Toward a Search-Theoretic Approach of Modern Economic Development, Financial Deepening, and Urbanization: A Cross-Country Perspective[1]

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Abstract

This paper focuses on the relationship between economic development, urbanization, and financial deepening. The discussion is developed based on a second-generation money-search model. The model is extend by including bankers in the matching process. This extension enables us to generate interest rates (e.g., deposit rate and financial service charge) endogenously. In the discussion, we suppose the urbanization increases the arrival rate (frequency of matching). Then, we will find that the velocity of money is positively associated with the urbanization as partial effect. In addition, we will confirm that the velocity of money is negatively associated with the economic development and the economic development is positively correlated with the urbanization as total effect (or in a reduced form manner). The theoretical model is further used to understand the endogeneity structure. Then, using 2SLS estimations, we confirm that urban population share (urbanization) increases both velocity of money and GDP per capita as partial effect; hence, urbanization reverses financial deepening when it is measured by money per income (Marshall's k or the inverse of velocity). This result also confirms both the McKinnon-Shaw and the Schumpeterian hypotheses.

1 Introduction

Financial deepening and urbanization are at the core of modern economic growth. Urbanization stimulates the exchange of knowledge, matches entrepreneurs with investors, employers with workers (skills), and sellers with consumers. These factors promote the innovation that creates economic growth (*a la* Schumpeter [27]): for example, exchange of knowledge creates new knowledge; matching entrepreneurs with investors creates new industries; appropriately matched employers and workers increase productivity and income; and a larger frequency of sellers and consumers increases the chances of deals through the coincidence of wants (see, for example, Salop [26], Helsley and Strange [13, 14], Berliant et al. [4], and Zhang [35]).[1] Naturally, money is needed to materialize the chances of these activities.

When urbanization proceeds, does the circulation speed of money increase? It should increase if the money stock is fixed, because urbanization increases the chances of trade activities such as investment, hiring, and consumption. However, our heuristics predict a negative correlation between the velocity of money and urbanization: it is known that the velocity of money is negatively correlated with economic development (i.e., GDP per capita) and that economic development is positively correlated with urbanization. Thus, the first column of Table 1 shows the regression in money velocity (M2) on urban population share and per-capita GDP using unbalanced panel data between 1961 and 2008 for 169 countries (all values are logarithm values, and the errors are heteroskedasticity-robust). The financial deepening measured by per-capita money (M2) shows positive correlations with economic development and urbanization. The negative correlation between economic development and the velocity of money is explained by the McKinnon-Shaw hypothesis, which suggests that the development of financial products increases saving and reduces the circulation speed (McKinnon [22] and Shaw [28]).[2] The third column of Table 1 confirms a positive correlation between economic development and urbanization.

A negative correlation between urbanization and circulation speed would imply that the money supply is increased in advance of urbanization. If that is true, expansionary monetary policies

Table 1: Relationships between financial deepening and urbanization and GDP per capita

	Velocity of Money		Money per Cap.		Urban Pop.	
	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.
Urban Pop.	-0.7644**	(0.0494)	0.4506**	(0.1000)	—	—
GDP per Cap.	-0.2274**	(0.0318)	1.3629**	(0.1297)	0.2272**	(0.0200)
R-sq.	0.2137		0.1472		0.1231	
N	5,607		5,568		5,607	

Significance level: ** 1%; * 5%; † 10%
(Source: World Development Indicators 2009)

stimulate economic growth. However, if this is not supported after resolving empirical problems such as endogeneity, we cannot say that monetary policies stimulate economic growth. A positive correlation between urbanization and circulation speed would imply that urbanization creates a shortage of money despite being an engine of growth. In such a case, the shortage of money would induce developments in financial technologies; hence, urbanization would create innovation in the financial sector.

In the first part of this paper, I extend the second-generation money-search model (Trejos and Wright [31, 32]) to include the banking sector. In this model, using an analogous framework of Li [21] in a context of taxation by government and a government agent (tax collector), the banking sector is represented by the probability of meeting a banker. If a money holder is paired with a banker, he or she can make a deposit. The deposit rate is supposed to be determined via bilateral bargaining. The collected deposit is used to provide financial services for producers. Then, at the end of the period, the deposit is reimbursed to the depositor. With this extension, we can derive the deposit rate endogenously (with an equilibrium between financial services and real consumption markets). Since the derived deposit rate and financial service charge are very complex, they are examined numerically. We also draw a comparison with the interest rate derived in the third-generation model (Lagos and Wright [18]).

In the second part of this paper, I examine cross-country data (World Development Indicators 2009) to see the relationship among the velocity of money, the GDP per capita, and the urban population share. The theory is applied to resolve the endogeneity structure and derive the inferences. In the empirical section, after resolving the endogeneity problem, we will find positive correlations between GDP per capita and urbanization and between the velocity of money and urbanization. We will also confirm a negative correlation between GDP per capita and the velocity of money. As we also find positive correlations between urbanization and interest rates (e.g., deposit and lending rates), we conclude that urbanization creates shortage of money though it stimulates economic development. This result is consistently explained by the theory.

A focus of this paper is economic development. However, the difference from the currently growing literature on the third-generation search-theoretic approach to the development, such as Aruba and Wright [1], Aruba et al. [2], and Lagos and Rocheteau [19], is that the inclusion of the banking sector in the matching market. In addition, by applying the second-generation model, it is easily translated into an econometric model, as there is no classification between centralized and decentralized markets: for example, if we regard the traded goods are composite goods including the capital assets, we cannot say the consumption goods are raded in the decentralized market and the capital assets are in the centralized market (see, for example, Wang and Shi [34] for such a classification in the calibration).

The discussion is developed as follows. In Section 2, a standard second-generation money-search model is extended by including the probability of meeting bankers.. We then solve the dual-market model (the banking and consumption goods markets). The model is verified algebraically and then by numerical simulations. Empirical results are shown in the same section (2.5). Using the theoretical model, we resolve the endogeneity problem by running two-stage least square regressions (2SLS) in Section 3 and interpreting the result by applying the theoretical model of Section 2. We then conclude the discussion in Section 4. Some technical arguments are offered in the Appendix.

2 Money-Search Model

2.1 The Use of Second-Generation Model

There are three major kinds of money-search models. The first-generation model is the original Kiyotaki-Wright model (Kiyotaki and Wright [17]), which uses indivisible money and goods. The second-generation model is the extension of original Kiyotaki-Wright model and allows goods to be perfectly divisible (Trejos and Wright, [31]). The third-generation model is a further extension of the original Kiyotaki-Wright model and allows both money and goods to be perfectly divisible (Lagos and Wright, [18]). In this paper, we use the second-generation model. To pre-assess the use of the second-generation model, let q_i and m_j be the quantities and amounts of good and money exchanged by individuals i and j (if trade occurred).[3] In addition, we let $\bar{m}_j$ be the total money holding of individual j. In the first generation model, by definition, $q_i \equiv 1$ and $\bar{m}_j \equiv m_j \equiv 1$. In the second-generation model, by definition, $q_i > 0$ and $\bar{m}_j \equiv m_j \equiv 1$. In the third-generation model, by definition, $q_i > 0$ and $\bar{m}_j \geq m_j > 0$.

Let p_{ij} be the bargained price between agents i and j. Price is generally defined by the amount of money to obtain a unit of good; hence, $p_{ij} = m_j/q_i$. Without loss of generality, we can redefine the quantity unit to be the quantity of trade per transaction, that is, $Q_i = m_j q_i$. Then the price is also redefined as $p_{ij} = 1/Q_{ij}$. From these arguments, therefore, we can easily find the critical difference between Trejos and Wright [31] (second-generation model), and Shi [29] and Lagos and Wright [18] (third-generation model) is the inclusion of heterogeneity in individual money holdings more than the seller-buyer classification (in the third-generation model, the heterogeneity is dissolved in the centralized market, after which the model is treated as if homogeneous model, without the classifications of agents).

In the third-generation model, we can deal with savings endogenously, as agents do not spend all their money holdings. In the second-generation model, money holders must spend all their money holdings when they make transactions. This indicates that composite goods are inclusive of financial assets and that varieties of goods are the varieties of combinations of goods including financial assets (e.g., portfolios). The interpretation of composite goods becomes a key point in verifying the empirical results in a later section. In this sense, if there are two choices, M1 and M2, M2 becomes more suitable than M1 as a measurement of money in the second-generation model.[4]

The third-generation model is readily persuasive, as money and goods are divisible. However, we can redefine the quantity of composite goods and include financial assets in the composition of composite goods to make the second-generation model closer to the third-generation one. A weakness in the third-generation model is its classifications of the decentralized market (or day market) and the centralized market (or night market). In empirical applications, this flaw becomes a critical concern: for example, Wang and Shi [34] regard the centralized market as a financial market in their calibration. Our paper avoids this critical problem with the third-generation model and focuses on the second-generation model.

2.2 The Basic Framework

We use a standard second-generation money-search model *a la* Trejos and Wright [31, [32]] with an extension to include a financial accessibility factor in the cost function. In our model, there are $N \geq 2$ traditional agents classified into seller and buyer and $N' \geq 1$ agents classified into financial service providers (i.e., bankers, securities dealers, etc.). Then we define $b = N'/(N+N')$ to be the population share of bankers and $1-b$ to be the population share of the real-sector population.

In the beginning of the process, $\mu \times 100\%$ of agents are randomly endowed a unit of money; hence, the money supply of this economy is $m = \mu N$. The money is costly to store (as discussed later) and agents are not allowed to hold more than one unit of money. In the matching process, letting τ be the length of each period, agents are randomly matched in accordance with the Poisson process of arrival rate $\lambda_\tau > 0$, where $\tau > 0$ indicates the length of each period. Without loss of generality, τ can be defined to be small enough to have a sufficiently small λ_τ. When τ is sufficiently close to zero, using the Maclaurin series, we find $\lambda_\tau = \tau\lambda_0$, where λ_0 is the arrival rate per moment. Then $p_b = b\lambda_0$ is the probability of meeting with a banker.

2.2.1 Real Production Sector

Firstly, we consider the "traditional" agents. As usual, buyers are money holders but the seller does not hold money. Money is costly to store, and the periodical storage cost $\gamma < 0$ is borne by the buyers (money holders) at the beginning of each period. These agents are capable of producing a type of product denoted by i and deriving utility from the consuming product $j \neq i$. The range of product from which an agent can derive utility generates the corresponding probability of coincidence of want(s) in the matching process. For example, if the variety of products is in a compact set and the agent likes all other goods, the self product type is measure-zero and the probability of a coincidence of wants is one. If the agent likes a part of the varieties (either in a compact or finite set), the probability of a coincidence of want(s) is between zero and one. We let the probability of coincidence be $\alpha \in (0,1)$. As the social optimum quantity is traded in barter trade (see Appendix B.1), we assume that monetary transactions take place only in single coincidence pairs.

If real-sector agents are paired, the probability of non-coincidence is $(1-\alpha)^2$ and that of double coincidence is α^2; thus, the probability of single coincidence is computed as

$$1-(1-\alpha)^2-\alpha^2 = 2\alpha(1-\alpha). \tag{1}$$

For monetary trade, the agent who likes the paired partner's product must have money. Thus, the probability of monetary trade for the money holder (buyer) is

$$p_1 = \alpha(1-\alpha)(1-b)(1-\mu)\lambda_0, \tag{2}$$

and that of the non-money holder (seller) is

$$p_0 = \alpha(1-\alpha)(1-b)\mu\lambda_0. \tag{3}$$

If two agents are appropriately matched for monetary trade, the Nash bargaining rule determines the quantity of that trade. If the coupled agents like each other's products but neither is a money holder, barter trade takes place. The probability of such matching for both the non-money holder and the money holder is $p_2 = \alpha^2(1-b)\lambda_0$.

The utility of consuming q^d units of goods is uniformly represented by utility function $u(q^d)$. The utility function satisfies $u(0)=0$, $u'(q^d)>0$ with $u'(0)\to\infty$, and $u''(q^d)<0$. The cost function to produce q^s units of goods is also uniformly represented by $c(q^s,z)$, where z is the cost parameter. This cost function is derived from the cost minimization problem as explained below.

We suppose all real-sector agents are endowed with $k_0>0$ units of total production factor (i.e., per-capita capital stock). Then agents produce ordered quantities using the endowed input factor, their labor ℓ, and the other production factor z. The endowed total production factor is magnified by the labor input as $k=\ell k_0$. In this paper, z is the use of financial services. Let the production technology be represented by production function $q^s=F(k,z)$. As k is owned by each agent, the cost of using k, denoted by $\chi(k)$, is self-paid utility cost (e.g., disutility). The use of financial services is supposed to increase productivity as in the form

$$F(k,z) = \phi(z)f(k), \tag{4}$$

where f is the production function satisfying $f(0)=0$, $f'(k)>0$, and $f''(k)<0$; and $\phi(z)$ is the total factor productivity associated with z satisfying $\phi(0) \geq 0$, $\phi'(z) > 0$, and $\phi''(z) < 0$.

Let ρ be the service charge for financial services paid to the financial service provider by product. Then the disutility (cost) minimization problem is given by

$$\min_{z,k} \chi(k) \quad \text{subject to} \quad \phi(z) f(k) \geq q^d + \rho z, \tag{5}$$

where r becomes q^d after optimization (because of the homogeneity of the production function). The above minimization problem is merely to minimize the total factor input; hence, the problem is identical to that of maximizing the product with respect to z and solving the inequality constraint with equality with respect to k. The first order condition for the interior solution with respect to z therefore gives

$$\rho = \phi'(z) f(k) \quad \Rightarrow \quad \rho(z) = q^d \hat{\phi}(z), \tag{6}$$

where $\hat{\phi}(z) \equiv \phi'(z)/\phi(z)$. Consequently the cost function to supply $q^s = q^d = q$ is given by

$$c(q,z) = \chi \circ f^{-1}\left(\frac{q + q\hat{\phi}(z)}{\phi(z)}\right). \tag{7}$$

Specifically, if f and χ are degree $\kappa_1 \in (0,1]$ and $\kappa_2 \geq 1$ functions, respectively, the cost function is written as

$$c(q,z) = c_0 q^{\kappa_2/\kappa_1}\left(\frac{1 + \hat{\phi}(z)}{\phi(z)}\right)^{\kappa_2/\kappa_1}, \tag{8}$$

where $\kappa_2/\kappa_1 \geq 1$ and $c_0 > 0$ is a cost coefficient associated with $\chi \circ f$. Note that there are two possible cases—monetary trade (single coincidence) and barter trade (double coincidence). In these cases, corresponding pairs of z and q^d are applied to satisfy (6) with unique ρ.

Proposition 1 *Suppose $\hat{\phi}(z)$ is decreasing in z, so that $z\phi''(z)/\phi'(z) < z\phi'(z)/\phi(z)$. Then, the demand for financial services is increasing in q^d and decreasing in ρ. In addition, if $\hat{\phi}(z)$ is decreasing in z, as $\phi(z)$ is increasing in z, the cost function is decreasing in z (and the cost function is increasing in q^d).*

Proof. $\hat{\phi}(z)$ is decreasing in z if and only if

$$\frac{d\hat{\phi}}{dz} = \frac{\phi''(z)\phi(z) - \phi'(z)^2}{\phi(z)^2} < 0 \quad \Leftrightarrow \quad \frac{z\phi''(z)}{\phi(z)} < \frac{z\phi'(z)}{\phi(z)}. \tag{9}$$

It therefore immediately follows from (6) that the demand for financial services is increasing in q^d and decreasing in ρ. ■

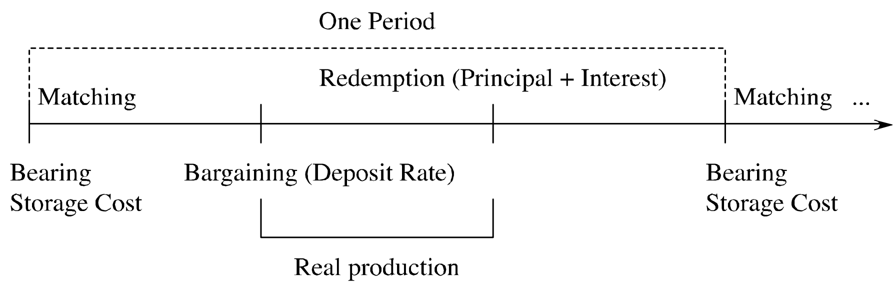

Figure 1: Money holder's banking timeline

2.2.2 Banking Sector

Next, we consider the financial service providers (the bankers) whose role in the process is fixed; thus, the bankers are bankers forever. The role of these agents is financial intermediation. In the matching process, bankers are randomly matched with other agents randomly. If a banker is matched with a money holder, the banker makes an offer on a financial product yielding a strictly positive utility flow $\delta > 0$ by the end of the current period; hence, the money holder receives utility flow δ and the principal by the end of the current period. The expected utility flow from money holding is $-\gamma = p_b\delta - \gamma_0$ when the money is banked, where $p_b = b\lambda_0$ is the probability of meeting a banker in the matching process (the deposit rate δ is supposed to be determined through a Nash bargaining process analogous to the traditional agents' matching and trading procedures). Through this process, bankers collect money from money holders to produce financial services for good production. The goods producers who are sellers and are matched with preferable buyers purchase financial services for processing orders. After providing all their financial services, bankers pay the contracted yield to the banks' depositors and consume all of the remaining revenue. The banking sector is owned by the bankers, and all bankers are treated equally. The details are discussed in the following.

Let Z^s be the aggregate supply of financial services. The financial services are equally accessible to all sellers. As we do not consider long-term lending in this model (eliminating the risk of management problems), we consider financial services to be provided as instant banking services, such as financial settlements. We therefore consider a financial service to be produced by labor and deposits; thus, the aggregate production function of financial services is

$$Z^s = G(H, D), \tag{10}$$

where D is the total amount of deposits, and H is the bankers' inputs: $H \equiv hN'$, where $h \geq 0$ is the average working hours. Then, the profit maximization problem of the banking sector is given by

$$\max_{H,D} \rho Z^s - \delta D - w(h) N', \tag{11}$$

where $w(h)$ is the total wage payment made to each banker. Then,

$$\frac{\partial G}{\partial D} = \frac{\delta}{\rho} \quad \text{and} \quad \frac{\partial G}{\partial H} = \frac{w'(h)}{\rho}. \tag{12}$$

In order to obtain an explicit solution, we suppose G is

$$G(H,D) = BH^{\zeta} D^{1-\zeta}, \tag{13}$$

where B is the total factor productivity of the banking sector. Then, we find the relationship between labor hours and the banking parameters as

$$\frac{\partial G/\partial H}{\partial G/\partial D} = \frac{\zeta}{1-\zeta}\frac{D}{H} \quad \Rightarrow \quad w'(h) h = \frac{\zeta}{1-\zeta}\frac{\delta D}{N'}. \tag{14}$$

In addition, from the first-order condition, the corresponding lending service rate is

$$\rho = \frac{\delta (D/N')^{\zeta} h^{-\zeta}}{(1-s) B}. \tag{15}$$

From the production function,

$$Z = BH^{\zeta} D^{1-\zeta} \quad \Rightarrow \quad h = \frac{D}{N'}\left(\frac{Z}{BD}\right)^{1/\zeta}, \tag{16}$$

and the service charge is

$$\rho Z = \frac{\delta D}{1-\zeta}. \tag{17}$$

Let $\tilde{Z} = E[Z]$ be the expectation of supply of financial services and $\tilde{D} = E[D]$ be the expectation of deposit. Let $\tilde{h} = E[h]$ be the average working hours of bankers computed from substituting $\tilde{Z}$ and $\tilde{D}$ into (16):

$$\tilde{h} = \frac{\tilde{D}}{N'}\left(\frac{\tilde{Z}}{B\tilde{D}}\right)^{1/\zeta} \quad \Rightarrow \quad \tilde{h} = \frac{\tilde{d}}{bB^{1/\zeta}}\left(\frac{\tilde{z}}{\tilde{d}}\right)^{1/\zeta}, \tag{18}$$

where $\tilde{z} = \tilde{Z}/(N+N')$ is the expected quantity of financial services per capita and $\tilde{d} = \tilde{D}/(N+N')$ is the expected per capita deposit. Then, the wage schedule to compensate for the anticipated working hours is given by $\tilde{w} = w(\tilde{h})$.

Note that, in this model, the bankers' working hours are an indicator of financial intermediation activities such as deposit accounts for money holders and financial services for real sector producers. Therefore, an increase of $\tilde{h}$ indicates an increase in financial intermediations (e.g., a financial deepening).

Since currency exchanges are always one currency unit, the expected amount of per capita deposit is equal to the expected number of matching between bankers and money holders; hence,

$$\tilde{d} = b(1-b)\mu\lambda_0. \tag{19}$$

Similarly, the expected demand for financial services is equal to the sum of demands from double-coincidence matching for baryer trade and single-coincidence matching for monetary trade. The derivation of $\tilde{Z}$ is as follows. The individual inverse demand function is given by (6). Let $\hat{\phi}^{-1}$ be the inverse function of $\hat{\phi}$. Then the demand function is given by

$$z = \hat{\phi}^{-1}(\rho/q^d). \tag{20}$$

Then the expected aggregate demand for financial services is given by $\tilde{z} = \tilde{z}(\rho)$ for

$$\tilde{z}(\rho) = \left\{\alpha^2\hat{\phi}^{-1}(\rho/\bar{q}) + 2\left(\alpha-\alpha^2\right)\left(\mu-\mu^2\right)\hat{\phi}^{-1}(\rho/q^{\circ})\right\}(1-b)^2\lambda_0, \tag{21}$$

where $\bar{q}$ and q° are the quantities of transactions in barter and monetary transactions, respectively.[5] Thus, the relative quantity of expected per capita financial services and expected per capita deposits, $\tilde{z}/\tilde{d}$, is computed from (19) and (21) as

$$\frac{\tilde{z}}{\tilde{d}} = \left(\frac{\alpha^2\hat{\phi}^{-1}(\rho/\bar{q})}{\mu} + 2\left(\alpha-\alpha^2\right)(1-\mu)\hat{\phi}^{-1}(\rho/q^{\circ})\right)\frac{1-b}{b}. \tag{22}$$

Subsequently, the wage function $\tilde{w}$ is rewritten in a reduced form as $\tilde{w} = \tilde{w}(\tilde{\rho})$, where $\tilde{\rho}$ is the financial service charge computed from expected values, $D = \tilde{D}$ and $Z = \tilde{Z}$.

Remark 1 *By chain rule, if $\hat{\phi}$ satisfies the condition for $\hat{\phi}'(z) < 0$, its inverse $\hat{\phi}^{-1}$ is decreasing in ρ and increasing in $q^d \in \{q^{\circ}, \bar{q}\}$. Therefore, $\tilde{z}$ and $\tilde{z}/\tilde{d}$ are decreasing in ρ and increasing in q^d.*

Proof. By the chain rule formula, we find

$$\frac{d\hat{\phi}^{-1}}{d(\rho/q^d)} = \frac{1}{\hat{\phi}' \circ \hat{\phi}^{-1}(\tilde{z})} < 0. \tag{23}$$

Therefore, $\hat{\phi}^{-1}$ is decreasing in ρ/q^d, which indicates $\hat{\phi}^{-1}$ is decreasing in ρ and increasing in q^d.

Consequently, (21) indicates ρ/q^d is decreasing in ρ and increasing in q^d and so is $\tilde{z}/\tilde{d}$. ■

The second statement of Remark 1 is an analogue of Proposition 1. That is, a higher financial service charge causes a decline in demand for financial services, and an increase in production causes a larger demand for financial services.

Since the net continuation gains from banking for the money holder is δ and the remaining banking profits are distributed to all bankers, the Nash bargaining problem between the individual banker and money holder is then given by

$$\max_{\delta} \; \delta^{\eta} \left(\frac{\tilde{\rho}\tilde{Z}}{N'} - \frac{\delta \tilde{D}}{N'} - \tilde{w} \right)^{1-\eta}, \tag{24}$$

where $\eta \in (0,1)$. The first order condition gives

$$\eta \left(\tilde{\rho}\tilde{Z} - \delta\tilde{D} - \tilde{w}N' \right) = (1-\eta)\,\delta\tilde{D}. \tag{25}$$

After substituting the market clearing condition (17) for (25) and rearranging terms, the Nash bargaining problem is

$$\delta = \frac{\tilde{w}N'}{\tilde{D}} \cdot \frac{\eta\,(1-\zeta)}{\eta+\zeta-1} \quad \Rightarrow \quad \delta = \frac{b\tilde{w}}{\tilde{d}} \cdot \frac{\eta\,(1-\zeta)}{\eta+\zeta-1}. \tag{26}$$

To satisfy the rationality condition of the money holder strictly $\delta > 0$, we must have a relatively large bargaining power for the depositor relative to the importance of the deposit in the financial service production process (e.g., $\eta > 1-\zeta$), and we do so (i.e., $\zeta = 0.5$ and $\eta = 1$ satisfy the condition). The lower bound of the depositor's bargaining power can decline as the importance of the deposit in the financial sector grows. Subsequently, after collecting all deposits, the service charge is determined by

$$\rho = \frac{\tilde{w}N'(D/\tilde{D})}{Z} \cdot \frac{\eta}{\eta+\zeta-1}. \tag{27}$$

Therefore, $\tilde{\rho}$ satisfies

$$\tilde{\rho} = \frac{\tilde{w}N'}{\tilde{Z}} \cdot \frac{\eta}{\eta+\zeta-1} \quad \Rightarrow \quad \tilde{\rho} = \frac{b\tilde{w}(\tilde{\rho})}{\tilde{z}(\tilde{\rho})} \cdot \frac{\eta}{\eta+\zeta-1}. \tag{28}$$

Note that $\eta > 1-\zeta$ means the bargaining power of depositors is greater than the share of input of deposit. Since the owner of a deposit is the depositor, it implies that the lower bound of the bargaining power of the depositor is subject to the input share of the deposit, and a larger input share must increase the lower bound of the depositor's bargaining power for a strictly positive deposit rate ($\delta > 0$).

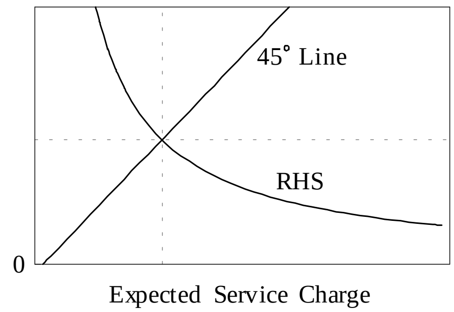

Figure 2: Equilibrium of banking sector (intersection)

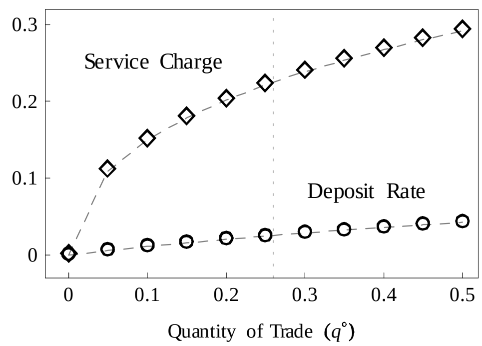

Figure 3: Service charge and deposit rate per quantity of monetary trade (no barter)

The equilibrium financial service charge is determined by solving the above fixed point problem: see Figure 2, where the right-hand-side of (28) is denoted as RHS. To ensure the existence of equilibrium, the right-hand-side of (28) must be decreasing in $\tilde{\rho}$. The necessary and sufficient condition for existence is that the elasticity of wage in financial services provision is greater than one. From (18), the sufficient condition for existence is that the wage function is weakly concave in the average working hours of bankers: $\partial^2\tilde{w}/\partial\tilde{h}^2 \geq 0$. Thus, in the following analysis, we assume the wage function is concave in the average working hours of bankers.

Remark 2 *An increase of* $q^d = \{q^{\circ}, \bar{q}\}$ *increases* $\tilde{\rho}$ *and* $\tilde{\delta}$.

Proof. Consistent with Remark 1 and (6), we see that an increase of q^d creates an upward shift in RHS. Then, we find positive correlations between $\tilde{\rho}$ and q^d and between $\tilde{\delta}$ and q^d. ■

For reference, Figure 3 shows a numerical example of the relationship between q^d and $\tilde{\rho}$ and $\tilde{\delta}$.[6] This example confirms Remark 2. To see the model fully, we consider in the next section how q^d is determined in the equilibrium.

2.3 Nash Bargaining Solution in Monetary Trade

Now we can compute the value functions and the Nash bargaining solution in monetary transactions. Let us write $\gamma(q,\bar{q}) = \gamma_0 - p_b\delta^*$. Then, the value functions of real-sector agents satisfy the following (see Appendix A for the derivations of these value functions):

$$\beta V_0 = p_1 \{V_1 - V_0 - c(q)\} + p_2 \tilde{v} + \dot{V}_0, \tag{29}$$

$$\beta V_1 = p_0 \{u(q) + V_0 - V_1\} + p_2 \tilde{v} + \gamma(q,\bar{q}) + \dot{V}_1, \tag{30}$$

where V_m is the value function of the agent having $m = \{0,1\}$ unit of money and $\dot{V}_m = dV_m/dt$; hence, V_0 and V_1 are the value functions for the non-money holder (seller) and the money holder (buyer). From (30) minus (29), we find

$$\dot{V}_1 - \dot{V}_0 = (\beta + p_0 + p_1)(V_1 - V_0) - p_1 c(q) - p_0 u(q) - \gamma(q,\bar{q}). \tag{31}$$

Let $\theta \in [0,1]$ be the bargaining power of the bargaining power of the buyer. From the Nash bargaining solution q° (see Appendix B.2), we find

$$V_1 - V_0 = (1-\theta) u(q^\circ) + \theta c(q^\circ), \tag{32}$$

$$\dot{V}_1 - \dot{V}_0 = \dot{q}^\circ \left\{ (1-\theta) \frac{du}{dq^\circ} + \theta \left(\frac{\partial c}{\partial q^\circ} + \frac{\partial c}{\partial z^\circ} \frac{dz^\circ}{dq^\circ} \right) \right\}. \tag{33}$$

By substituting (32) and (33) for (31), we find

$$\dot{q}^\circ = \frac{\mathscr{F}(q^\circ, z^\circ, \omega)}{\mathscr{G}(q^\circ, z^\circ, \omega)}, \tag{34}$$

where $\omega = \{\beta, \tilde{\gamma}, \theta, \mu, \lambda\} \in \Omega$ is the set of parameters. Then $\mathscr{F}(q^\circ, \omega)$ is the function given by

$$\begin{aligned} \mathscr{F}(q^\circ, z^\circ, \omega) &= \{(1-\theta)\beta - \lambda(\theta - \mu)\} u(q^\circ) \\ &\quad + \{\theta\beta + \lambda(\theta - \mu)\} c(q^\circ, z^\circ) - \gamma_0 + b\lambda_0 \delta(q^\circ, \bar{q}), \end{aligned} \tag{35}$$

$$\mathscr{G}(q^\circ, z^\circ, \omega) = (1-\theta) \frac{du}{dq^\circ} + \theta \left(\frac{\partial c}{\partial q^\circ} + \frac{\partial c}{\partial z^\circ} \frac{dz^\circ}{dq^\circ} \right). \tag{36}$$

where $\lambda \equiv \alpha(1-\alpha)(1-b)\lambda_0$ is the arrival rate of single coincidence; thus, the probability of monetary trade for the seller is $p_0 \equiv \mu\lambda$ and is $p_1 \equiv (1-\mu)\lambda$ for the buyer.

Remark 3 $\mathscr{G}(q^\circ, z^\circ, \omega) > 0$.

Proof. dc/dq is given by

$$\frac{dc}{dq} = \frac{\partial c}{\partial q} + \frac{\partial c}{\partial z} \frac{dz}{dq}, \tag{37}$$

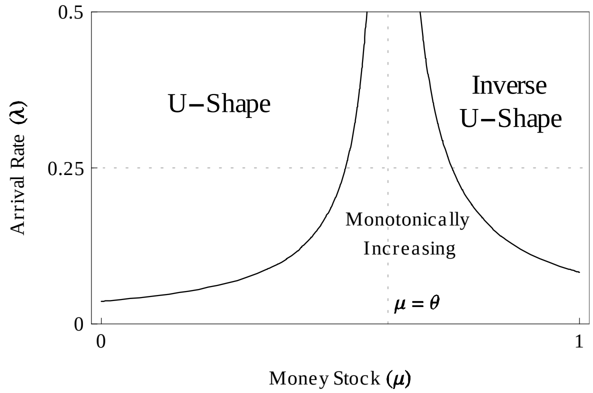

Figure 4: Classification of $\mathscr{F}$ by μ and λ

where dz/dq is computed from ϕ^{-1} as (23) and it is negative. In addition, $\partial c/\partial z$ is computed from (8) as

$$\frac{\partial c}{\partial z} = c_0 q^{\kappa_2/\kappa_1} \frac{\kappa_2}{\kappa_1} \left(\frac{1+\hat{\phi}(z)}{\phi(z)} \right)^{\kappa_2/\kappa_1 - 1} \frac{\hat{\phi}'(z)\phi(z) - \left\{1+\hat{\phi}(z)\right\}\phi'(z)}{\phi(z)^2} < 0, \tag{38}$$

where the inequality follows from $\phi'(z) > 0$ and $\hat{\phi}(z) < 0$. Therefore $dc/dz > 0$ holds and then $\mathscr{G}(q^\circ, z^\circ, \omega) > 0$. ■

Remark 3 posits that $\dot{q}^\circ \gtreqless 0$ is equivalent to $\mathscr{F}(q^\circ, z^\circ, \omega) \gtreqless 0$ pursuant to Trejos and Wright's original discussion [31]. The Nash bargaining solution is then given by $\dot{q}^\circ = 0$ for each parameter set ω; hence, q^*_ω solves $\mathscr{F}(q^\circ, z^\circ; \omega) = 0$. The next remark shows the shape of $\mathscr{F}(q^\circ, z^\circ; \omega)$ and the existence of equilibria for each ω.

Remark 4 *$\mathscr{F}(q^\circ, z^\circ; \omega)$ is $\cup$-shape in q° iff $\mu < \theta$ and $\lambda > (\theta - \mu)^{-1}(1-\theta)\beta$; $\cap$-shape in q° iff $\mu > \theta$ and $\lambda > (\mu - \theta)^{-1}\theta\beta$; and otherwise monotonically increasing in q°.*

Proof. See Appendix D.1. ■

The summary of Remark 4 is depicted in Figure 4. The intercept of $\mu = 0$, which is $(1-\theta)\beta/\theta$, is decreasing in θ, and the intercept of $\mu = 1$, which is $\theta\beta/(1-\theta)$, is increasing in θ. As it shows, an increase in the buyer's bargaining power (θ) expands the area of being $\cup$-shape $\mathscr{F}$ and vice versa for a decline of θ. We can then confirm that the dictatorial buyer model known to have a $\cup$-shape $\mathscr{F}$ corresponds to the case where the money supply is relatively small (e.g., $\mu < \theta$) and that the dictatorial buyer model known to have a $\cap$-shape $\mathscr{F}$ corresponds to the case where the money supply is relatively large (e.g., $\mu > \theta$).

The stability of equilibria is determined by $\mathscr{F}(q^\circ; \omega) - b\lambda_0\delta^*(q^\circ, \bar{q}) \gtrless 0$ around the steady state. Specifically, for a small perturbation $\varepsilon > 0$, the stability at q^*_ω requires $\mathscr{F}(q^*_\omega + \varepsilon; \omega) -$

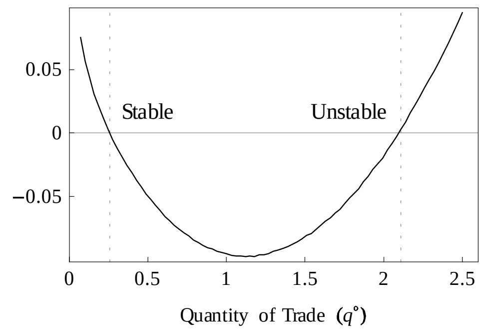

Figure 5: Stable Nash bargaining solution and banking rates

$b\lambda_0\delta^*(q^\circ,\bar{q}) < 0$ and $\mathscr{F}(q^*_\omega - \varepsilon;\omega) - b\lambda_0\delta^*(q^\circ,\bar{q}) > 0$. Otherwise q^*_ω is unstable. For example, Figure 5 depicts the stable and unstable equilibria for the $\cup$-shape $\mathscr{F}$ function. Note, because $\tilde{\delta}(q^\circ,\bar{q})$ is non-decreasing in q's (Remark 2), $\mathscr{F}(q^\circ;\omega) - b\lambda_0\delta^*(q^\circ,\bar{q})$ is $\cup$-shape if $\mathscr{F}$ is $\cup$-shape, as per the conventional Trajos-Wright model [31].

Proposition 2 *If $\mu > \theta$ (e.g., $\cap$-shape $\mathscr{F}$) and the money is costly to store (e.g., $\gamma_0 < 0$), the social rationality condition, $u(q^\circ) \geq c(q^\circ)$, is violated at the steady state equilibrium. If transactions are socially rational, $\mu < \theta$ must hold (e.g., $\cup$-shape $\mathscr{F}$) and then $\mathscr{F}$ is partially decreasing in λ and θ and increasing in μ.*

Proof. See Appendix D.2. ■

Suppose $\mathscr{F}$ is $\cup$-shape and there is a stable equilibrium q^*_ω for a given parameter set ω (i.e., Figure 5). Using Figure 5, we can then characterize the Nash bargaining solutions in terms of some key parameters (e.g., λ, μ, and θ). The initial impact of urbanization raising λ reduces q^*_ω, but the subsequent secondary effect in the financial service provision raises q^*_ω as the service charge and the deposit rate decrease. If the initial effect is larger than the secondary effect, urbanization reduces the quantity of trade in each match, and the periodical volume of production (i.e., GDP per capita) is ambiguous. If the secondary effect is larger than the initial effect, the urbanization increases the quantity of trade in each match, and it increases the periodical volume of production unambiguously. In contrast, the initial impact of an increase of money μ increases q^*_ω but the second effect may decrease q^*_ω, as the financial service charge and the deposit rate are affected: for example, an increase of μ will reduce the deposit rate, but it may raise the financial service rate. Hence, an increase of μ reinforces the increase of q^*_ω, but the total effect is again ambiguous. An example will be shown through numerical simulations later.

2.4 Velocity of Money and Per-Capita GDP

The velocity (circulation speed) of money is observed as $\mathscr{V} = PY/M$, where PY denotes nominal GDP and M money supply (i.e., M1, M2, etc.). The velocity of money is also represented as $\mathscr{V} = y/m$ using real GDP per capita $y = Y/N$ and real money stock per capita $m = (M/P)/N$, where P is the GDP deflator, Y is the real GDP, and N is the population size. In a money-search model, m is represented by the population share of money holders μ, and y is represented by aggregate production per capita in one period, so that y is represented by

$$y = (1-b)^2 \lambda_0 \left\{ \alpha^2 \bar{q} + 2\alpha\mu (1-\alpha)(1-\mu) q_\omega^* \right\}. \tag{39}$$

In this calculation, ρz (payment to the financial service) is not included since it is an intermediate input. Then the velocity of money is given by

$$\mathscr{V} = \frac{(1-b)^2 \lambda_0 \left\{ \alpha^2 \bar{q} + 2\alpha\mu (1-\alpha)(1-\mu) q_\omega^* \right\}}{\mu}. \tag{40}$$

If we can ignore α^2 (or simply exclude barter trade, as it is empirically unobservable), we find

$$y = 2\alpha\mu (1-\alpha)(1-\mu)(1-b)^2 \lambda_0 q_\omega^*, \tag{41}$$

$$\mathscr{V} = 2\alpha (1-\alpha)(1-\mu)(1-b)^2 \lambda_0 q_\omega^*. \tag{42}$$

In the following discussions, we use the above definitions to observe GDP per capita and the velocity of money.

2.5 Numerical Simulations

There are three markets to solve the equilibrium: (1) barter trade, (2) monetary trade, and (3) banking. These three markets are mutually dependent on each other. In the model, endogenous variables are $q^d = \{q^\circ, \bar{q}\}$, $\tilde{z}$, $\tilde{\rho}$, and $\tilde{\delta}$. We solve these variables with respect to q° and then find the stable Nash bargaining solution. To run actual simulations, we need to provide utility function u and financial service productivity function ϕ. The utility function is provided in a CRRA form:

$$u(q) = \frac{u_0 q^{1-\rho}}{1-\rho}, \tag{43}$$

where u_0 is a constant coefficient and ρ is the rate of relative risk aversion. These parameters are set to be $u_0 = 100$ (long-run model) or 150 (short-run model) and $\rho = 0.15$. The financial service

Table 2: Environmental parameters

Money Supply (μ)$^\sharp$	0.2
Arrival Rate (λ_0)$^\sharp$	0.8
Bargaining Power	
Real goods trade (θ)	1
Banking for deposit (η)	1
Inflation Rate (γ)	0.05
Discount Rate (β)	0.05
Banking Sector Size (b)	0.3
Preference-Match Rate (α)	0.1

$^\sharp$ If the parameter is fixed.

factor productivity ϕ is provided by

$$\phi(z) = 1 - \exp(-z). \tag{44}$$

Thus, $\hat{\phi}(z) = \phi'(z)/\phi(z)$ is given by

$$\hat{\phi}(z) = \frac{\exp(-z)}{1 - \exp(-z)}. \tag{45}$$

Using these functional forms, we consider two models: the short-run model and the long-run model classified by the wage function $\tilde{w}(\tilde{h})$.

The wage function for the short-run model is given as a linear function such as $\tilde{w} = w_0\tilde{h}$, where w_0 is the base hourly wage of bankers. In this case, for example, we are looking at a sticky wage rate model or the neighborhood of an equilibrium. Then we find

$$\tilde{\rho} = \frac{w_0}{B^{1/\zeta}} \left(\frac{\tilde{z}}{\tilde{d}}\right)^{(1-\zeta)/\zeta} \frac{\eta}{\eta + \zeta - 1}. \tag{46}$$

Subsequently, from (17), we also find the deposit rate being

$$\tilde{\delta} = \frac{w_0}{B^{1/\zeta}} \left(\frac{\tilde{z}}{\tilde{d}}\right)^{1/\zeta} \frac{\eta(1-\zeta)}{\eta + \zeta - 1}, \tag{47}$$

where $\tilde{\delta}$ is the deposit rate computed from the expected values, $D = \tilde{D}$ and $Z = \tilde{Z}$. During simulations, we assume $B = 1$ and $\zeta = 0.5$. Other parameter values are given as listed in Table 2 (these values imply a $\cup$-shape $\mathscr{F}$).

For $\lambda_0 = 0.2, 0.3, \ldots, 0.9$, corresponding stable Nash bargaining solutions are depicted in Figure 6 (left). In this simulation, $w_0 = 1/2$ is applied. The corresponding velocity and deposit rate

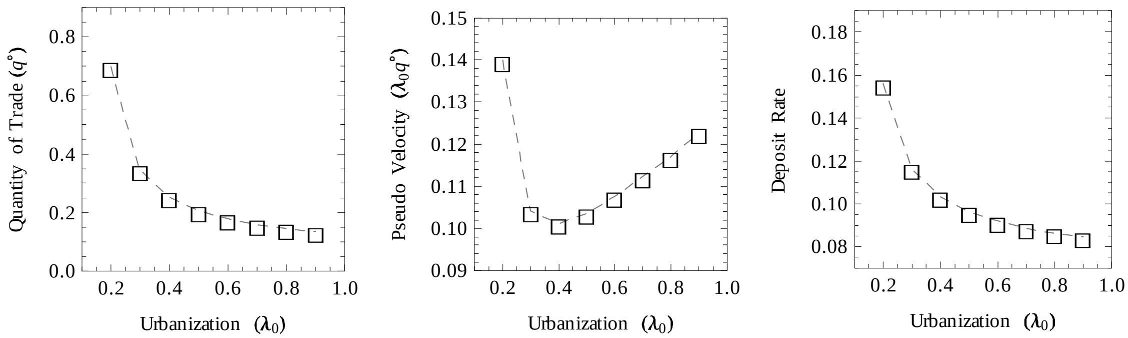

Figure 6: Short-run model ($w_0 = 0.5$, $0.2 \leq \lambda_0 \leq 0.9$)

are also shown in Figure 6 (center and right), where the velocity of money (42) is alternatively represented by $\lambda_0 q^*_\omega$, as its coefficient $2\alpha(1-\alpha)(1-\mu)(1-b)^2$ is constant (hence, it is marked as "pseudo" velocity).

From the charts in Figure 6, we find the quantity of trade in the stable Nash bargaining solution is decreasing in the arrival rate (i.e., urbanization) and that a part of the velocity of money (e.g., $0.2 \leq \lambda_0 \leq 0.4$) and the deposit rate are also decreasing in the arrival rate. Therefore, urbanization reduces the velocity of money and interest rates in the short-run. Note, the decline in the quantity of trade in the stable equilibrium is also seen in the non-extended Trejos-Wright model [31] (e.g., an increase in the circulation speed of money reduces its value of money). In addition, Lagos and Wright [18] imply a similar result, as they show the real interest rate is deflated by the arrival rate.

Proposition 3 *In short-run equilibrium, production level, the velocity of money, and real interest rates are negatively correlated with urbanization.*

For the long-rung model, the wage function is given as a convex form such as $\tilde{w} = w_0 \tilde{h}^2$; hence, the service charge and deposit rate are

$$\tilde{\rho} = \frac{w_0 \tilde{d}}{bB^{2/\zeta}} \left(\frac{\tilde{z}}{\tilde{d}}\right)^{(1-\zeta)/\zeta} \frac{\eta}{\eta + \zeta - 1} \quad \Rightarrow \quad \tilde{\delta} = \frac{w_0 \tilde{d}}{bB^{2/\zeta}} \left(\frac{\tilde{z}}{\tilde{d}}\right)^{1/\zeta} \frac{\eta(1-\zeta)}{\eta + \zeta - 1}. \tag{48}$$

For our simulation, we use the same parameter values, except w_0, so as to show an interesting case. As discussed in Section 2.2.2, the wage function affects the outcome whether or not an increase of arrival rate increases or decreases the interest rates, so that, we set $w_0 = 3$ to clarify the impact of the wage function. Then, for $\lambda_0 = 0.1, 0.2, \ldots, 0.8$, the result is as shown in Figure 7.

In the charts in Figure 7, we find cases opposite to the short-run (and the non-extended Trejos-Wright model [31]) as shown in Figure 7 (left). In such cases, an increase in the arrival rate

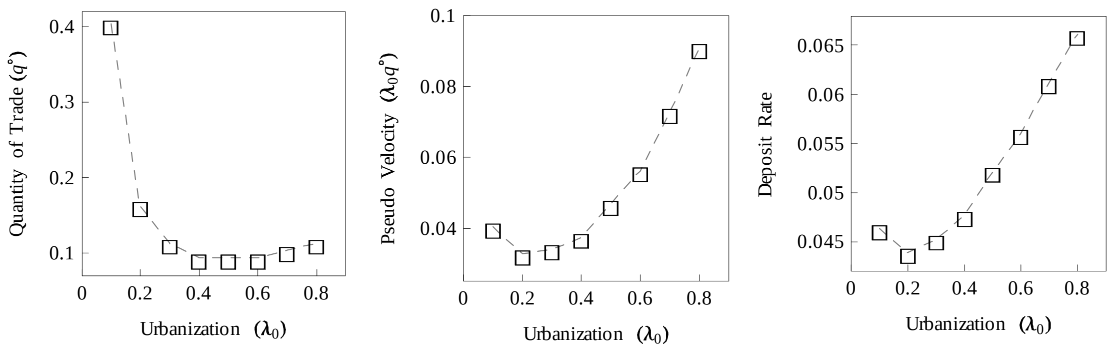

Figure 7: Long-run model ($w_0 = 3$, $0.1 \leq \lambda_0 \leq 0.8$)

increases the quantity of transaction in a stable Nash bargaining solution. Accordingly, the velocity of money and deposit rate are increased by urbanization, as shown in Figure 7 (center and right). Therefore, urbanization "can" increase the velocity of money, the quantity of transaction per trade, and interest rates. Note that the increase in the interest rate is caused by a smaller supply of deposit rate relative to the demand for financial services. This result cannot be derived from the Lagos-Wright model [18]. Since $\tilde{h}$ is an indicator of financial intermediation activities, the next proposition summarizes the result for a long-run equilibrium.

Proposition 4 *In a long-run equilibrium, production level, velocity of money, and real interest rates are positively correlated with urbanization if there are enough financial intermediation activities.*

For reference, the columns in Table 3 show estimations for the deposit rate and lending rate based on equations (47), (46), and (17), where (17) is solved as $\tilde{\delta} = \tilde{\rho}\,(1-\zeta)\,\tilde{z}/\tilde{d}$. In these regressions, the deposit is measured by percent of saving in GDP, and the provision of financial services is measured by percent of quasimoney in GDP. All values are in logarithm values. The deposit and lending rates are nominal rates, so that, inflation (logarithm of $\mathrm{CPI}_t/\mathrm{CPI}_{t-1}$) is included; hence, the rest of the right-hand-side variables estimate the real interest rates. The time indicator estimates the averaged residuals for each period (i.e., B, ζ, η, etc.). In the regressions, saving is not independently included as quasimoney, and M1 preclude it; hence, $\tilde{z}$ and $\tilde{d}$ are considered as functions of these monetary variables, and estimations are reduced forms.

The results in Table 3 show that the urban population share (an urbanization indicator) is positively correlated with the deposit and lending rates. In our model, that implies that $\tilde{w}$ is convex in banker's working hours (in the long-run model) and its impact is large. Interestingly, the significance of the urbanization indicator is held in the estimations based on equations (47) and (46),

Table 3: Reduced form fixed effect estimations of deposit and lending rates

	Deposit Rate		Lending Rate		Deposit Rate	
Urban Pop.	0.8618**	(0.2658)	0.4642**	(0.1643)	0.2878	(0.1802)
Inflation	1.3480**	(0.1801)	0.9555**	(0.1518)	0.0068	(0.0694)
M1[a]	-0.0591†	(0.0356)	-0.2838**	(0.0449)	0.0994†	(0.0539)
Quasimoney[b]	0.5184**	(0.1218)	0.0816	(0.0638)	0.2299**	(0.0856)
Lending Rate	—	—	—	—	1.1294**	(0.0771)
Time	-1.7311**	(0.2388)	-0.2291	(0.1388)	-1.3740**	(0.1577)
Intercept	4.4966**	(0.4305)	4.8816**	(0.3105)	-0.2767	(0.5130)
Adj. R-sq.	0.4621		0.4654		0.6965	
N	3,477		3,224		3,029	

[a] Log of M1 per capita (2005 USD); [b] Log of % of quasimoney in M2
(Source: World Development Indicators 2009)

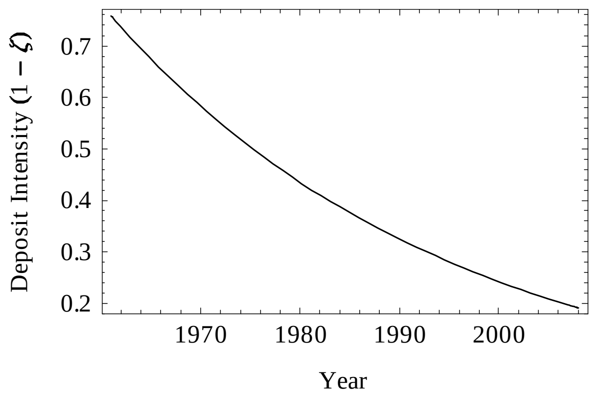

Figure 8: Estimated deposit intensity and its evolution

which are the first and second columns, whereas it is lost in the estimation based on equation (17). Furthermore, the results suggest statistically significant negative correlations between saving and interest rates. However, we can find a positive correlation between interest rates and financial services (quasimoney) only in the relationship with the deposit rate. There are two possible reasons for this (unless the model is incorrect): one is the power of quasimoney as a proxy for financial services and the other is the variance in the lending rate with respect to lenders.

For reference, the estimated time indicator and intercept in the third equation in Table 3 correspond to the logarithm of $1-\zeta$ in (17). Then, we can find the locus of deposit intensity, which is computed as $1-\zeta = \exp[\ln(1-\zeta)]$, as depicted in Figure 8. Since $1-\zeta$ declines over time (e.g., from 0.8 in 1961 to 0.2 in 2008), we see the importance of other production factors (e.g., physical capital, human capital, information technology, etc.) increasing in the banking sector. Hence, smaller deposits than before are now required to provide the same degree of financial service. Here, there is no role for urbanization, as is also shown by our theory and estimate.

Next, we see the relationship between GDP per capita (y) and the velocity of money ($\mathscr{V}$). If

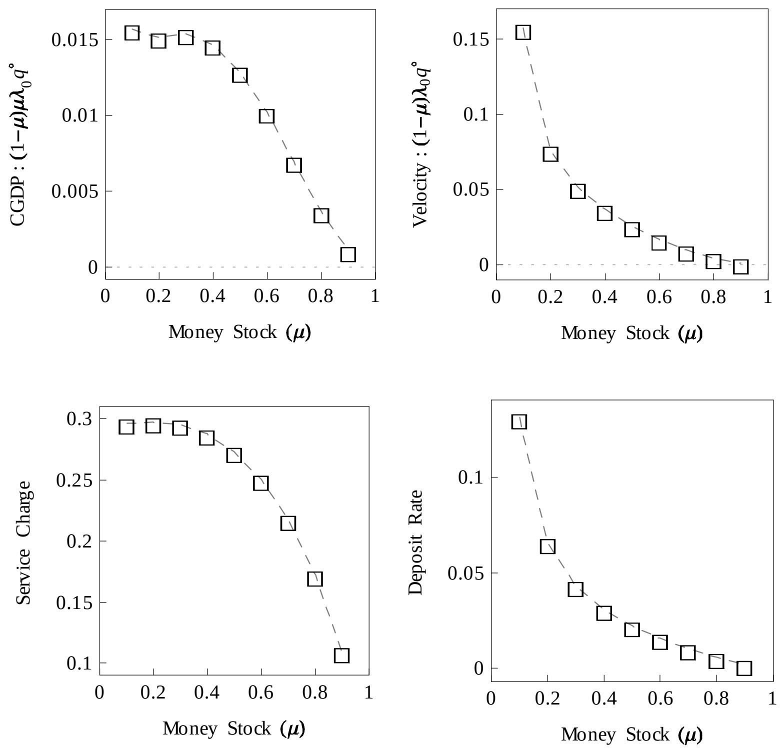

Figure 9: Changes in key variables to μ

μ is fixed, as (41) and (42) show, the directions of the changes in y and $\mathscr{V}$ are the same, so we alter μ. For the parameter values shown in Table 2, the result in the long-run model ($w_0 = 3$) is shown by Figure 9 (upper row). As Figure 9 (lower row) shows, the financial service charge and deposit rate are hump-shaped and monotonically decreasing in μ, respectively. As it is also shown later in regressions, especially the result that shows negative correlation between money supply and interest rates is consistent with the actual observation (e.g., the negative correlation between money supply, velocity of money, and interest rate).

From these numerical simulations, in long-run, we can see that a progress of urbanization increases velocity of money, per-capita GDP, and interest rates. Since an increase in the per-capita GDP is associated with an increase in the per-capita money holding, as our heuristics. The simulation also show that an increase in money reduces GDP per capita in most part and velocity of money. These previous results are guides for empirical studies to see partial effects of each variable (i.e. 2SLS). If λ_0 and μ is correlated, which will correspond to a reduced form estimation, the result will be more complex. From the WDI, a between-effect estimation[7] provides $\hat{\mu} = 1.78\hat{\lambda}_0$, where $\hat{\mu} = d\mu/\mu$ and $\lambda_0 = d\lambda_0/\lambda_0$ are percent-change in M2 per capita and urban population share,

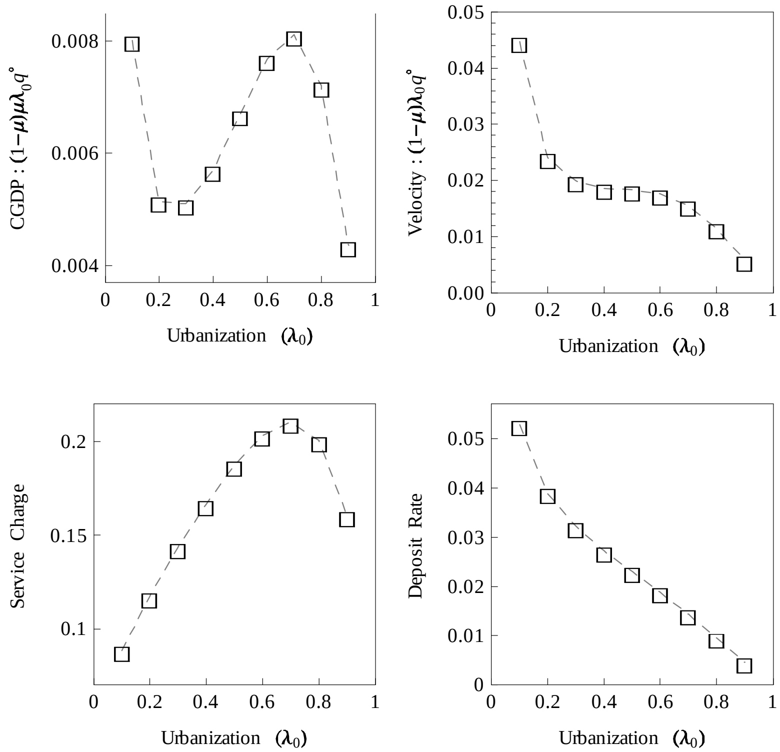

Figure 10: Changes in key variables to λ_0 with μ positively correlated with λ_0

respectively. Using this estimate, we assume

$$\mu = 0.15 \exp(1.78\lambda_0) \in [0.15, 0.89]. \tag{49}$$

The simulation result using the relationship between μ and λ_0 as (49) is shown in Figure 10. The first row of the figure shows that a progress of urbanization can raise GDP per capita while it reduces velocity of money. Then, the second row shows that a progress of urbanization initially increases financial service charge while it reduces deposit rate. The results in the first row are very close to our real world observation in reduced form: urbanization is positively correlated with GDP per capita and negatively correlated with velocity of money. The simulation gives unclear effects of interest rates (deposit rate is decreasing while financial service charge is hump-shaped). However, its intuition is clear: a progress of urbanization initially increases frequency of all matching to reduce the deposit rate and to increase the demand for financial services to increase the service charge. If the urbanization exceeds a certain level, a decline in quantity of trade reduces demand for financial services to reduce both deposit and financial service charge.

3 Empirical Investigations in Velocity

3.1 Empirical Strategy and Results

We consider the simultaneous equations model to estimate the velocity of money, $\mathscr{V} = y/\mu$, using the World Development Indicators 2009 (WDI) published by the World Bank (cross-country unbalanced panel data between 1961 and 2009). The model is estimated by the two-stage least square method (2SLS). In the model, real GDP per capita y and real money supply (M1) per capita, which is the proxy for μ, are endogenous variables; and capital stock and money supply in the previous period, current price level, inflation rate (logarithm of $\mathrm{CPI}_t/\mathrm{CPI}_{t-1}$), and current population are exogenous variables. The M1 per capita of country i in period t is converted into per-capita real USD (2005 basis) as

$$\mu_{it} = \frac{e_{it}\pi_t M_{it}}{N_{it}}, \tag{50}$$

where e_{it} is the exchange rate (local currency unit per dollar), and π_t is the consumer price index of the United States in period t. In order to include the impact of quasimoney, we also include the share of quasimoney in M2 as an endogenous variable. In addition, we include deposit rate and lending rate in one of two models, but the inclusion of interest rates reduces the sample size from ca. 3,300 to ca. 2,200 (we refer to the model including the interest rates as Model 2 and the other as Model 1). If deposit rate and lending rate are included, deposit rate is regarded as endogenous and regressed on lending rate based on a reduced form expression (17). If deposit and lending rates are not included, we use the growth rate of capital stock for the proxy of those interest rates (e.g., another reduced form). To gain efficiency in estimation, the total factor productivity is given by a time indicator instead of a year-dummy. The detailed model structure and result are as stated as follows.

For the first stage regression, taking a logarithm of (41), we obtain

$$\ln y = \ln 2\alpha\left(1-\alpha\right)\left(1-b\right)^2 + \ln\mu\left(1-\mu\right) + \ln\lambda_0 + \ln q_\omega^*, \tag{51}$$

where $\ln 2\alpha\left(1-\alpha\right)\left(1-b\right)^2$ is considered as a part of the constant term because these parameters are unobservable in the data. From (4), $\ln q_\omega^*$ is given by

$$\ln q_\omega^* = \ln\phi\left(z\right) + \ln f\left(k\right), \tag{52}$$

where z is endogenously determined by the financial service market. For simplicity, we consider a reduced form for the financial service market; hence, z is a function of relevant variables, $z = z\left(\bullet\right)$, such as money supply, current price level, inflation rate, and interest rate. Quasimoney (share of quasimoney in M2) and M1 per Cap. are endogenous variables depending on previous period's

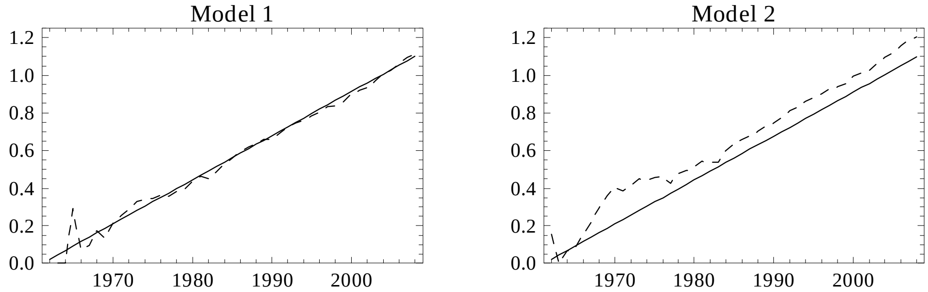

Figure 11: Estimated total factor productivities of the world

quasimoney and M1 per capita, and real side economy variables of current and previous periods. The inflation rate is given by $\ln \mathrm{CPI}_{it} / \ln \mathrm{CPI}_{it-1}$,where CPI_{it} is the price level (consumer price index) of country i in period t. All variables are self-explanatory logarithm values extracted from the WDI. Then we obtain Tables 4 and 5 (L1 indicates one-period lag variable and D1 one-period difference). In Table 4 (Model 1), interest rates (lending and deposit rates) are not included in the estimation so as to maintain the sample size. In Table 5 (Model 2), deposit rate is included as an endogenous variable (reduced form $z = z(\bullet)$ is applied). Note that saving is not included as a right-hand-side variable of the first stage regression of Model 2 because Quasimoney precludes it (M1 also includes deposit account). For reference, Figure 11 shows the estimates of logarithm of TFPs of the respective models (linear line) and corresponding estimates using a year-dummy (dashed lines), where the TFP in 1961 is one (thus its logarithm is zero). As this figure shows, we find fair correspondences between the TFPs estimated from the time-indicator and year-dummy estimations.

The second stage regression is based on the definition of the velocity of money. By taking the logarithm of $\mathscr{V} = y/\mu$, we find

$$\ln \mathscr{V} = \ln \bar{y} - \ln \mu, \tag{53}$$

where $\bar{y}$ is the estimate of y that is theoretically given by

$$\bar{y} = 2\alpha\mu (1-\alpha)(1-\mu)(1-b)^2 \lambda_0 \bar{q}^*_\omega. \tag{54}$$

Thus there are direct and indirect effects of μ and λ_0 (α and b are given), where the indirect effect theoretically emanates from $\bar{q}^*_\omega$. Since $\bar{q}^*_\omega$ is unobservable, we take $\bar{y}$ as the proxy for $\bar{q}^*_\omega$. Then, taking the direct effect into account, we obtain the regression results shown in Table 6 for M1 and M2 money measures (GDP per Cap., M1 per Cap., Quasimoney and Deposit Rate are endogenous variables and are the first stage estimates).

In the first-stage regressions tables (Tables 4 and 5), Angrist-Pischke chi-squared and F sta-

Table 4: First stage IV estimations for Model 1

	GDP per Cap.		Quasimoney		M1 per Cap.	
	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.
Urban Pop.	0.3126**	(0.0420)	-0.0188	(0.0260)	-0.0084	(0.0578)
CPI	-0.0174**	(0.0025)	0.0065**	(0.0017)	0.0019	(0.0052)
Inflation	-0.0010	(0.0115)	0.0153	(0.0226)	-0.0270	(0.1684)
M1 per Cap.	—	—	—	—	—	—
(L1)	0.0166**	(0.0045)	-0.0050	(0.0059)	0.8884**	(0.0846)
Quasimoney	—	—	—	—	—	—
(L1)	0.0304	(0.0372)	0.8457**	(0.0315)	0.0287	(0.0291)
Cap. Sock	—	—	—	—	—	—
(L1)	0.2816**	(0.0190)	0.0374**	(0.0109)	0.0727	(0.0553)
(D1)	0.0369	(0.1067)	-0.0012	(0.0215)	0.1626**	(0.0457)
Population	-0.9562**	(0.0343)	0.0855**	(0.0328)	-0.4220†	(0.2260)
Time	1.1001**	(0.0591)	-0.1483**	(0.0319)	0.3237*	(0.1582)
Adj. R-sq.	0.6384		0.8220		0.8371	
N	3,288		3,288		3,288	
F	487.56**		502.71**		800.45**	
AP Chi-sq.	1145.65**		1727.42**		121.06**	
AP F	571.19^{a}		861.25^{a}		60.36^{a}	

Table 5: First stage IV estimations for Model 2

	GDP per Cap.		Quasimoney		M1 per Cap.		Deposit Rate	
	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.
Urban Pop.	0.3294**	(0.0504)	-0.0146	(0.0319)	0.0536	(0.0504)	0.1073	(0.1068)
CPI	-0.0163**	(0.0045)	0.0051	(0.0048)	-0.0178*	(0.0075)	0.0164	(0.0113)
Inflation	0.0014	(0.0187)	-0.0172	(0.0536)	-0.1443*	(0.0606)	-0.0321	(0.0650)
M1 per Cap.	—	—	—	—	—	—	—	—
(L1)	0.0952**	(0.0158)	0.0188†	(0.0101)	0.8819**	(0.0207)	0.1574**	(0.0259)
Quasimoney	—	—	—	—	—	—	—	—
(L1)	0.1469**	(0.0137)	0.8537**	(0.0370)	0.0447	(0.0298)	0.1224**	(0.0405)
Cap. Sock	—	—	—	—	—	—	—	—
(L1)	0.1696**	(0.0172)	0.0239*	(0.0094)	0.0155	(0.0159)	0.0777*	(0.0313)
Population	-0.7839**	(0.0509)	0.1335**	(0.0496)	-0.2244*	(0.0896)	0.4858**	(0.1569)
Lending Rate	-0.0139	(0.0107)	0.0220	(0.0148)	-0.0716**	(0.0230)	1.1640**	(0.0421)
Time	1.0968**	(0.0544)	-0.2194**	(0.0488)	0.4689**	(0.0849)	-1.8730**	(0.1782)
Adj. R-sq.	0.7543		0.7522		0.8658		0.7064	
N	2,190		2,190		2,190		2,190	
F	258.11**		204.29**		1260.33**		231.14**	
AP Chi-sq.	264.85**		804.18**		1175.01**		777.87**	
AP F	131.84^{a}		400.33^{a}		584.93^{a}		387.23^{a}	

tistics (abbreviated as AP Chi-sq. and AP F, respectively) provide underidentification and weak identification tests for each endogenous regressors. The AP Chi-sq. is compared with an ordinary corresponding chi-squared value, and its significance of rejection is as reported. The AP F is to be compared with the corresponding critical value of Stock and Yogo [30, Tables 1 and 2, pp. 58-59] (compare the values when the number of the included regressor, n, is one). Both AP Chi-sq. and AP F reject the null hypotheses, and we can say that these identification tests are clear for all endogenous regressors. In addition, Tables 4 and 5 report an adjusted R-squared and F test statistic for the joint significance of instrument variables. The adjusted R-squared scores of all first stage regressions are not small, and all F test statistics that reject the instruments are jointly insignificant; hence, all regressions and excluded instrument variables are meaningful in their espective first-stage regressions.

In the second-stage regressions table (Table 6), the Kleibergen-Paap Lagrange multiplier statistics (KP LM) tests underidentification jointly across endogenous regressors. According to the table, all second-stage regressions are not underidentified. Weak identifications are tested by the Cragg-Donald Wald and Kleibergen-Paap F statistics (abbreviated as CD F and KP F, respectively). The critical values again rely on Stock and Yogo [30, Tables 1 and 2, pp. 58-59]. All the obtained values for testing weak-identification will reject the null hypotheses both in a 5% relative IV bias and a 10% maximal size (the significance level is 5%). The robustness of the endogenous regressors to weak-identification is jointly tested by the Anderson-Rubin chi-squared (or alternatively F) and the Stock-Wright S statistics (abbreviated as AR Chi-sq., AR F, and SW LM, respectively). These statistics show the robustness of the endogenous regressors to the existence of weak-identification. Overidentification is tested by Hansen's J statistic (abbreviated by Hansen J). Its null hypothesis is the non-existence of overidentification, and reported values cannot reject the null hypothesis; hence, the estimated models cannot be said to be overidentified.

To see the difference between IV estimation and OLS estimation, Table 7 shows the reduced form OLS estimations for Models 1 and 2. In these OLS regressions, especially, we cannot find any significance of urban population share. Table 8 shows the total effect of urban population share to examine the consistency between OLS and IV estimations. We can find consistency only in M2 velocity. Note that Hromcová [15] uses a cash-in-advance model to consider a positive correlation between the M1 velocity and GDP observed in the United States and suggests the positive correlation is caused by the accumulation of human capital in the banking sector. Ireland [16] proposes a model that shows a positive correlation between the mney velocity and GDP, and Benk et al. [3] considers a model that supports Hromcová's argument [15].

Table 6: Second stage IV estimations

	Model 1				Model 2			
	M1 Velocity		M2 Velocity		M1 Velocity		M2 Velocity	
	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.
Urban Pop.	0.1922**	(0.0579)	0.1968**	(0.0556)	0.0451	(0.0651)	0.1977**	(0.0790)
CPI	0.0372**	(0.0033)	0.0138**	(0.0030)	0.0382**	(0.0105)	0.0210	(0.0142)
Inflation	-0.0195	(0.0280)	-0.0148	(0.0348)	-0.0684	(0.0562)	0.0185	(0.0707)
Cap. Stock	—	—	—	—	—	—	—	—
(D1)	-0.0152	(0.0336)	-0.0061	(0.0448)	—	—	—	—
GDP per Cap.	-0.1268**	(0.0414)	-0.4280**	(0.0409)	0.2434**	(0.0874)	-0.5588**	(0.0949)
M1 per Cap.	-0.0660**	(0.0105)	-0.0447**	(0.0082)	-0.2965**	(0.0315)	-0.0793*	(0.0309)
Quasimoney	0.3576**	(0.0270)	-0.2175**	(0.0326)	0.2729**	(0.0373)	-0.3218**	(0.0368)
Deposit Rate	—	—	—	—	0.0648**	(0.0174)	-0.0008	(0.0212)
Time	-0.4877	(0.0530)	-0.4188	(0.0542)	-0.5772**	(0.0781)	-0.3496**	(0.0971)
Adj. R-sq.	0.3167		0.3807		0.6025		0.3954	
N	3,288		3,288		2,190		2,190	
KP LM	338.228**		338.228**		168.74**		168.74**	
CD F	384.023^{b}		384.023^{b}		129.18^{b}		129.18^{b}	
KP F	221.075^{c}		221.075^{c}		49.91^{c}		49.91^{c}	
AR F	77.97**		67.44**		75.33**		117.49**	
AR Chi-sq.	312.77**		270.54**		378.28**		590.04**	
SW S	260.67**		236.49**		264.00**		284.45**	
Hansen J	1.632		0.119		0.667		0.016	

Table 7: Non-IV estimations

	Model 1				Model 2			
	M1 Velocity		M2 Velocity		M1 Velocity		M2 Velocity	
	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.
Urban Pop.	0.1758	(0.1827)	0.1787	(0.1764)	0.0032	(0.1550)	0.0859	(0.1639)
GDP per Cap.	0.0646	(0.0620)	-0.1728**	(0.0651)	0.5465**	(0.1225)	0.0667	(0.1059)
CPI	0.0432**	(0.0120)	0.0222**	(0.0084)	0.0394†	(0.0205)	0.0260	(0.0254)
Inflation	-0.0065	(0.0416)	0.0021	(0.0521)	-0.0465	(0.0608)	0.0220	(0.0766)
Cap. Stock	—	—	—	—	—	—	—	—
(D1)	0.0068	(0.0246)	0.0226	(0.0293)	—	—	—	—
M1 per Cap.	-0.0792†	(0.0414)	-0.0643*	(0.0251)	-0.3797**	(0.0760)	-0.2216**	(0.0617)
Quasimoney	0.3583**	(0.0586)	-0.2270**	(0.0800)	0.2460**	(0.0797)	-0.3906**	(0.0754)
Deposit Rate	—	—	—	—	0.0485*	(0.0230)	0.0040	(0.0304)
Time	-0.6729**	(0.1465)	-0.6629**	(0.1312)	-0.7853**	(0.1323)	-0.7532**	(0.1500)
Adj. R-sq.	0.3636		0.4375		0.6464		0.5225	
N	3,288		3,288		2,190		2,190	

Table 8: Total effect of urban population share

	M1 Velocity		M2 Velocity	
	Total Effect	S.E.	Total Effect	S.E.
Without Deposit Rate	0.1464**	(0.0492)	0.0675	(0.0514)
(Smaller Sample)	0.1797**	(0.0572)	0.0210	(0.0648)
With Deposit Rate	0.1124*	(0.0573)	0.0140	(0.0663)

3.2 Inference

To derive inferences, we rely on regressions for M2 velocity because there are inconsistencies in the regressions for M1 velocity in Models 1 and 2 (Table 6) as well as in total effect and OLS results (Tables 7 and 8). The use of M2 is also supported by the initial assessment of the use of the second-generation money-search model; hence, a reasonable explanation of the loss of robustness in the M1 velocity regressions is the incomprehensive inclusion of bank accounts.

In the first-stage regressions, urban population share affects only GDP per capita in a statistically significant way. The impact of an increase in urban population share is then positive. In the second-stage regressions, in both Models 1 and 2, urban population share affects the M2 velocity statistically significantly. The impact of an increase in urban population share is then positive. These results (the partial effects of urbanization) can be explained by the theoretical long-run model, as shown in Figure 7.[8]

The impact of an increase in GDP per capita in M2 velocity is negative. This is consistent with our heuristics. In the theory, the negative correlation between the velocity of money and GDP per capita can occur only when the money stock changes. According to the first-stage regressions, GDP per capita is positively associated with the money stocks (M1 per capita and quasimoney-M2 ratio). In the long-run model, we can find a segment where GDP per capita and the velocity of money are negatively correlated with each other when the money stock increases. For example, an increase in money from $\mu = 0.2$ to 0.3 or 0.4 in Figure 9 produces an increase in GDP per capita, a decrease in velocity of money, and a negative correlation between GDP per capita and the velocity of money.

From these observations, we can say that urbanization increases both GDP per capita and the velocity of money and that GDP per capita reduces the velocity of money. The positive correlation between economic development (GDP per capita) and urbanization can be explained by the Schumpeterian hypothesis (Schumpeter [27]): urbanization stimulates interactions among entrepreneurs and investors to create innovations (see, for example, Salop [26], Helsley and Strange [13, 14], Berliant et al. [4], and Zhang [35]), which is an engine of growth. In addition, urbanization can increase the number of participants in the consumption market, and it increases the chance of a coincidence of wants (for example, Ellison et al. [8] and Virág [33] consider a similar situation in auctioning markets). On one level, therefore, these increased transaction activities increase the velocity of money (e.g., the positive effect of urban population share in M2 velocity).

Urbanization affects M2 velocity indirectly via GDP per capita, which is the cause of the ambiguous result in the reduced form estimation (Tables 7 and 8). According to the theory, (41) and (42), the only source of negative correlation between GDP per capita and velocity of money is a change of real money per capita. If $\mu < 0.5$, an increase of per-capita money (a form of financial deepening) increases the per-capita GDP while it reduces the velocity of money. This mechanism works in the theoretical model *as if* the McKinnon-Shaw hypothesis in the advanced stage of modern economic development (McKinnon [22] and Shaw [28]).[9] In that case, the empirical result and the theory become consistent with each other. Along with the positive direct effect of urbanization on velocity, therefore, financial deepening accompanied by economic development reduces M2 velocity, which results in the negative impact of urbanization on M2 velocity.

In these two major results, we find a coexistence of the Schumpeterian and McKinnon-Shaw hypotheses: urbanization stimulates entrepreneur-investor matching, employer-worker matching, and seller-buyer matching. The increase in interactions stimulates economic development. However, stimulated economic development faces a shortage of money; the financial service fee and deposit rate will thus increase. In theory, this is an increase in the banker's wage rate caused by a smaller input of deposit. In the real world, an increased deposit rate will attract more deposits (note that, in the theoretical model, the deposit rate is determined by bargaining). In addition, the shortage of money will stimulate developments in financial products. The economic development (stimulated by urbanization) induces developments in financial products and more deposit and saving; therefore, the velocity declines, as implied by McKinnon [22] and Shaw [28].

In the previous section concerning theory, Table 3 has shown positive correlations between urban population share and nominal interest rates. The theoretical model suggests that urbanization creates a shortage of money and accelerates circulation speed (*a la* Figure 7). This proposition is reinforced by the negative correlation between M1 per capita and nominal interest rates in the same regression table. The positive correlations between quasimoney and nominal interest rates indicate that an increase in the interest rate resulting from a larger demand for money increases the supply of quasimoney.

3.3 Robustness

We now consider the robustness of the regressions. Although the inferences are derived from the regressions on M2 velocity, we also see the robustness of regressions on M1. By doing so, we can check the robustness of rejecting the use of M1 velocity as well as finding the cause of the inconsistency between the M1 velocity estimations of Models 1 and 2.

First, we consider the robustness of the reduction in sample size in Model 1. The sample size of Model 1 is 3,288 and that of Model 2 is 2,190. The aim of this robustness check is to see the difference in results between Model 1 and Model 2 and to check if the sign and significance of GDP per capita and urban population share emanate from the choice of variable or the sample size. The results are shown in Table 9. According to Table 9, the reduction of the sample size in Model 1 generates results similar to the Model 2 regressions except for an increase in the significance of urban population share in M1 velocity. This implies that the results in M1 velocity are unstable. However, we also find that the result in M2 velocity is quite robust to the sample size.

Next, we consider an inclusion of population density. As stated in the introduction, the definition of urban area differs by country. When including population density in the regression, we should note that (the logarithms of) population density and population have a one-to-one correlation:

$$\underset{(S.E.)}{\text{Pop. Density}} = \underset{(0.0004)}{1.0001}^{**}\text{Population} - \underset{(0.0065)}{11.68}^{**} + a_i + \varepsilon_{it}, \tag{55}$$

where a_i is the country-specific term, ε_{it} is the error (reported standard errors are heteroskedasticity robust errors). All values are logarithm values; R-squared is 0.9994, and N is 5,816. The relationship and significance levels (e.g., p-values and R-squared) are not altered by the change in samples. In addition, we have to be careful to include population density in place of population, as the first stage regression is based on the aggregate production function. Thus, population density is included as an endogenous regressor. The second stage regressions are shown in Table 10. As this figure shows, the inclusion of population density does not alter the overall result.[10] In addition, population density has less impact than urban population share. Therefore, the benchmark results are not altered by the inclusion of population density.

The argument assumes that the relationship between (the logarithm of) velocity of money and GDP per capita is linear. However, the McKinnon-Shaw hypothesis predicts a hump-shape relationship between the two parameters (see, for example, Bordo et al. [7] for such a hump-shape result in a much earlier period's data set); hence, the simplest relationship is given as a quadratic form. The methodology of instrumental variable estimation of quadratic forms is not well established. Thus, we cannot solely rely on the IV estimation of the quadratic endogenous variable, though it is easily estimated. However, we can refer to such estimation when discussing the feasibility of linearity assumption.

To estimate the quadratic form, we consider the centered logarithm of GDP per capita. The use of a centered variable reduces the correlation between first order and second order polynomials; hence, the correlation between the two endogenous variables. In the first regression, we include second order and cross terms of all exogenous and instrument variables, except for the time indicator, to estimate endogenous variables, including the second order term of GDP per capita. Then

Table 9: IV estimations of Model 1 using the same sample (smaller sample) as Model 2

	First Stage Regressions						*Second Stage Regressions*			
	GDP per Cap.		Quasimoney		M1 per Cap.		M1 Velocity		M2 Velocity	
	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.
Urban Pop.	0.3156**	(0.0505)	-0.0084	(0.0332)	0.0237	(0.0517)	0.1278*	(0.0658)	0.1866**	(0.0777)
CPI	-0.0185**	(0.0044)	0.0060	(0.0045)	-0.0214**	(0.0077)	0.0384**	(0.0104)	0.0197	(0.0140)
Inflation	-0.0028	(0.0147)	0.0040	(0.0395)	-0.2045**	(0.0469)	0.0014	(0.0393)	0.0159	(0.0589)
Cap. Sock	—	—	—	—	—	—	—	—	—	—
(L1)	0.1948**	(0.0205)	0.0207*	(0.0101)	0.0526**	(0.0178)	—	—	—	—
(D1)	0.0963**	(0.0334)	-0.0088	(0.0133)	0.1257**	(0.0294)	0.0004	(0.0300)	0.0307	(0.0311)
Population	-0.8210**	(0.0499)	0.1716**	(0.0522)	-0.3417**	(0.0903)	—	—	—	—
GDP per Cap.	—	—	—	—	—	—	0.1950*	(0.0885)	-0.5265**	(0.0919)
M1 per Cap.	—	—	—	—	—	—	-0.3034**	(0.0331)	-0.0919**	(0.0305)
(L1)	0.0897**	(0.0149)	0.0161	(0.0099)	0.8839**	(0.0203)	—	—	—	—
Quasimoney	—	—	—	—	—	—	0.2988**	(0.0377)	-0.3276**	(0.0356)
(L1)	0.1463**	(0.0136)	0.8554**	(0.0372)	0.0375	(0.0303)	—	—	—	—
Time	1.1116**	(0.0550)	-0.2590**	(0.0502)	0.5550**	(0.0860)	-0.6719**	(0.0751)	-0.3610**	(0.0897)
Adj. R-sq.	0.7649		0.7512		0.8645		0.5917		0.4134	
N	2,146		2,146		2,146		2,146		2,146	
F	283.25**		255.55**		1520.99**		—		—	
AP Chi-sq.	251.99**		993.40**		1140.17**		—		—	
AP F	125.44[a]		494.50[a]		567.56[a]		—		—	
KP LM	—		—		—		129.70**		129.70**	
CD F	—		—		—		190.15[b]		190.15[b]	
KP F	—		—		—		56.74[c]		56.74[c]	
AR F	—		—		—		89.67**		146.97**	
AR Chi-sq.	—		—		—		360.27**		590.48**	
SW S	—		—		—		249.77**		276.78**	
Hansen J	—		—		—		0.335		0.254	

Table 10: IV estimations including population density

	Model 1				*Model 2*			
	M1 Velocity		M2 Velocity		M1 Velocity		M2 Velocity	
	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.
Urban Pop.	0.2501**	(0.0712)	0.2175**	(0.0679)	0.0838	(0.0762)	0.1924*	(0.0923)
CPI	0.0355**	(0.0035)	0.0130**	(0.0034)	0.0365**	(0.0105)	0.0211	(0.0140)
Inflation	-0.0204	(0.0282)	-0.0158	(0.0350)	-0.0685	(0.0569)	0.0155	(0.0715)
Cap. Sock	—	—	—	—	—	—	—	—
(D1)	-0.0203	(0.0376)	-0.0079	(0.0465)	—	—	—	—
Pop. Density	-0.0978	(0.0817)	-0.0233	(0.0838)	-0.0813	(0.1147)	0.0202	(0.1335)
GDP per Cap.	-0.1815**	(0.0617)	-0.4464**	(0.0637)	0.1863†	(0.1078)	-0.5508**	(0.1311)
M1 per Cap.	-0.0649**	(0.0105)	-0.0441**	(0.0084)	-0.2923**	(0.0323)	-0.0781*	(0.0330)
Quasimoney	0.3684**	(0.0296)	-0.2142**	(0.0367)	0.2860**	(0.0416)	-0.3230**	(0.0429)
Deposit Rate	—	—	—	—	0.0651**	(0.0180)	0.0003	(0.0219)
Time	-0.3719**	(0.1113)	-0.3841**	(0.1113)	-0.4711**	(0.1557)	-0.3678*	(0.1828)
Adj. R-sq.	0.3060		0.3754		0.5970		0.3951	
N	3,271		3,271		2,163		2,163	
KP LM	168.931**		168.931**		121.813**		121.813	
CD F	198.265^{b}		198.265^{b}		79.333^{b}		79.333^{b}	
KP F	42.193^{c}		42.193^{c}		18.323^{c}		18.323^{c}	
AR F	78.43**		68.38**		75.34**		116.98**	
AR Chi-sq.	314.63**		274.32**		378.39**		587.49**	
SW S	262.35**		238.29**		264.26**		281.11**	
Hansen J	Exactly Identified		Exactly Identified		Exactly Identified		Exactly Identified	

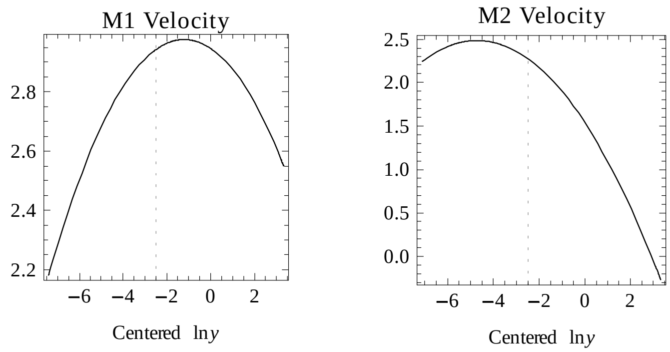

Figure 12: Quadratic form estimation for GDP per capita for Model 1

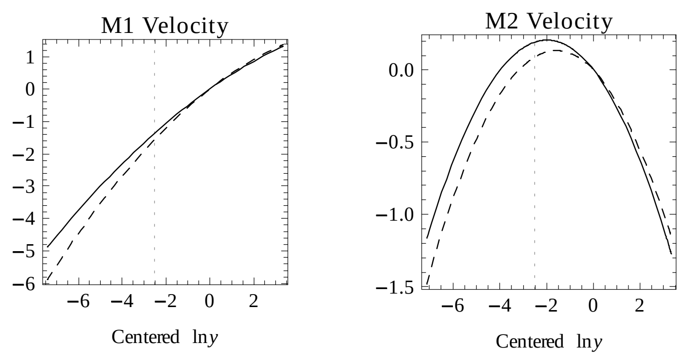

Figure 13: Quadratic form estimation for GDP per capita for Model 2

we estimate the second stage regression using the estimates of endogenous variables. The results for the quadratic form in Models 1 and 2 are shown in Figures 3.3 and 12, respectively,[11] where the dotted grid lines indicate respective 1% percentile values of centered logarithm of per-capita GDP (see Table 11 for 1%, 5%, and 10% percentile values in each sample). In addition, the dashed curves in Figure 12 indicates the estimates of quadratic forms for Model 1 using the same sample as Model 2.

From the quadratic form estimations, we find the following. There is a hump-shape relationship between economic development (per-capita GDP) and M2 velocity both in Models 1 and 2, which is consistent with the McKinnon-Shaw hypothesis. However, we can approximate the relationship by linearity assumption because most observations note the downward sloping segment in the quadratic form (e.g., 99%). We also find a similar relationship between economic development and M1 velocity if we take Model 1. If we take Model 2, however (as also found in the linear model), we can find only a positive correlation between economic development and M1 velocity. Hence, the robustness in M1 velocity is lost also in the quadratic form estimation by the deletion of some observations.

Table 11: Quantiles of centered logarithm of GDP per capita

Percentiles	Model 1	Model 2
Min.	*-7.4158*	*-7.1107*
1%	-2.4939	-2.5725
5%	-1.9857	-2.0122
10%	-1.6833	-1.5936

Table 12 shows estimations of Models 1 and 2 excluding the 1% quantile observations (e.g., the centered logarithm value of per-capita GDP being larger than -2.4939). According to this table, the overall results are not altered by the deletion of those 1% quantile observations except for the score of Hansen's J test in Model 2 for M2 velocity. When Hansen's J test gives a lower score, it implies that there is an overidentification. From all other Model 2 regressions for M2 velocity, the deposit rate is consistently insignificant. In addition, the endogeneity test to treat the deposit rate as exogenous gives $\chi^2(1) = 0.014$ and its p-value is 0.9062. Then, Model 3 in Table 12 estimates an alternative model in which the deposit rate is included as an exogenous variable (in the first-stage regression, it is replaced with the lending rate). In Model 3, we can find a sufficiently high score in Hansen's J test, and signs of coefficients are consistent with Model 2.

4 Conclusion

In this paper, the second-generation money-search model is extended by including the banking sector to derive interest rate endogenously. The derived interest rate shows a relationship with the arrival rate (e.g., interpreted as a form of urbanization). This result is similar to the third-generation money-search model (Lagos and Wright [18]) that suggests a negative correlation between real interest rate and urbanization, but the direction of the change may be different in my model. For instance, the real interest rate may increase as urbanization proceeds. The difference emanates from the endogenous determination of the interest rate via the financial service market. If urbanization raises the production level, the demand for financial services will increase, which will raise the interest rate if the supply of deposit does not increase sufficiently. Then urbanization raises the velocity of money as well as the production level and interest rate.

In the theoretical model, we find a positive correlation between GDP per capita and the velocity of money when the money stock is fixed. However, we generally find a negative correlation between GDP per capita and the velocity of money. Such a negative correlation can be found if we change the money stock. In the real world, the money supply is actually adjusted to the GDP level.

The theoretical model is applied to the empirical investigation, clarifying the endogenous structure and deriving the inferences. In the empirical investigation, the focus is on the velocity of

Table 12: IV estimations with deletion of 1% quatile observations

	Model 1				*Model 2*				*Model 3*	
	M1 Velocity		M2 Velocity		M1 Velocity		M2 Velocity		M2 Velocity	
	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.
Urban Pop.	0.1837**	(0.0579)	0.2155**	(0.0540)	0.0356	(0.0603)	0.1782**	(0.0691)	0.2575**	(0.0689)
CPI	0.0368**	(0.0033)	0.0146**	(0.0029)	0.0305**	(0.0098)	0.0113	(0.0127)	0.0203**	(0.0065)
Inflation	-0.0188	(0.0299)	-0.0134	(0.0370)	-0.0775	(0.0616)	0.0214	(0.0727)	0.0431	(0.0543)
Cap. Sock	—	—	—	—	—	—	—	—	—	—
(D1)	-0.0123	(0.0351)	0.0537	(0.0325)	—	—	—	—	—	—
GDP per Cap.	-0.1279**	(0.0415)	-0.4158**	(0.0399)	0.3866**	(0.0801)	-0.3180**	(0.0681)	-0.5662**	(0.0459)
M1 per Cap.	-0.0661**	(0.0108)	-0.0486**	(0.0084)	-0.3625**	(0.0345)	-0.1819**	(0.0225)	-0.0372**	(0.0101)
Quasimoney	0.3676**	(0.0281)	-0.2383**	(0.0294)	0.2226**	(0.0335)	-0.4034**	(0.0313)	-0.1893**	(0.0415)
Deposit Rate	—	—	—	—	0.0496**	(0.0188)	-0.0270	(0.0210)	-0.0073	(0.0135)
Time	-0.4820**	(0.0533)	-0.4368**	(0.0521)	-0.6420**	(0.0710)	-0.4822**	(0.0837)	-0.4690**	(0.0683)
Adj. R-sq.	0.3247		0.4173		0.6464		0.5043		0.3935	
N	3,254		3,254		2,142		2,142		2,467	
KP LM	386.918**		386.918**		226.142**		226.142**		266.118**	
CD F	853.974^b		853.974^b		196.360^b		196.360^b		693.882^b	
KP F	295.743^c		295.743^c		81.126^c		81.126^c		148.160^c	
AR F	78.12**		71.52**		73.08**		134.87**		85.02**	
AR Chi-sq.	313.37**		286.90**		367.01**		677.33**		341.41**	
SW LM	259.93**		248.57**		266.63**		298.30**		237.45**	
Hansen J	1.417		0.089		0.028		3.087†		1.033	

money (M2) and urbanization. The empirical study stands on the assumption that urbanization (measured by urban population) increases the arrival rate. Then, the obtained result is as follows. Urbanization raises GDP per capita and the velocity of money. This result confirms the Schumpeterian hypothesis and the proposition that increased transactions increase the circulation speed of money. In addition, we also find a negative correlation between GDP per capita and the velocity of money. According to the theory, such a negative correlation results from an increase in per-capita money supply (a form of financial deepening). In addition, we find a positive correlation between urbanization and interest rate, both in theory and in the empirical evidence: an increase of money, therefore, reduces the interest rate. This implies that urbanization faces a shortage of money. Then the higher interest rate attracts more deposits, and the shortage of money stimulates developments in financial products. Thus, the empirical evidence also supports the McKinnon-Shaw hypothesis.

The thesis obtained from this study also determines whether financial deepening or urbanization stimulates modern economic development. The answer is urbanization. Were this answer not true, we would see a negative correlation between interest rate and urbanization because the interest rate is determined by the production level.

In further empirical investigation, it would be interesting to see data for individual countries, as that would enable us to include more and more specific variables than those in the cross-country data. For example, the size of the banking sector is not included in the regressions of this study, but it is available for some countries. In addition, the development of slums is a major problem with urban population growth, and slum population data are available for some countries. We might then find a new research theme concerning the relationship between financial deepening and the development of slums. These examples are feasible as long as we do not stick to the cross-country data. In a country-specific study, however, the sample size would decline. A cross-country study (like this one) could then be referenced for asymptotic properties. As a theoretical extension, it would be interesting to see the impact of risk management on the money-search model, as the storage cost of money is also affected by the risks in banking activities. We could also compare the second-generation and third-generation models directly, which may enable us to integrate these two approaches.

Appendix

A Bellman Equations

Let $p_1 = 2\alpha(1-\alpha)(1-b)(1-\mu)\lambda_0$ be the probability of meeting a non money holder (seller) and $p_0 = 2\alpha(1-\alpha)(1-b)\mu\lambda_0$ be that of meeting a money holder (buyer). We consider a sufficiently small τ and appropriately defined $\lambda \in (0,1)$. Let $V_0(t)$ be the value function of the seller at period t. The value function must satisfy the next Bellman equation:

$$V_0(t) = \frac{\tau p_1 \{V_1(t+\tau) - c(q^s;A)\} + (1-\tau p_1) V_0(t+\tau) + \tau p_2 \tilde{v} + o(\tau)}{1+\tau\beta}, \tag{56}$$

where $o(\tau)$ is the counting error within length τ such that $o(\tau)/\tau \mapsto 0$. This Bellman equation is rearranged to get

$$\begin{aligned} \tau\beta V_0(t) &= \tau p_1 \{V_1(t+\tau) - V_0(t+\tau) - c(q^s;A)\} + \tau p_2 \tilde{v} \\ &\quad + V_0(t+\tau) - V_0(t) + o(\tau), \end{aligned} \tag{57}$$

which is further rearranged to get (29):

$$\begin{aligned} \beta V_0(t) &= p_1 \{V_1(t+\tau) - V_0(t+\tau) - c(q^s;A)\} + p_2 \tilde{v} + \frac{V_0(t+\tau) - V_0(t)}{\tau} + \frac{o(\tau)}{\tau} \\ &\mapsto p_1 \{V_1(t) - V_0(t) - c(q;A)\} + \tau p_2 \tilde{v} + \dot{V}_0(t) \quad (\tau \mapsto 0), \end{aligned} \tag{58}$$

where the last correspondence follows from $o(\tau)/\tau \mapsto 0$ and the definition of time-derivative:

$$\lim_{\tau \to 0} \frac{V_m(t+\tau) - V_m(t)}{\tau} \equiv \dot{V}_m(t). \tag{59}$$

For the buyer, similarly, its Bellman equation must satisfy

$$V_1(t) = \frac{\tau p_0 \left\{u(q^d) + V_0(t+\tau)\right\} + (1-\tau p_0) V_1(t+\tau) + \tau p_2 \tilde{v} + \tau\gamma + o(\tau)}{1+\tau\beta}, \tag{60}$$

which is rearranged to get

$$\begin{aligned} \tau\beta V_1(t) &= \tau p_0 \left\{u(q^d) + V_0(t+\tau) - V_1(t+\tau)\right\} + \tau p_2 \tilde{v} \\ &\quad + V_1(t+\tau) - V_1(t) + \tau\gamma + o(\tau). \end{aligned} \tag{61}$$

For $\tau \mapsto 0$, (61) is further rearranged to get (30):

$$\beta V_1(t) = p_0 \left\{u(q^d) + V_0(t+\tau) - V_1(t+\tau)\right\} + p_2 \tilde{v} + \gamma + \dot{V}_1(t). \tag{62}$$

B Nash Bargaining Solution

B.1 Barter Trade

Suppose two agents, indexed by $i = \{1,2\}$, are paired and both of them have no money whereas both of them like each other's goods. Then they bargain the quantities of transactions in accordance with the Nash bargaining problem such as

$$\max_{q_i} \{u(q_1) - c(q_2)\}\{u(q_2) - c(q_1)\}, \tag{63}$$

where q_i is the offer of agent i. For agent $i = 1$, the first order condition is given by

$$\frac{u'(q_1)}{c'(q_1)} = \frac{u(q_1) - c(q_2)}{u(q_2) - c(q_1)}. \tag{64}$$

Similarly, for agent $i = 2$, that is

$$\frac{u'(q_2)}{c'(q_2)} = \frac{u(q_2) - c(q_1)}{u(q_1) - c(q_2)}. \tag{65}$$

These conditions are simultaneously satisfied when $q_1 = q_2 = \bar{q}$ solves the social optimum: $\bar{q} = \arg\max\{u(q) - c(q)\}$. Therefore, the quantity of trade in barter is the quantity of trade in the social optimum.

B.2 Monetary Trade

The Nash bargaining problem in monetary trade is to maximize the joint net gains of buyer and seller (Nash product), hence

$$\max_{q} \{u(q) + V_0 - V_1\}^{\theta}\{V_1 - c(q,z) - V_0\}^{1-\theta}. \tag{66}$$

In the above problem, the value function is not known yet, and agents cannot differentiate the Nash product. However, it is known that splitting net gains is in accordance with respective agents' bargaining power; hence, the bargaining rule is determined by solving

$$\{u(q) + V_0 - V_1\} : \{V_1 - c(q,z) - V_0\} = \theta : (1-\theta). \tag{67}$$

This equation provides the bargaining rule that is always conformed by the seller and buyer:

$$V_1 - V_0 = (1-\theta)u(q) + \theta c(q,z). \tag{68}$$

Thus, the monetary equilibrium conforms to the above Nash bargaining rule.

C Measurement of Money in Regressions

In the regressions, we use (the logarithm of) real M1 per capita and quasimoney-M2 ratio as measurements of money. The reason is as follows.

We consider M2 as a measurement of money. However, M2 is equal to M1 plus quasimoney, and M1 and quasimoney may take different roles in the empirical model. Then we want to consider M1 per capita and quasimoney per capita in the regression separately. However, we find higher correlation rates between (the logarithm of) M1 per capita and quasimoney per capita, as shown in Tables 13 and 14. To reduce the correlation rate, we consider (the logarithm of) Quasimoney-M2 ratio as a regressor in place of quasimoney per capita. Then the correlation rate declines from 0.4-0.5 to around 0.15.

For reference, Table 15 shows regressed relationships between logarithm of M2 per capita and its decompositions. According to these regressions, we confirm the decomposed variables well reproduce the M2 measure.

D Proofs

D.1 Proof of Remark 4

Because u and c are monotonic, the shape of $F\left(q^{\circ};\omega\right)$ is characterized by examining slopes of u and c at $q^{\circ}=0$ and $q^{\circ}\mapsto\infty$. For instance, in q°, $F\left(q^{\circ};\omega\right)$ is (i) $\cup$-shape if and only if $F'\left(0;\omega\right)<0$ and $F'\left(\infty;\omega\right)>0$; (ii) $\cap$-shape if and only if $F'\left(0;\omega\right)>0$ and $F'\left(\infty;\omega\right)<0$; (iii) monotonically increasing if and only if $F'\left(0;\omega\right)>0$ and $F'\left(\infty;\omega\right)>0$; and (iv) monotonically decreasing if and only if $F'\left(0;\omega\right)<0$ and $F'\left(\infty;\omega\right)<0$. As $u'\left(0\right)\gg c'\left(0,A\right)$ and $u'\left(\infty\right)\ll c'\left(\infty,A\right)$, the slope of $F\left(q^{\circ};\omega\right)$ is determined by the coefficients of u at $q=0$ and of c at $q^{\circ}\mapsto\infty$ that are given by $\left(1-\theta\right)\beta-\lambda\left(\theta-\mu\right)$ and $\theta\beta+\lambda\left(\theta-\mu\right)$, respectively.

(i) If $F\left(q^{\circ};\omega\right)$ is $\cup$-shape, we must have

$$\begin{cases} \left(1-\theta\right)\beta-\lambda\left(\theta-\mu\right)<0 \\ \theta\beta+\lambda\left(\theta-\mu\right)>0. \end{cases} \tag{69}$$

Let $\mu<\theta$. In this case, the second inequality in (69) is automatically satisfied. Then the require-

Table 13: Correlations among measurements of money (Model 1)

	(1)	(2)	(3)	(4)
(1) M2 per Capita	1.0000			
(2) M1 per Capita	0.9613	1.0000		
(3) Quasimoney per Capita	0.6514	0.4725	1.0000	
(4) Quasimoney-M2 Ratio	0.3700	0.1526	0.8016	1.0000

N = 2,190

Table 14: Correlations among measurements of money (Model 2)

	(1)	(2)	(3)	(4)
(1) M2 per Capita	1.0000			
(2) M1 per Capita	0.9688	1.0000		
(3) Quasimoney per Capita	0.5851	0.4126	1.0000	
(4) Quasimoney-M2 Ratio	0.3481	0.1552	0.8191	1.0000

N = 3,288

Table 15: Relationship between M2, M1 and quasimoney

	Model 1		Model 2	
M2 per Capita	Coef.	S.E.	Coef.	S.E.
M1 per Capita	1.0127**	(0.0107)	0.9623**	(0.0218)
Quasimoney-M2 Ratio	0.7071**	(0.0939)	0.7759**	(0.1116)
Intercept	1.3642**	(0.0823)	1.7589**	(0.1191)
R-squared	0.9678		0.9007	
N	3,288		2,190	

ment is only the first inequality of (69), which is rearranged as

$$\lambda > \frac{(1-\theta)\beta}{\theta-\mu} > 0. \tag{70}$$

Alternatively, let $\theta \leq \mu$. The first inequality of (69) cannot hold if $\mu = \theta$. If $\mu > \theta$, the first inequality of (69) requires

$$\lambda < \frac{(1-\theta)\beta}{\theta-\mu} < 0, \tag{71}$$

which contradicts to the definition of $\lambda > 0$. Therefore, F is $\cup$-shape if and only if $\mu < \theta$ and $\lambda > (\theta-\mu)^{-1}(1-\theta)\beta$ hold.

(ii) If $F(q^{\circ};\omega)$ is $\cap$-shape, we must have

$$\begin{cases} (1-\theta)\beta - \lambda(\theta-\mu) > 0 \\ \theta\beta + \lambda(\theta-\mu) < 0. \end{cases} \tag{72}$$

Let $\theta \geq \mu$. The second inequality of condition (72) cannot hold if $\mu = \theta$. If $\mu < \theta$, the second inequality of condition (72) requires

$$\lambda < \frac{-\theta\beta}{\theta-\mu} < 0, \tag{73}$$

which contradicts the definition of $\lambda > 0$. Alternatively, let $\mu > \theta$. In this case, the first inequality of (72) is automatically satisfied. Then the requirement is only the second inequality of (72), which is rearranged as

$$\lambda > \frac{-\theta\beta}{\theta-\mu} > 0. \tag{74}$$

Therefore, F is $\cap$-shape if and only if $\mu > \theta$ and $\lambda > (\mu-\theta)^{-1}\theta\beta$ hold.

(iii) If $F(q^{\circ};\varepsilon)$ is monotonically increasing, we must have

$$\begin{cases} (1-\theta)\beta - \lambda(\theta-\mu) > 0 \\ \theta\beta + \lambda(\theta-\mu) > 0. \end{cases} \tag{75}$$

The requirements in condition (75) are actually the complements of conditions (69) for $\mu < \theta$ and (72) for $\mu > \theta$. Therefore, F is monotonically increasing if and only if F is neither $\cup$-shape nor $\cap$-shape.

(iv) If $F(q^{\circ};\varepsilon)$ is monotonically decreasing, we must have

$$\begin{cases} (1-\theta)\beta - \lambda(\theta-\mu) < 0 \\ \theta\beta + \lambda(\theta-\mu) < 0. \end{cases} \tag{76}$$

However, as it is also implied by the proof of (iii), the requirements in condition (76) cannot be satisfied because these inequalities actually require that λ be in an infeasible range such as

$$(1-\theta)\beta < \lambda(\theta-\mu) < -\theta\beta. \tag{77}$$

Therefore, $F(q^{\circ};\omega)$ is not a monotonically decreasing function.

D.2 Proof of Proposition 2

Using (29) and (30), we consider $V_1 - V_0$ at the steady state (e.g., $\dot{V}_0 = \dot{V}_1 = 0$) that is given by

$$V_1 - V_0 = \frac{\lambda(1-\mu)u(q^{\circ}) + \lambda\mu c(q^{\circ}) + (1-b)\lambda_0\gamma(q^{\circ},\bar{q})}{\beta + (1-b)\lambda}. \tag{78}$$

At the equilibrium, the value given by (78) is equal to the value that is given by the Nash bargaining solution (32); hence, we find

$$\frac{\lambda(1-\mu)u(q^{\circ}) + \lambda\mu c(q^{\circ}) + \gamma(q^{\circ},\bar{q})}{\beta+\lambda} = (1-\theta)u(q^{\circ}) + \theta c(q^{\circ}). \tag{79}$$

This equation is further arranged as

$$u(q^{\circ}) - c(q^{\circ}) = \frac{\beta u(q^{\circ}) - \gamma(q^{\circ},\bar{q})}{\theta\beta + (\theta-\mu)(1-b)\lambda}. \tag{80}$$

Therefore, for $\gamma \leq 0$, $u(q^{\circ}) - c(q^{\circ}) > 0$ if and only if $\theta\beta + (\theta-\mu)\lambda > 0$ and that implies F is a $\cup$-shape function (Remark 4).

Let F be a $\cup$-shape function (hence, $\mu < \theta$). Then we consider partial derivatives of $F(q^{\circ},\omega)$ with respect to $\{\lambda,\mu,\theta\}$. The partial derivative with respect to λ is given by

$$\frac{\partial\mathscr{F}}{\partial\lambda_0} = -2\alpha(1-\alpha)(1-b)(\theta-\mu)\{u(q^{\circ}) - c(q^{\circ},z)\} < 0, \tag{81}$$

where the inequality follows from $\mu < \theta$ and $u(q^{\circ}) \geq c(q^{\circ})$. The partial derivative with respect to b is given by

$$\frac{\partial\mathscr{F}}{\partial b} = 2\alpha(1-\alpha)(\theta-\mu)\lambda_0\{u(q^{\circ}) - c(q^{\circ},z)\} > 0. \tag{82}$$

The partial derivative with respect to α is given by

$$\frac{\partial\mathscr{F}}{\partial\alpha} = -2(1-2\alpha)(1-b)(\theta-\mu)\lambda_0\{u(q^{\circ}) - c(q^{\circ},z)\}, \tag{83}$$

which is negative for $\alpha \leq 1/2$ and positive for $\alpha \geq 1/2$. Therefore, λ_0 and $\alpha \leq 1/2$, shift $\mathscr{F}$

downward and b and $\alpha \geq 1/2$ shift $\mathscr{F}$ upward. In other words, λ shifts $\mathscr{F}$ upward.

The partial derivatives with respect to μ and θ are given by

$$\frac{\partial \mathscr{F}}{\partial \mu} = \lambda \{u(q^\circ) - c(q^\circ, z)\} > 0, \tag{84}$$

$$\frac{\partial \mathscr{F}}{\partial \theta} = -(\beta + \lambda)\{u(q^\circ) - c(q^\circ, z)\} < 0, \tag{85}$$

where the inequalities follows from $u(q^\circ) \geq c(q^\circ)$. Therefore, μ shifts $\mathscr{F}$ upward and θ shifts $\mathscr{F}$ downward.

Notes

[1]See also Ellison et al. [8] and Virág [33].

[2]Because the velocity of money is the inverse of income per money (Marshall's *k*), we can also say the negative correlation between GDP per capita and the velocity of money is a positive correlation between economic development and financial deepening (measured by Marshall's *k*).

[3]In this argument, individual i is supposed to be a buyer and individual j is supposed to be a seller.

[4]Other alternative measures, such as M3 and MZM, are also interesting to consider in this context, but I use M2 (and M1) because of the availability of sufficient cross-country data.

[5]The quantities traded in barter and monetary transactions are generally different.

[6]The parameters are the same as those for the later numerical analysis in Section 2.5 (see Table 2).

[7]The sample size is 5,801 and the R-squared is 0.2398.

[8]In that model, velocity and GDP per capita move in the same direction when the money stock is fixed.

[9]An additional argument is shown as a robustness check in the next subsection.

[10]See Stock and Yogo [30, Tables 1 and 2, pp. 58-59] for the critical values for the weak-instrument tests. As $n = K_2 = 4$ in Model 1 and $n = K_2 = 5$ in Model 2, the weak identification tests, both in relative bias and size are significantly positive.

[11]In Model 1, the p-values for the first and second order terms in M1 velocity are 0.1369 and 0.0011, respectively, and those in M2 velocity are 0.0000 and 0.0000, respectively. Similarly, in Model 2, we see 0.0000 and 0.0279 in M1 velocity, respectively, and 0.0013 and 0.0000 in M2 velocity, respectively. These p-values are computed from heteroskedasticity robust errors.

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