Robust control, informational frictions and international consumption correlations

by Luo, Nie and Young

Discussion by Anastasios Karantounias
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What this paper is doing.

- Puzzle: Models predict international consumption correlations are *larger* than income correlations.
- Data: international consumption correlations are *smaller* than income correlations.

This paper

- Build a small open economy LQ Permanent Income model.
  1. Introduce doubts about the model and evaluate the model consumption correlations $\Rightarrow$ Correlations become smaller but not sufficiently enough.
  2. Introduce Rational Inattention (RI) into a robust PI model $\Rightarrow$ The gradual response to shocks helps international consumption correlations to come closer to the data.

Virtues of the paper

- Tractability: explicit solutions by remaining in the LQ framework.
- Calibrating doubts about the model seriously by using detection error probabilities.

Discussion

- Highlight the exact mechanism that is introduced by doubts about the model.
- Offer some suggestions about the RB-RI formulation.

Permanent income model with RE

- Small open economy with $\beta R = 1$.

$$\max_{c_t, b_{t+1}} E_t \sum_{t=0}^{\infty} \beta^t u(c_t)$$

s.t.

$$c_t + b_{t+1} = Rb_t + y_t$$

- Quadratic utility $u(c) = -\frac{1}{2}(c - \bar{c})^2$.
- Recast the problem in terms of Permanent Income (PI):

$$s_t \equiv b_t + \frac{1}{R} E_t \sum_{i=0}^{\infty} \frac{1}{R^i} y_{t+i}$$

- Budget constraint in terms of PI:

$$s_{t+1} = Rs_t - c_t + \zeta_{t+1}$$

where $\zeta_{t+1} \equiv (E_{t+1} - E_t) \sum_{i=1}^{\infty} \frac{1}{R^i} y_{t+i}$: innovation in PV of future labor income.

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$$c_t = E_t c_{t+1}$$

- Optimal consumption

$$c_t = (R-1)s_t$$

- Consumption and PI dynamics

$$\begin{aligned} c_{t+1} - c_t &= (R-1)\zeta_{t+1} \\ s_{t+1} - s_t &= \zeta_{t+1} \end{aligned}$$

International correlations

- Assume $y_t^* = \rho^* y_{t-1}^* + \epsilon_{t+1}^*$, $Corr(\epsilon_t, \epsilon_t^*) = \eta$.
- $corr(y_t, y_t^*) = \Pi_y \eta, \quad \Pi_y < 1.$

International correlations

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- $corr(y_t, y_t^*) = \Pi_y \eta, \quad \Pi_y < 1.$
- International correlation of consumption changes Δc

$$corr(\Delta c_t, \Delta c_t^*) = corr(\epsilon_t, \epsilon_t^*) = \frac{1}{\Pi_y} corr(y_t, y_t^*) > corr(y_t, y_t^*)$$

- Consumption correlations are *larger* than income correlations.

Permanent income model with doubts about the model

$$v(s_t) = \max_{c_t}\Big\{u(c_t) + \beta \min_{m_{t+1}} \big[E_t m_{t+1} v(s_{t+1}) + \theta E_t m_{t+1} \ln m_{t+1}\big]\Big\}$$

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- $\theta > 0$ penalty parameter that captures doubts about the model.

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$$s_{t+1} = R s_t - c_t + \zeta_{t+1}$$

- $\theta > 0$ penalty parameter that captures doubts about the model.
- Perform minimization $\Rightarrow$

$$m_{t+1} = \frac{\exp(-\frac{1}{\theta}) v(s_{t+1})}{E_t \exp(-\frac{1}{\theta} v(s_{t+1}))}$$

- Assign high probability to low utility events.

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- Assign high probability to low utility events.
- Euler equation ($\beta R = 1$)

$$u'(c_t) = E_t m_{t+1} u'(c_{t+1})$$

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- Consumption function

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- Worst-case conditional mean

$$\tilde{\mu}_t = A + B_1 s_t$$

with $A < 0$ and $B_1 > 0$ ($A \equiv B_1 \equiv 0$ for no doubts about the model).

$$c_t = A + (R - 1 + B_1) s_t$$

  - $A < 0$: precautionary savings
  - $B_1 > 0$ extra sensitivity to a shock in s_t.

Consumption dynamics and correlations

- Evolution of s_t and c_t

$$
\begin{aligned}
s_{t+1} - s_t &= \zeta_{t+1} - \tilde{\mu}_t \\
c_{t+1} - c_t &= (R - 1 + B_1)(\zeta_{t+1} - \tilde{\mu}_t)
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c_{t+1} &= A(1 - R) + (1 - B_1)c_t + (R - 1 + B_1)\zeta_{t+1} \\
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- Consumption correlations

$$Corr(c_t, c_t*) = \frac{\Pi_s}{\Pi_y} corr(y_t, y_t^*)$$

- Potential to reduce consumption correlations.

Rational inattention and robustness

- RI: DM have finite information capacity $\Rightarrow$ choose optimally to allocate their attention to higher utility activities.

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- Theorem(Sims): In LQG models the optimal $\pi(c|x)$ is gaussian $\Rightarrow$ DM acts as if he observes a signal with *endogenous* noise $s = x +$ noise and sets the action as function of the signal $c(s)$.

Rational inattention and robustness

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- Theorem(Sims): In LQG models the optimal $\pi(c|x)$ is gaussian $\Rightarrow$ DM acts as if he observes a signal with *endogenous* noise $s = x +$ noise and sets the action as function of the signal $c(s)$.
- RB-RI problem that LNY setup:

$$\hat{v}(\hat{s}_t) = \max_{c_t} \min_{\nu_t} \Big\{ u(c) + \beta E_t \big(\hat{v}(\hat{s}_{t+1}) + \theta \nu_t^2 \big) \Big\}$$

s.t.

$$s_{t+1} = R s_t - c_t + \zeta_{t+1} + \nu_t$$

$$\hat{s}_{t+1} = (1 - \Theta)(R\hat{s}_t - c_t + \nu_t) + \Theta \underbrace{(s_{t+1} + \xi_{t+1})}_{s^*_{t+1}}$$

Comments and questions

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- Do the LQG theorems hold?
- In the particular formulation: Given the RI endogenous signal extraction problem, LNY assume that the are misspecified state dynamics only and no misspecification doubts in the signal dynamics.
- A more natural formulation would consider misspecification in both state and signal dynamics.

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- Do the LQG theorems hold?
- In the particular formulation: Given the RI endogenous signal extraction problem, LNY assume that the are misspecified state dynamics only and no misspecification doubts in the signal dynamics.
- A more natural formulation would consider misspecification in both state and signal dynamics.
- More generally: Why not attacking the problem with a variant of robust filtering?
- For example, the income process can consist of transitory and persistent components and the agents is trying to filter, considering mispecification in his state-signal dynamics.
- Clarify also the connections of the LNY setup with the robust filtering setup of Kasa(2006).

A RI formulation from first principles

- x: state, y: control, U: return function, e.g. $U = -(y-x)^2$

$$\max_{\pi(y|x)} \sum_x \pi(x) \sum_y \pi(y|x) U(x,y)$$

s.t.

$$\sum_y \pi(y|x) = 1$$

$$I(X,Y) = \sum_x \sum_y \pi(x,y) \ln \frac{\pi(x,y)}{\pi(x)\pi(y)} \le \kappa \quad (\mu)$$

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- optimality condition for conditional density

$$\frac{\pi(y|x)}{\pi(y)} = \frac{\exp(\frac{1}{\mu}U(x,y))}{\sum_y \pi(y)\exp(\frac{1}{\mu}U(x,y))}$$

- Agent assigns more attention to events with high utility.
- $\mu \to \infty \Rightarrow \pi(y|x) = \pi(y)$, so y becomes independent of x.

Potential formulation of RI with RB I: Doubts about the exogenous state but <u>no</u> doubts about the capacity channel.

$$\max_{\pi(y|x)} \min_{m(x)\geq 0} \sum_x m(x)\pi(x) \sum_y \pi(y|x)U(x,y) + \theta \sum_x \pi(x)m(x)\ln m(x)$$

s.t.

$$\begin{aligned}
\sum_y \pi(y|x) &= 1 \\
I(X,Y) &\leq \kappa \\
\sum_x \pi(x)m(x) &= 1
\end{aligned}$$

- Worst-case model of x

$$m(x) = \frac{\exp(\frac{-1}{\theta}\sum_y \pi(y|x)U(x,y))}{\sum_x \pi(x)\exp(\frac{-1}{\theta}\sum_y \pi(y|x)U(x,y))}$$

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- Agent wants to allocate attention to high utility events, but adjusts the likelihood of these events in a conservative way due to doubts about the distribution of x.

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- Potential formulation II: Include doubts about the model in the capacity constraint:

$$\tilde{I}(X,Y) \equiv \sum_x \sum_y \pi(y,x) \ln \frac{\pi(y,x)}{\pi(y)\tilde{\pi}(x)}$$

where $\pi(y,x) = \pi(y|x)\tilde{\pi}(x)$ and $\tilde{\pi}(x) = \pi(x)m(x)$.

- lose convenient risk-sensitive adjustment but more consistent with the spirit of model uncertainty.