

Inflation and output in New Keynesian models with a transient interest rate peg

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Abstract

Recent monetary policy experience suggests a simple diagnostic for models of monetary non-neutrality. Suppose the central bank pegs the nominal interest rate below steady state for a reasonably short period of time. Familiar intuition suggests that this should be modestly inflationary, and a reasonable model should deliver such a prediction. We pursue this simple diagnostic in several variants of the familiar Dynamic New Keynesian (DNK) model. Some variants of the model produce counterintuitive inflation reversals where the effect of the interest rate peg can switch from highly inflationary to highly deflationary for only modest changes in the length of the interest rate peg. Curiously, this unusual behavior does not arise in a sticky information model of the Phillips curve.

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1 Introduction

This paper examines a very simple question: what is the behavior of inflation if a central bank credibly announces a fixed interest rate for an extended period of time? To be precise, we examine the perfect foresight path of sticky price models in which the nominal interest rate is pegged for a finite period of time. It is of course well known that if the peg is perpetual, then there is local equilibrium indeterminacy and thus sunspot equilibria. But if the peg is finite, and the subsequent monetary policy is consistent with a unique equilibrium, then this subsequent behavior provides the needed terminal condition that implies determinacy along the entire path.

Our analysis is motivated by the use of such monetary policies in the aftermath of the 2008 financial crisis. But we do not view this paper as an investigation of policy during the financial crisis. Nor do we see this as a data-matching exercise. Instead we see a transient interest rate peg as an experiment for investigating the reasonableness of monetary models. That is, a transient interest rate peg is a reasonable policy experiment, and monetary models should make reasonable predictions of the impact of such a policy.

Our experiments consider a fully anticipated and unconditional lowering of the monetary policy rate for a finite number of periods. Throughout most of the analysis, we do not explicitly motivate why such peg is pursued. Our paper is simply conducting positive economics: what would a model predict as the inflation dynamics from such an interest rate peg? As noted, this is a useful model diagnostic in that a reasonable model should give a reasonable answer. In a robustness check we add shocks to the natural rate of interest to the model that drive policy to the zero lower bound. We find that all our main results from the exogenous peg assumption carry over to this setup of an endogenously constant rate at the lower bound.

Our principle results include the following. First, we find that the workhorse [3] model of time-dependent pricing delivers reasonable responses of inflation and output in response to such a policy. However, if the model is extended to include endogenous state variables such as (but not limited to) indexation, the [3] model implies that the initial responses can become arbitrarily large as the duration of the fixed interest rate regime approaches some critical value and then switch sign and become arbitrarily negative as this critical value is exceeded slightly. For empirically realistic models such as the [13] model, the critical duration for which these asymptotes occur is around eight quarters. These reversals are typically periodic, so that the sign of the initial inflation response changes as the length of the extended period expands. We view these "reversals" as a particularly troubling and counterintuitive aspect of these models. Second, we demonstrate that these reversals are robust along several dimensions. For example, they continue to arise in a more sophisticated DSGE model such as [13], and they also arise in the nonlinear version of the Calvo model with indexation. Further, this peculiar behavior also arises in the model if there are natural rate shocks that push the economy to the zero bound. Third, we find that these reversals do not arise in the baseline sticky-information model of [11]. Below we demonstrate that the reason for the different dynamics in the Calvo vs. the Mankiw-Reis frameworks, is that there are no endogenous state variables in the latter. Taken together, we believe these results imply that: (i) the Calvo class of sticky price models should be used with great caution when studying the effects of

exogenous interest rate paths, and (ii) the [11] model should be investigated more fully as a possible framework to be used for analyzing such policy scenarios.

Several other authors have examined issues of this type. The closest precursor to our analysis is [9]. They provide a convenient mechanism for solving monetary models in which nominal rates are set exogenously for a finite period of time. These authors pointed out inflation reversals, what they call "unusual equilibria." However, these authors do not discuss the economic mechanism that generates these equilibria, nor did they demonstrate their periodicity, nor the fact that they survive in the nonlinear version of the model. What is the cause of these reversals? [9] suggest that the unusual equilibria arise whenever a reduction in the nominal rate goes along with a rise in the real rate. This mechanical explanation is clearly correct, but it is problematic for at least two reasons. First, it does not explain the periodicity of the reversals. Second, it does not aid in understanding why reversals arise only if there are endogenous states in the model. That is, in the simple Calvo model with no state variables, there are never reversals. Instead we demonstrate that the source of these reversals can arise from an interaction between the initial inflation rate and the subsequent path of inflation. The initial inflation rate (the inflation rate at the beginning of the extended period) is a function of the future path of inflation. This effect is familiar. But there is another effect operative in the experiment. The future path of inflation is affected by the initial inflation rate. This backward-looking force becomes significant if there are structural elements in the model that introduce a backward element, e.g., the endogenous state arising from inflation indexation. This backward force is also increasing in the length of the extended period. We show that if this backward element becomes large enough, then the model delivers inflation reversals that are periodic. Other endogenous state variables (such as consumption in the case of habit formation) can generate similar reversals of inflation and output for much the same reasons.

Other previous work includes [1] and [8] who suggests novel ways to generate reasonable forecasts in models with fixed interest rates over finite horizons. Relatedly, [2] and [5] have examined the size of the fiscal multiplier when the nominal rate is pegged for a finite time period.

Section 2 examines the issue in the simplest New Keynesian model. Section 3 adds inertia in the Phillips curve and Taylor rule and provides conditions for the existence of asymptotes. This section also provides sensitivity analysis and shows that such reversals also obtain in the [13] model at their estimated parameter values. In addition, we document that reversals are not an artifact of the linearization but are inherent in the fully nonlinear model as well. The peculiar predictions also arise in the model with natural rate shocks that lead to an interest rate peg because of the zero bound. Section 4 analyzes the baseline sticky information model of and demonstrates that reversals do not arise there. Section 5 concludes. The appendix contains proofs and some sensitivity analysis.

2 The DNK model with no endogenous states

The canonical DNK model is given by the following:

$$\pi_t = \kappa y_t + \beta \pi_{t+1} \quad (1)$$

$$y_t = y_{t+1} - \frac{1}{\sigma} (i_t - \pi_{t+1}) \quad (2)$$

where π_t , y_t , and i_t , denote inflation, the output gap, and the nominal rate, respectively, all measured as deviations from the steady state. We dispense with expectations notation as we are focusing on the perfect foresight path. We analyze this DNK model under two assumptions about the exit from the interest rate peg that turn out to have important implications for inflation dynamics. In the deterministic exit case, date agents know the date of the exit from the peg for sure. With a stochastic exit, agents know ex-ante that in any given period, the peg could continue with a certain probability or end for good.

Throughout this section, we assume that the interest rate is pegged for exogenous reasons in order to isolate the role of the peg itself. We study the role of other shocks that may drive the economy to the zero lower bound and thus create a natural explanation for why rates are fixed in a subsequent section.

2.1 Deterministic exit

With a deterministic exit, we assume the central bank announces an interest rate rule given by the following:

$$i_t = \begin{cases} i^* & t = 1, 2, \dots, T \\ \phi_\pi \pi_t + \phi_y y_t & t = T+1, T+2, \dots \end{cases}$$

Under standard assumptions on ϕ_y and ϕ_π there is a unique equilibrium for $t > T$. Since there are no state variables nor exogenous shocks, the unique equilibrium for $t > T$ is given by $\pi_t = y_t = 0$. For the first T periods, the constant interest rate suggests that there is equilibrium indeterminacy. But since this policy regime is finite, the subsequent uniqueness for $t > T$ serves as a terminal condition and thus ensures uniqueness of equilibrium along the entire path¹. During the constant interest rate policy, the inflation dynamics are governed by

$$\pi_t = -\frac{\kappa}{\sigma} i^* + \left(\frac{\kappa}{\sigma} + 1 + \beta \right) \pi_{t+1} - \beta \pi_{t+2} \quad (3)$$

with the two terminal conditions:

$$\pi_T = -\frac{\kappa}{\sigma} i^* \quad (4)$$

$$\pi_{T-1} = -\left(\frac{\kappa}{\sigma} + 2 + \beta \right) \frac{\kappa}{\sigma} i^* \quad (5)$$

¹In a very early precursor to this work, Hall and Kennally (1988) also use the subsequent policy regime as a terminal condition for the unstable early regime.

It is convenient to invert this system, running time backwards from the end of the extended period. That is, let z_s denote the value of inflation s -periods before time T : $z_s \equiv \pi_{T+1-s}$. The difference equation (3) can now be written as:

$$z_s = \frac{\kappa}{\sigma} i^* + \left(\frac{\kappa}{\sigma} + 1 + \beta \right) z_{s-1} - \beta z_{s-2}$$

with the two *initial* conditions:

$$z_1 = \pi_T = -\frac{\kappa}{\sigma} i^* \quad (6)$$

$$z_2 = \pi_{T-1} = -\left(\frac{\kappa}{\sigma} + 2 + \beta \right) i^* \quad (7)$$

The two initial conditions (7)-(8) imply that there is a unique solution to the difference equation (6). The solution has a simple form:

Proposition 1 *The inflation rate during the extended period is given by*

$$\pi_{T+1-s} \equiv z_s = i^* + m_1 e_1^s + m_2 e_2^s \quad \text{for } s = 0, 1, \dots, T$$

for $s = 1, 2, \dots, T$, where $e_1 > 1, e_2 < 1$, are the eigenvalues of the difference equation, and m_1 and m_2 come from the two restrictions (6)-(7). *Proof: see Appendix B.*

Several remarks are in order. First, the solution to the difference equation is independent of T . That is, m_1 and m_2 are independent of T . This arises because there are only restrictions on inflation at the end of the extended period, and not at the beginning of the extended period. Second, the inflation path during the extended period is entirely independent of the Taylor rule that commences at the end of the extended period. But this subsequent policy is necessary as it provides a terminal condition to the indeterminate difference equation. These first two remarks will not be true if there are endogenous or exogenous state variables. Third, the inflation behavior given in Proposition 1 is explosive since $e_1 > 1$. That is, if we run time backwards from the end of the extended period, the initial inflation rate explodes exponentially in T :

$$\pi_1 \equiv z_T = i^* + m_1 e_1^T + m_2 e_2^T$$

This unstable root is a manifestation of the familiar local indeterminacy induced by an interest rate peg. This explosive behavior is already evident in (4)-(5):

$$\pi_{T-1} = -\left(\frac{\kappa}{\sigma} + 2 + \beta \right) i^* = \left(\frac{\kappa}{\sigma} + 2 + \beta \right) \pi_T > \pi_T$$

This expression also demonstrates that the simple DNK model never delivers inflation reversals in that the terminal and penultimate inflation rates always have the anticipated sign. The previous inflation rates then also have the appropriate sign via (4). As for explosiveness, the unstable eigenvalue (e_1) during the extended period has a fairly simple form if we assume $\beta = 1$. It is given by:

$$e_1 = x + \sqrt{x^2 - 1} \quad \text{where: } x = 1 + \frac{\kappa}{2\sigma}$$

The key variable determining inflation during the extended period is $\frac{\kappa}{2\sigma}$. A smaller value for σ magnifies the inflation response, because it makes spending more interest rate sensitive. A larger value for κ increases the inflation response, because the more flexible prices are (the higher κ) the more the real rate interest rate falls. This stimulates demand and puts even further upward pressure on prices.

Figure 1 provides a quantitative example of these effects. The calibration is standard: $\kappa = 0.025$, $\sigma = 1$, $\beta = .99$. As noted, the policy rule subsequent to the extended period is irrelevant (as long as it produces determinacy). We consider a policy of pegging the annualized nominal rate at 1% below steady state for T periods.

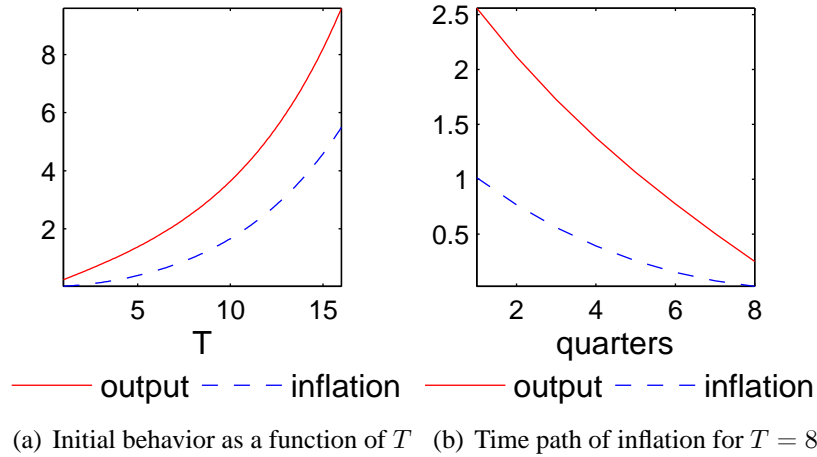


Figure 1: Inflation dynamics in New Keynesian model with fixed interest rates, $\sigma = 1$

The appendix provides sensitivity analysis to parameter values. Quantitatively, the results seem quite reasonable. For all but extreme parameter combinations, a peg for 8 quarters is modestly inflationary. We thus conclude that the simplest DNK model delivers reasonable answers to the interest rate peg experiment.

2.2 Stochastic exit

This subsection considers a stochastic exit from the fixed-rate regime. This is motivated by the influential papers of [6] and [5] that assume Markov process for the shocks that drive the economy to the zero lower bound. We assume that as long as no exit has occurred previously, there is a probability p that the interest rate remains pegged, and probability $(1 - p)$ that an exit to an active interest rate regime occurs. The mean duration of the extended period is $T = 1/(1 - p)$, or $p = (T - 1)/T$, so that there is a natural mapping in these linear models between the deterministic peg and the stochastic exit. Exit is an absorbing state.

We demonstrate that the initial inflation rate is an exponential function of T , but that if T rises above a critical value, initial inflation switches from unboundedly positive to mildly negative, ie., there is a sharp inflation reversal. This inflation behavior is quite peculiar and

surely unreasonable, but never arises in the simple DNK model with a deterministic interest rate peg. In this model with no endogenous states, when the exit occurs $\pi_t = y_t = 0$. Hence, the equilibrium is given by two numbers: the levels of inflation and gap during the extended period. The model is given by equations (1) and (2) in the previous subsection. The solution for inflation is given by:

$$\pi_t = -\frac{\frac{\kappa}{\sigma}i^*}{(1-p)(1-\beta p) - p\frac{\kappa}{\sigma}}$$

The mean duration of the peg is given by $T = 1/(1-p)$, or $p = (T-1)/T$, so that we can write (??) as

$$\pi_t = -\frac{\frac{\kappa}{\sigma}i^*T^2}{\beta + T(1-\beta) + T(T-1)\frac{\kappa}{\sigma}}$$

A simple case is to set $\beta = 1$ so that we have

$$\pi_t = \frac{-\frac{\kappa}{\sigma}i^*T^2}{1 - T(T-1)\frac{\kappa}{\sigma}}$$

The normal case is where the denominator is positive, or

$$T_{crit} = \left(1 + \sqrt{1 + 4\frac{\sigma}{\kappa}}\right) / 2$$

There is an asymptote and then sign switch at T_{crit} . If we use $\kappa = .025$, and $\sigma = 1$, we have that $T_{crit} = 6.84$. Thus, asymptotes and sign switches are possible in the simple DNK model as long as the exit is stochastic.

But with a stochastic exit, there is also the possibility of equilibrium indeterminacy whenever the expected duration of the fixed-rate regime is too large (recall that there is no certain end date). It is thus of interest to see whether the sign switch occurs in the determinacy region of the parameter space. In the case of stochastic exit, equation (3) becomes

$$\beta p^2 \pi_{t+2} - \left[\frac{\kappa}{\sigma} + (1+\beta)\right] p \pi_{t+1} + \pi_t + \frac{\kappa}{\sigma} i^* = 0 \quad (8)$$

Proposition 2 *The determinacy region of the stochastic exit model given by (8) is identical to the region for which $T < T_{crit}$, i.e there are no reversals whenever the model is determinate.*

Proof: see Appendix B.

What is the mechanism that generates this inflation reversal? As proposition 2 states formally, the reversal arises at exactly the same T in which the stationary equilibrium is indeterminate. But indeterminacy means that the stationary equilibrium includes lagged inflation as an endogenous state variable. To see this, consider the general solution for inflation along a dynamic path where no exit occurs. This is given by

$$\pi_t = ca^t + b \quad (9)$$

where a and b will vary with T . As the appendix shows, when $T < T_{crit}$, a is outside the unit circle and so imposing non-explosiveness requires $c = 0$ and $\pi_t = b$. But when $T > T_{crit}$,

a is inside the unit circle and c is free. Inflation now becomes an endogenous state variable, which can be seen from re-writing (9) for $t > 1$ as

$$\pi_t = a\pi_{t-1} + b(1 - a) \quad (10)$$

where b is negative for $T > T_{crit}$. Initial inflation π_1 , of course, is free. A natural equilibrium selection device is to impose that inflation is determined by a unique stationary decision rule at all times, effectively imposing (10) also for $t = 1$. Under this assumption and assuming $\pi_0 = 0$, initial inflation is negative in the region of indeterminacy (and is subsequently governed by (10)).²

To summarize, reversals of initial inflation as the duration of the peg is extended arise if and only if the equilibrium contains an endogenous state. Why are endogenous states a necessary condition for reversals? Recall that a change in the nominal interest rate must always be consistent with its two determinants, a change in the real rate and a change in expected inflation. In the deterministic experiment, we can solve backwards for the equilibrium inflation path from the end of the peg in period T . If there are no endogenous states, then π_{T+1} is unaffected by previous outcomes. In the present context with no future shocks and an active Taylor rule, $\pi_{T+1} = y_{T+1} = 0$. Thus, the real rate must fall in T to achieve the peg. But this stimulates spending and pushes up π_T and contributes to a further fall in the real rate in period $T - 1$. This (explosive) behavior continues back to the beginning of the extended period so that there cannot be a reversal. But this argument breaks down if there is an endogenous state. Time- T expected inflation (π_{T+1}) is now directly linked to past inflation π_T . Hence, if π_T is below steady-state, then pushes π_{T+1} below steady state. This implies that it is now possible for the nominal rate to be low in period T because of a Fisher effect which signals expected deflation. But an expected deflation during the extended period allows for positive real rates and low output, which in turn generates the deflationary pressures, etc.. None of this is possible in a model without endogenous states. Below we will pursue this link between endogenous states and inflation reversals but in environments in which there is equilibrium determinacy because the peg is deterministic.

3 Adding state variables to the model

Many have argued that the simple DNK model is a poor description of macro behavior because of its inability to generate inflation persistence, see [7]. In this section we explore the effect of adding inflation indexation to the model as in [4] and [13]. Indexation introduces inflation inertia into the model and is of interest for two contrasting reasons. First, the initial inflation response may be dampened if initial inflation is linked to inflation in the previous period. Second, indexation introduces an endogenous state to the model, and the stochastic exit results suggest that this endogenous state may deliver inflation reversals. We pursue these issues here.

²If instead we choose the equilibrium where inflation is constant before exit ($c = 0$), then inflation is constant at b , which is unboundedly negative in the neighborhood of T_{crit} .

The basic model is the same as before, but the Phillips curve is now given by:

$$\pi_t = \frac{\kappa}{1 + \beta\lambda} y_t + \frac{\beta}{1 + \beta\lambda} \pi_{t+1} + \frac{\lambda}{1 + \beta\lambda} \pi_{t-1} \quad (11)$$

where $0 \leq \lambda \leq 1$ is the degree of inflation indexation. Similarly, the policy rule subsequent to the extended period is characterized by inertia $0 \leq \rho < 1$, so that for $t > T$ we have

$$i_t = \rho i_{t-1} + (1 - \rho) (\phi_\pi \pi_t + \phi_y y_t) \quad (12)$$

Again, under standard assumptions on the policy rule, there is a unique equilibrium for $t > T$. We denote these decision rules for $t > T$ by:

$$\pi_t = b_1 \pi_{t-1} + b_2 i_{t-1} \quad (13)$$

$$i_t = d_1 \pi_{t-1} + d_2 i_{t-1} \quad (14)$$

$$y_t = f_1 \pi_{t-1} + f_2 i_{t-1} \quad (15)$$

During the extended period of time, lagged inflation is the only state variable. The difference equation governing dynamics is given by:

$$\begin{aligned} \pi_t = & \frac{-\kappa i^*}{\sigma[1 + \lambda(1 + \beta)]} + \frac{\lambda}{[1 + \lambda(1 + \beta)]} \pi_{t-1} + \frac{\sigma[1 + \beta(1 + \lambda)] + \kappa}{\sigma[1 + \lambda(1 + \beta)]} \pi_{t+1} \\ & - \frac{\beta}{1 + \lambda(1 + \beta)} \pi_{t+2} \end{aligned} \quad (16)$$

We will find it convenient to write this as:

$$\pi_t = c + \gamma_0 \pi_{t-1} + \gamma_1 \pi_{t+1} + \gamma_2 \pi_{t+2} \quad (17)$$

The state variables imply that we have one initial and two terminal conditions:

$$\begin{aligned} \pi_0, \\ \pi_T = c + \gamma_0 \pi_{T-1} + \gamma_1 (b_1 \pi_T + b_2 i^*) + \gamma_2 [b_1 (b_1 \pi_T + b_2 i^*) + b_2 (d_1 \pi_T + d_2 i^*)], \\ \pi_{T-1} = c + \gamma_0 \pi_{T-2} + \gamma_1 \pi_T + \gamma_2 (b_1 \pi_T + b_2 i^*) \end{aligned}$$

As before, the system can be expressed in z -form:

$$z_s = c + \gamma_0 z_{s+1} + \gamma_1 z_{s-1} + \gamma_2 z_{s-2} \quad (18)$$

With two initial and one terminal condition in z -form:

$$z_{T+1} = \pi_0, \quad (19)$$

$$z_1 = c + \gamma_0 z_2 + \gamma_1 (b_1 z_1 + b_2 i^*) + \gamma_2 [b_1 (b_1 z_1 + b_2 i^*) + b_2 (d_1 z_1 + d_2 i^*)], \quad (20)$$

$$z_2 = c + \gamma_0 z_3 + \gamma_1 z_1 + \gamma_2 (b_1 z_1 + b_2 i^*) \quad (21)$$

The solution to this system is given by the following.

Proposition 3 *The system in (18) has one eigenvalue e_1 inside the unit circle and two eigenvalues (e_2 and e_3) outside the unit circle. The inflation rate during the extended period is then given by*

1. *If e_2 and e_3 are real:*

$$\pi_{T+1-s} \equiv z_s = i^* + m_1 e_1^s + m_2 e_2^s + m_3 e_3^s \quad (22)$$

2. *If e_2 and e_3 are complex, $(e_2, e_3) = a \pm bi$:*

$$\pi_{T+1-s} \equiv z_s = i^* + m_1 e_1^s + m_2 r^s \cos(\theta s) + m_3 r^s \sin(\theta s)$$

where $r = \sqrt{a^2 + b^2} > 1$, $\theta = \tan^{-1}(\frac{b}{a})$, and m_1 , m_2 , and m_3 , come from the three restrictions (21)-(23). These constants do depend upon T because of the condition (21).

Proof: See Appendix B.

Proposition 3 demonstrates that once again we have explosive inflation behavior as we increase the length of the extended period. An additional implication of Proposition 3 is that we can get inflation reversals. Suppose that we are in a parameter region with complex roots. The initial inflation behavior is then given by

$$\pi_1 \equiv z_T = i^* + m_1 e_1^T + m_2 r^T \cos(\theta T) + m_3 r^T \sin(\theta T)$$

Since $r > 1$, the initial inflation rate is explosive in T . But the sign of this effect may switch between positive and negative. Numerically we find that this switch is marked by an asymptote, where initial inflation goes from arbitrarily positive to arbitrarily negative as we increase T . Strictly speaking there are no asymptotes when T is restricted to take on integer values, but sign reversals will still occur. Figure 2 plots initial inflation against the duration of the fixed-rate regime and shows these reversals. The calibration is given by: $\kappa = 0.025$, $\sigma = 1$, $\beta = 0.99$, $\lambda = 1$, $\rho = 0.80$, $\phi_\pi = 1.5$, $\phi_y = 0.5$. The plot somewhat masks the asymptote and extreme behavior that first arises between $T = 6$ and $T = 7$ (and periodically thereafter). But note that as we approach the asymptote, initial inflation goes from unboundedly positive to unboundedly negative.

Figure 8 shows dynamics of the model for two different values of T on either side of the asymptote. The figures plot the inflation behavior for $T = 6$, and $T = 8$ periods. Because these are trigonometric functions, we of course have further sign reversals as we increase T . Along with these peculiar reversals, we also have enormous inflation rates from very modest extended periods. This is in sharp contrast to the model without state variables. Hence, adding state variables has two peculiar implications: (i) the existence of enormous inflation rates for modest extended periods, and (ii) sign reversals so that a low nominal rate peg for six periods can have the opposite effect of a low nominal rate for eight periods.

Panel (a) of figure shows that for $T = 6$ a lower path for the nominal rate has the usual expansionary effect on inflation and output. But if rates are pegged below steady state

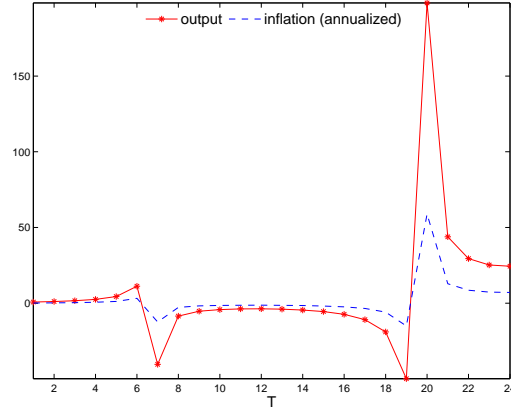


Figure 2: Initial output and inflation as a function of the duration T

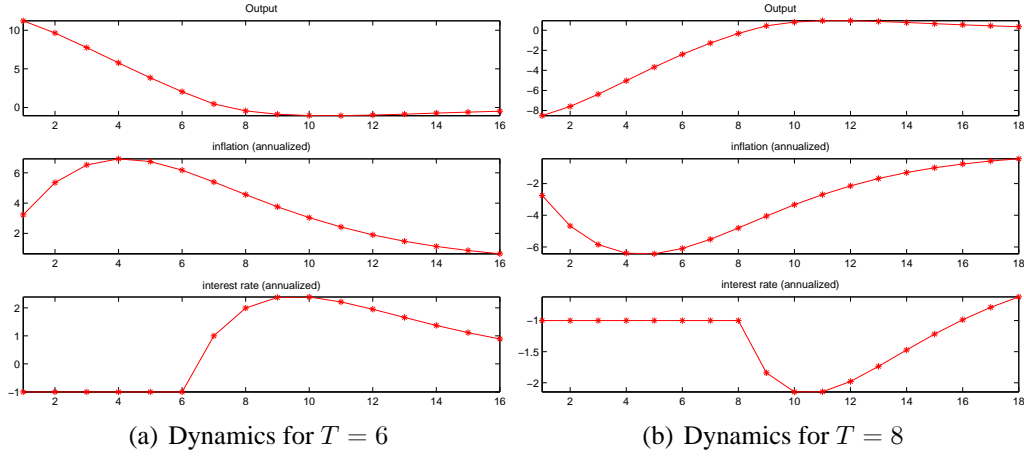


Figure 3: Dynamics in New Keynesian model with inflation inertia

for seven periods, output and inflation fall. We do not show a figure for this, because the nominal interest rate violates the zero lower bound after the exit from the peg. This suggests that according to the model, it is simply not possible for the central bank to keep the rate pegged at zero for 7 quarters and then revert to a Taylor rule. Panel(b) in figure shows a similar collapse that does not violate the lower bound for $T = 8$.

We find that numerically a necessary condition for this perverse sign-switch behavior is the existence of complex roots to the characteristic equation that defines behavior during the extended period of low rates. For $\beta = 1$, the condition for the existence of complex roots has a particularly simple form. There are complex roots if and only if $\Delta < 0$, with

$$\Delta \equiv -4 \frac{\kappa}{\sigma} \lambda^3 + \left[12 \frac{\kappa}{\sigma} - 8 \left(\frac{\kappa}{\sigma} \right)^2 \right] \lambda^2 - 4 \left[\left(\frac{\kappa}{\sigma} \right)^3 + 5 \left(\frac{\kappa}{\sigma} \right)^2 + 3 \left(\frac{\kappa}{\sigma} \right) \right] \lambda + \left(\frac{\kappa}{\sigma} \right)^2 + 4 \left(\frac{\kappa}{\sigma} \right) \quad (23)$$

This relationship is a convex and decreasing mapping between $\frac{\kappa}{\sigma}$ and λ . For example, for $\kappa = 0.025$, there are complex roots if and only if $\lambda > 0.57$. For $\frac{\kappa}{\sigma} = 0.25$, there are complex roots if $\lambda > 0.29$. The existence of complex roots implies that there is an asymptote at some value of T . Note from (23), that the coefficients from the exiting interest rate rule (12) have no effect on the existence of complex roots (and thus asymptotes). Finally, if $\lambda = 0$, so that there are no state variables as in the previous section, there are never reversals, regardless of the value of $\frac{\kappa}{\sigma}$.

3.1 Some intuition

A peculiarity of the DNK model with inflation inertia is the possibility of counterintuitive inflation behavior, ie., inflation may fall in response to a lower path for the nominal interest rate, what Laseen and Svensson (2011) call an ‘unusual equilibrium’. Since this unusual inflation response never arises in the simple DNK model, it is not purely a manifestation of the calibration of price stickiness κ or interest rate sensitivity σ . As discussed earlier, endogenous states are a necessary condition for the existence of inflation reversals. But why do we get asymptotic reversals (from positive to negative infinity) of initial inflation and output as the duration of the peg is extended? We provide some intuition here. Combining the bond Euler equation and Phillips curve we can solve for initial inflation as a function of the weighted future inflation path:

$$\pi_1 = FI + \frac{\kappa}{\sigma(1 + \beta\lambda)}(\sigma f_2 + b_2 - T)i^* + \frac{\lambda}{(1 + \beta\lambda)}\pi_0 \quad (24)$$

where we define future inflation, FI , as:

$$FI \equiv \frac{\kappa}{\sigma(1 + \beta\lambda)} \sum_{t=2}^T \pi_t + \frac{\beta}{(1 + \beta\lambda)}\pi_2 + \frac{\kappa}{\sigma(1 + \beta\lambda)}(\sigma f_1 + b_1)\pi_T \quad (25)$$

Expression (24) emphasizes the familiar forward looking aspect of inflation determination. Setting lagged inflation to steady state, it directly links initial inflation to the subsequent inflation path (FI) and the size of the monetary expansion. But initial inflation affects all future inflation rates via indexation and thus must also be compatible with this endogenous pass-through to future inflation. Using Proposition 2, we can express FI as a function of the initial inflation rate. For the case of real roots (the complex case is similar), we have

$$FI = \Omega\pi_1 \quad \text{with: } \Omega \equiv \left[\frac{\kappa}{\sigma(1 + \beta\lambda)} \sum_{t=2}^T \theta_{j,T} + \frac{\beta}{(1 + \beta\lambda)}\theta_{2,T} + \frac{\kappa}{\sigma(1 + \beta\lambda)}(\sigma f_1 + b_1)\theta_{T,T} \right] \quad (26)$$

where $\theta_{j,T} \equiv \frac{\pi_j}{\pi_1} = \left[\frac{i^* + m_1 e_1^{T+1-j} + m_2 e_2^{T+1-j} + m_3 e_3^{T+1-j}}{i^* + m_1 e_1^T + m_2 e_2^T + m_3 e_3^T} \right]$ The key issue is the behavior of Ω . Since the slope in (24) is unity, sharp reversals of initial inflation occur whenever changes in the duration of the peg imply that Ω crosses unity. Intuitively, this term is large if there is strong pass through of current inflation to the sum of future inflation because of endogenous

states (larger λ), or if the peg is maintained for more periods (larger T). It is important to recall that the constants (m_1, m_2, m_3) depend on T implying that Ω is a complicated function of structural parameters and T . Hence, we proceed numerically. If the indexation parameter λ is small, there are real roots and we can show numerically that Ω converges asymptotically from below to unity as T increases to infinity. This implies explosive inflation behavior as T increases without bound but no inflation reversals. If the indexation parameter λ is large enough, we have complex roots and thus sign switches in $\theta_{j,T}$ as we increase T . Thus, there exist periodic crossings of Ω with unity and sharp inflation reversals as T increases. At each of these reversals, the only way initial inflation can remain consistent with its endogenous pass through to future inflation is for initial inflation to switch sign as the peg is extended one more period.

We provide a convenient graphical exposition below. Figure 4 depicts (24) and (26) as two linear functions in (FI, π_1) space.

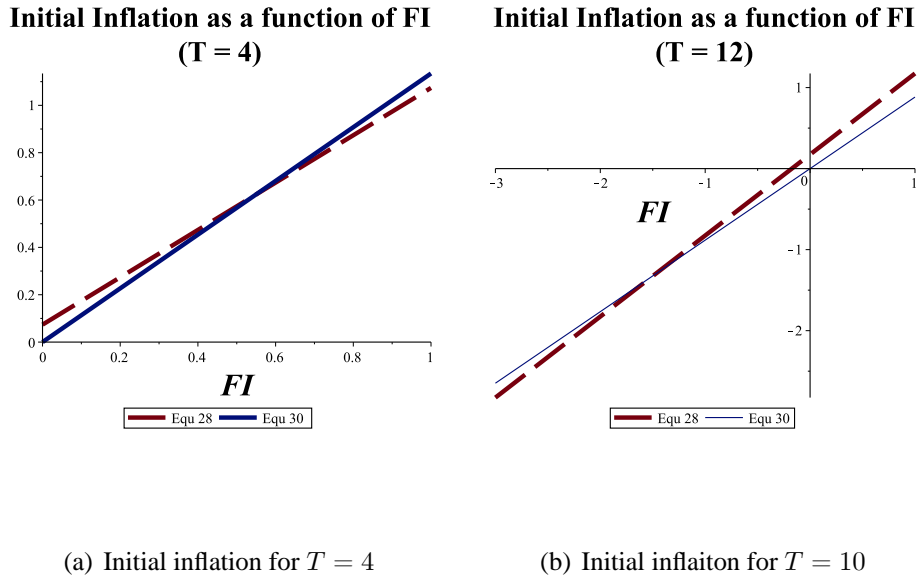


Figure 4: Intuition

The figure considers the same calibration as before ($\kappa = 0.025$, $\sigma = 1$, $\beta = 0.99$, $\lambda = 1$, $\rho = 0.80$, $\phi_\pi = 1.5$, $\phi_y = 0.5$), which implies that there are complex roots, and we have a sign switch as T increases. For $i^* = 0$, (24) and (26) cross at the origin (0,0). But with $i^* < 0$, (24) shifts up and the sign of the new intersection depends upon the size of Ω . If the backward element is small, $\Omega < 1$, then we have an intersection in the positive orthant so that initial and subsequent inflation is above steady state. This is the intuitive outcome. But as the backward element becomes more important, eg., higher levels of inflation indexation (λ) or a longer extended period (T), Ω increases and we can abruptly shift from the positive orthant to the negative orthant, ie., an inflation reversal. This is the case considered in Figure

5. Note that for $T = 4$, we have a usual equilibrium in which the extended period of low rates generates current and future inflation. But as T increases, Ω increases and crosses unity implying a sign switch and inflation reversal. The companion figure looks at the case of $T = 12$. As we increase T further, the size of Ω crosses back and forth over unity implying periodic inflation reversals.

3.2 Does the linear approximation drive our results?

We have treated the linear equilibrium conditions as the model economy. But it is well known from [2] and others that the linear approximation can be inaccurate in circumstances where interest rates are fixed because of the zero lower bound. This section therefore studies whether our key findings carry over to the nonlinear model that these equations are derived from.

We outline this nonlinear model only briefly, as it has been extensively described in the literature. The economy is populated by household with separable preferences in consumption and labour. The risk aversion parameter is σ and the inverse Frisch elasticity of labour supply is given by θ . We assume that the Phillips curve is obtained from a Calvo price-setting problem. Firms are allowed to update their prices with probability $1 - \alpha$. Non-optimising firms index their prices to past inflation with parameter λ . The price-setting problem is described in detail by [12]. We refer the reader to their paper for a full exposition of the price-setting problem and a complete derivation of the equilibrium conditions.

The price index implies

$$1 = \alpha \pi_t^{\eta-1} \pi_{t-1}^{\lambda(1-\eta)} + (1 - \alpha) \tilde{p}_t^{1-\eta}$$

where (in a slight abuse of notation) π_t denotes gross inflation in this non-linear case. Here, η is the elasticity in the Dixit-Stiglitz aggregator of differentiated goods and \tilde{p}_t is the relative price of optimizing firms. Price dispersion d_t evolves as

$$d_t = (1 - \alpha) \tilde{p}_t^{-\eta} + \alpha \left(\frac{\pi_t}{\pi_{t-1}^\lambda} \right)^\eta d_{t-1}$$

With subsidies that make the steady state efficient, the first-order condition for price-setting can be written as $x_{1,t} = x_{2,t}$. Here, the auxiliary variables $x_{1,t}$ and $x_{2,t}$ have a recursive representation given by

$$x_{1,t} = y_t m c_t \tilde{p}_t^{-\eta-1} + \alpha \beta \left(\frac{y_{t+1}}{y_t} \right)^{-\sigma} \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right)^{-\eta-1} \left(\frac{\pi_t^\lambda}{\pi_{t+1}} \right)^{-\eta} x_{1,t+1} \quad (27)$$

$$x_{2,t} = y_t \tilde{p}_t^{-\eta} + \alpha \beta \left(\frac{y_{t+1}}{y_t} \right)^{-\sigma} \left(\frac{\pi_t^\lambda}{\pi_{t+1}} \right)^{1-\eta} \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right)^{-\eta} x_{2,t+1} \quad (28)$$

We assume that the production function is linear in labour such that marginal cost is given by $m c_t = y_t^{\theta+\sigma} d_t^\theta$. The Bond Euler equation is

$$y_t^{-\sigma} = \beta y_{t+1}^{-\sigma} \frac{1 + i_t}{\pi_{t+1}}$$

The interest rate rule is given by

$$i_t = \begin{cases} i^* & t = 1, 2, \dots, T \\ (\frac{1}{\beta} \pi_t^{\phi_\pi} y_t^{\phi_y})^{1-\rho} (i_{t-1})^\rho & t = T+1, T+2, \dots \end{cases}$$

where (in another abuse of notation) i_t denotes the gross nominal interest rate. When linearized around a steady state with $\pi = y = d = p = 1$, the Phillips curve collapses to that given in (11).

We compute the solution via stacking the set of nonlinear equilibrium conditions for $t = 1, 2, \dots, K$. As a terminal condition, we impose that the model is in the deterministic steady state in period $K+1$. Initial conditions assume $\pi_0 = d_0 = 1$. We set $K = T+100$, the results do not depend on the choice of K as long as K is sufficiently large. This system is then solved via a nonlinear equation solver. Note that under the assumption of perfect foresight, the solution is exact up to machine precision without the need for any approximation. Our calibration sets $\eta = 11, \sigma = \theta = 1, \beta = 0.99, \phi_\pi = 1.5, \phi_y = 0.5$. We choose α such that the linearized model has a slope of the output gap version of the Phillips curve as in our previous analysis where $\kappa = 0.025$.

We first consider the nonlinear analog to the no-indexation model in Section 2, we thus set $\lambda = 0$ and $\rho = 0$. Panel (a) in Figure 5 plots initial inflation against the duration of the fixed-rate regime in the nonlinear model and in the linearized model.

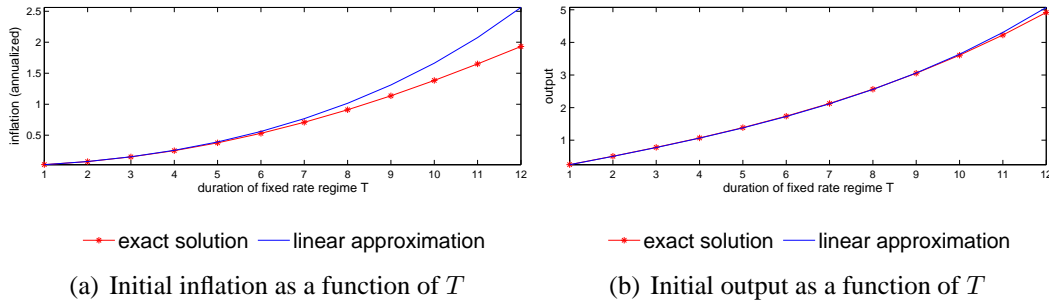


Figure 5: Exact versus linear solution: no inflation inertia

The initial inflation behavior in the nonlinear model is tracked reasonably well by the solution from the linearized model for durations of up to about six quarters. For larger durations the approximation becomes increasingly bad and it grossly overstates the rise in initial inflation for large durations. The linear solution for output is very accurate as Panel (b) in Figure 5 shows.

Turning to the model with inflation inertia, we find the same qualitative behavior for inflation in the nonlinear model as in the linearized model. In particular, initial inflation and output rise for certain values of the duration of the fixed-rate regime, but then there are sign reversals where inflation and output fall as the duration becomes larger. Figure 6 considers the model with indexation, setting $\lambda = 1$ and $\rho = 0.8$.

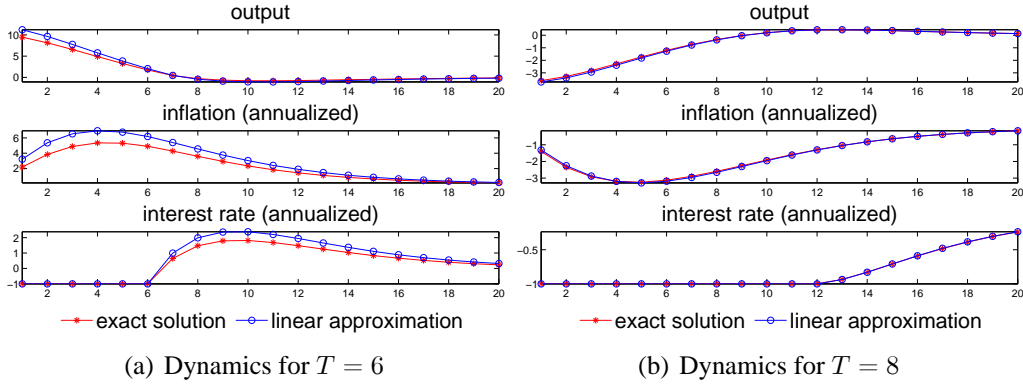


Figure 6: Exact versus linear solution: inflation inertia model

Panel (a) in Figure 6 assumes a duration of the fixed interest rate regime of six quarters. Inflation and output rise for both solutions techniques, but the magnitude is much bigger for the linear approximation. To illustrate that reversals are not an artifact of the linear approximation, we also show a fixed-rate regime of duration $T = 8$ quarters. We have chosen this value for T , because the nominal rate after the exit does not violate the lower bound at this value. Panel (b) in Figure 6 shows that sign reversals occur for both solution techniques. For $T = 8$, the linear approximation performs reasonably well quantitatively.

We note that our nonlinear analysis leaves interesting questions untouched. The nonlinear model has parameters that drop out of the linearized equations, such as η . Similarly, the Rotemberg model of quadratic price adjustment is isomorphic to the Calvo model in its linearized form, but the two differ in their nonlinear aspects. Exploring these issues is beyond the scope of this paper. We conclude that while the linear approximation technique sometimes significantly distorts the quantitative results, our key qualitative result is unchanged once we allow for a full nonlinear model. Reversal of the initial inflation and output response as the duration of the extended period is increased modestly can again be found.

3.3 Do other shocks matter?

Interest rates are most likely to remain pegged for an extended period due to the zero lower bound. In our previous analysis we have ignored other shocks that could have driven rates to the zero lower bound. One may therefore speculate whether omitting other shocks unduly influences our previous findings and the unusual behavior of inflation and output under a transient peg would cease to exist once these shocks are added.

With shocks to the natural rate, the Euler equation now becomes

$$y_t = y_{t+1} - \frac{1}{\sigma} (i_t - \pi_{t+1} - r_t^n) \quad (29)$$

The literature on natural rate shocks has modeled them in different ways. We first consider shocks to natural real interest rate r_t^n that are fully anticipated and last for T periods and revert

to zero thereafter, similar to the setup in section IV in [5]. We also consider a deterministic evolution of this disturbance that geometrically declines towards zero, similar to the autoregressive shock in [10]. To ensure that the mean disturbance is equal across these scenarios, we set the autoregressive parameter δ such that $T = 1/(1 - \delta)$

As a baseline model, we assume full indexation $\lambda = 1$ and drop interest rate inertia for simplicity, thus $\rho = 0$. The natural rate shock is known ex-ante to last for $T = 8$ periods and its value is -0.02 (quarterly) during this period. This calibration is roughly in line with the great recession style shock of [10]. We assume that the central bank responds to this shock by announcing that it will keep rates fixed at the zero lower bound for periods $1, \dots, K$ and revert to a Taylor rule thereafter. We compute the model solution for various values of K . Note that we do not impose that the central bank reverts to a Taylor rule as soon as such a rule would predict positive interest rates. This is in line with the idea of forward guidance, namely that keeping rates low for longer may improve outcomes during the period where the zero bound is binding.

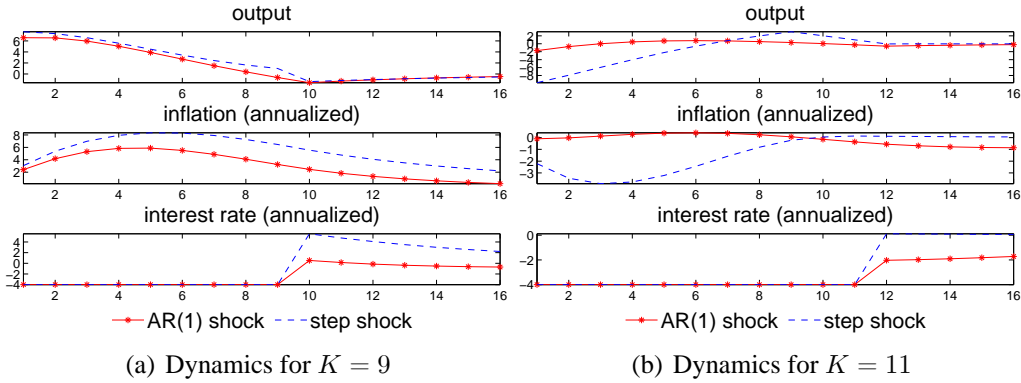


Figure 7: Natural rate shock that lasts for 8 periods

When the natural rate shocks lasts for $T = 8$ periods, holding rates constant for $K < 8$ violates the zero lower bound after the exit and is not an equilibrium. Setting $K = T = 8$ is identical to our previous analysis without a natural rate shock but pegging the nominal interest 100 basis points *above* steady state. We know from Figure 3 that this results in very *large* initial inflation and output. Figure 7 plots the model dynamics with forward guidance, i.e. when agents expect interest rates to be held fixed at the lower bound for $K > T$, periods. The figure shows that even the qualitative behavior of inflation and output is very sensitive to the timing of the exit. If the exit is expected to occur in period 9, there is a boom in inflation and output. One would have thought that keeping rates low for one more period such that $K = 10$ would increase inflation and output even more, but instead output declines. And the decline is even greater when rates are held low for $K = 11$ periods.

The rationale for this unusual behavior can be inferred from the discussion above. By itself, an 8-period negative demand shock that pushes the economy to the zero bound is expansionary (see Figure 3b and recall that $i^* - r^n = +100 \text{ bp}$). By itself, keeping rates low

for $K = 8, 9, 10$, periods is contractionary. Combining the two we find that the longer rates are kept low after the natural rate shock has died out, the more of a contraction is generated. None of this accords with the conventional view about the effects of monetary policy and most would probably regard this as a highly troubling implication of the DNK model in this environment.

Qualitatively, the autoregressive model has similar properties. This is despite the fact that some of the fall in the natural rate now occurs after the exit from the fixed rate regime. The lower bound is violated if the peg lasts for less than 8 periods, but an 8 period peg results in a sharp rise in output and inflation. Similarly, pegging rates sufficiently long for reduces output and inflation below steady in the autoregressive model as well, as can be seen in panel (b) of Figure 7.

Finally, we examine the model dynamics under the assumption that the

3.4 Does an empirical model deliver reversal?

In this subsection we investigate whether the perverse effects outlined earlier continue to arise in a more complete model economy. A classic reference here is the small-scale estimated DSGE model of [13] that includes habit persistence in consumption, capital accumulation, investment adjustment costs, nominal wage indexation, and nominal price indexation. We will use their benchmark model and estimated parameter values to investigate the existence of perverse behavior in a more realistic model of the economy. The reader is referred to the paper by [13] for the derivation of the model and the estimated parameter values.

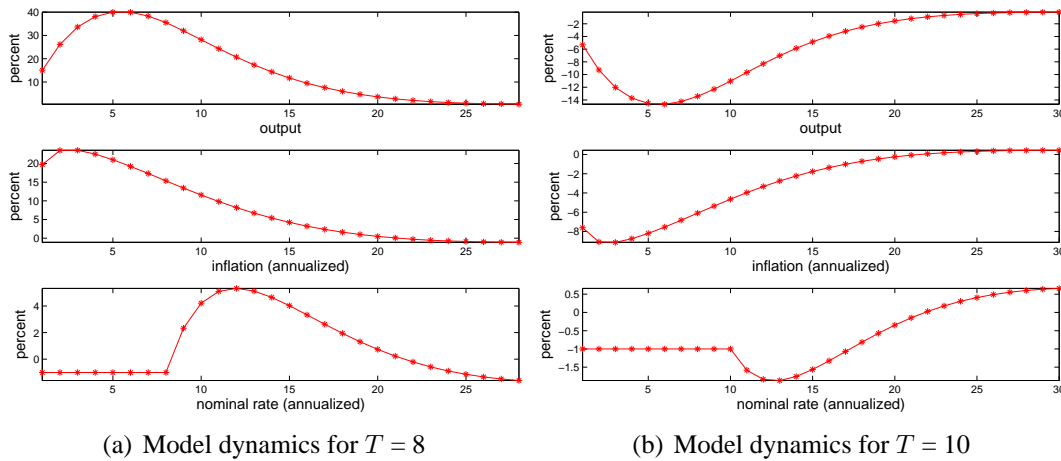


Figure 8: Dynamics in Smets Wouters model with fixed interest rates

Remarkably, we again observe sign switches in the behavior for inflation and output for reasonably short durations of the fixed-rate regime. Apparently, the additional frictions and features introduced in the empirically relevant model leave our main conclusion unchanged.³

³In this more complicated environment, there is still a link between complex eigenvalues and the appearance

When the peg lasts eight periods, initial inflation and output rise. But for a duration of $T = 9$, there is already a contraction in output and inflation. However, the nominal interest rate violates the zero lower bound after the exit from the peg. Thus, we depict in panel (b) of Figure 8 a duration for the peg of $T = 10$ periods for which the lower bound is never violated.

When studying natural rate shocks in the Smets and Wouters model, we find similar behavior as in the simple model with inflation inertia, if we drop the assumption of interest rate smoothing in the Taylor rule. That is, without smoothing and a duration of the natural rate shock of 9 periods, we have that keeping rates at the lower bound for 9 quarters generates a boom in inflation and output. Longer durations of the peg reduce inflation and output below steady state for the reasons outlined earlier. With interest rate smoothing, we find that a peg of 8 periods is an equilibrium that results in a severe contraction. But keeping rates low for longer only makes the contraction more severe.

4 An alternative Phillips Curve model: Sticky information

Although a workhorse model for monetary analysis, the DNK model with inflation indexation has the unusual implication that inflation reversals arise under plausible calibration of the model. Are there alternative models of nominal rigidities that deliver inertia in the endogenous variables, but do not suffer from the unusual inflation behavior under an interest rate peg? Our answer is yes: the class of sticky information models fulfills both requirements.

We conduct the same monetary policy experiment in the limited information model of [11]. This model was originally proposed as a way of generating additional inflation inertia but does so in a manner quite distinct from inflation indexation. The DNK model of section 3 exhibits inflation inertia by adding lagged inflation as an endogenous state variable to the Phillips curve. In contrast, the Mankiw-Reis model creates inertia via the slow evolution of information to firms in the model economy. This mechanism creates inflation inertia via the exogenous state variable that tracks the number of firms that have updated prices since the announcement of the extended period of low rates. A key difference compared to the DNK model with indexation is that the state variable in Mankiw-Reis is exogenous to the other variables in the model. Hence, one would anticipate, and we demonstrate here, that the benchmark Mankiw-Reis model is not subject to the unusual equilibria that arise in the DNK model with endogenous states.

Before proceeding to the more complicated environment of [11], we anticipate our results by examining a simpler sticky information model with deterministic information revelation. Consider a world with N types of firms, each of whom becomes attentive and re-sets their entire price path every N periods. The extended period is assumed to be shorter than the price stickiness ($T < N$). In log deviations, the firm's optimal price path (p_k^*) is to set the planned price proportional to nominal marginal cost:

$$p_k^* = p_k + \psi y_k \tag{30}$$

of reversals. But numerically we do not observe periodicity, but typically only two reversals.

where ψ links output to real marginal cost. Given the nature of the extended period experiment, the price level (p_k) is then given by

$$p_k = \frac{k}{N} p_k^* \quad (31)$$

for $k \leq N$. We thus have a simple expression for output

$$y_k = \frac{N - k}{\psi k} p_k \quad (32)$$

for $k \leq N$. The Fisher equation can be expressed as

$$i_k = \pi_{k+1} + \frac{\sigma}{\psi(k+1)} \left[(N - k - 1) \pi_{k+1} - \frac{N}{k} p_k \right] \quad (33)$$

for $k \leq N - 1$. The subsequent monetary policy is given by an aggressive Taylor-type rule based on current inflation ($i_t = \phi_\pi \pi_t$), so that we have $\pi_N = 0$. The model equilibrium is given by the price path that satisfies (33), with output then given by (32). The model can easily be solved backwards. We focus on just a few features of the equilibrium. First, during the extended period ($i_k = i^*$) there are no endogenous states in the model and (33) can be expressed as

$$p_k = \left\{ \frac{[1 + \frac{\sigma}{\psi(k+1)}(N - k - 1)]}{[1 + \frac{\sigma}{\psi(k+1)}(\frac{N}{k} + N - k - 1)]} \right\} p_{k+1} - \frac{i^*}{[1 + \frac{\sigma}{\psi(k+1)}(\frac{N}{k} + N - k - 1)]} \quad (34)$$

The key variable driving price stickiness is the time index k , but time is entirely exogenous. Second, the price level at the end of the price stickiness is given by:

$$p_N = p_{N-1} = \frac{p_{N-2}}{1 + \frac{\sigma}{\phi_\pi \psi(k+1)} \frac{N}{k}} < p_{N-2} \quad (35)$$

This deflationary dynamic continues as we move backwards, so that prices decline from the end of the extended period (period $T+1$) until period N when prices stabilize. The pace of this deflation is inversely related to the size of ϕ_π . For example, if ϕ_π is very large, then prices stabilize at the end of the extended period ($p_T = p_{T+k}$ for $k > 1$), while this subsequent deflation is more dramatic if ϕ_π is closer to one. The size of this subsequent deflation then feeds back onto price behavior during the extended period via p_{T+1} in (34). To summarize: prices rise and peak during the extended period, and then decline until stabilizing at the end of the price stickiness. Although inflation overshoots the steady-state of zero, there are never initial inflation reversals in this sticky information model. Evidently this is because of the absence of endogenous state variables.

These same mechanisms are at work in the more sophisticated framework of [11]. We describe the model briefly, for a full derivation, see [11]. Along with inattentiveness in price setting, the model assumes that there is inattentiveness in wage setting and consumption planning. The population shares of agents having last planned k periods ago is geometrically

declining in k and may differ across each of these three planning choices. Output of firm i is a linear function of labor at firm(i): $y_t(i) = \beta l_t(i)$ with $\beta \in (0, 1]$. Demand for goods variety j depends on aggregate output and the firm's relative price with elasticity ν : $y_t(j) = y_t - \nu(p_t(j) - p_t)$. Similarly, demand for labor variety h depends on aggregate labor and the worker's relative wage with elasticity γ : $l_t(h) = l_t - \gamma(w_t(h) - w_t)$. Households have separable preferences over consumption and leisure with coefficient of relative risk aversion θ^{-1} and Frisch elasticity of labor supply given by ψ .

The price Phillips curve states that the price level is a weighted sum of past expectations of each cohort's firm-specific nominal marginal cost

$$p_t = \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j E_{t-j} \left[p_t + \frac{\beta(w_t - p_t) + (1 - \beta)y_t}{\beta + \nu(1 - \beta)} \right]$$

where λ denotes the constant probability that a firm will become attentive. The wage Phillips curve is given by

$$w_t = \omega \sum_{j=0}^{\infty} (1 - \omega)^j E_{t-j} \left[p_t + \frac{\gamma(w_t - p_t)}{\gamma + \psi} + \frac{l_t}{\gamma + \psi} - \frac{\theta}{\theta(\gamma + \psi)} R_t \right]$$

where R_t is the long term real interest rate and ω is the attentiveness probability. Finally, aggregate output is given by

$$y_t = -\delta \sum_{j=0}^{\infty} (1 - \delta)^j E_{t-j} \theta R_t \quad (36)$$

where δ is the attentiveness probability. The model is closed by a simple Taylor rule as in equation (12) and calibrated as in that subsection.

The key parameters are the attentiveness probabilities for price setters, wage setters and consumption planners λ, ω and δ . We assume that all plans are updated once per year on average, but our results are not quantitatively very sensitive to that calibration. Figure 9 shows a time path for a duration of the peg equation to eight quarters.⁴ The model generates substantial humps in inflation and output due to the slow dissemination of information to agent decision making.

But as Figure 10 shows neither the initial levels of inflation and output nor their peaks show reversals as we increase the duration of the peg.

5 Conclusion

This is not an econometric test of DNK models. Nor is it a statement about other possible shocks hitting the system. It is instead a question of *prima facie* plausibility. Our results demonstrate that these models can produce implausible behavior under reasonable policy scenarios. Again, the existence of inflation reversals is particularly problematic: is it reasonable

⁴The sticky information model is solved by truncating the infinite sums at 1000.

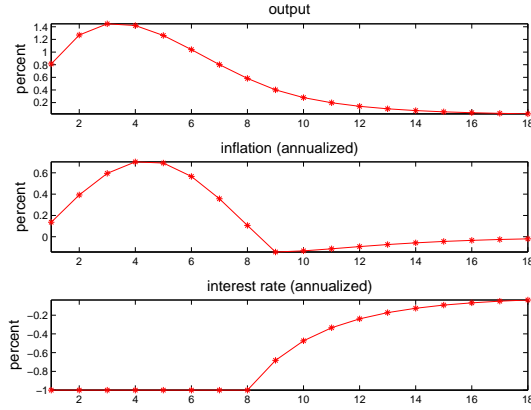


Figure 9: Sticky information model: Time path for $T = 8$

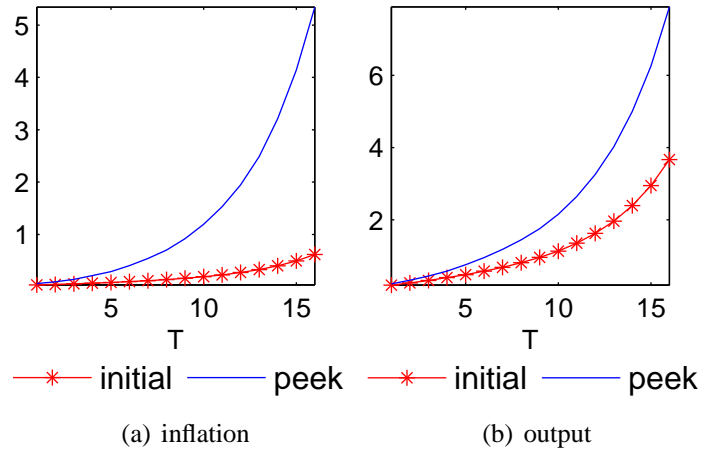


Figure 10: Sticky information: model behavior as a function of T

to think that keeping rates low for 8 periods is dramatically different than keeping rates low for 9 periods? Evidently the combination of an exogenous interest rate path and endogenous inflation inertia creates the possibility of highly unusual outcomes. Our sensitivity analysis indicates that this is not an artifact of linearization, nor of the complexity of the model as it also arises in the medium-scale estimated model of [13]. Instead it is endemic in typical DNK models.

As we have argued, the key ingredient for inflation reversals is the presence of endogenous state variables that enhance the feedback from past inflation on to current inflation. We have emphasized the role of inflation indexation. But other endogenous state variables can have a similar effect. For example, inflation reversals also arise in the DNK model without inflation indexation but with habit in consumption (so that lagged consumption is an endogenous state). The assumption of habit preferences leads to inertial behavior in consumption.

This inertial aggregate demand component then implies, through the Phillips curve, inertia in inflation. But this implies a strong link between past inflation and current inflation. If this effect is strong enough, inflation reversals will occur.

This analysis suggests several conclusions. First, DNK models should be used with great caution for exogenous interest rate scenarios as the models tend to deliver unreasonable answers. Second, Bayesian estimation of these DNK models may be improved by penalizing the parameter space that is consistent with inflation reversals. If it is our prior that inflation reversals are perverse, then should not the set of prior parameter values reflect this? Finally, the baseline sticky information model of [11] should be investigated more systematically as it does not deliver these peculiar outcomes. This is noteworthy since [?] has shown that the Calvo model with indexation and the sticky information model do equally well in delivering the conventional reaction to an unanticipated interest rate movement. In contrast, we show that these models behave very differently in our policy experiment of an anticipated interest rate peg. Evidently, an exogenous interest rate path is best examined in a model with an exogenous evolution of price stickiness.

Appendix

A Sensitivity analysis

The magnitude of the inflation response depends crucially on the ratio $\frac{\kappa}{\sigma}$ and our baseline calibration assumes conservative values for this. But, for example, [14] argues that in a model without capital one should use a lower value for σ as that makes spending more interest rate sensitive, partly compensating for the absence of investment which is more interest rate sensitive. Hence, as a form of sensitivity analysis we consider a range of values for κ and σ , respectively. For an extended period of $T = 8$, panel (a) of Figure 11 graphs the initial inflation rate as a function of κ and panel (b) does the same as a function of σ (in both cases we set the other parameter to its previous calibration). The time path subsequent to this initial inflation rate is similar to Figure 1. Qualitatively, the behavior in Figure 11 is not surprising: the interest rate peg has a larger effect on inflation and output if prices are more flexible or if spending is more sensitive to interest rates. Curiously, the model's prediction does not converge to the flexible price outcome as $\kappa \rightarrow \infty$. Figure 11 would lead one to anticipate that the output effect becomes unboundedly positive. But in a flexible price model there is no output effect. The discrepancy arises because the interest rate peg experiment is not possible in the flexible price model. That is, following the discussion in Section 2.2, $\pi(T + 1) = 0$, so that the nominal rate can be pegged in period T only by varying the real rate. But this is not possible in a flexible price model.

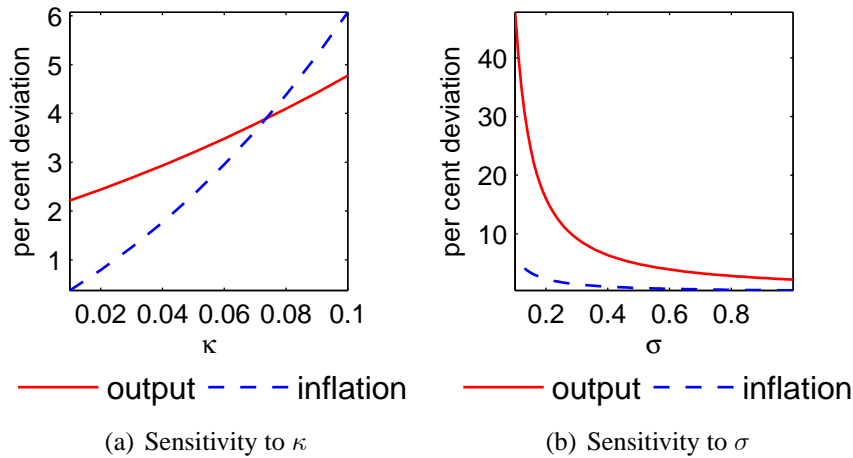


Figure 11: Sensitivity of initial inflation and output to parameters

B Proofs

Proposition 4 *The inflation rate during the extended period is given by*

$$\pi_{T+1-s} \equiv z_s = i^* + m_1 e_1^s + m_2 e_2^s \quad \text{for } s = 0, 1, \dots, T$$

for $s = 1, 2, \dots, T$, where $e_1 > 1, e_2 < 1$, and m_1 and m_2 come from the two restrictions (6)-(7).

Proof. The difference equation is given by

$$z_s = -\frac{\kappa}{\sigma} i^* + \left(\frac{\kappa}{\sigma} + 1 + \beta \right) z_{s-1} - \beta z_{s-2} \equiv c + \gamma_1 z_{s-1} + \gamma_2 z_{s-2}$$

with the two initial conditions:

$$\begin{aligned} z_1 &= c \\ z_2 &= c + \gamma_1 c \end{aligned}$$

The particular solution is given by: $m = c + \gamma_1 m + \gamma_2 m$. Solving this we have $m = i^*$. The characteristic equation of the homogeneous system is given by

$$q^2 - \gamma_1 q - \gamma_2 = 0$$

or

$$h(q) \equiv q^2 - \left(\frac{\kappa}{\sigma} + 1 + \beta \right) q + \beta$$

Since h is convex, $h(0) > 0$, and $h(1) < 0$, the difference equation has two real eigenvalues denoted by $e_1 > 1$ and $e_2 < 1$. The general solution is thus given by $z_s = m + m_1 e_1^s + m_2 e_2^s$. ■

Proposition 5 *The determinacy region of the stochastic exit model given by (8) is identical to the region for which $T < T_{crit}$, i.e there are no reversals whenever the model is determinate.*

Proof. To compute the determinacy region, we postulate a candidate equilibrium with unknown coefficients a, b , and c :

$$\pi_t = ca^t + b$$

Combining the Phillips curve with the IS curve, we have:

$$\beta p^2 \pi_{t+2} - \left[\frac{\kappa}{\sigma} + (1 + \beta) \right] p \pi_{t+1} + \pi_t + \frac{\kappa}{\sigma} i^* = 0 \quad (\text{B.1})$$

Put into (B.1):

$$\beta p^2 (ca^{t+2} + b) - \left[\frac{\kappa}{\sigma} + (1 + \beta) \right] p (ca^{t+1} + b) + ca^t + b + \frac{\kappa}{\sigma} i^* = 0$$

The value of b is the particular solution and is given by the solution to:

$$\beta p^2 b - \left[\frac{\kappa}{\sigma} + (1 + \beta) \right] p b + b + \frac{\kappa}{\sigma} i^* = 0$$

But this is just the solution given by (8).

$$b = \frac{\frac{\kappa}{\sigma} i^*}{p \frac{\kappa}{\sigma} - (1 - p)(1 - \beta p)}$$

The value of a is given by the root of the characteristic equation:

$$h(a) \equiv \beta p^2 a^2 - \left[\frac{\kappa}{\sigma} + (1 + \beta) \right] p a + 1$$

This equilibrium is stationary if a is in the unit circle. Note that h is quadratic, convex, $h(0) > 1$, $h'(0) < 0$. The key issue then is the value of $h(1)$. The value of a is in the unit circle if $h(1) < 0$.

$$h(1) = \beta p^2 - \left[\frac{\kappa}{\sigma} + (1 + \beta) \right] p + 1 < 0$$

But this is equivalent to the issue of whether there are reversals. That is, if $h(1) < 0$, then $a < 1$ and there are equilibrium of the form $??$. But c is free, there are an infinite number of equilibria. ■

Proposition 6 *The system in (18) has one eigenvalue e_1 inside the unit circle and two eigenvalues (e_2 and e_3) outside the unit circle. The inflation rate during the extended period is then given by*

1. *If e_2 and e_3 are real:*

$$\pi_{T+1-s} \equiv z_s = i^* + m_1 e_1^s + m_2 e_2^s + m_3 e_3^s$$

2. *If e_2 and e_3 are complex, $(e_2, e_3) = a \pm bi$:*

$$\pi_{T+1-s} \equiv z_s = i^* + m_1 e_1^s + m_2 r^s \cos(\theta s) + m_3 r^s \sin(\theta s)$$

where $r = \sqrt{a^2 + b^2}$, $\theta = \tan^{-1}(\frac{b}{a})$, and m_1 , m_2 , and m_3 , come from the three restrictions (21)-(23). These constants do depend upon T because of the condition (21).

Proof. The difference equation (18) is given by

$$z_s = c + \gamma_0 z_{s+1} + \gamma_1 z_{s-1} + \gamma_2 z_{s-2}$$

The particular solution to this system is given by the m that satisfies

$$m = c + \gamma_0 m + \gamma_1 m + \gamma_2 m$$

or

$$m = \frac{c}{1 - \gamma_0 - \gamma_1 - \gamma_2} = i^*$$

The characteristic equation of the homogeneous system is

$$g(q) \equiv \frac{\lambda}{[1 + \lambda(1 + \beta)]} q^3 - q^2 + \left[\frac{\sigma(1 + \beta\lambda + \beta) + \kappa}{\sigma[1 + \lambda(1 + \beta)]} \right] q - \frac{\beta}{[1 + \lambda(1 + \beta)]}$$

We can express this more conveniently as

$$g(q) \equiv \lambda q^3 - [1 + \lambda(1 + \beta)] q^2 + \left[(1 + \beta\lambda + \beta) + \frac{\kappa}{\sigma} \right] q - \beta$$

The product of the three roots is $\frac{\beta}{\lambda} > \beta$. We also have $g(0) < 0$, $g(\beta) > 0$, so there is a real root in $(0, \beta)$. But since the product of the roots exceeds β , the other two roots must be outside the unit circle. The solution to the difference equation is then given by the sum of the particular solution and the homogeneous solution. ■

References

- [1] A.P. Blake. Fixed interest rates over finite horizons. Bank of England Working Paper No. XXX, 2012.
- [2] R.A Braun and L.M. Korber. New keynesian dynamics in a low interest rate environment. FRB Atlanta Working Paper No. 2011-10, 2011.
- [3] Guillermo A. Calvo. Staggered price setting in a utility-maximizing framework. *Journal of Monetary Economics*, 12:383–398, 1983.
- [4] Lawrence J. Christiano, Martin Eichenbaum, and Charles L. Evans. Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, 113:1–45, 2005.
- [5] Lawrence J. Christiano, Martin Eichenbaum, and Sergio Rebelo. When is the government spending multiplier large? *Journal of Political Economy*, 119:78–121, 2011.
- [6] Gauti Eggertsson and Michael Woodford. The zero interest rate bound and optimal monetary policy. *Brookings Papers on Economic Activity*, pages 139–211, 2003.
- [7] J. C. Fuhrer and G.R. Moore. Inflation persistence. *Quarterly Journal of Economics*, 85:163–190, 1995.
- [8] Jordi Galí. Constant interest rate projections without the curse of indeterminacy: a note. *International Journal of Economic Theory*, 5:61–68, 2009.

- [9] S. Laseen and L.E.O. Svensson. Anticipated alternative policy rate paths in policy simulations. *International Journal of Central Banking*, 7(3):1–35, 2011.
- [10] Andrew Levin, David Lopez-Salido, Edward Nelson, and Tack Yun. Limitations on the effectiveness of forward guidance at the zero lower bound. *International Journal of Central Banking*, 6:143–185, 2010.
- [11] Gregory N. Mankiw and Ricardo Reis. Pervasive stickiness. *American Economic Review*, 96(2):164–169, 2006.
- [12] Stephanie Schmitt-Grohé and Martin Uribe. Optimal fiscal and monetary policy in a medium-scale macroeconomic model: Expanded version. NBER Working Paper 11417, 2005.
- [13] Frank Smets and Raf Wouters. Shocks and frictions in us business cycles: a bayesian dsge approach. *American Economic Review*, 97:586–606, 2007.
- [14] Michael Woodford. *Interest and prices: foundations of a theory of monetary policy*. Princeton University Press, 2003.