

Simple Analytics of the Government Expenditure Multiplier

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 - Question especially salient when, as recently, further **interest-rate cuts** not possible
- Much public discussion based on quite old-fashioned models: unlike contemporary discussions of monetary policy
- Recent years have seen development of a **theory of stabilization policy** that integrates consequences of **price/wage stickiness** for output determination with **intertemporal optimization**
— implications for fiscal stimulus?

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 - lump-sum taxation
 - taxes guarantee intertemporal solvency
 - monetary policy independent of public debt

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 - Focus on models with:
 - representative household
 - lump-sum taxation
 - taxes guarantee intertemporal solvency
 - monetary policy independent of public debt
 - Hence path of public debt irrelevant, focus on implications of alternative paths for government purchases

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 - the degree of price or wage stickiness?
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 - the degree of economic slack?
 - whether the federal funds rate has reached the zero bound?
- Also: does countercyclical government spending increase **welfare**?

A Neoclassical Benchmark

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- Production technology (**capital stock fixed**):

$$Y_t = f(H_t), \quad f' > 0, \quad f'' < 0$$

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- note this is also FOC for welfare-maximizing output
- can solve for Y_t as function of **current** G_t only

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- Multiplier is seen to be:

$$\frac{dY}{dG} = \Gamma \equiv \frac{\eta_u}{\eta_u + \eta_v} < 1$$

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- Necessarily less than 1 (government purchases crowd out private spending)
 - substantially less than 1, unless $\eta_u \gg \eta_v$
 - e.g., Eggertsson (2009) parameters: $\Gamma = 0.4$

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- How much the “labor wedge” changes, under any given hypothesis about sticky prices, sticky wages, or sticky information, depends on **degree of monetary accommodation**

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 - a useful benchmark because the answer is independent of the details of price or wage adjustment (within that broad family)
 - corresponds to the textbook “multiplier” calculation, that determines the size of the rightward shift of “IS curve”

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 - can be achieved, for example, by Taylor rule with suitably time-varying intercept (to be determined)

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- Note result is **independent** of details of stickiness of prices, wages or information

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- According to Hall (2009), the ability of NK models to explain such effects depends on prediction of **counter-cyclical markups**, for which evidence is weak (Nekarda and Ramey, 2009)
- In fact, we can obtain a multiplier of 1 regardless of wage-price block of model
 - can easily specify to be consistent with the procyclical markups found by Nekarda and Ramey: sticky wages and prices, procyclical labor productivity due to overhead labor

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- labor/supply demand factors that determine Γ still matter, but **only** to determine **how inflationary** the hypothesized policy is

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- Doesn't the multiplier depend on the degree of “slack”?
 - Not if real interest rate is held constant! But again, degree of slack may affect **plausibility** of assuming that central bank will take actions required to hold it constant

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- But while the NK model implies that the multiplier **can** be higher than the neoclassical prediction, it **need** not be
 - low multipliers also possible, under other assumptions about monetary policy

Alternative Degrees of Monetary Accommodation

- Suppose, instead, that CB enforces a **strict inflation target**:
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— hence multiplier the same as in the neoclassical model
- In any of these models: larger multiplier requires **inflation**

Alternative Degrees of Monetary Accommodation

- A common monetary policy specification: interest rate determined by a **Taylor rule**
- Simple case (again consistent with zero-inflation steady state):

$$i_t = \bar{r} + \phi_\pi \pi_t + \phi_y (\hat{Y}_t - \Gamma \hat{G}_t)$$

where $\phi_\pi > 1$, $\phi_y > 0$ as proposed by Taylor (1993)

— here “output gap” is interpreted as output in excess of **flex-price equilibrium output**

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- Consider path for government purchases of form $G_t = G_0 \rho^t$, for some $0 \leq \rho < 1$.

— then forward path is **same function** of current G_t at all times

Monetary Policy Follows a Taylor Rule

- In the Calvo model (**purely forward-looking**), this implies that equilibrium Y_t, π_t, i_t are all time-invariant functions of G_t
- Can again define a static “multiplier” dY/dG :

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$$\frac{dY}{dG} = \frac{1 - \rho + \psi\Gamma}{1 - \rho + \psi},$$

where

$$\psi \equiv \sigma \left[\phi_y + \frac{\kappa}{1 - \beta\rho} (\phi_\pi - \rho) \right] > 0.$$

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- Note this implies that

$$\Gamma < \frac{dY}{dG} < 1$$

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- Note that multiplier is **smaller** if
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- In each of these **limiting cases** ($\kappa \rightarrow \infty$, $\phi_\pi \rightarrow \infty$, $\phi_y \rightarrow \infty$, or $\rho \rightarrow 1$), neoclassical multiplier is recovered

Monetary Policy Follows a Taylor Rule

- Arguably more realistic specification:

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— note that central banks' measures of “potential output” aren't typically adjusted in response to government spending

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- Multiplier in this case

$$\frac{dY}{dG} = \frac{1 - \rho + (\psi - \sigma\phi_y)\Gamma}{1 - \rho + \psi}$$

is necessarily **smaller**; for large enough ϕ_y , can even be **smaller than the neoclassical multiplier!**

— e.g. Eggertsson (2009) parameters: $\Gamma = 0.4$, but multiplier for Taylor rule with $\phi_\pi = 1.5$, $\phi_y = 0.25$ is only 0.3

Fiscal Stimulus at the Zero Lower Bound

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Fiscal Stimulus at the Zero Lower Bound

- A case of particular interest: effects of increased government purchases, when central bank's policy rate is at **zero lower bound**:
 - currently relevant case in many countries
 - interest in fiscal stimulus especially great, because further interest-rate cuts not possible
 - monetary accommodation especially plausible: even if central bank wishes to implement strict inflation target, or follow Taylor rule, it may be constrained by lower bound on interest rate, and this **should not change due to modest increase in government purchases**

Fiscal Stimulus at the Zero Lower Bound

- How ZLB may sometimes be binding constraint: extend model to allow for a **credit spread** Δ_t between the CB policy rate i_t and the interest rate that is relevant to aggregate demand determination
- Log-linearized Euler equation then becomes

$$\hat{Y}_t - \hat{G}_t = E_t[\hat{Y}_{t+1} - \hat{G}_{t+1}] - \sigma(i_t - E_t\pi_{t+1} - r_t^{net})$$

where

$$r_t^{net} \equiv -\log \beta - \Delta_t$$

decreases if a disruption of credit markets increases Δ_t
(here, exogenously)

— Cúrdia and Woodford (2009) provide more detailed microfoundations

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- ZLB more likely to bind when r_t^{net} (real policy rate required to maintain expenditure at steady-state level \bar{C}) is temporarily low, due to **elevated credit spreads**
- Simple example (Eggertsson, 2009):
 - In normal state (low credit spreads), $r_t^{net} = \bar{r} > 0$
 - Shock at date zero lowers r_t^{net} to $r_L < 0$
 - Each period, probability μ that credit spread remains high ($r_t^{net} = r_L$) another period, if still high in last period; with probability $1 - \mu$, reversion to normal level
 - Once r_t^{net} reverts to normal level \bar{r} , remains there forever after

Fiscal Stimulus at the Zero Lower Bound

- Assume CB follows Taylor rule when consistent with ZLB:

$$i_t = \max \{ \bar{r} + \phi_\pi \pi_t + \phi_y \hat{Y}_t, 0 \}$$

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- Policies to consider: $G_t = G_L$ for all $t < T$ (random date at which credit spreads revert to normal), $G_t = \bar{G}$ for all $t \geq T$
— consider effects of varying G_L (fiscal stimulus during crisis)

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- Markovian structure implies equilibrium in which

$$\begin{aligned} \pi_t = \pi_L, Y_t = Y_L, i_t = i_L \text{ for all } t < T; \text{ and} \\ \pi_t = 0, Y_t = \bar{Y}, i_t = \bar{r} \text{ for all } t \geq T. \end{aligned}$$

Fiscal Stimulus at the Zero Lower Bound

- Solution: $i_L = 0$ (ZLB continues to bind) for all $G_L \leq G^{crit}$, while $i_L > 0$ (Taylor rule applies) for all $G_L > G^{crit}$

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- Solution: $i_L = 0$ (ZLB continues to bind) for all $G_L \leq G^{crit}$, while $i_L > 0$ (Taylor rule applies) for all $G_L > G^{crit}$
- For $G_L \leq G^{crit}$,

$$\hat{Y}_L = \vartheta_r r_L + \vartheta_G \hat{G}_L$$

where

$$\vartheta_r \equiv \frac{\sigma(1 - \beta\mu)}{(1 - \mu)(1 - \beta\mu) - \kappa\sigma\mu} > 0$$

$$\vartheta_G \equiv \frac{(1 - \mu)(1 - \beta\mu) - \kappa\sigma\mu\Gamma}{(1 - \mu)(1 - \beta\mu) - \kappa\sigma\mu} > 1$$

Fiscal Stimulus at the Zero Lower Bound

- Solution: $i_L = 0$ (ZLB continues to bind) for all $G_L \leq G^{crit}$, while $i_L > 0$ (Taylor rule applies) for all $G_L > G^{crit}$

- For $G_L \leq G^{crit}$,

$$\hat{Y}_L = \vartheta_r r_L + \vartheta_G \hat{G}_L$$

where

$$\vartheta_r \equiv \frac{\sigma(1 - \beta\mu)}{(1 - \mu)(1 - \beta\mu) - \kappa\sigma\mu} > 0$$

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- For $G_L > G^{crit}$, equilibrium same as above for Taylor rule: $dY/dG < 1$, possibly less than Γ

Fiscal Stimulus at the Zero Lower Bound

- Eggertsson (2009) parameter values:

β	0.997
κ	0.00859
σ	0.862
Γ	0.425

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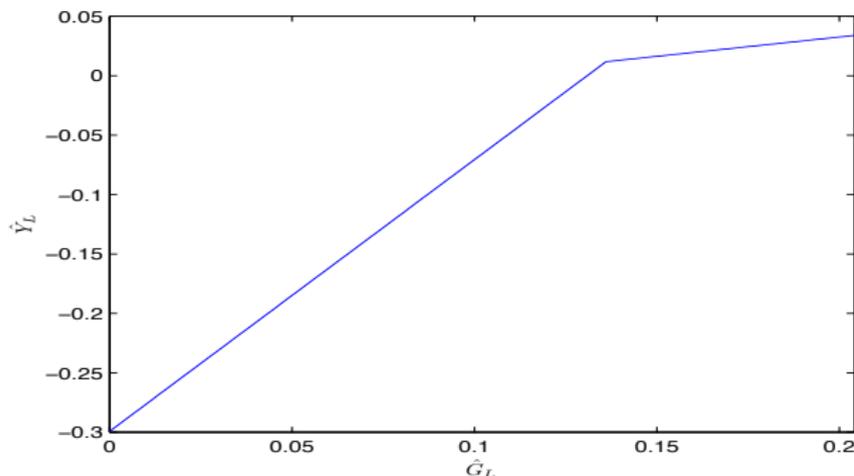
Implications: multiplier = 2.29 for $G < G^{crit}$, 0.32 for $G > G^{crit}$

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- Here $\hat{G}^{crit} = 13.6$ percent of steady-state GDP

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- For large enough value of μ , multiplier can be **much** greater!
 - unboundedly large as $\mu \rightarrow \bar{\mu}$
- This is precisely the case in which risk of **output collapse** is greatest in absence of fiscal stimulus: for dY/dr becomes very large as well
 - so fiscal stimulus highly effective exactly in case where most badly needed (“Great Depression” case)

Fiscal Stimulus at the Zero Lower Bound

- Why do Cogan *et al.* (2009), Erceg and Lindé (2009) find much **smaller** multipliers, in simulations using empirical NK models, despite assuming a situation in which **ZLB initially binds**?

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- Why do Cogan *et al.* (2009), Erceg and Lindé (2009) find much **smaller** multipliers, in simulations using empirical NK models, despite assuming a situation in which **ZLB initially binds**?
- The main difference is **not** their use of more complex models: Christiano *et al.* (2009) find **multiplier can be 2 or more**, using closely related empirical NK model
- Important difference: Cogan *et al.*, Erceg and Lindé assume increase in government purchases that extends **beyond** the time when **ZLB ceases to bind**, interest rates set by Taylor rule
 - Expectation of higher government purchases **after** period for which ZLB binds can **reduce** output when it does!

Fiscal Stimulus: The Importance of Duration

- Why expectation that high government spending will continue after ZLB ceases to bind can **reduce** output during the crisis:
 - if Taylor Rule determines monetary policy post-crisis (or inflation target), higher G then will **crowd out private spending**
⇒ higher expected marginal utility of income ⇒ less desired spending during crisis

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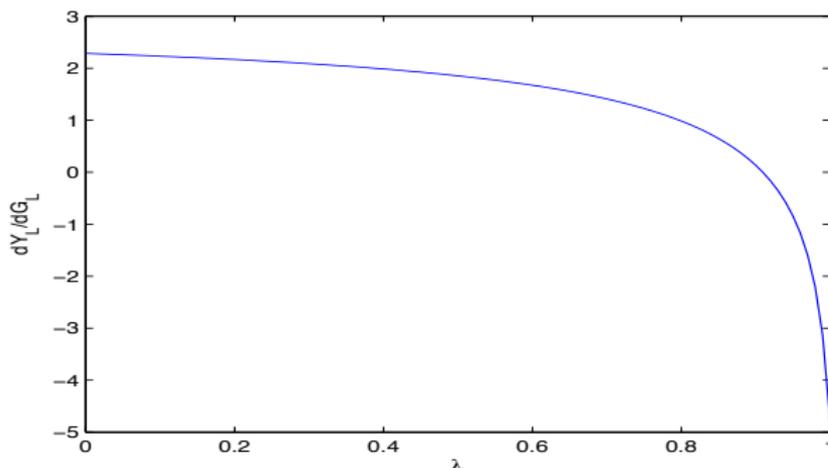
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 - higher G then can also **reduce inflation then** ⇒ lower expected inflation ⇒ zero nominal rate implies **higher real interest rate**
⇒ less desired spending during crisis

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- Multiplier below 1 for $\lambda > 0.8$, negative for $\lambda > 0.91$

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— **additive separability** implicit in previous calculations

— $\eta_g \equiv -g''\bar{G}/g' \geq 0$ a measure of degree of **diminishing returns** to government expenditure

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- Simple principle: choose government purchases to ensure **efficient composition** of aggregate expenditure: maximize $u(Y_t - G_t) + g(G_t)$, for given aggregate expenditure Y_t

— Note this principle requires **no consideration** of effects of government purchases on economic activity

Government Purchases and Welfare

- **Sticky prices or wages:** if increasing G_t increases Y_t , welfare is increased iff

$$(u' - \tilde{v}') \frac{dY}{dG} + (g' - u') > 0$$

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- But: effective **monetary policy** should minimize the importance of this additional consideration!

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Government Purchases and Welfare

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- Then optimal monetary policy maintains **zero inflation** at all times (**assuming ZLB not a problem**)
 - this achieves the flex-price equilibrium allocation, which is efficient, regardless of path $\{G_t\}$
- So optimal choice of $\{G_t\}$ is **same as in neoclassical model!**
 - determined purely by **principle of efficient composition**

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Fiscal Stabilization at the Zero Lower Bound

- But result is different if financial disturbance causes **ZLB to bind**, preventing complete stabilization through monetary policy
- 2-state Markov example: assume that \bar{G} is **optimal steady-state level**, and that central bank targets **zero inflation** except when constrained by ZLB
- Quadratic approximation to expected utility varies inversely with

$$\begin{aligned} E_0 \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda_y (\hat{Y}_t - \Gamma \hat{G}_t)^2 + \lambda_g \hat{G}_t^2] \\ = \frac{1}{1 - \beta\mu} [\pi_L^2 + \lambda_y (\hat{Y}_L - \Gamma \hat{G}_L)^2 + \lambda_g \hat{G}_L^2] \end{aligned}$$

— choose \hat{G}_L to minimize this

Fiscal Stabilization at the Zero Lower Bound

- Optimal level:

$$\hat{G}_L = - \frac{\tilde{\zeta}(\vartheta_G - \Gamma)\vartheta_r}{\tilde{\zeta}(\vartheta_G - \Gamma)^2 + \lambda_g} \quad r_L > 0$$

where

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- Optimal to choose $\hat{G}_L > 0$, even though principle of efficient composition would require $\hat{G}_L < 0$ (since $\hat{C}_L < 0$)

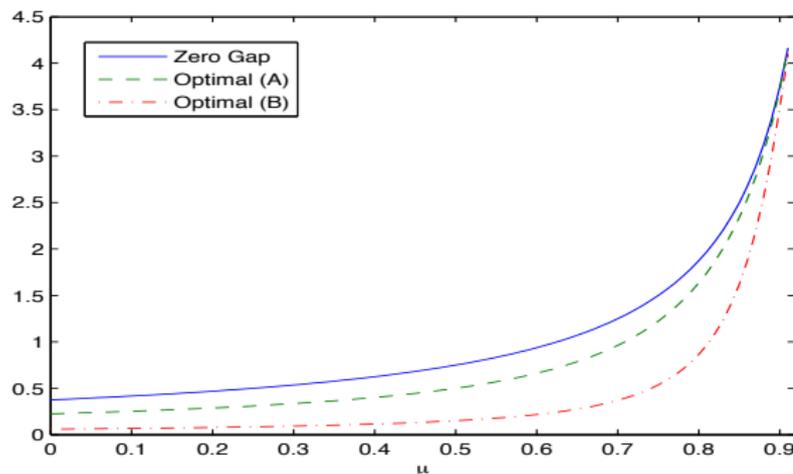
— but optimal \hat{G}_L is **less** than the level required to “fill the output gap” (ensure that $\hat{Y}_L - \Gamma\hat{G}_L = 0$)

Fiscal Stabilization at the Zero Lower Bound

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- Case (A): $\eta_g = 0$; Case (B): same diminishing returns as for private expenditure

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- Here the case for fiscal stabilization policy again depends on assuming a **suboptimal** monetary policy
 - optimal policy would instead involve **commitment to subsequent reflation** (Eggertsson and Woodford, 2003)

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 - optimal policy would instead involve **commitment to subsequent reflation** (Eggertsson and Woodford, 2003)
- But the sub-optimality is of a plausible kind: inability to commit to history-dependent policy
 - becomes much more problematic when ZLB binds

Conclusions

- Under “Great Depression” circumstances (ZLB reached, μ large), multiplier should be **large**, and it is optimal to increase government purchases **aggressively**, nearly to extent required to “fill the output gap”

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- If ZLB reached, but μ is small, multiplier should still be **greater than 1**, and it is optimal to increase G **beyond point consistent with efficient composition**, though probably only a small fraction of what would “fill the gap”
- When ZLB is not a constraint, output-gap stabilization should largely be left to **monetary policy**; decisions about government purchases governed by the principle of **efficient composition of aggregate expenditure**

Conclusions

- When ZLB binds, effective fiscal stimulus (and welfare-maximizing policy) require that government purchases be increased **for as long as ZLB still binds, but not longer**