Repos, Fire Sales, and Bankruptcy Policy

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Abstract

The events from the 2007-2009 financial crisis have raised concerns that the failure of large financial institutions can lead to destabilizing fire sales of assets. The risk of fire sales is related to exemptions enjoyed by many financial contracts, such as repo, from an automatic stay on collateral of a bankrupt firm. An automatic stay essentially prevents the holder of a defaulting debtor's collateral from immediately liquidating it. In this paper, we construct a model of repo transactions, and consider the effects of changing bankruptcy rule regarding automatic stays on the activity in the repo market. We find that exempting repos from the automatic stay of bankruptcy is beneficial for creditors who hold the borrowers' collateral. Although the exemption may increase the size of the repo market by enhancing the liquidity of collateral, it can also lead to damaging fire sales, which may reduce real investment activity. Hence, policy makers must tradeoff the benefits of investment activity against the benefits of liquid markets for collateral.

1 Introduction

An institution that suffers large losses may be forced to sell assets at distressed or *fire-sale* prices. If other institutions revalue their assets at these temporarily low market values, then the initial sale can set off a cascade of fire sales that inflicts losses on many institutions, (French *et al.*, (2010)). A number of commentators have identified fires sales as depleting the balance sheets of financial institutions and aggravating the fragility of the financial system in the recent financial crisis, (Shleifer and Vishny, 2011). Therefore, via defaults and fire sales, one troubled institution can damage another and, as a result, reduce the financial system's capacity efficiently allocate resources. Many commentators have identified the freezing of the repurchase agreement (repo) market as a key contributor to the recent financial crisis.

This paper develops a model of a repo market. We examine the implications of different policy rules on the activity in this market, as well as in the market where the collateral of a defaulted borrowers can be resold. An important feature of the repo market is that the repo lender (or creditor) holds collateral of the borrower. Under current bankruptcy rules, the lender can liquidate the collateral if the borrower defaults. In effect, the repo contract is exempt from a standard bankruptcy procedure, a stay, which prevents creditors from liquidating the collateral of the defaulting party. This exemption from the stay, which is afforded to many financial contracts, has raised concerns that the default of a large financial institution could trigger destabilizing fire sales of assets. Such fears are based on the failure, or near failure of Bear Stearns and Lehman brothers in 2008.[1]

In our model, the possibility of borrower default motivates lenders to request collateral from buyers. The insurance function of the collateral is imperfect. Bankruptcy rules imposing a stay on the liquidation of the borrowers' collateral effectively impose a cost on holders of collateral. This cost can be associated with not having access to its funds during the liquidation process, or with the risk that the collateral could lose value. We model this as a reduction in the value of the collateral.

If, instead, bankruptcy rules allow the lender to liquidate the borrower's collateral, then that collateral can be sold in a secondary market. Depending on the depth of that market, sales of collateral in the market can have important effects on other participants in that market. We focus on the effect of lenders' sales of collateralized assets as increasing the cost of real investments taken on by competing investors. We model this through a fire sale externality. With this assumption, lenders do not take into account the effect they have on investors when choosing their activity in the repo market. Absent this assumption, lenders would internalize all the effects of their sales of assets on the economy and make efficient investment decisions in the repo market.

The externality creates a trade-off when considering bankruptcy rules. On the one hand, exempting repo transactions from the bankruptcy stay is desirable since it increases the value of the borrower's collateral to the lender since it can be liquidated in case of default. In this sense, exemption from the bankruptcy stay makes the repo market more "liquid." On the other hand, this increased activity results in more pronounced fire sales in case of borrower default, which reduces the investment activity, which is not desirable. The relative size of these two effects depend on the parameter values of the model and, thus, so does the optimal bankruptcy rule.

Under the US Bankruptcy Code, most collateral is subject to an automatic stay when a debtor files for bankruptcy. This stay delays the ability of a creditor to realize value through a sale of the collateral.[2] Over the decades since the current framework was established, an ever-increasing set of qualified

[1] It is worth noting that these fears were, for the most part, not realized in part because Bear Stearns was purchased by JP Morgan Chase, and the US broker dealer unit of Lehman did not declare bankruptcy.

[2] The purpose of this stay in general is to prevent the destruction of value that could occur in a competitive rush by creditors to seize the assets the bankrupt firm. To the extent that the assets used as collateral are financial assets rather than real assets, the destructiveness of this "grab race" is less of a consideration, and so in this paper we focus instead on the effects of a rush to *sell* these assets in a less-than-perfectly liquid market.

financial contracts ("QFC's"), including repos, has been exempted from the stay. (In the case of banks taken over by the FDIC or systemically important financial institutions under Dodd-Frank, there may be a stay for a limited time even for QFC's.)

Our paper focuses on the trade-off between the liquidity of the repo market and the potential for fire sales related to the exemption from the stay. This trade-off is discussed in the legal literature; see Roe (2011). Duffie and Skeel (2012) outline a number of costs and benefits associated with safe harbors from the automatic stay in bankruptcy, including a variety of ways in which the stay can decrease the value of the collateral contract, and on the other side, the potential for costs in a fire sale. (They note that in particular money market mutual funds holding repos may be forced by regulation to sell the collateral in the case of the bankruptcy of a counterparty).

Bolton and Oehmke argue against priveleged positions for derivatives in bankruptcy, because it inefficiently undermines the position of other creditors. The paper that is most closely related to our is Acharya, Anshuman, and Viswanathan (2012) which focuses on a trade-off similar to ours. Our paper does not address some of the benefits of the exemption from the stay associated with close-out netting. An analysis of these benefits is provided in McAndrews and Roberds (2003).

The paper is organized as follows. The basic model, without default, is presented in the next section. The basic model is generalized to allow for defaults in section 3. Section 4 examines the nature of a government policy intervention and carefully analyzes the trade-offs that the government faces. Section 5 offers some concluding comments.

2 The Basic Model

The economy has 3 dates—1, 2, and 3—and 2 goods—a and c. Good a is durable—that is it can be costlessly stored from one period to the next. Good c is perishable.

There are 4 types of agents: lenders, L, borrowers, B, investors, I, and traders, T. The measure of each type of agent is n^i, where $i \in \{L, B, I, T\}$.

Lenders and borrowers are born at the beginning of date 1. Borrowers live at dates 1 and 2, and lenders live at dates 1, 2 and 3. Investors and traders are born at the beginning of date 3 and live only at date 3.

Borrowers like to consume good a at date 2. They possess a costless technology that instantaneously converts good c into good a one-for-one in date 1 or date 2. They can also produce good c, but only at date 2, at a cost of 1 unit of effort per unit of good. The preferences of a borrower, U^B, are given by

$$U^B = a_2 - c_2.$$

(Subscripts indicate when the goods are produced or consumed.)

Lenders want to consume goods a and c at dates 2 and 3; they like good c more than good a. Lenders can produce good c only at date 1, where one unit of costly effort produces one unit of good c. The preferences of a lender, U^L, are given by

$$U^L = u(c_2) + c_3 + \gamma a_2 + \gamma a_3 - c_1,$$

where u is increasing and strictly concave, and $0 < \gamma < 1$.

Traders are endowed with $\bar{c}$ units of good c at date 3. They like to consume goods a and c at date 3, and their preferences, U^T, are given by

$$U^T = a_3 + c_3.$$

Investors are endowed with $\bar{a}$ units of good a at date 3, where $\bar{a} < \bar{c}$.[3] They like to consume goods a and c, and their preferences, U^I, are

$$U^I = a_3 + c_3.$$

Investors have a technology, f, that produces good c using good c as an input at date 3. f is increasing, strictly concave and $f'(\bar{a}) > 1$. The last assumption implies that f is a productive technology in the sense that if the investor could exchange his endowment of good a for $\bar{a}$ units of good c, then marginal return is strictly greater than one for all levels of input $c \in (0, \bar{a}]$.

Agents trade in pairs; that is, they are bilaterally matched. Agents are matched at the beginning of date 1 and at the beginning of date 3. The date 1 and date 3 matching processes are independent of one another. Since investors and traders are not alive at date 1, only lenders and borrowers enter the matching process at that time.

Some bilateral matches can generate surplus for the agents in the match. For example, borrowers and lenders can benefit from trading good c at date 1 for good c at date 2. In particular, a matched lender can produce good c at date 1 and give it to the borrower, (who converts it into good a). In return, the borrower can produce good c for the lender at date 2. Let this trading arrangement be compactly represented by the "contract" (c_1, c_2). Note that since the good c that is produced at date 1 is converted to good a one-for-one, $a_1 = c_1$; and since good a is durable, $a_2 = a_1$. Implicitly embedded in contract (c_1, c_2) is a promise: the borrower promises to produce good c for the lender at date 2. We will assume that agents can commit to any (feasible) promise they make when matched.

Traders and investors can benefit from exchanging good a for good c at date 3. In particular, a matched investor can exchange some of his endowment of good a for some the the trader's endowment of good c. The trading arrangement between investors and traders can be represented by the contract (a_3, c_3), i.e., the investor gives up a_3 units of his endowment of good a and receives c_3 units of the trader's endowment of good c.

The date-1 contract, (c_1, c_2), between a matched lender and borrower is determined by bargaining. The lender's payoff (and surplus) associated with contract (c_1, c_2) is $u(c_2) - c_1$. Since technology implies that $c_1 = a_2$, the borrower's surplus is $c_1 - c_2$. Total match surplus generated by contract (c_1, c_2)

[3] We consider the case where $\bar{a} > \bar{c}$ in footnotes.

is

$$S^{BL} = u\left(c_2\right) - c_2.$$

A borrower accepts contract (c_1, c_2) only if $c_1 \geq c_2$ and a lender accepts only if $u\left(c_2\right) \geq c_1$. For simplicity, we assume that the lender has all of the bargaining power and makes a take-it-or-leave-it offer to the borrower. This bargaining protocol implies that the lender will choose $c_2 = c_1$ and, hence, receives the entire match surplus.[4] The lender offers contract (c^*, c^*) to the borrower, where $u'\left(c^*\right) = 1$, since this offer maximizes match surplus. The borrower will accept this offer.

Let m^{ij} represent the probability that agent i is matched with agent j at date 1, and let m represent the measure of productive date 1 matches. We will assume that the matching technology is Leontief in nature and takes the form $m = \min\left\{n^L, n^B\right\}$, $m^{LB} = m/n^L$, $m^{BL} = m/n^B$, and $m^{BB} = m^{LL} = 0$. For this matching technology one can interpret agents as directing their search to a productive partner, where the "short side" of the market determines the number of matches.

Lenders have no incentive to enter the date 3 matching process, independent of being matched or not at date 1, since they have nothing to offer in a date 3 match that could generate a match surplus. Therefore, the expected payoff to a lender—measured before agents are matched at date 1—is $m^{LB}\left[u\left(c^*\right) - c^*\right]$. Since the lender has all of the bargaining power in a date 1 match with a borrower, the expected payoff to a borrower is zero.

Only investors and traders enter the date 3 matching process. In an investor-trader match, the investor's payoff associated with contract (a_3, c_3) is $f\left(c_3\right) + \bar{a} - a_3$. The surplus that the investor receives is $f\left(c_3\right) + \left(\bar{a} - a_3\right) - \bar{a} = f\left(c_3\right) - a_3$. The trader's payoff associated with contract (a_3, c_3) is $a_3 + \bar{c} - c_3$, and the surplus he receives is $a_3 + \bar{c} - c_3 - \bar{c} = a_3 - c_3$. Hence the total match surplus is

$$S^{IT} = f\left(c_3\right) - c_3.$$

The investor will accept contract (a_3, c_3) only if $f\left(c_3\right) > a_3$, and the trader will accept the offer (a_3, c_3) only if $a_3 \geq c_3$. We assume that the investor has all of the bargaining power. The investor will offer contract (a_3, c_3) to the trader, where $a_3 = c_3 = \min\left\{\bar{a}, \bar{c}\right\} = \bar{a}$, which implies that the match surplus is $f\left(\bar{a}\right) - \bar{a}$.

Let M^{ij} represent the probability that agent i is matched with agent j at date 3, and M represent the measure of date-3 productive matches. For a Leontief matching function, $M = \min\left\{n^I, n^T\right\}$, $M^{IT} = M/n^I$, $M^{TI} = M/n^T$, and $M^{II} = M^{TT} = 0$. Since the investor has all of the bargaining power, his payoff is $M^{IT}\left(f\left(\bar{a}\right) - \bar{a}\right) + \bar{a}$, and the expected payoff to the trader is $\bar{c}$.[5]

Let p_a represent the value to an investor of having an additional unit of good a, measured in terms of good c, at the beginning of date 3 before matching takes

[4] Dividing surplus between the bargainers will not significantly affect our results).

[5] If $\bar{a} \geq \bar{c}$, then the investor will offer (a_3, c_3), where $a_3 = c_3 = \min\left\{\bar{a}, \bar{c}\right\} = \bar{c}$. In this case the expected payoff to the investor is $M^{IT}\left(f\left(\bar{c}\right) - \bar{c}\right) + \bar{a}$, and the expected payoff to the trader is $\bar{c}$.

place. Then

$$p_a = M^{IT} f'(\bar{a}) + \left(1 - M^{IT}\right),$$

i.e., the investor is indifferent between receiving p_a units of good c for sure, and receiving an additional unit of good a.[6]

Consider the problem of a planner whose objective is to maximize total social surplus, S, where

$$\begin{aligned} S &= m(u(c_2) - c_1) + m(a_2 - c_2) + M[(\bar{c} - c_3) + a_3 - \bar{c}] + \\ &\quad M[(\bar{a} - a_3) + f(c_3) - \bar{a}] \\ &= m(u(c_2) - c_2) + M(f(c_3) - c_3), \end{aligned}$$

since $c_1 = a_2$. Assuming that the planner must respect agent participation constraints, total social surplus will be maximized at $c_2 = c^*$ and $c_3 = \bar{a}$, the take-it-or-leave-it offers made by the lender and investor, respectively.[7] The planner can implement this surplus as long as $u(c_2) \geq a_2 \geq c_2$ and $f(c_3) \geq a_3 \geq c_3$, i.e., agent participation constraints are satisfied. Although the planner can redistribute surplus from the lender to the borrower (by increasing a_2 from c^*) and from the investor to the trader (by increasing a_3 from $\bar{a}$), he cannot increase total surplus compared to the equilibrium outcome.[8]

The equilibrium in the basic model is Pareto efficient. The basic model lacks frictions that are needed to generate contracts that resemble repo contracts or something that looks like a "fire sale." In addition, since agents do not default on the their contracts, the basic model can say nothing about bankruptcy or bankruptcy policy. In the next section we introduce a borrower default friction and examine how this affects optimal contracts, and the relationship between bankruptcy policy and fire sales.

3 A Model with Borrower Default

We extend the basic model by introducing the possibility of exogenous default by borrower. Default is modeled by having the borrower probabilistically die between dates 1 and 2. With probability δ a fraction Δ of borrowers die, and with probability $1 - \delta$ no one dies.[9] We will refer to the former outcome as the

[6] If $\bar{a} > \bar{c}$, then the price of good $\bar{a}$ is 1 since if the investor is given an additional unit good a he will simply consume it. The *average* price of good a, however, is

$$M^{IT} \frac{f(\bar{a})}{\bar{a}} + \left(1 - M^{IT}\right) > 1.$$

We would argue that in this case, the average price is the relevant statistic when thinking about gains from trade.

[7] The planner also takes as given the matching techologies and the bargaining protocol of the agents.

[8] If $\bar{a} > \bar{c}$, ...

[9] We can assume that with probability $1 - \delta$, a finite number, i.e., a set of measure zero, of borrowers die. This way there can be defaults even in "good" times, but these defaults are essentially unimportant for the economy. This would correspond to situations (in the real world) where there are "fails" or defaults and these have no significant implications for asset prices or economic activity.

default state, and the latter as the no-default state. From an ex ante date 1 perspective, the probability that a borrower dies is $\delta\Delta$. We use two parameters to describe default so that we can model a rare event, a 'small' δ, such as a major financial meltdown, a 'big' Δ.

The above contract can be interpreted as an unsecured (by collateral) loan since it is only the borrower's promise that supports the date 2 payment. In practice, it is not at all unusual for unsecured creditors to receive nothing in the event that the borrower defaults. We model this outcome by assuming that when a matched borrower—holding c_1 units of good a and promising to produce c_2 units of good c at date 2—dies in between dates 1 and 2, the good a he is holding "disintegrates," and, as a result, the lender receives nothing. Although there is little the lender can do about a borrower's broken promise to supply good c at date 2—the borrower is dead after all—the lender can secure his claim against the borrower by contractually preventing the borrower from holding good a between dates 1 and 2. Specifically, the contract can specify that the lender produces good c at date 1 and gives it to a borrower; the borrower then converts good c into good a, *and gives good a back to the lender to hold as collateral*. At date 2, the collateral—good a—is transferred back to the borrower if he produces good c for the lender; if the borrower does not produce at date 2—because he has died—the collateral becomes the property of the lender. This sort of contract partially insures the lender against a borrower default: If the borrower dies, then the lender has the collateral which is valuable to him at both dates 2 and 3.

Whether one interprets the above contract as a collateralized loan or a repo contract depends on *when* the lender is able to use the collateral. In practice, bankruptcy law specifies when collateral can be used by the lender. Under the US Bankruptcy Code, virtually all collateral is subject to an automatic stay when a debtor files for bankruptcy. This means that a secured (by collateral) creditor is unable to access and use the collateral for a certain period of time after a debtor defaults. However, some financial assets, such as derivatives and repo contracts, are exempt from the automatic stay, which implies that a secured creditor can immediately access and use the collateral as he sees fit. In terms of the model, if the bankruptcy policy dictates an automatic stay in the event of a debtor default, then the contract described above is a *collateralized loan*. In this situation, in the event of a debtor default, the earliest that collateral can by used by the lender is at date 3 after the matching process has been completed. If, instead, the bankruptcy policy exempts the collateral from an automatic stay, then the above contract is *repo*, and the lender can use the collateral as he sees fit starting at date 2, when it becomes known that the debtor has (died and) defaulted on his contractual payment of c_2. In the next section, we analyze the implications of a bankruptcy policy that exempts the collateralized contracts from an automatic stay. In the subsequent section, we examine the implications of a bankruptcy policy that imposes an automatic stay on collateral.

Repo contracts

A repo contract is represented compactly by $(\tilde{c}_1, \tilde{c}_2)$. Let the 'tilde' denote the optimal decisions made by the lender regarding the "initial loan" size, $\tilde{c}_1$, the amount of collateral, $a_1 = \tilde{c}_1$, and the "loan repayment," $\tilde{c}_2$. If the borrower does not default, then the lender receives $\tilde{c}_2$ units of good c from the borrower, and the lender transfers the collateral, $\tilde{c}_1$ units of good a, to the borrower at date 2. If the borrower defaults, then, at date 2 the lender owns the collateral, a, which can be used by him starting at date 2.

Suppose a matched borrower dies. The lender can consume the collateral at either dates 2 or 3, and his payoff is $\gamma\tilde{c}_1$. Alternatively, the lender can enter the date 3 matching process with his collateral. If he is matched with a trader, then there are gains from trade because the lender's relative valuation of good a to good c is γ and the trader's is 1. Hence, the lender's payoff can exceed $\gamma\tilde{c}_1$ if he is matched with a trader.[10]

Denote the terms of trade in a lender-trader match by the contract $(\tilde{a}_3, \tilde{c}_3)$, where the lender gives $\tilde{a}_3$ units of good a to the trader in exchange for $\tilde{c}_3$ units of good c. The payoff to the lender associated with contract $(\tilde{a}_3, \tilde{c}_3)$ is $\gamma(\tilde{a}_2 - \tilde{a}_3) + \tilde{c}_3$, where $\tilde{a}_2$ represents the amount of good a that the lender brings into the match, and the payoff to the trader is $\tilde{a}_3 + \bar{c} - \tilde{c}_3$. Total *surplus* in a lender-trader match, S^{LT}, is

$$S^{LT} = (1-\gamma)\,\tilde{a}_3.$$

Assume the lender has all of the bargaining power. The trader will accept the lender's offer only if $\tilde{a}_3 \geq \tilde{c}_3$, and the take-it-or-leave-it assumption implies that $\tilde{a}_3 = \tilde{c}_3$. The payoff to a matched lender holding collateral equal to $\tilde{a}_2$ is

$$\gamma(\tilde{a}_2 - \tilde{a}_3) + \tilde{c}_3 = \begin{cases} \tilde{a}_2 & \text{if } \tilde{a}_2 \leq \bar{c} \\ \gamma(\tilde{a}_2 - \bar{c}) + \bar{c} & \text{if } \tilde{a}_2 > \bar{c} \end{cases},$$

and the payoff to the trader is $\bar{c}$. Since the lender's expected payoff associated with entering the date 3 matching process is strictly greater than $\gamma\tilde{a}_2 = \gamma\tilde{c}_1$, he will always enter the date 3 matching process holding collateral $\tilde{a}_2$ when his borrower defaults.

The repo contract $(\tilde{c}_1, \tilde{c}_2)$ that the lender offers the borrower in a date 1 match is obviously affected by the possibility that his borrower defaults. Since the lender has all of the bargaining power in the date 1 match, $\tilde{c}_1 = \tilde{c}_2 = \tilde{a}_1 = \tilde{a}_2$. Denote the probability that the lender is matched with a trader in the event that his borrower dies as M_d^{LT}, and M_d as the measure of matches between traders and either lenders or investors at date 3 in the default state. For the Leontief matching technology lenders and investors direct their search to traders, and, therefore, $M_d = \min\left\{n^I + \Delta m, n^T\right\}$,

$$M_d^{LT} = \frac{M_d}{\Delta m + n^I},$$

[10] If the lender is matched with an investor, there are no gains from trade—since both agents have good a—and his payoff will be $\gamma\tilde{c}_1$.

and $M^{ii} = 0$ for $i \in \{I, L, T\}$.

We can characterize the optimal date 1 repo contract, $\{\tilde{c}_1, \tilde{c}_2\}$, by considering the following two maximization problems. The first problem describes a situation where a lender's collateral—which is c_1^1 units of good a—is less than the trader's endowment of $\bar{c}$. In this situation a lender, whose borrower has defaulted, will be able to exchange all of his collateral for good c if he is matched with a trader at date 3. The lender's date-1 problem, which is to choose the amount of good c to produce at date 1, denoted c_1^1, is

$$\max_{c_1^1} -c_1^1 + (1 - \delta\Delta) u \left(c_1^1\right) + \delta\Delta \left[M_d^{LT} c_1^1 + \left(1 - M_d^{LT}\right) \gamma c_1^1\right], \tag{1}$$

i.e., the lender produces c_1^1 units of good c_1, which the borrower converts into c_1^1 units of good a. If the borrower does not die, then the lender consumes c_1^1 units of good c at date 2. If the borrower dies, the lender is able to enter the date-3 matching process since there is an exemption on the automatic stay; he consumes c_1^1 units of good c if he is matched, and c_1^1 units of good a if he is not.

The second problem describes the situation where the lender's collateral—which is c_1^2 units of good a—is greater than the trader's endowment. In this is situation, the lender will only be able to exchange part of his collateral for good c if he is matched with a trader at date 3. The lender's date-1 problem is to choose the amount of good c to produce at date 1, denoted c_1^2,

$$\max_{c_1^2} \; -c_1^2 + (1 - \delta\Delta) u \left(c_1^2\right) + \delta\Delta \left[M_d^{LT} \left(\gamma \left(c_1^2 - \bar{c}\right) + \bar{c}\right) + \left(1 - M_d^{LT}\right) \gamma c_1^2\right] \tag{2}$$

In both problems, if the borrower defaults, the lender enters the date-3 matching process, and is matched with a trader with probability M_d^{LT}. The first-order conditions, *with equality*, for each of these problems are

$$(1 - \delta\Delta) u' \left(c_1^1\right) + \delta\Delta \left(M_d^{LT} + \left(1 - M_d^{LT}\right) \gamma\right) = 1$$

and

$$(1 - \delta\Delta) u' \left(c_1^2\right) + \delta\Delta\gamma = 1,$$

respectively. Note that the first-order condition associated with problem (2) does not depend on the date-3 matching probability. Since $\gamma < 1$, note that $c_1^1 > c_1^2$.

Note that if $c_1^1 > \bar{c}$, then problem (1) does not make economic sense since it assumes that the lender can trade c_1^1 units of good a for c_1^1 units of good c if he is matched with a trader at date 3. But this outcome is infeasible. Similarly, if $c_1^2 < \bar{c}$, then problem (2) does not make economic sense since it assumes that the lender can only trade a fraction of the c_1^2 units of good a for good c if he is matched with a trader at date 3. But the lender will be able to trade all of his units of good a. Hence, if, $c_1^1 < \bar{c}$, then problem (1) is the solution to the lender's date-1 contract offer to the borrower, i.e., $\tilde{c}_1 = \tilde{c}_2 = c_1^1$.[11] This solution to the problem is consistent with the lender trading all of the collateral

[11] If $c_1^1 < \bar{c}$, problem (2) is not a relevant economic problem since $c_1^2 < \bar{c}$.

for good c at date 3 if he is matched with a trader. And, if $c_1^2 > \bar{c}$, then problem (2) is the solution to the lender's date-1 contract offer to the borrower, i.e., $\tilde{c}_1 = \tilde{c}_2 = c_1^2$.[12] This solution is consistent with the lender only trading a fraction of the collateral for good c.

Finally, if $c_1^1 > \bar{c}$ and $c_1^2 < \bar{c}$, then neither problem solves the lender's date-1 contract offer. Problem (1) assumes that if the lender is matched with a trader he is able to exchange all of his holdings of good a for good c; but this is inconsistent with $c_1^1 > \bar{c}$. Problem (2) assumes that the lender is able to exchange only part of his holdings of good a for good c; but this is inconsistent with $c_1^2 < \bar{c}$. Note that there is a discrete decrease in the *marginal* benefit associated with having an additional unit of good c at $c = \bar{c}$; the expected marginal benefit falls from $\delta\Delta\left[M^{LT} + \left(1 - M^{LT}\right)\gamma\right]$ to $\delta\Delta\gamma$. Since $(1-\delta\Delta)\, u'(\bar{c}) + \delta\Delta\left[M^{LT} + \left(1 - M^{LT}\right)\gamma\right] > 1$ and $(1-\delta\Delta)\, u'(\bar{c}+\varepsilon) + \delta\Delta\gamma < 1$ for any $\varepsilon > 0$, the lender's optimal date-1 contract offer to the borrower is $\tilde{c}_1 = \tilde{c}_2 = \bar{c}$ when $c_1^1 > \bar{c}$ and $c_1^2 < \bar{c}$.

Suppose that the default state is realized and, as a result, Δn^B borrowers die in between dates 1 and 2. Then, at date 3, traders, investors and lenders will enter the matching process. Denote the probability that an investor is matched with a trader in the default state as M_d^{IT}, where $M_d^{IT} = M_d^{LT}$. The terms of trade between a matched investor and trader is not a function of the matching probability M_d^{LT}. Hence, the investor exchanges $\min\{\bar{a}, \bar{c}\} = \bar{a}$ units of good a for $\bar{a}$ units of good c with the trader. From the investor's date 3 perspective, the price of good a, measured before agents are matched at date 3, p_a^δ, is

$$p_a^\delta = M_d^{IT} f'(\bar{a}) + \left(1 - M_d^{IT}\right).$$

It is important to emphasize that $p_a^\delta \leq p_a$, since $M^{IT} \geq M_d^{IT}$.[13] When $M^{IT} > M_d^{IT}$, $p_a > p_a^\delta$, and the lower price in the default state will be referred to as a "fire sale" of asset a. The value of asset a decreases to investors because lenders' enter the date-3 matching process to sell their collateral and this reduces the probability that the investors are matched with traders. (In the no-default state, an event that occurs with probability $1-\delta$, the price of asset a is p_a.) There are real affects associated with the fire sale since the total amount of real investment falls, compared to the situation where borrowers do not default.

4 Government policy

In the basic no-default model, the government cannot increase total social surplus, compared to the equilibrium allocation. When borrowers can default, however, a government may be able to increase total social surplus, compared to the equilibrium allocation, by affecting the flow of lenders that enter the date-3 matching process. In particular, the government policy instrument is the specification of automatic stay provisions or exclusions on collateral. Let θ represent

[12] If $c_1^2 > \bar{c}$, then $c_1^1 > \bar{c}$, and problem (1) is not a relevant economic problem.

[13] As above, M^{IT} represents the probability that an investor is matched with a trader in the no-default state. In the no-default, lenders do not enter the date-3 matching process.

the fraction of lenders that are allowed to use the collateral of their defaulting borrower as they see fit starting at the beginning of date 2. An exemption from an automatic stay on collateral for *all* lenders implies that $\theta = 1$, and an automatic stay on all collateral, where lenders are only able to access their collateral in date 3 after the matching process is completed, implies that $\theta = 0$. When $\theta = 1$, the $(\tilde{c}_1, \tilde{c}_2)$ is a repo contract; when $\theta = 0$, it is a collateralized loan contract. Note that $\theta \in (0, 1)$ can be interpreted as a partial exemption from an automatic stay in the sense that some lenders, $\theta \Delta m$ of them, are exempt from an automatic say and others, $(1 - \theta) \Delta m$ of them, are not. It is important to emphasize that if a lender's collateral is subject to an automatic stay, then he (and his collateral) cannot participate in the date-3 matching process.

Government policy, through its effect on θ, can affect the payoffs and behavior of the various agents in the economy. The expected payoff to a borrower, W_B, is

$$W_B = m^{BL} (1 - \delta\Delta) (\tilde{a}_1 - \tilde{c}_2),$$

where m^{BL} is the probability that a borrower is matched with a lender at date 1. The behavior of the borrower can be affected by government policy since policy can affect $\tilde{c}_1$, which, in turn, affects $\tilde{a}_1$ and $\tilde{c}_2$. The payoff to the borrower is unaffected by government policy since the lender has all of the bargaining power, $\tilde{c}_2 = \tilde{a}_1$, which implies that $W_B = 0$.

The payoff to the lender, W_L, is given by

$$\begin{aligned} W_L \;\; = \;\; & m^{LB}\{-\tilde{a}_1 + (1 - \delta\Delta) u(\tilde{c}_2) + \delta[\Delta\theta(M_d^{LT} (\tilde{c}_3 + \gamma\tilde{a}_2 - \gamma\tilde{a}_3) + \\ & \left(1 - M_d^{LT}\right) \gamma\tilde{a}_2)] + \Delta (1 - \theta) \gamma\tilde{a}_2\}. \end{aligned}$$

Government policy can the the payoff of the lender directly—since θ appears in W_L—and indirectly through $\tilde{c}_1$ and $\tilde{c}_2$—and, as a result, through $\tilde{a}_1$,$\tilde{a}_2$, $\tilde{c}_3$, and $\tilde{a}_3$.

The expected payoff to a trader, W_T, is

$$\begin{aligned} W_T \;\; = \;\; & (1 - \delta) \left[M^{TI} (\hat{a}_3 - \hat{c}_3 + \bar{c}) + \left(1 - M^{TI}\right) \bar{c}\right] + \\ & \delta \left[M_d^{TI} (\hat{a}_3 - \hat{c}_3 + \bar{c}) + M_d^{TL} (\tilde{a}_3 - \tilde{c}_3 + \bar{c}) + \left(1 - M_d^{TI} - M_d^{TL}\right) \bar{c}\right] \\ = \;\; & \left[(1 - \delta) M^{TI} + \delta M_d^{TI}\right] (\hat{a}_3 - \hat{c}_3) + \delta M_d^{TL} (\tilde{a}_3 - \tilde{c}_3) + \bar{c}, \end{aligned}$$

where the 'hat' over the a_3 and c_3 represents (optimal) offers made by the investor to the trader,

$$M_d^{TL} = \frac{M_d}{n^T} \frac{\Delta\theta m}{\Delta\theta m + n^I}$$

and

$$M_d^{TI} = \frac{M_d}{n^T} \frac{n^I}{\Delta\theta m + n^I}.$$

Since investors and lenders have all of the bargaining power in their matches with traders, $\hat{a}_3 = \hat{c}_3$ and $\tilde{a}_3 = \tilde{c}_3$, which implies that $W_T = \bar{c}$. Hence, the payoff to the trader is unaffected by government policy θ. In fact, $\hat{c}_3 = c_3^* = \min\{\bar{c}, \bar{a}\} = \bar{a}$, which is the trade allocation in a trader-investor match in a world without default. Note, however, that $\tilde{c}_3$ and $\tilde{a}_3$ can be affected by government policy.

Finally, the payoff to the investor, W_I, is

$$\begin{aligned} W_I &= (1-\delta)\left[M^{IT}\left(f\left(\hat{c}_3\right)-\hat{a}_3+\bar{a}\right)+\left(1-M^{IT}\right)\bar{a}\right]+ \\ &\quad \delta\left[M_d^{IT}\left(f\left(\hat{c}_3\right)-\hat{a}_3+\bar{a}\right)+\left(1-M_d^{IT}\right)\bar{a}\right] \\ &= \left[(1-\delta)M^{IT}+\delta M_d^{IT}\right]\left(f\left(\hat{c}_3\right)-\hat{a}_3\right)+\bar{a}. \end{aligned}$$

Although the behavior of the investor is unaffected by government policy—since $\hat{a}_3 = \hat{c}_3 = c_3^* = \min\{\bar{a}, \bar{c}\} = \bar{a}$—his payoff is affected since the matching probability M_d^{IT} is a function of θ.

In order to evaluate various government policies, we must understand how the behavior of a lender—which is simply his choice of $\tilde{c}_1$—is influenced by changes in θ. As in section 3, consider two maximization problems, which generalize problems (1) and (2) to take account of government policy, θ. The problems are,

$$\max_{c_1^1} -c_1^1+(1-\delta\Delta)u\left(c_1^1\right)+\delta\Delta\left\{\theta\left[M_d^{LT}c_1^1+\left(1-M_d^{LT}\right)\gamma c_1^1\right]+(1-\theta)\gamma c_1^1\right\} \tag{G1}$$

and

$$\begin{aligned} \max_{c_1^2} \quad & -c_1^2+(1-\delta\Delta)u\left(c_1^2\right)+\delta\Delta\{[M_d^{LT}\left(\gamma\left(c_1^2-\bar{c}\right)+\bar{c}\right) \\ & +\left(1-M_d^{LT}\right)\gamma c_1^2]+(1-\theta)\gamma c_1^2\}\ . \end{aligned} \tag{G2}$$

The first problem assumes that, in the event of a borrower default, the lender can exchange all of his collateral (one-for-one) for the consumption good if he is matched with a trader at date 3. The second problem assumes that the lender will be unable to exchange all of his collateral. The first-order conditions for these problems are,

$$(1-\delta\Delta)u'\left(c_1^1\right)+\delta\Delta\left(\gamma+(1-\gamma)\theta M_d^{LT}\right)=1 \tag{3}$$

and

$$(1-\delta\Delta)u'\left(c_1^2\right)+\delta\Delta\gamma=1, \tag{4}$$

respectively. As in section 3, $c_1^1 > c_1^2$. Qualitatively speaking, the characterization of the lender's behavior regarding his choice of $\tilde{c}_1$ is identical to that in section 3. In particular, if $c_1^1 < \bar{c}$, then the lender's choice of his date-1 production of good c, $\tilde{c}_1 = \tilde{c}_1^1$, where $\tilde{c}_1^1$is given by the solution to (3); if $c_1^2 > \bar{c}$, then $\tilde{c}_1 = c_1^2$, where c_1^2 is given by the solution to (4); and if $c_1^1 > \bar{c}$ and $c_1^2 < \bar{c}$, then $\tilde{c}_1 = \bar{c}$.

We first characterize how government policy can affect the behavior of agents. In particular, proposition 1 demonstrates how loan size, $\tilde{c}_1$, for the contract $(\tilde{c}_1, \tilde{c}_2)$ is affected by a change in θ.

Proposition 1 *$\tilde{c}_1$ is weakly increasing in θ.*

Proof. If $c_1^1 < \bar{c}$, then $\tilde{c}_1 = c_1^1$. From (3), we have

$$\frac{\partial \tilde{c}_1}{\partial \theta} = \frac{\partial c_1^1}{\partial \theta} = -\frac{\delta \Delta (1-\gamma) \partial \left(\theta M_d^{LT}\right) / \partial \theta}{(1-\delta\Delta) u'' \left(c_1^1\right)}. \tag{5}$$

Since

$$\theta M_d^{LT} = \begin{cases} \theta & \text{if} \quad \Delta\theta m + n^I < n^T \\ \frac{\theta n^T}{\Delta\theta m + n^I} & \text{if} \quad \Delta\theta m + n^I > n^T \end{cases},$$

and

$$\frac{\partial \left(\theta M_d^{LT}\right)}{\partial \theta} = \begin{cases} 1 & \text{if} \quad \Delta\theta m + n^I < n^T \\ \frac{n^I n^T}{(\Delta\theta m + n^I)^2} & \text{if} \quad \Delta\theta m + n^I > n^T \end{cases}, \tag{6}$$

we get that $\partial c_1^1 / \partial \theta > 0$. If $c_1^2 > \bar{c}$, then $\tilde{c}_1 = c_1^2$ and, from (4), $\partial \tilde{c}_1 / \partial \theta = \partial c_1^1 / \partial \theta = 0$. Finally, if $c_1^1 > \bar{c}$ and $c_1^2 < \bar{c}$, then $\tilde{c}_1 = \bar{c}$ and $\partial \tilde{c}_1 / \partial \theta = 0$. ■

The intuition behind this proposition is straightforward. Having access to the date-3 matching process is valuable for the lender. Suppose that $\tilde{c}_1 < \bar{c}$. One can interpret an increase in θ as providing the lender with better insurance against borrower default in the sense that an increase in θ increases the probably that lender will be able to exchange good a—which he values "a little"—for good c—which he values "a lot"—if the borrower defaults. Since the cost associated with borrower default declines as θ increases, the lender has an incentive to increase his date 1 loan, $\tilde{c}_1$, and the collateral $\tilde{a}_1 = \tilde{c}_1$ that he holds. Suppose now that $\tilde{c}_1 \geq \bar{c}$. In this situation, the lender has no incentive to increase his date-1 loan size $\tilde{c}_1$ when θ increases since, independent the lender being is matched or not at date 3, the value of an additional unit collateral, conditional on the borrower defaulting, is unchanged and equal to $\gamma < 1$.

The government seeks to maximize total social surplus, S, which is given by

$$S = n^B W_B + n^L W_L + n^I (W_I - \bar{a}) + n^T (W_T - \bar{c}).$$

The assumed bargaining conventions implies that the expression for total social surplus can be simplified to

$$S = n^L W_L + n^I (W_I - \bar{a}),$$

which means we only need to focus on the behavior of and payoffs to lenders and investors.

We now characterize how government policy affects total social surplus. Since the surplus to investors is

$$W_I - \bar{a} = \left[(1-\delta) M^{IT} + \delta M_d^{IT}\right] (f(\bar{a}) - \bar{a}), \tag{7}$$

where $\hat{c}_3 = c_3^* = \min\{\bar{a}, \bar{c}\} = \bar{a}$, the government policy θ affects the investor's surplus only through the matching probability, M_d^{IT}.

Proposition 2 *The investor's payoff is weakly decreasing in θ.*

Proof. Note that

$$\frac{\partial M_d^{IT}}{\partial \theta} = \begin{cases} 0 & \text{if} \quad n^T > \Delta\theta m + n^I \\ -\frac{\Delta m n^T}{(\Delta\theta m + n^I)^2} & \text{if} \quad n^T \leq \Delta\theta m + n^I \end{cases}$$

and, since $M^{IT} = \min\left\{n^I, n^T\right\}/n^I$, $\partial M^{IT}/\partial\theta = 0$. Therefore, $\partial W_I/\partial\theta = \delta\left(\partial M_d^{IT}/\partial\theta\right)\left(f\left(\bar{a}\right) - \bar{a}\right)$ or

$$\frac{\partial W_I}{\partial \theta} = \begin{cases} 0 & \text{if} \quad n^T > \Delta\theta m + n^I \\ -\delta \frac{\Delta m n^T}{(\Delta\theta m + n^I)^2}\left(f\left(\bar{a}\right) - \bar{a}\right) & \text{if} \quad n^T \leq \Delta\theta m + n^I \end{cases}. \tag{8}$$

■

This proposition accords with intuition. If the measure of traders is relatively large—in the sense that $n^T > \Delta\theta m + n^I$—then increasing access to the date-3 matching process for lenders has no effect on the investors' surpluses since investors are matched with probability one at date 3. If, however, the number of traders is not relatively large—in the sense that $n^T \leq \Delta\theta m + n^I$—then increasing access to the date-3 matching process to lenders will reduce the probability that investors are matched with traders and, hence, reduces the payoffs to lenders.

Turning to lenders, since, $\tilde{c}_1 = \tilde{c}_2 = \tilde{a}_2 = \tilde{a}_1$ and $\tilde{a}_3 = \min\{\tilde{c}_1, \bar{c}\}$, the surplus function for a lender can be simplified to

$$W_L = m^{LB}\left\{-\tilde{c}_1 + (1 - \delta\Delta)\, u\left(\tilde{c}_1\right) + \delta\Delta\gamma\tilde{c}_1 + \delta\Delta\theta M_d^{LT}\tilde{a}_3\left(1 - \gamma\right)\right\}. \tag{9}$$

To assess how its policy affects total social surplus, the government must understand how W_L is affected by a change in θ.

Proposition 3 *The lender's payoff is strictly increasing in θ.*

Proof. The derivative of (9) with respect to θ is

$$\begin{aligned} \frac{\partial W_L}{\partial \theta} &= m^{LB}\left\{\frac{\partial \tilde{c}_1}{\partial \theta}\left[-1 + \delta\Delta\gamma + (1 - \delta\Delta)\, u'\left(\tilde{c}_1\right)\right] + m^{LB}\delta\Delta\left(1 - \gamma\right)\frac{\partial \tilde{a}_3}{\partial \theta}\theta M_d^{LT}\right\} + \\ & \quad m^{LB}\delta\Delta\left(1 - \gamma\right)\left\{\frac{\partial\left(\theta M_d^{LT}\right)}{\partial \theta}\tilde{a}_3 +\right\} \end{aligned} \tag{10}$$

The first line of (10) is equal to zero. When $\tilde{c}_1 < \bar{c}$, this is implied by (3), recognizing that $\partial\tilde{c}_1/\partial\theta = \partial\tilde{a}_3/\partial\theta$. When $\tilde{c}_1 \geq \bar{c}$, (4) implies that $\partial\tilde{c}_1/\partial\theta = \partial\tilde{a}_3/\partial\theta = 0$. Therefore

$$\begin{aligned} \frac{\partial W_L}{\partial \theta} &= m^{LB}\delta\Delta\left(1 - \gamma\right)\left\{\frac{\partial\left(\theta M_d^{LT}\right)}{\partial \theta}\tilde{a}_3\right\} \\ &= m^{LB}\delta\Delta\left(1 - \gamma\right)\begin{cases} \tilde{a}_3 & \text{if} \quad \Delta\theta m + n^I < n^T \\ \frac{n^I n^T}{(\Delta\theta m + n^I)^2}\tilde{a}_3 & \text{if} \quad \Delta\theta m + n^I > n^T \end{cases} \\ &> 0 \end{aligned}$$

■

The intuition behind proposition 2 is straightforward. Holding $\tilde{c}_1$ constant, an increase in θ increases the chance that the lender will be able to participate in the date-3 matching process. This unambiguously increases the surplus of the lender because, in the event of a borrower default, the value of either part or all of the lender's collateral a increases from γa to a. As well, if $\tilde{c}_1 < \bar{c}$, then, holding the date-3 matching probability constant, an increase in θ optimally increases $\tilde{c}_1$ and, by construction, the lender's collateral holdings. Since an increase in $\tilde{c}_1$ is an optimal response to an increase in θ, the lender's surplus must also increase.

Propositions 2 and 3 identify the trade-off that the government faces when choosing its policy. An increase in θ (weakly) lowers the probability that an investor will be matched with a trader and, hence, (weakly) lowers the level (productive) investment. But an increase in θ strictly increases the probability that a lender will be matched with a trader, in the event of a borrower default, and this enhances the "liquidity" of a lender's collateral. (Collateral becomes more "liquid" in the sense that it can be converted into the consumption good with a higher probability.) To assess a government policy that changes the value of θ, one simply has to compare the "investment effect" with the "liquidity effect." Generally speaking, the net effect can go either way as the magnitudes of the two effects depend upon model parameters.

Consider first the situation where $n^T > \Delta m + n^I$. One can interpret this situation as one where the date-3 market is "liquid" for both investors and lenders since they are always matched with probability one. In this situation, the optimal government policy is obvious.

Proposition 4 *When , then the optimal government policy provides an exemption from a bankruptcy stay for all lenders.*

Proof. From (8) and (10), when $n^T > \Delta m + n^I$

$$\frac{\partial W}{\partial \theta} = n^L \frac{\partial W_L}{\partial \theta} + n^I \frac{\partial W_I}{\partial \theta} = m\delta\Delta\left(1-\gamma\right)\tilde{a}_3 > 0,$$

for all θ. Hence, the government should choose θ "as high as possible," i.e., $\theta = 1$. ■

Consider now the interesting case where the date-3 market is illiquid from the investor's perspective in the sense that $n^I > n^T$. For this case, again using (8) and (10), we obtain

$$\frac{\partial W}{\partial \theta} = \frac{1}{\left(\Delta\theta m + n^I\right)^2}\left[\left(1-\gamma\right)\tilde{a}_3\left(\theta\right) - \left(f\left(\bar{a}\right) - \bar{a}\right)\right]. \tag{11}$$

When $n^I > n^T$, the optimal government policy is determined by comparing the value of $f(\bar{a}) - \bar{a}$—which is proportional to the investment effect—with that of $(1-\gamma)\tilde{a}_3(\theta)$—which is proportional to the liquidity effect—for various values of θ. More formally,

Proposition 5 *Suppose $n^I > n^T$. If*

$$(1-\gamma)\,\tilde{a}_3(0) > (f(\bar{a}) - \bar{a}),$$

then the optimal government policy provides an exemption from a bankruptcy stay of all lenders, i.e., $\theta = 1$. If

$$(1-\gamma)\,\tilde{a}_3(1) < (f(\bar{a}) - \bar{a}),$$

then is the optimal government policy requires a bankruptcy stay for all lenders, i.e., $\theta = 0$.

Proof. Since $\tilde{a}_3 = \min\{\tilde{c}_1, \bar{c}\}$, Proposition 1 implies that $\partial\tilde{a}_3/\partial\theta \geq 0$. If $(1-\gamma)\,\tilde{a}_3(0) > (f(\bar{a}) - \bar{a})$, then from (11) $\partial W/\partial\theta > 0$ for all $\theta \in [0,1]$, and setting $\theta = 1$ is optimal. If $(1-\gamma)\,\tilde{a}_3(1) < (f(\bar{a}) - \bar{a})$, then from (11) $\partial W/\partial\theta < 0$ for all $\theta \in [0,1]$, and setting $\theta = 0$ is optimal. ■

This proposition is quite interesting. Even though the date-3 market is illiquid from the perspective of investors—even if lenders are not permitted to participate—it may be optimal for the government to exempt defaulted lenders from a bankruptcy stay. This may happen when, intuitively speaking, the "liquidity value" of allowing lenders to have access to traders is greater than the "investment value" associated with investor-trader matches. It is true that when $\theta = 1$, lenders will displace economy-wide investment when $n^I > n^T$. However, the value of the liquidity, $(1-\gamma)\,\tilde{a}_3$, generated by lenders exceeds that of the displaced investment.

The next proposition fully characterizes optimal government policy when $n^I > n^T$.

Proposition 6 *When $n^I > n^T$, an optimal government policy either provides an exemption from a bankruptcy stay for all lenders, $\theta = 1$, or imposes a bankruptcy stay on all lenders, $\theta = 0$.*

Proof. When $W(\theta)$ is strictly monotonic in θ, the optimal θ is either $\theta = 0$ or $\theta = 1$, see Proposition 5.

Suppose that $W(\theta)$ is weakly monotonic in θ. There are two cases to consider. If $(1-\gamma)\,\tilde{a}_3(0) = (f(\bar{a}) - \bar{a})$ and $(\partial\tilde{a}_3(\theta)/\partial\theta)_{\theta=0} = 0$, then $\theta = 1$ and $\theta = 0$ are optimal policies, (and, in fact, any $\theta \in [0,1]$ is an optimal policy). If $(1-\gamma)\,\tilde{a}_3(0) < (f(\bar{a}) - \bar{a})$, $(\partial\tilde{a}_3(\theta)/\partial\theta)_{\theta=0} > 0$, and $\partial\tilde{a}_3(\theta)/\partial\theta = 0$ for all $\theta \in \left[\hat{\theta}, 1\right]$, $\hat{\theta} > 0$, then the unique optimal policy is $\theta = 0$.

Suppose that $W(\theta)$ is not monotonic in θ. $W(\theta)$ is not monotonic only if: (i) $(1-\gamma)\,\tilde{a}_3(0) < (f(\bar{a}) - \bar{a})$; (ii) $(1-\gamma)\,\tilde{a}_3\left(\hat{\theta}\right) = (f(\bar{a}) - \bar{a})$, for some $\hat{\theta} < 1$; and (iii) $\tilde{a}_3(\theta) = \tilde{c}_1(\theta) < \bar{c}$ at $\theta = \hat{\theta}$. In this situation, a global maximum of $W(\theta)$ will occur at either $\theta = 0$ or $\theta = 1$ (or both). That is, a global maximum cannot occur at $\theta \in (0,1)$. The optimal government policy is unique if and only if $W(0) \neq W(1)$. ■

When $n^I > n^T$, the optimal government policy is either imposes a bankruptcy stay on all lenders or an exemption from a bankruptcy stay for all lenders.

Although a partial exemption, i.e., $\theta \in (0,1)$, is permitted, it is never optimal, except for the knife edge case where $W'(\theta) = 0$ for all $\theta \in [0,1]$.[14] But even in this knife-edge case, $\theta = 1$ or $\theta = 0$ is an optimal policy. In spite of the relative illiquidity of the date-3 market, i.e., $n^I > n^T$, an exemption from a bankruptcy stay for all lenders is optimal when the liquidity value associated with providing access to the date 3 market for lenders is greater than the displacement of real investment opportunities.

The final case in terms of date-3 market liquidity to consider is when $n^I < n^T$ and $\Delta n^L + n^I > n^T$. Here, there exists a θ, say θ^*, such that $\theta^* \Delta n^L + n^I = n^T$. Clearly, it would never be optimal for the government to choose a $\theta < \theta^*$. Optimal government policy here somewhat mirrors the case where $n^I > n^T$, except now the lower bound of optimal government policy is θ^* instead of $\theta = 0$. In particular

Proposition 7 *When $n^I < n^T$ and $\Delta n^L + n^I > n^T$ an optimal government policy either provides an exemption from a bankruptcy stay for all lenders, $\theta = 1$, or imposes a bankruptcy stay on fraction $1 - \theta^*$ of lenders.*

Proof. The proof follows those of Propositions 5 and 6. ■

NOTE: We should make some remarks here, in terms of giving a possible interpretation is the optimal government policy turns out to be θ^*, where $0 < \theta^* < 1$.

5 Final Remarks

To be completed. One rather important thing to point out is that some people have proposed policies that provide an exemption from a bankruptcy stay *only if* the market for collateral is extremely liquid, (Duffie, Skeel); if the market for collateral is not extremely liquid, then an stay should be imposed on collateral. In terms of the our model, these people would allow for an exemption from a bankruptcy stay, i.e., $\theta = 1$, whenever $n^T > \Delta n^L + n^I$; if $n^T \leq \Delta n^L + n^I$, then they would impose $\theta = 0$. But note that these people are only focusing on the investment effect associated bankruptcy stays and completely ignore the liquidity effect. If the liquidity effect is relatively large, then an exemption for bankruptcy stay, i.e., $\theta = 0$, can be an optimal policy even when the market for collateral is not extremely liquid.

6 (Some) References

Acharya, Viral V., V. Ravi Anshuman, and S. Viswanathan. 2012. "Bankruptcy Exemption of Repo Markets: Too Much Today for Too Little Tomorrow?" Manuscript.

Bolton, Patrick and Martin Oehmke. 2011. "Should Derivatives be Privileged in Bankruptcy?" NBER working paper 17599.

[14] Among other things, this knife-edge case requires that $\tilde{c}_1 > \bar{c}$.

Duffie, Darell, and David Skeel. 2012. "A Dialogue on the Costs and Benefits of Automatic Stays for Derivatives and Repurchase Agreements." Rock Center for Corporate Governance at Stanford University Working Paper No. 108.

McAndrews, James J., and William Roberds. 2003. "Payment Intermediation and the Origins of Banking" Manuscript.

Roe, Mark. 2011. "The Derivatives Players' Payment Priorities as Financial Crisis Accelerator." Stanford Law Review 63.