# Risk, Returns, and Multinational Production*

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**Abstract**

This paper starts by unveiling a new empirical regularity: multinational corporations systematically tend to exhibit higher stock market returns and earnings yields than non-multinational firms. Within non-multinationals, exporters tend to exhibit higher earnings yields and returns than firms selling only in their domestic market. To explain this pattern, we develop a real option value model where firms are heterogeneous in productivity, and have to decide whether and how to sell in a foreign market where demand is risky. Firms can serve the foreign market through trade or foreign direct investment, thus becoming multinationals. Multinational firms are more exposed to risk: following a negative shock, they are reluctant to exit the foreign market because they would forgo the sunk cost that they paid to start investing abroad. We calibrate the model to match U.S. export and FDI dynamics, and use it to explain cross-sectional differences in earnings yields and returns.

**Keywords:** Multinational firms, option value, cross-sectional returns

**JEL Classification** : F12, F23, G12

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# 1 Introduction

Multinational firms tend to exhibit higher stock market returns and earnings yields than non-multinational firms. Among non-multinationals, exporters tend to exhibit higher returns and earnings yields than firms selling only in their domestic market. Many studies in the new trade literature have documented features distinguishing firms that sell into foreign markets from firms that do not: exporters and multinational firms tend to be larger, more productive, to employ more workers, and sell more products than firms selling only domestically.[1] However, none of this literature has addressed the question of whether the international status of the firm matters for its investors. Similarly, in the financial literature, explanations of the cross section of returns disregarded the role of the international status of the firm.[2]

In this paper we attempt to fill this gap in the literature. We develop a real option value model where firms' heterogeneity, aggregate uncertainty and sunk costs provide the missing link between firms' international status and their returns on the stock market.

The fact that exporters and multinational firms give higher yields and returns than domestic firms does not constitute an anomaly *per se*. It just indicates that these firms are "riskier" than firms that do not serve foreign markets: if this was not the case, rational agents would not hold shares of domestic firms in equilibrium. The purpose of our structural model is to identify a plausible channel that delivers differential exposure to risk of firms with different "international status". The mechanism of the model is simple: suppose a firm decides to enter a foreign markets where aggregate demand is subject to fluctuations, and entry involves a sunk cost. In "good times", when prospects of growth make entry profitable, a firm may decide to pay the sunk cost and enter. If – after entry – the shock reverses, the firm will be reluctant to exit immediately because of the sunk cost it paid to enter, and may prefer to bear losses for a while, hoping for better times to come again. If sunk costs of establishing a foreign affiliate are larger than the sunk costs of starting to export, then the exposure to demand fluctuations and possible negative profits will be higher for multinational firms than for exporters, and will command a higher return in equilibrium.

The choice of whether to serve the foreign market and how (via export or foreign invest-

[1] See, among others, Bernard, Jensen, and Schott (2009).
[2] One notable exception is Fatemi (1984).

ment, henceforth FDI) is endogenous, and we model it following the literature on heterogeneous firms, namely the influential contribution by Helpman, Melitz, and Yeaple (2004). Exports are characterized by low sunk costs and high variable costs, due to the necessity of shipping goods every period, while FDI entails high sunk costs of setting up a plant and starting production abroad, but low variable costs, since there is no physical separation between production and sales. The model in Helpman, Melitz, and Yeaple (2004) is static, hence the value of a firm coincides with its profits, and earnings yields are constant across firms. A dynamic but deterministic model, or a dynamic and stochastic model with idiosyncratic shocks, share the same feature, with earnings-to-price ratios simply given by the discount rate. The same is true for the returns, which are given by the earning yields plus the expected change in the valuation of the firm (this last term being zero). To generate heterogeneity in these variables across firms, we extend the basic Helpman, Melitz, and Yeaple (2004) framework to a dynamic and stochastic environment characterized by persistent shocks, using Dixit (1989) as a benchmark to model entry decision under uncertainty.

Firms choose whether to export or invest abroad based on their productivity and on prospects of growth of foreign demand. Larger sunk costs of investment compared to export imply that multinational firms' behavior displays more persistence than exporters' behavior, and multinationals may experience larger losses if the economy is hit by a negative shock.

How does this behavior generate heterogeneity in earnings yields and returns? Sunk costs of exports and FDI can be interpreted as the premia to be paid to exercise the option of entering the foreign market. The value of this option is an important component of the valuation of the firm. Hence profit flow and firm value are not proportional due to this extra component: the option value of entering/exiting the market, which differs across firms. To generate heterogeneity in stock market returns, we model risk-averse consumers who own shares of the firms, and discount future consumption streams with a stochastic discount factor dependent on the aggregate shocks. Firms' heterogeneity and endogenous status choices imply that different firms will differ in the covariance of their earnings yields with the aggregate uncertainty, which affects consumers' marginal utility. As a result, the model endogenously determines cross-sectional differences in earnings-to-price ratios and returns, and provides a complementary explanation for the cross section of returns exploiting the production side from an international point of view.

The solution of the model delivers a series of predictions relating firms' productivity and the realization of the shocks to the pattern of trade and FDI dynamics. First, more productive firms need smaller positive shocks to enter a foreign market, and larger negative shocks to exit. Second, a larger positive shock is needed to induce a domestic firm to become multinational with respect to the one needed to become an exporter. Third, a larger negative shock is needed to induce a multinational to exit the market with respect to the one needed to induce an exporter to exit. The model is consistent with qualitative features of the data on trade and FDI dynamics, like the imperfect sorting of productivity into international statuses, and the fact that also relatively large firms exhibit changes of status. We calibrate the theory to match these facts quantitatively, and with the parameterized model we compute moments of the financial variables from simulated data. We show that the model is able to generate the rankings of earnings yields and returns, which were not targeted in the calibration.

Why are we interested in the cross section of returns and earnings yields? Historically, average returns vary across stocks. Fama and French (1996) is a comprehensive description of the cross-sectional picture of returns. In this paper we address the risk-return trade-off regarding multinational and non-multinational firms. We focus on cash flow dynamics of the firm and how these are determined by endogenous decisions and exogenous risk. Multinational firms are exposed to foreign demand risk for longer due to the higher persistence in their status. This risk must be rewarded by a higher asset returns in equilibrium. Investors will be willing to hold these companies if the returns are high enough to compensate for the risk. We find that this risk is not fully captured by the multifactor model in Fama and French (1993).

The existing financial literature that focuses on cross-sectional differences in earnings-to-price ratios and returns abstracts from the international organization of the firm. Various attempts to explain cross sectional differences in expected returns are based either on different specifications of preferences, or on the presence of persistent shocks to the endowments, or both.[3] We contribute to the financial literature by endogenizing the exposure of cash-flows to these types of shocks. Exposure is directly linked to the decision of when and how to serve the foreign market, which is ultimately driven by the interaction between productivity and cost structure.

[3]Yogo (2006) and Piazzesi, Schneider, and Tuzel (2007) are examples of non-separable goods in the utility function. Campbell and Cochrane (1999) use internal habits specifications. Bansal and Yaron (2004) and Hansen, Heaton, and Li (2008) use recursive preferences with persistent shocks to the endowment.

Our work is related to the literature on trade and FDI under uncertainty, mainly to Rob and Vettas (2003), Russ (2007), and Ramondo and Rappoport (2008). Rob and Vettas (2003) developed a model of trade and FDI with uncertain demand growth. In their framework FDI is irreversible, so it can generate excess capacity, but has lower marginal cost compared to export. The authors show that uncertainty implies existence of an interior solution where export and FDI coexist. Besides the different focus of the exercise, our work generalizes their model to one with many heterogeneous firms and a more general process for demand growth. Russ (2007) also formulates a problem of foreign investment under uncertainty to study the response of FDI to exchange rate fluctuations. Her model features firm heterogeneity, but does not allow trade as a way to serve foreign markets. Ramondo and Rappoport (2008) introduce idiosyncratic and aggregate shocks in a model where firms can locate plants both domestically and abroad. Multinational production allows firms to match domestic productivity and foreign shocks, and works as a mechanism for risk sharing. Our framework allows for risk sharing and diversification in addition to the risk exposure driven by the combination of aggregate shocks and sunk costs. We allow for country-specific shocks with various correlation patterns. Moreover, we model both trade and multinational production as different modes of dealing with uncertainty in foreign markets.

This paper is also related to a growing body of literature on trade dynamics with sunk costs. Particularly, Alessandria and Choi (2007) and Irarrazabal and Opromolla (2009) model entry and exit into the export market in a world with idiosyncratic productivity shocks and sunk costs. Our model is closer to the framework in Irarrazabal and Opromolla (2009) for the use of the real option value analogy in solving the firm's optimization problem. While Irarrazabal and Opromolla (2009) concentrate their attention on the impact of idiosyncratic productivity shocks for firm dynamics, we model aggregate demand shocks that affect firms differently only through their endogenous choice of international status. Alessandria and Choi (2007) study the impact of firms' shocks and sunk costs on the business cycle. While the objective of our exercise is different from their paper, we follow their calibration methodology. Both papers analyze the decision to export, but do not consider the possibility of FDI sales. Roberts and Tybout (1997) and Das, Roberts, and Tybout (2007) address empirically the issue of market participation for export. Our model has similar predictions for both exports and FDI sales, and can be calibrated using information from trade and FDI data. In general, we contribute to the trade dynamics literature both empirically and theoretically: we document features of trade and FDI dynamics for large firms, and we incorporate in the model the mode of entry (*i.e.*, the decision between export and FDI sales).

While individual elements of our framework are found in other work, to our knowledge this paper is the first to propose a dynamic industry equilibrium model where risk affects firms' international strategies and their financial variables in the stock market. The remainder of the paper is organized as follows. Section 2 presents empirical evidence establishing the ranking in earnings-to-price ratios and returns according to the firms' international status. Sections 3 and 4 develop the model and illustrate its analytical properties. Section 5 brings the model to the data: we parameterize the model using aggregate trade and FDI data and report the quantitative results on the earnings yields and returns predicted by the calibrated version of the model. Section 6 is devoted to robustness checks, and Section 7 concludes.

## 2 Motivating Evidence

In this section we document an empirical regularity distinguishing firms that sell only in their domestic market from exporters and multinational firms. Multinational firms tend to exhibit significantly higher earnings yields and returns than exporters, and exporters in turn have higher earnings yields and returns than firms selling only in their domestic market.

Our sample consists of US-based manufacturing firms in the Compustat Segments database, and tracks about 5,300 firms from 1979 to 2006. We define a firm to be a multinational (MN) if it reports the existence of a foreign geographical segment associated with positive sales. Similarly, we define a firm to be an exporter if it reports a positive level of export sales.[4,5] According to this definition, on average, 34.81% of firms sell only

[4]Multinational and exporter dummies are constructed based on Compustat geographic and operating segments data. According to the Statement of Financial Accounting Standards (FAS) No. 131, *"segments are components of an enterprise about which separate financial information is available"*. Firms must report information about profits, revenues and assets for each segment. For geographical segments, this information includes revenues perceived and assets held in foreign countries. The FAS is not explicit in defining an ownership threshold for reporting, but the existence of accounting standards for the segments themselves leads us to think that the parent (U.S.-based) firm must have a control stake in the foreign entity. Moreover, one of the Financial Accounting Standards Board (FASB)'s roles is to *"require significant disclosures about the separate operating segments of an entity's business so that investors can evaluate the differing risks in the diverse operations"*. Appendix A contains a summary of the FAS Statement, and more details about the construction of the sample.

[5]6% of firms in the sample report both positive exports and FDI sales. We classify these firms as

Table 1: **Summary Statistics**.

| | Domestic | Exporters | Multinationals |
|---|---|---|---|
| domestic sales (millions $) | 274.18 | 301.03 | 1562.39 |
| export sales (millions $) | 0 | 32.25 | 127.36 |
| FDI sales (millions $) | 0 | 0 | 937.52 |
| number of employees (thousands) | 1.71 | 2.43 | 12.27 |
| capital/labor ratio (millons $ per worker) | 0.11 | 0.05 | 0.06 |
| book value per share ($) | 5.26 | 6.78 | 9.43 |
| market capitalization (billions $) | 0.16 | 0.16 | 1.82 |
| book/market | 0.68 | 0.73 | 0.61 |
| earnings per share ($) | 0.23 | 0.5 | 0.88 |
| share price ($) | 8.38 | 10.31 | 17.47 |
| annual return | 0.06 | 0.1 | 0.11 |
| number of firms | 2580 | 2164 | 2667 |

in the U.S. market, 29.2% also export to foreign countries, and the remaining 35.99% have positive levels of FDI sales.[6] Table 1 reports descriptive statistics of the sample we use.

In line with the numbers reported by other papers, exporters and multinationals have a size advantage with respect to domestic firms, both in terms of sales and number of employees. Particularly, the size advantage of multinationals is extremely large: on average, multinational firms hire about five times more workers than exporters, and have sales about eight times larger. Consistently with previous evidence, export sales are a small percentage (10%) of exporters' total sales.[7] The novel facts that Table 1 highlights are that the same ordered differences hold for financial variables like book value, earnings, share prices, and returns.

Differences in earnings and market prices do not cancel out: at the contrary, also earnings yields (or earnings-to-price ratios) are ordered. Figure 1 shows earnings-to-price ratios

multinationals, based on the criterium that the existence of FDI sales is sufficient to make them subject to the risk of owning a plant in a foreign market. For robustness, however, we run all the regressions contained in this section also excluding those firms from the sample. The results are unchanged.

[6] Notice that the shares of firms belonging to each group are very different from what reported in other papers that use different data. Particularly, the share of multinational firms is disproportionately large. This is due to the fact that Compustat collects data for publicly listed firms only, which tend to be the largest firms in the population.

[7] Bernard et al. (2003) report an average ratio of export over total sales of 10% for a sample of manufacturing firms in the OECD countries.

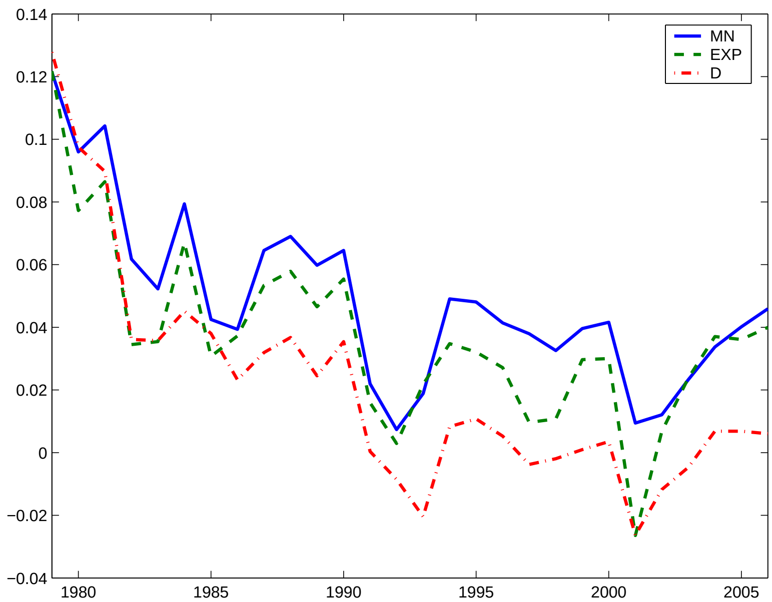


Figure 1: Earnings-to-price ratios, portfolios of firms in each group.

over time for three portfolios of firms. Each portfolio is composed by firms with the same international status (only domestic sales, exporters, multinationals).[8] The solid line represents earnings-to-price ratios of multinational firms, the dashed line the ones of exporters, and the dash-dotted line the ones of firms selling only domestically. Multinational firms exhibit higher earnings-to-price ratios than non-multinational firms, consistently over the entire time period. Similarly, exporters exhibit higher earnings-to-price ratios than firms selling only in their domestic market. The average earnings-to-price ratios for the three groups are 4.85% for multinational firms, 4.1% for exporters, and 2.25% for firms selling only domestically.

Figure 1 plots the raw data, but the ordering is robust to controlling for the effect of other variables related to size and industry. To separate the effects of the international status from other firm characteristics, the left panel of Table 2 displays the results of the

[8]The portfolios are constructed as follows. For each firm $i$, determine its status $S$ ($S = D, EXP, MN$) at the end of year $t-1$, and collect data on earnings ($e_t^i$) and market capitalization ($p_t^i$) in year $t$. Portfolio earnings $E_t^S$ and portfolio value $P_t^S$ are constructed as equally weighted averages of individual values:

$$E_t^S = \frac{1}{N_t^S}\sum_{i\in S} e_t^i \quad , \quad P_t^S = \frac{1}{N_t^S}\sum_{i\in S} p_t^i \ , \ \ \forall\ S$$

where $N_t^S$ denotes the number of firms in status $S$ at time $t$. Portfolio earnings yields are given by $E_t^S/P_t^S$.

following firm-level regression:

$$(e/p)_{it} = \alpha + \gamma_1 D_{it}^{MN} + \gamma_2 D_{it}^{EXP} + \gamma_3 \beta_i^{MKT} + \gamma_4 X_{it} + \delta_{NAICS} + \delta_t + \varepsilon_{it}. \qquad (1)$$

The dependent variable $(e/p)_{it}$ is the earnings-to-price ratio of firm $i$ in year $t$. $D_{it}^{MN}$ and $D_{it}^{EXP}$ are multinational and exporter dummies, respectively, $\beta_i^{MKT}$ is the market *beta* of the primary security of firm $i$,[9] $X_{it}$ is a set of controls, including capital/labor ratio, sales per employee (our measure of productivity), book-to-market ratio, total revenues and market capitalization (measures of size). $\delta_{NAICS}$ and $\delta_t$ are 4-digit industry and year fixed effects, respectively, and $\varepsilon_{it}$ is an orthogonal error term.

The coefficients associated to exporters and MN dummies are positive and significant. Moreover, the coefficient associated to multinationality is significantly larger than the one associated to export status, identifying a further difference between these two groups. We reject the null hypothesis that the two dummies' coefficients are the same, confirming the difference in earnings yields of multinational firms *versus* exporters. While Table 2 only contains sales per employee and total revenues as additional controls, we run the regression also adding capital intensity and market capitalization.[10] The purpose of adding the market *betas* is to control for aggregate market risk and to highlight the contribution of the international status to the magnitude of earnings yields once market risk is accounted for. Similarly, measures of size (revenues, market capitalization) and book-to-market ratio are meant to control for other potential sources of risk.

Earnings-to-price ratios carry information about returns on the firms' stocks. The right panel of Table 2 reports the results of a regression analogous to (1), but with annual firm-level returns as the dependent variable.[11]

The coefficients on the multinational and exporter dummies are positive and significant, which confirms that firms selling in foreign markets tend to have higher returns than firms selling only domestically. The coefficient on the multinational dummy is significantly higher than the one on the exporter dummy, indicating even larger excess returns for multinational

[9] The market *betas* have been computed by running a regression of individual security returns on the market aggregate returns (NYSE, AMEX, and Nasdaq) for the entire sample period.

[10] In the data capital intensity appears to be highly correlated with sales per employee, while market capitalization is highly correlated with total revenues. For this reason, we run the regression with all the combinations of the two controls that are not strongly correlated with each other. The results are basically identical across specifications.

[11] We identify firm-level returns with the returns of the firm's common equity. Data on returns are taken from CRSP.

Table 2: **Earnings-to-Price and Returns Regressions.** Firm-level regressions of earnings-to-price ratios and stock returns on multinational and exporter dummies, market *betas* and other controls, with year and industry fixed effects. Standard errors clustered by firm and status. (Top and bottom one percent of earnings-to-price sample excluded, top and bottom five percent of returns sample excluded. All dollar values are expressed in billions).

| | Earnings-to-Price | | | Returns | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (1) | (2) | (3) |
| MN dummy | 0.092 | 0.093 | 0.098 | 0.05 | 0.05 | 0.046 |
| | (0.005)*** | (0.005)*** | (0.005)*** | (0.005)*** | (0.005)*** | (0.005)*** |
| EXP dummy | 0.061 | 0.062 | 0.062 | 0.035 | 0.035 | 0.037 |
| | (0.006)*** | (0.006)*** | (0.006)*** | (0.006)*** | (0.006)*** | (0.006)*** |
| $\beta^{MKT}$ | | -0.004 | -0.004 | | 0.008 | 0.005 |
| | | (0.001)*** | (0.001)*** | | (0.002)*** | (0.002)*** |
| sales per emp. | 5.23e-05 | 5.24e-05 | 5.24e-05 | 5.24e-05 | 4.95e-05 | 4.77e-05 |
| | (2.92e-05)* | (2.99e-05)* | (2.99e-05)* | (3.9e-05)* | (2.97e-05)* | (2.88e-05) |
| total revenue | 8.68e-07 | 8.38e-07 | 8.02e-07 | 4.77e-07 | 6.07e-07 | 3.36e-07 |
| | (4.04e-07)** | (4.07e-07)** | (4.03e-07)** | (3.07e-07) | (3.27e-07)* | (2.65e-07) |
| book/market | | | -0.001 | | | -0.104 |
| | | | (0.007) | | | (0.011)*** |
| constant | -0.095 | -0.094 | -0.093 | 0.021 | 0.007 | 0.081 |
| | (0.005)*** | (0.006)*** | (-0.007)*** | (0.005)*** | (0.005) | (0.01)*** |
| Prob > $F$: | | | | | | |
| $H_0$: MN=EXP | 0 | 0 | 0 | 0 | 0 | 0.06 |
| No. of obs. | 47691 | 45979 | 45979 | 47547 | 45975 | 45858 |
| $R^2$ | 0.073 | 0.073 | 0.072 | 0.134 | 0.134 | 0.161 |

firms. The ranking holds controlling for market risk, sales per employee, total revenues, and book-to-market.[12] Any cross-sectional differences in returns generated by exposure to aggregate risk is captured by cross sectional differences in their market *betas*. Hence, the significant coefficients on the multinational and exporter dummies identify a separate source of risk.

After an exploration of earnings-to-price and returns across the three groups of firms, it seems natural to explore the source of higher returns. Higher returns do not constitute a puzzle by themselves; they simply indicate that multinational firms are riskier. From a CAPM point of view, higher returns must be explained by higher *betas*, or co-movements

[12]The discussion about the correlations across controls in footnote 10 still applies.

with the aggregate risks. Beyond the one-factor CAPM model, Fama and French (1993) introduced a multifactor extension to the original CAPM. Fama and French (1993) argue that a unique source of risk is not able to explain the cross section of returns. Instead, a three-factor model explains a higher fraction of the variation in expected returns. Higher returns must be explained by higher exposure to either of these three factors: market excess returns, high-minus-low book-to-market, or small-minus-big portfolio.[13] Each of the three factors is assumed to mimic a macroeconomic aggregate risk. Therefore, any asset is represented as a linear combination of the three Fama-French factors.

Columns (1)-(3) in Table 3 show the results of running one time-series Fama-French regression for each of the three portfolios of firms characterized by the same international status.[14] The risk to which multinationals and exporters are exposed, and the corresponding higher returns that they provide to investors, are not explained by the three existing Fama-French factors. On the contrary, we find that the market *betas* are lower than those of domestic firms. Multinationals and exporters' exposure to the HML factor, related to the value premium, is also significantly lower than the exposure of domestic firms to the same factor. In columns (4)-(6) we enlarge the set of factors by considering the excess returns on an "international market" portfolio that serves as a market benchmark for firms with foreign operations. Data on the excess returns on this global market portfolio are obtained from Kenneth French's data library on international indexes.[15] The coefficients on the international market *betas* are not significant. If the exposure to the three factors does not explain the higher reward that multinational stocks provide, it must be reflected in the pricing errors of the model. In fact, the *alpha* of the portfolio of multinational firms is significantly higher than the one of the exporters' portfolio, which in turn is higher than the *alpha* of the portfolio of domestic firms. The results are unchanged when we include among

[13] The small-minus-big (SMB) and high-minus-low (HML) factors are constructed upon 6 portfolios formed on size and book-to-market. The portfolios are the intersection of 2 portfolios formed on size (small and big) and 3 portfolios formed on book equity to market equity (from higher to lower: value, neutral, and growth.) This generates 6 portfolios: small-value, small-neutral, small-growth, big-value, big-neutral, and big-growth. SMB is the average returns on the three small portfolios minus the average returns on the three big portfolios. HML is the average return on the two value portfolios minus the average return on the two growth portfolios. The third factor, the excess return on the market, is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks minus the one-month Treasury bill rate. For more details see Fama and French (1993).

[14] Every year portfolios are formed by equally-weighting firms belonging to each of the three categories (see footnote 8). We run the Fama-French regressions also at the firm-status level: the results are consistent with the ones of the portfolio regressions, albeit noisier due to the even smaller number of observations.

[15] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/int_index_port_formed.html.

the regressors the returns on the global market portfolio. GRS tests on the null hypothesis that the *alphas* are jointly equal to zero strongly reject the hypothesis.

Table 3: **Portfolio Regressions**. Time-series coefficient estimates of Fama-French 3-factor regressions for the three equally-weighted portfolios based on international status. Portfolio annual excess returns are regressed on the three Fama-French factors (market excess return, Small Minus Big, and High Minus Low) for specification (1)-(3). In Columns (4)-(6) we add excess returns from an international index portfolio. The $\alpha$ coefficients capture the pricing errors of the three-factor model.

| | (1) DOM | (2) EXP | (3) MN | (4) DOM | (5) EXP | (6) MN |
|---|---|---|---|---|---|---|
| $R^{mkt}$ | 0.695 | 0.611 | 0.671 | 0.693 | 0.636 | 0.66 |
| | (0.07)*** | (0.075)*** | (0.071)*** | (0.084)*** | (0.09)*** | (0.086)*** |
| $R^{SMB}$ | 0.735 | 0.744 | 0.529 | 0.735 | 0.74 | 0.531 |
| | (0.081)*** | (0.087)*** | (0.083)*** | (0.083)*** | (0.089)*** | (0.085)*** |
| $R^{HML}$ | 0.267 | 0.172 | 0.188 | 0.267 | 0.171 | 0.189 |
| | (0.069)*** | (0.074)** | (0.07)** | (0.07)*** | (0.075)** | (0.071)** |
| $R^{INT}$ | | | | 0.002 | -0.031 | 0.015 |
| | | | | (0.056) | (0.06) | (0.058) |
| $\alpha$ | -0.079 | -0.029 | -0.02 | -0.079 | -0.028 | -0.02 |
| | (0.013)*** | (0.014)** | (0.013) | (0.013)*** | (0.0144)* | (0.013) |
| Prob $> F$: | | | | | | |
| $H_0$: $\alpha_{DOM} = \alpha_{EXP} = \alpha_{MN} = 0$ | | | 0 | | | 0 |
| Obs. | 28 | 28 | 28 | 28 | 28 | 28 |
| $R^2$ | 0.903 | 0.878 | 0.87 | 0.903 | 0.879 | 0.87 |

Tables 2 and 3 consistently convey the message that the three groups of firms differ significantly in their financial variables, and that these differences cannot be accounted for either by differences in productivity and size or by traditional risk-factor explanations. The exposure to a global index does not explain the differences in returns either. The excess return of multinational firms is not explained by the common factors widely used in the finance literature. Our goal is to understand what is the underlying risk driver that generates multinationals returns in excess of those of exporters or domestic firms. In the next section we develop a dynamic structural model in which productivity differences determine the selection of firms into the three statuses, and the presence of aggregate shocks and sunk costs gives rise to the observed pattern in their financial variables.

# 3 Model

## 3.1 Preferences and Technology

The economy is composed of two countries, Home and Foreign. Variables related to consumers and firms from the foreign country are marked with an asterisk (*). In both countries, agents are infinitely lived, and have preferences defined by:

$$U = \int_0^\infty e^{-\vartheta t} \frac{Q(t)^{1-\gamma}}{1-\gamma} dt$$

where $\theta > 0$ is the subjective discount factor, and $\gamma > 1$ denotes risk aversion. $Q$ is a CES aggregate of differentiated varieties:

$$Q(t) = \left( \int q_i(t)^{1-1/\eta} di \right)^{\eta/(\eta-1)}$$

where $\eta > 1$ denotes the elasticity of substitution across varieties.

Agents maximize $U$ subject to their budget constraint. Labor supply is perfectly inelastic. Income is given by the wage plus the profit shares derived from ownership of firms incorporated in the country where the agents live. In each country, aggregate consumption of the differentiated good is hit by random shocks. $Q$ and $Q^*$ evolve according to geometric Brownian motions:

$$\frac{dQ}{Q} = \mu dt + \sigma dz \tag{2}$$

$$\frac{dQ^*}{Q^*} = \mu^* dt + \sigma^* dz^* \tag{3}$$

where $\mu, \mu^* \geq 0$, $\sigma, \sigma^* > 0$ and $dz$, $dz^*$ are the increments of two standard Wiener processes with correlation $\rho \in [-1, 1]$:

$$dz = dW \tag{4}$$

$$dz^* = \rho dW + \sqrt{1-\rho^2} dW^2. \tag{5}$$

$dW$ is the increment of a standard Wiener process, hence $E(dz) = E(dz^*) = 0$, and $E(dz^2) = E(d(z^*)^2) = dt$, $E(dzdz^*) = \rho$.

Agents are risk averse. Separability of the intratemporal utility function implies that agents in each country discount future utility with stochastic discount factors described by the following geometric Brownian motions:

$$\frac{dM}{M} = -rdt - \sigma_M dz \tag{6}$$

$$\frac{dM^*}{M^*} = -r^* dt - \sigma_M^* dz^* \tag{7}$$

where $r = \vartheta + \gamma\mu$ ($r^* = \vartheta + \gamma\mu^*$) is the risk-free rate, $\sigma_M = \gamma\sigma$ ($\sigma_M^* = \gamma\sigma^*$) and $dz$, $dz^*$ are the increments of the Brownian motions ruling the evolution of $Q$ and $Q^*$.[16] For convergence of the risk-adjusted expected values, we will assume that $r > \mu + \sigma\sigma_M$ and $r > \mu^* + \rho\sigma^*\sigma_M$ (and similarly $r^* > \mu^* + \sigma^*\sigma_M^*$ and $r^* > \mu + \rho\sigma\sigma_M^*$).

Labor is the only factor of production. Each country is populated by a continuum of firms of total mass $n$ ($n^*$), which operate under a monopolistically competitive market structure. Each firm produces a differentiated variety with a linear technology defined by a unit labor requirement $a$, which is a random draw from a distribution $G(a)$ ($G^*(a)$). $a$ indicates the number of units of labor that a firm employs in order to produce one unit of a differentiated variety. Differentiated varieties $q_i$ are tradeable; hence, a firm may sell its own variety only in its domestic market or both in the domestic and in the foreign market.[17]

Let us now turn to the description of the production costs in the differentiated sector. We assume that there are no fixed costs associated to production for the domestic market, so every firm makes positive profits from domestic sales, and always sells in its domestic market.[18] Besides producing for its domestic market, firms can produce also for the foreign country. Production in the foreign market involves fixed operating costs, to be paid every period, and sunk costs of entry. If a firm decides to sell in the foreign market, it can do so

[16] The stochastic discount factor is equal to the intertemporal marginal rate of substitution. Marginal utility of consumption is: $M = e^{-\vartheta t}Q(t)^{-\gamma}$. Hence:

$$\frac{dM}{M} = -\vartheta dt - \gamma\frac{dQ}{Q} = -(\vartheta + \gamma\mu)dt - \gamma\sigma dz.$$

[17] Since consumers own shares of firms incorporated in the country where they live, when firms enter foreign markets, the fluctuations of both domestic and foreign demand have effects on consumers' incomes via the profit shares.

[18] We could have introduced positive fixed costs of domestic production, and modeled the initial decision of entry in the domestic market, like in Helpman, Melitz, and Yeaple (2004) and Irarrazabal and Opromolla (2009). This would have introduced additional complications in solving for firms' optimal dynamics, without any gains for our empirical analysis. Compustat includes only publicly listed firms, so when a firm enters or exits Compustat we do not have any information about whether the firm is in fact entering or exiting the market.

either via exports or via foreign direct investment. We call multinationals those firms that decide to serve the foreign market through FDI sales.

We model the choice between trade and FDI along the lines of Helpman, Melitz, and Yeaple (2004): exports entail a relatively small sunk cost of entry, $F_X$, but a per-unit iceberg transportation cost $\tau$ to be paid every period.[19] Instead, FDI is associated to a larger sunk cost, $F_I$ ($F_I > F_X$), but there are no transportation costs to be covered every period, as both production and sales happen in the foreign market.[20]

Both exports and FDI also entail fixed operating costs to be paid every period, that we denote with $f_X$ and $f_I$ for exports and FDI, respectively. After entering in the foreign market, a firm can exit at no cost. However, if it decides to re-enter, it will have to pay the sunk cost again.[21] Sunk costs and stochastic demand imply that firms decide to enter when their expected profits are well above zero, and are reluctant to exit even in case of losses due to negative shocks. We show that this dynamic behavior, labeled "hysteresis" in the literature (see Dixit and Pindyck (1994)), is more severe for multinational firms than for exporters, due to the larger sunk costs of FDI. Notice also that the cost structure and the nature of uncertainty imply that if a firm decides to enter the foreign market, it will do so either as an exporter or as a multinational firm, but it will never adopt the two strategies at the same time.[22]

[19] $\tau > 1$ units of good need to be shipped for one unit of good to arrive to the destination country.

[20] The assumption that $F_I > F_X$ is key for our results on hysteresis and risk exposure of profit flows. It seems intuitive to us that the costs of starting operations in a new production facility are higher than the costs of establishing an export channel. FDI entails a series of one-time costly activities, like acquiring licenses, dealing with local institutions (often in a foreign language), searching for qualified local labor, arrange relationships with suppliers, and so on. When the investment is greenfield, these costs are added to the cost of actually building a foreign plant. When the investment takes the form of a merger, the firm has to pay the initial acquisition cost. Clearly in both cases foreign plants can be sold, so part of the initial cost may not be sunk. However, all those activities related to starting production in a foreign country are.

[21] Roberts and Tybout (1997) report evidence on the fact that previous exporting experience matters as long as firms do not exit the foreign market. They find that the costs of entry for first-time exporters are not statistically different from the costs of entry for second-time exporters, *i.e.* firms that were once selling in the foreign market, exited, and decided to re-enter. Our assumptions on the structure of sunk costs are motivated by these findings.

[22] This feature of the model is the same as in Helpman, Melitz, and Yeaple (2004). Rob and Vettas (2003) obtain the existence of an equilibrium where firms can optimally choose to adopt simultaneously the two strategies because in their model firms choose the amount of the foreign investment, and given the structure of demand there may be the possibility of overinvestment. In their framework, FDI can be adopted to cover certain demand, while exports are used to serve the additional random excess demand without incurring the cost of a larger investment that could be underutilized. In the data we do observe firms that both export and have FDI sales (about 6% of the total). This fact can be rationalized within our framework by having multiproduct firms that choose different strategies for different product lines, or in a multi-country model where firms choose different strategies to enter different countries. Unfortunately, there is not enough information in the Compustat Segments data to check whether any of these is the case. Explaining the

Hence for a given realization of $(Q, Q^*)$, a firm with productivity $1/a$ must choose its optimal status $S$ ($S \in \{D, X, I\}$, *i.e.* domestic, exporter, or multinational), the current selling price $p_S(a)$, and an updating rule (how to change the optimal price and status following changes in aggregate demand).

The CES aggregation over individual varieties implies that individual pricing rules are independent of $(Q, Q^*)$. However, marginal costs of production and optimal pricing rules vary with the status of the firm. Let $w$, $w^*$ denote the wages in the home and foreign countries, respectively. We describe here the pricing problem of Home country firms. Prices charged by Foreign country firms are determined in the same way.

The marginal cost of domestic production is given by the labor requirement times the domestic wage, $MC_D = aw$. The marginal cost of exporting is augmented by the iceberg transportation cost: $MC_X = \tau aw$. When the firm serves the foreign market through FDI, firm-specific productivity is transferred to the foreign country and the firm employs foreign labor: $MC_I = aw^*$. CES preferences across varieties of the differentiated good imply that the optimal prices are $p_S(a) = \frac{\eta}{\eta - 1} MC_S(a)$.

Let $\pi_D(a; Q)$, $\pi_X(a; Q^*)$ and $\pi_I(a; Q^*)$ denote the per-period profits from domestic sales, from exports and from FDI sales abroad, respectively, for a Home country firm with productivity $1/a$, given a realization of the aggregate quantity demanded $(Q, Q^*)$:

$$
\begin{aligned}
\pi_D(a; Q) &= B(aw)^{1-\eta} P^{\eta} Q && (8) \\
\pi_X(a; Q^*) &= B(\tau aw)^{1-\eta} P^{*\eta} Q^* - f_X && (9) \\
\pi_I(a; Q^*) &= B(aw^*)^{1-\eta} P^{*\eta} Q^* - f_I && (10)
\end{aligned}
$$

where $B \equiv \eta^{-\eta}(\eta - 1)^{\eta - 1}$, and $P$ ($P^*$) is the aggregate price of the differentiated good in the Home (Foreign) country, that firms take as given while solving their maximization problem.

## 3.2 Value Functions

We solve the model along the lines of Dixit (1989). The state of the economy is described by the vector $\Sigma = (Q, Q^*, \Omega, \Omega^*)$, where $\Omega = (\omega_X, \omega_I)$ ($\Omega^* = (\omega_X^*, \omega_I^*)$) describes the distribution of firms from the Home (Foreign) country into the three statuses.[23] In the

choice of firms to adopt both entry strategies would probably need a differently tailored framework, and is beyond the scope of this paper.

[23] $\omega_D = 1 - \omega_X - \omega_I$.

following, we omit the dependence of the value functions on $\Omega$ and $\Omega^*$ to ease the notation.

Let $\mathcal{V}_S(a, Q, Q^*)$ denote the expected net present value of a Home country firm whose productivity is $1/a$, starting in status $S$ ($S = D, X, I$) when the realization of aggregate demand is $(Q, Q^*)$, and following optimal policy. Since there are no fixed costs to sell domestically, all firms are active in their domestic market and make positive profits $\pi_D(a; Q)$ from domestic sales. Domestic activities are not affected by the realization of foreign demand $Q^*$. Similarly, the decision of whether to sell in the foreign market is not affected by the realization of domestic demand $Q$. For this reason, we can express the value function as:

$$\mathcal{V}_S(a, Q, Q^*) = \mathcal{S}(a, Q) + V_S(a, Q^*) \tag{11}$$

where $\mathcal{S}(a, Q)$ is the expected present discounted value of profits from domestic sales, which is independent on firm status, and $V_S(a, Q^*)$ is the expected present discounted value of profits from foreign sales for a firm in status $S$.

Over a generic time interval $\Delta t$, the two components of the value function for a firm that is currently selling only in its domestic market can be expressed as:

$$\mathcal{S}(a, Q) = \pi_D(a, Q) M\Delta t + E[M\Delta t \cdot \mathcal{S}(a, Q')|Q]. \tag{12}$$

$$V_D(a, Q^*) = \max\left\{E[M\Delta t \cdot V_D(a, Q^{*\prime})|Q^*]\ ;\ V_X(a, Q^*) - F_X\ ;\ V_I(a, Q^*) - F_I\right\}. \tag{13}$$

While (12) simply tracks the evolution of domestic profits, the right hand side of (13) expresses the firm's possibilities. If it sells only domestically, it gets the continuation value from not changing status, equal to the expected discounted value of the firm conditional on the current realization of foreign demand $Q^*$. If it decides to switch to export (FDI) it gets the value of being an exporter, $V_X$ (multinational, $V_I$) minus the sunk cost of entry $F_X$ ($F_I$). Similarly, the present discounted value of profits from foreign sales for an exporter is:

$$V_X(a, Q^*) = \max\left\{\pi_X(a, Q^*) M\Delta t + E[M\Delta t \cdot V_X(a, Q^{*\prime})|Q^*]\ ;\ V_D(a, Q^*)\ ;\ V_I(a, Q^*) - F_I\right\} \tag{14}$$

and for a multinational:

$$V_I(a, Q^*) = \max\left\{\pi_I(a, Q^*) M\Delta t + E[M\Delta t \cdot V_I(a, Q^{*\prime})|Q^*]\ ;\ V_D(a, Q^*)\ ;\ V_X(a, Q^*) - F_X\right\}. \tag{15}$$

Notice that the continuation value of an exporter (a multinational) also includes the profit flow from sales in the foreign market $\pi_X(a, Q^*)M\Delta t$ ($\pi_I(a, Q^*)M\Delta t$). There are no costs of exiting the foreign market: if a firm decides to exit, its value is simply that of a domestic firm: $V_D(a, Q^*)$.

In Appendix B, we show that the solution of the value functions $\mathcal{S}(a, Q)$, $V_S(a, Q^*)$ for $S \in \{D, X, I\}$ takes the form:

$$\begin{aligned}
\mathcal{S}(a, Q) &= \frac{\pi_D(a, Q)}{r - (\mu + \sigma\sigma_M)} &\quad (16)\\
V_D(a, Q^*) &= A_D(a)Q^{*\alpha} + B_D(a)Q^{*\beta} &\quad (17)\\
V_X(a, Q^*) &= A_X(a)Q^{*\alpha} + B_X(a)Q^{*\beta} + \frac{B(\tau a w)^{1-\eta}P^{*\eta}Q^*}{r - (\mu^* + \rho\sigma^*\sigma_M)} - \frac{f_X}{r} &\quad (18)\\
V_I(a, Q^*) &= A_I(a)Q^{*\alpha} + B_I(a)Q^{*\beta} + \frac{B(a w^*)^{1-\eta}P^{*\eta}Q^*}{r - (\mu^* + \rho\sigma^*\sigma_M)} - \frac{f_I}{r} &\quad (19)
\end{aligned}$$

where $\alpha$ and $\beta$ are the negative and positive values of $\xi$:[24]

$$\xi = \frac{(1-m) \pm \sqrt{(1-m)^2 + 4\bar{r}}}{2}$$

and $m = \frac{2(\mu^* + \rho\sigma^*\sigma_M)}{(\sigma^*)^2}$, $\bar{r} = \frac{2r}{(\sigma^*)^2}$. $A_S(a)$ and $B_S(a)$ ($S \in \{D, X, I\}$) are firm-specific, time-varying parameters to be determined.[25]

Notice that since there are no fixed or sunk costs associated to domestic production, there is no option value associated to future domestic profits. The value function $\mathcal{S}(a, Q)$ is simply equal to the discounted flow of domestic profits. Conversely, the option value of changing status is a component of the expected present discounted value of foreign profits. In particular, the option value is the only component of the present discounted value of foreign profits for domestic firms. For exporters and multinationals, the value is given by the sum of the discounted foreign profit flow from never changing status plus the option value of changing status. The terms $\mu + \sigma\sigma_M$, $\mu^* + \rho\sigma^*\sigma_M$ in the discount are the risk-adjusted drifts, result of taking expectations of the value function under the risk-neutral measure.

Equations (17), (18), and (19) describe the value of foreign profits in the firms' contin-

[24] $\alpha < 0$, $\beta > 1$.

[25] Remember that we are not making explicit the dependence of the value functions on the distribution of firms in the three statuses. The parameters $A_D(a)$ and $B_D(a)$ are time-varying because they also depend on the distribution of firms.

uation regions. We still need to solve for the updating rule, which in this case consists of thresholds in the realizations of $Q^*$ that induce firms to change status. Let $Q^*_{RS}(a)$ denote the quantity threshold at which a firm with productivity $1/a$ switches from status $R$ to status $S$, for $R, S \in \{D, X, I\}$.[26] In order to find the six quantity thresholds $Q^*_{SR}(a)$ and the six value function parameters $A_S(a)$, $B_S(a)$, for $S \in \{D, X, I\}$, we impose the following value-matching and smooth-pasting conditions:

$$
\begin{aligned}
V_D(a, Q^*_{DX}(a)) &= V_X(a, Q^*_{DX}(a)) - F_X && (20) \\
V_D(a, Q^*_{DI}(a)) &= V_I(a, Q^*_{DI}(a)) - F_I && (21) \\
V_X(a, Q^*_{XD}(a)) &= V_D(a, Q^*_{XD}(a)) && (22) \\
V_X(a, Q^*_{XI}(a)) &= V_I(a, Q^*_{XI}(a)) - F_I && (23) \\
V_I(a, Q^*_{ID}(a)) &= V_D(a, Q^*_{ID}(a)) && (24) \\
V_I(a, Q^*_{IX}(a)) &= V_X(a, Q^*_{IX}(a)) - F_X && (25) \\
V'_R(a, Q^*_{RS}(a)) &= V'_S(a, Q^*_{RS}(a)) \ , \ for \ S, R \in \{D, X, I\}. && (26)
\end{aligned}
$$

For each $a$, equations (20)-(26) are a system of twelve equations in twelve unknowns (the six quantity thresholds $Q^*_{SR}(a)$ and the six parameters $A_S(a)$, $B_S(a)$, for $S, R \in \{D, X, I\}$). The system is highly nonlinear, and as such is associated to multiple solutions. To get an economically sensible solution, we follow Dixit (1989) and impose a series of restrictions on the parameters $A_S(a)$, $B_S(a)$. Since $\alpha < 0$, $\beta > 1$, the terms in $Q^{*\alpha}$ ($Q^{*\beta}$) are large for low (high) realizations of $Q^*$. For low realizations of $Q^*$, entry is a remote possibility for a firm selling only in its domestic market, hence the value of the option of entering must be nearly worthless: $A_D(a) = 0$, $\forall a$. It must then be that $B_D(a) \geq 0$ to insure non-negativity of $V_D(a, Q^*)$.

Similarly, for high realizations of $Q^*$, the option of quitting FDI for another strategy is nearly worthless, hence $B_I(a) = 0$. Moreover, a multinational firm has expected value $\frac{B(aw^*)^{1-\eta}(P^*)^\eta Q^*}{r-(\mu^*+\rho\sigma^*\sigma_M)} - \frac{f_I}{r}$ from the strategy of never changing status, hence the optimal strategy must yield a no lesser value: $A_I(a) \geq 0$.

Finally, an exporter has expected value $\frac{B(\tau aw)^{1-\eta}(P^*)^\eta Q^*}{r-(\mu^*+\rho\sigma^*\sigma_M)} - \frac{f_X}{r}$ from the strategy of never changing status, hence its optimal strategy must yield a no lesser value for any realization

[26] The quantity thresholds $Q^*_{RS}(a)$ also depend on the distribution of firm statuses $\Omega^*$, which affects the equilibrium price index, and are hence time-varying.

of $Q^*$: $A_X(a), B_X(a) \geq 0$.

As a result of these restrictions, the value function of a domestic firm $V_D$ is increasing on the entire domain, indicating the fact that, as the realized aggregate demand in the foreign market $Q^*$ increases, the value of the option of entering the foreign market (either through trade or FDI) increases. The value functions of an exporter and of a multinational ($V_X$ and $V_I$ respectively) are U-shaped: for low levels of $Q^*$, the term with the negative exponent $\alpha$ dominates, and the value is high due to the option of leaving the market. Conversely, for high levels of $Q^*$, the value is high due to the profit stream that the firm derives from staying in the market and, for exporters, due to the additional option value of becoming a multinational firm (the term with the positive exponent $\beta$).

Value functions and quantity thresholds for Foreign country firms are derived in an analogous manner.

The system of value-matching and smooth-pasting conditions includes among its variables the aggregate price index $P^*$. $P^*$ is an endogenous variable and – as will be clearer in the next section – it depends on the realization of $Q^*$. For this reason, one should write each condition taking into account the equilibrium price at that specific realization of $Q^*$ (*i.e.*, if $Q^* = Q^*_{DX}$, then $P^* = P^*(Q^*_{DX})$). However, we appeal to the result developed in Leahy (1993) and Dixit and Pindyck (1994), Chapters 8-9, whereby a firm can ignore the effects of the actions of other firms when solving for the optimal thresholds triggering investment, and use the market equilibrium price $P^*$ in all the value-matching and smooth-pasting equations. The intuition behind this results is as follows: the actions of other firms affect the problem of the individual firm via the price index in two ways: more firms entering the foreign market reduce a) the profit flows from foreign sales (export or FDI), and b) the option value of waiting to start selling abroad. It can be shown that these two effects exactly offset each other; hence, taking into account the effect of the actions of other firms on the price index is immaterial for the determination of the thresholds.[27]

[27] Dixit and Pindyck (1994) and Leahy (1993) show this results for a perfectly competitive industry with free entry and CRS production technologies, where the shocks follow a general diffusion process. Leahy (1993) also shows that free entry is unnecessary to obtain the result. Our economy differs from the ones they study in that firms' technologies exhibit increasing returns to scale and the market structure is monopolistically competitive. However, we argue that the result still applies for the following reasons. The potential problem with increasing returns is that they may induce "too large" investment by the firms. This does not apply to our framework, where firms only decide whether to entry or not, and not the amount to invest. Imperfect competition in turn may invalidate the result if firms display some type of strategic behavior, which is clearly not the case for monopolistic competition with a continuum of firms.

### 3.3 Equilibrium

The price indexes in the two countries are the solution of the following system of two equations, where each price index is an aggregate of prices of domestic sales, prices of imports, and prices of FDI sales of multinational firms from the other country:

$$P^{1-\eta} = n \int \left(\frac{\eta a w}{\eta - 1}\right)^{1-\eta} dG(a) + ...$$

$$...n^* \left[\int_{\omega_X^*(Q)} \left(\frac{\eta \tau a w^*}{\eta - 1}\right)^{1-\eta} dG^*(a) + \int_{\omega_I^*(Q)} \left(\frac{\eta a w}{\eta - 1}\right)^{1-\eta} dG^*(a)\right] \tag{27}$$

$$(P^*)^{1-\eta} = n^* \int \left(\frac{\eta a w^*}{\eta - 1}\right)^{1-\eta} dG^*(a) + ...$$

$$...n \left[\int_{\omega_X(Q^*)} \left(\frac{\eta \tau a w}{\eta - 1}\right)^{1-\eta} dG(a) + \int_{\omega_I(Q^*)} \left(\frac{\eta a w^*}{\eta - 1}\right)^{1-\eta} dG(a)\right] \tag{28}$$

where $n$ ($n^*$) is the mass of firms from the Home (Foreign) country selling differentiated varieties, and $\omega_X^*(Q)$, $\omega_I^*(Q)$ ($\omega_X(Q^*)$, $\omega_I(Q^*)$) are the shares of these firms that export or have multinational sales when the realization of aggregate demand is $(Q, Q^*)$.

The equations describing aggregate prices depend – via the integration limits – on the distribution of firms into statuses, which in turn depends on the quantity thresholds $Q_{RS}^*$. On the other hand, quantity thresholds themselves depend on aggregate prices (as evident from the value functions). In solving the firm's problem, we appeal to the equivalence result shown in Leahy (1993): when finding the quantity thresholds, each firm takes aggregate prices and the firms' distribution into statuses as given, and does not take into account the effect of its own entry and exit decisions on these variables. This result simplifies considerably the computation of the equilibrium. The computational algorithm is described in Appendix C.

Since we abstract from the problem of entry in the domestic market, we take the mass of firms $n$ ($n^*$) as given.[28] The initial values of the processes ruling the evolution of the state, $Q(0)$ and $Q^*(0)$, are also taken as given.

[28]Since we do not impose free-entry, we set the masses of firms to $n = n^* = 1$, and present the results in terms of shares of the total number of firms.

### 3.4 Earnings-to-Price Ratios and Returns

The solution of the model delivers quasi-closed form solutions (up to multiplicative parameters) for the value functions $\mathcal{V}_S(a, Q, Q^*)$ ($S \in \{D, X, I\}$), and allows us to compute easily the earnings-to-price ratios and returns generated by the model.

Our earnings yields measure in the model is given by the ratio $\pi_t/\mathcal{V}_t$, where $\pi_t$ represents per-period profits and $\mathcal{V}_t$ is the market value of the firm. In a static model, or in a dynamic but deterministic model, or in a dynamic model with non-persistent shocks, $\pi_t/\mathcal{V}_t$ is constant and independent on the firm's status, since per-period profits and value of the firm are proportional. Dynamics and uncertainty introduce a wedge between these two magnitudes, which reflects the option value.

Let $ep_S(a, Q, Q^*)$ denote the earnings yields of a firm with productivity $1/a$ in status $S$ when the realization of aggregate demand is $(Q, Q^*)$. Earnings yields in the model are given by:

$$ep_D(a, Q, Q^*) = \frac{\pi_D(a, Q)}{\mathcal{V}_D(a, Q, Q^*)} \tag{29}$$

$$ep_X(a, Q, Q^*) = \frac{\pi_D(a, Q) + \pi_X(a, Q^*)}{\mathcal{V}_X(a, Q, Q^*)} \tag{30}$$

$$ep_I(a, Q, Q^*) = \frac{\pi_D(a, Q) + \pi_I(a, Q^*)}{\mathcal{V}_I(a, Q, Q^*)}. \tag{31}$$

The empirical evidence presented in Section 2 suggests the following ordering in aggregate earnings yields across groups:

$$\int_{\omega_D(Q^*)} ep_D(a, Q, Q^*)dG(a) < \int_{\omega_X(Q^*)} ep_X(a, Q, Q^*)dG(a) < \int_{\omega_I(Q^*)} ep_I(a, Q, Q^*)dG(a). \tag{32}$$

While is not possible to prove analytically that the model generates this ordering, the results of our numerical simulations confirm that the calibrated model is consistent with it, and that the ordering is robust to a number of variations in the calibration.

Returns in the model are given by the earnings yields plus the expected change in the valuation of the firm:

$$ret_S(a, Q, Q^*) = ep_S(a, Q, Q^*) + \frac{E[d\mathcal{V}_S(a, Q, Q^*)]}{\mathcal{V}_S(a, Q, Q^*)}, \ for\ S \in \{D, X, I\}. \tag{33}$$

Also in this case, the model does not have clear-cut analytical predictions for the ordering of $\frac{E[d\mathcal{V}_S(a,Q,Q^*)]}{\mathcal{V}_S(a,Q,Q^*)}$. The value of this object depends on the curvature of the value functions and it also critically depends on the calibration, since different firms exhibit different value functions and respond differently to the same realizations of the shocks. For the model to reproduce the ordering found in the data:

$$\int_{\omega_D(Q^*)} ret_D(a,Q,Q^*)dG(a) < \int_{\omega_X(Q^*)} ret_X(a,Q,Q^*)dG(a) < \int_{\omega_I(Q^*)} ret_I(a,Q,Q^*)dG(a) \tag{34}$$

we need the differences in $\frac{E[d\mathcal{V}_S(a,Q,Q^*)]}{\mathcal{V}_S(a,Q,Q^*)}$ not to overturn the ordering of the earnings yields, which is what our baseline calibration delivers. The robustness exercises in Section 6 highlight the sensitivity of this component of the returns to selected parameters of the model.

The results of the calibrated model are presented in Section 5. Before moving to the quantitative results, in the next section we show a series of qualitative properties of the model that illustrate its amenability to reproduce features of the trade dynamics data.

# 4 Qualitative Properties of the Solution

In this section we illustrate the workings of the model with a series of analytical properties of the solution. These properties highlight the potential of the model to account for export and FDI dynamics of heterogeneous firms, and the departure from the deterministic model which allows to match the ranking in the financial variables.

## 4.1 Ordering of the Quantity Thresholds

The relationship between the sunk costs of exporting and FDI, $F_I > F_X$, implies a precise ordering of the quantity thresholds that are solution of (20)-(26).

**Theorem 1.** *If $F_I > F_X$, $f_I = f_X$, and $\tau w = w^*$, the quantity thresholds $Q^*_{RS}(a)$, for $R, S \in \{D, X, I\}$ and for a given productivity level $1/a$, satisfy the following ordering:*

$$Q^*_{IX}(a) < Q^*_{ID}(a) < Q^*_{XD}(a) < Q^*_{DX}(a) < Q^*_{DI}(a) < Q^*_{XI}(a). \tag{35}$$

**Proof:** See Appendix B.

The ordering in (35) is easy to prove in the extreme case in which operating costs are equal across modes of serving the foreign market. However, the ordering holds in our numerical results for a wide range of parameterizations. As long as the differences in operating costs of exports *versus* FDI are "not too large" the thresholds will be ordered according to (35).

The intuition behind this property of the model is straightforward. Like in Dixit (1989), the pure presence of sunk costs implies that entry thresholds are higher than exit thresholds: $Q^*_{DX} > Q^*_{XD}$, $Q^*_{DI} > Q^*_{ID}$, and $Q^*_{XI} > Q^*_{IX}$. A higher quantity demanded $Q^*$ is needed to induce a firm to pay the sunk entry cost of FDI with respect to the quantity necessary to induce the firm to export: $Q^*_{DI} > Q^*_{DX}$. An even larger positive shock is needed to induce and exporter to become a multinational, since it is already serving the foreign market with exports and already paid a sunk cost: $Q^*_{XI} > Q^*_{DI}$. Similarly when considering exit, a larger negative shock is needed to induce a multinational to exit the foreign market with respect to the shock needed to induce an exporter to exit: $Q^*_{ID} < Q^*_{XD}$. Finally, an even larger negative shock is needed to induce a multinational to divest but still serve the foreign market as an exporter: $Q^*_{IX} < Q^*_{ID}$.

The ordering of expression (1) applies for a given productivity level $1/a$. The following subsection illustrates the behavior of the value functions and quantity thresholds for different values of firm-level productivity.

### 4.2 Comparative Statics: Value and Productivity

System (20)-(26) makes clear that both the quantity thresholds and the parameters of the value functions depend on the productivity level $1/a$. Figure 2 shows the value function of a domestic firm as a function of the aggregate quantity demanded in the foreign market $Q^*$ and of productivity $1/a$. $V_D$ is increasing in $Q^*$, as the option value of entering the foreign market is increasing in the quantity demanded. $V_D$ is also increasing in firm's productivity, as more productive firms can get higher profits from entering the foreign market.

Figure 3 shows the value functions of an exporter and of a multinational firm as functions of $Q^*$ and $1/a$. $V_X$ and $V_I$ are U-shaped functions of $Q^*$, indicating the high option value of exiting for low realizations of $Q^*$ and the high option value of not changing status for high realizations of $Q^*$.[29] The behavior of the value functions for $Q^* \to 0$ does not vary much across the productivity dimension: when $Q^*$ is low, the value is high as firms of

[29]Notice that for $Q^* \to \infty$, the value function of an exporter is steeper than the one of a multinational, because the exporter gets high value both from staying in the market as an exporter and from the option value of becoming a multinational.

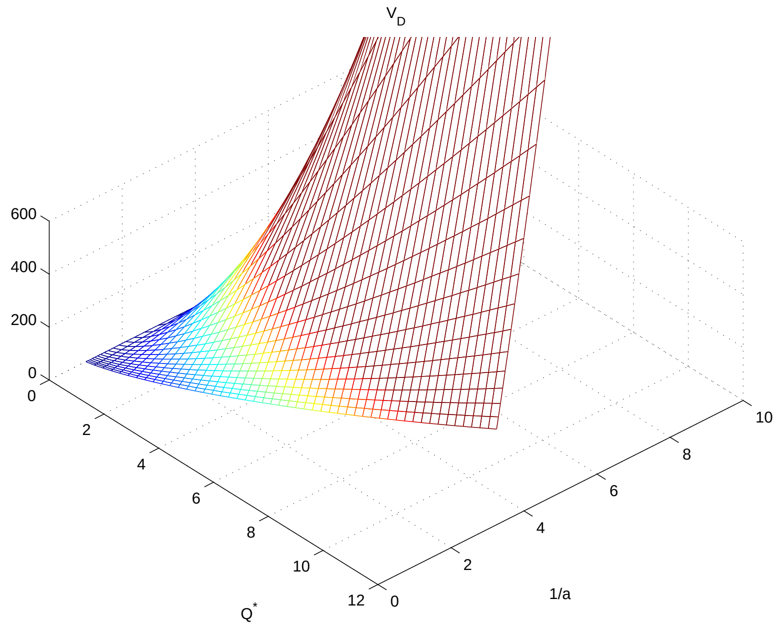


Figure 2: Value function of a domestic firm.

all productivity levels associate a high value to the option of exiting. Conversely, the behavior of the value functions when $Q^*$ is "large" varies with individual productivity: the value function is steeper for higher productivity firms, indicating that more productive firms obtain higher returns from staying in the foreign market when the realized aggregate demand is high.

From Figure 3, the qualitative behavior of $V_X$ and $V_I$ appears very similar. Figure 4 plots the difference between the value functions of firms serving the foreign market and firms selling only domestically, $V_X - V_D$ and $V_I - V_D$. For each productivity level $1/a$, each plot has two stationary points, a local maximum and a local minimum. The value matching and smooth pasting conditions imply that the local maxima correspond to the "entry" thresholds ($Q^*_{DX}$ and $Q^*_{DI}$ in the left and right plot respectively), while the local minima correspond to the "exit" thresholds ($Q^*_{XD}$ and $Q^*_{ID}$). The picture shows that both entry and exit thresholds are decreasing in $1/a$, indicating that more productive firms enter the foreign market for lower realizations of aggregate demand $Q^*$ with respect to less productive firms. Similarly, more productive firms need larger negative shocks to demand to be induced to exit the foreign market with respect to less productive firms. Notice that for $Q^* \to 0$, $V_X - V_D$ and $V_I - V_D$ tend to infinity, because the option value of exiting the

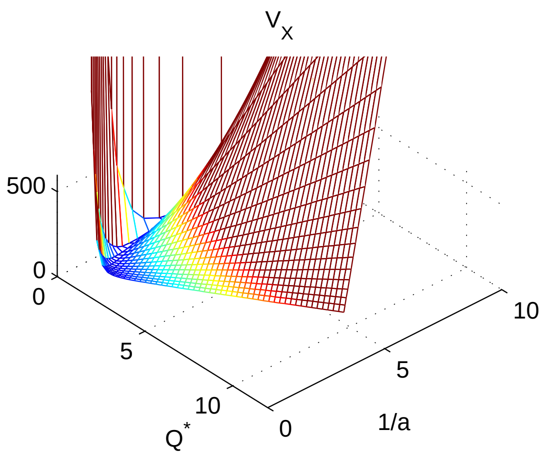


Figure 3: Value functions of an exporter and of a multinational firm.

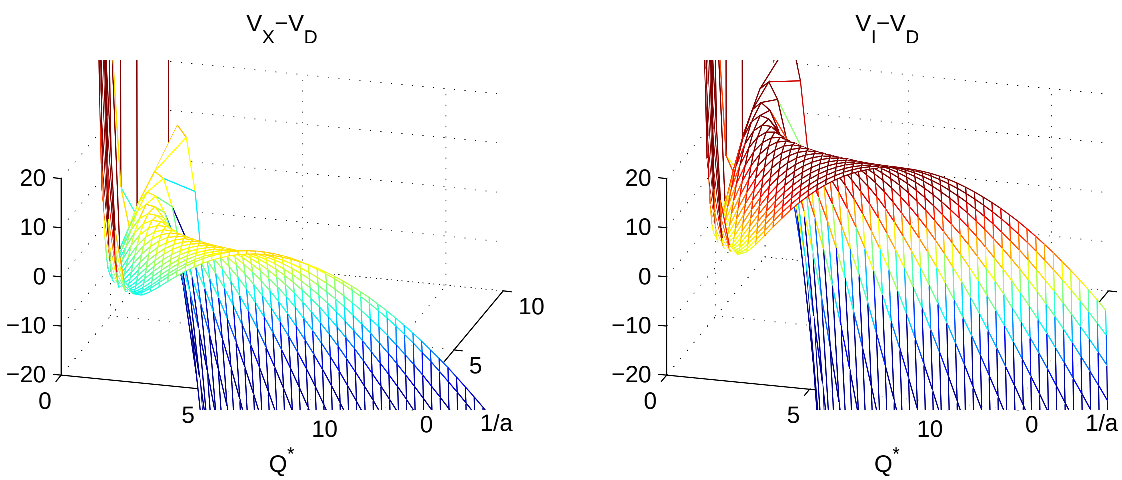


Figure 4: Difference between the value functions of exporters and multinationals and the value function of domestic firms.

foreign market is extremely high for very low realizations of $Q^*$ (and irrespective of firm's productivity). Conversely, for $Q^* \to \infty$, $V_X - V_D$ and $V_I - V_D$ tend to negative infinity, because the domestic firms' option value of entering the foreign market is extremely high, compared to the flow profits of staying for firms that are already serving that market. The difference between the value functions of a multinational firm and of an exporter displays similar properties.

Figure 4 suggests a systematic relationship between the quantity thresholds $Q^*_{RS}$ and the firm productivity level $1/a$. Theorem 2 establishes this result.

**Theorem 2.**

$$\frac{\partial Q^*_{RS}(a)}{\partial a} > 0, \quad for \quad R, S \in \{D, X, I\}, \ \forall a. \tag{36}$$

**Proof:** See Appendix B.

Theorem 2 establishes that the six thresholds $Q^*_{RS}(a)$ are decreasing in productivity $1/a$, indicating that more productive firms need smaller positive shocks to demand to enter the foreign market, and larger negative shocks to exit. The one-to-one correspondence between productivities and quantity thresholds established by Theorem 2 implies that the functions $Q^*_{RS}(a)$ are invertible, hence for each realization of aggregate foreign demand $Q^*$ we can compute six productivity thresholds $a_{RS}(Q^*)$, for $R, S \in \{D, X, I\}$, that determine the selection of heterogeneous firms into the three statuses and their likelihood of switching across statuses. This redefinition of the thresholds in terms of productivity is extremely helpful to compute the model numerically. The shares of firms belonging to each status can be written as functions of the productivity thresholds $a_{RS}$, so the law of motion of the status distribution is given by:

$$\omega_{Dt+1} = \omega_{Dt} \cdot [1 - G(a_{DX})] + \omega_{Xt} \cdot [1 - G(a_{XD})] + \omega_{It} \cdot [G(a_{IX}) - G(a_{ID})] \tag{37}$$

$$\omega_{Xt+1} = \omega_{Dt} \cdot [G(a_{DX}) - G(a_{DI})] + \omega_{Xt} \cdot [G(a_{XD}) - G(a_{XI})] + \omega_{It} \cdot [1 - G(a_{IX})] \tag{38}$$

$$\omega_{It+1} = \omega_{Dt} \cdot G(a_{DI}) + \omega_{Xt} \cdot G(a_{XI}) + \omega_{It} \cdot G(a_{ID}) \tag{39}$$

where we omitted the dependence of $\omega_{St}$ and $a_{RS}$ on $Q^*$ to ease the notation. For a given initial distribution $\Omega_0$, equations (37) - (39) describe its evolution depending on the productivity thresholds ruling firms' allocation into statuses. Notice that the sets $\omega_{St}$ vary with the realization of $Q^*$, as firms may switch status, but only depend on the firms' status in the previous period, due to the Markov property of Brownian motions.

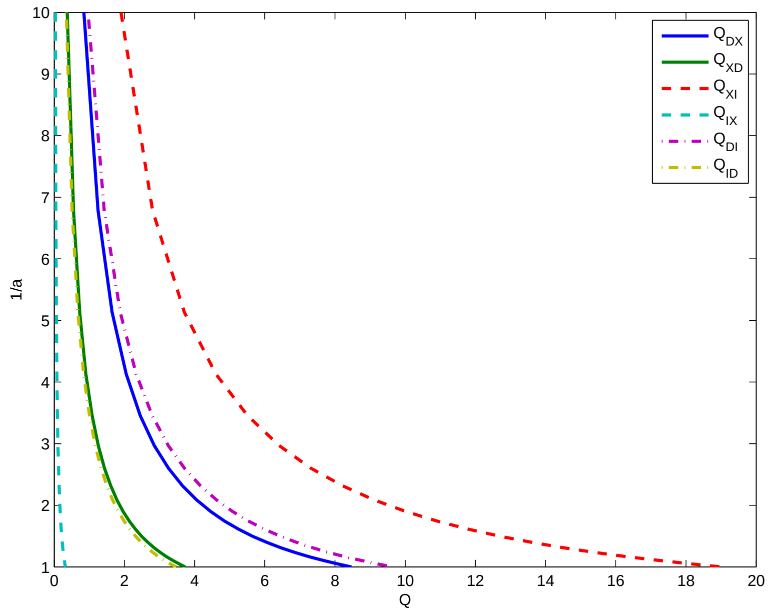


Figure 5: Quantity thresholds as functions of firm's productivity.

Figure 5 illustrates Theorem 2. For an arbitrary parametrization, we plot the six quantity thresholds as functions of firm-level productivity. The picture also shows an additional property of the thresholds: hysteresis, defined as the horizontal distance between entry and exit thresholds, is also decreasing in productivity, indicating that – for the same choice of status – more productive firms suffer less from being locked into a market by the presence of sunk entry costs, and exhibit less hysteresis than less productive firms. On the other hand, more productive firms self-select into the status ($I$) which is associated with more hysteresis.[30] This ambiguous result generates an imperfect sorting of productivities into status, which is a well-documented feature of the data.

## 5 Empirical Results

The objective of this section is an evaluation of the model performance in matching qualitatively and quantitatively features of the data on trade and FDI dynamics, and the pattern of earnings yields and returns across firms. In Section 5.1 we calibrate the parameters of the model to quantitatively match the switching pattern and the relative presence of the

[30] Dixit and Pindyck (1994) show that hysteresis is increasing in the sunk cost of investment: entry (exit) thresholds are increasing (decreasing) in the sunk cost, hence the difference between entry and exit thresholds is also increasing in the sunk cost.

three types of firms in the data. In Section 5.2 we use the calibrated version of the model to compute earnings yields and returns, which are not targeted moments in the calibration. We show that the quantitative theory is able to replicate the observed ranking in financial variables that we documented in Section 2.

### 5.1 Calibration

The calibration exercise that we present in this section is designed to match a series of facts on exports and FDI sales dynamics. We do not make any use of financial variables in this exercise. In the next section we show that the model calibrated by targeting trade facts only performs well also in matching non-targeted moments like earnings yields and returns.

We present a bilateral calibration exercise, that describes export and FDI activity between the U.S. and an aggregate set of trading partners. Due to data availability, we impose a series of symmetry assumptions. In particular, we assume that preferences and productivity distributions are identical in the U.S. and in the other countries, and that the cost structure (the values of $\tau$, $f_X$, $F_X$, $f_I$, $F_I$) is also the same across countries.[31]

To calibrate the model, we need to choose a functional form for the cost distribution $G(a)$, and assign values to its parameters. We need to parameterize the Brownian motions, and choose values for preference parameters and parameters describing trade and FDI costs. We refer to the literature to assign parameters to the preferences and to the firms' productivity distributions. The parameters ruling the Brownian motions are chosen to match data on standard deviations and correlation of GDP growth between the U.S. and its major trading partners. We choose the remaining parameters so that the model matches a series of moments computed from the data. We start describing the calibration with the parameters we adopt from the literature.

Several studies document that the tail of the empirical firm size distribution is well approximated by a Pareto distribution (see for example Luttmer (2007)). Since firm size (sales) is linked to the productivity distribution in the model, we assume that firms' productivities $1/a$ are distributed according to a Pareto law with location parameter $b$ and shape parameter $k$.[32] We normalize $b = 1$, and choose $k$ to match the coefficient of the empirical

[31] Compustat records data of firms with activities in the U.S., among which there are both U.S. based firms and foreign firms. However, only data of foreign firms with activities in the U.S. are reported (in other words, we have no data about foreign firms with activities only in their domestic market), which implies that we cannot construct shares of foreign firms in each status or their dynamic behavior.

[32] The Pareto distribution is also a convenient choice for computational reasons, since it allows to solve

sales distribution: if productivity is Pareto-distributed with shape parameter $k$, sales in the model are also Pareto-distributed with shape parameter $k/(\eta-1)$. By regressing firm rank on firm size, Luttmer (2007) finds that $k/(\eta-1) = 1.06$. We then choose $k$ accordingly, given a value for $\eta$. There is little agreement in the literature on the value to attribute to the elasticity of substitution across differentiated varieties, $\eta$. Many papers that focus on long-run macroeconomic predictions use a standard value of 2. Other papers that focus on matching data at business cycle frequencies choose much higher values. Alessandria and Choi (2007), for example, set $\eta \approx 10$ (to match markups of about 11%). We set $\eta = 2.65$, equal to the median value in Broda and Weinstein (2006) SITC 5-digit estimates.[33] This choice implies $k = 1.749$. Finally, we set the risk aversion parameter to $\gamma = 20$, as an intermediate value among those proposed by the literature.[34]

We abstract from labor cost and market size differences, and set both wages to $w = w^* = 1$ and the mass of firms in each country to $n = n^* = 1$.

We impose that the drifts of the Brownian motions ruling the evolution of $Q, Q^*$ have value $\mu = \mu^* = 0$. The need to impose zero expected demand growth arises from the fact that we abstract from firms' productivity growth.[35] We compute $\sigma$ and $\sigma^*$ as the standard deviations of consumption growth for the U.S. and for a set of OECD countries:

$$\begin{aligned} \sigma &= st.dev.(g_t^{us}) = 0.031 \\ \sigma^* &= st.dev.(g_t^{oecd}) = 0.023 \end{aligned}$$

explicitly for the aggregate prices $P$, $P^*$ as functions of the productivity thresholds $a_{RS}$ and of the other parameters of the model.

[33] Broda and Weinstein (2006) estimate sectoral elasticities of substitution from price and volume data on U.S. consumption of imported goods. By using data at the 10-digit Harmonized System, they estimate how much demand shifts between 10-digit varieties when relative prices vary, within each 3-digit SITC sector.

[34] Assigning a value to $\gamma$ is a difficult choice to make in this setting. In their seminal contribution, Mehra and Prescott (1985) report evidence from several micro and macro studies suggesting a value of $\gamma$ between 1 and 4. They also show that a model with CRRA preferences and such a low value of $\gamma$ can match the risk-free rate, but generates returns that are too low compared with the data (the equity premium puzzle). Hansen and Singleton (1982) estimate the value of $\gamma$ that matches returns, obtaining a value of $\gamma \approx 60$. This high value is implausible based on the empirical evidence, and generates too high a risk-free rate. Our model features CRRA preferences for the differentiated good, and is hence subject to the same problem: we are not able to match quantitatively both the risk-free rate and the returns. For this reason, we choose an "intermediate" value in our baseline calibration. The choice of $\gamma = 20$ is consistent with empirical estimates of the Sharpe ratio: $S \equiv \frac{E(ret-r)}{\sigma_{ret}}$. It is possible to prove that $S \leq (1+r)(\gamma\sigma_{\Delta C})$. Estimates of the Sharpe ratio for the U.S. found $S \approx 0.4$. Our chosen value of $\gamma$ is in line with these estimates, generating a Sharpe ratio of 0.47.

[35] If $\mu > 0$, $E(dQ/Q)$ would be increasing over time and for $t \to \infty$ all firms would become multinationals. By setting $\mu = 0$, we are implicitly assuming that $Q$, $Q^*$ and $b$ grow at a rate such that the distribution of firms in the three groups does not degenerate over time.

where $g_t^{oecd}$ ($g_t^{us}$) is the average growth rate of consumption across OECD countries (in the U.S.) in year $t$ ($t$=1979-2006).[36] We set the risk-free rate in both economies ($r$ and $r^*$) to match a long-run average of 3-month T-bills rate of 2%. For the calibration of $\sigma_M$ and $\sigma_M^*$, notice that CRRA preferences with risk-aversion coefficient $\gamma$ imply that $\sigma_M = \gamma\sigma$ ($\sigma_M^* = \gamma\sigma^*$).[37] Hence $\sigma_M = 0.46$ and $\sigma_M^* = 0.62$. The correlation coefficient between the two Brownian motions, $\rho$, is a key parameter in this exercise. To select a value for $\rho$, we computed correlations in GDP growth rates between the U.S. and a set of countries that include the U.S. major trading partners (Canada, Mexico, France, Germany, the U.K., Spain, Japan, Hong Kong, Korea, Malaysia and Taiwan) and the largest developing economies (China, Brazil and India). For the sample period (1979-2006) correlation vary from a minimum of -0.02 (between U.S. and Brazil) to a maximum of 0.786 (between U.S. and Canada). Brazil is the only country in the sample displaying a negative correlation. The mean correlation is 0.19, and the median is 0.12. Based on this numbers, for our baseline calibration we choose a value of $\rho = 0.12$.

It remains to calibrate the variable trade cost $\tau$, fixed operating costs $f_X$, $f_I$, sunk costs $F_X$, $F_I$ (that we assume to be equal in the U.S. and in its aggregate trading partners), and the initial values for the aggregate demand levels, $Q(0)$ and $Q^*(0)$. We follow the methodology of the calibration in Alessandria and Choi (2007), and select values for these parameters to match a set of moments related to trade and FDI dynamics. We target data on firms' persistence in the same status and on the shares of the three types of firms in the data.

We compute all the moments from Compustat data. In terms of status persistence, on average, 93.11% of domestic firms remain domestic the following year, while 3.73% of them become exporters, and the remaining become multinationals. 90.27% of exporters continue exporting the following year, while 5.96% of them become multinationals, and the remaining exit the foreign market to sell domestically only. Multinational firms exhibit even higher persistence, with 98.31% of them continuing being multinationals the following year, and only 0.82% of them becoming exporters the following year. Domestic firms' and exporters' persistence moments are close to the ones reported in Alessandria and Choi (2007), but we are unaware of other papers computing moments related to persistence in multinational activity.

[36] Data source: OECD Statistics.
[37] See footnote 16.

Table 4: **Summary of Calibrated Parameters.**

| Parameter | Definition | Value | Source |
|---|---|---|---|
| Brownian motions | | | |
| $\mu$, $\mu^*$ | drift of $Q$, $Q^*$ | 0 | no productivity growth |
| $\sigma$ | variance of $Q$ | 0.023 | st.dev. of U.S. cons. growth |
| $\sigma^*$ | variance of $Q^*$ | 0.031 | st.dev. of OECD cons. growth |
| $\rho$ | correlation of $Q$, $Q^*$ | 0.12 | GDP growth correlations |
| $r$, $r^*$ | risk-free rate | 0.02 | 3-month T-bills rate |
| $\sigma_M$ | variance of s.d.f. | 0.46 | model restriction ($\sigma_M = \gamma\sigma$) |
| $\sigma_M^*$ | variance of s.d.f. | 0.62 | model restriction ($\sigma_M^* = \gamma\sigma^*$) |
| Pareto distribution | | | |
| $b$ | lower bound | 1 | normalization |
| $k$ | shape parameter | 1.749 | Luttmer (2007) |
| Preferences | | | |
| $\eta$ | el. of substitution | 2.65 | Broda and Weinstein (2006) |
| $\gamma$ | risk aversion | 20 | Sharpe ratio |
| Trade and FDI costs | | | |
| $\tau$ | iceberg export cost | 1.32 | to match data |
| $f_X$ | fixed export cost | 100 | to match data |
| $f_I$ | fixed FDI cost | 210 | to match data |
| $F_X$ | sunk export cost | 600 | to match data |
| $F_I$ | sunk FDI cost | 1500 | to match data |
| Aggregate demand | | | |
| $Q(0)$, $Q^*(0)$ | initial demand | 0.5 | to match data |

Next, we look at the average share of firms in each status. In Compustat, the average share of firms selling only domestically is 34.81% of the sample, while exporters are 29.2% of the sample, and multinational firms account for the remaining 35.99% of the sample. As previously noted, these numbers reflect selection of the largest firms in the data set.[38] Despite the divergence of our moments with the ones reported in other papers, we decided to match Compustat data in this exercise to be internally consistent. While not representative of the entire population of U.S. firms, Compustat offers a detailed portrait of the largest firms in the economy, which are the major players in determining volumes of trade and

[38]Bernard et al. (2007), among others, report that the average share of manufacturing firms that export is about 18%, while Bernard and Jensen (2007) report that multinational firms represent only 1% of manufacturing firms.

FDI.[39]

Table 4 summarizes the calibrated parameters. The calibrated iceberg cost is 32%, consistent with a medium-range estimate in Eaton and Kortum (2002). Sunk costs of export and FDI equal to 600 and 1500, respectively, indicate that a domestic firm must spend on average 3.68 times its per-period revenue to enter the foreign market as an exporter, and about 9.2 times its per-period revenue to start FDI operations there. Fixed costs of export and FDI equal to 100 and 210, respectively, indicate that an exporter must spend on average 39.9% of its per-period revenue in operating costs, and that a multinational firms must spend on average 6.54% of its per-period revenue in operating costs. Aggregate demand parameters are set at $Q = Q^*(0) = 0.5$.

Table 5: **Moments.** Comparison of the moments, model *versus* data. (Source: Compustat).

| | Data | Model | | Data | Model |
|---|---|---|---|---|---|
| $D \to D$ (%) | 93.11 | 95.06 | $X$ (%) | 29.2 | 26.73 |
| $D \to X$ (%) | 3.73 | 2.68 | $I$ (%) | 35.99 | 29.74 |
| $X \to X$ (%) | 90.27 | 90.91 | | | |
| $X \to I$ (%) | 5.96 | 5.31 | | | |
| $I \to I$ (%) | 98.31 | 97.74 | | | |
| $I \to X$ (%) | 0.82 | 2.26 | | | |

Table 5 displays jointly the moments computed from the data and the moments generated by the calibrated model. Overall, the model performs well in matching the moments from the data: the calibration error, computed as the sum of squared differences between the model-generated moments and the moments computed from the data, is equal to 0.0053. In particular the persistence parameters are precisely matched, while the shares of exporters and multinational are slightly smaller than in the data.

[39]The choice of a Pareto distribution for firms' productivity is robust to the selection problem associated with dealing with Compustat data. The Pareto distribution is invariant to lower truncations, hence if we assume that the entire productivity distribution is Pareto with parameters $(b, k)$, the distribution of firms in Compustat will also be Pareto, with parameters $(b', k)$, $b' > b$. As the lower bound of the distribution does not enter the computation of the moments, we normalize it to one.

## 5.2 Quantitative Results: Earnings Yields and Returns

With the calibrated version of the model, we compute earnings yields and returns across the three groups of firms. In our calculations, we follow the construction of the portfolios we used in the data analysis presented in Section 2. We generate an artificial dataset of 100 firms with productivities drawn from a Pareto distribution with parameters $(b, k) = (1, 1.749)$, and we simulate a 20-period economy 100 times.[40] In each simulation, we initialize the firms' distribution into status by assuming that all firms start domestic, and we generate a sample process for $(Q, Q^*)$. Given the process of the shocks, we simulate the economy, recording the distributions of firms into status in each period. For each firm and period, we compute earnings, equilibrium value (our model-based measure of market capitalization), and variation in the equilibrium value of the firm. For each year we create three portfolios of domestic firms, exporters, and multinationals, and we compute portfolio earnings, prices, value changes, earnings yields (earnings-to-price ratios), and returns (earnings-to-price ratios plus average percentage changes in value). For each simulation, we compute the mean and standard deviation of earning yields and returns over time.[41] We repeat this process for the 100 Monte Carlo simulations, and we average the results across simulations.

Table 6: **Earnings Yields**. Summary statistics of earnings yields computed from simulated data, and comparison with real data. All values are in percentage terms.

| | Mean (model) | Mean (data) | Std. Dev. (data) |
|---|---|---|---|
| DOM | 0.909 | 2.12 | 3.54 |
| EXP | 0.978 | 3.7 | 2.85 |
| MN | 1.217 | 4.85 | 2.76 |

Table 6 reports the results for earnings yields. The model generates average earnings yields of 1.22% for multinational firms, 0.98% for exporters, and 0.91% for firms selling only domestically, which are consistent with the ordering we found in the data, albeit lower in levels.

Table 7 reports the results for the returns. The model generates average returns of 1.3%

[40] The small number of Monte Carlo simulations is justified by the fact that for $\mu = 0$ there are small differences across simulations. The entire computation of the model for a given parametrization takes about 3 hours on a cluster of 8 CPUs.

[41] When averaging over time, we discard the first three periods of each simulation to reduce the importance of the fact that we initialize the firms' distribution into status at $\Omega_0 = \Omega_0^* = (0, 0)$.

Table 7: **Returns**. Summary statistics of returns computed from simulated data, and comparison with real data. All values are in percentage terms.

| | Mean (model) | Mean (data) | Std. Dev. (data) |
|---|---|---|---|
| DOM | 0.916 | 7.37 | 15.03 |
| EXP | 1.057 | 10.69 | 15.34 |
| MN | 1.3 | 11.62 | 12.86 |

for multinational firms, 1.057% for exporters, and 0.916% for firms selling only domestically, which are also consistent with the ordering we found in the data.

While the model succeeds in matching the order of earnings yields and returns across group, it fails in matching their magnitudes. As we mentioned previously, this comes at no surprise in light of the well known inability of models based on CRRA preferences to match quantitatively stock market excess returns for reasonable values of the parameter ruling risk aversion. Our robustness exercises will shed light on this aspect of the computation by showing counterfactual calculations for different values of $\gamma$.

# 6 Robustness

The objective of this section is to illustrate how the numerical results of the model change when letting vary the value of some relevant parameters. In particular, we focus our attention on alternative values of $\rho$ (the correlation between the Brownian motions ruling the evolution of $(Q, Q^*)$) and $\gamma$ (the risk aversion).

## 6.1 Negative correlation between aggregate shocks across countries

A model with stochastic demand in both countries implies that the income of domestic agents depends on both the realization of foreign and domestic demand. Fluctuations in domestic demand $Q$ enter directly into consumers' utility. Fluctuations in foreign demand $Q^*$ imply fluctuations in the profit shares that consumers receive from the stocks they hold; hence, they affect their income and their consumption of the homogeneous good $H$, which also enters the utility function. Fluctuations in $Q$ and $Q^*$ have similar qualitative effects on domestic consumers' utility (and symmetrically on foreign consumers).

The baseline value of $\rho = 0.12$ was chosen as the median correlation between the GDP growth of the U.S. and a set of major trading partners and other large countries. The choice of the value of $\rho$ is important because it determines the relative strengths of two channels that operate in the model. On the one hand, selling in foreign markets increases the riskiness of a firm's profit flows via the combination of aggregate persistent shocks and sunk costs, as we emphasize throughout the paper. On the other hand, one could think of exporters and multinational firms entering foreign markets to hedge against the fluctuations of demand in their home market. If this second channel prevailed, exporters and multinationals should exhibit lower returns in equilibrium, as the international status actually reduces their riskiness. The correlation between the processes for $(Q, Q^*)$ is tightly linked to the relative strength of these two channels: a positive (negative) $\rho$ reinforces the first (second) channel.

Consistent with the channel emphasized in the paper, the data show that most GDP correlations between the U.S. and its major trading partners are positive and high. However, to illustrate the potential of the hedging mechanism, we present the results of the model for $\rho = -0.02$, equal to the correlation between U.S. and Brazil (the lowest in the sample used in the calibration). The other parameters are left unchanged with respect to the baseline scenario.

The model with negative $\rho$ fits the data almost equally well as the baseline case: the calibration error is 0.0062. The results for earning yields and returns are displayed in Table 8. While the ordering of the earnings yields is preserved and the magnitudes are very similar to the baseline case, the same is not true for the returns. Domestic firms exhibit the highest returns, consistent with the idea that a negative $\rho$ strengthens the hedging motive, and exporters and multinationals are “safer” than firms selling only in their domestic market. However, comparing the returns of exporters and multinationals one can still see the effect of the sunk costs at work: albeit lower than the returns of domestic firms, multinationals exhibit higher returns than exporters, indicating the higher riskiness driven by higher sunk costs.

### 6.2 Higher Risk Aversion

The baseline value of $\gamma = 20$ was chosen to be consistent with empirical estimates of the Sharpe ratio within a CRRA framework. As our baseline results show, the calibrated model generates earnings yields and returns that are much lower than the ones observed in the

Table 8: **Baseline *versus* Negative Correlation**. Summary statistics of earnings yields and returns computed from simulated data, baseline scenario and counterfactual scenario ($\rho = -0.02$). All values are in percentage terms.

| | Baseline | | $\rho = -0.02$ | |
|---|---|---|---|---|
| | Earnings Yields | Returns | Earnings Yields | Returns |
| DOM | 0.909 | 0.916 | 0.938 | 1.789 |
| EXP | 0.978 | 1.057 | 1.111 | 0.872 |
| MN | 1.217 | 1.3 | 1.255 | 1.295 |

data. In light of the extensive literature on the equity premium puzzle,[42] this hardly comes at a surprise. In this section we present the results of the model for a slightly higher value of $\gamma$ and highlight the effect of the increase in $\gamma$ for the returns. We run the simulation for $\gamma = 30$.[43]

The match of the model generated moments with the data is hardly affected by the change in $\gamma$: the calibration error is 0.0077. The results for earning yields, value changes and returns are displayed in Table 9. The orders of both earnings yields and returns are not affected by the change in $\gamma$. Earnings yields are lower, because a higher $\gamma$ correspond to a lower discount term. The changes in the value functions ($d\mathcal{V}/\mathcal{V}$) are more pronounced than in the baseline scenario: this second effect is driven by the impact of $\gamma$ on the concavity of the value functions. On aggregate, the level effect on the earnings yields dominates, and returns are lower than in the baseline scenario. This happens because in the calibration we set $\mu = \mu^* = 0$, so that $\gamma$ has no effect on the risk-adjusted drift, but only on the variance. Simulating the model for a positive value of $\mu$ would reduce the level effect on the earnings yields and magnify the concavity effect on $d\mathcal{V}/\mathcal{V}$. However, the choice of a positive value of $\mu$ would not be appropriate for the calibration, as it would imply computing the model-generated moments as time-series averages of non-stationary data. For this reason we do not follow this direction.

Addressing the challenge of replicating earnings yields and returns of magnitudes consistent with the data would require different assumptions about preferences and the growth process, and is beyond the scope of this paper.

[42] See Mehra and Prescott (1985) and Hansen and Singleton (1982) among others.

[43] The choice of $\gamma$ for this experiment is constrained by the fact that the discount factors $r - (\mu + \sigma\sigma_M)$ and $r - (\mu^* + \rho\sigma^*\sigma_M)$ must be positive, and $\sigma_M = \gamma\sigma$. For this reason we cannot run the exercise for $\gamma$ as high as – say – in Hansen and Singleton (1982).

Table 9: **Baseline *versus* Higher Risk Aversion**. Summary statistics of earnings yields, value changes and returns computed from simulated data, baseline scenario and counterfactual scenario ($\gamma = 30$). All values are in percentage terms.

| | Baseline | | | $\gamma = 30$ | | |
|---|---|---|---|---|---|---|
| | Earnings Yields | Value Change | Returns | Earnings Yields | Value Change | Returns |
| DOM | 0.909 | 0.009 | 0.916 | 0.413 | 0.127 | 0.54 |
| EXP | 0.978 | 0.081 | 1.057 | 0.454 | 0.107 | 0.559 |
| MN | 1.217 | 0.078 | 1.3 | 0.647 | 0.088 | 0.736 |

# 7 Conclusions

This paper started by presenting a novel fact distinguishing multinational firms from exporters and from firms selling only in their domestic market. Multinational corporations tend to exhibit higher earnings yields and returns than non-multinational firms. Within non-multinationals, exporters tend to have higher earnings yields than firms selling only in their domestic market. To explain this fact, we presented a real option value model where firms choose optimally whether to produce only domestically, export, or serve the foreign market through FDI. In equilibrium, firms are imperfectly sorted into the three statuses according to their productivity and to the realization of the shocks. The option value of changing status introduces a systematic deviation between a firm's profits and its valuation, which generates heterogeneity in earnings yields. The endogenous choice of international status generates heterogeneity in the covariance of profit flows with aggregate shocks, which affects the returns required by risk-averse stockholders.

While being consistent with a number of facts about trade and FDI dynamics, the model provides a complementary explanation for the cross section of returns by exploiting the production side from an international point of view. Firms selling in foreign markets are more exposed to aggregate risk. If these firms exit after a negative shock, they forgo the sunk cost that they paid to enter. This generates status persistence, and the possibility of negative profits. Moreover, exiting is a more expensive strategy for multinationals than for exporters, who paid a lower premium; hence, exporters are less exposed and less risky: the difference in sunk costs generates a difference in exposure and in excess returns between exporters and multinational firms.

The solution of the model delivers a series of predictions relating firm productivity and the realizations of the shocks to the pattern of status changes. We calibrate the model to match quantitatively relevant moments related to firm dynamics and selection into export/FDI. With the parameterized model we compute moments of the financial variables from simulated data. We show that in addition to matching fairly well the overall aggregate dynamics of trade and FDI, the model is also able to reproduce the ordering in earnings yields and returns that we observe in the data.

We see this paper as the first step in a novel research agenda linking trade and FDI dynamics to asset pricing. Interesting extensions include a firm-level analysis of the dynamics of financial variables at the times of status changes, and the study of differential exit patterns of exporters *versus* multinational firms. We think this is a promising avenue for research in finance and international trade that we plan to pursue in future work.

# Appendix

## A Accounting Standards and Data Selection

The empirical analysis contained in this paper is based on annual, firm-level data. We limit the present study to the universe of publicly traded, US based, manufacturing firms included in the Standard & Poors Compustat Segments Database.[1] Compustat data is comprised of key components from annual regulatory filings. Information on firms' foreign operations is included in the Segments Data. Segments data categorize a firm's operations along a particular business division and report sales, asset, and other information based on these groups.[2]

The Financial Accounting Standards Board (FASB), in its Statement No. 131, sets the standards for the way in which public businesses report information about operating segments in their annual financial statements. Operating segments are defined by the FASB as *"components of an enterprise about which separate financial information is available that is evaluated regularly by the chief operating decision maker in deciding how to allocate resources and in assessing performance"*.

FAS 131 establishes standards for the way firms should disclose data about products and services, geographic areas, and major customers. The FAS 131 determines that firms should

[1] The NAICS code for manufacturing firms contain the 2 digit prefix 31, 32, or 33.
[2] The four segment classifications are business, geographic, operating, and state.

report data about revenues derived from the firm's *products or services*, *countries* in which they earn revenues and hold assets, and about major *customers* regardless of whether that information is used in making operating decisions. However, the statement does not require firms to disclose the information on all the different segment types if it is not prepared for internal use and reporting would be impracticable. Therefore, the firms decide how to report the data, disaggregated in several different ways: either by product, geography, legal entity, or by customer, but they do not necessarily have to report all of them. This method is referred to as the management approach. The statement establishes a minimum threshold to report separately information about an operating segment: either revenues of the segment are 10% or more of the combined revenue of all operating segments, or profits or losses are 10% or more of the combined reported profit or losses, or its assets are 10% or more of the combined assets of all operating segments. Hence, if a given firm considers best practice to aggregate the information upstream to the management level by customer, it may elect not to disclose geographical segments information. That contrasts with the previous FAS No. 14, superseded by FAS No. 131 in 1998, in which firms were required that the financial statements of a business include information about the enterprise's operations in different industries, its foreign operations by geographical area and export sales, and its major customers.

According to the new FAS 131, when an enterprise reports the existence of a geographical segment, it must report revenues and holdings of long-lived assets. This information may or may not be disaggregated by individual foreign country. In a sense, the new regulation goes towards a major disaggregation of the information, provided that it does not contrast with the normal management of the firm.

Faced with the potential measurements problems associated with the loose reporting requirements of the Compustat Segments, we had two options to select our dataset: 1) include in the dataset only those firms that reported the existence of operating segments and drop all the others, or 2) include all firms in Compustat and impute as Domestic the status for those firms that did not report the existence of operating segments. The data analysis reported in Section 2 corresponds to the first selection criterium, which we prefer, because it generates a cleaner, albeit smaller, dataset. For robustness, we run all the regressions also using the dataset constructed with the second selection criterium, and the results on the ranking of earnings yields and returns are unchanged.

The relevant segment for our classification of firms by status division is the geographic segment. 96% of the firms that reported the existence of operating segments also report the existence of a geographic segment. Segments that report only domestic sales are classified as domestic, those that report positive export sales are classified as exporters, and those that report positive foreign sales are classified as multinational firms. For the remaining firms, we aggregate data from the business segment and assume the firm's operations are entirely domestic. However, due to reporting errors, misclassifications, and multiple classifications, a few notes are required.

As is typical when a data point is unreliable, unreported, or otherwise a break from the traditional definition, the provider will report codes in place of an interpretable value. Compustat employs a similar methodology. In these instances we assume the segment to be of negligible importance and consider associated sales and exports to be null. As mentioned above our implementation of segment data relies entirely on the classification provided in the data. However, in a few instances sales for the non-domestic segments indicate the market of operation as the United States. In these cases we assume the reported classification was in error and re-classify the segment as domestic.

The data is then aggregated by firm-year. For many firms this aggregation requires combining multiple segments and may result in competing classifications for a firm in a particular year. In these instances we classify the firm by the most "globally engaged" reported segment (for example domestic firms with exports are classified as exporters, while exporters with foreign sales are considered multinational). Once firms have been appropriately classified by their international status we are able to observe the dynamics of this classification.

Examining a firm's international classification over time reveals what we believe to be reporting errors. These cases are characterized by a one-year "downward" status change, which results in a return to the original status. We believe this transient status change is a result of inaccurate segment reporting. As such we re-classify the observation to ensure continuity in the series. However, the opposite is not true: when a firm enters into an international market only to return to a less advanced geographic segment the following year, that firm retains the reported classification. The logic for this is evident - it is far easier to omit classification in a given year than to report segment details that were nonexistent.

Another dimension of selecting the data involves which criteria to use to establish the unit of observation and to eliminate outliers. The data span 28 years, from 1979 to 2006,[3] but many firms have observations only for a part of this time interval. We impose a lower bound of 6 yearly observation for a firm to be included in our sample. Additionally, we remove extreme observations in each international classification by dropping the top and bottom 1% of earnings-to-price ratios by group and the top and bottom 5% of returns by group.

# B Derivations and Proofs

## B.1 Derivation of the Value Functions

In this section we present the details of the derivation of the value functions in Section 3.

We start by solving for the value function of a firm that is currently selling only in its domestic market. From (12),(13), in the continuation region:

$$\pi_D(a,Q)M\Delta t + E[M\Delta t \cdot \mathcal{S}(a,Q')|Q] - \mathcal{S}(a,Q) + E[M\Delta t \cdot V_D(a,Q^{*\prime})|Q^*] - V_D(a,Q^*) = 0.$$

For $\Delta t \to 0$:

$$\pi_D(a,Q)Mdt + E[d(M \cdot \mathcal{S}(a,Q))] + E[d(M \cdot V_D(a,Q^*))] = 0.$$

The terms in the expectations can be written as:

$$\begin{aligned} E[d(M \cdot \mathcal{S})] &= E[dM \cdot \mathcal{S} + M \cdot d\mathcal{S} + dM \cdot d\mathcal{S}] \\ &= M \cdot S \cdot E\left[\frac{dM}{M} + \frac{d\mathcal{S}}{\mathcal{S}} + \frac{dM}{M} \cdot \frac{d\mathcal{S}}{\mathcal{S}}\right] \\ &= M \cdot S\left[-rdt + E\left(\frac{d\mathcal{S}}{d\mathcal{S}}\right) + E\left(\frac{dM}{M} \cdot \frac{d\mathcal{S}}{\mathcal{S}}\right)\right] \\ &= Mdt\left[-r\mathcal{S} + E\left(\frac{d\mathcal{S}}{dt}\right) + E\left(\frac{dM}{M} \cdot \frac{d\mathcal{S}}{dt}\right)\right] \end{aligned} \tag{A.1}$$

[3] All variables have been deflated to constant 1984 dollars.

$$
\begin{aligned}
E[d(M\cdot V_D)] &= E[dM\cdot V_D + M\cdot dV_D + dM\cdot dV_D] \\
&= M\cdot V_D\cdot E\left[\frac{dM}{M}+\frac{dV_D}{V_D}+\frac{dM}{M}\cdot\frac{dV_D}{V_D}\right] \\
&= M\cdot V_D\left[-rdt+E\left(\frac{dV_D}{V_D}\right)+E\left(\frac{dM}{M}\cdot\frac{dV_D}{V_D}\right)\right] \\
&= Mdt\left[-rV_D+E\left(\frac{dV_D}{dt}\right)+E\left(\frac{dM}{M}\cdot\frac{dV_D}{dt}\right)\right] \qquad \text{(A.2)}
\end{aligned}
$$

where the dependence of $\mathcal{S}$ on $(a, Q)$ and the dependence of $V_D$ on $(a, Q^*)$ have been suppressed to ease the notation. Plugging (A.1) and (A.2) into the no-arbitrage condition:

$$
\pi_D - r\mathcal{S} + E\left(\frac{d\mathcal{S}}{dt}\right) + E\left(\frac{dM}{M}\cdot\frac{d\mathcal{S}}{dt}\right) - rV_D + E\left(\frac{dV_D}{dt}\right) + E\left(\frac{dM}{M}\cdot\frac{dV_D}{dt}\right) = 0. \quad \text{(A.3)}
$$

By applying Ito's Lemma and using the expressions for the Brownian motions ruling $Q$ and $Q^*$, we can derive expressions for some of the terms in (A.3):

$$
\begin{aligned}
d\mathcal{S} &= \mathcal{S}'dQ + \frac{1}{2}\sigma^2Q^2\mathcal{S}''dt = \mathcal{S}'[\mu Qdt + \sigma Qdz] + \frac{1}{2}\sigma^2Q^2\mathcal{S}''dt \\
E[d\mathcal{S}] &= \mu Q\mathcal{S}'dt + \frac{1}{2}\sigma^2Q^2\mathcal{S}''dt \\
dV_D &= V_D'dQ^* + \frac{1}{2}(\sigma^*)^2(Q^*)^2V_D''dt = V_D'[\mu^*Q^*dt + \sigma^*Q^*dz^*] + \frac{1}{2}(\sigma^*)^2(Q^*)^2V_D''dt \\
E[dV_D] &= \mu^*Q^*V_D'dt + \frac{1}{2}(\sigma^*)^2(Q^*)^2V_D''dt\ .
\end{aligned}
$$

Using these results and the expression for the evolution of $M$, we can rewrite (A.3) as:

$$
\begin{aligned}
&\pi_D - r\mathcal{S} + \mu Q\mathcal{S}' + \frac{1}{2}\sigma^2Q^2\mathcal{S}'' + E\left[(-rdt+\sigma_M dz)\cdot\left(\mu Q\mathcal{S}' + \sigma Q\mathcal{S}'\frac{dz}{dt} + \frac{1}{2}\sigma^2Q^2\mathcal{S}''\right)\right] - ... \\
&...rV_D + \mu^*Q^*V_D' + \frac{1}{2}(\sigma^*)^2(Q^*)^2V_D'' + ... \\
&...E\left[(-rdt+\sigma_M dz)\cdot\left(\mu^*Q^*V_D' + \sigma^*Q^*V_D'\frac{dz^*}{dt} + \frac{1}{2}(\sigma^*)^2(Q^*)^2V_D''\right)\right] = 0
\end{aligned}
$$

and multiplying every term for $dt$:

$$\begin{aligned}
&\pi_D dt - r\mathcal{S}dt + \mu Q\mathcal{S}'dt + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}''dt + ...\\
&E\left[(-rdt + \sigma_M dz)\cdot\left(\mu Q\mathcal{S}'dt + \sigma Q\mathcal{S}'dz + \frac{1}{2}\sigma^2 Q^2\mathcal{S}''dt\right)\right] - ...\\
&...rV_D dt + \mu^* Q^* V_D' dt + \frac{1}{2}(\sigma^*)^2 (Q^*)^2 V_D'' dt + ...\\
&...E\left[(-rdt + \sigma_M dz)\cdot\left(\mu^* Q^* V_D' dt + \sigma^* Q^* V_D' dz^* + \frac{1}{2}(\sigma^*)^2 (Q^*)^2 V_D'' dt\right)\right] = 0. \qquad \text{(A.4)}
\end{aligned}$$

For the properties of Brownian motions, the terms in expectation can be reduced to:

$$\begin{aligned}
& E\left[(-rdt + \sigma_M dz)\cdot\left(\mu Q\mathcal{S}'dt + \sigma Q\mathcal{S}'dz + \frac{1}{2}\sigma^2 Q^2\mathcal{S}''dt\right)\right] = ...\\
... &= E\Bigg[ - r\mu Q\mathcal{S}'dt^2 - r\sigma Q\mathcal{S}'dtdz - r\frac{1}{2}\sigma^2 Q^2\mathcal{S}''dt^2 + \sigma_M \mu Q\mathcal{S}'dzdt + ...\\
& ...\sigma_M \sigma Q\mathcal{S}'dz^2 + \sigma_M\frac{1}{2}\sigma^2 Q^2\mathcal{S}''dzdt\Bigg] = ...\\
... &= \sigma_M \sigma Q\mathcal{S}'E(dz^2) = ...\\
... &= \sigma\sigma_M Q\mathcal{S}'dt
\end{aligned}$$

and:

$$\begin{aligned}
& E\left[(-rdt + \sigma_M dz)\cdot\left(\mu^* Q^* V_D' dt + \sigma^* Q^* V_D' dz^* + \frac{1}{2}(\sigma^*)^2 (Q^*)^2 V_D'' dt\right)\right] = ...\\
... &= E\Bigg[-r\mu^* Q^* V_D' dt^2 - r\sigma^* Q^* V_D' dtdz^* - r\frac{1}{2}(\sigma^*)^2 (Q^*)^2 V_D'' dt^2 + \sigma_M \mu^* Q^* V_D' dzdt + ...\\
& ... + \sigma_M \sigma^* Q^* V_D' dzdz^* + \sigma_M \frac{1}{2}(\sigma^*)^2 (Q^*)^2 V_D'' dzdt\Bigg] = ...\\
... &= \sigma^* \sigma_M Q^* V_D' E(dzdz^*) = ...\\
... &= \rho\sigma^*\sigma_M Q^* V_D' dt.
\end{aligned}$$

So we can rewrite (A.4) as:

$$\begin{aligned}
\pi_D - r\mathcal{S} + \mu Q\mathcal{S}' + \frac{1}{2}\sigma^2 Q^2\mathcal{S}'' + \sigma\sigma_M Q\mathcal{S}' - rV_D + \mu^* Q^* V_D' + \frac{1}{2}(\sigma^*)^2 (Q^*)^2 V_D'' + \rho\sigma^*\sigma_M Q^* V_D' &= 0\\
\pi_D - r\mathcal{S} + (\mu + \sigma\sigma_M)Q\mathcal{S}' + \frac{1}{2}\sigma^2 Q^2\mathcal{S}'' - rV_D + (\mu^* + \rho\sigma^*\sigma_M)Q^* V_D' + \frac{1}{2}(\sigma^*)^2 (Q^*)^2 V_D'' &= 0.
\end{aligned}$$

One possible solution of this equation is such that:

$$
\begin{aligned}
\pi_D(a,Q) - r\mathcal{S}(a,Q) + (\mu + \sigma\sigma_M)Q\mathcal{S}'(a,Q) + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}''(a,Q) &= 0 \qquad \text{(A.5)}\\
-rV_D(a,Q^*) + (\mu^* + \rho\sigma^*\sigma_M)Q^* V_D'(a,Q^*) + \frac{1}{2}(\sigma^*)^2(Q^*)^2 V_D''(a,Q^*) &= 0. \qquad \text{(A.6)}
\end{aligned}
$$

We start by deriving the value function $\mathcal{S}(a,Q)$, solution of (A.5). Let $\hat{\mu} \equiv \mu + \sigma\sigma_M$ and recall that $\pi_D(a,Q) = B(aw)^{1-\eta}P^\eta Q \equiv \mathcal{P}_D(a)Q$:

$$
-r\mathcal{S} + \hat{\mu}Q\mathcal{S}' + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}'' = -\mathcal{P}_D(a)Q.
$$

This differential equation admits a solution of the form $\mathcal{S}(a,Q) = Q^\chi + cQ$, where $\chi$ is the solution of the following quadratic equation:

$$
\begin{aligned}
-r + \left(\hat{\mu} - \frac{1}{2}\sigma^2\right)\chi + \frac{1}{2}\sigma^2\chi^2 &= 0\\
-\bar{r}_s + (m_s - 1)\chi + \chi^2 &= 0
\end{aligned}
$$

where $m_s = \frac{2\hat{\mu}}{\sigma^2}$ and $\bar{r}_s = \frac{2r}{\sigma^2}$. Hence:

$$
\chi = \frac{(1-m_s) \pm \sqrt{(1-m_s)^2 + 4\bar{r}_s}}{2}.
$$

Similarly, $c$ is the solution of:

$$
\begin{aligned}
-rc + c\hat{\mu} &= -\mathcal{P}_D\\
c &= \frac{\mathcal{P}_D}{r - \hat{\mu}}.
\end{aligned}
$$

Hence the value function describing the expected present discounted value of domestic profits takes the form:

$$
\mathcal{S}(a,Q) = A_{\mathcal{S}}(a)Q^{\alpha_s} + B_{\mathcal{S}}(a)Q^{\beta_s} + \frac{\mathcal{P}_D Q}{r - \hat{\mu}}
$$

where $\alpha_s$ and $\beta_s$ are the negative and positive value of $\chi$, respectively, and $A_{\mathcal{S}}(a)$ and $B_{\mathcal{S}}(a)$ are firm-specific parameters to be determined. Since there is no option value associated to domestic profits, we can impose: $A_{\mathcal{S}}(a) = B_{\mathcal{S}}(a) = 0$, so that the solution is simply given by the value of profits discounted with the risk-adjusted measure:

$$\mathcal{S}(a,Q) = \frac{\mathcal{P}_D Q}{r-\hat{\mu}} = \frac{B(aw)^{1-\eta}P^{\eta}Q}{r-(\mu+\sigma\sigma_M)} \ . \tag{A.7}$$

In the same way we can derive the value function $V_D(a,Q^*)$, solution of (A.6). Let $\tilde{\mu} \equiv \mu^* + \rho\sigma^*\sigma_M$:

$$-rV_D + \tilde{\mu}Q^*V_D' + \frac{1}{2}(\sigma^*)^2(Q^*)^2V_D'' = 0.$$

This differential equation admits a solution of the form $V_D(a,Q^*) = (Q^*)^{\xi}$, where $\xi$ is the solution of the following quadratic equation:

$$\begin{aligned} -r + \left(\tilde{\mu} - \frac{1}{2}(\sigma^*)^2\right)\xi + \frac{1}{2}(\sigma^*)^2\xi^2 &= 0 \\ -\bar{r} + (m-1)\xi + \xi^2 &= 0 \end{aligned}$$

where $m = \frac{2\tilde{\mu}}{(\sigma^*)^2}$ and $\bar{r} = \frac{2r}{(\sigma^*)^2}$. Hence:

$$\xi = \frac{(1-m) \pm \sqrt{(1-m)^2 + 4\bar{r}}}{2}.$$

The value function describing the expected present discounted value of foreign profits of a domestic firm takes the form:

$$V_D(a,Q^*) = A_D(a)(Q^*)^{\alpha} + B_D(a)(Q^*)^{\beta} \tag{A.8}$$

where $\alpha$ and $\beta$ are the negative and positive value of $\xi$, respectively.

By following the same procedure, we can solve for the expected value of foreign profits of an exporter, $V_X(a,Q^*)$, and of a multinational, $V_I(a,Q^*)$:

$$V_X(a,Q^*) = A_X(a)(Q^*)^{\alpha} + B_X(a)(Q^*)^{\beta} + \frac{B(\tau aw)^{1-\eta}(P^*)^{\eta}Q^*}{r-(\mu^*+\rho\sigma^*\sigma_M)} - \frac{f_X}{r} \tag{A.9}$$

$$V_I(a,Q^*) = A_I(a)(Q^*)^{\alpha} + B_I(a)(Q^*)^{\beta} + \frac{B(aw^*)^{1-\eta}(P^*)^{\eta}Q^*}{r-(\mu^*+\rho\sigma^*\sigma_M)} - \frac{f_I}{r}. \tag{A.10}$$

## B.2 Proof of Theorem 1: Ordering of the Quantity Thresholds

**Theorem 1.** *If $F_I > F_X$, $f_I = f_X$, and $\tau w = w^*$, the quantity thresholds $Q^*_{RS}(a)$, for $R, S \in \{D, X, I\}$ and for a given productivity level $1/a$, satisfy the following ordering:*

$$Q^*_{IX}(a) < Q^*_{ID}(a) < Q^*_{XD}(a) < Q^*_{DX}(a) < Q^*_{DI}(a) < Q^*_{XI}(a). \tag{A.11}$$

*Proof:* If $f_X = f_I$ and $\tau w = w^*$, the operating costs of a firm with productivity $1/a$ that decides to sell abroad are the same regardless of status, hence the choice of status is uniquely determined by the sunk costs. From Dixit and Pindyck (1994), Chapter 7, we know that entry (exit) thresholds are increasing (decreasing) in both the sunk costs of entry and exit, hence $Q_{DI} > Q_{DX}$ and $Q_{ID} < Q_{XD}$.

To determine the relative position of $Q_{XI}$, $Q_{IX}$, it is sufficient to notice that the problem of a firm that switches from exports to FDI and viceversa can be written as a problem where $F_I$ is the sunk cost of entry and $F_X$ is the sunk cost of exit. Hence the switch from export to FDI is characterized by the same entry cost and a higher exit cost than the change from domestic sales only to export, hence $Q_{XI} > Q_{DI}$ and $Q_{IX} < Q_{ID}$.

The pure presence of sunk costs implies $Q_{XD} < Q_{DX}$, which completes the result. □

## B.3 Proof of Theorem 2: Thresholds and Productivity

**Theorem 2.**

$$\frac{\partial Q^*_{RS}(a)}{\partial a} > 0, \quad for \quad R, S \in \{D, X, I\}, \ \forall a. \tag{A.12}$$

*Proof:* The proof closely follows Appendix B of Dixit (1989). We show the result for $Q_{DX}$ only; the proof for the other thresholds follows exactly the same steps.

The value-matching conditions for $Q^*_{DX}$, $Q^*_{XD}$ are:

$$\begin{aligned} A_X {Q^*_{DX}}^{\alpha} + (B_X - B_D){Q^*_{DX}}^{\beta} + \frac{B(\tau a w)^{1-\eta} P^{*\eta} Q^*_{DX}}{r - (\mu^* + \rho\sigma^*\sigma_M)} - \frac{f_X}{r} &= F_X \\ A_X {Q^*_{XD}}^{\alpha} + (B_X - B_D){Q^*_{XD}}^{\beta} + \frac{B(\tau a w)^{1-\eta} P^{*\eta} Q^*_{XD}}{r - (\mu^* + \rho\sigma^*\sigma_M)} - \frac{f_X}{r} &= 0. \end{aligned}$$

Differentiating and using the smooth-pasting conditions:

$$Q^{*}_{DX}{}^{\alpha}dA_X + Q^{*}_{DX}{}^{\beta}d(B_X - B_D) + \frac{(1-\eta)B(\tau w)^{1-\eta}a^{-\eta}P^{*\eta}Q^{*}_{DX}}{r-(\mu^{*}+\rho\sigma^{*}\sigma_M)}da = 0 \tag{A.13}$$

$$Q^{*}_{XD}{}^{\alpha}dA_X + Q^{*}_{XD}{}^{\beta}d(B_X - B_D) + \frac{(1-\eta)B(\tau w)^{1-\eta}a^{-\eta}P^{*\eta}Q^{*}_{XD}}{r-(\mu^{*}+\rho\sigma^{*}\sigma_M)}da = 0. \tag{A.14}$$

Dividing (A.13) by $Q^{*}_{DX}$ and (A.14) by $Q^{*}_{XD}$ and combining them:

$$dA_X = \left(\frac{Q^{*}_{DX}{}^{\beta-1} - Q^{*}_{XD}{}^{\beta-1}}{Q^{*}_{XD}{}^{\alpha-1} - Q^{*}_{DX}{}^{\alpha-1}}\right)d(B_X - B_D). \tag{A.15}$$

Plugging (A.15) into (A.13):

$$d(B_X - B_D) = \left(\frac{Q^{*}_{XD}{}^{\alpha-1} - Q^{*}_{DX}{}^{\alpha-1}}{Q^{*}_{DX}{}^{\beta-1}Q^{*}_{XD}{}^{\alpha-1} - Q^{*}_{DX}{}^{\alpha-1}Q^{*}_{XD}{}^{\beta-1}}\right)\cdot\left(-\frac{(1-\eta)B(\tau w)^{1-\eta}a^{-\eta}P^{*\eta}}{r-(\mu^{*}+\rho\sigma^{*}\sigma_M)}da\right) \tag{A.16}$$

and plugging (A.16) into (A.15):

$$dA_X = \left(\frac{Q^{*}_{DX}{}^{\beta-1} - Q^{*}_{XD}{}^{\beta-1}}{Q^{*}_{DX}{}^{\beta-1}Q^{*}_{XD}{}^{\alpha-1} - Q^{*}_{DX}{}^{\alpha-1}Q^{*}_{XD}{}^{\beta-1}}\right)\cdot\left(-\frac{(1-\eta)B(\tau w)^{1-\eta}a^{-\eta}P^{*\eta}}{r-(\mu^{*}+\rho\sigma^{*}\sigma_M)}da\right). \tag{A.17}$$

The smooth-pasting condition for $Q^{*}_{DX}$ is:

$$\alpha A_X Q^{*}_{DX}{}^{\alpha-1} + \beta(B_X - B_D)Q^{*}_{DX}{}^{\beta-1} + \frac{B(\tau a w)^{1-\eta}P^{*\eta}}{r-(\mu^{*}+\rho\sigma^{*}\sigma_M)} = 0.$$

Let $G_{DX}(\cdot) = V_X(\cdot) - V_D(\cdot)$. Differentiating the condition above:

$$G''_{DX}(\cdot)dQ^{*}_{DX} + \alpha Q^{*}_{DX}{}^{\alpha-1}dA_X + \beta Q^{*}_{DX}{}^{\beta-1}d(B_X - B_D) + \frac{(1-\eta)B(\tau w)^{1-\eta}a^{-\eta}P^{*\eta}}{r-(\mu^{*}+\rho\sigma^{*}\sigma_M)}da = 0. \tag{A.18}$$

Let $\Delta \equiv Q^{*}_{DX}{}^{\beta-1}Q^{*}_{XD}{}^{\alpha-1} - Q^{*}_{DX}{}^{\alpha-1}Q^{*}_{XD}{}^{\beta-1}$. Substituting in the expressions for $dA_X$ and $d(B_X - B_D)$, equation (A.18) can be rewritten as:

$$\begin{aligned}\underbrace{-G''_{DX}}_{>0}(\cdot)dQ^{*}_{DX} \quad &= \quad \underbrace{\frac{(\eta-1)B(\tau w)^{1-\eta}a^{-\eta}P^{*\eta}}{\Delta[r-(\mu^{*}+\rho\sigma^{*}\sigma_M)]}}_{>0}da\cdot ...\\ &\quad ...\cdot\left[\alpha\left(Q^{*}_{DX}{}^{\alpha+\beta-2} - Q^{*}_{DX}{}^{\alpha-1}Q^{*}_{XD}{}^{\beta-1}\right) + \beta\left(Q^{*}_{DX}{}^{\alpha+\beta-2} - Q^{*}_{DX}{}^{\beta-1}Q^{*}_{XD}{}^{\alpha-1}\right) + \Delta\right].\end{aligned}$$

In order to show that $\frac{\partial Q^*_{DX}(a)}{\partial a} > 0$, we must show that the last term of the expression above is positive:

$$\begin{aligned}
\left[\alpha\left({Q^*_{DX}}^{\alpha+\beta-2} - {Q^*_{DX}}^{\alpha-1}{Q^*_{XD}}^{\beta-1}\right) + \beta\left({Q^*_{DX}}^{\alpha+\beta-2} - {Q^*_{DX}}^{\beta-1}{Q^*_{XD}}^{\alpha-1}\right) + \Delta\right] &= \ldots \\
\frac{1}{{Q^*_{DX}}^{\alpha+\beta-2}}\left[\alpha\left(1 - \left(\frac{Q^*_{XD}}{Q^*_{DX}}\right)^{\beta-1}\right) + \beta\left(\left(\frac{Q^*_{XD}}{Q^*_{DX}}\right)^{\alpha-1} - 1\right) - \left(\frac{Q^*_{XD}}{Q^*_{DX}}\right)^{\alpha-1} + \left(\frac{Q^*_{XD}}{Q^*_{DX}}\right)^{\beta-1}\right] &= \ldots \\
\underbrace{\frac{1}{{Q^*_{DX}}^{\alpha+\beta-2}}}_{>0}\left[\alpha\left(1 - \left(\frac{Q^*_{DX}}{Q^*_{XD}}\right)^{1-\beta}\right) + \beta\left(\left(\frac{Q^*_{DX}}{Q^*_{XD}}\right)^{1-\alpha} - 1\right) - \left(\frac{Q^*_{DX}}{Q^*_{XD}}\right)^{1-\alpha} + \left(\frac{Q^*_{DX}}{Q^*_{XD}}\right)^{1-\beta}\right]&.
\end{aligned}$$

Let $z \equiv \frac{Q^*_{DX}}{Q^*_{XD}} > 1$ and let $\phi(z) \equiv \left[\alpha\left(1 - z^{1-\beta}\right) + \beta\left(z^{1-\alpha} - 1\right) - z^{1-\alpha} + z^{1-\beta}\right]$. Then $\phi(1) = 0$ and $\phi'(z) = (1-\alpha)(\beta-1)(z^{-\alpha} - z^{-\beta}) > 0$, which proves the result. $\square$

## C Algorithm for the Computation of the Equilibrium

In this section we report the algorithm used to compute the equilibrium of the model described in Section 3.

At $t = 0$:

1. Initialize the firms' distribution into statuses: $\Omega_0 = \Omega^*_0 = (0, 0)$ (all firms start by selling in their domestic market only).

2. Compute $P_0$, $P^*_0$ using the shares of firms in each status from $\Omega_0$, $\Omega^*_0$.

3. Compute the optimal thresholds $Q_{RS}(0)$, $Q^*_{RS}(0)$ using $P_0$, $P^*_0$ as the price indexes.

For $t = 1, \ldots T$:

4. Draw a realization of $(Q(t), Q^*(t))$.

5. Compare $(Q(t), Q^*(t))$ with the thresholds $Q_{RS}(t-1)$, $Q^*_{RS}(t-1)$ and compute the new firm distributions into statuses $\Omega_t$, $\Omega^*_t$.

6. Compute the equilibrium price indexes $P_t$, $P^*_t$ using the shares of firms in each status from $\Omega_t$, $\Omega^*_t$.

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