Robust Control, Informational Frictions, and International Consumption Correlations*

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Abstract

In this paper we examine the effects of two types of information imperfections, robustness (RB) and finite information-processing capacity (called rational inattention or RI), on international consumption correlations in an otherwise standard small open economy model. We show that in the presence of capital mobility in financial markets, RB lowers the international consumption correlations by generating heterogeneous responses of consumption to income shocks across countries facing different macroeconomic uncertainty. However, the calibrated RB model cannot explain the observed consumption correlations quantitatively. We then show that introducing RI into the RB model is capable of matching the behavior of international consumption quantitatively via two channels: (1) the gradual response to income shocks that increases the correlations and (2) the presence of the common noise shocks that reduce the correlations.

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1 Introduction

A common assumption in international business cycles models is that world financial markets are complete in the sense that individuals in different countries are able to fully insure country-specific income risks using international financial markets. Under this assumption, the models predict that consumption (or consumption growth) is highly correlated across countries, and in some cases the international consumption correlation is equal to 1 regardless of income or output correlations. The intuition is that since consumers are risk averse they will choose to smooth consumption over time by trading in international financial markets. However, in the data cross-country consumption correlations are very low and are even lower than corresponding income correlations in many countries. For example, Backus, Kehoe, and Kydland (1992) solve a two-country real business cycles model and argue that the puzzle that empirical consumption correlations are actually lower than output correlations is the most striking discrepancy between theory and data.[1] In the literature, the empirical low international consumption correlations have been interpreted as indicating international financial markets imperfections – for examples, see Kollman (1996), Baxter and Crucini (1995), and Lewis (1996).

Other extensions have been proposed to make the models better fit the data. For example, Devereux, Gregory, and Smith (1992) show that in the perfect risk-sharing model nonseparability between consumption and leisure has the potential to reduce the cross-country consumption correlation. Stockman and Tesar (1995) show that the presence of nontraded goods in the complete-market model can also improve the model's prediction. Fuhrer and Klein (2006) show that habit formation has important implications for international consumption correlations. In particular, they show that with a shock to the interest rate habit formation *by itself* can generate positive consumption correlations across countries even in the absence of international risk sharing and common income shocks. They then argue that if habit is a good characterization of consumers' behavior, the absence of international risk sharing is *even more striking* than standard tests suggest; that is, existing studies may overstate the extent to which common consumption movements across countries reflect international risk sharing because some of them are due to habit.[2]

In this paper, we examine how introducing two types of information imperfection, preference for robustness (RB, a concern for model misspecification) and information-processing constraints (rational inattention, RI) into an otherwise standard small open economy (SOE) model can im-

[1]Pakko (1996) shows that, in the presence of complete asset markets, consumption should be more highly correlated with total world income than with domestic income, while the data shows the opposite. This result provides an alternative standard for evaluating models of international business cycles.

[2]Baxter and Jermann (1997) also argue that the international diversification puzzle is *"worse than you think"* due to nontraded labor income being correlated with the return to domestic assets.

prove the model's predictions on the international consumption correlations puzzle we discussed above. Hansen and Sargent (1995, 2007) first introduce robustness into economic models. In robust control problems, agents are concerned about the possibility that their model is misspecified in a manner that is difficult to detect statistically; consequently, they make their decisions as if the subjective distribution over shocks was chosen by a malevolent nature in order to minimize their expected utility (the solution to a robust decision-maker's problem is the equilibrium of a max-min game between the decision-maker and nature).[3] Robustness models produce precautionary savings but remain within the class of LQ-Gaussian models, which leads to analytical simplicity.[4] After introducing RB into the standard SOE model and solving it explicitly, we find that RB can help improve the model's consistency with the empirical evidence on international consumption correlations.[5] Specifically, we show that in the presence of capital mobility in international financial markets, RB lowers the international consumption correlations by introducing heterogenous responses of consumption to income shocks across countries facing different macroeconomic uncertainty. The reason is that in the model in which agents are concerned about model misspecification, given heterogeneity in fundamental macroeconomic uncertainty, the effects of RB on consumption adjustments are different for the countries we study; consequently, it leads to lower consumption correlations across these countries. After calibrating the RB parameter using the detection error probabilities, we find that although the calibrated RB-SOE model can better match the data on international consumption correlations, it is still not enough to explain the observed consumption correlations quantitatively. Luo and Young (2009a) show that RB by itself fails to generate the hump-shaped impulse responses of consumption to income shocks, and introducing another informational friction, rational inattention (RI), can help produce the realistic consumption dynamics. We therefore introduce RI into our RB-SOE model and examine whether the interaction of the two important informational frictions can further improve the model's performance on international consumption correlations.

Sims (2003) first introduced rational inattention into economics and argued that it is a plausible method for introducing sluggishness, randomness, and delay into economic models. In his formulation agents have finite Shannon channel capacity, limiting their ability to process signals

[3]We interpret fear of model misspecification as an information imperfection because it implies that the true data-generating process is unknown.

[4]A second class of models that produces precautionary savings but remains within the class of LQ-Gaussian models is the risk-sensitive model of Hansen, Sargent, and Tallarini (1999). See Hansen and Sargent (2007a) and Luo and Young (2009a) for detailed comparisons of the two models.

[5]Recently, some papers have incorporated model uncertainty into small open economy models and examined the effects of robustness on monetary policy. See Justiniano and Preston (2010), Leitemo and Söderström (2008), and Dennis, Leitemo, and Söderström (2009).

about the true state of the world. As a result, an impulse to the economy induces only gradual responses by individuals, as their limited capacity requires many periods to discover just how much the state has moved. In contrast, the RE hypothesis implicitly assumes that consumers are endowed with infinite information-processing capacity, allowing them to respond immediately and completely to changes in the economy without making systematic errors. However, individuals in reality do not seem to have unlimited mental capacity: they do not respond swiftly or thoroughly to all available information related to their economic decisions. Sims (2003) and Luo (2008) use this model to explore anomalies in the consumption literature, particularly the well-known *excess sensitivity* and *excess smoothness* puzzles. Since RI induces additional uncertainty about the state of the economy and the state is a part of the model, it could be regarded as another source of model uncertainty. Agents in the model can reduce this kind of model uncertainty (the uncertainty about the current state) by learning via finite channels. The two hypotheses actually capture two important aspects of informational frictions. Specifically, agents with finite capacity cannot observe the current state but know what the innovation is and how it affects the transition of the state, while agents with a preference for RB do not know the probability model driven by the innovation but can observe the current state perfectly.

In this paper after introducing RI into the RB-SOE model and solving it explicitly, we show that introducing RI affects international consumption correlations via two channels: the gradual response of consumption growth to income shocks and the presence of the common noises due to limited capacity. Just like the habit formation or sticky expectations hypotheses (infrequent updating as in Bacchetta and van Wincoop 2010), the first channel increases cross-country consumption correlations. In contrast, the second channel reduces the correlations because the common noise generated by RI increases consumption volatility but has no impact on the covariance of consumption across countries. Using the same calibration procedure, we find that introducing RI can further improve the model's quantitative predictions on the observed consumption correlations. Specifically, we show that consumption correlations can be quite small if agents have low channel capacity and their noise shocks are highly correlated across individuals. Both conditions seem *ex ante* plausible: households would allocate very little of their scarce attention to aggregate movements in income (as they are not costly) and often would use common sources to obtain information (such as newspapers or TV).

The remainder of the paper is organized as follows. Section 2 reviews the standard full-information rational expectations small open economy (SOE) model and discuss the puzzling implications for international consumption correlations in the model. Section 3 presents the RB model, calibrates the model misspecification parameter, and show to what extent RB can improve the model's performance. Section 4 introduces RI into the RB model and shows that RI can further improve the model's predictions. Section 5 concludes.

2 A Full-information Rational Expectations Small Open Economy Model

2.1 Model Setup

In this section we present a full-information rational expectations (RE) version of a small open economy (SOE) model and will discuss how to incorporate two informational frictions: robustness (RB) and rational inattention (RI) into this stylized model in the next sections. Following the literature, we assume that the model economy is populated by a continuum of identical infinitely-lived consumers, and the only asset that is traded internationally is a risk-free bond.

The full-information RE-SOE model, the small-open economy version of Hall's permanent income model, can be formulated as

$$\max_{\{c_t\}} E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] \tag{1}$$

subject to the flow budget constraint

$$b_{t+1} = Rb_t - c_t + y_t, \tag{2}$$

where $u(c_t) = -\frac{1}{2}\left(\overline{c} - c_t\right)^2$ is the utility function, $\overline{c}$ is the bliss point, c_t is consumption, R is the exogenous and constant gross world interest rate, b_t is the amount of the risk-free world bond held at the beginning of period t, and y_t is net income in period t and is defined as output less than investment and government spending. The model assumes perfect capital mobility in that domestic consumers have access to the bond offered by the rest of the world and that the real return on this bond is the same across countries. In other words, the world risk-free bond provides a mechanism for risk sharing of domestic households who can use the international capital market to smooth consumption. Finally we assume that the no-Ponzi-scheme condition is satisfied.

A similar problem can be formulated for the rest of the world (ROW). We use an asterisk ("$*$") to represent the rest of world variables. For example, we assume that y_t^* is the average endowment (net income) of the rest of the world (G-7, OECD, or EU). Furthermore, we assume that domestic endowment and the ROW endowment are correlated. We will specify the structure of the income processes later.

Let $\beta R = 1$; optimal consumption is then determined by permanent income:

$$c_t = (R-1)\, s_t \tag{3}$$

where

$$s_t = b_t + \frac{1}{R}\sum_{j=0}^{\infty} R^{-j} E_t\left[y_{t+j}\right] \tag{4}$$

is the expected present value of lifetime resources, consisting of financial wealth (the risk free foreign bond) plus human wealth. In order to facilitate the introduction of robustness and rational inattention we follow Luo (2008) and Luo and Young (2010) and reduce the multivariate model with a general income process to a univariate model with iid innovations to permanent income s_t that can be solved in analytically.[6] Letting s_t be defined as a new state variable, we can reformulate the PIH model as

$$v(s_0) = \max_{\{c_t, s_{t+1}\}_{t=0}^{\infty}} \left\{ E_0\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right]\right\} \tag{5}$$

subject to

$$s_{t+1} = Rs_t - c_t + \zeta_{t+1}, \tag{6}$$

where the time $(t+1)$ innovation to permanent income can be written as

$$\zeta_{t+1} = \frac{1}{R}\sum_{j=t+1}^{\infty}\left(\frac{1}{R}\right)^{j-(t+1)}\left(E_{t+1} - E_t\right)\left[y_j\right]; \tag{7}$$

$v(s_0)$ is the consumer's value function under RE. Under the RE hypothesis, this model with quadratic utility leads to the well-known random walk result of Hall (1978),

$$\begin{aligned}\Delta c_t &= \frac{R-1}{R}\left(E_t - E_{t-1}\right)\left[\sum_{j=0}^{\infty}\left(\frac{1}{R}\right)^j \left(y_{t+j}\right)\right] \\ &= (R-1)\,\zeta_t,\end{aligned} \tag{8}$$

which relates the innovations to consumption to income shocks.[7] In this case, the change in consumption depends neither on the past history of labor income nor on anticipated changes in labor income. In addition, certainty equivalence holds, and thus uncertainty has no impact on optimal consumption.

We close the model by specifying domestic and foreign income processes as follows:

$$y_{t+1} = \rho y_t + \varepsilon_{t+1}, \tag{9}$$

$$y^*_{t+1} = \rho^* y^*_t + \varepsilon^*_{t+1}, \tag{10}$$

[6] See Luo (2008) for a formal proof of this reduction. Multivariate versions of the RI model are numerically, but not analytically, tractable, as the variance-covariance matrix of the states cannot generally be obtained in closed form.

[7] Under RE the expression of the change in individual consumption is the same as that of the change in aggregate consumption.

where ε_{t+1} and ε_{t+1}^* are Gaussian innovations to domestic and ROW income with mean 0 and variance ω^2 and ω^{*2}, respectively. To model the observed income correlations across countries, we assume that the correlation between ε_{t+1} and ε_{t+1}^*,

$$\text{corr}\left(\varepsilon_{t+1}, \varepsilon_{t+1}^*\right) = \eta. \tag{11}$$

Given the income processes, (9) and (10), the income correlation can be written as

$$\text{corr}\left(y_{t+1}, y_{t+1}^*\right) = \Pi_y \eta, \tag{12}$$

where $\Pi_y = \frac{\sqrt{(1-\rho^2)(1-\rho^{*2})}}{1-\rho\rho^*}$. Note that $\Pi_y = 1$ when $\rho = \rho^*$ and $\Pi_y < 1$ when $\rho \neq \rho^*$. In this case, permanent income and the innovation to it are

$$s_t = b_t + \frac{y_t}{R-\rho}; \tag{13}$$

$$\zeta_t = \frac{\varepsilon_t}{R-\rho}, \tag{14}$$

respectively. Similarly, for the average country in the international organization, $s_t^* = b_t^* + \frac{y_t^*}{R-\rho^*}$ and $\zeta_t^* = \frac{\varepsilon_t^*}{R-\rho^*}$.

2.2 Implications for Consumption Correlations

In the full-information RE model proposed in Section 2.1, consumption growth can be written as

$$\Delta c_t = \frac{R-1}{R-\rho}\varepsilon_t, \tag{15}$$

which means that consumption growth is white noise and the impulse response of consumption to the income shock is *flat* with an immediate upward jump in the initial period that persists indefinitely. (See the solid line in Figure 1.) However, as well documented in the consumption literature, the impulse response of aggregate consumption to aggregate income takes a *hump-shaped* form, which means that aggregate consumption growth reacts to income shocks gradually. Similarly, for the rest of the world,

$$\Delta c_t^* = \frac{R-1}{R-\rho^*}\varepsilon_t^*.$$

The international consumption correlation can thus be written as

$$\text{corr}(\Delta c_t, \Delta c_t^*) = \text{corr}(\varepsilon_t, \varepsilon_t^*) = \frac{1}{\Pi_y}\,\text{corr}\left(y_t, y_t^*\right). \tag{16}$$

Note that when income processes in both countries are unit roots, i.e., $\rho = \rho^* = 1$, the correlation reduces to $\text{corr}(\Delta c_t, \Delta c_t^*) = \text{corr}(\Delta y_t, \Delta y_t^*) = \eta$. In this case the consumption correlation across countries is the same as the corresponding income correlation.

It is clear that if the estimated income persistence parameters, ρ and ρ^*, are different and less than 1, $\Pi_y < 1$ and $\operatorname{corr}(c_t, c_t^*) > \operatorname{corr}(y_t, y_t^*)$. This prediction contradicts the empirical evidence, as international consumption correlations are lower than income correlations for most pairs of countries.

3 The Effects of RB on Consumption Correlations

3.1 The RB Version of the SOE Model

Robust control emerged in the engineering literature in the 1970s and was introduced into economics and further developed by Hansen, Sargent, and others. A simple version of robust optimal control considers the question of how to make decisions when the agent does not know the probability model that generates the data. Specifically, an agent with a preference for robustness considers a range of models surrounding the given approximating model, (6), and makes decisions that maximize lifetime expected utility given the worst possible model. Following Hansen and Sargent (2007a), a simple robustness version of the SOE model proposed in Section 2.1 can be written as

$$v(s_t) = \max_{c_t} \min_{\nu_t} \left\{ -\frac{1}{2} (\bar{c} - c_t)^2 + \beta \left[\vartheta \nu_t^2 + E_t \left[v(s_{t+1}) \right] \right] \right\} \tag{17}$$

subject to the distorted transition equation (i.e., the worst-case model):

$$s_{t+1} = R s_t - c_t + \zeta_{t+1} + \omega_\zeta \nu_t, \tag{18}$$

where ν_t distorts the mean of the innovation and $\vartheta > 0$ controls how bad the error can be.[8] As shown in Hansen, Sargent, and Tallarini (1999) and Hansen and Sargent (2007a), this class of models produces precautionary behavior while maintaining tractability within the LQ-Gaussian framework.

When domestic income follows an AR(1) process, (9), solving this robust control problem yields the following proposition:

Proposition 1 *Under RB, the consumption function is*

$$c_t = \frac{R-1}{1-\Sigma} s_t - \frac{\Sigma \bar{c}}{1-\Sigma}, \tag{19}$$

[8] Formally, this setup is a game between the decision-maker and a malevolent nature that chooses the distortion process ν_t. $\vartheta \geq 0$ is a penalty parameter that restricts attention to a limited class of distortion processes; it can be mapped into an entropy condition that implies agents choose rules that are robust against processes which are close to the trusted one. In a later section we will apply an error detection approach to estimate ϑ in small open economies.

the mean of the worst-case shock is

$$\nu_t \omega_\zeta = \frac{(R-1)\Sigma}{1-\Sigma} s_t - \frac{\Sigma}{1-\Sigma}\bar{c}, \tag{20}$$

and $s_t \left(= b_t + \frac{y_t}{R-\rho}\right)$ *is governed by*

$$s_{t+1} = \rho_s s_t + \frac{\Sigma}{1-\Sigma}\bar{c} + \zeta_{t+1}, \tag{21}$$

where $\zeta_{t+1} = \frac{\varepsilon_{t+1}}{R-\rho}$, $\Sigma = R\omega_\zeta^2/(2\vartheta) > 0$ *measures the effect of robustness on consumption, and* $\rho_s = \frac{1-R\Sigma}{1-\Sigma} \in (0,1)$.

Proof. See Appendix 6.1. ■

The consumption function under RB, (19), shows that the preference for robustness, ϑ, affects the precautionary savings increment, $-\frac{\Sigma}{1-\Sigma}\bar{c}$. The smaller the value of ϑ the larger the precautionary saving increment, provided $\Sigma < 1$.

Proposition 2 $\Sigma < 1$.

Proof. The second-order condition for a minimization by nature can be rearranged into

$$\vartheta > \frac{1}{2} R^2 \omega_\zeta^2. \tag{22}$$

Using the definition of Σ we obtain

$$1 > R\Sigma.$$

Since $R > 1$, we must have $\Sigma < 1$. ■

The expression for consumption also implies that the stronger the preference for robustness, the more consumption responds initially to changes in permanent income; that is, under RB consumption is more sensitive to unanticipated income shocks. This response is referred to as "making hay while the sun shines" in van der Ploeg (1993), and reflects the precautionary aspect of these preferences. Note that for the average country in the international organization, we have analogous results: we can just replace s_t and ζ_t with s_t^* and ζ_t^*, respectively.

3.2 Implications of RB for Consumption Correlations

Combining (19) with (21), consumption dynamics under RB in the domestic and average country are

$$c_t = \rho_s c_{t-1} + \frac{(R-1)\Sigma\bar{c}}{1-\Sigma} + \frac{R-1}{1-\Sigma}\zeta_t, \tag{23}$$

and

$$c_t^* = \rho_s^* c_{t-1}^* + \frac{(R-1)\,\Sigma^* \overline{c}}{1-\Sigma^*} + \frac{R-1}{1-\Sigma^*}\zeta_t^*, \tag{24}$$

respectively. Figure 1 also illustrates the response of aggregate consumption to an income shock ε_{t+1} in the domestic country; comparing the solid line (RE) with the dash-dotted line (RB), it is clear that RB raises the sensitivity of consumption to unanticipated changes in income. Given these two expressions, we have the following proposition about the cross-country consumption correlation:

Proposition 3 *Under RB, the consumption correlation between the home country and the ROW can be written as*

$$\operatorname{corr}(c_t, c_t^*) = \frac{\Pi_s}{\Pi_y} \operatorname{corr}(y_t, y_t^*)\,; \tag{25}$$

where

$$\Pi_s = \frac{\sqrt{\left(1-\rho_s^2\right)\left(1-\rho_s^{*2}\right)}}{1-\rho_s \rho_s^*}, \tag{26}$$

$\Pi_y = \frac{\sqrt{(1-\rho^2)(1-\rho^{*2})}}{1-\rho\rho^*}$, $\rho_s = \frac{1-R\Sigma}{1-\Sigma}$, $\rho_s^* = \frac{1-R\Sigma^*}{1-\Sigma^*}$, *and we use the facts that* $\operatorname{corr}(\zeta_t, \zeta_t^*) = \operatorname{corr}(\varepsilon_t, \varepsilon_t^*) = \eta$ *and* $\operatorname{corr}(y_t, y_t^*) = \Pi_y \eta$.

Proof. See Appendix 6.2. ■

Expression (25) clearly shows that the degrees of preference for robustness (RB), ρ_s and ρ_s^* (Σ and Σ^*) , affect the consumption correlation across countries. The value of Π_s defined in (26) measures to what extent RB changes consumption correlations across countries. It is straightforward to show that when the effects of RB, Σ, are the same in the two economies, ($\rho_s = \rho_s^*$ or $\Sigma = \Sigma^*$), $\Pi_s = 1$. In this case, RB has *no* impact on the consumption correlation: $\operatorname{corr}(c_t, c_t^*) = \operatorname{corr}(y_t, y_t^*)/\Pi_y$, which is just the correlation obtained in the standard RE-SOE model, (16). Note that Σ can be written as $\Sigma^* + \Delta_\Sigma$ where Δ_Σ is defined as the difference between the domestic country and the ROW. Therefore, both the degree of RB in the ROW and the difference between the degrees of RB in the two economies affects the consumption correlation across countries.[9]

Figure 2 shows that the impact of RB on consumption correlations, Π_s, is increasing with Σ^*, the degree of RB in the ROW, for different values of Δ_Σ. (Here we set $R = 1.04$.) Note that holding Δ_Σ constant, Π_s is also increasing with Σ. Consumption is more sensitive to income shocks when Σ is larger, which by itself increases consumption correlations across countries when

[9] In the next section, we will calibrate the RB parameter Σ and Σ^* using the detection error probabilities and show that the values of Σ and Σ^* are different.

the difference in RB is fixed. Figure 2 also shows that Π_s is decreasing in the difference between Σ and Σ^*, Δ_Σ, for given values of Σ^*. For example, when $\Sigma^* = 0.05$, $\Pi_s = 0.84$ if $\Sigma - \Sigma^* = 0.1$, and $\Pi_s = 0.69$ if $\Sigma - \Sigma^* = 0.2$. After calibrating the model we will examine the net effect of RB on the consumption correlations.

3.3 Calibrating the RB Parameter

In this subsection we follow Hansen, Sargent, and Wang (2002) and Hansen and Sargent (2007a) to calibrate the RB parameter (ϑ and Σ). Specifically, we calibrate the model by using the model detection error probability that is based on a statistical theory of model selection (the approach will be precisely defined below). We can then infer what values of the RB parameter ϑ imply reasonable fears of model misspecification for empirically-plausible approximating models. In other words, the model detection error probability is a measure of how far the distorted model can deviate from the approximating model without being discarded; standard significance levels for testing are then used to determine what reasonable fears entail.

3.3.1 The Definition of the Model Detection Error Probability

Let model A denote the approximating model and model B be the distorted model. Define p_A as

$$p_A = \text{Prob}\left(\log\left(\frac{L_A}{L_B}\right) < 0 \middle| A\right), \tag{27}$$

where $\log\left(\frac{L_A}{L_B}\right)$ is the log-likelihood ratio. When model A generates the data, p_A measures the probability that a likelihood ratio test selects model B. In this case, we call p_A the probability of the model detection error. Similarly, when model B generates the data, we can define p_B as

$$p_B = \text{Prob}\left(\log\left(\frac{L_A}{L_B}\right) > 0 \middle| B\right). \tag{28}$$

Following Hansen, Sargent, and Wang (2002) and Hansen and Sargent (2007a), the detection error probability, p, is defined as the average of p_A and p_B:

$$p(\vartheta) = \frac{1}{2}(p_A + p_B), \tag{29}$$

where ϑ is the robustness parameter used to generate model B. Given this definition, we can see that $1-p$ measures the probability that econometricians can distinguish the approximating model from the distorted model. Now we show how to compute the model detection error probability in the RB model.

3.3.2 Calibrating the RB Parameter in the SOE Model

Let's first consider the model with the robustness preference. In the domestic country, under RB, assuming that the approximating model generates the data, the state, s_t, evolves according to the transition law

$$\begin{aligned} s_{t+1} &= Rs_t - c_t + \zeta_{t+1}, \\ &= \frac{1 - R\Sigma}{1 - \Sigma} s_t + \frac{\Sigma}{1 - \Sigma} \bar{c} + \zeta_{t+1}. \end{aligned} \tag{30}$$

In contrast, assuming that the distorted model generates the data, s_t evolves according to

$$\begin{aligned} s_{t+1} &= Rs_t - c_t + \zeta_{t+1} + \omega_\zeta \nu_t, \\ &= s_t + \zeta_{t+1}. \end{aligned} \tag{31}$$

In order to compute p_A and p_B, we use the following procedure:

1. Simulate $\{s_t\}_{t=0}^T$ using (30) and (31) a large number of times. The number of periods used in the simulation, T, is set to be the actual length of the data for each individual country.
2. Count the number of times that $\log\left(\frac{L_A}{L_B}\right) < 0 \Big| A$ and $\log\left(\frac{L_A}{L_B}\right) > 0 \Big| B$ are each satisfied.
3. Determine p_A and p_B as the fractions of realizations for which $\log\left(\frac{L_A}{L_B}\right) < 0 \Big| A$ and $\log\left(\frac{L_A}{L_B}\right) > 0 \Big| B$, respectively.

In practice, given Σ, to simulate the $\{s_t\}_{t=0}^T$ we need to know a) the volatility of ζ_t in (30) and (31), and b) the value of $\bar{c}$. For a), we can compute it from $\mathrm{sd}(\zeta) = \frac{\sqrt{1-\rho^2}}{R-\rho}\mathrm{sd}(y)$, where $\mathrm{sd}(y)$ is the standard deviation of the net output.[10] For b), we use the local coefficient for relative risk aversion $\gamma = -\frac{u''(c)c}{u'(c)} = \frac{c}{\bar{c}-c}$ to recover $\bar{c} = \left(1 + \frac{1}{\gamma}\right) c$ where c is mean consumption. Here we set $\gamma = 2$ as the benchmark case. The gross interest rate R is set to be 1.04 in all calibrations. For the average country in the international organization, we can use the same procedure to calibrate the RB parameter.

3.4 Data

To implement the calibration procedure we describe in the previous section, we have to first define our empirical counterpart of the domestic country and the rest of the world (ROW) in the model. Then we can compute the required input statistics from the data used in the calibration process.

[10] Net output is defined as $GDP - I - G$ where I and G are investment and government expenditure, respectively.

First, we define ROW as the average of G-7.[11] Second, we choose one of the relatively small countries in G-7 as the domestic country. In practice, we have chosen 5 different countries, Canada, Italy, UK, France and Germany as the domestic country and report the results in the next section.[12] For example, if we choose Canada as the domestic country, the domestic income, y_t, is the net output of Canada at time t, and the average income of the risk sharing group, y_t^*, corresponds to the average level of net output for G-7 countries at time t.

The annual data we use come from World Development Indicators.[13] The data cover the period from year 1970 to year 2006 which are the common periods of the variables that we use in the data set for different countries. Net output (y) is constructed as $GDP - I - G$, where I is Gross Fixed Capital Formation and G is General Government Final Consumption Expenditure. Consumption (c) in defined as Household Final Consumption Expenditure, and the risk-free bond in the model (b) corresponds to Net Foreign Assets. All the variables are measured in the US currency of year 2000. We apply the HP filter to the time series before computing the statistics.

3.5 Main Findings

Table 1 reports the calibration results and some key data statistics (the stochastic properties of net output and consumption and net output correlations across countries) for the five countries we set as the domestic country in turn. In the calibration exercise, following Hansen and Sargent (2007), we set the detection error probability, p, to be a plausible value, 10% (i.e., with 90% probability consumers can distinguish the approximating model from the distorted model). The last column in Table 1 reports the model misspecification parameter ϑ relative to the one estimated for Canada.[14]

Given these estimation and calibration results, Table 2 compares the implications for the consumption correlations between the RE and RB models. Our key result here is that RB improves

[11] In this paper we follow Crucini (1999) to use G-7 countries. Pakko (1998) used 15 OECD countries to study cross-country correlations. In one of our robustness checks, we also use an alternative definition which defines ROW as the average of the G-7 excluding the country we choose as the domestic country. For instance, when we choose Canada as the domestic country, we use the other six countries in G-7 to define ROW. However, we find this alternative definition does not alter our main findings.

[12] We do not model the two largest countries in G-7, US and Japan, as the domestic country because they are too large to be modeled as small open economies.

[13] For the US data, since the consumption variable (Household Final Consumption Expenditure) contains missing values, we replace it with the Personal Consumption Expenditure (CEA) from the FAME economic databases maintained by Board of Governors of Federal Reserve System.

[14] As shown in section 2.2, consumption in RE models follows a random walk process which does not allow us to explicitly write down the expression for $\operatorname{corr}(c, c^*)$ in that case. Thus, we approximate the value of $\operatorname{corr}(c, c^*)$ by choosing a value of Σ extremely close to zero to approximate the predictions of the RE model.

the performance of the model in terms of the cross-correlations of consumption; at the estimated Σ each country displays a lower correlation with the foreign aggregate. Furthermore, the cross-correlations of consumption are uniformly below the income correlations under RB, whereas the opposite holds for RE. Quantitatively, however, the improvements are relatively small for most of the countries we consider here. One exception is the United Kingdom; however, since the UK had the largest deviation under RE, even the relatively large improvement still leaves the RB model with a lot of ground to cover to reconcile theory and data. There are two clear patterns apparent in Tables 1 and 2. The reduction in the consumption correlation is decreasing in Π_s and increasing in Σ. The large improvement in the UK correlation is therefore due to the unusually low value of Π_s and unusually high value of Σ. (We will provide more explanations on this in the next paragraph.) These findings are consistent with our theoretical results obtained in Section 3.2.

`[Insert Tables 1-2 Here]`

To examine whether the findings are robust, we do sensitivity analysis by using different values of the detection error probabilities (p). Specifically, when p is set to 5% and 15%, Tables 3 and 4 clearly show that our main findings that the presence of RB reduces consumption correlations across countries and the prediction that consumption correlations under RB are consistently lower than the corresponding income correlations is robust to different values of the detection error probability. In addition, it is also clear from the tables that as p varies from 10% to 5% or 15%, RB only has a small impact on $\operatorname{corr}(c_t, c_t^*)$. For example, when p falls from 10% to 5%, $\operatorname{corr}(c_t, c_t^*)$ decreases from 0.323 to 0.322 in Canada; from 0.502 to 0.501 in Italy; and from 0.454 to 0.452 in the UK.

As p decreases, it generally leads to a lower calibrated ϑ (and ϑ^*) and a higher calibrated Σ (and Σ^*); this combination generates two competing effects on consumption correlations. First, the increase of Σ^* will increase Π_s (as we can see from Figure 2), so consumption correlations will increase. Second, reducing p not only increases Σ^* but also increases the difference $\Sigma - \Sigma^*$ (which means the increase of Σ is more than that of Σ^* as shown in Table 1). This increase of $\Sigma - \Sigma^*$ decreases the consumption correlation, as we can see from Figure 2 as well. Hence, these two offsetting effects imply that consumption correlations do not change much as p varies.

`[Insert Tables 3-4 Here]`

4 Extension to Incorporate RI

Since the model with only RB failed to produce a sufficiently-large decrease in the consumption correlations across the board, we modify the model to include RI. In Luo, Nie, and Young (2010) we show that the RB-RI model generates a significant improvement in the model's ability to replicate the joint dynamics of current accounts, consumption, and net income. We now turn to investigating the impact of RI on the consumption correlations, and will show that it can further reduce them relative to RB.

4.1 Information-processing Constraints

Under RI, consumers in the economy face both the usual flow budget constraint and information-processing constraint due to finite Shannon capacity first introduced by Sims (2003). As argued by Sims (2003, 2006), individuals with finite channel capacity cannot observe the state variables perfectly; consequently, they react to exogenous shocks incompletely and gradually. They need to choose the posterior distribution of the true state after observing the corresponding signal.[15]

Following Sims (2003), the consumer's information-processing constraint can be characterized by the following inequality:

$$\mathcal{H}\left(s_{t+1}|\mathcal{I}_t\right) - \mathcal{H}\left(s_{t+1}|\mathcal{I}_{t+1}\right) \leq \kappa, \tag{32}$$

where κ is the consumer's channel capacity, $\mathcal{H}\left(s_{t+1}|\mathcal{I}_t\right)$ denotes the entropy of the state prior to observing the new signal at $t+1$, and $\mathcal{H}\left(s_{t+1}|\mathcal{I}_{t+1}\right)$ is the entropy after observing the new signal.[16] The concept of *entropy* is from information theory, and it characterizes the uncertainty in a random variable. The right-hand side of (32), being the reduction in entropy, measures the amount of information in the new signal received at $t+1$. Hence, as a whole, (32) means that the reduction in the uncertainty about the state variable gained from observing a new signal is bounded from above by κ. Since the *ex post* distribution of s_t is a normal distribution, $N\left(\widehat{s}_t, \sigma_t^2\right)$, (32) can be reduced to

$$\log|\psi_t^2| - \log|\sigma_{t+1}^2| \leq 2\kappa \tag{33}$$

where $\sigma_{t+1}^2 = \text{var}\left[s_{t+1}|\mathcal{I}_{t+1}\right]$ and $\psi_t^2 = \text{var}\left[s_{t+1}|\mathcal{I}_t\right]$ are the posterior variance and prior variance of the state variable, respectively. To obtain (33), one needs to use the fact that the entropy of a

[15] More generally, agents choose the joint distribution of consumption and permanent income subject to restrictions about the transition from prior (the distribution before the current signal) to posterior (the distribution after the current signal). The signal is current consumption (equivalently future permanent income); the interpretation is that nature picks current consumption but must draw from the distribution specified by the agent.

[16] We regard κ as a technological parameter. If the base for logarithms is 2, the unit used to measure information flow is a 'bit', and if we use the natural logarithm e, the unit is a 'nat'. 1 nat is equal to $\log_2 e = 1.433$ bits.

Gaussian random variable is equal to half of its logarithm variance plus a constant term.

It is straightforward to show that in the univariate case (33) has a steady state σ^2.[17] In steady state consumers in the domestic country behave as if observing a noisy measurement which is $s^*_{t+1} = s_{t+1} + \xi_{t+1}$, where ξ_{t+1} is the endogenous noise and its variance $\alpha_t^2 = \text{var}[\xi_{t+1}|\mathcal{I}_t]$ is determined by the usual updating formula of the variance of a Gaussian distribution based on a linear observation:

$$\sigma_{t+1}^2 = \psi_t^2 - \psi_t^2\left(\psi_t^2 + \alpha_t^2\right)^{-1}\psi_t^2. \tag{34}$$

In the steady state $\sigma^2 = \psi^2 - \psi^2\left(\psi^2 + \alpha^2\right)^{-1}\psi^2$, which can be solved as $\alpha^2 = \left[\left(\sigma^2\right)^{-1} - \left(\psi^2\right)^{-1}\right]^{-1}$. Note that (34) implies that in the steady state $\sigma^2 = \left(\frac{1}{R-\rho}\right)^2 \frac{\omega_\zeta^2}{\exp(2\kappa)-R^2}$ and $\alpha^2 = \text{var}[\xi_{t+1}] = \frac{\left[\omega_\zeta^2/(R-\rho)^2 + R^2\sigma^2\right]\sigma^2}{\omega_\zeta^2/(R-\rho)^2 + (R^2-1)\sigma^2}$. Analogous expressions for the information-processing constraints and the steady state conditional variance exist for agents in ROW.

4.2 The RB-RI Version of the SOE Model

Following Hansen and Sargent (2007a) and Luo and Young (2010), the robust PIH problem with inattentive consumers can be written as

$$\widehat{v}(\widehat{s}_t) = \max_{c_t}\min_{\nu_t}\left\{-\frac{1}{2}(c_t - \overline{c})^2 + \beta E_t\left[\vartheta\nu_t^2 + \widehat{v}(\widehat{s}_{t+1})\right]\right\} \tag{35}$$

subject to the flow budget constraint

$$s_{t+1} = Rs_t - c_t + \zeta_{t+1} + \omega_\zeta \nu_t \tag{36}$$

and the Kalman filter equation

$$\widehat{s}_{t+1} = (1-\theta)\left(R\widehat{s}_t - c_t + \omega_\zeta\nu_t\right) + \theta\left(s_{t+1} + \xi_{t+1}\right), \tag{37}$$

where $\widehat{s}_t = E[s_t|\mathcal{I}_t]$ is the conditional mean of s_t, $\sigma^2 = \text{var}[s_t|\mathcal{I}_t]$ is the steady state conditional variance of s_t, ξ_{t+1} is the iid endogenous noise with $\alpha^2 = \text{var}[\xi_{t+1}] = \frac{\left[\omega_\zeta^2/(R-\rho)^2 + R^2\sigma^2\right]\sigma^2}{\omega_\zeta^2/(R-\rho)^2 + (R^2-1)\sigma^2}$, $\theta = \sigma^2/\alpha^2 = 1 - 1/\exp(2\kappa) \in [0,1]$ is the constant optimal weight on any new observation, and $s_0 \sim N\left(\widehat{s}_0, \sigma^2\right)$ is fixed.[18] The following proposition summarizes the solution to the RB-RI model.

Proposition 4 *Given ϑ and θ, the consumption function of inattentive consumers is*

$$c_t = \frac{R-1}{1-\Sigma}\widehat{s}_t - \frac{\Sigma}{1-\Sigma}\overline{c}, \tag{38}$$

[17] Convergence requires that $\kappa > \log(R) \approx R - 1$. See Luo and Young (2010).

[18] That is, θ measures how much new information is transmitted each period or how much uncertainty is removed upon the receipt of a new signal.

the mean of the worst-case shock is

$$\nu_t \omega_\zeta = \frac{(R-1)\Sigma}{1-\Sigma}\widehat{s}_t - \frac{\Sigma}{1-\Sigma}\overline{c}, \tag{39}$$

where $\Sigma = R\omega_\zeta^2/(2\vartheta)$, *and the dynamics of* s_t *and* $\widehat{s}_t$ *are governed by the following system:*

$$s_{t+1} = Rs_t - \frac{R-1}{1-\Sigma}\widehat{s}_t + \zeta_{t+1} + \frac{\Sigma}{1-\Sigma}\overline{c}, \tag{40}$$

$$\widehat{s}_{t+1} = (1-\theta)\frac{1-R\Sigma}{1-\Sigma}\widehat{s}_t + \theta(s_{t+1}+\xi_{t+1}) + \frac{(1-\theta)\Sigma}{1-\Sigma}\overline{c}. \tag{41}$$

Finally, $\Sigma < 1$.

Proof. To obtain the solution to the RI-RB model we simply need to replace s_t with $\widehat{s}_t$ in the consumption function under robustness and the mean of the worst-case shock. Substituting (38) into the actual budget constraint, $s_{t+1} = Rs_t - c_t + \zeta_{t+1}$, and the Kalman filter equation, $\widehat{s}_{t+1} = (1-\theta)(R\widehat{s}_t - c_t) + \theta(s_{t+1}+\xi_{t+1})$, yields (40) and (41). Our previous result that $\Sigma < 1$ holds unchanged. ■

In the RB-RI model individual dynamics are not identical to aggregate dynamics. Combining (38) with (41) yields the change in individual consumption in the RI-RB economy; in the RB-RI model individual dynamics are not identical to aggregate dynamics. Combining (38) with (41) yields the change in individual consumption in the RI-RB economy:

$$\Delta c_t = \frac{(1-R)\Sigma}{1-\Sigma}(c_{t-1}-\overline{c}) + \frac{R-1}{1-\Sigma}\left(\frac{\theta\zeta_t}{1-(1-\theta)R\cdot L} + \theta\left(\xi_t - \frac{\theta R\xi_{t-1}}{1-(1-\theta)R\cdot L}\right)\right), \tag{42}$$

where L is the lag operator and we assume that $(1-\theta)R < 1$.[19] This expression shows that consumption growth is a weighted average of all past permanent income and noise shocks. Since it permits exact aggregation, we can obtain the change in aggregate consumption as

$$\Delta c_t = \frac{(1-R)\Sigma}{1-\Sigma}(c_{t-1}-\overline{c}) + \frac{R-1}{1-\Sigma}\left(\frac{\theta\zeta_t}{1-(1-\theta)R\cdot L} + \theta\left(\overline{\xi}_t E^i[\xi_t] - \frac{\theta R\overline{\xi}_{t-1}}{1-(1-\theta)R\cdot L}\right)\right), \tag{43}$$

where i denotes a particular individual, $E^i[\cdot]$ is the population average, and $\overline{\xi}_t = E^i[\xi_t]$ is the common component of the noise.[20] This expression shows that even if every consumer only faces the common shock ζ, the RI economy still has heterogeneity since each consumer faces

[19] This assumption is innocuous, since it is weaker than the condition needed for convergence of the filter (it requires that $\kappa > \frac{1}{2}\log(R) \approx \frac{R-1}{2}$). The condition implies that consumption is responsive enough to the state to 'zero out' the effect of the explosive root in the Euler equation; see Sims (2003).

[20] For simplicity, here we use the same notation c for aggregate consumption.

the idiosyncratic noise induced by finite channel capacity.[21] Assume that ξ_t consists of two independent noises: $\xi_t = \overline{\xi}_t + \xi_t^i$, where $\overline{\xi}_t = E^i[\xi_t]$ and ξ_t^i are the common and idiosyncratic components of the error generated by ζ_t, respectively. A single parameter,

$$\lambda = \frac{\text{var}\left[\overline{\xi}_t\right]}{\text{var}\left[\xi_t\right]} \in [0,1],$$

can be used to measure the common source of coded information on the aggregate component (or the relative importance of $\overline{\xi}_t$ vs. ξ_t).[22] Note that (43) can be written in the following AR(1) process:

$$c_t = \rho_s c_{t-1} + \frac{(R-1)\,\Sigma\overline{c}}{1-\Sigma} + \frac{R-1}{1-\Sigma}\left(\frac{\theta\zeta_t}{1-(1-\theta)R\cdot L} + \theta\left(\overline{\xi}_t - \frac{\theta R\overline{\xi}_{t-1}}{1-(1-\theta)R\cdot L}\right)\right), \tag{44}$$

which can be written as the following ARMA$(2,1)$ process:

$$\left(1-\phi_1\cdot L-\phi_2\cdot L^2\right)c_t = \theta\frac{R-1}{1-\Sigma}\left(\zeta_t+\overline{\xi}_t-R\overline{\xi}_{t-1}\right), \tag{45}$$

where

$$\phi_1 = \rho_s + \rho_\theta = \rho_s + (1-\theta)R,$$
$$\phi_2 = -\rho_s\rho_\theta = -\rho_s(1-\theta)R.$$

Figure 1 also shows how RI can help generate the smooth and hump-shaped impulse response of consumption to the income shock, which, as argued in Sims (2003), fits the VAR evidence better. Similarly, in the rest of the world, we have

$$\left(1-\phi_1^*\cdot L-\phi_2^*\cdot L^2\right)c_t^* = \theta^*\frac{R-1}{1-\Sigma^*}\left(\zeta_t^*+\overline{\xi_t^*}-R\overline{\xi_t^*}\right), \tag{46}$$

where

$$\phi_1^* = \rho_s^* + \rho_\theta^* = \rho_s^* + (1-\theta^*)R,$$
$$\phi_2^* = -\rho_s^*\rho_\theta^* = -\rho_s^*(1-\theta^*)R.$$

[21] As argued in Sims (2003), although the randomness in an individual's response to aggregate shocks will be idiosyncratic because it arises from the individual's own information-processing constraint, there is likely a significant common component. The intuition is that people's needs for coding macroeconomic information efficiently are similar, so they rely on common sources of coded information.

[22] The special case that $\lambda = 1$ can be viewed as a representative agent model in which the aggregation issue does not arise.

4.3 Consumption Correlations under RB and RI

Given (45) and (46), we have the following proposition about the cross-country consumption correlation under RB and RI:

Proposition 5 *The consumption correlation between the two economies under RB and RI is*

$$\operatorname{corr}\left(c_t, c_t^*\right) = \frac{\operatorname{cov}\left(c_t, c_t^*\right)}{\sqrt{\operatorname{var}\left(c_t\right)\operatorname{var}\left(c_t^*\right)}} = \frac{\Pi}{\Pi_y}\operatorname{corr}\left(y_t, y_t^*\right), \tag{47}$$

where

$$\Pi = \frac{\sum_{k=0}^{\infty}\left\{\left[\sum_{j=0,j\leq k}^{k}\left(\rho_\theta^j\rho_s^{k-j}\right)\right]\left[\sum_{j=0,j\leq k}^{k}\left(\rho_\theta^{*j}\rho_s^{*k-j}\right)\right]\right\}}{\sqrt{\Xi_1\Xi_2}}, \tag{48}$$

$$\Xi_1 = \frac{\left(1+\rho_s\rho_\theta\right)}{\left(1-\rho_s\rho_\theta\right)\left[\left(1+\rho_s\rho_\theta\right)^2-\left(\rho_s+\rho_\theta\right)^2\right]} +$$
$$\frac{\lambda^2}{\left(1/\left(1-\theta\right)-R^2\right)\theta}\left\{\frac{1}{1-\rho_s^2}+\left(\theta R\right)^2\frac{\left(1+\rho_s\rho_\theta\right)}{\left(1-\rho_s\rho_\theta\right)\left[\left(1+\rho_s\rho_\theta\right)^2-\left(\rho_s+\rho_\theta\right)^2\right]}\right\},$$
$$\Xi_2 = \frac{\left(1+\rho_s^*\rho_\theta^*\right)}{\left(1-\rho_s^*\rho_\theta^*\right)\left[\left(1+\rho_s^*\rho_\theta^*\right)^2-\left(\rho_s^*+\rho_\theta^*\right)^2\right]} +$$
$$\frac{\lambda^{*2}}{\left(1/\left(1-\theta^*\right)-R^2\right)\theta^*}\left\{\frac{1}{1-\rho_s^{*2}}+\left(\theta^* R\right)^2\frac{\left(1+\rho_s^*\rho_\theta^*\right)}{\left(1-\rho_s^*\rho_\theta^*\right)\left[\left(1+\rho_s^*\rho_\theta^*\right)^2-\left(\rho_s^*+\rho_\theta^*\right)^2\right]}\right\}.$$

Proof. See Appendix 6.3. ■

We assume that the RI parameters are the same in the domestic country and the rest of the world: $\theta^* = \theta$, $\rho_\theta^* = \rho_\theta$, and $\lambda^* = \lambda$.[23] Π will converge to Π_s, (26) defined in Section 3.2, as θ converges to 1. It is worth emphasizing that the presence of endogenous noises, $\overline{\xi}_{t-j}$ and $\overline{\xi}_{t-j}^*$ $(j \geq 0)$, in the expressions for the dynamics of aggregate consumption does not affect the covariance between under RI and RB, $\operatorname{cov}\left(c_t, c_t^*\right)$, as all noises are iid and are also independent of the exogenous income shocks $(\zeta_{t-j},\ j \geq 0)$. Therefore, the presence of the common noise shocks will further reduce the consumption correlations across countries as they increase the variances of both c_t and c_t^*.

[23] It is straightforward to show that allowing for the heterogeneity in θ will further reduce the cross-country correlations. This case may be of interest, however, since Luo and Young (2009) show that it can imply infrequent updating as in Bacchetta and van Wincoop (2010). Since it lies beyond our purposes here and poses calibration challenges, we leave it for future work.

4.3.1 A Special Case: No Common Noise or $\lambda = 0$

In the case without common noises ($\lambda = 0$), (47) can thus be reduced to

$$\operatorname{corr}\left(c_t, c_t^*\right) = \frac{\Pi}{\Pi_y} \operatorname{corr}\left(y_t, y_t^*\right), \tag{49}$$

where

$$\Pi = \frac{\sum_{k=0}^{\infty}\left\{\left[\sum_{j=0,j\leq k}^{k}\left(\rho_\theta^j \rho_s^{k-j}\right)\right]\left[\sum_{j=0,j\leq k}^{k}\left(\rho_\theta^j \rho_s^{*k-j}\right)\right]\right\}}{\sqrt{\frac{(1+\rho_s\rho_\theta)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]}\frac{(1+\rho_s^*\rho_\theta)}{(1-\rho_s^*\rho_\theta)\left[(1+\rho_s^*\rho_\theta)^2-(\rho_s^*+\rho_\theta)^2\right]}}}. \tag{50}$$

Note that here we have assumed that $\theta^* = \theta$ and $\rho_\theta^* = \rho_\theta$. Figures 3 and 4 illustrate how the interaction between RB and RI affects consumption correlations across countries in this special case when $\Sigma = 0.7$ and $R = 1.04$. Note that here we use Π to measure to what extent RB and RI can affect the correlation because Π converges to 1 as Σ^* and Σ reduces to 0 and θ increases to 1. They show that given the level of finite capacity measured by θ, the consumption correlation is increasing with the degree of Σ^* (RB in the ROW) and is decreasing with the difference of RB in the two economies, $\Sigma - \Sigma^*$. These results are the same as those obtain the RB model in which $\theta = 1$ (channel capacity, κ, is infinite). In addition, it is also clear that given the Σ^* or $\Sigma - \Sigma^*$, the correlation is decreasing with the degree of attention (θ). That is, the gradual response of consumption to income shocks due to finite capacity by itself increases the consumption correlation.

The effect of RI on cross-country consumption correlations in this special case is similar to that of habit formation, because habit formation also leads to slow adjustments in consumption.[24] As shown in Fuhrer and Klein (2006), the presence of habit formation increases the correlation of consumption across countries and the empirical evidence of high consumption correlations might reflect habit persistence rather than common income risks or risk sharing. In addition, this special case can also be compared to the sticky expectations (SE) model. The idea of SE is to relax the assumption that all consumers' expectations are completely updated at every period and assume that only a fraction of the population update their expectations on permanent income and re-optimize in any given period.[25] As shown in Carroll and Slacalek (2006), SE can also generate the same predictions on aggregate consumption dynamics as habit formation. Consequently, it is straightforward to show that SE generates the same predictions on international consumption correlations as habit formation and the special case of $\lambda = 0$ do.

[24] See Luo (2008) for a detailed proof for the observational equivalence between this special RI case and habit formation in the sense that they generate the same consumption dynamics.

[25] Reis (2006) uses "inattentiveness" to characterize the infrequent adjustment behavior of consumers.

4.3.2 General Cases ($\lambda > 0$)

In the general cases in which $\lambda > 0$, the aggregate endogenous noise due to finite capacity plays an important role in determining the consumption correlations. Some recent papers have shown the importance of noise shocks for aggregate fluctuations. For example, Angeletos and La'O (2009) show how dispersed information about the underlying aggregate productivity shock contributes to significant noise in the business cycle and helps explain cyclical variations in observed Solow residuals and labor wedges in the RBC setting. Lorenzoni (2009) examines how demand shocks (noisy news about future aggregate productivity) contribute to business cycles fluctuations in a new Keynesian model. Here we will show that an aggregate noise component can improve the model's predictions on consumption correlation across countries.

To examine the effects of the aggregate noise, λ, on the consumption correlation in the RB-RI model, we set the degree of attention, θ, to be 60% and the strength of RB in the domestic country to be 0.7 in this subsection. Figure 5 illustrates how the consumption correlation is increasing with Σ^* for every given λ, and is decreasing with λ for every given Σ^*. As in the previous section, Figure 6 illustrates how the consumption correlation is decreasing with $\Sigma - \Sigma^*$. (It is also clear that the correlation is decreasing with λ for any given $\Sigma - \Sigma^*$.) Note that given the RI parameters, the effects of Σ^* and $\Sigma - \Sigma^*$ on the correlation are the same as in the RB model.

The intuition that the consumption correlation is decreasing with λ is as follows. Given the degree of attention (θ), λ has *no* impact on the covariance between the two consumption processes but increases the variance of consumption, which in turn reduces the consumption correlation. It is obvious that RI and RB have the most significant impacts on the cross-country consumption correlation in the representative agent case ($\lambda = 1$) because the impact of the noises due to RI on the variances of consumption that appear in the denominator of (48) is largest in this case.

The effect of θ on the consumption correlation differs in sign for $\lambda = 0$ and λ (sufficiently) positive; Figure 3 shows that the first case yields the correlation as an increasing function of θ, while Table 5 (to be discussed below) shows that the second case has the correlation as a decreasing function of θ. The intuition is that when $\lambda = 0$ there is a missing effect. Aggregate consumption in the model is affected only by the common noise shock $\bar{\xi}$; if $\lambda = 0$ this shock has zero variance, and the variance is increasing in λ. Thus, the effect of increasing θ when $\lambda > 0$ decreases the variances of c and c^* and thus reduces the correlation; if λ is large enough, this effect dominates the positive effect identified above and the correlation is increasing in θ.

4.4 Main Findings

To illustrate the quantitative implications of the RB-RI model on the consumption correlation, we fix the the RB parameter at the same levels we obtain in Section 3.5 and vary the two RI parameters, λ and θ. As in Section 3.5, we set the detection error probability, p, to be a plausible value, 10%. Table 5 reports the implied consumption correlations (between the domestic country and ROW) between the RE, RB, and RB-RI models. There are two interesting observations in the table. First, given the degrees of RB and RI (θ), $\operatorname{corr}(c_t, c_t^*)$ decreases with the aggregation factor (λ). Second, when λ is positive (even if it is very small, e.g., 0.1 in the table), $\operatorname{corr}(c_t, c_t^*)$ is decreasing with the degree of inattention (i.e., increasing with θ). The intuition is that when there are common noises, the effect of the noises could dominate the effect of gradual consumption adjustments on cross-country consumption correlations. This contrasts with the results in Section 4.3.1. That is, when there is no common noise, $\operatorname{corr}(c_t, c_t^*)$ is decreasing with the degree of attention (θ), and the effect of RI on $\operatorname{corr}(c_t, c_t^*)$ is similar to that of habit formation or sticky expectations.

As we can see from Table 5, for all the countries we consider here, introducing RI into the RB model can make the model better fit the data on consumption correlations at many combinations of the parameter values. For example, for Italy, when $\theta = 60\%$ (60% of the uncertainty is removed upon receiving a new signal about the innovation to permanent income) and $\lambda = 1$, the RB-RI model predicts that $\operatorname{corr}(c_t, c_t^*) = 0.27$, which is very close to the empirical counterpart, 0.25. For France, when $\theta = 90\%$ and $\lambda = 0.5$, the RB-RI model predicts that $\operatorname{corr}(c_t, c_t^*) = 0.46$, which exactly matches the empirical counterpart.

To examine whether the findings are robust, we do sensitivity analysis using different values of the detection error probabilities (p) in the calibration. As in the last section, here we also set p to be 5% and 15%. Tables 6 and 7 show that our main findings in the benchmark RB-RI model are very robust; varying the value of p only has tiny effects on the consumption correlations.

A single bit of information transmitted every quarter is a very low number and is probably well below the total information-processing ability of human beings. However, it is not implausible for ordinary investors/consumers to allocate such small amounts of attention to macroeconomic variables because they also face many other competing demands on capacity. For an extreme case, a young worker who accumulates balances in his retirement savings account might pay no attention to the behavior of the stock market until he retires, particularly if the account is highly illiquid. In addition, in our models for simplicity we only consider one shock to permanent income, while in reality consumers face substantial idiosyncratic shocks that we do not model in this paper. Maćkowiak and Wiederholt (2009) solve an optimal attention allocation problem and find that a much larger fraction of capacity is devoted to monitoring the most important idiosyncratic shock. For example, they show that given total capacity $\kappa = 3$ bits, firms only allocate 6% of their capacity to track aggregate conditions.[26] Therefore, the exogenous capacity given in our model can be regarded as a shortcut to small fractions of households' total capacity used to monitor their permanent income hit by an aggregate shock. In addition, low capacity devoted to processing macroeconomic information could be rationalized by examining the welfare effects of finite channel capacity.[27]

`[Insert Tables 5-7 Here]`

5 Conclusion

In this paper we provide further evidence that movements in consumption across countries can be understood easily when viewed through the lens of a permanent income model that incorporates robust decision-making and limited information-processing capacity (robustness and rational inattention); combined with the results in Luo, Nie, and Young (2010) on the model's ability to capture the dynamics of the current account, we can safely say that rational inattention and robustness have a role in future open-economy macro studies. The model used here has many virtues – it is analytically tractable (leaving nothing hidden behind numerical computations), it displays precautionary savings, and it resolves the classic *excess sensitivity* and *excess smoothness* puzzles in aggregate consumption. However, it does have some shortcomings, such as reliance on a constant return to savings, linear-quadratic functional forms, and a univariate source of risk.[28] The absence of shocks to the interest rate may be of particular importance, given the results in Neumeyer and Perri (2005) regarding the importance of such disturbances. We are working to relax these limitations currently in order to confront the model with more aspects of small open economy behavior, such as portfolio choice.[29]

[26]There are some existing estimation and calibration results in the literature. For example, Adam (2005) found $\theta = 40\%$ or $\kappa = 0.255$ bits based on the response of aggregate output to monetary policy shocks. Luo (2008) found that if $\theta = 50\%$, the otherwise standard permanent income model can generate realistic relative volatility of consumption to labor income.

[27]See Luo and Young (2010) for details about the welfare losses due to RI in the RB model; they are uniformly small.

[28]As noted elsewhere, we can drop this univariate assumption at the cost of losing the ability to solve analytically for the variance-covariance matrix of the RI-induced noise shocks. We are currently pursuing an extension with capital to verify the robustness of our results.

[29]Kraay and Ventura (2000,2003) and Tille and van Wincoop (2010) discuss the 'new view' of the current account as primarily a portfolio allocation issue, while Luo (2010) applies rational inattention to a single agent portfolio problem. Merging the two literatures using our existing model is the subject of current work.

6 Appendix (Not for Publication)

6.1 Solving the Robust Model

To solve the Bellman equation (17), we conjecture that

$$v(s_t) = -As_t^2 - Bs_t - C, \tag{51}$$

where A, B, and C are undetermined coefficients. Substituting this guessed value function into the Bellman equation gives

$$-As_t^2 - Bs_t - C = \max_{c_t} \min_{\nu_t} \left\{ -\frac{1}{2}(\overline{c} - c_t)^2 + \beta E_t \left[\vartheta \nu_t^2 - As_{t+1}^2 - Bs_{t+1} - C \right] \right\}. \tag{52}$$

We can do the min and max operations in any order, so we choose to do the minimization first. The first-order condition for ν_t is

$$2\vartheta\nu_t - 2AE_t\left[\omega_\zeta \nu_t + Rs_t - c_t\right]\omega_\zeta - B\omega_\zeta = 0,$$

which means that

$$\nu_t = \frac{B + 2A(Rs_t - c_t)}{2\left(\vartheta - A\omega_\zeta^2\right)}\omega_\zeta. \tag{53}$$

Substituting (53) back into (52) gives

$$-As_t^2 - Bs_t - C = \max_{c_t} \left\{ -\frac{1}{2}(\overline{c} - c_t)^2 + \beta E_t \left[\vartheta \left[\frac{B + 2A(Rs_t - c_t)}{2\left(\vartheta - A\omega_\zeta^2\right)}\omega_\zeta \right]^2 - As_{t+1}^2 - Bs_{t+1} - C \right] \right\}, \tag{54}$$

where

$$s_{t+1} = Rs_t - c_t + \zeta_{t+1} + \omega_\zeta \nu_t.$$

The first-order condition for c_t is

$$(\overline{c} - c_t) - 2\beta\vartheta\frac{A\omega_\zeta}{\vartheta - A\omega_\zeta^2}\nu_t + 2\beta A\left(1 + \frac{A\omega_\zeta^2}{\vartheta - A\omega_\zeta^2}\right)(Rs_t - c_t + \omega_\zeta\nu_t) + \beta B\left(1 + \frac{A\omega_\zeta^2}{\vartheta - A\omega_\zeta^2}\right) = 0.$$

Using the solution for ν_t the solution for consumption is

$$c_t = \frac{2A\beta R}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}s_t + \frac{\overline{c}\left(1 - A\omega_\zeta^2/\vartheta\right) + \beta B}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}. \tag{55}$$

Substituting the above expressions into the Bellman equation gives

$$
\begin{aligned}
& - As_t^2 - Bs_t - C \\
&= -\frac{1}{2}\left(\frac{2A\beta R}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}s_t + \frac{-2\beta A\overline{c} + \beta B}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}\right)^2 \\
&+ \frac{\beta\vartheta\omega_\zeta^2}{\left(2\left(\vartheta - A\omega_\zeta^2\right)\right)^2}\left(\frac{2AR\left(1 - A\omega_\zeta^2/\vartheta\right)}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}s_t + B - \frac{2\overline{c}\left(1 - A\omega_\zeta^2/\vartheta\right)A + 2\beta AB}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}\right)^2 \\
&- \beta A\left(\left(\frac{R}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}s_t - \frac{-B\omega_\zeta^2/\vartheta + 2c + 2B\beta}{2\left(1 - A\omega_\zeta^2/\vartheta + 2\beta A\right)}\right)^2 + \omega_\zeta^2\right) \\
&- \beta B\left(\frac{R}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}s_t - \frac{-B\omega_\zeta^2/\vartheta + 2c + 2B\beta}{2\left(1 - A\omega_\zeta^2/\vartheta + 2\beta A\right)}\right) - \beta C.
\end{aligned}
$$

Given $\beta R = 1$, collecting and matching terms, the constant coefficients turn out to be

$$A = \frac{R\,(R-1)}{2 - R\omega_\zeta^2/\vartheta}, \tag{56}$$

$$B = -\frac{R\overline{c}}{1 - R\omega_\zeta^2/\,(2\vartheta)}, \tag{57}$$

$$C = \frac{R}{2\left(1 - R\omega_\zeta^2/2\vartheta\right)}\omega_\zeta^2 + \frac{R}{2\left(1 - R\omega_\zeta^2/2\vartheta\right)(R-1)}\overline{c}^2. \tag{58}$$

Substituting (56) and (57) into (55) yields the consumption function (19) in the text.

We impose parameter restrictions so that $A > 0$, implying the value function is concave; these restrictions amount to requiring that ϑ not be too small and are shown in the text to imply $\Sigma < 1$.

6.2 Deriving International Consumption Correlations under RB

Given the AR(1) expressions for c_t and c_t^*, (23) and (24), the consumption correlation between the home country and the rest of the world (ROW) can be written as:

$$c_t = \rho_s c_{t-1} + \frac{(R-1)\,\Sigma\overline{c}}{1-\Sigma} + \frac{R-1}{1-\Sigma}\zeta_t, \tag{59}$$

$$c_t^* = \rho_s^* c_{t-1}^* + \frac{(R-1)\,\Sigma^*\overline{c}}{1-\Sigma^*} + \frac{R-1}{1-\Sigma^*}\zeta_t^*, \tag{60}$$

$$
\begin{aligned}
&\operatorname{corr}(c_t, c_t^*) \\
&= \frac{\operatorname{cov}(c_t, c_t^*)}{\sqrt{\operatorname{var}(c_t)\operatorname{var}(c_t^*)}}, \\
&= \frac{\sqrt{(1-\rho_s^2)(1-\rho_s^{*2})}}{1-\rho_s\rho_s^*}\frac{\operatorname{cov}(\zeta_t, \zeta_t^*)}{\omega_\zeta\omega_{\zeta^*}},
\end{aligned}
$$

which is just (25) in the main text. Note that here we use the following facts:

$$
\begin{aligned}
\operatorname{var}(c_t) &= \left(\frac{R-1}{1-\Sigma}\right)^2 \frac{\omega_\zeta^2}{1-\rho_s^2}; \\
\operatorname{var}(c_t^*) &= \left(\frac{R-1}{1-\Sigma^*}\right)^2 \frac{\omega_\zeta^2}{1-\rho_s^{*2}}; \\
\operatorname{cov}(c_t, c_t^*) &= \operatorname{cov}\left(\frac{R-1}{1-\Sigma}\frac{\zeta_t}{1-\rho_s\cdot L}, \frac{R-1}{1-\Sigma^*}\frac{\zeta_t^*}{1-\rho_s^*\cdot L}\right) \\
&= \frac{R-1}{1-\Sigma}\frac{R-1}{1-\Sigma^*}\frac{1}{1-\rho_s\rho_s^*}\operatorname{cov}(\zeta_t, \zeta_t^*).
\end{aligned}
$$

6.3 Deriving International Consumption Correlations under RB and RI

Given the AR(2) expressions for c_t and c_t^*, (45) and (46), the consumption correlation between the home country and the rest of the world can be written as:

$$
\begin{aligned}
&\operatorname{corr}(c_t, c_t^*) \\
&= \frac{\operatorname{cov}(c_t, c_t^*)}{\sqrt{\operatorname{var}(c_t)\operatorname{var}(c_t^*)}} \\
&= \frac{1+(\rho_\theta+\rho_s)(\rho_\theta^*+\rho_s^*)+\left(\rho_\theta^2+\rho_s\rho_\theta+\rho_s^2\right)\left(\rho_\theta^{*2}+\rho_s^*\rho_\theta^*+\rho_s^{*2}\right)+\cdots}{\sqrt{\begin{pmatrix} \frac{(1+\rho_s\rho_\theta)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]} \\ +\frac{\lambda^2}{(1/(1-\theta)-R^2)\theta}\left[\frac{1}{1-\rho_s^2}+(\theta R)^2\frac{(1+\rho_s\rho_\theta)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]}\right]\end{pmatrix}\times\begin{pmatrix} \frac{\left(1+\rho_s^*\rho_\theta^*\right)}{\left(1-\rho_s^*\rho_\theta^*\right)\left[\left(1+\rho_s^*\rho_\theta^*\right)^2-\left(\rho_s^*+\rho_\theta^*\right)^2\right]} \\ +\frac{\lambda^{*2}}{(1/(1-\theta^*)-R^2)\theta^*}\left[\frac{1}{1-\rho_s^{*2}}+(\theta^* R)^2\frac{\left(1+\rho_s^*\rho_\theta^*\right)}{\left(1-\rho_s^*\rho_\theta^*\right)\left[\left(1+\rho_s^*\rho_\theta^*\right)^2-\left(\rho_s^*+\rho_\theta^*\right)^2\right]}\right]\end{pmatrix}}}\frac{\operatorname{cov}(\zeta_t, \zeta_t^*)}{\omega_\zeta\omega_{\zeta^*}} \\
&= \frac{\sum_{k=0}^{\infty}\left\{\left[\sum_{j=0,j\le k}^{k}\left(\rho_\theta^j\rho_s^{k-j}\right)\right]\left[\sum_{j=0,j\le k}^{k}\left(\rho_\theta^{*j}\rho_s^{*k-j}\right)\right]\right\}}{\sqrt{\Xi_1\Xi_2}},
\end{aligned}
$$

which is just (47) in the text. Note that here we use the following facts:

$$\operatorname{var}(c_t) = \left(\frac{R-1}{1-\Sigma}\right)^2 \theta^2 \left(\begin{array}{c} \frac{(1+\rho_s\rho_\theta)\omega_\zeta^2}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]} \\ +\frac{\operatorname{var}\left[\bar{\xi}_t\right]}{1-\rho_s^2} + (\theta R)^2 \frac{(1+\rho_s\rho_\theta)\operatorname{var}\left[\bar{\xi}_t\right]}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]} \end{array} \right)$$

$$= \left(\frac{R-1}{1-\Sigma}\right)^2 \theta^2 \left(\begin{array}{c} \frac{(1+\rho_s\rho_\theta)\omega_\zeta^2}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]} \\ + \left[\frac{1}{1-\rho_s^2} + (\theta R)^2 \frac{(1+\rho_s\rho_\theta)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]}\right] \operatorname{var}\left[\bar{\xi}_t\right] \end{array} \right)$$

$$= \left(\frac{R-1}{1-\Sigma}\right)^2 \theta^2 \left(\begin{array}{c} \frac{(1+\rho_s\rho_\theta)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]} \\ + \left[\frac{1}{1-\rho_s^2} + (\theta R)^2 \frac{(1+\rho_s\rho_\theta)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]}\right] \frac{\lambda^2}{(1/(1-\theta)-R^2)\theta} \end{array} \right) \omega_\zeta^2,$$

$$\operatorname{var}(c_t^*) = \left(\frac{R-1}{1-\Sigma^*}\right)^2 \theta^{*2} \left(\begin{array}{c} \frac{\left(1+\rho_s^*\rho_\theta^*\right)}{\left(1-\rho_s^*\rho_\theta^*\right)\left[\left(1+\rho_s^*\rho_\theta^*\right)^2-\left(\rho_s^*+\rho_\theta^*\right)^2\right]} \\ +\frac{\lambda^2}{(1/(1-\theta^*)-R^2)\theta^*} \left[\frac{1}{1-\rho_s^{*2}} + (\theta^* R)^2 \frac{\left(1+\rho_s^*\rho_\theta^*\right)}{\left(1-\rho_s^*\rho_\theta^*\right)\left[\left(1+\rho_s^*\rho_\theta^*\right)^2-\left(\rho_s^*+\rho_\theta^*\right)^2\right]}\right] \end{array} \right) \omega_{\zeta^*}^2,$$

and

$$\operatorname{cov}(c_t, c_t^*) = \operatorname{cov}\left(\frac{R-1}{1-\Sigma}\frac{\theta\left(\zeta_t+\bar{\xi}_t-R\bar{\xi}_{t-1}\right)}{(1-\rho_s\cdot L)(1-(1-\theta)R\cdot L)}, \frac{R-1}{1-\Sigma^*}\frac{\theta^*\left(\zeta_t^*+\overline{\xi_t^*}-R\overline{\xi_t^*}\right)}{(1-\rho_s^*\cdot L)(1-(1-\theta^*)R\cdot L)}\right)$$

$$= \theta\theta^*\frac{R-1}{1-\Sigma}\frac{R-1}{1-\Sigma^*}\operatorname{cov}\left(\frac{\zeta_t}{(1-\rho_s\cdot L)(1-(1-\theta)R\cdot L)}, \frac{\zeta_t^*}{(1-\rho_s^*\cdot L)(1-(1-\theta^*)R\cdot L)}\right)$$

$$= \theta\theta^*\frac{R-1}{1-\Sigma}\frac{R-1}{1-\Sigma^*}\operatorname{cov}\left(\begin{array}{c} \zeta_t + (\rho_\theta+\rho_s)\zeta_{t-1} + \left(\rho_\theta^2+\rho_s\rho_\theta+\rho_s^2\right)\zeta_{t-2} \\ +\left(\rho_\theta^3+\rho_s\rho_\theta^2+\rho_s^2\rho_\theta+\rho_s^3\right)\zeta_{t-3}+\cdots, \\ \zeta_t^* + (\rho_\theta^*+\rho_s^*)\zeta_{t-1}^* + \left(\rho_\theta^{*2}+\rho_s^*\rho_\theta^*+\rho_s^{*2}\right)\zeta_{t-2}^* \\ +\left(\rho_\theta^{*3}+\rho_s^*\rho_\theta^{*2}+\rho_s^{*2}\rho_\theta^*+\rho_s^{*3}\right)\zeta_{t-3}^*+\cdots \end{array} \right)$$

$$= \theta\theta^*\frac{R-1}{1-\Sigma}\frac{R-1}{1-\Sigma^*}\left(1+(\rho_\theta+\rho_s)(\rho_\theta^*+\rho_s^*)+\left(\rho_\theta^2+\rho_s\rho_\theta+\rho_s^2\right)\left(\rho_\theta^{*2}+\rho_s^*\rho_\theta^*+\rho_s^{*2}\right)+\cdots\right)\operatorname{cov}(\zeta_t,\zeta_t^*)$$

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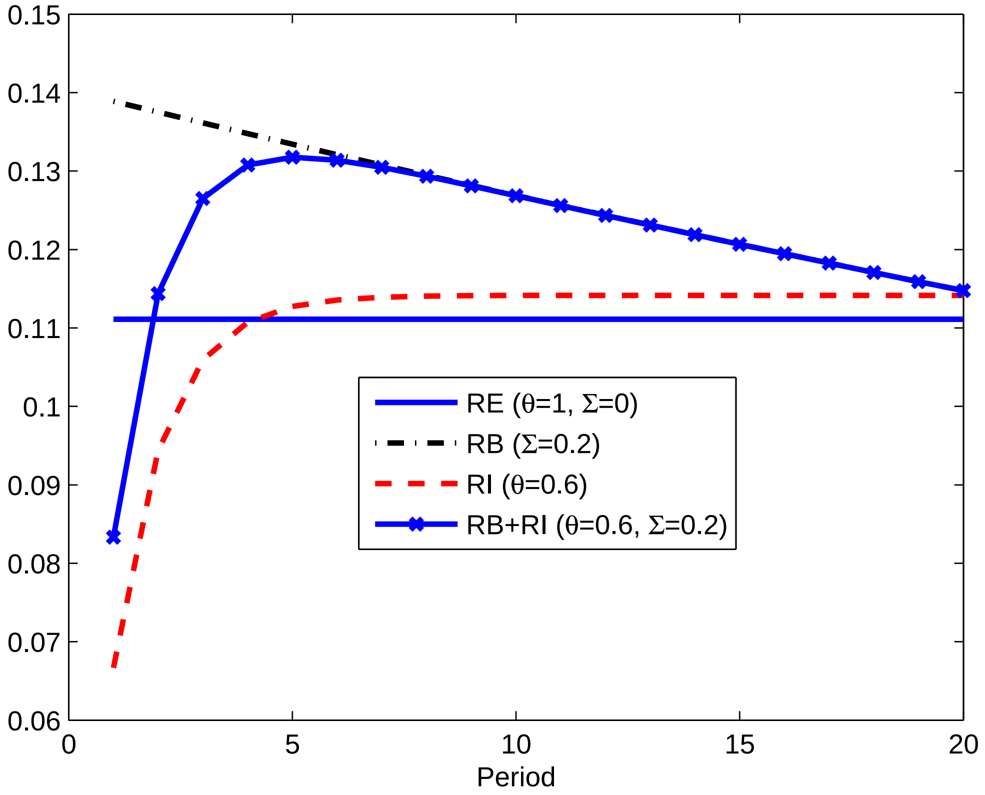

Figure 1: Impulse Responses of Consumption to Income Shocks

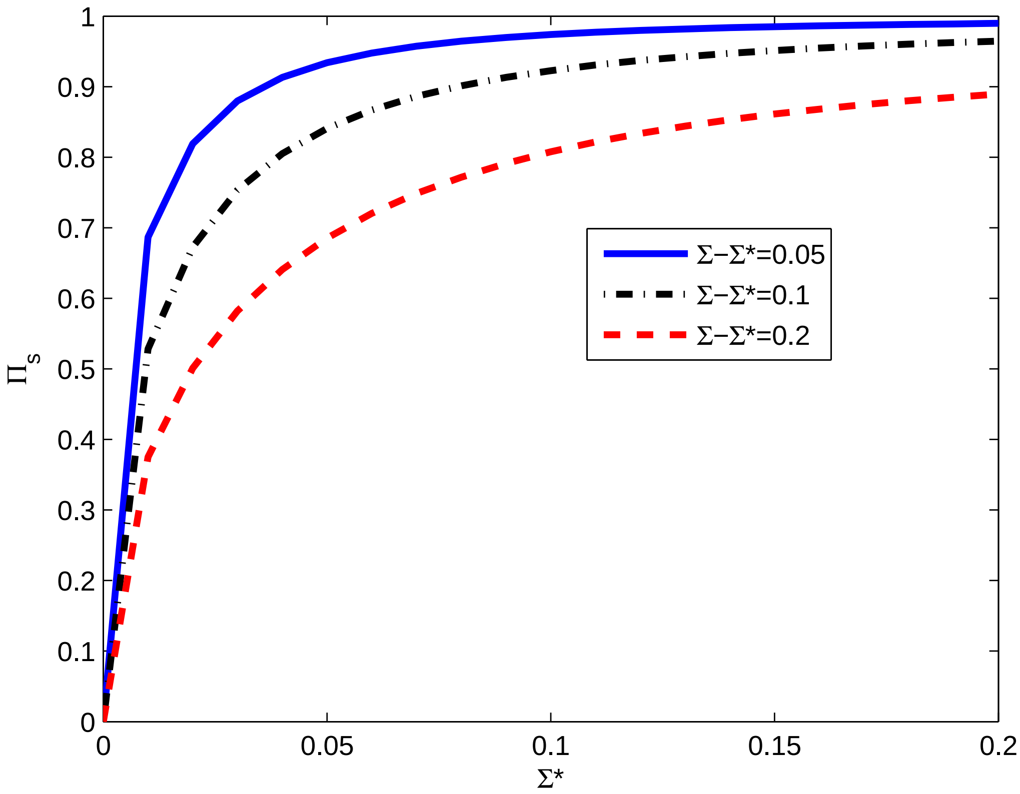

Figure 2: International Consumption Correlation under RB

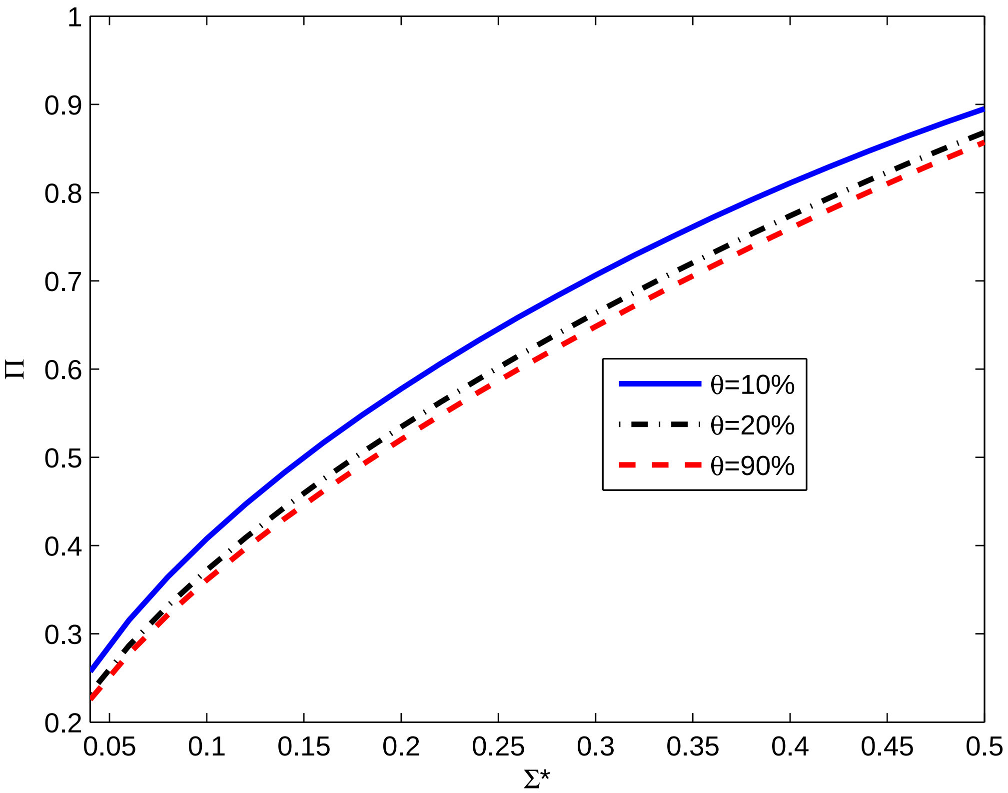

Figure 3: International Consumption Correlation under RB and RI when $\lambda = 0$.

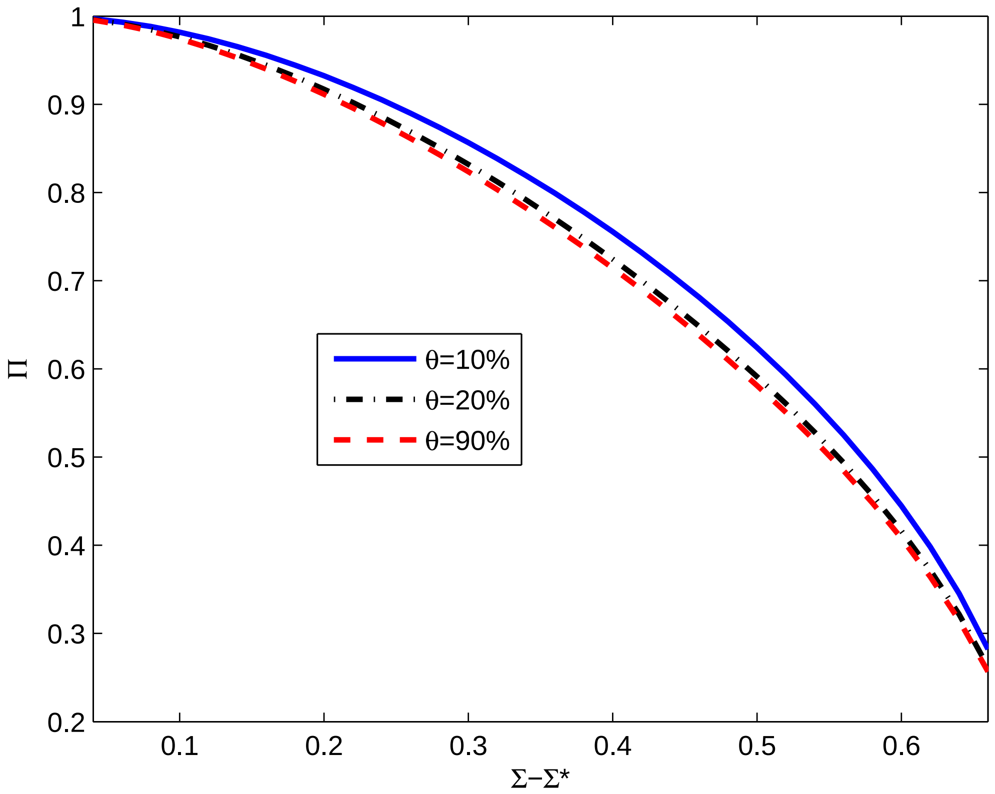

Figure 4: International Consumption Correlation under RB and RI when $\lambda = 0$.

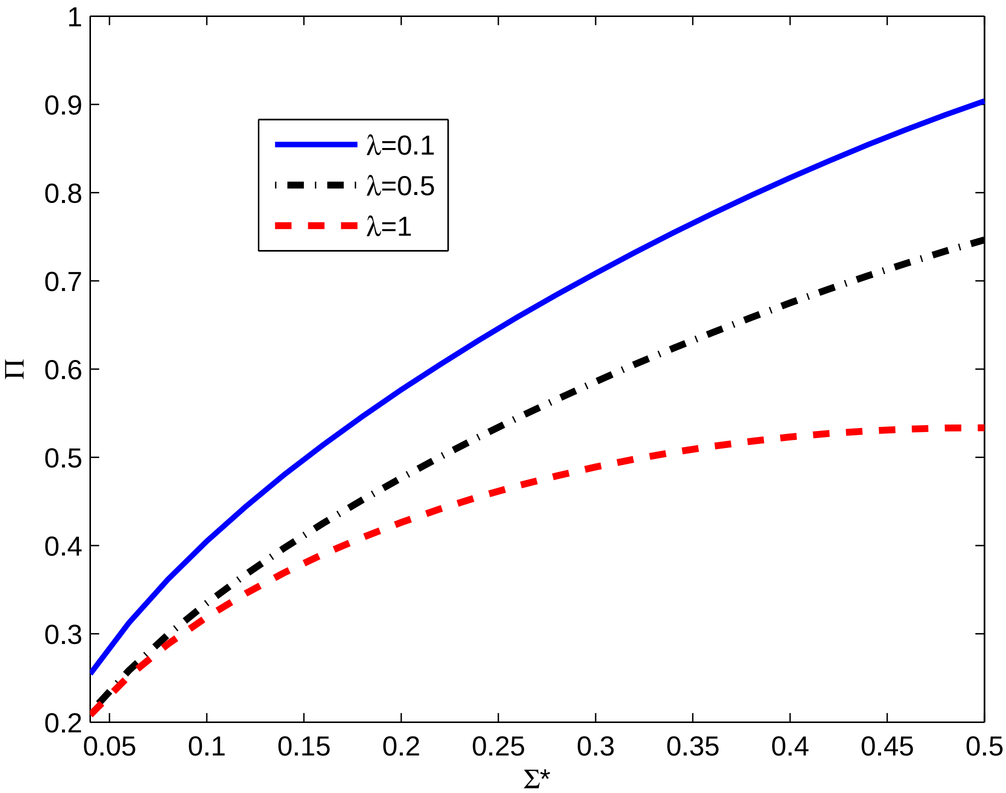

Figure 5: International Consumption Correlation under RB and RI when $\lambda \neq 0$.

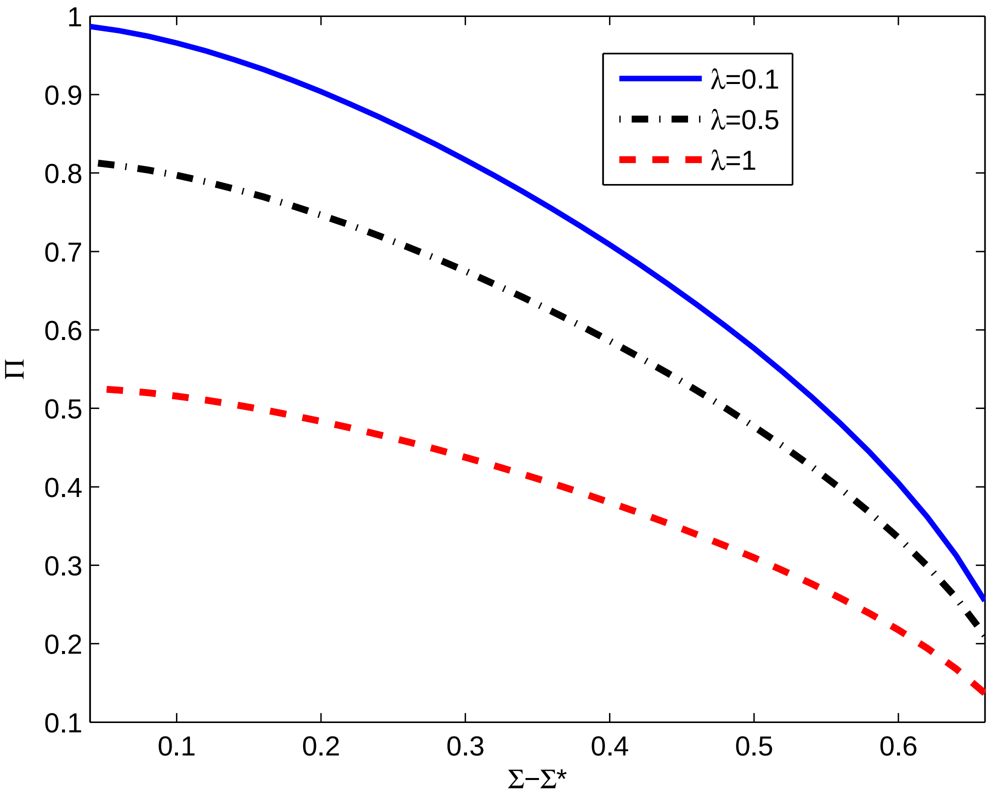

Figure 6: International Consumption Correlation under RB and RI when $\lambda \neq 0$.

Table 1: Estimation and Calibration Results for Different Countries ($p = 0.1$)

	ρ	ρ^*	$\sigma(y)/\sigma(y^*)$	Π_y	Π_s	Σ	Σ^*	$corr(y, y^*)$	$corr(c, c^*)$	ϑ
Canada	0.674	0.581	0.621	0.988	0.785	0.093	0.024	0.403	0.382	1.000
Italy	0.499	0.581	0.582	0.993	0.935	0.050	0.025	0.534	0.246	1.029
UK	0.683	0.581	1.062	0.986	0.666	0.150	0.025	0.683	0.207	1.871
France	0.666	0.581	0.588	0.990	0.950	0.045	0.024	0.509	0.460	1.821
Germany	0.629	0.581	1.031	0.997	0.890	0.061	0.024	0.451	0.040	3.681

Table 2: Comparing the Model Performance ($p = 0.1$)

	Data (corr (y, y^*))	Data (corr (c, c^*))	RE (corr (c, c^*))	RB (corr (c, c^*))
Canada	0.403	0.382	0.408	0.323
Italy	0.534	0.246	0.538	0.502
UK	0.683	0.207	0.693	0.454
France	0.509	0.460	0.514	0.490
Germany	0.451	0.040	0.453	0.402

Table 3: Calibration Results and Model Comparison ($p = 0.05$)

	Π_s	Σ	Σ^*	Data corr (y, y^*)	Data corr (c, c^*)	ϑ	RE corr (c, c^*)	RB corr (c, c^*)
Canada	0.788	0.118	0.031	0.403	0.382	1.000	0.408	0.322
Italy	0.931	0.066	0.031	0.534	0.246	0.995	0.538	0.501
UK	0.652	0.189	0.031	0.683	0.207	1.880	0.693	0.452
France	0.953	0.057	0.031	0.509	0.460	1.823	0.514	0.489
Germany	0.884	0.081	0.031	0.451	0.040	3.534	0.453	0.400

Table 4: Calibration Results and Model Comparison ($p = 0.15$)

	Π_s	Σ	Σ^*	corr (y, y^*)	corr (c, c^*)	ϑ	RE	RB
Canada	0.796	0.077	0.020	0.403	0.382	1.000	0.408	0.325
Italy	0.929	0.042	0.020	0.534	0.246	1.010	0.538	0.500
UK	0.657	0.126	0.020	0.683	0.207	1.829	0.693	0.455
France	0.954	0.036	0.020	0.509	0.460	1.864	0.514	0.490
Germany	0.892	0.051	0.020	0.451	0.040	3.632	0.453	0.404

Table 5: Theoretical corr (c, c^{*}) from Different Models $(p = 0.1)$

	Data	RE	RB	RB+RI ($\theta = 0.9$)	RB+RI ($\theta = 0.6$)	RB+RI ($\theta = 0.3$)
Canada						
($\lambda = 1$)	0.38	0.41	0.33	0.27	0.17	0.12
($\lambda = 0.5$)	0.38	0.41	0.33	0.31	0.27	0.23
($\lambda = 0.1$)	0.38	0.41	0.33	0.32	0.32	0.32
Italy						
($\lambda = 1$)	0.25	0.54	0.50	0.42	0.27	0.19
($\lambda = 0.5$)	0.25	0.54	0.50	0.48	0.41	0.36
($\lambda = 0.1$)	0.25	0.54	0.50	0.50	0.50	0.49
UK						
($\lambda = 1$)	0.21	0.69	0.45	0.36	0.23	0.16
($\lambda = 0.5$)	0.21	0.69	0.45	0.41	0.36	0.31
($\lambda = 0.1$)	0.21	0.69	0.45	0.43	0.43	0.43
France						
($\lambda = 1$)	0.46	0.51	0.49	0.40	0.26	0.18
($\lambda = 0.5$)	0.46	0.51	0.49	0.46	0.40	0.34
($\lambda = 0.1$)	0.46	0.51	0.49	0.48	0.48	0.48
Germany						
($\lambda = 1$)	0.04	0.45	0.40	0.33	0.21	0.15
($\lambda = 0.5$)	0.04	0.45	0.40	0.38	0.33	0.28
($\lambda = 0.1$)	0.04	0.45	0.40	0.40	0.40	0.39

Table 6: Theoretical corr (c, c^*) from Different Models ($p = 0.05$)

	Data	RE	RB	RB+RI ($\theta = 0.9$)	RB+RI ($\theta = 0.6$)	RB+RI ($\theta = 0.3$)
Canada						
($\lambda = 1$)	0.38	0.41	0.32	0.27	0.17	0.12
($\lambda = 0.5$)	0.38	0.41	0.32	0.31	0.26	0.23
($\lambda = 0.1$)	0.38	0.41	0.32	0.32	0.32	0.32
Italy						
($\lambda = 1$)	0.25	0.54	0.50	0.41	0.27	0.19
($\lambda = 0.5$)	0.25	0.54	0.50	0.48	0.41	0.35
($\lambda = 0.1$)	0.25	0.54	0.50	0.50	0.50	0.49
UK						
($\lambda = 1$)	0.21	0.69	0.45	0.37	0.24	0.17
($\lambda = 0.5$)	0.21	0.69	0.45	0.43	0.37	0.32
($\lambda = 0.1$)	0.21	0.69	0.45	0.45	0.45	0.44
France						
($\lambda = 1$)	0.46	0.51	0.49	0.40	0.26	0.18
($\lambda = 0.5$)	0.46	0.51	0.49	0.46	0.40	0.35
($\lambda = 0.1$)	0.46	0.51	0.49	0.49	0.49	0.48
Germany						
($\lambda = 1$)	0.04	0.45	0.40	0.33	0.22	0.15
($\lambda = 0.5$)	0.04	0.45	0.40	0.38	0.33	0.28
($\lambda = 0.1$)	0.04	0.45	0.40	0.40	0.40	0.39

Table 7: Theoretical corr (c, c^*) from Different Models ($p = 0.15$)

	Data	RE	RB	RB+RI ($\theta = 0.9$)	RB+RI ($\theta = 0.6$)	RB+RI ($\theta = 0.3$)
Canada						
($\lambda = 1$)	0.38	0.41	0.33	0.27	0.17	0.12
($\lambda = 0.5$)	0.38	0.41	0.33	0.31	0.27	0.23
($\lambda = 0.1$)	0.38	0.41	0.33	0.32	0.32	0.32
Italy						
($\lambda = 1$)	0.25	0.54	0.50	0.41	0.27	0.19
($\lambda = 0.5$)	0.25	0.54	0.50	0.47	0.41	0.35
($\lambda = 0.1$)	0.25	0.54	0.50	0.50	0.50	0.49
UK						
($\lambda = 1$)	0.21	0.69	0.45	0.38	0.24	0.17
($\lambda = 0.5$)	0.21	0.69	0.45	0.43	0.37	0.32
($\lambda = 0.1$)	0.21	0.69	0.45	0.45	0.45	0.45
France						
($\lambda = 1$)	0.46	0.51	0.49	0.40	0.26	0.18
($\lambda = 0.5$)	0.46	0.51	0.49	0.47	0.40	0.35
($\lambda = 0.1$)	0.46	0.51	0.49	0.49	0.49	0.48
Germany						
($\lambda = 1$)	0.04	0.45	0.40	0.33	0.22	0.15
($\lambda = 0.5$)	0.04	0.45	0.40	0.38	0.33	0.29
($\lambda = 0.1$)	0.04	0.45	0.40	0.40	0.40	0.40