International recessions*

Fabrizio Perri
University of Minnesota and Federal Reserve Bank of Minneapolis

Vincenzo Quadrini
University of Southern California

October 18, 2012

Abstract

The 2007-2009 crisis was characterized by an unprecedented degree of international synchronization as all major industrialized countries experienced large macroeconomic contractions around the date of Lehman bankruptcy. During the same period countries also experienced large and synchronized tightening of credit conditions. We present a two-country model with financial market frictions where a credit tightening can emerge as a self-fulfilling equilibrium caused by pessimistic but fully rational expectations. As a result of the credit tightening and the associated lack of liquidity, countries experience large and *endogenously* synchronized declines in asset prices and economic activity (international recessions). We also show that the economic contraction is more severe after a prolonged period of credit expansion.

Keywords: Credit shocks, global liquidity, international co-movement.

JEL classification: F41, F44, G01.

*We thank seminar participants at Atlanta Fed, Boston University, Central Bank of Turkey, Chicago Booth, Columbia University, Dallas Fed, Federal Reserve Board, Harvard University, Indiana University, New York Fed, San Francisco FED, UC Berkeley, UCLA, University of Colorado, University of Notre Dame, University of Toulouse and attendees at the Advances in International Macroeconomics conference in Brussels, Asset Prices and the Business Cycle conference in Barcelona, Bank of Japan International Conference, Bundesbank Spring Conference in Eltville, Challenges in Open Economy Macroeconomics after the Financial Crisis conference at the St Louis FED, NBER IFM Meeting, NBER Summer Institute, Philadelphia Workshop on Macroeconomics, SED meeting in Ghent, Stanford SITE conference, The Financial Crisis: Lessons for International Macroeconomics conference in Paris. We also thank Philippe Bacchetta, Ariel Burstein, Fabio Ghironi, Jean Imbs, Thomas Laubach, Enrique Martinez-Garcia, Paolo Pesenti, Xavier Ragot, Etsuro Shioji and Raf Wouters for excellent discussions.

1 Introduction

One of the most striking features of the 2007-2009 recession is that in the midst of the crisis—the quarter after the Lehman bankruptcy—all major industrialized countries experienced extraordinarily large and synchronized contractions in both real and financial aggregates. In this paper we show that the crisis and its international synchronization can arise as the outcome of a self-fulfilling equilibrium characterized by global liquidity shortage when financial markets are highly integrated. We show this by developing a two-country incomplete-markets model where firms use credit to finance hiring and investment. Credit is constrained by the option to default. If firms are up against the constraint, equilibrium employment and investment is affected by the shadow cost of credit, which in turn depends upon the tightness of the credit constraint.

Our first result is that, if countries are financially integrated, the shadow cost of credit is equalized across countries. Hence, an 'exogenous' tightening of credit affects employment and output in all countries, regardless of where the tightening originates. This result suggests a channel for the international transmission of credit shocks but does not deal with the more fundamental question of what *causes* a credit shock.

Our second result provides an 'endogenous' mechanism for credit tightening. We show that tighter/looser credit constraints can emerge endogenously as multiple self-fulfilling equilibria. In 'bad' equilibria, markets expect low resale prices for the assets of defaulting firms. Because of the expected low value of liquidated assets, firms face tight borrowing limits and are liquidity constrained. But because firms are liquidity constrained, there are no firms that can purchase the assets of liquidated firms and, as a result, the resale price of firms' assets is low. This rationalizes the expectation of low prices leading to bad equilibria characterized by depressed economic activity, reduced financial intermediation and low asset prices. On the other hand, in 'good' equilibria, markets expect high resale prices for the assets of defaulting firms, which allows for looser borrowing constraints. As a result of the high borrowing capacity, firms are not liquidity constrained and ex-post there are firms with the required liquidity to purchase the assets of liquidated firms. This, in turn, keeps the price of the liquidated assets high and rationalizes, ex-post, the ex-ante expectation of high prices leading to equilibria with sustained economic activity, financial intermediation and asset prices.

The distinction between 'exogenous' and 'endogenous' credit shocks does not only provide a more interesting theory of the recession. More importantly, it also explains one central feature of the recent crisis, that is, the international co-movement in financial intermediation. Although exogenous credit shocks can generate co-movement in real economic activities, they do not generate co-movement in financial flows. Instead, endogenous credit shocks generate international co-movement in both real and financial flows. This is because endogenous credit shocks are determined by the expected resale price of firms' assets. But in a financially integrated economy, the expected resale price is common across countries. Hence, credit contractions are also common across countries and generate co-movement in all variables, real and financial. Modeling fluctuations in credit tightness as endogenous outcomes has also important policy implications. It suggests that changes in the structural features of the economy, such as financial integration or the public provision of liquidity, can change the volatility and international correlation of shocks, which are usually treated as exogenous processes.

Our third result relates to the depth of the crisis. We show that 'ordinary' credit shocks, that is, shocks that would cause a mild contraction under normal circumstances, can generate 'extra-ordinary' recessions if they arise after a long period of credit expansion. To illustrate this result in the context of our model, we characterize an equilibrium path in which credit constraints are not binding for a long period of time. During this period both economies undergo a persistent expansion of economic activity (gradual) and of credit (rapid). If constraints become binding after this long expansionary phase, firms are forced to de-leverage quickly, which causes a sharp contraction in real economic activity. This happens even if agents fully anticipate the possibility of the reversal. The asymmetry between expansions and contractions captures the macroeconomic developments of advanced economies during the recent cycle and other episodes of financial crises (see, for example, Reinhart and Rogoff (2009) and Schularick and Taylor (2011)).

One important observation concerning the international dimension of the recent crisis is that, although real GDP experienced similar contractions in the US and in the rest of the G7 countries, employment was hit particularly hard in the US but not in the remaining G7 countries (see Ohanian (2010)). As a consequence, labor productivity increased in the US but declined in the rest of the G7 countries. Our baseline model with integrated credit markets and symmetric labor markets does not capture these cross-country differences. However, in the final section of the paper we show that the heterogeneous response of employment is not necessarily inconsistent with the idea of a credit shock, once we allow for cross-country differences in the characteristics of national labor markets (more flexibility in the US and less flexibility in other G7 countries). With this extension, the emergence of a bad self-fulfilling equilibrium has the potential to explain the similar cross-country responses of GDP and financial markets, and the heterogeneous responses of employment, productivity and the labor wedge.

Of course, the theory proposed in this paper is not the only explanation for the international recession. Conceivably, one could potentially develop other theories of common global shocks in which credit contraction is only a consequence and not a cause of the crisis. We view the comparative evaluation of different theories of global crises as an interesting direction for future research.

The role of credit shocks for macroeconomic fluctuations has been recently investigated primarily in closed economy models where the shocks follow purely exogenous processes.[1] In this paper, instead, we study the international implications of these shocks

[1]Examples are Christiano, Motto and Rostagno (2009), Gertler and Karadi (2009), Goldberg (2010),

and provide a micro foundation which is based on self-fulfilling expectations. Our theory is in line with the idea of liquidity crises resulting from multiple equilibria outcomes as discussed in Lucas and Stokey (2011) and it shares some similarities with the multiple equilibria property of the model studied in Kocherlakota (2009). Finally, the idea that multiple equilibria can emerge in models with credit limited by the value of collateral has been first proposed by Shleifer and Vishny (1992) and, more recently, by Benmelech and Bergman (2012) but in closed economy models. Our paper, instead, focuses on the international implications of these shocks. In this respect, our paper is related to the literature studying the sources of macroeconomic co-movement and international transmission of shocks, starting with Backus, Kehoe & Kydland (1992).

In the literature there are two predominant explanations for international co-movement. The first explanation is based on the existence of global or common shocks, that is, exogenous disturbances that are correlated across countries. The second explanation is based on the international transmission of country-specific shocks (for example through investment). In this paper we show that credit shocks generate co-movement for both reasons: *exogenous* credit shocks spill over from one country to the other, and *endogenous* credit shocks will appear to the econometrician like a common-shock or a global factor. Recent contributions that analyze directly the strong international co-movement during the 2007-2009 crisis include Dedola & Lombardo (2010), Devereux & Yetman (2010), Devereux & Sutherland (2011), and Kollmann, Enders & Müller (2011). All of these studies focus on the international transmission of shocks in models with financial market frictions but do not consider the possibility of endogenously generated fluctuations in credit. Finally, the idea that multiple equilibria can generate international co-movement has also been proposed recently by Bacchetta and Van Wincoop (2011).

Central to the multiplicity of equilibria is that financial constraints are 'occasionally binding'. This leads to another important difference between our paper and other studies that investigate the macroeconomic impact of financial shocks. Most of these contributions limit the analysis to equilibria with always binding constraints and the quantitative properties are studied using linear approximation techniques. In our model, instead, borrowing constraints are only occasionally binding which is key to generate multiple equilibria. This is also important to generate the asymmetry between long and gradual credit driven booms and sharp credit driven recessions. Mendoza (2010) also studies an economy with occasionally binding constraints but does not investigate the importance of financial shocks. Furthermore, Mendoza (2010) focuses on a small open economy and does not address the issue of international co-movement which is central in our paper. Occasionally binding constraints are also central to Brunnermeier and Sannikov (2010) but their analysis is limited to productivity shocks and in a closed economy.

The paper is organized as follows. In Section 2 we provide some macroeconomic and

Jermann and Quadrini (2011), Khan and Thomas (2010), Liu, Wang and Zha (2012), Lorenzoni and Guerrieri (2010).

financial evidence about the recent crisis. We then present the theoretical framework gradually, starting in Section 3 with a version of the model where capital is fixed and credit shocks are exogenous. Section 4 makes the credit shocks endogenous and describes the conditions for multiple equilibria. Section 5 adds capital accumulation and Section 6 conducts a quantitative analysis. In Section 7 we extend the model by allowing for cross-country heterogeneity in domestic labor markets. Section 8 concludes.

2 Macroeconomic evidence

We first present some facts about international co-movement during the 2007-2009 crisis and then some evidence on the dynamics of credit and employment.

2.1 International co-movement

Figure 1 plots the GDP dynamics for the G7 countries during the six most recent US recessions. In each panel we plot the percent deviations of GDP for each country from the level of GDP in the quarter preceding the start of the US recession (based on the NBER business cycle dating committee). Comparison of the bottom right panel of the figure with the other panels suggests how the 2007-2009 period and, in particular, the period following the Lehman crisis (marked by the vertical line), stands out both in terms of depth and in terms of macroeconomic synchronization among the G7 countries. In none of the previous recessions GDP fell so much and in all countries.

Another way to illustrate the increased international co-movement associated with the recent crisis is provided by Figure 2. This figure plots the average correlation of 10 years rolling windows of quarterly GDP growth between all G7 countries. Two standard deviation confidence bands are also plotted. The dates in the graph correspond to the end points of the window used to compute the correlation. We can see from the figure that during the last two quarters of 2008 (the vertical line marks the third quarter), the average correlation jumped from 0.3 to 0.7 and the sample standard deviation of the correlations fell from 0.19 to 0.09. This confirms that the 2007-2009 stands out in the post-war era as a period of extraordinarily high co-movement for all developed countries, a point also emphasized in Imbs (2010).

The high degree of international co-movement between the US and other major industrialized countries is also observed in other real and financial variables. Figure 3 plots GDP, consumption, investment and employment in the period 2005-2010 for the US and an aggregate of the other countries in the G7 group (from now on G6). The figure highlights that GDP, consumption and investment were all hit hard in both the US and the G6 countries. This is especially noticeable after the Lehman crisis, marked by the vertical line. Employment also declined in the US and abroad, even though the US decline is much larger than the decline in the G6, a feature emphasized by Ohanian

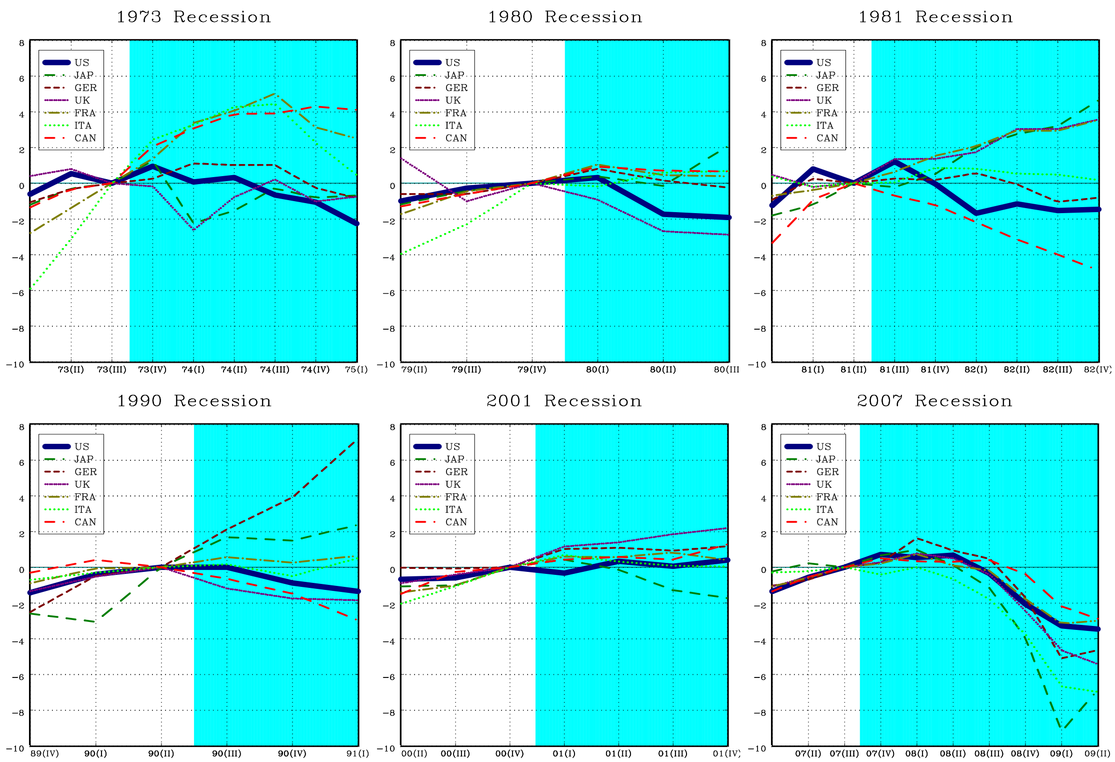

Figure 1: The dynamics of GDP in the G7 countries during the six most recent US recessions. The series have been normalized in the quarter preceding the start of the recession. The outlined area denotes the official NBER recessions dates.

(2010). We will return to this issue in the last part of the paper where we will propose a possible explanation for the heterogeneous dynamics of the labor market.

Figure 4 plots the dynamics of some financial variables. The top left panel plots the growth rate of stock prices for the US and for the G6 and it documents the massive and synchronous decline in stock prices that took place during the crisis.[2]

The top right panel reports the growth in total gross debt for the non-financial business sector which also dropped during the crisis.[3] Indicators of credit market conditions

[2]Stock prices for the US are the MSCI BARRA US stock market index, while stock prices in the G6 countries are computed using the MSCI BARRA EAFE+Canada index which is an average of stock prices in advanced economies except the US.

[3]The series for the US real debt is from the Flows of Funds Accounts and for the whole nonfinancial business sector. The series for the G6 is the sum of net debt (in constant PPP dollars) of the corporate non-financial sector for the Euro Area, Japan and Canada. Debt is defined as credit market instruments minus liquid assets i.e. the sum of foreign deposits, checkable deposits and currency, time and savings deposits, money market funds, securities RPs, commercial paper, treasury securities, agency and GSE backed securities, municipal securities and mutual fund shares

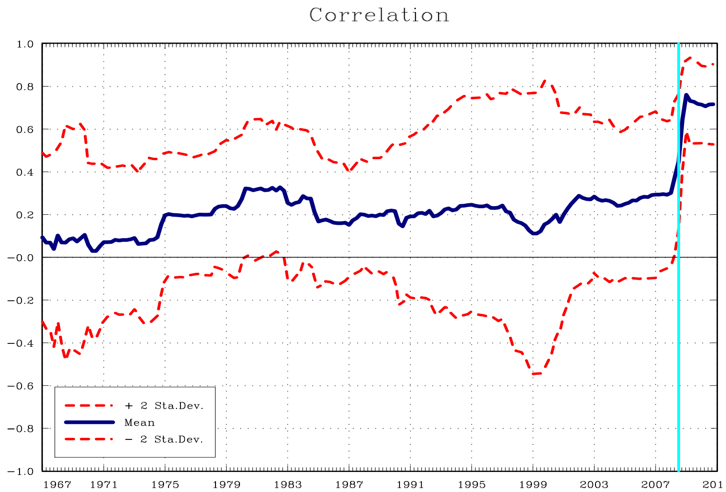

Figure 2: Rolling correlations of quarterly GDP growth among G7 countries. The continuous line is the mean correlation. The dashed lines denote ±2 standard deviation band. The vertical line denotes the third quarter of 2008 when the Lehman's bankruptcy took place.

based on credit volumes have been criticized as they do not take into account that a credit crunch might induce firms to draw on existing credit lines, so the distress does not immediately show up in quantities. See, for example, Gao and Yun (2009). For this reason the bottom left panel reports a different indicator of credit market conditions. The indicator is not based on volumes of credit but on opinion surveys of senior loan officers of banks. The plotted index is the percentage of banks that relaxed the standards to approve commercial and industrial loans minus the percentage of banks that tightened the standards. Thus, a negative number represents an overall tightening of credit.[4]

As can be seen from the third panel of Figure 4, the index shows a credit tightening that starts before the decline in credit growth. To take both types of evidence into account, the bottom right panel constructs a credit index that is a simple average of the two previous measures, with each series normalized by its own standard deviation.

The key lesson we learn from Figure 4 is that, right around 2008, credit conditions moved from strongly loose/expansionary to strongly tight/contractionary both in the US and in the other G7 countries. This evidence will be particularly important in the

[4]The series for the US is released by the Federal Reserve Board (Senior Loan Officers Opinions Survey). The series for the G6 is based on similar surveys released by the European Central Bank (ECB Bank Lending Survey), Bank of Japan (Senior Loan Officer Opinion Survey) and Bank of Canada (Senior Loan Officers Opinions Survey). It is computed as the weighted (by overall debt) average of the indices for the Euro area, Japan and Canada. Therefore, the average series does not correspond exactly to the series for the G6 countries because data for the UK is not available and it includes Euro countries that are not part of the G7 group. The indices are typically reported with the inverted sign (tightening credit standards instead of relaxing credit standards).

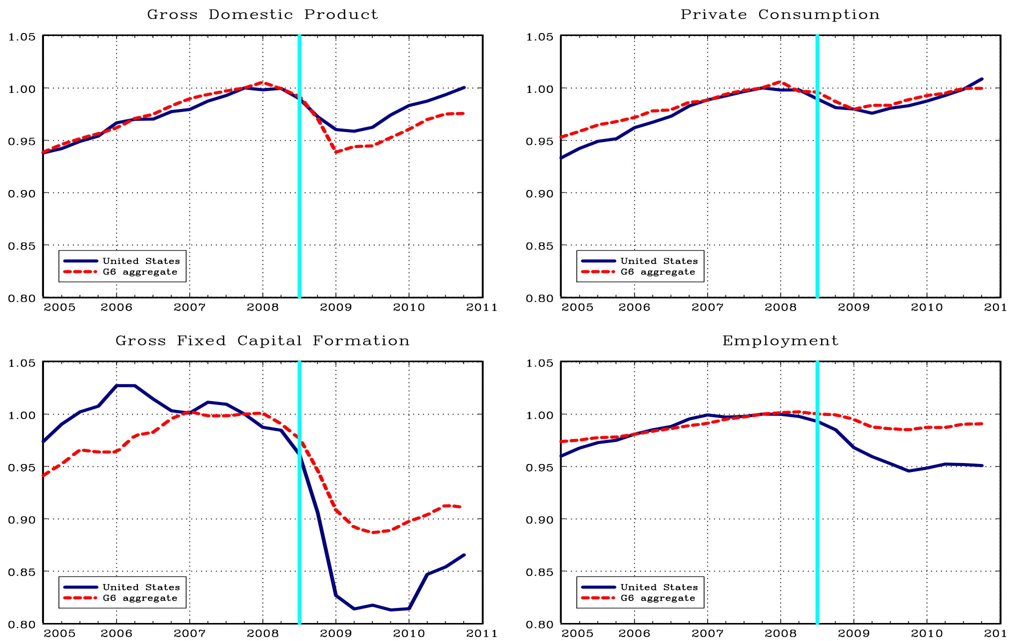

Figure 3: GDP, Consumption, Investment and Employment in US and G6 aggregate: 2005-2010. Data for GDP, consumption and investment are from OECD Quarterly National Accounts in PPP constant dollars. Data for employment are from OECD Main Economic Indicators. All series are normalized to 1 in the first quarter of 2007. The vertical line denotes the third quarter of 2008 when the Lehman's bankruptcy took place.

second part of the paper as it allows us to identify more precisely the nature of the crisis.

2.2 Domestic co-movement between credit and employment

As discussed in the introduction, our main hypothesis is that tight credit affects economic activity and especially employment. Here we provide some empirical support for this idea by plotting the growth rates of employment, GDP, and business credit during the crisis in the US and in the G6 countries. Figure 5 shows that in the quarters following the Lehman crisis (indicated by the vertical line), both credit and employment slowdown significantly in the US and in the G6. Interestingly, GDP also declines initially but recovers more quickly than employment and credit. For example, in the first quarter of 2009, credit and employment are still depressed (experiencing negative growth) in the US and in the G6. However, GDP has already recovered (experiencing positive growth) in both countries. We view this evidence as consistent with our basic hypothesis: tight credit reduces employment and as employment falls labor productivity increases so that the decline in GDP is not as severe as the decline in employment.

A final observation relates to the asymmetry between real and financial variables in

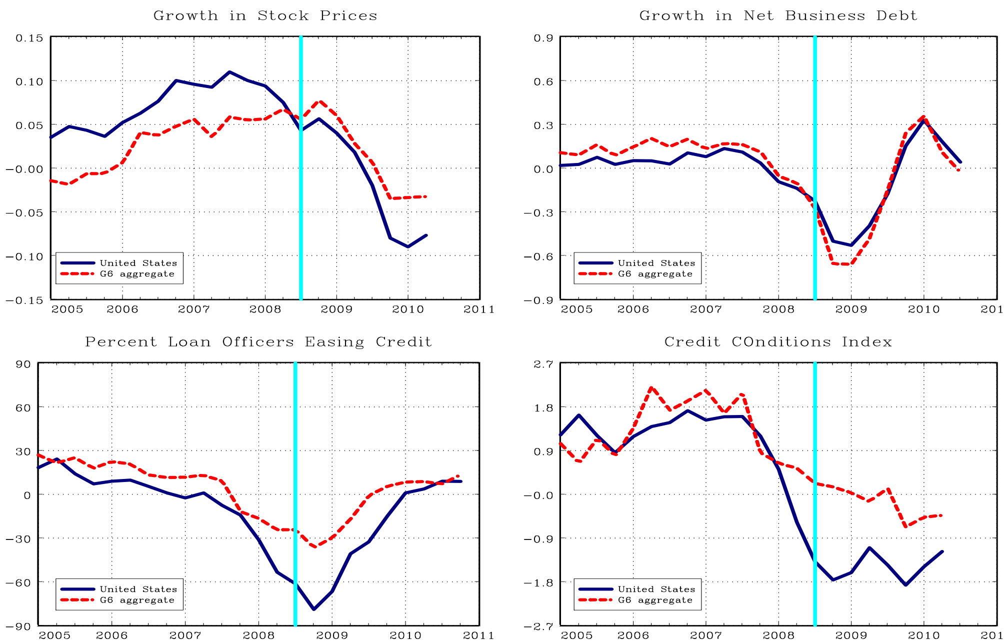

Figure 4: GDP, Stock markets and credit conditions in US and G6 aggregate: 2005-2010. The vertical line denotes the third quarter of 2008 when the Lehman's bankruptcy took place.

the expansion phase before the crisis and the collapse during the crisis. Figure 5 shows that debt experienced rapid growth (about 6% per year in the US and 4% per year in the G6) in the years preceding the crisis, while the growth in real variables has been moderate (GDP grew about 2% per year both in the US and the G6). In the crisis period, instead, all variables, real and financial, contracted sharply. This feature is not unique to the 2007-2009 crisis. Several authors have observed that many historical episodes of credit booms are not associated with much faster growth in real economic activity. However, when the credit booms reach a sudden stop, their reversals are often associated with sharp macroeconomic contractions. See, for example, Reinhart and Rogoff (2009), Classens, Kose and Terrones (2011), Schularick and Taylor (2011).

The facts presented in this section—high international co-movement in real and financial variables during the crisis, the large employment (for the US) and stock markets collapse, and the asymmetry between the pre-crisis phase and post-crisis phase—cannot be easily explained with a standard workhorse international business cycle model (see, for example, Heathcote and Perri (2004)). In the next sections we propose a theoretical framework with credit disturbances that helps us understanding these facts.

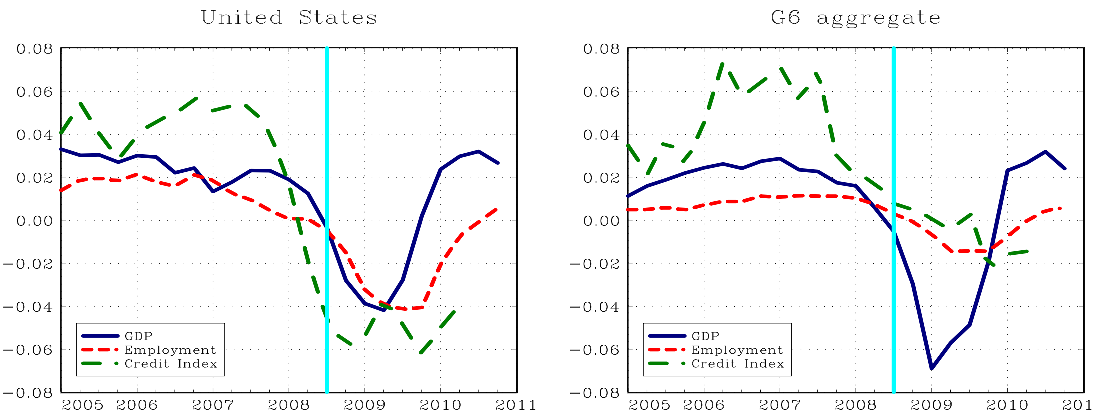

Figure 5: Domestic co-movement of credit, employment and GDP: 2005-2010. The credit series have been rescaled (by a 0.033 factor) such that they are visually comparable to employment and GDP series. The vertical line denotes the third quarter of 2008 when the Lehman's bankruptcy took place.

3 Model with fixed capital and exogenous credit shocks

We start with a simple model without capital accumulation and with exogenous credit shocks. This allows us to provide analytical intuitions for the key transmission mechanism of these shocks. After the presentation of the simple model, we will make the credit shock endogenous as the outcome of self-fulfilling equilibria. We then generalize the model with capital accumulation.

There are two types of atomistic agents, investors and workers. Based on the assumption of market segmentation, only investors have access to the ownership of firms while workers can only save in the form of bonds. We further assume that they have different discount factors: β for investors and $\delta > \beta$ for workers. As we will see, the higher discounting of investors implies that in equilibrium firms borrow from workers.[5] To facilitate the presentation we first describe the closed-economy version of the model.

3.1 Investors and firms

Investors have lifetime utility $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$. They are the owners of firms and can trade shares with other investors. Since investors are homogeneous and they earn only capital incomes from the ownership of firms' shares, their consumption is equal to the

[5]There are several approaches proposed in the literature that introduce a borrowing incentive for firms: tax deductability of interest payments, uninsurable idiosyncratic risks, bargaining of wages, etc. For our purpose, however, the distinction between these more micro-founded approaches is not important. Therefore, we simply assume different discount factors which we interpret as capturing, in reduced form, all of these mechanisms.

dividends paid by firms.[6] Denoting by d_t the dividends, the effective discount factor for investors is $m_{t+1} = \beta u_c(d_{t+1})/u_c(d_t)$. This is also the discount factor for firms since they maximize shareholders' wealth. As we will see, fluctuations in the effective discount factor play a central role for the transmission of shocks.

Firms operate the production function $F(h_t) = \bar{k}h_t^{\nu}$, where $\bar{k}$ is a fixed input of capital and h_t is the variable input of labor. The parameter ν is smaller than 1 implying decreasing returns to scale in the variable input. In this version of the model without capital accumulation we can think of $\bar{k}$ as a normalizing constant.

Firms start the period with *intertemporal* debt b_t. Before producing they choose labor input h_t, dividends d_t, and next period debt b_{t+1}. The budget constraint is

$$b_t + w_t h_t + d_t = F(h_t) + \frac{b_{t+1}}{R_t}, \tag{1}$$

where R_t is the gross interest rate.

The payments of wages, $w_t h_t$, dividends, d_t, and current debt net of the new issue, $b_t - b_{t+1}/R_t$, are made before the realization of revenues. This implies that the firm faces a cash flow mismatch during the period. The cash needed at the beginning of the period is $w_t h_t + d_t + b_t - b_{t+1}/R_t$. To cover the cash flow mismatch, the firm contracts the intra-period loan $l_t = w_t h_t + d_t + b_t - b_{t+1}/R_t$ which is repaid at the end of the period, after the realization of revenues. From the budget constraint (1) we can see that the intra-period loan l_t is equal to the revenue $F(h_t)$.[7]

Debt contracts are not perfectly enforceable as the firm can default. Default takes place at the end of the period before repaying the intra-period loan. At this stage the firm holds the revenue $F(h_t)$ which is equal to the intra-period loan l_t. The revenue represents liquid funds that can be easily diverted in the event of default. Default gives the lender the right to liquidate the firm's assets. But after the diversion of $l_t = F(h_t)$, the only remaining asset is the physical capital $\bar{k}$. Suppose that the liquidation value of capital is $\xi_t \bar{k}$, where ξ_t is stochastic. Since default arises at the end of the period, the total liabilities of the firm are $l_t + b_{t+1}/R_t$. To ensure that the firm does not default, the total liabilities are subject to the enforcement constraint

$$l_t + \frac{b_{t+1}}{R_t} \leq \xi_t \bar{k}. \tag{2}$$

A formal derivation of this constraint is provided in Appendix A and it is based on similar assumptions as in Hart and Moore (1994).

[6] The assumption that investors only hold firms' shares and they cannot borrow or save in the form of bonds is without loss of generality. Borrowing and/or savings will be done on their behalf by firms.

[7] In alternative to using intra-period loans we could assume that firms carry to the next period the cash accrued from the sale of their products. Although the model would have similar properties, the explicit consideration of cash would complicate the numerical solution by adding another state variable.

Fluctuations in ξ_t affect the ability to borrow and, as we will see, they generate pro-cyclical movements in real and financial variables. Our ultimate goal is to derive the variable ξ_t endogenously from liquidity considerations, as we will do in Section 4. In that section, the variable ξ_t represents the equilibrium liquidation price of capital which fluctuates as the economy switches from one self-fulfilling equilibrium to the other. For the moment, however, we treat ξ_t as an exogenous stochastic variable.[8]

To illustrate the role played by fluctuations in ξ_t, consider a pre-shock equilibrium in which the enforcement constraint is binding. Starting from this equilibrium, suppose that ξ_t decreases. This forces the firm to reduce either the dividends and/or the input of labor. To see this, let's start with the case in which the firm does not change the input of labor h_t, which implies that the intra-period loan also does not change because $l_t = w_t h_t + d_t + b_t - b_{t+1}/R_t = F(h_t)$. Consequently, the only way to satisfy the enforcement constraint, equation (2), is by reducing the intertemporal debt b_{t+1}. We can then see from the budget constraint, equation (1), that the reduction in b_{t+1} requires a reduction in dividends. Thus, the firm is forced to substitute debt with equity.

Alternatively, the firm could keep the dividends unchanged and reduce the intra-period loan $l_t = F(h_t)$. This would also ensure that the enforcement constraint is satisfied but it requires the reduction in the input of labor. Therefore, after a reduction in ξ_t, the firm faces a trade-off: paying lower dividends or cutting employment. The optimal choice depends on the relative cost of changing these two margins which, as we will see, depends on the stochastic discount factor for investors $m_{t+1} = \beta u_c(d_{t+1})/u_c(d_t)$.

Firm's problem: The optimization problem of the firm can be written recursively as

$$V(\mathbf{s}; b) = \max_{d,h,b'} \left\{ d + Em'V(\mathbf{s}'; b') \right\} \tag{3}$$

subject to:

$$b + d = F(h) - wh + \frac{b'}{R} \tag{4}$$

$$F(h) + \frac{b_{t+1}}{R_t} \leq \xi \bar{k}, \tag{5}$$

where $\mathbf{s}$ are the aggregate states, including the shock ξ, and the prime denotes the next period variable. The enforcement constraint takes into account that the intra-period loan is equal to the firm's output, that is, $l_t = w_t h_t + d_t + b_t - b_{t+1}/R_t = F(h_t)$.

[8]Movements in the liquidation price are consistent with Eisfeldt and Rampini (2006) who provide evidence that the liquidity of capital must be procyclical to match the observed reallocation of capital.

The firm takes as given all prices and the first order conditions are

$$F_h(h) = \frac{w}{1-\mu}, \tag{6}$$

$$REm' = 1 - \mu, \tag{7}$$

where μ is the Lagrange multiplier for the enforcement constraint. These conditions are derived under the assumption that dividends are always positive, which will be the case if the investors' utility satisfies $u_c(0) = \infty$. The detailed derivation is in Appendix B.

We can see from condition (6) that there is a wedge in the demand for labor if the enforcement constraint is binding, and therefore, $\mu > 0$. This derives from the fact that labor needs to be financed and part of the financing has to come from equity (lower payment of dividends). As long as the cost of equity, $1/Em'$, is greater than the cost of debt, R, expanding the input of labor is costly in the margin because the firm needs to substitute debt with equity. It is then the return differential $1/Em' - R$ that determines the labor wedge as can be seen from condition (7).[9] The wedge is strictly increasing in μ and disappears when $\mu = 0$, that is, when the enforcement constraint is not binding.

Some partial equilibrium properties: The characterization of the firm's problem in partial equilibrium provides helpful insights about the property of the model once extended to a general equilibrium set-up. For partial equilibrium we mean the allocation achieved when the interest and wage rates are both exogenously given and constant.

A decrease in ξ makes the enforcement constraint tighter. Because all firms reduce the payment of dividends (since they are homogeneous), the aggregate consumption of investors decreases. This induces a decline in the discount factor $m' = \beta u_c(d')/u_c(d)$ and an increase in the multiplier μ (from equation (7)). From condition (6) we can then see that the demand for labor declines.

Intuitively, when the credit conditions become tighter, firms need to rely more on equity financing and less on debt. This requires investors to cut consumption (dividends) which is costly due to the concavity of the utility function. Because of this, in the short-term firms do not raise enough equity needed to keep the pre-shock production scale and cut employment. If investors' utility were linear (risk-neutrality), the discount factor would be equal to $Em' = \beta$ and the credit shock would not affect employment. This also requires that the interest rate does not change, which is the case in the partial equilibrium considered here.

In the general equilibrium, of course, prices do change. In particular, movements in the demand of credit and labor affect the interest rate R and the wage rate w. To derive

[9] We can term the differential 'equity premium'. However, the equity premium is not only determined by the price of risk (risk premium) since shareholders and bondholders are different agents.

the aggregate effects we need to close the model and characterize the general equilibrium but, as we will see, the mechanism described here is not offset by movements in prices.

3.2 Closing the model and general equilibrium

There is a representative household/worker with lifetime utility $E_0 \sum_{t=0}^{\infty} \delta^t U(c_t, h_t)$, where c_t is consumption, h_t is labor and δ is the intertemporal discount factor. It will be convenient to assume that the period-utility takes the form

$$U(c_t, h_t) = \log(c_t) - \alpha \frac{h_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}.$$

The worker's budget constraint is $w_t h_t + b_t = c_t + \frac{b_{t+1}}{R_t}$, and the first order conditions for labor, h_t, and next period bonds, b_{t+1}, are

$$\alpha h_t^{\frac{1}{\eta}} c_t = w_t, \tag{8}$$

$$\delta R_t E_t \left\{ \frac{U_c(c_{t+1}, h_{t+1})}{U_c(c_t, h_t)} \right\} = 1. \tag{9}$$

We can now define a competitive general equilibrium. The aggregate states, denoted by $\mathbf{s}$, are given by the credit conditions ξ and the aggregate stock of bonds B.

Definition 3.1 (Recursive equilibrium) *A recursive competitive equilibrium is defined by a set of functions for (i) workers' policies $h_w(\mathbf{s})$, $c_w(\mathbf{s})$, $b_w(\mathbf{s})$; (ii) firms' policies $h(\mathbf{s}; b)$, $d(\mathbf{s}; b)$, $b(\mathbf{s}; b)$; (iii) firms' value $V(\mathbf{s}; b)$; (iv) aggregate prices $w(\mathbf{s})$, $R(\mathbf{s})$, $m(\mathbf{s}')$; (v) law of motion for the aggregate states $\mathbf{s}' = \Psi(\mathbf{s})$; such that: (i) household's policies satisfy the optimality conditions (8)-(9); (ii) firms' policies are optimal and $V(\mathbf{s}; b)$ satisfies the Bellman's equation (3); (iii) the wage and interest rates are the clearing prices in the markets for labor and bonds, and the discount factor for firms is $m(\mathbf{s}') = \beta u_c(d_{t+1})/u_c(d_t)$; (iv) the law of motion $\Psi(\mathbf{s})$ is consistent with the aggregation of individual decisions and the stochastic processes for z and ξ.*

To illustrate some of the key properties of the model, we look at the special case without uncertainty, that is, ξ is constant. In this economy the enforcement constraint binds in a steady state equilibrium. To see this, consider the first order condition for the bond, equation (9), which in a steady state becomes $\delta R = 1$. Using this condition to eliminate R in (7) and taking into account that in a steady state $Em' = \beta$, we get $\mu = 1 - \beta/\delta > 0$ (since $\delta > \beta$). Firms would like to borrow as much as possible because the interest rate is smaller than their discount rate.

With uncertainty, however, the enforcement constraint may be binding only occasionally. In particular, after a large and unexpected decline in ξ. In this case firms will be forced to cut dividends inducing a change in the discount factor Em'. Furthermore, the change in the demand for credit affects the equilibrium interest rate. Using condition (7) we can see that these changes affect the multiplier μ, which in turn impacts on the demand for labor (see equation (6)). On the other hand, an increase in ξ may leave the enforcement constraint non-binding without direct effects on the demand of labor. Therefore, the responses to credit shocks could be asymmetric: negative shocks induce large falls in employment and output while the impact of positive shocks is moderate.

3.3 Capital mobility

Let's consider now two countries, domestic and foreign, with the same size, preferences and technology as described in the previous section. Although we consider the case with only two symmetric countries, the model can be easily extended to any number of countries and with different degrees of heterogeneity. For the moment we continue to assume that ξ_t is an 'exogenous' stochastic variable, specific to each country.

Once we allow for cross-country capital mobility, we have to specify what agents can do in an integrated financial market. We continue to assume that there is market segmentation in the ownership of firms, that is, workers are unable to purchase shares of firms. However, in addition to domestic bonds they can purchase foreign bonds. Furthermore, investors are now able to purchase shares of foreign firms.

Investors/firms: Because firms are subject to country-specific shocks, investors would gain from diversifying the cross-country ownership of shares. Therefore, in an economy that is financially integrated, investors choose to own the worldwide portfolio of shares (representative 'worldwide' investor).[10] Because domestic and foreign firms are owned by the same representative shareholder, they will use the same discount factor $m_{t+1} = \beta u_c(d_{t+1} + d^*_{t+1})/u_c(d_t + d^*_t)$, where investors' consumption is the sum of dividends paid by domestic firms, d_t, plus the dividends paid by foreign firms, d^*_t. From now on we will use the star superscript to denote variables pertaining to the foreign country.

Besides the common discount factor, firms continue to solve problem (3) and the first order conditions are given by equations (6) and (7). Let's focus on condition (7), which we rewrite here for both countries,

$$\begin{aligned} R_t E m_{t+1} &= 1 - \mu_t, \\ R^*_t E m^*_{t+1} &= 1 - \mu^*_t. \end{aligned}$$

[10]Perfect diversification of portfolios is optimal because investors' utility depends only on consumption. If they derived utility also from leisure, perfect diversification would not be necessarily optimal.

Since the discount factor is common to domestic and foreign firms, that is, $Em_{t+1} = Em^*_{t+1}$, and the interest rate is equalized across countries, $R_t = R^*_t$, the lagrange multipliers are also equalized, that is, $\mu_t = \mu^*_t$. Therefore, independently of which country is hit by a shock, if the enforcement constraint is binding for domestic firms, it will also be binding for foreign firms. We state this property formally in the following proposition.

Proposition 3.1 *In a financially integrated economy, the enforcement constraint is either binding in both countries or non-binding in both countries.*

Proof 3.1 *The proof follows directly from the first order conditions above.*

The equalization of the multiplier also implies that the labor wedges in the two countries are equal. In fact, condition (6) is still the optimality condition for the choice of labor in both countries, that is,

$$F_h(h_t) = w_t\left(\frac{1}{1-\mu_t}\right),$$
$$F_h(h^*_t) = w^*_t\left(\frac{1}{1-\mu^*_t}\right).$$

This property is crucial for understanding the cross-country impact of a credit shock.

Households/workers: Although workers are still prevented from accessing the market for the ownership of firms, with capital mobility they can lend to both domestic and foreign firms. Furthermore, they can engage in international financial transactions with foreign workers. In particular, we assume that domestic workers can trade state-contingent claims with foreign workers. However, we continue to assume that firms cannot trade state contingent claims with workers, that is, the repayment of bonds is unconditional. This assumption is essential to retain market incompleteness.

Denote by $n_{t+1}(\mathbf{s}_{t+1})$ the units of consumption goods received at time $t+1$ by domestic workers if the aggregate states are $\mathbf{s}_{t+1}$. These are worldwide states, and therefore, they include the aggregate states of both countries as will be made precise below. Of course, in equilibrium, the consumption units received by workers in the domestic country must be equal to the consumption units paid by workers in the foreign country, that is, $n_{t+1}(\mathbf{s}_{t+1}) + n^*_{t+1}(\mathbf{s}_{t+1}) = 0$ for all possible realizations of the aggregate states $\mathbf{s}_{t+1}$.

The budget constraint of a worker in the domestic country is

$$w_t h_t + b_t + n_t = c_t + \frac{b_{t+1}}{R_t} + \int_{\mathbf{s}_{t+1}} n_{t+1}(\mathbf{s}_{t+1})q(\mathbf{s}_{t+1})/R_t,$$

where $q_t(\mathbf{s}_{t+1})/R_t$ is the unit price of the contingent claims.

Given the specification of the utility function, the first order conditions for the choice of labor, h_t, next period bonds, b_{t+1}, and foreign claims, $n_{t+1}(\mathbf{s}_{t+1})$, are

$$\alpha h_t^{\frac{1}{\eta}} c_t = w_t, \tag{10}$$

$$\delta R_t E_t \left(\frac{c_t}{c_{t+1}} \right) = 1, \tag{11}$$

$$\delta R_t \left(\frac{c_t}{c_{t+1}(\mathbf{s}_{t+1})} \right) p(\mathbf{s}_{t+1}) = q(\mathbf{s}_{t+1}), \qquad \text{for all } \mathbf{s}_{t+1}, \tag{12}$$

where $p(\mathbf{s}_{t+1})$ is the probability (or probability density) of the aggregate states in the next period for the world economy.

Since in equilibrium the prices and probabilities of the contingent claims are the same for domestic and foreign workers, condition (12) implies

$$\frac{c_t}{c_t^*} = \frac{c_{t+1}(\mathbf{s}_{t+1})}{c_{t+1}^*(\mathbf{s}_{t+1})}. \tag{13}$$

Therefore, the ratio of consumptions for domestic and foreign workers remains constant over time. We denote this constant ratio by χ. This is a well known property in environments with a full set of state-contingent claims. In our environment the constancy of the consumption ratio is among workers (and among investors) but not between workers and investors because of the assumption of market segmentation.

Before continuing we would like to clarify that the assumption of contingent claims among workers is not essential for the results of the paper. We could simply assume that workers can engage in international non-contingent lending and borrowing. However, the availability of contingent claims greatly simplifies the characterization of the equilibrium because it allows us to reduce the number of 'sufficient' state variables. As we will see, this property will be convenient in the extension of the model with capital accumulation.

Aggregate states and equilibrium: We can now define the equilibrium for the open-economy version of the model. The aggregate states $\mathbf{s}$ are given by the variables ξ and ξ^*, the financial liabilities of firms, B_t and B_t^*, and the net foreign asset position of the domestic country, N_t. Since in equilibrium the net foreign asset position of the domestic country is the negative of the asset position of the foreign country, once we know B_t, B_t^* and N_t we also know the total wealth of domestic workers, $B_t + N_t$, and foreign workers, $B_t^* - N_t$. Therefore, the sufficient set of state variables is $\mathbf{s}_t = (\xi, \xi^*, B_t, B_t^*, N_t)$.

Definition 3.2 (Recursive equilibrium) *A recursive competitive equilibrium is defined by a set of functions for: (i) households' policies* $h_w(\mathbf{s})$, $c_w(\mathbf{s})$, $b_w(\mathbf{s})$, $n_w(\mathbf{s};\mathbf{s}')$,

$h_w^*(\mathbf{s})$, $c_w^*(\mathbf{s})$, $b_w^*(\mathbf{s})$, $n_w^*(\mathbf{s};\mathbf{s}')$; (ii) firms' policies $h(\mathbf{s};b)$, $d(\mathbf{s};b)$, $b(\mathbf{s};b)$, $h^*(\mathbf{s};b)$, $d^*(\mathbf{s};b)$, $b^*(\mathbf{s};b)$; (iii) firms' values $V(\mathbf{s};b)$ and $V^*(\mathbf{s};b)$; (iv) aggregate prices $w(\mathbf{s})$, $w^*(\mathbf{s})$, $R(\mathbf{s})$, $m(\mathbf{s},\mathbf{s}')$, $q(\mathbf{s};\mathbf{s}')$; (v) law of motion for the aggregate states $\mathbf{s}' = \Psi(\mathbf{s})$; such that: (i) household's policies satisfy the optimality conditions (8)-(12); (ii) firms' policies are optimal and satisfy the Bellman's equation (3) for both countries; (iii) the wages clear the labor markets; the interest rates and the price for contingent claims clear the worldwide financial markets; the discount rate of firms satisfies $m(\mathbf{s},\mathbf{s}') = \beta u_c(d_{t+1}+d^*_{t+1})/u_c(d_t + d^*_t)$; (iv) the law of motion $\Psi(\mathbf{s})$ is consistent with the aggregation of individual decisions and the stochastic process for ξ and ξ^*.

The only difference with respect to the equilibrium in the closed economy is that there is the additional market for foreign claims and the discount factor for firms is given by the worldwide representative investor. The market clearing condition for the foreign claims is $N(\mathbf{s}') + N^*(\mathbf{s}') = 0$. This is in addition to the clearing conditions for the bond market (lending to firms).

Although the general definition of the recursive equilibrium is based on the set of state variables $\mathbf{s}_t = (\xi_t, \xi_t^*, B_t, B_t^*, N_t)$, we can use some of the properties derived above and characterize the equilibrium with a smaller set of states. Let $W_t = B_t + B_t^*$ be the worldwide wealth of households/workers. This is the sum of bonds issued by domestic firms, B_t, and foreign firms, B_t^*. Using the fact that the consumption ratio of domestic and foreign workers is constant at χ and the employment policy of firms does not depend on the individual debt, the recursive equilibrium can be characterized by the state vector $\mathbf{s}_t = (\xi_t, \xi_t^*, W_t)$. The assumption of cross-country risk-sharing among workers and investors (but not between workers and investors) allows us to reduce the number of *endogenous* states to only one variable.

Intuitively, by knowing W_t, we know the worldwide liability of firms, but not the distribution between domestic and foreign firms. However, to characterize the firms' policies, we only need to know the worldwide debt, which is equal to W_t. Since investors own an internationally diversified portfolio of shares, effectively there is only one representative global investor. It is as if there is a representative firm with two productive units: one unit located in the domestic country and the other in the foreign country. Since both units have a common owner, it does not matter how the debt is distributed between the two units. What matters from the perspective of the investor is the total debt and the total payment of dividends.[11]

Total workers' wealth is also a sufficient statistic for the characterization of the workers' policies since the consumption ratio between domestic and foreign households

[11]This is similar to the problem solved by a multinational firms that faces demand uncertainty in different countries as studied in Goldberg and Kolstad (1995). There is also some similarity with the problem faced by multinational banks that own subsidiaries in different countries. Cetorelli and Goldberg (2010) provide evidence that multinational banks do reallocate financial resources internally in response to country-specific shocks.

remains constant at χ. This property limits the computational complexity of the model, making the use of non-linear approximation methods practical.

We are now ready to characterize the macroeconomic impact of a credit shock.

Proposition 3.2 *An unexpected change in ξ_t (domestic credit shock) has the same impact on employment and output of domestic and foreign countries.*

Proof 3.2 *We have already shown that the Lagrange multiplier μ_t is common for domestic and foreign firms. If the ratio of wages in the two countries does not change, the first order conditions imply that all firms choose the same employment. To complete the proof we have to show that the cross-country wage ratio stays constant. Because firms in both countries have the same demand for labor and the ratio of workers' consumption remains constant, the first order condition for the supply of labor from workers (equation (10)) implies that the wage ratio between the two countries does not change.*

Thus, independently of whether a credit shock hits the domestic or foreign markets, both countries experience the same macroeconomic consequences. In this way the model generates international co-movement in response to country-specific credit shocks even if these shocks are not correlated across countries.

4 Endogenous credit shocks

Although exogenous credit shocks can explain co-movement in GDP and other real variables, there are two limitations. The first is that treating changes to the availability of credit as exogenous does not help us understanding the causes of these changes and the policies that could reduce the frequency of the credit shocks. The second and more specific limitation is that with exogenous credit shocks the model predicts that financial flows tend to move in opposite directions across countries.

To show that financial flows are negatively correlated across countries, consider an initial equilibrium in which the enforcement constraint is not binding in either countries. Starting from this equilibrium, suppose that only the domestic economy is hit by a credit contraction (reduction in ξ_t but not in ξ_t^*) inducing binding enforcement constraints in both countries. Since ξ_t is lower only in the domestic country, the debt of domestic firms contracts but the debt of foreign firms increases. Foreign firms increase their borrowing to pay more dividends to shareholders in order to offset the reduction in dividends paid by domestic firms. Therefore, the model with 'exogenous' credit shocks tends to generate negative cross-country co-movement in credit.

This feature of the model is inconsistent with Figure 4 which shows a high degree of cross-country co-movement in credit during the recent crisis. However, once we make fluctuations in ξ_t and ξ_t^* endogenous, the model also generates positive co-movement in financial flows, introducing a second source of real macroeconomic synchronization. Let's start by stating the two key assumptions under which the model can generate endogenous fluctuations in the availability of credit.

Assumption I *In case of liquidation, the firm's capital* $\bar{k}$ *is perfectly divisible and can be sold to households or firms. Households have the ability to transform one unit of reallocated capital in* $\underline{\xi}$ *units of consumption goods. Firms have the ability to transform (produce) one unit of reallocated capital in* $\overline{\xi} > \underline{\xi}$ *units of consumption goods.*

Thus, in the event of liquidation, the reallocation of capital to other firms, as opposed to households, is more efficient. We also assume that $\overline{\xi}$ is sufficiently small so that the value of a firm is always bigger than the liquidation value of its capital, $\overline{\xi}\bar{k}$.

Before proceeding it is important to clarify the nature of this assumption and specifically that one unit of capital transferred to another firm can be transformed in $\overline{\xi}$ units of consumption. Perhaps, a more plausible assumption would be that in case of liquidation only a fraction $\overline{\xi}$ of capital can be reinstalled for future production in other firms. In this case the value or price of the reinstalled capital would be the marginal increase in the value of the firm with respect to the added capital, that is,

$$\xi_t = \overline{\xi}\left(\frac{\partial E m_{t+1} V(\mathbf{s}_{t+1}; b_{t+1})}{\partial \bar{k}}\right).$$

By assuming that firms have the ability to transform one unit of reallocated capital in $\overline{\xi}$ units of consumption we are effectively assuming that the term in parenthesis is 1.

The reason we did not use this alternative assumption is because it would complicate the characterization of the general equilibrium but would not change the key qualitative properties of the model (something that we checked numerically). However, when we extend the model with capital accumulation in Section 5, the term in parenthesis would actually be 1 if there are not convex adjustment costs in investment. It would also be 1 if we assume that only the instalment of newly produced investment is subject to the convex adjustment cost, while the cost to re-install previously accumulated capital is the loss of the fraction $1 - \overline{\xi}$. With this in mind, the simplifying assumption made here does not alter in important ways the plausibility of the model.

Assumption II *The purchase of liquidated capital needs to be financed.*

What this means is that firms can buy the capital liquidated by other firms only if they have liquid funds, that is, they have the ability to borrow more. To better understand this assumption, consider the enforcement constraint

$$\xi_t \bar{k} \geq F(h_t) + \frac{b_{t+1}}{R_t}, \tag{14}$$

where now ξ_t is the expected end-of-period value of price of liquidated capital.

If at the beginning of the period firms choose to borrow less than the limit, that is, the enforcement constraint is not binding, they have the option to raise additional funds at the end of the period to purchase the capital of defaulting firms. Therefore, ex-post there will be firms that have the ability to purchase the capital of a defaulting firm. In this case the market price of liquidated capital is $\overline{\xi}$. However, if firms choose to borrow up to the limit at the beginning of the period, at the end of the period there will not be any firm with liquidity (unused credit) to purchase the capital of defaulting firms. Thus, the liquidated capital can only be sold to households and the price will be $\underline{\xi}$.[12]

Under assumptions I and II, the value of liquidated capital depends on the financial choice of firms, which in turn depends on the expected liquidation value. This interdependence allows the model to generate multiple self-fulfilling equilibria. To provide the intuition for the multiplicity of equilibria, consider the following example.

Suppose that the expected liquidation price is $\xi_t = \underline{\xi}$. The low price makes the enforcement constraint (14) tighter, which may induce firms to borrow up to the limit in order to contain the reduction in dividends and/or employment. Then, if all firms borrow up to the limit, there will not be any firm, ex-post, that has the liquidity to purchase the capital of defaulting firms. Thus, the ex-post liquidation price will be $\underline{\xi}$, fulfilling the market expectation.

Now suppose that the expected liquidation price is $\overline{\xi}$. Because the enforcement constraint (14) is not tight in the current period but could become tight in the future, firms may choose not to borrow up to the limit. But then, in case of liquidation, there will be firms with the liquidity to purchase the liquidated capital and the market price will be $\overline{\xi}$. So also in this case we have that the ex-ante expectation of a high liquidation price will be fulfilled by the firms' borrowing choice.

Whether both equilibria with tight and loose credit are possible depends on the aggregate state of the economy and on the particular capital regime (autarky versus financial integration). We thus proceed by first characterizing the equilibria in autarky.

4.1 Autarky regime

In this version of the model the aggregate states $\mathbf{s}_t$ are simply equal to the aggregate stock of debt B_t. Three cases are possible:

1. The liquidation price is $\underline{\xi}$ with probability 1. This arises if we are in a state $\mathbf{s}_t$ in which firms borrow up to the limit independently of the expected price ξ_t.

[12] The purchased capital cannot act as a collateral since the firm transforms it in consumption goods which are then sold for liquid funds. Since liquid funds are divertible, creditors have not viable means to force the borrower to pay back these funds.

2. The liquidation price is $\overline{\xi}$ with probability 1. This arises if we are in a state $\mathbf{s}_t$ in which firms do not borrow up to the limit independently of the expected price ξ_t.

3. The liquidation price is $\underline{\xi}$ with some probability $\bar{p} \in (0,1)$. This arises if we are in a state $\mathbf{s}_t$ in which firms borrow up to the limit when the expected price is $\xi_t = \underline{\xi}$ but they do not borrow up to the limit when the expected price is $\xi_t = \overline{\xi}$.

The third case allows for sunspot equilibria, and therefore, fluctuations in ξ_t. Denote by $\varepsilon \in \{0,1\}$ a non-fundamental shock (sunspot). This variable takes the value of zero with probability $\bar{p} \in (0,1)$ and 1 with probability $1-\bar{p}$, and it is serially uncorrelated. We then define $p(\mathbf{s}_t)$ the probability of an equilibrium with binding enforcement constraints and low liquidation price $\xi_t = \underline{\xi}$ (tight credit equilibrium).

Definition 4.1 (Recursive equilibria for given $\bar{p}$) *A recursive competitive equilibrium for given $\bar{p} \in (0,1)$ is defined as a set of functions for: (i) households' policies $h_w(\mathbf{s},\xi)$, $c_w(\mathbf{s},\xi)$, $b_w(\mathbf{s},\xi)$; (ii) firms' policies $h(\mathbf{s},\xi;b)$, $d(\mathbf{s},\xi;b)$, $b(\mathbf{s},\xi;b)$; (iii) firms' value $V(\mathbf{s},\xi;b)$; (iv) aggregate prices $w(\mathbf{s},\xi)$, $R(\mathbf{s},\xi)$, $m(\mathbf{s},\xi,\mathbf{s}',\xi')$; (v) liquidation price $\xi(\mathbf{s},\varepsilon) \in \{\underline{\xi},\overline{\xi}\}$ and probability of low price equilibria $p(\mathbf{s})$; (vi) law of motion for the aggregate states $\mathbf{s}' = \Psi(\mathbf{s},\xi)$; such that: (i) household's policies satisfy the optimality conditions (8)-(12); (ii) firms' policies are optimal and satisfy the Bellman's equation (3); (iii) the wage and interest rate clear the labor and credit markets; the discount factor used by firms satisfies $m(\mathbf{s},\varepsilon,\mathbf{s}',\varepsilon') = \beta u_c(d_{t+1})/u_c(d_t)$; (iv) the probability of low price equilibria is consistent with individual firms' policies and liquidity requirement, that is,*

$$p(\mathbf{s}) = \begin{cases} 0, & \text{if} \quad F(h(\mathbf{s},\xi;B)) + \frac{b(\mathbf{s},\xi;B)}{R(\mathbf{s},\xi)} < \xi\bar{k} \quad \text{for all} \quad \xi \in \{\underline{\xi},\overline{\xi}\} \\ \bar{p}, & \text{if} \quad F(h(\mathbf{s},\overline{\xi};B)) + \frac{b(\mathbf{s},\overline{\xi};B)}{R(\mathbf{s},\overline{\xi})} < \overline{\xi}\bar{k} \quad \text{and} \quad F(h(\mathbf{s},\underline{\xi};B)) + \frac{b(\mathbf{s},\underline{\xi};B)}{R(\mathbf{s},\underline{\xi})} = \underline{\xi}\bar{k} \\ 1, & \text{if} \quad F(h(\mathbf{s},\xi;B)) + \frac{b(\mathbf{s},\xi;B)}{R(\mathbf{s},\xi)} = \xi\bar{k} \quad \text{for all} \quad \xi \in \{\underline{\xi},\overline{\xi}\} \end{cases};$$

(v) the law of motion $\Psi(\mathbf{s})$ is consistent with the aggregation of individual decisions and the stochastic process for ε.

The next proposition establishes the existence of sunspot equilibria, that is, the existence of states $\mathbf{s}$ for which the prices $\underline{\xi}$ and $\overline{\xi}$ could both emerge in equilibrium.

Proposition 4.1 *Denote by ε a shock that takes the value of 0 with probability $\bar{p} \in (0,1)$ and 1 with probability $1-\bar{p}$. If $\overline{\xi} - \underline{\xi}$ is sufficiently large, the economy displays sunspot equilibria: it never converges to a steady state with a unique liquidation price but switches stochastically between equilibria with $\xi_t = \underline{\xi}$ and equilibria with $\xi_t = \underline{\xi}$.*

Proof 4.1 *See Appendix C.*

The proof of the proposition, provided in the appendix, has a simple intuition. If sunspot equilibria do not exist, then $p(\mathbf{s}_t)$ is either always equal to 1 (implying that the liquidation price is always $\xi_t = \underline{\xi}$) or always equal to 0 (implying that the liquidation price is always $\xi_t = \overline{\xi}$). But if the liquidation price is constant, the economy is deterministic and converges to a steady state. In the steady state the enforcement constraint is binding (because of the higher discounting of firms) and the price of liquidated capital must be $\xi_t = \underline{\xi}$ for all t. Starting from this equilibrium, if agents expect $\xi_t = \overline{\xi}$, the enforcement constraint becomes non-binding (provided that $\overline{\xi} - \underline{\xi}$ is sufficiently large) and the expectation of $\xi_t = \overline{\xi}$ moves the economy away from the steady state with $\xi_t = \underline{\xi}$. On the other hand, an equilibrium with enforcement constraints that are never binding is not feasible. In fact, if the enforcement constraint is never binding, $\xi_t = \overline{\xi}$ for all t and the economy converges to a steady state since there is not uncertainty. But in a steady state the enforcement constraint is binding (again because of the higher discounting of firms), violating the hypothesis that the enforcement constraint is never binding. Therefore, the economy must reach a set of states where the enforcement constraint becomes binding stochastically. The stochastic switch is caused by the draw of the non-fundamental shock ε_t. The only case in which the economy can converge to a deterministic steady state is when $\bar{p} = 1$.

4.2 Financial integration

As in the closed economy, different values of ξ_t are associated with self-fulfilling expectations. We assume that each country receives a draw of the non-fundamental shock: $\varepsilon \in \{0, 1\}$ for the domestic country and $\varepsilon^* \in \{0, 1\}$ for the foreign country. These non-fundamental shocks are independent and identically distributed across countries and over time, and the probability of drawing a value of zero is $\bar{p}$ in both countries.

If countries were in financial autarky, equilibria would be as described in the previous section. Since the draws of ε_t and ε^*_t are independent across countries, the liquidation price ξ_t in the domestic country could be different from the price ξ^*_t in the foreign country. With financial autarky, however, ξ_t cannot be different from ξ^*_t.

Lemma 4.1 *In equilibria with integrated financial markets, ξ_t is always equal to ξ^*_t*

Proof 4.1 *Suppose that the equilibrium is characterized by $\xi_t = \underline{\xi}$ and $\xi^*_t = \overline{\xi}$. To have $\xi_t = \underline{\xi}$ we need $\mu_t > 0$ and to have $\xi^*_t = \overline{\xi}$ we need $\mu^*_t = 0$. However, as stated in Proposition 3.1, with integrated financial markets $\mu_t = \mu^*_t$. Using the same argument we can exclude the possibility of an equilibrium with $\xi_t = \overline{\xi}$ and $\xi^*_t = \underline{\xi}$.*

Hence, financial integration implies perfect cross-country co-movement in the liquidation prices ξ_t and ξ_t^*, which introduces a second channel of real macroeconomic synchronization: not only a change in one country ξ affects the real sector of the other country but movements in the liquidation prices become perfectly correlated across countries. As a result, also the financial flows become internationally correlated. This is an important theoretical result of the paper and suggests that the Lehman's default could have been the trigger that switched the world economy from an equilibrium with globally loose credit to an equilibrium with tight credit and shortage of liquidity, causing widespread contraction in economic and financial activities.

What we have not shown yet is whether sunspot equilibria could also arise in the economy with integrated financial markets (for the closed economy the existence of sunspot equilibria was established in Proposition 4.1). This boils down to showing the existence of states $\mathbf{s}$ under which the liquidation prices $\underline{\xi}$ and $\bar{\xi}$ are both possible in equilibrium. As a result, the economy switches stochastically between binding and non-binding enforcement constraints. Before doing so, let's introduce some notation.

The probability of tight credit equilibria with $\xi_t = \xi_t^* = \underline{\xi}$ can still be expressed as a function of the aggregate states, that is, $p(\mathbf{s}_t)$. Now, however, the emergence of equilibria with different liquidations prices can be induced by a change in expectations in only one of the two countries. Depending on the state $\mathbf{s}_t$, we have three cases:

1. The liquidation price is $\underline{\xi}$ with probability 1. This arises if we are in a state in which firms borrow up to the limit independently of the expected price.

2. The liquidation price is $\bar{\xi}$ with probability 1. This arises if we are in a state in which firms do not borrow up to the limit independently of the expected price.

3. The liquidation price is $\underline{\xi}$ only with some probability. Two cases are possible:

    (a) The liquidation price is $\underline{\xi}$ with probability $\bar{p}^2$. This arises if we are in a state $\mathbf{s}_t$ in which firms choose to borrow up to the limit only if the liquidation prices are expected to be low in both countries, that is, $\xi_t = \underline{\xi}$ and $\xi_t^* = \underline{\xi}$. This case is associated with the draws of $\varepsilon_t = 0$ and $\varepsilon_t^* = 0$. Since this happens with probability $\bar{p}$ in each country, the probability of this event is $\bar{p}^2$.

    (b) The liquidation price is $\underline{\xi}$ with probability $2\bar{p}(1-\bar{p}) + \bar{p}^2$. This arises if we are in a state $\mathbf{s}_t$ in which firms borrow up to the limit even if the liquidation price is expected to be low in only one country (for example, $\xi_t = \underline{\xi}$ and $\xi_t^* = \bar{\xi}$ which is associated with $\varepsilon_t = 0$ and $\varepsilon_t^* = 1$). The probability of this event is $\bar{p}(1-\bar{p})$. Cross country symmetry implies that this is also the case if the expectation of a low price arises in the other country. Therefore, the probability that one of the two countries obtain a low realization of ε is $2\bar{p}(1-\bar{p})$. Of course, if firms choose to borrow up to the limit whenever the price is low in one of the two countries, they will also borrow up to the limit if the price is low in both countries, that is, $\xi_t = \underline{\xi}$ and $\xi_t^* = \underline{\xi}$. The probability of this event is $\bar{p}^2$. Therefore, the total probability that firms borrow up to the limit if the liquidation price is low in at least one country is $2\bar{p}(1-\bar{p})+\bar{p}^2$.

The definition of equilibria is similar to the definition provided for the autarky regime with some small adjustments. In particular, individual policies are now functions of both liquidation prices, ξ and ξ^*. For example, the borrowing policy of domestic firms will be denoted as $b(\mathbf{s}, \xi, \xi^*; B)$. Even if in equilibrium the liquidation prices will be equalized across countries, the use of this notation allows for (out of equilibrium) prices that are different across countries. Besides the using of this extended notation, the only change we need to make to Definition 4.1 is in the probability of tight credit equilibria $p(\mathbf{s})$. When countries are financially integrated, this probability is given by

$$p(\mathbf{s}) = \begin{cases} 0, & \text{if } \left\{ \begin{array}{l} F(h(\mathbf{s},\xi,\xi^*;B)) + \frac{b(\mathbf{s},\xi,\xi^*;B)}{R(\mathbf{s},\xi,\xi^*)} < \xi\bar{k} \quad \text{and/or} \\ F(h^*(\mathbf{s},\xi,\xi^*;B)) + \frac{b^*(\mathbf{s},\xi,\xi^*;B)}{R(\mathbf{s},\xi,\xi^*)} < \xi^*\bar{k}, \quad \text{for all } \xi,\xi^* \in \{\underline{\xi},\overline{\xi}\} \end{array} \right. \\ \\ \bar{p}^2, & \text{if } \left\{ \begin{array}{l} F(h(\mathbf{s},\xi,\xi^*;B)) + \frac{b(\mathbf{s},\xi,\xi^*;B)}{R(\mathbf{s},\xi,\xi^*)} = \xi\bar{k} \quad \text{and} \\ F(h^*(\mathbf{s},\xi,\xi^*;B^*)) + \frac{b^*(\mathbf{s},\xi,\xi^*;B^*)}{R(\mathbf{s},\xi,\xi^*)} = \xi^*\bar{k}, \quad \text{for only } \xi,\xi^* = \underline{\xi} \end{array} \right. \\ \\ 2\bar{p}(1-\bar{p})+\bar{p}^2, & \text{if } \left\{ \begin{array}{l} F(h(\mathbf{s},\xi,\xi^*;B)) + \frac{b(\mathbf{s},\xi,\xi^*;B)}{R(\mathbf{s},\xi,\xi^*)} < \xi\bar{k} \quad \text{and/or} \\ F(h^*(\mathbf{s},\xi,\xi^*;B^*)) + \frac{b^*(\mathbf{s},\xi,\xi^*;B^*)}{R(\mathbf{s},\xi,\xi^*)} < \xi^*\bar{k}, \quad \text{for only } \xi,\xi^* = \overline{\xi} \end{array} \right. \\ \\ 1, & \text{if } \left\{ \begin{array}{l} F(h(\mathbf{s},\xi,\xi^*;B)) + \frac{b(\mathbf{s},\xi,\xi^*;B)}{R(\mathbf{s},\xi,\xi^*)} = \xi\bar{k} \quad \text{and} \\ F(h^*(\mathbf{s},\xi,\xi^*;B^*)) + \frac{b^*(\mathbf{s},\xi,\xi^*;B^*)}{R(\mathbf{s},\xi,\xi^*)} = \xi^*\bar{k}, \quad \text{for all } \xi,\xi^* \in \{\underline{\xi},\overline{\xi}\} \end{array} \right. \end{cases}$$

Proposition 4.2 *Denote by ε and ε^* country-specific independent stochastic variables that take the value of 0 with probability $\bar{p}$ and 1 with probability $1-\bar{p}$. If $\overline{\xi} - \underline{\xi}$ is sufficiently large, the economy displays sunspot equilibria: it never converges to a steady state but switches stochastically between equilibria with $\xi_t = \xi_t^* = \underline{\xi}$ and equilibria with $\xi_t = \xi_t^* = \overline{\xi}$.*

Proof 4.2 *It follows the same steps of the proof of Proposition 4.1.*

If we start from a state $\mathbf{s}_t$ in which the liquidation price is unique, then initially the economy does not admit multiple equilibria. This is equivalent to say that initially $p(\mathbf{s}_t)$ is either 0 or 1. However, after a finite number of periods, the state $\mathbf{s}_t$ enters a set where multiple equilibria arise, that is, the economy switches between equilibria with $\xi_t = \xi_t^* = \underline{\xi}$ and equilibria with $\xi_t = \xi_t^* = \overline{\xi}$ with some probability $p(\mathbf{s}_t) \in (0,1)$. The proposition does not establish whether this probability is $2\bar{p}(1-\bar{p})+\bar{p}^2$ or $\bar{p}^2$ since this depends on parameters. The intuition why the economy reaches states with multiple equilibria is analogous to the proof of Proposition 4.1. Because of the higher discounting of investors, firms become constrained at some point in the future even if they are not constrained now. Likewise, if they are constrained, the expectation of a high liquidation price $\xi_t = \xi_t^* = \overline{\xi}$ is sufficient to make the enforcement constraint non-binding, moving the economy to the equilibrium with $\xi_t = \xi_t^* = \overline{\xi}$.

5 Model with capital accumulation

We now relax the assumption that the input of capital is fixed. This introduces additional state variables that increase the computational complexity of the model. Since the enforcement constraint is only occasionally binding, we need to use global approximation techniques. These techniques are computationally intensive and become quickly impractical when we have a large number of state variables. Therefore, in order to reduce the sufficient set of state variables, we will make special assumptions about the production technology which takes the form

$$y_t = (K_t + K_t^*)^{1-\theta} k_t^\theta h_t^\nu.$$

The variable K_t is the 'aggregate' capital in the domestic country and K_t^* in the foreign country. The variables k_t and h_t are, respectively, the 'individual' input of capital and the 'individual' input of labor. It is further assumed that $\theta + \nu < 1$.

The dependence of the production function from the worldwide stock of capital, $K_t + K_t^*$, introduces positive externalities. The purpose of the externalities is to have constant returns in reproducible factors (AK technology), maintaining the competitive structure of the model, that is, each producer runs a production technology with non-increasing returns. As we will see, the AK structure simplifies the computation of the equilibrium and this is the only motivation for this assumption.

Given i_t the flow of investment, the stock of capital evolves according to

$$k_{t+1} = (1-\tau)k_t + \Upsilon\left(\frac{i_t}{k_t}\right)k_t,$$

where τ is the depreciation rate and the function $\Upsilon(.)$ is strictly increasing and concave, capturing adjustment costs in investment. Adjustment costs in investment are common in international macro models to avoid excessive volatility of investments.

The budget constraint of the firm is

$$b_t + d_t + i_t = (K_t + K_t^*)^{1-\theta} k_t^\theta h_t^\nu - w_t h_t + \frac{b_{t+1}}{R_t},$$

and the enforcement constraint

$$\xi_t k_{t+1} \geq (K_t + K_t^*)^{1-\theta} k_t^\theta h_t^\nu + \frac{b_{t+1}}{R_t}.$$

We now take advantage of the AK structure and normalize the model by the worldwide capital $K_t + K_t^*$. Using the tilde sign to denote normalized variables, we rewrite the budget constraint, the law of motion for capital and the enforcement constraint as

$$\tilde{b}_t + \tilde{d}_t + \tilde{i}_t = \tilde{k}_t^\theta h_t^\nu - \tilde{w}_t h_t + \frac{g_t \tilde{b}_{t+1}}{R_t}, \tag{15}$$

$$g_t \tilde{k}_{t+1} = (1-\tau)\tilde{k}_t + \Upsilon\left(\frac{\tilde{i}_t}{\tilde{k}_t}\right)\tilde{k}_t, \tag{16}$$

$$\xi_t g_t \tilde{k}_{t+1} \geq \tilde{k}_t^\theta h_t^\nu + \frac{g_t \tilde{b}_{t+1}}{R_t}. \tag{17}$$

The variable $g_t = (K_{t+1} + K_{t+1}^*)/(K_t + K_t^*)$ is the gross growth rate of worldwide capital and $\tilde{k}_t = k_t/(K_t + K_t^*)$ the normalized individual capital. We will denote by $s_t = K_t/(K_t + K_t^*)$ the aggregate share of capital owned by domestic firms. Since in equilibrium $k_t = K_t$, we also have that $\tilde{k}_t = s_t$.

As in the model without capital accumulation, investors hold an internationally diversified portfolio of shares and firms use the common discount factor $m_{t+1} = \beta[(d_{t+1} + d_{t+1}^*)/(d_t + d_t^*)]^{-\sigma}$. In normalized variables, the discount factor can be rewritten as

$$m_{t+1} = g_t^{-\sigma}\beta\left(\frac{\tilde{d}_{t+1} + \tilde{d}_{t+1}^*}{\tilde{d}_t + \tilde{d}_t^*}\right)^{-\sigma} = g_t^{-\sigma}\tilde{m}_{t+1}.$$

The optimization problem solved by an individual firm is

$$\tilde{V}(\tilde{\mathbf{s}}; \tilde{k}, \tilde{b}) = \max_{\tilde{d}, \tilde{h}, \tilde{i}, \tilde{b}'} \left\{ \tilde{d} + g^{1-\sigma} E\tilde{m}'\tilde{V}(\tilde{\mathbf{s}}'; \tilde{k}', \tilde{b}') \right\} \tag{18}$$

$$\text{subject to (15), (16), (17),}$$

where $\tilde{V}$ is the firm's value normalized by aggregate worldwide capital $K + K^*$, and $\tilde{\mathbf{s}}$ denotes the normalized aggregate states as specified below.

We can now see the analytical convenience of the AK structure. Thanks to this feature of the production technology, we can write the firm's value function as $V_t = (K_t + K_t^*) \cdot \tilde{V}_t$ and rescale the problem of the firm by worldwide capital. By doing so, we do not need to keep track of the aggregate stock of capital. Of course, because we are looking at a general equilibrium, we also need to make sure that the supply of labor does not grow over time. This is the case with the workers' utility specified earlier.

Appendix D derives the first order conditions for the firm. After imposing the equilibrium conditions $k_t = K_t$ and $\tilde{k}_t = s_t$, they can be written as

$$\nu s_t^{\theta} h_t^{\nu-1} = \frac{\tilde{w}_t}{1-\mu_t}, \tag{19}$$

$$g_t^{-\sigma} R_t E\tilde{m}_{t+1} = 1-\mu_t, \tag{20}$$

$$Q_t \Upsilon'\left(\tilde{i}_t\right) = 1, \tag{21}$$

$$Q_t = \xi_t \mu_t + \bar{g}_t^{-\sigma} E\tilde{m}_{t+1}\Big\{(1-\mu_{t+1})\theta s_{t+1}^{\theta-1} h_{t+1}^{\nu} - \tilde{i}_{t+1} + \Big[1-\tau+\Upsilon\left(\tilde{i}_{t+1}\right)\Big] Q_{t+1}\Big\}. \tag{22}$$

Here μ_t is the Lagrange multiplier associated with the enforcement constraint and Q_t is the Lagrange multiplier associated with the law of motion for capital (Tobin's q). We can verify that the stock of capital does not enter any of these equations, which validates the conjecture that the optimal policies are independent of the stock of capital.

The property that the Lagrange multipliers and the labor wedge $1/(1-\mu_t)$ are equalized across countries also applies to this extended version of the model. In fact, from condition (20) we can see that the common discount factor and the equalization of the interest rates across countries imply $\mu_t = \mu_t^*$. Therefore, if the enforcement constraint is binding in one country, it must also be binding in the other.

Aggregate states and equilibrium: Denote by $\tilde{W}_t = \tilde{B}_t s_t + \tilde{B}_t^*(1-s_t)$ the normalized worldwide wealth of households/workers. Because of the normalization described above, we only need to keep track of two 'endogenous' state variables: $\tilde{W}_t$ and s_t. Therefore, compared to the simpler model considered earlier, the introduction of capital accumulation adds only one state variable, that is, the share of worldwide capital owned by domestic firms, s_t.[13] The definition of equilibria with endogenous ξ_t and ξ_t^* and the existence of sunspot equilibria is similar to the model without capital accumulation. By having only two endogenous state variables, it becomes practical to solve the model numerically using global approximation methods. Appendix E reports the list of equilibrium conditions and describes the computational procedure.

[13] This additional state is necessary because of the adjustment cost in investment. In absence of adjustment costs, we could also ignore s_t.

6 Quantitative analysis

In the quantitative application we interpret country 1 as the US and country 2 as representative of the other countries in the group of the seven largest industrialized economies (Canada, Japan, France, Germany, Italy and UK). The period in the model is a quarter.

The discount factor for workers, δ, and the discount factor for investors, β, are set to target an average yearly interest rate of 1.6 percent and an average yearly return on equity of 7 percent. In the deterministic steady state the interest rate is equal to $1/\delta - 1$ and the return on equity is equal to $1/\beta - 1$. In the stochastic economy the relations between the intertemporal discount factors and the average returns are more complex. Therefore, to choose δ and β we follow an iterative procedure where we fix the parameters, solve the model, and check whether the average returns match the targets. The required values are $\delta = 0.996$ and $\beta = 0.986$. Therefore, there is a 1 percent difference between the two discount factors. This is smaller than the equity premium, $5.4\% \div 4 = 1.35\%$. The difference can be attributed to the compensation required by investors for holding risky equity (risk premium).

The utility function of workers takes the form $U(c, h) = \ln(c) - \alpha h^{1+1/\eta}/(1 + 1/\eta)$, where η is the Frisch elasticity of labor supply. We set the elasticity to 0.75 which is between the micro and macro estimates. The parameter α is set so that average working hours are equal to 0.3.

Next we parameterize the production function. The parameter ν is chosen to have an average labor income share of 0.7. Without uncertainty, the fraction of output going to workers in the form of wages is $\nu\beta/\delta$.[14] In the stochastic economy the average labor share is slightly different. Therefore, the choice of ν also requires an iterative procedure. The return to scale for an individual firm is set to $\theta + \nu = 0.9$ which implies $\theta = 0.9 - \nu$.

The stock of capital evolves according to $k' = (1 - \tau)k + \Upsilon(i/k)k$ with

$$\Upsilon\left(\frac{i}{k}\right) = \frac{\phi_1}{1-\zeta}\left(\frac{i}{k}\right)^{1-\zeta} + \phi_2.$$

This functional form is widely used in the literature (see, for example, Jermann (1998)). The parameters ϕ_1 and ϕ_2 are chosen so that in the deterministic steady state $Q = 1$ and $I = \tau K$, which requires $\phi_1 = \tau^{\zeta}$ and $\phi_2 = -\zeta\tau/(1 - \zeta)$. Therefore, there are two parameters that need to be calibrated, τ and ζ. The first is the depreciation rate which we set to $\tau = 0.02$. The second determines the sensitivity of the adjustment cost to investment and we set it to $\zeta = 0.5$.

[14] From the first order condition of labor, equation (6), we derive $wh/F(z, k, h) = \nu(1 - \mu)$, which provides an expression for the labor share. We now use condition (7) to derive an expression for μ. Taking into account that in a deterministic steady state $m' = \beta$ and $R = 1/\delta$, this condition becomes $\beta/\delta = 1 - \mu$. Substituting in the labor share $\nu(1 - \mu)$, we get the expression reported in the main text.

At this point we are left with the parameters that determine the liquidation price its stochastic properties. In addition to the values of $\underline{\xi}$ and $\overline{\xi}$, we have to pin down $\bar{p}$, that is, the probability with which each country draws $\varepsilon = 0$ (which could lead to pessimistic expectations $\xi = \underline{\xi}$ in states with multiple equilibria). We choose $\underline{\xi}$, $\overline{\xi}$ and $\bar{p}$ jointly to match three targets: (i) an average leverage (debt over capital) of 0.5; (ii) a standard deviation for debt-to-output ratio of ; (iii) a frequency of crisis (low liquidation price equilibria) of 4 percent.[15] The full list of parameter values are reported in Table 1. It turns out that for this parametrization, the economy always stay within the set of states for which multiple equilibria are admissible.

Table 1: List of parameters

Discount factor for households/workers, δ	0.996
Discount factor for entrepreneurs, β	0.986
Utility parameter, α	16.293
Labor elasticity, η	0.750
Production technology, θ	0.200
Production technology, ν	0.700
Depreciation rate, τ	0.020
Capital adjustment parameter, ζ	0.500
Low liquidation value, $\underline{\xi}$	0.550
High liquidation value, $\overline{\xi}$	0.650
Frequency of low liquidation value, $\bar{p}$	0.200

6.1 Results

Our first result follows directly from Proposition 4.1 which extends to this more general environment with capital accumulation: changes in ξ_t and ξ_t^* are perfectly correlated across countries and all variables (real and financial) are also perfectly correlated. Hence, the model can generate very strong international co-movement in real and financial variables, consistently with the property of the recent crisis as documented in Section 2.

In addition to the co-movement, we outline four key properties: (i) the asymmetric response to contractions and expansions; (ii) the counter-cyclicality of labor productivity; (iii) the severity of crises that arise after long periods of credit booms; (iv) the importance of credit for the volatility of labor and asset prices.

[15]Although the three parameters are chosen jointly, we can identify the primary parameter that affects each of the three targets. The average leverage is mostly determined by the average ξ. The standard deviation of debt is mostly determined by the difference between $\overline{\xi}$ and $\underline{\xi}$. The frequency of crisis is mostly determined by $\bar{p}$.

I - Asymmetry Figure 6 plots the impulse responses to a credit expansion and a credit contraction. Because of the symmetry, we report only the responses for one country. A credit expansion is generated starting from the limiting equilibrium in which the economy converges after a long sequence of $\xi_t = \underline{\xi}$. From this equilibrium we consider a sequence of $\xi_t = \overline{\xi}$ starting at $t = 1$. Therefore, a credit expansion is generated by a permanent switch from equilibria with $\xi = \underline{\xi}$ to equilibria with $\xi = \overline{\xi}$. Similarly, the impulse responses to a credit contraction are generated starting from the limiting equilibrium in which the economy converges after a long sequence of $\xi_t = \overline{\xi}$. Starting at $t = 1$ the economy experiences a sequence of draws $\xi_t = \underline{\xi}$.

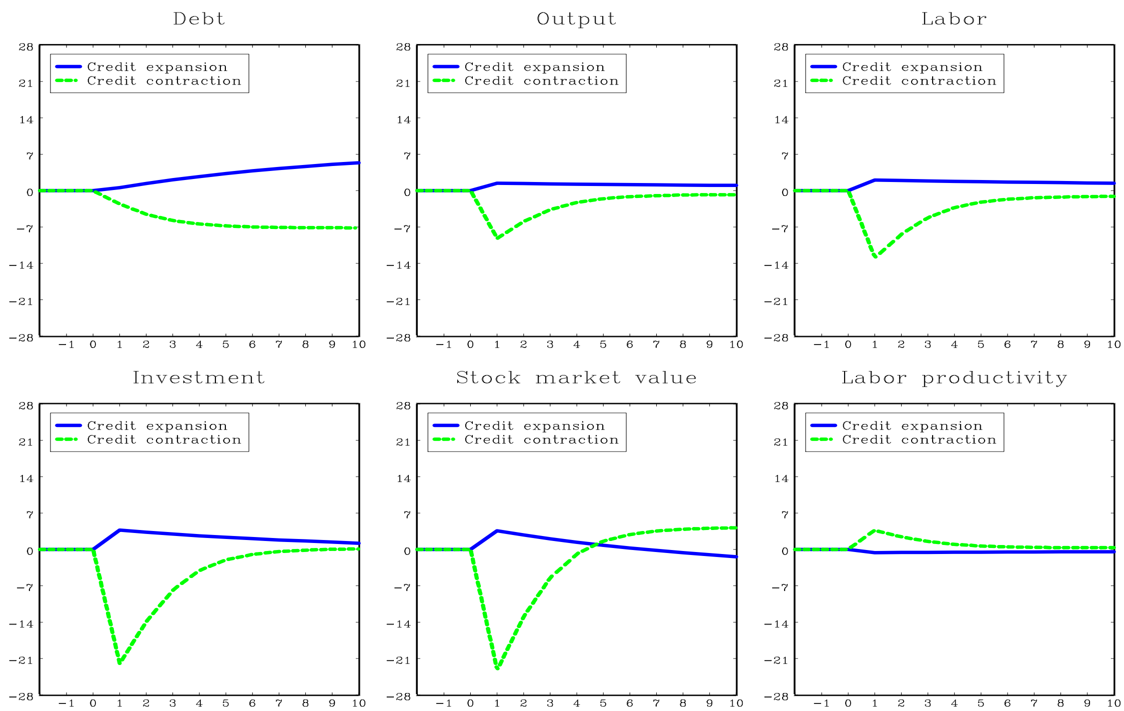

Figure 6: Impulse responses to credit expansions and contractions.

Two remarks are in order. First, the impulse responses take place in a range of states that admit multiple equilibria. Therefore, the selected sequences of ξ_t are possible equilibrium outcomes. Second, agents do not know in advance the actual sequence of liquidation prices. Therefore, they take into account the uncertainty induced by the stochastic distribution of ξ_t.

In response to the credit expansion, we see a gradual increase in the stock of debt and a persistent expansion in labor and output. The magnitude of the macroeconomic expansion, however, is not large at impact.[16] The response to a credit contraction,

[16]The macroeconomic expansion induced by the credit boom arises through the following mechanism.

instead, displays a very different pattern. The stock of debt declines more quickly and the response of labor, output and investment are much larger at impact. Therefore, the model generates a strong asymmetry in the responses to credit expansions and contractions.

The intuition for the asymmetry is best understood starting from a situation in which the enforcement constraint is not binding. If the constraint gets relaxed, the Lagrange multiplier cannot fall below zero and the expansionary effect on unemployment is mild (only through general equilibrium effects). Instead, if the constraints get tighter, the Lagrange multiplier goes from 0 to being positive and that causes a fall in employment and output (through equation (19)). As we discussed in Section 2, this asymmetry is consistent with the macroeconomic dynamics observed during the period 2005-2010.

II - Counter-cyclical labor productivity The last panel of Figure 6 plots the impulse responses of labor productivity, that is, the ratio between output and hours. As in the previous figure we see an asymmetry between credit expansions and credit contractions. More importantly, a credit expansion causes a decline in labor productivity while a credit contraction generates an increase in labor productivity. This is important for capturing the counter-cyclical dynamics of the US labor productivity during the recent crisis as documented in Section 2.

III - Credit booms and severity of recessions Figure 7 plots the impulse responses to credit expansions that later revert back to pre-expansion levels. Starting from an equilibrium to which the economy converges after a long sequence of $\xi_t = \underline{\xi}$, at time 1 the economy experiences a switch to $\xi_t = \overline{\xi}$ (credit expansion). The value of ξ stays at the higher level for several periods and then it reverts back to $\underline{\xi}$ permanently. Again, agents do not fully anticipate these particular sequences and form expectations based on their conditional distribution. We consider credit booms with duration of 4 quarters (left panels) and 20 quarters (right panels).

The key finding is that the macroeconomic impact of the credit contraction increases with the duration of the preceding credit expansion. After a protracted credit boom, the economy accumulates large volume of debt and when the credit reversal arrives, the required de-leveraging is more severe. This forces firms to implement larger hiring and investment cuts and generates a stronger macroeconomic contraction. In this way the

At impact the firm becomes unconstrained which eliminates the labor wedge. In addition to that and after the initial period, there is a second mechanism. As firms take on more debt, they pay more dividends, increasing the discount factor m'. Thanks to the lower discounting, firms invest more. At the same time, the higher borrowing from firms increases the equilibrium interest rate which in turn increases labor supply and output.

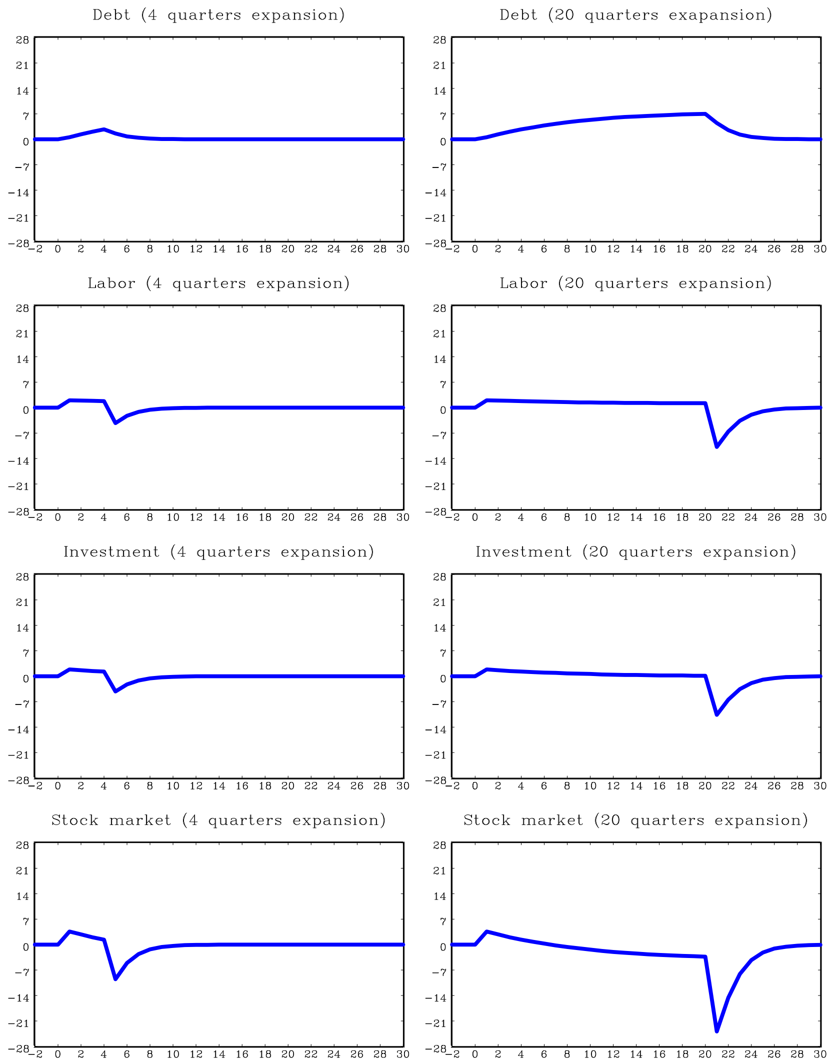

Figure 7: Duration of credit expansions and severity of contractions.

model explains why recessions that arise after long periods of financial expansions tend to be associated with more severe macroeconomic contractions.

IV - Volatility of labor and asset prices The first column of Table 2 reports simulation statistics computed after detrending the series with a band-pass filter that preserves cycles of 1.5-8 years (Baxter and King (1999)).

Two properties are especially noticeable. First, the model can generate a high volatility of labor, larger than the volatility of output. This is because fluctuations in the availability of credit cause autonomous movements in employment that, due to decreas-

Table 2: Business cycle statistics of key variables from detrended simulated series.

	Credit shocks only	Productivity shocks only	Both shocks
	Standard deviations		
Output	0.88	0.76	1.16
Consumption	0.68	0.44	0.77
Labor	1.26	0.26	1.26
Investment	2.27	0.77	2.36
Tobin's q	1.14	0.38	1.18
Stock market value	2.46	0.54	2.45
Interest rate	0.48	0.25	0.48
Return on equity	5.82	0.37	5.82
	Expected returns (% annualized)		
Interest rate	1.40	1.56	1.40
Return on equity	6.96	5.62	6.96
Equity risk premium	1.56	0.06	1.56
Nonbinding constraints, %	96.44	99.99	96.04

Notes: The standard deviations for the returns on stocks and bonds are calculated on unfiltered data.

ing returns, drive smaller movements in output. Second, the model also generates high volatility of asset prices. In particular, the stock market value (equity value of firms) is almost three times more volatile than output. This can also be seen in the bottom panel of Figure 6 which plots the impulse responses of the market value of equity. The reason for the high asset price volatility is due to mechanisms. The first is that credit fluctuations affect the dividends (consumption) received by investors which in turn affect their stochastic discount factor. The second mechanism is through the adjustment cost of capital. Since fluctuations in credit also affect investment, they impact on Tobin's q. The high volatility of asset prices generated by the model suggests that fluctuations in the availability of credit could contribute to explaining the large volatility of stock prices we have observed during the recent crisis (see Figure 4).

As a result of the higher volatility of asset prices and of the discount factor for investors, the model can also generate a non-negligible equity risk-premium.[17] This is

[17]We should be careful in defining the equity risk-premium. Since bond holders (workers) have a higher discounting than equity holders (investors), the difference between the expected return on equity

about 1.56 percent yearly. We also observe that the volatility of equity returns is quite high in the model but the volatility of the interest rate is small.

6.2 Productivity shocks

Before closing this section we also show how the model performs when we consider standard productivity shocks, alone (second column of Table 2) and together with credit shocks (third column of Table 2). To add productivity shocks we specify the production function as $y_t = z_t(K_t+K_t^*)^{1-\theta} k_t^\theta h_t^\nu$, where z_t denotes the stochastic level of productivity. The variable z_t is country-specific and follows the Markov process

$$\left(\begin{array}{c} \log(z_{t+1}^{US}) \\ \log(z_{t+1}^{G6}) \end{array} \right) = \left[\begin{array}{cc} \rho_z & 0 \\ 0 & \rho_z \end{array} \right] \left(\begin{array}{c} \log(z_t^{US}) \\ \log(z_t^{G6}) \end{array} \right) + \left(\begin{array}{c} \epsilon_{t+1}^{US} \\ \epsilon_{t+1}^{G6} \end{array} \right),$$

where ϵ_t^{US} and ϵ_t^{G6} are innovations with mean 0, common standard deviation σ_ϵ and correlation ρ_ϵ. We then pick standard values for these parameters and in particular we set $\rho_z = 0.98$, $\sigma_\epsilon = 0.006$ and $\rho_\epsilon = 0.15$.

The second column of Table 2 shows that the model with only productivity shocks generates much lower volatilities of hours and asset prices. It is also worth nothing that the enforcement constraint is basically never binding. Because of this, the labor wedge is (almost) always zero, which explains why labor and asset prices are not very volatile. The last column of Table 2 shows that the model with both shocks produces statistics very similar to those generated by the model with only credit shocks.

7 Global financial crisis with heterogeneous labor markets

In Section 2 we have shown that, although the decline in output during the recent crisis has been very similar across the G7 countries, the employment decline has been more severe in the US than in the other G7 countries, a fact also pointed out by Ohanian (2010). The heterogeneous dynamics of the labor market in the US and other G7 countries is also reflected in the dynamics of the 'labor wedge', that is, the difference between the marginal rate of substitution between consumption and leisure and the marginal product of labor. Formally, this is defined as $U_h(c_t, h_t)/U_c(c_t, h_t) - F_h(k_t, h_t)$, where U_h and U_c are the marginal utilities of leisure and consumption, respectively, and F_h is the marginal product of labor. With CES utility and Cobb-Douglas production

(for investors) and on bonds (for workers) is not the risk premium. In fact, even in absence of risk, the return on equity will be higher than the return on bonds. Given the calibration of $\delta = 0.996$ and $\beta = 0.986$, the return differential in absence of risk would be about 4 percent yearly. Given this, we define the equity risk-premium as the difference between the return differential between equity and bonds and the difference in discount rates between investors and workers.

function, the labor wedge is equal to

$$\text{Wedge} = \frac{\phi c_t}{1 - h_t} - (1 - \theta)\frac{y_t}{h_t}. \tag{23}$$

Using this formula, Ohanian and Raffo (2011) find that while in the US the labor wedge dropped dramatically during the recent crisis, other G7 countries experienced only a modest decline on average. In few countries like Germany it even increased. The goal of this section is to show that the different behavior of the labor market between the US and other G7 countries can be reconciled with the view of a global financial crisis if the structure of the labor market differs across countries.

In order to show this point we extend our model by adding two elements: variable labor utilization and heterogeneous labor rigidities. The role of variable labor utilization is to allow for a more powerful mechanism for endogenous fluctuations in measured labor productivity. The role of labor rigidities is to allow for the response of labor utilization to differ from the response of measured labor input. By further assuming that labor rigidities differ across countries, the model is then capable of generating heterogeneous responses of macroeconomic and labor market variables. The assumption of heterogeneous labor rigidities between the US and other G7 countries is consistent with anecdotal as well as more systematic evidence. For example, Ohanian and Raffo (2011) refer to indicators from the OECD Employment Outlook (2008) and report that the US is the country with the most flexible labor market. Instead, the labor market of many of the countries in continental Europe and Japan are ranked as the least flexible.

Let's start with labor utilization. The production function is specified as $F(k_t, n_t)$, where n_t is the 'effective' input of labor. This results from the combination of (measured) hours, h_t, and (unmeasured) utilization, e_t, according to the function

$$n_t = \left[h_t^{\frac{\varrho-1}{\varrho}} + e_t^{\frac{\varrho-1}{\varrho}}\right]^{\frac{\varrho}{\varrho-1}}.$$

The parameter ϱ is the elasticity of substitution between hours spent in the workplace and actual utilization. When $\varrho = 1$ we have $n_t = h_t^{0.5} \cdot e_t^{0.5}$, often used in the literature.

The utilization cost derives from workers disutility. Given the utility function $U(c_t, h_t + e_t)$, workers face higher disutility when they spent more hours in the working place and when their services are utilized more intensively. Under this specification, the utilization cost equals the wage w_t, and the total cost of labor is $(h_t + e_t)w_t$.

So far, the addition of labor utilization is inconsequential for the properties of the model. Given the CES aggregation and that the wage rate is the price for both h_t and e_t, firms always choose $e_t = h_t$. Thus, we can simply focus on h_t as in the original model. This equivalence no longer holds once we add some rigidities on working hours h_t.

Some authors interpret labor market rigidities as constraining the extensive margin (employment) rather than the intensive margin (per-worker hours). However, since our model does not distinguish these two margins, we interpret labor market rigidities as restricting total hours h_t. More specifically, we assume that firms incur the convex cost

$$\kappa(h_t - \bar{h})^2 w_t.$$

Ideally, we would like to use a more standard adjustment cost, such as $\kappa(h_t - h_{t-1})^2 w_t$. This alternative formulation, however, would introduce an additional state variable, h_{t-1}, which increases the computational complexity of the model. To avoid this, we specify the cost as deviation from the exogenous fixed target $\bar{h}$. The multiplication by the wage rate is motivated by economic and technical considerations. From an economic point of view it is likely that the direct cost of labor, which depends on the wage, also affects the cost of changing employment. An example is severance payments. From a technical point of view the presence of the wage allows us to apply the same normalization procedure used in the version of the model with capital accumulation.

The key parameter is κ. With a positive value of κ, the response of utilization e_t to shocks is bigger than the response of hours h_t. This generates a decline in measured TFP and, potentially, a decline in measured labor productivity y_t/h_t. These effects increase with the value of κ. Therefore, if in our model the first country (the US) is characterized by lower labor market rigidities than the second country (the other G7), the model could generate very different labor market dynamics in these two countries.

7.1 Simulation results

We describe here only the calibration of the parameters that need to be re-calibrated or were not present in the baseline model. We start with the elasticity of substitution between hours and utilization ϱ, which we set to 5. This value implies a high degree of substitutability between hours and utilization. The utility parameter α is chosen to have working hours to be 0.3 on average in the equilibrium without labor rigidities.

The remaining parameters are $\bar{h}$, κ^1 for country 1 and κ^2 for country 2. Given the values of κ^1 and κ^2, we choose $\bar{h}$ to have the desired differential in average employment between the two countries. We choose total hours in the US to be 5 percent higher than in other G7 countries. Unfortunately, to pin down the values of κ^1 and κ^2, we are unaware of any direct empirical measurement that can inform us about the value of these two parameters. Because of this, we take a more pragmatic approach. We assign $\kappa^1 = 0.3$ and $\kappa^2 = 1.5$ so that the model generates heterogeneous drops in labor wedges during a crisis (switch to the equilibrium with $\xi = \underline{\xi}$) similar to the drops observed during the recent crisis. Of course, the relevance of the exercise is only to show that the model 'could' generate the heterogeneous responses of the labor market observed in the US and in the other G7 countries. In this way we show that the idea of a global financial crisis as a driver of the recent recession cannot be written down by the observation of cross-country heterogeneity in labor market dynamics, although this is only suggestive.

Figure 8 plots the impulse responses to a permanent credit contraction. The impulse responses are constructed using the same methodology as in Figure 6. The responses of investment and output are very similar in the two countries. However, the responses of hours and the labor wedge are significantly smaller in country 2.[18] We also observe strong heterogeneity in the response of labor productivity which falls only slightly in country 1 but experiences a large drop in country 2. Therefore, the model could replicate the different dynamics of the labor market between the US and other G6 countries even if the dynamics of other macroeconomic variables are similar.

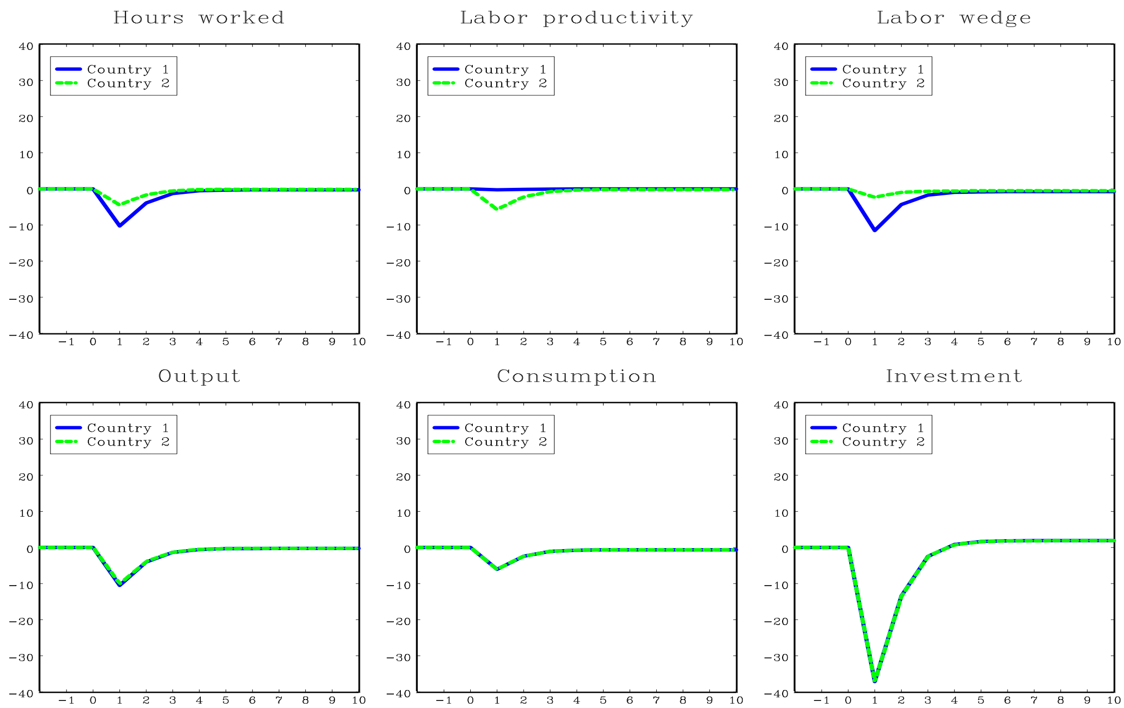

Figure 8: Impulse responses to a credit contraction with asymmetric labor markets.

Before closing, we would like to make some remarks about the nature of labor rigidities. Rigidities are typically interpreted as the consequence of institutional factors. Here, however, we give a broader interpretation. For example, it is well known that labor market rigidities are different across sectors. To the extent that in certain countries the crisis

[18]In our model, the labor wedge is slightly different from equation (23) because the production function is not constant returns. Furthermore, there is labor utilization and c_t is only the consumption of workers, not aggregate consumption. However, we measure the labor wedge as if the true model was the standard RBC since this is the approach used in the literature. After simulating the model and generating the series for c_t, h_t and y_t, we compute the wedge by plugging the series in equation (23). The values of the parameters are the same as those used in Ohanian and Raffo (2011), that is, $\beta = 0.99$, $\theta = 0.36$, $\delta = 0.0175$, $g = 0.005$ and ϕ is chosen so to have steady state hours of 0.33.

has impacted sectors with greater labor market flexibility, we may observe larger declines in employment and hours. For instance, the construction sector is typically characterized by greater hiring flexibility because of its cyclicality. Then, countries that experience large contractions in the real estate sector are also likely to experience larger drops in employment. This is the case for Spain, a country where the real estate sector experienced an abnormal boom before the crisis. In this sense, a country like Spain could be considered a country with a flexible labor market, simply because the more flexible sector was hit hard by the crisis.

8 Conclusion

We have documented that the recent financial crisis has been characterized by a historically high degree of international synchronization in real and financial variables. We have proposed a theoretical framework in which endogenous credit booms and credit crises can result from self-fulfilling expectations. These episodes affect the real sector of the economy through a credit channel: booms enhance the borrowing capacity of firms and lead to (mildly) higher employment and production. Crises curtail borrowing capacity and lead to sharp contractions in real activity and asset prices.

When countries are financially integrated, self-fulfilling credit booms/crises also generate large spillovers to the real and financial sectors of other countries. There are two channels of international transmission. The first channel is through the cost of capital which in an integrated financial market is equalized across countries. The second channel is based on the endogenous nature of credit market conditions. These conditions change when the economy switches from one self-fulfilling equilibrium to another self-fulfilling equilibrium. But in an integrated financial market, the shift in one country can only arise if the shift arises also in the other. Therefore, changing financial market conditions become highly synchronized.

This study does not exclude the possibility that other sources of business cycle fluctuations also generate international co-movement in real variables. Our interest in changing credit market conditions as a source of business cycle is motivated by their ability to generate large cross-country co-movement in the real sector of the economy together with large international co-movement in the flows of financing and asset prices. As far as the recent crisis is concerned, we do not claim that a credit contraction is the only cause of the crisis. We have shown, however, that changing credit market conditions can go a long way in capturing several features of the crisis and, especially, its unprecedented international synchronization.

Appendix

A Debt renegotiation

The enforcement constraint is derived from the following assumptions. Default arises at the end of the period before repaying the intra-temporal loan $l_t = F(h_t)$. In case of default the lender confiscates the physical capital, which is sold at price ξ_t, but not the liquidity l_t.

Define first the value of the firm recursively as

$$V_t(b_t) = d_t + E_t m_{t+1} V_{t+1}(b_{t+1}),$$

where m_{t+1} is the discount factor, taken as given by an individual firm. Since default takes place at the end of the period, after the payment of dividends, the value of not defaulting is $E_t m_{t+1} V_{t+1}(b_{t+1})$.

In the event of default the parties negotiate a repayment τ_t to the lender. If they reach an agrement, the firm continues operation and its value is $E_t m_{t+1} V_{t+1}(b_{t+1}) + l_t - \tau_t$. What this says is that the firm retains its continuation value $E_t m_{t+1} V_{t+1}(b_{t+1})$ plus the liquidity net of the bargained payment τ_t. Without an agrement the firm retains only the divertible liquidity l_t (threat value). The net value of an agrement is the difference between the value of renegotiation and the threat value, that is

$$E_t m_{t+1} V_{t+1}(b_{t+1}) - \tau_t. \tag{24}$$

Let's now consider the lender. With an agrement the lender gets $\tau_t + b_{t+1}/R_t$. The intertemporal debt is discounted since will be repaid next period. Without an agrement the lender receives the liquidation value of capital, $\xi_t \bar{k}$ (threat value). Thus, the net value of renegotiation is

$$\tau_t + \frac{b_{t+1}}{R_t} - \xi_t \bar{k}. \tag{25}$$

The net surplus is the sum of the net values for the firm, (24), and the lender, (25),

$$S_t(b_{t+1}) = E_t m_{t+1} V_{t+1}(b_{t+1}) + \frac{b_{t+1}}{R_t} - \xi_t \bar{k}. \tag{26}$$

Under the assumption that the firm has all the bargaining power, the value of defaulting is $l_t + S_t(b_{t+1})$. Incentive compatibility requires that the value of not defaulting is (weakly) bigger than the value of defaulting, that is,

$$E_t m_{t+1} V_{t+1}(b_{t+1}) \geq l_t + S_t(b_{t+1}).$$

Substituting the definition of the net renegotiation surplus $S_t(b_{t+1})$, Equation (26), and rearranging, we obtain the enforcement constraint $\xi_t \bar{k} = l_t + \frac{b_{t+1}}{R_t}$.

B First order conditions

Consider the optimization problem (3) and let λ and μ be the Lagrange multipliers associated with the two constraints. Taking derivatives we get

$$\begin{aligned} d: &\quad 1-\lambda=0 \\ h: &\quad \lambda[F_h(h)-w]-\mu F_h(h)=0 \\ b': &\quad Em'V_{b'}(\mathbf{s}';b')+\frac{\lambda}{R}-\frac{\mu}{R}=0. \end{aligned}$$

Using the envelope condition $V_b(\mathbf{s};b)=-\lambda$, the first order conditions become (6) and (7).

C Proof of Proposition 4.1

Sunspot equilibria arise when, for a given state $\mathbf{s}_t$, two types of equilibria are possible: the equilibrium with binding enforcement constraint and low liquidation price $\xi_t=\underline{\xi}$ (tight credit equilibrium) and the equilibrium with non-binding enforcement constraint and high liquidation price $\xi_t=\overline{\xi}$ (loose credit equilibrium). We have already shown that in equilibrium binding enforcement constraints are always associated with low liquidation price and non-binding enforcement constraints are always associated with high liquidation price. When the two types of equilibria are both possible, the selection is determined by the realization of the non-fundamental shock $\varepsilon\in\{0,1\}$. On the other hand, sunspot equilibria do not exist if the economy converges to one of the two types of equilibria (tight credit or loose credit) and changes in expectations about the liquidation price do not move the economy to the other type of equilibria (from tight credit to loose credit and viceversa). In what follows we show that neither tight credit equilibria nor loose credit equilibria can persist forever.

Suppose that the enforcement constraint is not initially binding. Furthermore, let's assume that the constraint is not binding also in future periods. Therefore, we start with the assumption that the economy remains in the loose credit equilibrium and the liquidation price is $\xi_t=\overline{\xi}$ for all t. Since the liquidation price remains constant over time, there is not uncertainty and the economy converges to a steady state. We now show that in the steady state the enforcement constraint must be binding and, therefore, we obtain a contradiction to the assumption that the loose credit equilibrium can persist forever.

From the workers' first order condition (9) evaluated at the steady state we obtain $\delta R=1$. From the firms' first order condition (7), also evaluated at the steady state, we obtain $\beta R=1-\mu$. Since $\delta>\beta$ by assumption, these two conditions imply that the multiplier associated with the enforcement constraint μ is strictly positive. Therefore, the enforcement constraint must be binding in a steady state. This contradicts the assumption that loose credit with $\xi_t=\overline{\xi}$ are the only possible type of equilibria.

Let's now show that also tight credit equilibria with binding enforcement constraint and $\xi_t=\underline{\xi}$ can not persist forever. Suppose on the contrary that they can persist forever. Since the model becomes deterministic, the economy converges to a steady state. Using the first order conditions for households and firms evaluated at the steady state (equations (9) and (7)) we can verify that the enforcement constraint is binding in the steady state and the liquidation price must be $\xi_t = \underline{\xi}$. The next step is to show that this steady state can not persist forever, that is, the expectation of a high liquidation price $\xi_t = \overline{\xi}$ would make the enforcement constraint non-binding if $\overline{\xi} - \underline{\xi}$ is sufficiently large.

Consider the enforcement constraint evaluated at the steady state associated with $\xi_t = \underline{\xi}$

$$F(h) + \frac{B'}{R} = \underline{\xi}\bar{k},$$

where h, B and R are steady state labor, debt and interest rate. Starting from this steady state, if the expectation for the liquidation price switches to $\xi_t = \overline{\xi}$, but firms do not change h and B', the enforcement constraint is no longer binding. Of course, firms will also change h and B' in response. In particular, they increase B'. From the budget constraint of the firm (4) we see that this implies a higher payment of dividends in the current period. Starting from the next period, however, the payment of dividends will be lower since a higher stock of debt implies higher interest payments. Therefore, the stochastic discount factor $m' = \beta u_c(d_{t+1})/u_c(d_t)$ increases compared to the steady state value. At the same time, using the budget constraint of workers $c + b'/R = b + wh$, we see that workers' consumption declines in the current period (since workers save more in the form of bonds) but will be higher in the next period when workers receive more interest payments. From condition (9) we see that this generates an increase in the interest rate R.

We now consider the first order condition for the firm, equation (7). Compared to the steady state, since both R and m' increase, the multiplier μ must decline. If the increase in debt is sufficiently large (which will happen if $\overline{\xi} - \underline{\xi}$ is sufficiently large), the multiplier μ becomes zero implying that the enforcement constraint is no longer binding. Therefore, the expectation of a high liquidation price is validated ex-post by the fact that firms become unconstrained. By assumption, this happens with probability $1 - \bar{p}$.

In the proof provided so far, we have kept h constant. The input of labor also changes but a similar argument applies even if h is allowed to change. We have thus proved that neither loose credit equilibria with $\xi_t = \overline{\xi}$ nor tight credit equilibria with $\xi_t = \underline{\xi}$ can persist in the long-run. Thus, sunspot equilibria in which the economy switches stochastically between tight and looses credit emerge in this economy.

D First order conditions for the model with capital accumulation

Differentiating the firm's problem (18) with respect to h_t, $\tilde{b}_{t+1}$, $\tilde{i}_t$, $\tilde{k}_{t+1}$, we get

$$\nu \tilde{k}_t^{\theta} h_t^{\nu-1} = \frac{\tilde{w}_t}{1-\mu_t},$$
$$\frac{1-\mu_t}{R_t} + g_t^{-\sigma} E\tilde{m}_{t+1}\tilde{V}_b(\tilde{\mathbf{s}}_{t+1}; \tilde{k}_{t+1}, \tilde{b}_{t+1}) = 0,$$
$$Q_t \Upsilon'\left(\frac{\tilde{i}_t}{\tilde{k}_t}\right) = 1,$$
$$Q_t = \xi_t \mu_t + g_t^{-\sigma} E\tilde{m}_{t+1}\tilde{V}_k(\tilde{\mathbf{s}}_{t+1}; \tilde{k}_{t+1}, \tilde{b}_{t+1}),$$

where μ_t is the lagrange multiplier associated with the enforcement constraint and Q_t is the lagrange multiplier associated with the law of motion of capital (Tobin's q). The multiplier associated with the budget constraint is 1. For the foreign country we have the same conditions with start superscripts. The envelope conditions are

$$\tilde{V}_b(\tilde{\mathbf{s}}_t; \tilde{k}_t, \tilde{b}_t) = -1,$$
$$\tilde{V}_k = (1-\mu_t)\theta \tilde{k}_t^{\theta-1} h_t^{\nu} + \left[1 - \tau + \Upsilon\left(\frac{\tilde{i}_t}{\tilde{k}_t}\right) - \Upsilon'\left(\frac{\tilde{i}_t}{\tilde{k}_t}\right)\frac{\tilde{i}_t}{\tilde{k}_t}\right] Q_t.$$

Substituting and imposing equilibrium conditions $k_t = K_t$ and $\tilde{k}_t = s_t$, we obtain (19)-(22).

E Dynamic system and solution approach

We will use the bar sign to denote aggregate worldwide variables normalized by the worldwide stock of capital. For example, $\bar{d}_t$ is the normalized worldwide dividend, defined as $\bar{d}_t = \frac{d_t + d_t^*}{K_t + K_t^*} \equiv \tilde{d}_t + \tilde{d}_t^*$. The full list of equilibrium conditions are:

$$1 = \delta g_t^{-1} R_t E_t \left(\frac{\bar{c}_{t+1}}{\bar{c}_t}\right)^{-1} \tag{27}$$

$$\tilde{c}_t^* = \chi \tilde{c}_t \tag{28}$$

$$\tilde{w}_t h_t + \tilde{w}_t^* h_t^* + \bar{b}_t = \bar{c}_t + \frac{g_t \bar{b}_{t+1}}{R_t} \tag{29}$$

$$\bar{b}_t + \bar{d}_t + \bar{i}_t = s_t^{\theta} h_t^{\nu} + (s_t^*)^{\theta}(h_t^*)^{\nu} - \tilde{w}_t h_t - \tilde{w}_t^* h_t^* + \frac{\bar{g}_t \bar{b}_{t+1}}{R_t} \tag{30}$$

$$g_t(\xi_t s_{t+1} + \xi_t^* s_{t+1}^*) \geq \frac{g_t \bar{b}_{t+1}}{R_t} + s_t^{\theta} h_t^{\nu} + (s_t^*)^{\theta}(h_t^*)^{\nu} \tag{31}$$

$$(1-\mu_t)\bar{d}_t^{-\sigma} = \beta g_t^{-\sigma} R_t E \bar{d}_{t+1}^{-\sigma} \tag{32}$$

$$\alpha h_t^{\frac{1}{\eta}} = \frac{\tilde{w}_t}{\tilde{c}_t} \tag{33}$$

$$\alpha (h_t^*)^{\frac{1}{\eta}} = \frac{\tilde{w}_t^*}{\tilde{c}_t^*} \tag{34}$$

$$g_t s_{t+1} = (1-\tau)s_t + \Upsilon\left(\frac{\tilde{i}_t}{s_t}\right) s_t \tag{35}$$

$$g_t s_{t+1}^* = (1-\tau)s_t^* + \Upsilon\left(\frac{\tilde{i}_t^*}{s_t^*}\right) s_t^* \tag{36}$$

$$\nu s_t^{\theta} h_t^{\nu-1} = \frac{\tilde{w}_t}{1-\mu_t} \tag{37}$$

$$\nu (s_t^*)^{\theta}(h_t^*)^{\nu-1} = \frac{\tilde{w}_t^*}{1-\mu_t} \tag{38}$$

$$Q_t \Upsilon'\left(\frac{\tilde{i}_t}{s_t}\right) = 1 \tag{39}$$

$$Q_t^* \Upsilon' \left(\frac{\tilde{i}_t^*}{s_t^*} \right) = 1 \tag{40}$$

$$Q_t = \xi_t \mu_t + \beta g_t^{-\sigma} E \left(\frac{\bar{d}_{t+1}}{\bar{d}_t} \right)^{-\sigma} \left\{ (1-\mu_{t+1}) \theta s_{t+1}^{\theta-1} h_{t+1}^{\nu} - \frac{\tilde{i}_{t+1}}{s_{t+1}} + \left[1 - \tau + \Upsilon \left(\frac{\tilde{i}_{t+1}}{s_{t+1}} \right) \right] Q_{t+1} \right\} \tag{41}$$

$$Q_t^* = \xi_t^* \mu_t + \beta g_t^{-\sigma} E \left(\frac{\bar{d}_{t+1}}{\bar{d}_t} \right)^{-\sigma} \left\{ (1-\mu_{t+1}) \theta (s_{t+1}^*)^{\theta-1} (h_{t+1}^*)^{\nu} - \frac{\tilde{i}_{t+1}^*}{s_{t+1}^*} + \left[1 - \tau + \Upsilon \left(\frac{\tilde{i}_{t+1}^*}{s_{t+1}^*} \right) \right] Q_{t+1}^* \right\} \tag{42}$$

Equations (27)-(42) form a dynamic system composed of 16 equations. Given the states $\xi_t, \xi_t^*, \bar{b}_t, s_t$, the unknown variables are h_t, h_t^*, c_t, c_t^*, w_t, w_t^*, i_t, i_t^*, Q_t, Q_t^*, g_t, μ_t, R_t, $\bar{d}_t$, $\bar{b}_{t+1}$, s_{t+1}. Therefore, we have a dynamic system of 16 equations in 16 unknowns.

The computational procedure is based on the approximation of five functions:

$$\begin{aligned}
\Gamma_1(\mathbf{s}_{t+1}) &= \bar{c}_{t+1}^{-1} \\
\Gamma_2(\mathbf{s}_{t+1}) &= \bar{d}_{t+1}^{-\sigma} \\
\Gamma_3(\mathbf{s}_{t+1}) &= \bar{d}_{t+1}^{-\sigma} \left\{ (1-\mu_{t+1}) \theta s_{t+1}^{\theta-1} h_{t+1}^{\nu} - \frac{\tilde{i}_{t+1}}{s_{t+1}} + \left[1 - \tau + \Upsilon \left(\frac{\tilde{i}_{t+1}}{s_{t+1}} \right) \right] Q_{t+1} \right\} \\
\Gamma_4(\mathbf{s}_{t+1}) &= \bar{d}_{t+1}^{-\sigma} \left\{ (1-\mu_{t+1}) \theta (s_{t+1}^*)^{\theta-1} (h_{t+1}^*)^{\nu} - \frac{\tilde{i}_{t+1}^*}{s_{t+1}^*} + \left[1 - \tau + \Upsilon \left(\frac{\tilde{i}_{t+1}^*}{s_{t+1}^*} \right) \right] Q_{t+1}^* \right\} \\
\Gamma_5(\mathbf{s}_{t+1}) &= p(\mathbf{s}_{t+1})
\end{aligned}$$

The procedure starts with a guess for the values of the approximated functions $\Gamma_1(\mathbf{s}_{t+1})$, $\Gamma_2(\mathbf{s}_{t+1})$, $\Gamma_3(\mathbf{s}_{t+1})$, $\Gamma_4(\mathbf{s}_{t+1})$ and $\Gamma_5(\mathbf{s}_{t+1})$. We construct first a two dimensional grid for the endogenous states $\bar{b}$ and s. Then for each realization of the price—ξ_t and ξ_t^*—we guess the values taken by the above functions over the grid points. Values outside the grid are obtained through bi-linear interpolation. Once we know the approximated functions and probabilities for ξ_{t+1}, we can solve for the 16 unknowns of the system (27)-(42) at each grid point and for each possible value of ξ_t and ξ_t^*. In finding the solutions we check whether the enforcement constraint is binding ($\mu_t > 0$) or not binding ($\mu_t = 0$). We then use the solutions found at each grip point to update the guesses for the five functions $\Gamma_1(\mathbf{s}_{t+1})$, $\Gamma_2(\mathbf{s}_{t+1})$, $\Gamma_3(\mathbf{s}_{t+1})$, $\Gamma_4(\mathbf{s}_{t+1})$ and $\Gamma_5(\mathbf{s}_{t+1})$. To update these probabilities we need to check whether multiple equilibria are feasible for all possible states. Effectively we check this on the grid points of the states. We keep iterating until the guesses for $\Gamma_1(\mathbf{s}_{t+1})$, $\Gamma_2(\mathbf{s}_{t+1})$, $\Gamma_3(\mathbf{s}_{t+1})$, $\Gamma_4(\mathbf{s}_{t+1})$ and $\Gamma_5(\mathbf{s}_{t+1})$, evaluated at the grid points, are equal to the values obtained by solving the dynamic system (also at the grid points).

References

Backus, D. K., Kehoe, P. J. and F. E. Kydland (1992). International Real Business Cycles. *Journal of Political Economy*, 100(4): 745-75.

Baxter, M. and R. G. King (1999). Measuring Business Cycles: Approximate Band-Pass Filters for Macroeconomic Time Series. *Review of Economics and Statistics*, 81(4): 575-93.

Benmelech E. and N. Bergman (2012) Credit Traps, *American Economic Review*, Forthcoming

Brunnermeier, M. and Y. Sannikov (2010). A Macroeconomic Model with a Financial Sector. Unpublished manuscript, Princeton University.

Cetorelli N. and L. S. Goldberg (2010). Banking Globalization and Monetary Transmission. Unpublished manuscript, Federal Reserve Bank of New York.

Christiano L., R. Motto and M. Rostagno (2009). Financial Factors in Economic Fluctuations. Unpublished manuscript, Northwestern University.

Claessens S. Kose, A and M. Terrones (2011). Financial Cycles: What? How? When?, CEPR Discussion Paper No. 8379.

Dedola L. and G. Lombardo (2010). Financial frictions, Financial integration and the international propagation of shocks. Unpublished manuscript, European Central Bank.

Devereux, M. B. and J. Yetman (2010). Leverage Constraints and the International Transmission of Shocks. *Journal of Money, Credit and Banking*, 42(s1): 71-105.

Devereux, M. B. and A. Sutherland (2011). Evaluating international financial integration under leverage constraints. *European Economic Review*, 55(3): 427-42.

Eisfeldt, A.L., and A. Rampini. (2006). Capital Reallocation and Liquidity. *Journal of Monetary Economics*, 53(3): 369-99.

Gao P. and H. Yun (2009). Commercial Paper, Lines of Credit, and the Real Effects of the Financial Crisis of 2008: Firm-Level Evidence from the Manufacturing Industry. Unpublished manuscript, University of Notre Dame.

Gertler, M. and P. Karadi (2009). A Model of Unconventional Monetary Policy. *Journal of Monetary Economics*, 58(1): 17-34.

Goldberg, J. (2010), A Model of Deleveraging, Unpublished manuscript, MIT

Goldberg, L. S. and C. D. Kolstad (1995). Foreign Direct Investment, Exchange Rate Variability and Demand Uncertainty. *International Economic Review*, 36(4), 855-73.

Hart, O. and J. Moore (1994). A Theory of Debt Based on the Inalienability of Human Capital. *Quarterly Journal of Economics*, 109(4), 841-79.

Heathcote, J. and F. Perri (2004). Financial Globalization and Real Regionalization. *Journal of Economic Theory*, 119(1), 207-43.

Imbs, J. (2010). The First Global Recession in Decades. *IMF Economic Review*, 58(2), 327-54.

Jermann, U. (1998). Asset Pricing in Production Economies. *Journal of Monetary Economics*, 41(2), 257-75.

Jermann, U. and V. Quadrini (2011). Macroeconomic Effects of Financial Shocks. *American Economic Review*, 102(1), 23871

Khan, A. and J. Thomas (2010). Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity. Unpublished manuscript, Ohio State University.

Kocherlakota, N. (2009). Bursting Bubbles: Consequences and Cures, Unpublished manuscript, Federal Reserve Bank of Minneapolis.

Kollmann, R., Z. Enders and G. J. Müller (2011). Global Banks and International Business Cycle. *European Economic Review*, 55(3), 407-26.

Liu, Z., P. Wang and T. Zha (2012). Land-Price Dynamics and Macroeconomic Fluctuations. *Econometrica*, forthcoming.

Lorenzoni, G. and V. Guerrieri (2010). Credit Crises, Precautionary Savings and the Liquidity Trap, Unpublished manuscript, Chicago Booth School of Business.

Lucas, R. and N. Stokey (2011). Liquidity Crises, Unpublished manuscript, University of Chicago.

Mendoza, E. (2010). Sudden Stops, Financial Crises and Leverage. *American Economic Review*, 100(5), 1941-66.

Ohanian, L. (2010) The Economic Crisis from a Neoclassical Perspective, *Journal of Economic Perspectives*, 24(4), Fall, 45-66.

Ohanian, L. and A. Raffo (2011). Hours Worked over the Business Cycle in OECD Countries, 1960-2010. Article prepared for the April 2011 *Carnegie-Rochester Conference Series on Public Policy*.

Reinhart, C. and K. Rogoff (2009). This Time is Different: Eight Centuries of Financial Folly. Princeton, N.J.: Princeton University Press.

Schularick, M. and A. M. Taylor (2011). Credit Booms Gone Bust: Monetary Policy, Leverage Cycles and Financial Crises, 1870-2008. *American Economic Review*, 102(2), 102961.

Shleifer, A., and R. W. Vishny. (1992) Liquidation Values and Debt Capacity: A Market Equilibrium Approach Journal of Finance, 47: 143-66.