Banking, Liquidity and Bank Runs in an Infinite Horizon Economy

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Abstract

We develop a variation of the macroeconomic model of banking in Gertler and Kiyotaki (GK2011) that allows for household liquidity risks and bank runs as in Diamond and Dybvig (DD1983). As in GK, because bank net worth fluctuates with aggregate production, the spread in the expected rates of return on bank credit and deposit fluctuates countercyclically. However, because bank assets have a longer maturity than deposits, bank runs are possible as in DD. Whether a bank run equilibrium exists depends on the condition of bank balance sheets and an equilibrium liquidation price for bank assets. Thus in normal times a bank run equilibrium may not exist, but the possibility can arise in a severe recession. Overall, the goal is to present a framework that synthesizes the macroeconomic and microeconomic approaches to banking and banking instability.

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1 Introduction

There are two complementary approaches in the literature to capturing the interaction between banking distress and the real economy. The first, summarized recently in Gertler and Kiyotaki (2011), emphasizes how the depletion of bank capital in an economic downturn hinders banks ability to intermediate funds. Due to agency problems (and possibly also regulatory constraints) a bank's ability to raise funds depends on its capital. Portfolios losses experienced in a downturn accordingly lead to losses of bank capital that are increasing in the degree of leverage. In equilibrium, a contraction of bank capital and bank assets raises the cost of bank credit, slows the economy and depresses asset prices further. The second approach, pioneered by Diamond and Dybvig (1983), focuses on how maturity mismatch in banking, i.e. the combination of short term liabilities and partially illiquid long term assets, opens up the possibility of bank runs. If they occur, runs lead to inefficient asset liquidation along with a general loss of banking services.

In the recent crisis, both phenomena were clearly at work. Depletion of capital from losses on sub-prime loans and related assets forced many financial institutions to contract lending and raised the cost of credit they did offer. (See, e.g. Adrian, Colla and Shin, 2012, for example.) Eventually, however, weakening financial positions led to classic runs on a number of the investment banks and money market funds, as emphasized by Gorton (2010) and Bernanke (2010). The asset firesale induced by the runs amplified the overall financial distress.

To date, macroeconomic models which have tried to capture the effects of banking distress have emphasized balance-sheet and financial accelerator effects, but not captured bank runs. Most models of bank runs, however, are typically quite stylized and not suitable for quantitative analysis. Further, often the runs are not connected to fundamentals. That is, they may be equally likely to occur in good times as well as bad.

Our goal is to develop a simple macroeconomic model of banking instability that features both balance-sheet and financial accelerator effects and bank runs. Our approach emphasizes the complementary nature of these mechanisms. Balance sheet conditions not only affect the cost of bank credit, they also affect whether runs are possible. In this respect one can relate the possibility of runs to macroeconomic conditions and in turn characterize how runs feed back into the macroeconomy.

For simplicity, we consider an infinite horizon economy with a fixed supply of capital, along with households and bankers. It is not difficult to map the framework into a more conventional macroeconomic model with capital accumulation. The economy with a fixed supply of capital, however, allows us to characterize in a fairly tractable way how banking distress and bank runs affect the behavior of asset prices. With capital accumulation, asset price movements translate into affects on investment and output.

As in Gertler and Karadi (2011) and Gertler and Kiyotaki (2011), endogenous procyclical movements in bank balance sheets lead to countercyclical movements in the cost of bank credit. At the same time, due to maturity mismatch, bank runs may be possible, following Diamond and Dybvig (1983). Whether or not a bank run equilibrium exists will depend on two key factors: the condition of bank balance sheets and an endogenously determined liquidation price. Thus, a situation can arise where a bank run cannot occur in normal times, but where a severe recession can open up the possibility.

Critical to the possibility of runs is that banks issues demandable short term debt. In our baseline model we simply assume this is the case. We then provide a stronger motivation for this scenario by introducing household liquidity risks, in the spirit of Diamond and Dybvig.

Section 2 presents the model, including both a no-bank run and a bank run equilibria, along with the extension to the economy with household liquidity risks. Section 3 presents a number of illustrative numerical experiments. We illustrate how bank balance sheet behavior affects the cost of credit in the no-bank run equilibrium and how it may open up the possibility of destructive bank run behavior. In our baseline model we restrict attention to unancitipated bank runs. In section 4 we describe the extension to the case of anticipated bank runs. Finally, in section 5 we conclude with a discussion of policies that can reduce the likelihood of bank runs. As in Diamond and Dybvig, there is a role for deposit insurance. However, other possibilities, including commitment by the central bank to lender of last resort activity can also be useful.

2 Basic Model

2.1 Key Features

The framework is a variation of the infinite horizon macroeconomic model with a banking sector and liquidity risks developed in Gertler and Kiyotaki

(2011). There are two classes of agents - households and bankers - with a continuum of measure unity of each type. Bankers intermediate funds between households and productive assets.

There are two goods, a durable asset "capital," and a nondurable good. Capital does not depreciate and is fixed in total supply which we normalize to be unity. We suppose that banks are more efficient than households at screening and monitoring capital, but we also allow households to directly hold capital as well. Let K_t^b be the total capital held by banks and K_t^h be the amount held by households. Then the allocation of capital must satisfy

$$K_t^b + K_t^h = 1. \tag{1}$$

When a banker intermediates K_t^b units of capital in period t, there is a payoff of $Z_{t+1}K_t^b$ units of the nondurable good in period $t+1$ plus the undepreciated leftover capital:

$$\begin{array}{ccc} \textit{date t} & & \textit{date t+1} \\ K_t^b \text{ capital}\} & \rightarrow & \left\{ \begin{array}{c} K_t^b \text{ capital} \\ Z_{t+1}K_t^b \text{ output} \end{array} \right. \end{array} \tag{2}$$

where Z_{t+1} is a multiplicative aggregate shock to productivity.

By contrast, we suppose that households that directly hold capital at t for a payoff at $t+1$ must pay a management cost of $f(K_t^h)$ units of the nondurable goods at t, as follows:

$$\begin{array}{ccc} \textit{date t} & & \textit{date t+1} \\ \left. \begin{array}{c} K_t^h \text{ capital} \\ f(K_t^h) \text{ goods} \end{array} \right\} & \rightarrow & \left\{ \begin{array}{c} K_t^h \text{ capital} \\ Z_{t+1}K_t^h \text{ output} \end{array} \right. \end{array} \tag{3}$$

The management cost is meant to reflect the lack of expertise relative to banks that households have in screening and monitoring investment projects. We suppose that for each household the management cost is increasing and convex in the quantity of capital held:

$$f(K_t^h) = \begin{cases} \frac{\alpha}{2}(K_t^h)^2, \text{ for } K_t^h \leq \overline{K}^h, \\ \alpha\overline{K}^h(K_t^h - \frac{\overline{K}^h}{2}), \text{ for } K_t^h > \overline{K}^h. \end{cases} \tag{4}$$

with $\alpha > 0$. Further, below some threshold $\overline{K}^h \in (0,1)$, $f(K_t^h)$ is strictly convex and then becomes linear after it reaches $\overline{K}^h$. We allow for the kink in the marginal cost to ensure that it remains profitable for households to absorb all the capital in the wake of a banking collapse.

In the absence of financial market frictions, bankers will intermediate the entire capital stock. In this instance, households save entirely in the form of bank deposits. If the banks are constrained in their ability to obtain funds, households will directly hold some of the capital. Further, to the extent that the constraints on banks tighten countercyclically, as will be the case in our model, the share of capital held by households will move countercyclically. In the extreme case of a bank run, households hold the entire capital stock (i.e., $K_t^h = 1$).

As with virtually all models of banking instability beginning with Diamond and Dybvig (1983), a key to opening up the possibility of a bank run is maturity mismatch. Banks issue non-contingent short term liabilities and hold imperfectly liquid long term assets. Within our framework, the combination of financing constraints on banks and inefficiencies in household management of capital will give rise to imperfect liquidity in the market for capital.

We proceed to describe the behavior of households, banks and the competitive equilibrium. We then describe the circumstances under which bank runs are possible. For expositional purposes, we begin by studying a benchmark model where we simply assume that banks issues short term debt. Within this benchmark model we can illustrate the main propositions regarding the possibility of bank runs and the connection to bank balance sheet strength. We then generalize the model to allow for household liquidity risks in the spirit of Diamond and Dybvig in order to motivate why banks issue demandable deposits.

2.2 Households

Each household consumes and saves. Households save by either by lending funds to competitive financial intermediaries or by holding capital directly. In addition to the returns on portfolio investments, each household also receives an endowment of nondurable goods, $Z_t W^h$, every period that varies proportionately with the aggregate productivity shock Z_t.

Intermediary deposits held from t to $t+1$ are one period bonds that pay the certain gross return R_{t+1} in the absence of a bank run. In the event of a bank run, a depositor may receive either the full promised return or nothing, depending on the timing of the withdrawal. Following Diamond and Dybvig (1983), we suppose that deposits are paid out according to a "sequential service constraint." Depositors form a line to withdraw and the bank meets the obligation sequentially until its funds are exhausted. If the bank has insufficient funds to meet its withdrawal requests, a fraction of depositors will be left with nothing. In Basic Model, we assume that bank runs are completely unanticipated events. Thus, we proceed to solve the household's choice problem as if it perceives no possibility of a bank run. Then in a subsequent section we characterize the circumstances under which an unanticipated run might be possible.

Let C_t^h be household consumption, D_t be household bank deposits, and Q_t be the market price of capital. Household utility U_t is given by

$$U_t = E_t \left(\sum_{i=0}^{\infty} \beta^i \ln C_{t+i}^h \right)$$

with $0 < \beta < 1$. The household then chooses consumption C_t^h, direct capital holdings K_t^h and bank deposits D_t to maximize expected utility subject to the budget constraint

$$C_t^h + D_t + Q_t K_t^h + f(K_t^h) = Z_t W^h + R_t D_{t-1} + (Z_t + Q_t) K_{t-1}^h,$$

where, again, we assume that the household assigns a zero probability of a bank run.

The first order conditions for deposits and direct capital holdings are given by

$$E_t(\Lambda_{t,t+1} R_{t+1}) = 1 \tag{5}$$

$$E_t(\Lambda_{t,t+1} R_{kt+1}^h) = 1 \tag{6}$$

where

$$\Lambda_{t,t+i} = \beta^i \frac{C_t^h}{C_{t+i}^h}$$

$$R_{t+1}^h = \frac{Z_{t+1} + Q_{t+1}}{Q_t + f'(K_t^h)} \tag{7}$$

and $f'(K_t^h) = \alpha K_t^h$ for $K_t^h \epsilon (0, \overline{K}^h]$ and $= \alpha \overline{K}^h$ for $K_t^h \epsilon [\overline{K}^h, 1]$. $\Lambda_{t,t+i}$ is the household's marginal rate of intertemporal substitution between consumption at date $t+i$ and t, and R_{t+1}^h is the household's gross marginal rate of return from direct capital holdings.

Observe that so long as the household has at least some direct capital holdings, the first order condition (6) will help determine the market price of capital. Further, the market price of capital tends to be decreasing in the share of capital held by households given that over the range $(0, \overline{K}^h]$, the marginal management cost $f'(K_t^h)$ is increasing. As will become clear, a banking crisis will induce asset sales by banks to households, leading a drop in asset prices. The severity of the drop will depend on the quantity of sales and the convexity of the management cost function. In the limiting case of a bank run households absorb all the capital from banks. Capital prices will reach minimum as the marginal cost reaches a maximum (at $\alpha \overline{K}^h$).

Given the utility specification, one can combine the first order conditions with the budget constraint to obtain the following solutions for household consumption

$$C_t^h = (1-\beta) \left[R_t D_{t-1} + (Z_t + Q_t) K_{t-1}^h + F_t \right] \tag{8}$$

where

$$F_t = Z_t W^h + f'(K_t^h) K_t^h - f\left(K_t^h\right) + E_t(\Lambda_{t,t+1} F_{t+1}). \tag{9}$$

Households consume the fraction $1-\beta$ of total wealth. Here, F_t is the present value of sum of the endowment of nondurable goods and 'profits' from direct capital holdings, measured as the gap between the marginal management cost times direct capital holdings and the total management cost.

2.3 Banks

Each banker manages a financial intermediary. Bankers fund capital investments by issuing deposits to households as well as by using their own equity, or net worth, n_t. Due to financial market frictions, bankers may be constrained in their ability to obtain deposits from households.

To the extent bankers may face financial market frictions, they will attempt to save their way out of the financing constraint by accumulating retained earnings in order to move toward one hundred percent equity financing. To limit this possibility, we assume that bankers have finite expected horizons. In particular, we suppose that each banker has an i.i.d probability

σ of surviving until the next period and a probability $1-\sigma$ of exiting. The expected horizon of a banker is then $\frac{1}{1-\sigma}$. Note that the expected horizon may be long. But it is critical that it is finite.

Every period new bankers enter: The number of entering bankers equals the number who exit, keeping the total population of bankers constant. Each new banker takes over the enterprise of an exiting banker and in the process inherits the skills of the exiting banker. The exiting banker removes his equity stake n_t in the bank. The new banker's initial equity stake consists an endowment w^b of nondurable goods received only in the first period of operation. As will become clear, this setup provides a simple way to motivate "dividend payouts" from the banking system in order to ensure that banks use leverage in equilibrium.

In particular, we assume that bankers are risk neutral and enjoy utility from consumption in the period they exit.[1] Let c_t^b be terminal consumption by an individual banker. Then the expected utility of a continuing banker at the end of period t is given by

$$V_t = E_t\left[\sum_{i=1}^{\infty}\beta^i(1-\sigma)\sigma^{i-1}c_{t+i}^b\right].$$

During each period t a bank finances its asset holdings $Q_t k_t^b$ with deposits d_t and net worth n_t :

$$Q_t k_t^b = d_t + n_t. \tag{10}$$

We assume that banks cannot issue new equity: They can only accumulate net worth via retained earnings. While this assumption approximately accords with reality, we do not explicitly model the agency frictions that underpin it.

The net worth of "surviving" bankers is the gross return on assets net the cost of deposits, as follows:

$$n_t = (Z_t + Q_t)k_{t-1}^b - R_t d_{t-1}. \tag{11}$$

For new bankers at t, net worth simply equals the initial endowment:

$$n_t = w^b.$$

[1]We could generalize to allow active bankers to receive utility that is linear in consumption each period. So long as the banker is constrained, it will be optimal to defer all consumption until the exit period.

Finally, exiting bankers no longer operate banks and simply use their net worth to consume:

$$c_t^b = n_t.$$

To motivate a limit on the bank's ability to obtain deposits, we introduce the following moral hazard problem: At the end of the period the banker can choose to divert the fraction θ of assets for personal use. (Think of the way a banker may divert funds is by paying unwarranted bonuses or dividends to his or her family members.) The cost to the banker is that the depositors can force the intermediary into bankruptcy at the beginning of the next period. The depositors can only recover the fraction of $1-\theta$ of the assets from the defaulting bank. Accordingly, for rational depositors to lend, it must be the case that the franchise value of the bank, i.e., the present discounted value of payouts from operating the bank, V_t, exceeds the gain to the bank from diverting assets. Accordingly, any financial arrangement between the bank and its depositors must satisfy the following incentive constraint[2]:

$$\theta Q_t k_t^b \leq V_t. \tag{12}$$

Given that bankers simply consume their equity when they exit, we can restate the bank's franchise value recursively as the expected discounted value of net worth at the time of exiting, as follows:

$$V_t = E_t[\beta(1-\sigma)n_{t+1} + \beta\sigma V_{t+1}]. \tag{13}$$

The banker's optimization problem then is to choose $\left(k_t^b, d_t\right)$ each period to maximize the franchise value (13) subject to the incentive constraint (12) and the flow of funds constraints (10,11).

We guess that the bank's franchise value is the linear function of assets and deposits as follows

$$V_t = \nu_{kt} k_t^b - \nu_t d_t,$$

and then subsequently verify this guess. Using the flow-of-funds constraint (10), we can express the franchise value as:

$$V_t = \mu_t Q_t k_t^b + \nu_t n_t$$

[2]Note that the incentive constraint embeds the constraint that n_t must be positive for the bank to operate since V_t will turn out to be linear in n_t. We will choose parameters and shock variances that keep n_t non-negative in a "no-bank run" equilibrium. An unanticipated bank run, however, will force n_t to zero, as we show later.

with

$$\mu_t \equiv \frac{\nu_{kt}}{Q_t} - \nu_t.$$

We can think of μ_t as the excess marginal dollar value of assets over deposits. In turn, we can rewrite the incentive constraint as

$$\theta Q_t k_t^b \leq \mu_t Q_t k_t^b + \nu_t n_t.$$

It follows that incentive constraint (12) is binding if and only if the excess marginal value from honestly managing assets μ_t is positive but less than the marginal gain from diverting assets θ, i.e.[3]

$$0 < \mu_t < \theta.$$

Assuming this condition is satisfied, the incentive constraint leads to the following limit on the scale of bank assets $Q_t k_t^b$ to net worth n_t :

$$\frac{Q_t k_t^b}{n_t} = \frac{\nu_t}{\theta - \mu_t} \equiv \phi_t. \tag{14}$$

We refer to ϕ_t as the maximum leverage ratio. It depends inversely on θ; An increase in the bank's ability to divert funds reduces the amount depositors are willing to lend. As the bank expands assets by issuing deposits, its incentive to divert funds increases. The constraint (14) limits the portfolio size to the point where the bank's incentive to cheat is exactly balanced by the cost of losing the franchise value. In this respect the agency problem leads to an endogenous capital constraint.

From equations (10) and (11), the recursive expression of franchise value (13) becomes

$$\mu_t Q_t k_t^b + \nu_t n_t = \beta E_t \left\{ \left[1 - \sigma + \sigma \left(\nu_{t+1} + \phi_{t+1} \mu_{t+1} \right) \right] \left[(R_{t+1}^b - R_{t+1}) Q_t k_t^b + R_{t+1} n_t \right] \right\},$$

where R_{t+1}^b is the realized rate of return on banks assets (i.e. capital intermediated by bank), and is given by

$$R_{t+1}^b = \frac{Z_{t+1} + Q_{t+1}}{Q_t}.$$

[3]In the numerical analysis in section 3, we choose parameters to ensure that the condition $0 < \mu_t < \theta$ is always satisfied in the no bank-run equilibrium.

Using the method of undetermined coefficients, we verify the conjecture that the franchise value is indeed linear in assets and deposits, with

$$\mu_t = \beta E_t[(R^b_{t+1} - R_{t+1})\Omega_{t+1}], \tag{15}$$

$$\nu_t = \beta R_{t+1} E_t (\Omega_{t+1}) \tag{16}$$

with

$$\Omega_{t+1} \equiv 1 - \sigma + \sigma \left(\nu_{t+1} + \phi_{t+1}\mu_{t+1}\right).$$

The coefficient μ_t is the discounted excess return per unit of assets intermediated. The coefficient ν_t is the discounted cost per unit of deposits. In each case the payoffs are adjusted by the variable Ω_{t+1} which takes into account that, if the bank is constrained, the shadow value of a unit of net worth to the bank may exceed unity. In particular, we can think of Ω_{t+1} as a probability weighted average of the marginal values of net worth to exiting and to continuing bankers at t+1. For an exiting banker at $t+1$ (which occurs with probability $1-\sigma$, the shadow value of an additional unit of net worth is simply unity, since he or she just consumes it. Conversely, for a continuing banker (which occurs with probability σ), the shadow value is $\partial V_t/\partial n_t = \nu_t + \mu_t(\partial Q_t k^b_t/\partial n_t) = \nu_t + \mu_t \phi_t$. In this instance an additional unit of net worth saves the banker ν_t in deposit costs and permits he or she to earn the excess value μ_t on an additional ϕ_t units of assets, the latter being the amount of assets he can lever with an additional unit of net worth.

When the incentive constraint is not binding, unlimited arbitrage by banks will push discounted excess returns to zero, implying $\mu_t = 0$. When the incentive constraint is binding, however, limits to arbitrage emerge that lead to positive expected excess returns in equilibrium, i.e., $\mu_t > 0$. Note that the excess return to capital implies that for a given riskless interest rate, the required return to capital is higher than would otherwise be and, conversely the price of capital is lower. Indeed, a financial crisis in the model will involve a sharp increase in the excess rate of returns on asset along with a sharp contraction in the price of asset. In this regards, a bank run will be an extreme version of a financial crisis.

2.4 Aggregation and Equilibrium without Bank Runs

Given that the maximum feasible leverage ratio ϕ_t is independent of individual-specific factors and given a parametrization where the incentive constraint is binding in equilibrium, we can aggregate across banks to obtain the relation between total assets held by the banking system $Q_tK_t^b$ and total net worth N_t :

$$Q_tK_t^b = \phi_t N_t. \tag{17}$$

Summing across both surviving and entering bankers yields the following expression for the evolution of N_t :

$$N_t = \sigma[(Z_t + Q_t)K_{t-1}^b - R_tD_{t-1}] + W^b \tag{18}$$

where $W^b = (1-\sigma)w^b$ is the total endowment of entering bankers. The first term is the accumulated net worth of bankers that operated at $t-1$ and survived to t, equal to the product of the survival rate σ and the net earnings on bank assets $(Z_t + Q_t)K_{t-1}^b - R_tD_{t-1}$. Conversely, exiting bankers consume the fraction $1-\sigma$ of net earnings on assets:

$$C_t^b = (1-\sigma)[(Z_t + Q_t)K_{t-1}^b - R_tD_{t-1}]. \tag{19}$$

Total output Y_t is the sum of output from capital, household endowment Z_tW^h and bank endowment W^b :

$$Y_t = Z_t + Z_tW^h + W^b. \tag{20}$$

Finally, output is either used for management costs, or consumed by households and bankers:

$$Y_t = f(K_t^h) + C_t^h + C_t^b. \tag{21}$$

2.5 Bank Runs

We now consider the possibility of an unexpected bank run. In particular, we maintain assumption that when households acquire deposits at $t-1$ that mature in t, they attach zero probability to a possibility of a run at t. However, we now allow for the chance of a run ex post, as deposits mature at t and households must decide whether to roll them over for another period.

An ex post "run equilibrium" is possible if individual depositors believe that if other households do not roll over their deposits with the bank, the bank may not be able to meet its obligations on the remaining deposits. As in Diamond and Dybvig (1983), the sequential service feature of deposit contracts opens up the possibility that a depositor could lose everything by failing to withdraw. In this situation two equilibria are possible: a "normal" one where households keep their deposits in banks, and a "run" equilibrium where households withdraw all their deposits, banks are liquidated, and households use their residual funds to acquire capital directly.

We begin with the standard case where each depositor decides whether to run, before turning to a more quantitatively flexible case where at any moment only a fraction γ of depositors consider running.

2.5.1 Conditions for a Bank Run Equilibrium

In particular, at the beginning of period t, before the realization of returns on bank assets, depositors decide whether to roll over their deposits with the bank. If they choose to "run", the bank liquidates its capital and turns the proceeds over to households who then acquire capital directly with their less efficient technology. Let Q_t^* be the price of capital in the event of a forced liquidation. Then a run is possible if the liquidation value of bank assets $(Z_t + Q_t^*)K_{t-1}^b$ is smaller than its outstanding liability to the depositors:[4]

$$(Z_t + Q_t^*)K_{t-1}^b < R_t D_{t-1}. \tag{22}$$

If condition (22) is satisfied, an individual depositor who does not withdraw sufficiently early could lose everything in the even of run. If any one depositor faces this risk, then they all do, which makes a run equilibrium feasible. If the inequality is reversed, banks can always meet their obligations to depositors, meaning that runs cannot occur in equilibrium.[5]

The condition determining the possibility of a bank run depends on two key endogenous variables, the liquidation price of capital Q_t^* and the condition of bank balance sheets. Combining the bank funding constraint (10) with (22) implies we can restate the condition for a bank run equilibrium as

$$(Z_t + Q_t^* - R_t Q_{t-1})K_{t-1}^b + R_t N_{t-1} < 0.$$

This condition states that a bank run is possible if depositors perceive that conditional on liquidation of assets, net worth of the bank system would be negative. We can rearrange this to obtain a simple condition for a bank run equilibrium in terms of just three variables:

[4] Since banks are homogenous in $n_,$ the conditions for a run on the system are the same as for a run on any individual bank.

[5] We assume that there is a small cost of running to the bank, and that households will not run if their bank will pay the promised deposit return for sure.

$$R_t^{b*} \equiv \frac{Z_t + Q_t^*}{Q_{t-1}} < R_t(1 - \frac{1}{\phi_{t-1}}) \tag{23}$$

where ϕ_{t-1} is the bank leverage ratio at $t-1$. A bank run equilibrium is more likely the lower the realized rate of return on bank assets R_t^{b*} relative to the gross interest rate on deposits R_t and the higher is the leverage ratio. Note that the expression $(1 - \frac{1}{\phi_{t-1}})$ is the ratio of bank deposits D_{t-1} to bank capital $Q_{t-1}K_{t-1}^b$, which is increasing in the leverage ratio.

Since R_t^{b*}, R_t and ϕ_t are all endogenous variables, the possibility of a bank run may vary with macroeconomic conditions. The equilibrium absent bank runs (that we described earlier) determines the behavior of R_t and ϕ_t. The behavior of R_t^{b*} is increasing in the liquidation price Q_t^*, which depends on the behavior of the economy.

Finally, we now turn to a more flexible case that we use in the quantitative analysis, where at each time t only a fraction γ of depositors consider running. Here the idea is that not all depositors are sufficiently alert to market conditions to contemplate running. This scenario is consistent with evidence that during a run only a fraction of depositors actually try to quickly withdraw.

In this situation, a run is possible if any individual depositor who is considering a run perceives the bank cannot meet the obligations of the group that could potentially withdraw. Thus, assuming individual depositors who might run know γ, a run is possible if the following condition is satisfied

$$(Z_t + Q_t^*)K_{t-1}^b < \gamma R_t D_{t-1}$$

which, proceeding as before, can be expressed as

$$R_t^{b*} < \gamma R_t(1 - \frac{1}{\phi_{t-1}}). \tag{24}$$

Thus, in the condition for the possibility of a bank run equilibrium, the deposit rate is adjusted by the fraction γ of depositors who could potentially run.

2.5.2 The Liquidation Price

To determine Q_t^* we proceed as follows. A depositor run at t induces all banks that carried assets from $t-1$ to fully liquidate their asset positions and go out of business. New banks do not enter. Given our earlier assumption that new bankers can operate only by taking over functioning franchises of exiting bankers, the collapse of existing banks eliminates the possibility of transferring the necessary skill and apparatus to new bankers.[6]

Accordingly, when banks liquidate, they sell all their assets to households. In the wake of the run at date t and for each period after, accordingly:

$$1 = K_{t+i}^{h}, \text{ for all } i \geq 0, \tag{25}$$

where, again, unity is the total supply of capital. Further, since banks no longer are operating, entering bankers simply consume their respective endowments:

$$C_t^b = W^b.$$

The consumption of households is then the sum of their endowment and the returns on their capital net of management costs:

$$C_t^h = Z_t W^h + Z_t - f(1) \tag{26}$$

where the last term on the right is household portfolio management costs, which are at a maximum in this instance given that the household is directly holding the entire capital stock.

Let $R_{t+1}^{h^*}$ be the household's marginal return on capital from t to $t+1$ when banks have collapsed at t. Then the first order condition for household direct capital holding is given by

$$E_t\{\Lambda_{t,t+1} R_{t+1}^{h*}\} = 1$$

with

$$R_{t+1}^{h*} = \frac{Z_{t+1} + Q_{t+1}^*}{Q_t^* + \alpha \overline{K}^h}$$

where $\alpha \overline{K}^h$ is the marginal portfolio management cost when households are absorbing all the capital (see equation (4)).[7] Rearranging yields the following

[6] This assumption is for purely technical reasons: It makes the liquidation price easy to calculate. If we allowed new banks to enter, (under any reasonable calibration), the banking system would eventually recover, but it would be a very long and slow process. Under either approach, the near term behavior of liquidation prices would be simiar.

[7] When there are bank runs at date t, consumption and saving on capital and deposit are different across households, depending upon the timing of withdrawal. But we consider

expression for the liquidation price in terms of discounted dividends net the marginal management cost.

$$Q_t^* = E_t \left[\sum_{i=1}^{\infty} \Lambda_{t,t+i} (Z_{t+i} - \alpha \overline{K}^h) \right] - \alpha \overline{K}^h. \tag{27}$$

Everything else equal, the higher the marginal management cost the lower the liquidation price. Note Q_t^* as well as ϕ_t will vary with cyclical conditions. Thus, even if a bank run equilibrium does not exist in the neighborhood of the no-run steady state, it is possible that a sufficiently negative disturbance to the economy could open up this possibility. We illustrate this point in Section 3 below.

Finally, we observe that within our framework the distinction between a liquidity shortage and insolvency is more subtle than is often portrayed in popular literature. If a bank run equilibrium exists, banks become insolvent, i.e. their liabilities exceed their assets if assets are valued at the fire-sale price Q_t^*. But if assets are valued at the price in the no-run equilibrium Q_t, the banks are all solvent. Thus whether banks are insolvent or not depends upon equilibrium asset prices which in turn depend on the liquidity in the banking system; and this liquidity can change abruptly in the event of a run. As a real world example of this phenomenon consider the collapse of the banking system during the Great Depression. As Friedman and Schwartz (1963) point out that, what was initially a liquidity problem in the banking system (due in part by inaction of the Fed), turned into a solvency problem as runs on banks led to a collapse in long-term security prices and in the banking system along with it.

2.6 Household liquidity risks

Up to this point we have simply assumed that banks engage in maturity mismatch by issuing non-contingent one period deposits despite holding risky long maturity assets. We now motivate why banks might issue liquid short term deposits. In the spirit of Diamond and Dybvig (1983), we do so by introducing idiosyncratic household liquidity risks, which creates a desire by households for demandable debt. We do not derive these types of deposits

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that each household pays the same management fee ($\alpha\overline{K}$) for every unit of capital purchase. The profit of management company $\alpha\overline{K} - f(1)$ is distributed to all the households lump sum as in (9) . Then the marginal rate of substitution $\Lambda_{t,t+i}$ is the same across households.

from an explicit contracting exercise. However, we think that a scenario with liquidity moves us one step closer to understanding why banks issue liquid deposits despite having partially illiquid assets.

As before, we assume that there is a continuum of measure unity of households. To keep the heterogeneity introduced by having independent liquidity risks manageable, we further assume that each household consists of a continuum of unit measure individual members.

Each member of the representative household has a need for emergency expenditures within the period with probability π. At the same time, because the household has a continuum of members, exactly the fraction π has a need for emergency consumption.

In particular, let c_t^m be emergency expenditures by an individual member, with $\pi c_t^m = C_t^m$ being the total emergency expenditures by the family. For an individual with emergency expenditures needs, period utility is given by

$$\log C_t^h + \kappa \log c_t^m,$$

where C_t^h is regular consumption. For family members that do not need to make emergency expenditures, period utility is given simply by

$$\log C_t^h.$$

Because they are sudden, we assume that demand deposits at banks are necessary to make emergency expenditures above a certain threshold.

The timing of events is as follows: At the beginning of period t, before the realization of the liquidity risk during period t, the household chooses C_t^h and the allocation of its portfolio between bank deposits D_t and directly held capital $K_{t.}^h$ subject to the flow-of fund constraint:

$$C_t^h + D_t + Q_t K_t^h + f(K_t^h) = R_t D_{t-1} + (Z_t + Q_t) K_{t-1}^h + Z_t W^h - \overline{C}_t^m,$$

where the last term $\overline{C}_t^m$ is the expected sales of household endowment to meet the emergency expenditure of the other households (which is not realized yet at the beginning of period). The household plans the date-t regular consumption $\left(C_t^h\right)$ to be the same for every member since all members of the household are identical ex ante and utility is separable in C_t^h and c_t^m. After choosing the total level of deposits, the household divides them evenly amongst its members. During period t, an individual has access only to his or her own deposits at the time the liquidity risk is realized. Those having to make emergency expenditures above some threshold $\underline{c}^m$ must finance them from their deposits accounts at the beginning of t,[8]

$$c_t^m - \underline{c}^m \leq D_t. \tag{28}$$

Think of $\underline{c}^m$ as the amount of emergency expenditure that can be arranged through credit as opposed to deposits.[9] After the realization of the liquidity shock, individuals with excess deposits simply return them to the household. Under the symmetric equilibrium, the expected sales of household endowment to meet the emergency expenditure of the other households $\overline{C}_t^m$ is equal to the emergency expenditure of the representative household πc_t^m, and the deposit at the end of period D_t' is

$$D_t' = \pi(D_t - c_t^m) + (1-\pi)D_t + \overline{C}_t^m = D_t,$$

and equal to the deposit at the beginning of period. Thus the budget constraint of the household is given simply by

$$C_t^h + \pi c_t^m + D_t + Q_t K_t^h + f(K_t^h) = R_t D_{t-1} + (Z_t + Q_t)K_{t-1}^h + Z_t W^h. \tag{29}$$

The next sequence of optimization then begins at the beginning of period $t+1$.

We can express the formal decision problem of the household with liquidity risks as follows:

$$U_t(D_{t-1}, K_{t-1}^h) = \max_{C_t^h, c_t^m, D_t, K_t^h} \{(\log C_t^h + \pi\kappa \log c_t^m + \beta E_t[U_{t+1}(D_t, K_t^h)]\}$$

subject to the budget constraint (29) and the liquidity constraint (28).

Let χ_t be the Lagrangian multiplier on the liquidity constraint. Then the first order conditions for deposits D_t and emergency expenditures are given by:

$$E_t\{\Lambda_{t,t+1}R_{t+1}\} + \pi\frac{\chi_t}{1/C_t^h} = 1, \tag{30}$$

[8]One can think each member carrying a deposit certificate of the amount D_t. Each further is unable to make use of the deposit certificates of the other members of the family for his or her emergency consumption.

[9]We allow for $\underline{c}^m$ so that households can make some emergency expenditures in a bank run equilibrium, which keeps the marginal utility of c^m from going to infinity in this case.

$$\frac{\kappa}{c_t^m} - \frac{1}{C_t^h} = \chi_t. \tag{31}$$

The multiplier on the liquidity constraint χ_t is equal to the gap between the marginal utility of emergency consumption and regular consumption for a household member who experiences a liquidity shock. Observe that if the liquidity constraint binds, there is a relative shortage of the liquid asset, which pushes down the deposit rate, everything else equal, as equation (30) suggests.

The first order condition for the households choice of direct capital holding is the same as in the case without liquidity risks (see equation (6)). The decision problem for banks is also the same, as are the conditions for aggregate bank behavior.

In the aggregate (and after using the bank funding condition to eliminate deposits), the liquidity constraint becomes:

$$C_t^m - \pi \underline{c}^m \leq \pi(Q_t K_t^b - N_t).$$

Given that households are now making emergency expenditures, the relation for uses of output becomes

$$Y_t = C_t^h + C_t^m + C_t^b + f(K_t^h). \tag{32}$$

Otherwise, the remaining equations that determine the equilibrium without liquidity risks (absent bank runs) also applies in this case.

Importantly the condition for a bank run (equation 23) also remains unchanged. The determination of the liquidation price is also effectively the same (see equation 27). There is one minor change, however: The calculation of Q_t^* is slightly different since households are now making emergency expenditures c_t^m, in addition to consuming C_t^h.

3 Numerical Examples

Our goal here is to provide some suggestive numerical examples to illustrate the workings of the model. Specifically we construct an example where a recession tightens bank balance sheet constraints, which leads to an "excessive" drop in asset prices and opens up the possibility of a bank run. We then illustrate the effects of an unanticipated run. We first present results for our baseline model and then do the same for the model with liquidity risks.

3.1 Parameter Choices

Table 1 lists the choice of parameter values for our baseline model, while Table 2 gives the steady state values of the endogenous variables. Overall there are nine key parameters in the baseline model and an additional two in the model with liquidity risks. Two parameters in the baseline are conventional: the quarterly discount factor β which we set at 0.99 and the serial correlation of the productivity shock Z_t which we set at 0.95. Seven parameters $(\sigma, \theta, W^b, \alpha, \overline{K}^h, \gamma, W^h)$ are specific to our model. Our choice of these parameters is meant to be suggestive. We set the banker's survival probability σ equal to 0.93 which implies an expected horizon of three and half years. We choose values for the seizure rate parameter θ, the banker's initial endowment W^b, and the "management cost parameter α to hit the following targets in the steady state absent bank runs: a bank leverage ratio ϕ of eight, the annual spread between the the expected return on capital and the riskless rate of 240 basis points, and a steady state allocation of capital holdings such direct holdings by the household are small, e.g., roughly five percent. We set $\overline{K}^h$ based on two considerations: It is high enough so that K_t^h within a local region of the steady state stays below it (so we can use loglinear numerical methods within a local region of the steady state in the no run case). At the same time it is low enough to ensure that households find it profitable to directly hold capital in the bank run equilibrium. Finally, we set the fraction of depositors who may run at any moment to 0.7, which makes it feasible to have a steady state without a bank run equilibrium with the possibility of a run equilibrium in the recession. We set the household steady state endowment ZW^h (which roughly corresponds to labor income) to three times steady state capital income Z. We also normalize the steady state price of a unit of capital Q_t at unity, which restricts the steady value of Z_t (which determined output stream from capital).

Finally, for the model with liquidity risks we use the same parametrization as in our baseline case. There are, however, three additional parameters $(\kappa, \pi, \underline{c}_m)$. We choose these parameters to ensure that (i) the steady spread between the households net return on capital R^h and the deposit R rate is fifty basis points at an annual level, and (ii) households can still makes some emergency expenditures in an bank run equilibrium. For all other parameters, we use the same values as in the baseline case with one exception: we adjust the fraction of depositors who may run so that the steady state cost per deposit to the bank in the event of a run, γR is the same as in the baseline case.[10] Roughly speaking, for a given set of parameter values, this makes the likelihood of a run within a local region of the steady state the same in both cases.

3.2 Recessions, Banking Distress and Bank Runs: Some Simulations

We have parametrized the model so that a bank run equilibrium does not exist in the steady state but could arise if the economy enters a recession. We begin by analyzing the response of the economy to a negative shock to Z_t assuming the economy stays in the "no bank run" equilibrium. We then examine the effects of an unanticipated bank run, once the economy enters a region where runs are possible. For each case we first examine the baseline model and then turn to the model with liquidity risks.

Figure 1 shows the response of the baseline model to an unanticipated negative five percent shock to productivity, Z_t. This leads to a drop in output of roughly five percent, a magnitude which is characteristic of a major recession. Though a bank run does not arise in this case, the recession induces financial distress that amplifies the contraction in assets prices and raises the cost of bank credit. The unanticipated drop in Z_t reduces net worth N_t which tightens bank balance sheets, leading to a contraction of bank deposits and a firesale of bank assets, which in turn magnifies the asset price decline. Households absorb some of the asset, but because this is costly for them, the amount they acquire is limited. The net effect is a substantial increase in the cost of bank credit: the spread between the required expected return to bank assets and the riskless rate increases one hundred and fifty basis points on impact. Overall, the recession induces the kind of bank balance sheet/financial accelerator mechanism prevalent in Gertler and Kiyotaki (2011) and other macroeconomic models of bank distress.

Figure 2 repeats the same experiment as in Figure 1, this time examining the model with liquidity risks. The overall impact on the asset price and the cost of bank credit is similar to the baseline case. One interesting difference, however, is that unlike the baseline, there is a sharper drop in the deposit interest rates relative to the baseline. In the baseline, the deposit interest rate drops one hundred basis points on impact. In this case, the drop is almost

[10] The liquidity premium makes R lower in the case with liquidity risks than in the baseline case, which everything else equal, makes the likliehood of a run lower.

four times as much as the baseline for two reasons. First, the contraction in bank deposits raises the liquidity premium for deposits, which increases the spread between the expected rate of return on household's capital $E_t(R^h_{t+1})$ and the deposit rate. Second, $E_t(R^h_{t+1})$ drops by more in the baseline model as households now smooth the decline in C^h_t as they cut back sharply on C^m_t.[11]

We now allow for the possibility of bank runs. To determine whether a bank equilibrium exists, we first define $\overline{Q}_t$ as the threshold value of the liquidation price below which a bank run equilibrium exists. It follows from equation (24) that $\overline{Q}_t$ is given by

$$\overline{Q}_t = \gamma R_t(1 - \frac{1}{\phi_{t-1}})Q_{t-1} - Z_t. \tag{33}$$

Note that $\overline{Q}_t$ is increasing in ϕ_{t-1}, which implies that everything else equal a bank run equilibrium is more likely the higher is bank leverage at $t-1$. We next construct a variable called "run" that is the difference between $\overline{Q}_t$ and the liquidation price Q^*_t :

$$run_t = \overline{Q}_t - Q^*_t. \tag{34}$$

A bank run equilibrium exists iff

$$run_t > 0.$$

In the steady of our model $run < 0$, implying a bank run equilibrium does not exist in this situation. However, the recession opens up the possibility of $run_t > 0$, by simultaneously raising $\overline{Q}_t$ and lowering Q^*_t.

Figure 3 revisits the recession experiment for the baseline model, this time allowing for a bank run ex post. The first panel of the middle row shows that the run variable becomes positive upon impact and remains positive for roughly ten quarters. An unanticipated bank run is thus possible at any point in this interval. The reason the bank run equilibrium exists is that the negative productivity shock reduces the liquidation price Q^*_t and leads to an increase in the bank's leverage ratio ϕ_t (as bank net worth declines relative to assets). Both these factors work to make the banking system vulnerable to a run, as equations (33) and (34) indicate.

[11] From the household's Euler equation, $E_t(R^h_{t+1})$ depends positively on the growth of C^h_t.

In Figure 3 we suppose an unanticipated run occurs in the second period after the shock. The solid line portrays the bank run while the dotted line tracks the no-bank run equilibrium for reference. The run produces a complete liquidation of bank assets as K_t^b drops to zero. The asset price falls to it liquidation price which is roughly forty percent below the steady state. Output net of household capital management costs drops roughly twenty percent. The high management costs arise because in the absence of the banking system, households are directly holding the entire capital stock. The reduction of net output implies that household consumption drops roughly ten percent on impact. Bankers consumption drops nearly to zero as existing bankers are completely wiped out and new bankers are only able to consume their endowment.

Finally, Figure 4 repeats the experiment for the model with liquidity risks. The behavior of the economy in the wake of a bank run is very similar to the baseline case. One difference is that the time interval over which a bank run equilibrium is possible is shorter. This occurs because the drop in the deposit rate following the recessionary shock is greater than in the baseline case (see Figure 2) which reduces the likelihood that the conditions for a bank run equilibrium will be met.

4 Anticipated Bank Runs

So far, we have analyzed the existence and properties of an equilibrium with a bank run when the run is not anticipated. We now consider what happens if people expect a bank run will occur with a positive probability in future.

Define the recovery rate in the event of a bank next period, x_{t+1},as the ratio of the realized return on the bank assets to the promised deposit return in the event of bank-run, as follows:

$$x_{t+1} = \frac{(Q_{t+1}^* + Z_{t+1})k_t^b}{\overline{R}_{t+1}d_t}$$

where as before Q_{t+1}^* is the liquidation price of capital during the run. The recovery rate can be rewritten as a function of the rate of return on bank assets during the run and the leverage ratio of the previous period:

$$\begin{aligned} x_{t+1} &= \frac{R^{b*}_{t+1}}{\overline{R}_{t+1}} \cdot \frac{Q_t k^b_t}{Q_t k^b_t - n_t} \\ &= \frac{R^{b*}_{t+1}}{\overline{R}_{t+1}} \cdot \frac{\phi_t}{\phi_t - 1}. \end{aligned} \tag{35}$$

The realized rate of return on deposit depends upon whether the run occurs as well as the depositor's position in during the run, as

$$R_{t+1} = \begin{cases} \overline{R}_{t+1} \text{ if no bank run} \\ \overline{R}_{t+1} \text{ with probability } x_{t+1} \text{ if run occurs} \\ 0 \text{ with probability } 1 - x_{t+1} \text{ if run occurs} \end{cases}$$

Because we assumed that depositors will not run if they always receive the same return, the equilibrium with run exists if and only if the recovery rate is less than one.

Continue to assume that each household consists of a continuum of members and that when a run occurs, exactly the fraction x_{t+1} of the members receives the promised return on deposit. Then, the first order conditions for the household's consumption and portfolio choices implies (6) and

$$1 = \overline{R}_{t+1} E_t \left[(1 - \iota_{t+1}) \Lambda_{t,t+1} + \iota_{t+1} x_{t+1} \Lambda^*_{t,t+1} \right] \tag{36}$$

where ι_{t+1} is the indicator function which is equal to 1 if the run occurs and equal to 0 otherwise. $\Lambda_{t,t+1}$ and $\Lambda^*_{t,t+1}$ are the marginal rates of substitution between consumption of dates $t+1$ and t in the equilibrium without and with a run: $\Lambda_{t,t+1} = \beta C^h_{t+1}/C^h_t$ and $\Lambda^*_{t,t+1} = \beta C^{h*}_{t+1}/C^h_t$. From (35) and (36), we get

$$\overline{R}_{t+1} = \frac{1 - E_t \left(\iota_{t+1} \Lambda^*_{t,t+1} R^{b*}_{t+1} \right) \frac{\phi_t}{\phi_t - 1}}{E_t \left[(1 - \iota_{t+1}) \Lambda_{t,t+1} \right]}. \tag{37}$$

Observe that the promised rate of return on deposit is a decreasing function of the leverage rate ϕ_t as the recovery rate is decreasing in the leverage rate.

The bank chooses the balance sheet $\left(k^b_t, d_t, n_t\right)$ to maximize the objective V_t subject to the existing constraints $(10, 11, 12, 13)$ and the constraint on the promised rate of return on deposits (37). Because the objective and constraints of the bank is constant returns to scale, we can rewrite the bank's problem to choose the leverage ratio ϕ_t to maximize the value per unit of net worth as

$$\begin{aligned}\psi_t &= \frac{V_t}{n_t} = \max_{\phi_t} \beta E_t \left\{ (1-\sigma+\sigma\psi_{t+1}) \frac{n_{t+1}}{n_t} \right\} \\ &= \max_{\phi_t} \beta E_t \{ (1-\sigma+\sigma\psi_{t+1})(1-\iota_{t+1})[(R^b_{t+1} - \overline{R}_{t+1})\phi_t + \overline{R}_{t+1}] \}\end{aligned}$$

subject to the incentive constraint $\psi_t \geq \theta\phi_t$.

Using (37) to eliminate $\overline{R}_{t+1}$ from the objective, we have

$$\begin{aligned}\psi_t &= \max_{\phi_t} \beta E_t \left\{ \Omega_{t+1}(1-\iota_{t+1}) \left(R^b_{t+1}\phi_t - \frac{\phi_t - 1 - \phi_t E_t(\iota_{t+1}\Lambda^*_{t,t+1}R^{b*}_{t+1})}{E_t[(1-\iota_{t+1})\Lambda_{t,t+1}]} \right) \right\} \\ &= \max_{\phi_t} (\nu_t + \mu_t\phi_t),\end{aligned}$$

where

$$\nu_t = \frac{\beta E_t\left[(1-\iota_{t+1})\Omega_{t+1}\right]}{E_t[(1-\iota_{t+1})\Lambda_{t,t+1}]}, \text{ and} \tag{38}$$

$$\mu_t = \beta E_t \left[\Omega_{t+1}(1-\iota_{t+1}) \left(R^b_{t+1} - \frac{1 - E_t(\iota_{t+1}\Lambda^*_{t,t+1}R^{b*}_{t+1})}{E_t[(1-\iota_{t+1})\Lambda_{t,t+1}]} \right) \right] \tag{39}$$

with $\Omega_{t+1} = 1-\sigma+\sigma\psi_{t+1}$. Note that we need to check whether the discounted excess return on asset μ_t remains positive with the anticipated bank run, in order to show the incentive constraint binds

$$\psi_t = \nu_t + \mu_t\phi_t = \theta\phi_t,$$

or

$$\phi_t = \frac{\nu_t}{\theta - \mu_t}. \tag{40}$$

In the Appendix, we examine the conditions under which banks maximize the leverage ratio subject to the incentive constraint.

In the aggregate equilibrium in which all the banks maximize the leverage ratio, we have

$$Q_t K^b_t = \phi_t N_t,$$

where the evolution of the net worth is

$$N_t = (1-\iota_t)\left\{\sigma\left[(Z_t+Q_t)K^b_{t+1} - \overline{R}_t D_{t-1}\right] + W^b\right\}. \tag{41}$$

In order to examine the equilibrium in which banks continue to maximize the leverage with an anticipated run, we consider a simple case in which the recovery rate with run will be less than 100% and the bank-run equilibrium will exist at date $t+1$ for any realization of aggregate productivity Z_{t+1} as long as all the banks maximize the leverage at date t. We also assume that the occurrence of the bank-run equilibrium follows an exogenous AR(1) process

$$Prob_t\left(\iota_{t+1}=1 \mid Z_{t+1}\right)=p_t \tag{42}$$

$$p_t=\rho_p p_{t-1}+\varepsilon_{pt}, \text{ where } \varepsilon_{pt} \text{ is iid.}$$

Under assumption (42) , the household condition for deposit (36) becomes

$$1=\overline{R}_{t+1}[(1-p_t)E_t(\Lambda_{t,t+1})+p_t E_t(\Lambda^*_{t,t+1}x_{t+1})].$$

Because the recovery rate during run x_{t+1} is always less than 100%, as long as the marginal rate of substitution in the bank-run equilibrium is not too large so that

$$E_t\left(\Lambda^*_{t,t+1}x_{t+1}\right)<E_t(\Lambda_{t,t+1}),$$

the promised rate of return on deposit is an increasing function of the likelihood of the bank run equilibrium in order to compensate for the depositor's loss in the bank-run equilibrium. Also the discounted excess value of bank asset becomes

$$\mu_t=\beta(1-p_t)E_t[\Omega'_{t+1}(R^b_{t+1}-\overline{R}_{t+1})].$$

where Ω'_{t+1} is the value of Ω_{t+1} in the equilibrium without bank-run. Thus μ_t is a decreasing function of likelihood of bank-run equilibrium of future periods.

Therefore an increase in the likelihood of bank-run contracts banking and aggregate production in two ways. First, the promised rate of return on deposits must increase and the aggregate net worth of banks will deteriorate even a the bank-run does not occur. Secondly, the franchise value and the leverage ratio decrease with a higher likelihood of bank-run in future, which leads to smaller intermediation and lower aggregate output.

5 Conclusion

We end with some remarks about government financial policy. As in Diamond and Dybvig (1983) the existence of the bank run equilibrium introduces a role for deposit insurance. The belief that deposits will be insured eliminates the incentive to run in our framework just as it does in the original Diamond/Dybvig setup. If all goes well, further, the deposit insurance need never be used in practice. By being available, it serves its purpose simply by ruling out beliefs that could lead to a bank run.

Of course, as is well understood, moral hazard considerations (not present in our current framework) could produce negative side effects from deposit insurance. The solution used in practice is to combine capital requirements with deposit insurance. As many authors have pointed out, capital requirements help offset the incentives for risk-taking that deposit insurance induces. Within our framework, capital requirements have an added benefit: they reduce the size of the region where bank run equilibria are feasible. This occurs because the possibility of a bank run equilibrium is increasing in the leverage ratio. If equity capital is costly for banks to raise, as appears to be the case in practice, then capital requirements may also have negative side effects. A number of authors have recently pointed that the "stabilizing" effects of capital requirements may be countered to some degree by the increased cost of intermediation (to the extent raising equity capital is costly).

Our model suggests that a commitment by the central bank to lender of last resort policies may also be useful. Within our framework the endogenously determined liquidation price of bank assets is a key determinant of whether a bank run equilibrium exists. In this regard, a commitment to use lender of last resort policies to support liquidation prices could be useful. For example, central bank asset purchases that support the secondary market prices of bank assets might be effective in keeping liquidation prices sufficiently high to rule out the possibility of runs. (Consider the recent purchases by the Federal Reserve of agency mortgaged-backed securities, in the wake of the collapse of the shadow banking system.) An issue to investigate is whether by signalling the availability of such a tool in advance of the crisis, the central bank might have been able to reduce the likelihood of runs on the shadow banks ex post. Gertler and Karadi (2011) and Gertler and Kiyotaki (2011) explore the use of this kind of policy tool in a macroeconomic model of banking distress but without bank runs.[12] It would be useful to extend the analysis of this policy tool and related lender of last resort policies to a

[12] See also Gertler, Kiyotaki and Queralto (2011) which takes into account the moral hazard effects of lender of last resort policies by endogenizing the risk of bank liability structure.

setting with bank runs.

6 Appendix

Under (42), (38) and (39) are simplified to

$$\nu_t = \frac{\beta E_t(\Omega'_{t+1})}{E_t(\Lambda_{t,t+1})}$$

$$\mu_t = \beta(1-p_t)E_t[\Omega'_{t+1}(R^b_{t+1} - \overline{R}_{t+1})].$$

where Ω'_{t+1} is the value of Ω_{t+1} in the equilibrium without bank-run.

In order to show that an individual bank chooses the maximal leverage, we also have to check the global conditions. Suppose first that an individual bank does not issue deposits. It just invests out of its net worth and thus is able to function in the event of a bank run:

$$\psi^o_t = \beta E_t\{(1-\iota_{t+1})(1-\sigma+\sigma\psi^o_{t+1})R^b_{t+1} + \iota_{t+1}(1-\sigma+\sigma\psi^{**}_{t+1})R^{b*}_{t+1}\}$$

where ψ^{**}_{t+1} is value conditional on a bank run. We suppose that the bank is "small", so that conditional on a bank run, it is still the case that $Q_t = Q^*_t$, with

$$\psi^{**}_t = \nu^{**}_t + \mu^{**}_t\phi^{**}_t = \theta\phi^{**}_t$$

$$\nu^{**}_t = \beta E_t(\Omega^{**}_{t+1}\overline{R}^{**}_{t+1})$$

$$\mu^{**}_t = \beta E_t\Omega^{**}_{t+1}(R^{b**}_{t+1} - \overline{R}^{**}_{t+1})$$

$$R^{b**}_{t+1} = \frac{Z_t + Q^*_{t+1}}{Q^*_t}$$

$$\overline{R}^{**}_{t+1} = 1/E_t(\Lambda^{**}_{t,t+1})$$

$$\phi^{**}_t = \frac{\nu^{**}_t}{\theta - \mu^{**}_t}$$

$$\Omega^{**}_{t+1} = 1-\sigma+\sigma\theta\phi^{**}_{t+1}$$

References

[1] Adrian, T., Colla, P., and Shin, H., 2012. Which Financial Frictions? Paring the Evidence from Financial Crisis of 2007-9. Forthcoming in NBER Macroeconomic Annual.

[2] Allen, F., and Gale, D., 2007. Understanding Financial Crises. Oxford University Press.

[3] Bernanke, B., 2010. Causes of the Recent Financial and Economic Crisis. Statement before the Financial Crisis Inquiry Commission, Washington, September 2.

[4] Bernanke, B., and Gertler, M., 1989. Agency Costs, Net Worth and Business Fluctuations. American Economic Review 79, 14-31.

[5] Bigio, S., 2012. Financial Risk Capacity. Mimeo, NYU.

[6] Brunnermeier, M. K., and Sannikov, Y., 2011. A Macroeconomic Model with a Financial Sector. Mimeo, Princeton University.

[7] Brunnermeier, M.K., and Oehmke, M., 2012.. Bubbles, Financial Crises and Systemic Risk. Mimeo, Princeton University.

[8] Calomiris, C., and Kahn, C., 1991. The Role of Demandable Debt in Structuring Banking Arrangements. American Economic Review 81, 497-513.

[9] Diamond, D., and Dybvig, P., 1983. Bank Runs, Deposit Insurance, and Liquidity. Journal of Political Economy 91, 401-419.

[10] Diamond, D., and Rajan, R., 2000. A Theory of Bank Capital, Journal of Finance..

[11] Friedman, M., and Schwartz, A., 1963. A Monetary History of the United States, 1867-1960. Princeton University Press.

[12] Gertler, M., and Karadi, P., 2011. A Model of Unconventional Monetary Policy, Journal of Monetary Economics, January.

[13] Gertler, M., and Kiyotaki, N., 2010. Financial Intermediation and Credit Policy in Business Cycle Analysis. In Friedman, B., and Woodford, M. (Eds.), Handbook of Monetary Economics. Elsevier, Amsterdam, Netherlands.

[14] Gertler, M., Kiyotaki, N., and Queralto, A., 2011. Financial Crises, Bank Risk Exposure and Government Financial Policy. Mimeo, NYU and Princeton University.

[15] Gilchrist, S., Yankov, V., and Zakrasjek, E., 2009. Credit Market Shocks and Economic Fluctuations: Evidence from Corporate Bond and Stock Markets. Mimeo, Boston University.

[16] Gorton, G., 2010. Slapped by the Invisible Hand: The Panic of 2007. Oxford University Press.

[17] He. Z. and A. Krishnamurthy, 2012. Intermediary Asset Pricing. Mimeo, Northwestern University.

[18] Holmstrom, B. and J. Tirole, 1997. Financial Intermediation, Loanable Funds, and the Real Sector. Quarterly Journal of Economics 112, 663-692.

[19] Kiyotaki, N., and Moore, J., 1997. Credit Cycles. Journal of Political Economy 105, 211-248.

[20] Martin, A., D. Skeie and E.L. Von Thadden. Repo Runs. Mimeo, Federal Reserve Bank of New York.

[21] Roch, Francisco and Harald Uhlig, 2012. The Dynamics of Sovereign Debt Crises in a Monetary Union. Mimeo, University of Chicago.

Table 1: Parameters

Baseline Model		
β	0.99	Discount rate
σ	0.95	Bankers survival probability
θ	0.35	Seizure rate
α	0.1	Household managerial cost
$\bar{K}^h$	0.096	Threshold capital for managerial cost
γ	0.72	Fraction of depositors that can run
ρ	0.95	Serial correlation of productivity shock
Z	0.0161	Steady state productivity
ω^b	0.0019	Bankers endowment
ω^h	0.045	Household endowment
Additional Parameters for Liquidity Model		
κ	62.67	Preference weight on c_m
$\bar{c}^m$	0.01	Threshold for c_m
π	0.03	Probability of a liquidity shock
γ_L	0.67	Fraction of depositors that can run

Table 2: Steady State Values

Steady State for No Bank-Run Equilibrium		
	Baseline	Liquidity
K	1	1
Q	1	1
C^h	0.0541	0.0184
C^m	0	0.0348
C^b	0.0087	0.0088
K^h	0.0594	0.0545
K^b	0.9406	0.9455
ϕ	8	8
R^b	1.0644	1.0624
R^h	1.0404	1.0404
R	1.0404	1.0384
Steady State for Bank-Run Equilibrium		
	Baseline	Liquidity
K	1	1
Q^*	0.6340	1
C^h	0.0520	0.0515
C^m	0	0.01
C^b	0.0019	0.0019
K^h	1	1
K^b	0	0
ϕ		
R^b	1.1016	1.1068
R^h	1.0404	1.0404
R		

Figure 1: A Recession in the Baseline Model: No Bank Run Case

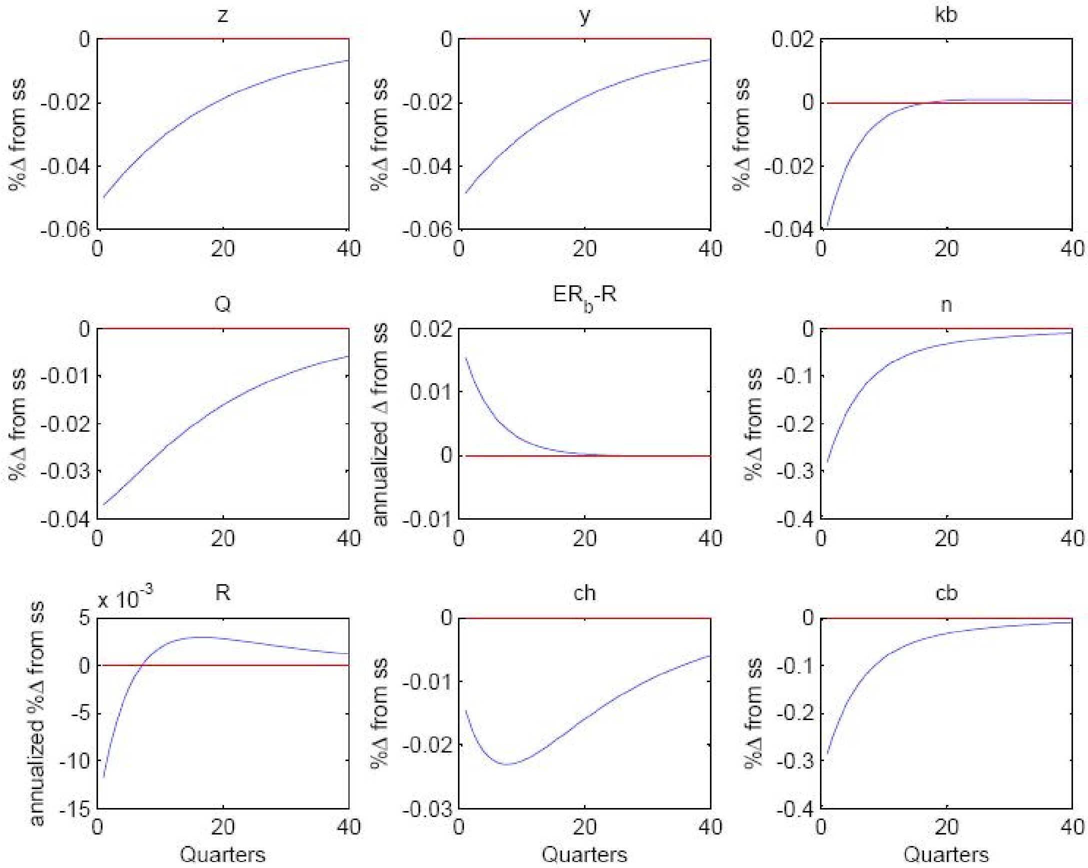

Figure 2: A Recession in the Liquidity Risk Model: No Bank Run Case

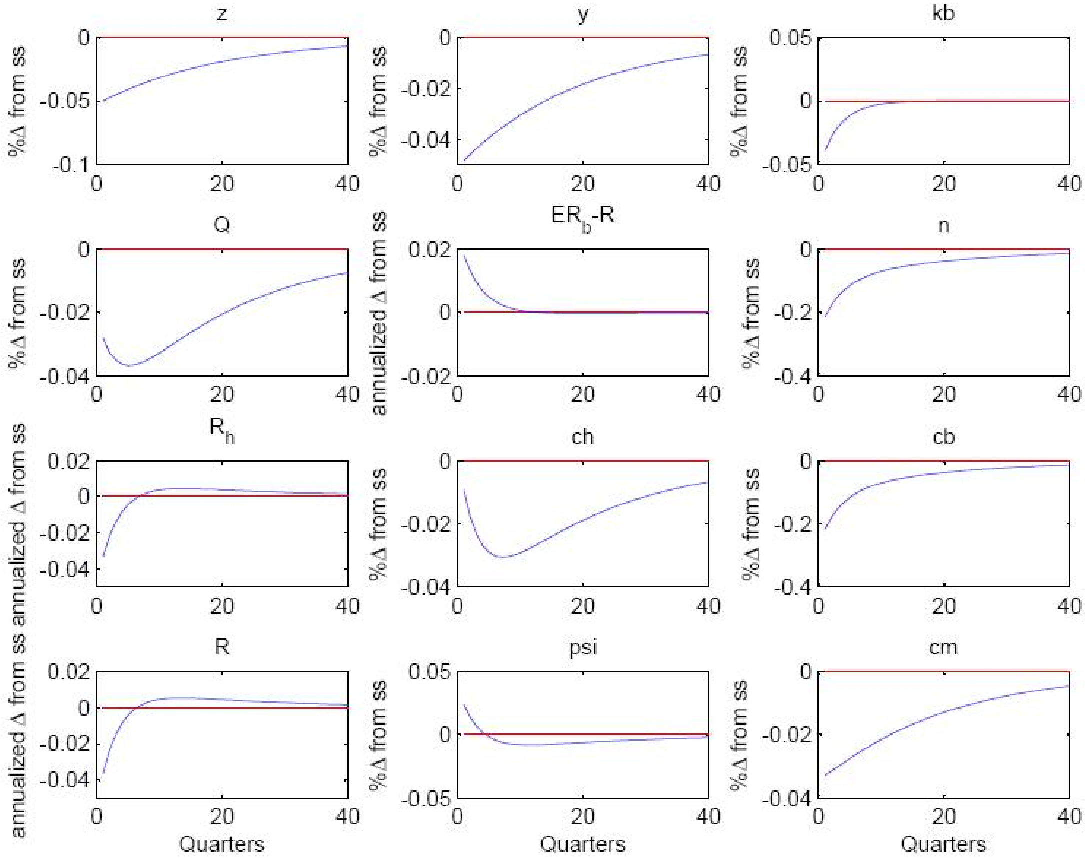

Figure 3: Ex Post Bank Run in the Baseline Model

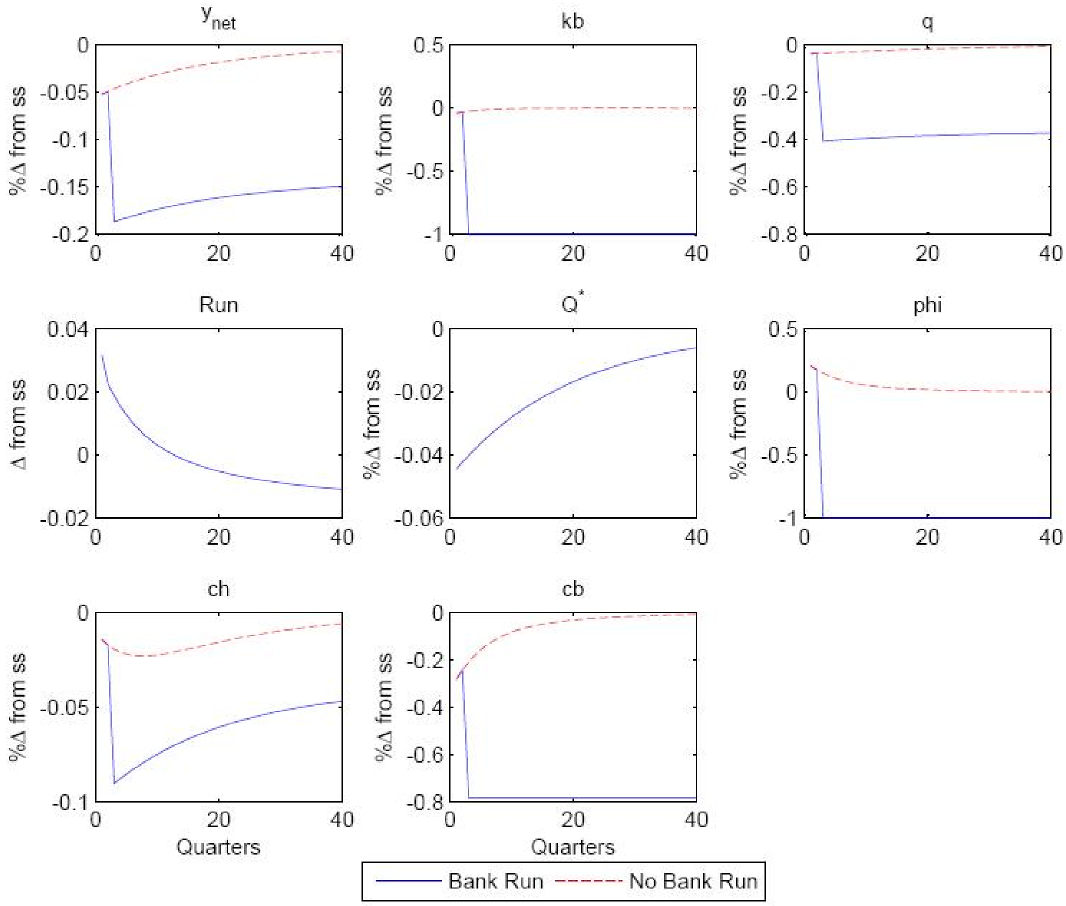

Figure 4: Ex Post Bank Run in the Liquidity Risk Model