)	(-0.14)
Repo	-0.08	1.47	0.08	0.96	0.46	-0.85	0.42
spread	(-1.59)	(1.67)	(-0.16)	(3.09)	(0.20)	(-1.01)	(0.23)
Credit	-0.05	-0.31	-0.25	-2.16	-1.42	-0.31	-1.15
spread	(-0.50)	(0.72)	(-0.62)	(-6.32)	(-0.73)	(-0.77)	(-1.26)
Term	0.50	0.58	0.27	-0.69	0.73	0.15	0.36
spread	(1.62)	(3.48)	(2.13)	(-3.65)	(1.17)	(0.76)	(0.97)
Leverage	-1.15 (-4.66)	-1.67 (-4.71)	-0.79 (-5.76)	-0.83 (-7.74)	-1.28 (-3.91)	-0.56 (-4.74)	-0.40 (-1.73)
Maturity	0.81	18.52	-3.78	-2.78	-1.89	-0.64	0.78
Mismatch	(-1.64)	(-1.59)	(0.41)	(-0.95)	(-0.03)	(0.01)	(0.01)
Book/Market	-10.55 (-3.65)	-15.67 (-3.00)	-6.71 (-3.85)	-10.12 (-6.98)	-1.24 (-0.22)	0.16 (0.09)	4.61 (0.70)

Average t-stats in parenthesis

It worth highlighting at least three findings. First, the VIX strongly affects the time-dynamics of all risk measures employed in this paper. The high average  $t$ -statistic reflects its high statistical significance. Second, the term spread coefficient is on average positive for  $CoVaR_{add,t}$ , but negative for  $CoVaR_{exp,t}$ . A higher term spread makes yield curve carry trades very profitable for financial institutions. Hence, each institution seems to contribute less to the systemic risk at that times. On the other hand, given that a systemic risk occurs, they are more exposed to risk (possibly because of increased asset-liability mismatch). Third, the 3 month short-term interest rate shows a similar pattern as the term spread. Times with high short-term interest rate lowers

the contribution  $CoVaR_{add,t}$ .

## 4 *CoVaR* and Institutions' Characteristics

As explained in Section 2, (time-varying) tail risk measure estimates can depend on few observations. We therefore try to relate them to variables that are more readily observable. In the next subsection we do so by relating the risk measure to maturity mismatch, leverage, book to market and institution's relative size. In the subsequent subsection, we show that these variables help us to predict future tail co-risk measures. Regulators and practitioners who have additional sources of data can find better explanatory variables on which financial regulation and internal control can be based.

Since most of these variables are only available at a quarterly basis, we aggregate weekly *CoVaR* measures to quarterly *CoVaRs* by taking the average of the weekly *CoVaR* within the same quarter.

## 4.1 Contemporaneous Relationship

TABLE 3: RISK MEASURES AND CHARACTERISTICS

	PANEL A: INSTITUTIONS				PANEL B: PORTFOLIOS			
	$\Delta CoVaR_{add,t}^i$		$\Delta CoVaR_{exp,t}^i$		$\Delta CoVaR_{add,t}^i$		$\Delta CoVaR_{exp,t}^i$	
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
	FE, TE	FE	FE, TE	FE	FE, TE	FE	FE, TE	FE
<i>VaR</i>	0.19*** (0.01)	0.17*** (0.01)	-0.10 (0.07)	0.15*** (0.03)	0.34*** (0.01)	0.24*** (0.01)	-0.55*** (0.03)	0.22*** (0.02)
Maturity	0.49 (0.30)	-0.94*** (0.32)	-1.92* (1.08)	-2.76*** (0.94)	-0.13 (0.24)	-1.94*** (0.28)	0.71 (0.48)	0.35 (0.55)
Leverage	-0.03*** (0.01)	-0.05*** (0.01)	-0.01 (0.02)	-0.01 (0.02)	-0.01** (0.00)	-0.06*** (0.01)	0.00 (0.01)	0.00 (0.01)
Book/Market	-0.03 (0.06)	-0.20*** (0.05)	0.07 (0.15)	0.05 (0.14)	0.05 (0.09)	-0.42*** (0.06)	-0.55*** (0.14)	-0.19 (0.12)
Weight	26.36*** (5.16)	31.91*** (5.92)	-13.57 (15.26)	-25.41* (13.48)	0.38 (0.93)	2.57** (1.04)	5.22*** (1.40)	1.37 (1.98)
Constant	0.24 (0.31)	1.36*** (0.30)	-3.88** (1.51)	-0.60 (0.93)	0.99*** (0.24)	2.93*** (0.23)	-8.99*** (0.46)	-1.74*** (0.47)
Observations	1539	1539	1539	1539	2668	2668	2668	2668
R-squared	0.73	0.57	0.59	0.49	0.78	0.58	0.73	0.37

Robust standard errors in parentheses

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

TE denotes time effects, FE denotes fixed effects.

Figures 1 and 2 already indicate that there is not strong cross-sectional relationship between VaR and CoVaR. On the other hand, Figure 3 suggests a strong link in the time-series. Our panel fixed effects regressions confirm these findings. However, they also show that other variables like institution specific characteristics are equally important in explaining our co-risk measures. This finding let to the main message of the paper: relying alone on *VaR* measures is not sufficient for regulating financial

institutions. Even though our funding liquidity measures are not very precise, they add valuable information and insight in analyzing institutions' contribution to systemic risk. More precise funding liquidity data would provide even a better guidance. Finally, it is worth mentioning that for  $\Delta CoVaR_{exp,t}^i$  the institutions'  $VaR^i$  is not even significant in the regression with fixed and time effects.

## 4.2 Predictive Relationship

Regulation is only countercyclical if it is tight during booms, i.e. before risk measures increase. Estimated risk measures often only increase at the onset of the crisis. Hence, in this subsection we try to identify variables which help to predict future CoVaR measures. Given our limited data source, we focus on the same institutions' characteristics as before.

TABLE 4: RISK MEASURES AND LAGGED CHARACTERISTICS

	PANEL A: INSTITUTIONS				PANEL B: PORTFOLIOS			
	$\Delta CoVaR_{add}^i$		$\Delta CoVaR_{exp}^i$		$\Delta CoVaR_{add}^i$		$\Delta CoVaR_{exp}^i$	
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
	FE, TE	FE	FE, TE	FE	FE, TE	FE	FE, TE	FE
<i>VaR</i>	0.15***	0.13***	-0.07	0.11***	0.27***	0.19***	-0.41***	0.15***
(lag)	(0.01)	(0.01)	(0.05)	(0.02)	(0.01)	(0.01)	(0.03)	(0.01)
Maturity	0.46	-0.77**	-2.07**	-2.81***	-0.03	-1.03***	1.87***	0.39
Mism.(lag)	(0.34)	(0.39)	(0.98)	(0.99)	(0.29)	(0.34)	(0.51)	(0.58)
Leverage	-0.03***	-0.06***	-0.01	-0.02	-0.01	-0.08***	0.01	-0.02
(lag)	(0.01)	(0.01)	(0.02)	(0.02)	(0.00)	(0.01)	(0.01)	(0.01)
Book to	-0.05	-0.14**	-0.04	0.13	-0.01	-0.22***	-0.52***	0.29***
Market (lag)	(0.06)	(0.05)	(0.17)	(0.14)	(0.08)	(0.06)	(0.14)	(0.10)
Weight	30.65***	34.57***	-14.30	-29.65**	-0.82	2.12*	4.96***	0.38
(lag)	(4.85)	(6.01)	(14.54)	(13.84)	(0.91)	(1.18)	(1.57)	(2.25)
Constant	-0.24	0.86**	-3.35***	-0.66	0.32	2.05***	-8.86***	-2.40***
	(0.37)	(0.36)	(1.23)	(0.88)	(0.27)	(0.26)	(0.49)	(0.47)
Observations	1544	1544	1544	1544	2668	2668	2668	2668
R-squared	0.69	0.46	0.59	0.48	0.74	0.43	0.69	0.30

Robust standard errors in parentheses

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

TE denotes time effects, FE denotes fixed effects.

The finding in Table 4 speak for themselves. Again the *VaR* is a useful indicator, but other variables are important as well and are not succumbed by the institutions' *VaR* estimate.

## 5 Conclusion

During financial crises or periods of financial intermediary distress, tail events tend to spill over across financial institutions. Such risk spillovers are important to understand

for portfolio managers, risk managers, and supervisors of financial institutions. The ability to monitor and potentially hedge risk spillovers can help to optimize portfolio performance, to set risk limits and margins, and to adequately regulate institutions. We find statistically and economically significant risk spillovers across institutions.

The financial market crisis of 2007-2009 has underscored fundamental problems in the current regulatory set-up. When regulatory capital and margins are set relative to *VaRs*, forced unwinding of one institution tends to increase market volatility, thus making it more likely that other institutions are forced to unwind and delever as well. In equilibrium, such unwinding gives rise to a margin/haircut spiral triggering an adverse feedback loop. An economic theory of such amplification mechanisms are provided by Brunnermeier and Pedersen (2009) and Adrian and Shin (2009). These “adverse feedback loops” were discussed by the Federal Open Market Committee in March 2008, and motivated Federal Reserve Chairman Ben Bernanke to call for regulatory reform.<sup>8</sup> Our *CoVaR* measure provides a potential remedy for the margin spiral, as the measure takes the volatility spillovers which give rise to adverse feedback loops explicitly into account. We propose to require institutions to hold capital not only against their *VaR*, but also against their *CoVaR*. “Crowded trades” such as the on-the-run/off-the-run trades that preceded the LTCM crisis, or the short-financials/long-oil trade of the spring of 2008, would be penalized by capital requirements.

For risk monitoring purposes, *CoVaR* is a parsimonious measure for the potential of systemic financial risk. Institutions that monitor systemic risk—for example, the Federal Reserve, other central banks around the world, the International Monetary Fund, and the Bank for International Settlement—have traditionally followed the evolution of *VaRs* of the financial sector. These institutions have also developed measures of sys-

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<sup>8</sup>See <http://www.federalreserve.gov/monetarypolicy/fomcminutes20080318.htm>. and <http://www.federalreserve.gov/newsevents/speech/bernanke20080822a.htm>.

temic risk based on time varying second moments, estimates of exposures to different risk factors, and financial system tail risk measures. The advantage of using *CoVaR* is that it is tightly linked to *VaR*, the predominant risk measure.

## A Appendix: Quantile Regressions

This appendix is a short introduction to quantile regressions in the context of a linear factor model. Suppose that returns  $R_t$  have the following (linear) factor structure:

$$R_t = \gamma_0 + X_t\gamma_1 + (\gamma_2 + X_t\gamma_3)\varepsilon_t \quad (4)$$

where  $X_t$  is a vector of risk factors. The error term  $\varepsilon_t$  is assumed to be i.i.d. with zero mean and unit variance and is independent of  $X_t$  so that  $E[\varepsilon_t|X_t] = 0$ . Returns are generated by a process of the "location-scale" family, so that both the conditional expected return  $E[R_t|X_t] = \gamma_0 + X_t\gamma_1$  and the conditional volatility  $Vol_{t-1}[R_t|X_t] = (\gamma_2 + X_t\gamma_3)$  depend on a set of factors. The coefficients  $\gamma_0$  and  $\gamma_1$  can be estimated consistently via OLS:<sup>9</sup>

$$\hat{\gamma}_0 = \alpha_{OLS} \quad (5)$$

$$\hat{\gamma}_1 = \beta_{OLS} \quad (6)$$

We denote the cumulative distribution function (cdf) of  $\varepsilon$  by  $F_\varepsilon(\varepsilon)$ , and the inverse cdf by  $F_\varepsilon^{-1}(q)$  for percentile  $q$ . It follows immediately that the inverse cdf of  $R_t$  is:

$$\begin{aligned} F_{R_t}^{-1}(q|X_t) &= \gamma_0 + X_t\gamma_1 + (\gamma_2 + X_t\gamma_3)F_\varepsilon^{-1}(q) \\ &= \alpha(q) + X_t\beta(q) \end{aligned} \quad (7)$$

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<sup>9</sup>The volatility coefficients  $\gamma_2$  and  $\gamma_3$  can be estimated using a stochastic volatility or GARCH model if distributional assumptions about  $\varepsilon$  are made, or via GMM. Below, we will describe how to estimate  $\gamma_2$  and  $\gamma_3$  using quantile regressions, which do not rely on a specific distribution function of  $\varepsilon$ .



where

$$\alpha(q) = \gamma_0 + \gamma_2 F_\varepsilon^{-1}(q) \quad (8)$$

$$\beta(q) = \gamma_1 + \gamma_3 F_\varepsilon^{-1}(q) \quad (9)$$

with quantiles  $q \in (0, 1)$ . We also call  $F_{R_t}^{-1}(q|X_t)$  the conditional quantile function and denote it by  $Q_{R_t}(q|X_t)$ . From the definition of VaR:

$$VaR_q|X_t = \inf_{VaR_q} \{\Pr(R_t \leq VaR_q|X_t) \geq q\} \quad (10)$$

follows directly that

$$VaR_q|X_t = Q_{R_t}(q|X_t) \quad (11)$$

the  $q$ -VaR in returns conditional on  $X_t$  coincides with conditional quantile function  $Q_{R_t}(q|X_t)$ . Typically, we are interested in values of  $q$  close to 0, or particularly  $q = 1\%$ . Note that by multiplying the (absolute value of the) VaR in return space the by hedge fund capitalization gives the VaR in terms of dollars.

We can estimate the quantile function via quantile regressions:

$$[\alpha_q, \beta_q] = \arg \min_{\alpha_q, \beta_q} \sum_t \theta_q(R_t - \alpha_q - X_t \beta_q) \quad \text{with } \theta_q(u) = (q - I_{u \leq 0})u \quad (12)$$

See Koenker and Bassett (1978), Koenker and Bassett (1978), and Chernozhukov and Umantsev (2001).

## B List of Financial Institutions

<b>PANEL A: BANK HOLDING COMPANIES</b>	<b>PERMCO</b>	<b>TIC</b>	<b>From</b>	<b>To</b>
BANK OF AMERICA CORP	3151	BAC	01/86	12/08
BANK OF NEW YORK MELLON CORP	20265	BK	01/86	09/08
BANK ONE CORP	606	ONE	01/86	03/04
BANKAMERICA CORP	437	BAC-old	01/86	06/98
BANKERS TRUST CORP	20266	BT	01/86	03/99
CHASE MANHATTAN CORP	20432	CMB-old	01/86	12/95
CITICORP	20456	C-old	01/86	09/98
CITIGROUP INC	20483	C	10/86	12/08
CONTINENTAL BANK CORP	20511	CBK	01/86	06/94
COUNTRYWIDE FINANCIAL CORP	796	CFC	01/86	03/08
FIRST CHICAGO CORP	20712	FNB	01/86	09/95
FIRST CHICAGO NBD CORP	3134	FCN	01/86	06/98
FIRST INTERSTATE BNCP	20720	I	01/86	12/95
JPMORGAN CHASE & CO	20436	JPM	01/86	12/08
MANUFACTURERS HANOVER CORP	21150	MHC	01/86	09/91
MORGAN (JP) & CO	21222	JPM-old	01/86	09/00
NORTHERN TRUST CORP	3275	NTRS	01/86	09/08
SECURITY PACIFIC CORP	4205	PAC	01/86	03/92
ZIONS BANCORPORATION	5057	ZION	01/86	09/08
<b>PANEL B: INVESTMENT BANKS</b>	<b>PERMCO</b>	<b>TIC</b>	<b>From</b>	<b>To</b>
BEAR STEARNS COMPANIES INC	20282	BSC	01/86	03/08
SALOMON BROTHERS/CITI GLOBAL MARKTS	21556	CGM	01/86	09/97
DISCOUNT CORP NY/DEL	1269	DCY	01/86	06/93
GOLDMAN SACHS GROUP INC	35048	GS	04/99	12/08
LEHMAN BROTHERS HOLDINGS INC	21606	LEH	05/87	09/08
MERRILL LYNCH & CO INC	21190	MER	01/86	12/08
MORGAN STANLEY	21224	MS	03/86	12/08
PAINE WEBBER GROUP	21359	PWJ	01/86	09/00
CREDIT SUISSE USA INC	30964	CS	10/95	09/00

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