Sovereign Risk Premia in the Eurozone [*]

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Abstract

This paper uses Bayesian methods to estimate the 'fiscal limit' distribution implied by a rational expectations framework using the data for Greece. We build a real business cycle model that allows interactions among fiscal policy instruments, stochastic 'fiscal limits,' and sovereign default risks. The fiscal policy specification takes into account government spending, lump-sum transfers, and distortionary taxation. A fiscal limit measures the debt level beyond which the government is no longer willing to finance, causing a partial default to occur. Using the particle filter to perform likelihood-based inference, we estimate the full nonlinear model with post-EMU data until 2010Q4. We find that Greek debt had a small, positive probability of default when Greece joined the EMU in 2001, but it fell quickly and remained close to zero until 2009, when it began to rise sharply to the range of 6% to 16% by the fourth quarter of 2010. The surge in the real interest rate in 2011, however, is generally outside of forecast confidence bands of our rational expectations model.

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Keywords: Sovereign default risk, sequential Monte Carlo, Bayesian estimation

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1. Introduction

In the past five years, there has been tremendous concern over the fiscal positions in several Eurozone nations. As seen in table 1, the long term interest rate spreads in the secondary market between German government bonds and several European countries' government bonds have risen markedly. The spread between Greek bonds and German bonds rose from 2.35 percentage points in 2009 to 19.2 in 2011, and the same spread between Portuguese and German bonds widened from 0.77 percentage points to 11.15 during the same period. Italy and Spain had notable increases as well. The aim of this paper is to discern the extent to which mounting interest rates and probabilities of default on sovereign debt in the Eurozone countries can be explained by a rational expectations model.

Bi and Traum (2012) shows how to use Bayesian methods and likelihood-based inference to estimate a real business cycle (RBC) model that allows for sovereign default. While Bi and Traum (2012) provides a coherent framework for estimating forward-looking 'fiscal limits,' their estimated model is unable to forecast the sharp rise in long term interest rates in 2011 in Greece, in part because their model estimates imply low historical probabilities of default. This paper extends the framework of Bi and Traum (2012) to a more realistic set-up and estimates a full nonlinear model using post-EMU data for Italy, Greece, Spain, and Belgium.[1] Using the estimated model, we identify country-specific probabilities of default, and discern the extent to which risk premia observed in European countries can be explained by macroeconomics fundamentals.

We consider a closed economy in which the government finances transfers and expenditures by collecting distortionary income taxes and issuing bonds. The bond contract is not enforceable and depends on the maximum level of debt that the government is *politically* able to service, a so-called 'fiscal limit.' We assume that the fiscal limit is stochastic and its distribution follows a logistical function. At each period, an effective fiscal limit is drawn from the distribution. If the level of government debt surpasses the effective limit, then the government reneges on a fraction of its debt. Based on the fiscal limit distribution, households can decide the quantity of government debts that they are willing to purchase and the price at which they are willing to pay.

The economy may switch between the default and no-default regimes endogenously, depending upon the level of government debt and the fiscal limit distribution. Therefore, the model cannot be solved using a first-order approximation; instead, it is solved using the monotone mapping method and estimated using Bayesian inference methods and a sequential Monte Carlo approximation of the likelihood [similar estimation methods are used in

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[1]The current draft provides results only for Greece.

Fernandez-Villaverde and Rubio-Ramirez (2007), Doh (2011), and Armisano and Tristani (2010)].

We estimate the model for Greece during the post-EMU period until 2010Q4. Using the estimated structural parameters, we compute the model-implied default probabilities for Greece's historical debt-to-GDP ratios. We find that Greek debt had a small, positive probability of default when Greece joined the EMU in 2001, but it fell quickly and remained close to zero until 2009, when it began to rise sharply to the range of 6% to 16% by the fourth quarter of 2010. Nevertheless, the surge in the real interest rate in Greece in 2011 is generally outside of forecast confidence bands of our rational expectations model. This suggests that the recent interest rate surge only can be explained by either a series of extremely bad structural shocks or by a fundamental change in the underlying economic structure, such as a change in investor sentiments.

In addition, after estimating the structural parameters we construct a model-implied distribution for the fiscal limit that is defined as the sum of the discounted maximum fiscal surplus in all future periods. This model-implied limit reflects the pure economic limit in raising tax revenue: the maximum fiscal surplus is obtained at the peak of the Laffer curve, beyond which a higher tax rate reduces the tax revenue. The estimated distribution for the fiscal limit, however, reflects both the political and the economic limit in raising tax revenue. By comparing the model-implied and the estimated distributions, we derive the 'political factor,' which measures the political willingness/ability to service its debt. We find that the political factor in Greece has been historically low, suggesting that unmodeled frictions have made the Greek government to be perceived as unable to sustain high levels of debt through large fiscal surpluses.

This paper is related to the large empirical literature that studies the determinants of sovereign default risk premia through reduced-form regressions.[2] This literature has found differences in sovereign risks across time and countries, suggesting the importance of country specific macroeconomic fundamentals to explain sovereign risk premia. In addition, related work by Ostry *et al.* (2010) estimate historical fiscal responses to construct 'debt limits,' although these limits are backward-looking by construction.

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[2]Recent examples include Lonning (2000), Lemmen and Goodhart (1999), Codogno *et al.* (2003), Alesina *et al.* (1992), Bernoth *et al.* (2006), Haugh *et al.* (2009), Bernoth and Erdogan (2011), Abmann and Hogrefe (2009), Maltritz (2011).

2. Model

Following Bi (2011), our model is a closed economy with linear production technology, whereby output depends on the level of productivity (A_t) and the labor supply (n_t).[3] Household consumption (c_t) and government purchases (g_t) satisfy the aggregate resource constraint,

$$c_t + g_t = A_t n_t. \tag{1}$$

Technological productivity A_t follows the $AR(1)$ process

$$A_t - A \;=\; \rho^A(A_{t-1} - A) + \varepsilon_t^A \qquad \varepsilon_t^A \sim \mathcal{N}(0, \sigma_A^2). \tag{2}$$

2.1 Government

The government finances lump-sum transfers to households (z_t) and exogenous and unproductive purchases by levying a tax (τ_t) on labor income and issuing one-period bonds (b_t). Let q_t be the price of the bond in units of consumption at time t. For each unit of the bond, the government promises to pay the household one unit of consumption in the next period. However, the bond contract is not enforceable. At each period, a stochastic fiscal limit, which is specified in terms of debt-over-GDP ratio and denoted as s_t^*, is drawn from its distribution, $s_t^* \sim \mathcal{S}^*$. We specify the cumulative density function of the fiscal limit distribution as a logistical function with parameters η_1 and η_2 dictating its shape.

$$p^* \equiv P(s_{t-1} \geq s_t^*) = \frac{\exp(\eta_1 + \eta_2 s_{t-1})}{1 + \exp(\eta_1 + \eta_2 s_{t-1})} \tag{3}$$

where s_t is defined as b_t/y_t. If the debt surpasses the fiscal limit, then it partially defaults. The default scheme can be summarized as,

$$\Delta_t = \begin{cases} 0 & \text{if } s_{t-1} < s_t^* \\ \delta & \text{if } s_{t-1} \geq s_t^* \end{cases}$$

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[3]Ceteris paribus, in our model the assumption that the economy is open and all debt is held by foreigners raises the observed risk premium relative to the closed economy environment,, as foreigners do not feel the negative wealth effects of debt and, in turn, have less incentive to hold debt. Thus, the estimates from our closed economy framework can be thought as the *lower* bound to estimates of the fiscal limit associated with certain probabilities of default.

The government's budget constraint is given by

$$\tau_t A_t n_t + b_t q_t = \underbrace{(1 - \Delta_t)b_{t-1}}_{b_t^d} + g_t + z_t. \tag{4}$$

The tax rate and government spending evolve according to the rules,

$$\tau_t = \underbrace{(1 - \rho^\tau)\tau + \rho^\tau \tau_{t-1} + \varepsilon_t^\tau}_{u_t^\tau} + \gamma^\tau\left(b_t^d - b\right) \qquad \varepsilon_t^\tau \sim \mathcal{N}(0, \sigma_\tau^2) \tag{5}$$

$$g_t = \underbrace{(1 - \rho^g)g + \rho^g g_{t-1} + \varepsilon_t^g}_{u_t^g} + \gamma^g\left(b_t^d - b\right) \qquad \varepsilon_t^g \sim \mathcal{N}(0, \sigma_g^2) \tag{6}$$

with $AR(1)$ components being denoted as u_t^τ and u_t^g. The non-distortionary transfers are modeled as a residual in the government budget constraint, exogenously determined by the AR(1) process,

$$z_t - z = \rho^z(z_{t-1} - z) + \varepsilon_t^z \qquad \varepsilon_t^z \sim \mathcal{N}(0, \sigma_z^2). \tag{7}$$

Since transfers are not an observable in our estimation, z_t can be thought of as capturing all movements in government debt that are not explained by the model.

2.2 Household

With access to the sovereign bond market, a representative household chooses consumption (c_t), labor supply (n_t), and bond purchases (b_t) by solving,

$$\max \quad E_0 \sum_{t=0}^{\infty} \beta^t \left(\log\left(c_t - h\bar{c}_{t-1}\right) + \phi\log(1 - n_t)\right) \tag{8}$$

$$s.t. \quad A_t n_t(1 - \tau_t) + z_t - c_t = b_t q_t - (1 - \Delta_t)b_{t-1} \tag{9}$$

The household's first-order conditions are,

$$\phi\frac{c_t - h\bar{c}_{t-1}}{1 - n_t} = A_t(1 - \tau_t) \tag{10}$$

$$q_t = \beta E_t\left((1 - \Delta_{t+1})\frac{c_t - h\bar{c}_t}{c_{t+1} - h\bar{c}_t}\right) \tag{11}$$

The bond price reflects the household's expectation about the probability and magnitude of sovereign default in the next period. The optimal solution to the household's maximization

problem must also satisfy the following transversality condition,

$$\lim_{j \to \infty} E_t \beta^{j+1} \frac{u_c(t+j+1)}{u_c(t)} (1 - \Delta_{t+j+1}) b_{t+j} = 0. \tag{12}$$

2.3 Model Solution

Other than the specifications for exogenous state variables, the core equilibrium equations
are,

$$q_t = \frac{b_t^d + z_t + g_t - \tau_t A_t n_t}{b_t} \tag{13}$$

$$q_t = \beta(c_t - hc_{t-1}) E_t \frac{1 - \Delta_{t+1}}{c_{t+1} - hc_t}. \tag{14}$$

The first equation is derived from the government budget constraint, while the second is from
the household's first-order conditions. We use the monotone mapping method (Coleman
(1991), Davig (2004)) to solve the decision rule of the bond price in terms of the state
vector. At time t, the state vector is $(b_t^d, c_{t-1}, A_t, u_t^g, z_t, u_t^\tau)$, and the decision rule of the bond
price can be written as $q_t = q(b_t^d, c_{t-1}, A_t, u_t^g, z_t, u_t^\tau)$.

In terms of computation, the most time-consuming part is the loop iterations of the
numerical integration in equation (14).

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\begin{aligned}
E_t \frac{1 - \Delta_{t+1}}{c_{t+1} - hc_t} &= \int_{\varepsilon_{t+1}^A} \int_{\varepsilon_{t+1}^g} \int_{\varepsilon_{t+1}^\tau} \int_{s_{t+1}^*} \frac{1 - \Delta_{t+1}}{c_{t+1} - hc_t} \\
&= \left(1 - \Phi(s_t \geq s_{t+1}^*)\right) \int_{\varepsilon_{t+1}^A} \int_{\varepsilon_{t+1}^g} \int_{\varepsilon_{t+1}^\tau} \frac{1}{c_{t+1} - hc_t}\Big|_{\text{no default}} \\
&+ \Phi(s_t \geq s_{t+1}^*) \int_{\varepsilon_{t+1}^A} \int_{\varepsilon_{t+1}^g} \int_{\varepsilon_{t+1}^\tau} \frac{1 - \delta}{c_{t+1} - hc_t}\Big|_{\text{default}}
\end{aligned}
\tag{15}
$$

Given the utility function, consumption is determined by,

$$c_t = \frac{\phi hc_{t-1} + (A_t - g_t)(1 - \tau_t)}{1 + \phi - \tau_t}. \tag{16}$$

Thus, the integration in Equation (15) can be re-written as

$$\int_{\varepsilon_{t+1}^A} \int_{\varepsilon_{t+1}^g} \int_{\varepsilon_{t+1}^\tau} \frac{1}{c_{t+1} - hc_t} = \int_{\varepsilon_{t+1}^A} \int_{\varepsilon_{t+1}^g} \int_{\varepsilon_{t+1}^\tau} \frac{1 + \phi - \tau_{t+1}}{(1 - \tau_{t+1})(A_{t+1} - g_{t+1} - hc_t)} \tag{17}$$

$$= \int_{\varepsilon_{t+1}^\tau} \frac{1 + \phi - \tau_{t+1}}{1 - \tau_{t+1}} \int_{\varepsilon_{t+1}^A} \int_{\varepsilon_{t+1}^g} \frac{1}{A_{t+1} - g_{t+1} - hc_t} \tag{18}$$

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The logarithmal utility function helps to reduce the 4-dimension integration into 1- and 2-dimension integrations. Appendix A discusses the solution procedure in details.

3. Estimation

The model is estimated for one countries at the moment: Greece (2001Q1-2010Q4). The start date represents the quarter in which Greece officially adopted the Euro, because the interest rates during the pre-Euro period are susceptible to exchange rate risk from which our model abstracts. Five observables are used for the estimation, including real output, government spending, tax revenue, government debt, and a 10-year real interest rate. Appendix B.1 provides a detailed description of the data.

3.1 Methodology

We estimate the model using Bayesian methods. The equilibrium system is written in the nonlinear state-space form:

$$x_t = f(x_{t-1}, \epsilon_t, \theta) \tag{19}$$

$$v_t = Ax_t + \xi_t, \tag{20}$$

where observables v_t are linked with model variables x_t via the matrix A, θ denotes model parameters, and ξ_t is a vector of measurement error distributed $N(0, \Sigma)$. We assume Σ is a diagonal matrix and calibrate the standard deviation of each measurement error to be 20% of the standard deviation of the corresponding observable variable.[4]

We use a particle filter to approximate the likelihood function. For a given sequence of observations up to time t, $v^t = [v_1, ..., v_t]$, the particle filter approximates the density $p(x_t|v^t, \theta)$ with a swarm of particles x_t^i ($i = 1, ..., N$), see appendix B.2 for more details. The particle filter is applicable for nonlinear and non-Gaussian distributions,[5] and it is increasingly used to estimate nonlinear DSGE models, to which class our model belongs. Recent examples include An and Schorfheide (2007), Fernandez-Villaverde and Rubio-Ramirez (2007), Armisano and Tristani (2010), Fernandez-Villaverde *et al.* (2011), and Doh (2011). Doucet *et al.* (2001) provide a textbook treatment.

We combine the likelihood $L(\theta|v^T)$ with a prior density $p(\theta)$ to obtain the posterior

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[4]Estimating measurement errors provides complications in nonlinear models. See Doh (2011) for more discussion on the role of measurement error in nonlinear DSGE model estimation.

[5]In addition, the particle filter is more robust than the unscented Kalman filter to sample initialization date, as the particle filter assumes a distribution for the unobserved initial state.

density kernel, which is proportional to the posterior density, $p(\theta|v^T) \propto p(\theta)L(\theta|v^T)$. We assume that parameters are independent a priori. However, we discard any prior draws that do not deliver a unique rational expectations equilibrium, as we restrict the analysis to the determinacy parameter subspace.[6] We construct the posterior distribution of the parameters using the random walk Metropolis-Hastings algorithm, see appendix B.3 for more details. In each estimation, we sample 75,000 draws from the posterior distribution and discard the first 15,000 draws.[7] The sample is thinned by every 25 draws, and the likelihood is computed using 60,000 particles.

3.2 Prior Distributions

We impose dogmatic priors over some parameters, which are listed in table C. The discount rate is 0.99, so that the deterministic net interest rate is 1%.[8] We calibrate the household's leisure preference parameter ϕ such that a household spends 25% of its time working at the steady state. We calibrate the deterministic debt to GDP ratio, government spending to GDP ratio, and tax rate to the mean values of the data sample.

The priors for the remaining parameters are listed in table C. The prior for habit persistence h is similar to those in the linear DSGE estimation literature, for instance Smets and Wouters (2007). For the remaining parameters, we first estimate using ordinary least squares an AR(1) process for GDP and processes for government spending, the tax rate, and transfers given by equations (5)-(7).[9] The results are used as general guidance for the region of the parameter space for the ρ, σ, and γ parameters.

For the responses of government spending and taxes to debt, we form priors for the long run responses in terms of percentage deviations from steady state, that is

$$\gamma^{g,L} \equiv \frac{\bar{b}\gamma^g}{\bar{g}(1-\rho^g)}, \quad \gamma^{t,L} \equiv \frac{\bar{b}\gamma^\tau}{\bar{\tau}(1-\rho^\tau)}$$

These values are more comparable to estimates in the literature. Since determinacy is sensitive to the combination of the $\gamma^{\tau,L}$ and $\gamma^{g,L}$ parameters, we restrict the lower bound of the $\gamma^{\tau,L}$ ($\gamma^{g,L}$) prior to a value that ensures determinacy when only $\gamma^{\tau,L}$ ($\gamma^{g,L}$) finances debt.

For the standard deviations of shocks, we form priors for the standard deviations relative

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[6]A technical appendix of the authors provides more discussion on this point.

[7]We use Fortran MPI code compiled in Intel Visual Fortran for the estimation. We use the computer server system at the Bank of Canada, each CPU of which uses Xeon CPU X5680 at 3.33GHz and has 23 processors with 64G RAM. One evaluation using the particle filter takes 10 seconds. These computational constraints limit the number of draws from the Metropolis-Hastings algorithm.

[8]The mean of our data is 0.8% for Greece.

[9]We back out the model-consistent tax rate and transfers series implied by our observables.

to relevant steady state variables: $\sigma_{k,p} \equiv \sigma_k/\bar{J}$ for $J = \{A, g, \tau, z\}$ and $k = \{a, g, \tau, z\}$. This gives standard deviations as percentage deviations, which provides more intuitive comparisons across values.

3.2.1 Fiscal Limit

We estimate one parameter from the fiscal limit distribution, which is represented by equation (3). Given two points on the distribution, $(\tilde{s}, \tilde{p})$ and $(\hat{s}, \hat{p})$, the parameters η_1 and η_2 can be uniquely determined by

$$\eta_2 = \frac{1}{\tilde{s} - \hat{s}} \log\left(\frac{\tilde{p}}{\hat{p}}\frac{1-\hat{p}}{1-\tilde{p}}\right), \quad \eta_1 = \log\frac{\tilde{p}}{1-\tilde{p}} - \eta_2\tilde{s}. \tag{21}$$

Since $(\tilde{s}, \tilde{p})$ and $(\hat{s}, \hat{p})$ provide a more intuitive description about the fiscal limit distribution than η_1 and η_2, we can fix $\tilde{p}$ and $\hat{p}$ at certain levels and estimate the corresponding $\tilde{s}$ and $\hat{s}$, instead of estimating η_1 and η_2 directly. We choose $\tilde{p} = 0.3$ and $\hat{p} = 0.999$. Unfortunately, given that defaults are not observed in our data sample, the data is unlikely to be informative about the upper bound of the distribution. Therefore, we estimate $\tilde{s}$ and fix the difference between $\tilde{s}$ and $\hat{s}$ to be 60% of steady-state output. This difference is chosen to capture the observation that once risk premia begin to rise, they do so rapidly.[10] Given the lack of guidance for the parameter $\tilde{s}$, we adopt a diffuse uniform prior over the interval 1.4 to 1.8.

3.2.2 δ Identification

To our knowledge, this paper is the first attempt to estimate a DSGE model of sovereign default. Thus, prior to estimating the model with real data, we performed several estimations with simulated data.[11] Unfortunately, the results revealed that we cannot jointly identify the rate of partial default δ and the fiscal limit parameter $\tilde{s}$ when the data excludes observed defaults. Parameters related to default affect observable variables through their influence on the risk premium. Since various combinations of δ and $\tilde{s}$ are consistent with the same risk premium, we cannot jointly identify the parameters. Given this limitation, we estimate our model for two different calibrations of δ: 0.05 and 0.075. These calibrations imply annualized rates of default δ^A of 20% and 30% respectively, which falls within the range of actual default rates in emerging market economies over the period 1983 to 2005, as documented by Bi (2011).

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[10]The difference of 60% of output, albeit ad-hoc, should not change the key estimation results as the data is unlikely to be informative about the upper bound of distribution.

[11]The results are available in a technical appendix from the authors.

3.3   Posterior Estimates

Table 3 compares the medians and 90% credible intervals of the posterior distributions estimated under the two specifications of partial default rate. The data are informative for all of the parameters, as the 90% credible intervals are smaller than those from the prior distributions.

The estimates of $\tilde{s}$ are very similar for the two specifications. If the rate of partial default δ^A is 30%, the debt to GDP ratio that is associated with 30% probability of default is between 1.43-1.54, with the median being 1.5. If the rate of partial default is 20%, on the other hand, the debt to GDP ratio that is associated with 30% probability of default is between 1.44-1.53, with the median being 1.48. Holding $\tilde{s}$ constant, a higher default rate implies a larger risk premium in the model. The response of tax rate to debt $\gamma^{\tau,L}$ would adjust to match the data. In the high δ^A calibration, the posterior for $\gamma^{\tau,L}$ has more values concentrated at higher levels than the posterior for the mid δ^A calibration. Ceteris paribus, a larger $\gamma^{\tau,L}$ implies a stronger response of tax rate to debt, which lowers the risk premium.

For comparison, we also list the estimates implied by a log-linearized version of our model without default. We use the Kalman filter to calculate the likelihood function and initialize the Metropolis-Hastings algorithm using the posterior mode and inverse Hessian at the posterior mode. The system of equations for the log-linearized model are listed in appendix C.

Interestingly, the estimates from the linear model suggest that the data is not informative about $\gamma^{g,L}$, as the 90% credible interval from the posterior distribution mirrors that from the prior distribution, shown in table (3). All the other parameters are comparable across different model specifications, although the 90% credible intervals are slightly tighter in the nonlinear models for parameters like ρ^g, ρ^z, and ρ^τ. The comparison suggests that allowing default in the standard RBC model may help to identify the fiscal policy responses in Greece.

4.   Analysis

4.1   Model Fit

To examine how well the model fits the data, we compute smoothed estimates of model variables using the sequential monte carlo approximation of the forward-backward smoothing recursion. Figures 1 compare the smoothed values from the various model specifications to the observable variables. For each specification, the fitted values are computed using the corresponding posterior mode. The fit for most variables is accurate, with output and tax

revenue being the least precise.

We also compute smoothed estimates of the measurement errors $E(\xi_t|v^T, \theta)$ and report their mean absolute values and relative standard deviations in table 4. The standard deviation of each measurement error was fixed to be 20% of the standard deviation of the respective observable variable. However, for most observables, the estimated relative standard deviation is less than 20%, which suggests that the measurement error did not introduce many constraints for the model fit. The exception is the measurement error for output and tax revenue, which is probably due to the reduced-form nature of the private sector in our model. Table 4 also shows that mean absolute values of measurement error are close to zero.

4.2 Default Probability and Interest Rate Dynamics

4.2.1 Default Risk

Figure 2 shows model-implied sovereign default probabilities for Greece, based upon the posterior estimates for the fiscal limit distribution when $\delta^A = 0.2$. Solid lines show the median and 90% confidence interval for historical debt-to-GDP ratios calculated from our debt and output observables for the estimated sample period.[12] Dashed lines denote the median and 90% confidence interval default probability for the out-of-sample debt-to-GDP ratio in 2011Q1.[13]

Figure 2 shows that initially when Greece joined the European Monetary Union in 2001, its sovereign debt had a small positive probability of default (1-3%). However, this probability quickly fell and for most of the 2000s, Greek debt had virtually zero probability of default. Starting in 2009 the probability of default rose steadily, with a sharp increase over 2010. The model-implied probability of default ranged from 6-16% in the end of the estimated period 2010Q4, and ranging from 8-22% in 2011Q1. Estimates from the model's fiscal limit distribution thus suggest the unsustainability of the Greek fiscal position and reflect large deterioration in confidence in Greek debt over 2010.

4.2.2 Out-of-Sample Interest Rate Forecasts

Over the course of 2011, the long term interest rate spread in the secondary market between Greek government bonds and German bonds rose from 9.1 percentage points in December

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[12] For the estimation we use data in terms of percentage deviations from the sample average. In contrast, we need level variables to back out model-implied probabilities of default. For model consistency, we convert the percentage deviations of the data to level variables using the steady state model variables.

[13] OECD government debt data for Greece is unavailable for later periods in 2011 and 2012, constraining our out-of-sample forecast to 2011Q1.

2010 to 19.21 in December 2011. Can the estimated model account for this sharp increase in the Greek interest rate?

To examine this issue, we simulate four quarters of time series 10,000 times starting from the fitted values for model variables in 2010Q4,[14] which gives a distribution for the forecasted path of the real interest rate in 2011. Figure 3 displays the median (blue, dotted line) and 90% interval (blue, dashed lines) of these model-implied interest rate forecasts for 2011, calculated using the posterior median parameter estimates (top panel) and the five percentile posterior estimates (bottom panel). The figure also plots the path of the real interest rate implied from the data (black solid line).

Figure 3 shows that the surge in the real interest rate in Greece is generally outside of forecast confidence bands of our rational expectations model. Using the posterior median parameter estimates, only the interest rate value in the first quarter of 2011 falls within the bands. The lower panel of the figure suggests that it is possible for model forecasts to be consistent with the 2011 interest rate path at all horizons, conditional upon extreme parameter values (note that this case is constructed based upon the posterior 5% estimates of *each* parameter, implying low responses of fiscal instruments to debt and a low debt-to-GDP ratio associated with a 30% probability of default). The model's difficulty in forecasting the interest rate path in 2011 suggest that the interest rate surge only can be explained by either a series of extremely bad structural shocks or by a fundamental change in the underlying economic structure, such as a change in investor preferences.

4.3   Laffer Curve and Fiscal Limit

In this section, we use the structural estimates to further explore how the market perceives the political willingness/ability to service its debt in Greece.

The proportional tax on labor income distorts a household's behavior as it lowers the after-tax wage and may induce households to work less. An increase in the tax rate can raise tax revenue when the existing tax rate is low, but it can reduce tax revenue when the existing tax rate is high, producing a Laffer curve. Laffer curves are usually dynamic in the sense that the shape of the Laffer curve depends on the state of the economy.[15] In our model, for given levels of productivity and government purchases (A_t, g_t), the government can collect the maximum level of tax revenue, denoted as $T^{max}(A_t, g_t)$, at the peak of the dynamic Laffer curve, denoted as $\tau^{max}(A_t, g_t)$. The maximum level of debt that the government can

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[14]See section ?? for details on the construction of fitted values.

[15]Trabandt and Uhlig (2011) compute Laffer curves for the United States and 15 European countries using a neoclassical model.

possibly pay back is the sum of the discounted maximum fiscal surplus in all future periods.

$$\mathcal{B}^{max} = \sum_{t=0}^{\infty} \beta^{t+1} \frac{u_c^{max}(A_{t+1}, g_{t+1})}{u_c^{max}(A_0, g_0)} (T^{max}(A_t, g_t) - g_t - z_t) \tag{22}$$

u_c^{max} represents the marginal utility of consumption when the tax rate is at the peak of the Laffer curve (τ^{max}). $\mathcal{B}^{max}$ is obtained, however, under the assumption that the government is willing to raise the tax at the peak of the Laffer curve, while angry protesters on Athen's streets illustrate the powerful political obstacles to achieve higher tax rates in reality. A reduced-form representation of the political economy perspective is to discount the fiscal surplus not only by a pure rate of time preference (β), but also by an additional political factor (β^{pol}).

$$\mathcal{B}^* = E \sum_{t=0}^{\infty} \beta^{t+1} \beta^{pol} \frac{u_c^{max}(A_{t+1}, g_{t+1})}{u_c^{max}(A_0, g_0)} (T^{max}(A_t, g_t) - g_t - z_t) \tag{23}$$

Given a particular set of parameter draws (θ_i), we can compute the model-implied distribution, $\mathcal{S}^{max}(\theta_i) = \mathcal{B}^{max}(\theta_i)/y_{T_0}^{max}(\theta_i)$, and the corresponding $\tilde{s}^{max}(\theta_i)$, at which the default probability is 0.3. $\tilde{b}_i$ is the corresponding draw for the debt threshold from our estimated fiscal distribution. Thus, the ratio between the estimated $\tilde{s}_i$ and the model-implied $\tilde{s}_i^{max}$ gives the political factor β_i^{pol}.

To compute the model-implied distribution, given the logarithm utility function, the tax revenue (T_t) can be written as,

$$
\begin{aligned}
T_t &= \tau_t \frac{A_t(1 - \tau_t) + \phi g_t + \phi h c_{t-1}}{1 + \phi - \tau_t} \\
&= (1 + 2\phi) A_t - \phi(g_t + h c_{t-1}) - \left(A_t(1 + \phi - \tau_t) + \frac{(1+\phi)\phi(A_t - h c_{t-1} - g_t)}{1 + \phi - \tau_t} \right). \tag{24}
\end{aligned}
$$

The tax revenue reaches to the maximum level (T_t^{max}) when the tax rate reaches the peak point of the Laffer curve (τ_t^{max}).

$$\tau_t^{max} = 1 + \phi - \sqrt{\frac{(1+\phi)\phi(A_t - g_t - h c_{t-1})}{A_t}} \tag{25}$$

$$\tag{26}$$

There exists a unique mapping between the exogenous state space (A_t, g_t) to τ_t^{max} and T_t^{max}. For a given set of parameter draws (θ_i), the distribution of fiscal limit ($\mathcal{S}^{max}(\theta_i)$) can be obtained using Markov Chain Monte Carlo simulation:

1. First, for each simulation j, we randomly draw the shocks for productivity (A_t^j), government purchases (g_t^j), and the transfers (z_t^j) for $T = 1500$ periods with the first $T_0 = 500$ as burn-in period. Assuming that the tax rate is always at the peak of the dynamic Laffer curves, we compute the paths of all other variables following the household first-order conditions and the budget constraints, and the discounted sum of maximum fiscal surplus is specified below.

$$B_j^{max}(\theta_i) = \sum_{t=T_0}^{t=T} \beta^{t+1-T_0} \frac{u_c^{max}(A_{t+1}^j, g_{t+1}^j)}{u_c^{max}(A_{T_0}^j, g_{T_0}^j)} \left(T^{max}(A_t^j, g_t^j) - g_t^j - z_t^j\right) \quad (27)$$

$$S_j^{max}(\theta_i) = \frac{B_j^{max}(\theta_i)}{y_{T_0}^{max,j}(\theta_i)} \quad (28)$$

2. Second, we repeat the simulation for 10000 times and obtain the distribution $\mathcal{S}^{max}(\theta_i)$ using the simulated $S_j^{max}(\theta_i)$ $(j = 1, ..., 10000)$.[16]

3. Finally, using Kernal estimation, we can derive the cumulative density function for the model-implied distribution $\mathcal{S}^{max}(\theta_i)$ for those particular parameter draws, and therefore obtain the model-implied debt level $\tilde{s}^{max}(\theta_i)$, at which the default probability is 0.3.

By repeating the above procedure to the posterior parameter draws (θ_i), we obtain a set of $\tilde{s}^{max}(\theta_i)$. The top row in table (5) shows the median and the 90% credible intervals for $\tilde{s}^{max}$ under various δ^A calibrations. The default rate specifications do not have much of an impact. For comparison, the second row in table (5) lists the median and the 90% credible confidence intervals for the estimated debt threshold $\tilde{s}$. The implied political factor β^{pol}, calculated as the ratio between the estimated $\tilde{s}$ and the model-implied $\tilde{s}^{max}$, is quite low in Greece, with the median of 0.3 under both δ cases. One interpretation is that the market perceives the Greek government is willing to raise taxes to the peak of Laffer curve with a probability of 30%.

5. Conclusion

This paper uses Bayesian methods to estimate the probability of sovereign default for Greece. We build a real business cycle model that allows the interactions among fiscal policy instruments, the stochastic fiscal limit, and sovereign default risks. The fiscal policy specification takes into account government spending, lump-sum transfers, and distortionary taxation. We

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[16]Increasing the number of simulations doesn't change the simulated distribution much.

model the fiscal limit distribution with a logistical function, which illustrates the market's belief about the government's ability to service its debt at various debt levels.

Using the particle filter to perform likelihood-based inference, we estimate the full non-linear model with post-EMU data. We find that Greek debt had a small, positive probability of default when Greece joined the EMU in 2001, but it fell quickly and remained close to zero until 2009, when it began to rise sharply to the range of 6% to 16% by the fourth quarter of 2010. Nevertheless, the surge in the real interest rate in Greece in 2011 is generally outside of forecast confidence bands of our rational expectations model.

Finally, we compute the dynamic Laffer curve for Greece and calculate the pure economic fiscal limit, that is the maximum level of debt that the government is able to service. We compare the difference between the estimated fiscal limit distributions and the pure economic fiscal limit and find that the Greek government has been perceived as willing to service its maximum level of debt with only about a 30% probability.

In current ongoing research, we are estimating the model for other European countries, so as to allow cross-country comparisons of default probabilities and political risk factors. Although our nonlinear model allows complex interactions among fiscal policy instruments and the fiscal limit, it is only a first step to understanding and estimating probabilities of default for developed countries. To understand fully the complexities associated with default risk, several other features are worthy of modeling attention, including the interaction of monetary and fiscal policies; the interaction of the financial sector and the government; and open economy issues including foreign holdings of debt and risks of contagion.

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A   Solving the Nonlinear Model

Other than the end-of-period government debt, all other variables are either exogenous or can be computed in terms of the current state $\psi_t = (b_t^d, c_{t-1}, A_t, u_t^g, z_t, u_t^\tau)$.

$$\tau_t = u_t^\tau + \gamma^\tau \left(b_t^d - b \right) \tag{A.1}$$

$$g_t = u_t^g + \gamma^g \left(b_t^d - b \right) \tag{A.2}$$

$$z_t = (1 - \rho^z)z + \rho^z z_{t-1} + \varepsilon_t^z \tag{A.3}$$

$$A_t = (1 - \rho^A)A + \rho^A A_{t-1} + \varepsilon_t^A \tag{A.4}$$

$$c_t = \frac{\phi h c_{t-1} + (A_t - g_t)(1 - \tau_t)}{1 + \phi - \tau_t} \tag{A.5}$$

$$\Delta_t = \begin{cases} 0 & \text{if } b_{t-1} < b_t^* \\ \delta & \text{if } b_{t-1} \geq b_t^* \end{cases}$$

The decision rule for government debt, $b_t = f^b(\psi_t)$, is solved in the following steps:

- Step 1: Define the grid points by discretize the state space ψ_t. Make an initial guess of the decision rule f_0^b over the state space.

- Step 2: At each grid point, solve the following core equation and obtain the updated rule f_i^b using the given rule f_{t-1}^b. The integral in the right-hand side is evaluated as described in Section 2.3 using numerical quadrature.

$$\frac{b_t^d + z_t + g_t - \tau_t A_t n(\psi_t)}{f_i^b(\psi_t)} = \beta(1 - \Delta_{t+1})E_t \frac{c(\psi_t) - hc_{t-1}}{c(\psi_{t+1}) - hc(\psi_t)} \tag{A.6}$$

where $\psi_{t+1} = \left(\underbrace{(f_{i-1}^b(\psi_t), \Delta_{t+1})}_{b_t^d}, c_t, A_{t+1}, u_{t+1}^g, z_{t+1}, u_{t+1}^\tau \right).$

- Step 3: Check the convergence of the decision rule. If $|f_i^b - f_{i-1}^b|$ is above the desired tolerance (set to $1e - 5$), go back to step 2; other wise, f_i^b is the decision rule and used to evaluate the particle filter as described below.

B   Estimation

B.1   Data Description

Data are for Italy and Greece. For each series, we transform the series into percentage deviations from the mean value of the sample and detrend each time series with its own linear trend.

Real GDP. Constructed by dividing the nominal quarterly gross domestic product from the OECD quarterly National Accounts (using the expenditure approach, B1_GE) by the gross domestic product deflator (constructed using the expenditure approach, B13).

Real Gov. Spending. Constructed using general government final consumption expenditure from the OECD quarterly National Accounts (P3S13) divided by the gross domestic product deflator (constructed using the expenditure approach, B13).

Real Tax Revenue. Using the annual nominal tax revenue (tax revenue consisting of indirect and direct taxes and social security contributions) from the OECD volume 90 (TIND + TY + SSRG), we interpolate the series to a quarterly frequency using the method of Chow and Lin (1971). For the interpolation, we construct a measure of total tax revenue by combining Eurostat quarterly series for tax receipts on income/wealth, production and imports, capital taxes, and social contributions. We seasonal adjustment this series using Demetra+ and the tramo-seat RSA4 specification. The seasonally adjusted quarterly Eurostat tax revenue series is then used for the interpolation to construct a quarterly real tax revenue series by dividing by the gross domestic product deflator (constructed using the expenditure approach, B13).

Real Gov. Debt. For Greece, using the annual nominal gross public debt series (under the Maastricht criterion) from the OECD volume 90, we interpolate the series to a quarterly frequency using the method of Chow and Lin (1971).[17] For the interpolation, we use the Eurostat quarterly series for nominal gross government consolidated debt. We seasonal adjustment this series using Demetra+ and the RSA4 specification. The seasonally adjusted quarterly Eurostat debt series is then used for the interpolation to construct a quarterly real debt series by dividing by the gross domestic product deflator (constructed using the expenditure approach, B13).

For Italy, we also interpolate a quarterly series. In this case, we use the annual nominal gross public debt series (under the Maastricht criterion) from the OECD volume 90 divided by the annual nominal gross domestic product (using the expenditure approach). We inter-

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[17]We use the quarterly real GDP series as a relative measure for the interpolation. Forni *et al.* (2009) use a similar approach.

polate this series to a quarterly frequency using the method of Chow and Lin (1971). For the interpolation, we use the Eurostat quarterly series for nominal gross government consolidated debt to GDP ratio. We seasonal adjustment this series using Demetra+ and the tramo-seat RSA4 specification. The seasonally adjusted quarterly Eurostat debt-to-GDP series is then used for the interpolation to construct a quarterly debt-to GDP series. We then multiply this series by the real quarterly gross domestic product (using the expenditure approach, B1_GE) from the OECD database to get a real quarterly government debt series.

Real Interest Rate. To construct a 10-year real interest rate measure, we use data for the nominal interest rate (taken from the BIS) and the expected inflation rate. Our measure of expected inflation for Italy comes from Consensus Economics, who ask a number of professional forecasters based in a variety of countries about their expectations of a wide range of economic variables. We use their long-term (five to ten year) forecast, which has been published biannually in April and October since the autumn of 1989.[18] For Greece, we use the expected inflation series from the Survey of Professional Forecasts EU-area five year ahead expected inflation series, which is a general euro-wide inflation series. The gross real interest rate is constructed using the relation

$$R_t = \frac{1 + i_t}{1 + \pi_t^e}$$

B.2   Particle Filter Algorithm

Let v^T denote $\{\hat{v}_t\}_{t=1}^T$, which evolves according to equations (19) and (20) in the text. To evaluate the likelihood function $L(\theta|v^T)$, we use a sequential Monte Carlo filter (specifically, the sequential importance resampling filter of Kitagawa (1996)). The algorithm is as follows:

- Step 1. Initialize the state variable x_0 by generating 40,000 values from the unconditional distribution $p(x_0|\theta)$. Denote these particles by x_0^i for $i = 1, ..., 40,000$. Draw 40,000 values from standard normal distributions for each of the structural shocks (ϵ^A, ϵ^g, ϵ^t, ϵ^z) and 40,000 values from a standard uniform distribution for fiscal limit probabilities. Denote the vector of these particles by u^i. By induction, in period t these are particles $u^{t|t-1,i}$.

- Step 2. Construct $x^{t|t-1,i}$ using equation (19) in the text. Assign to each draw ($u^{t|t-1,i}$,

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[18]This is the same method used in Upper and Worms (2003), and more details about the construction of real long term interest rates can be found therein.

$x^{t|t-1,i}$) a weight defined as:

$$w_t^i = \frac{1}{(2\pi)^{5/2}|\Sigma|^{1/2}} \exp\left[-\frac{1}{2}\left(y_t - Ax^{t|t-1,i}\right)' \Sigma \left(y_t - Ax^{t|t-1,i}\right)\right] \tag{B.1}$$

- Step 3. Normalize the weights:
$$\tilde{w}_t^i = \frac{w_t^i}{\sum_{i=1}^N w_t^i}$$
Update the values of $x^{t|t-1,i}$ by sampling with replacement 40,000 values of $x^{t|t-1,i}$ using the relative weights $\tilde{w}_t^i$ and the residual resampling algorithm.

- Repeat steps 2-3 for $t \leq T$.

The log-likelihood function is approximated by

$$L(\theta|v^T) \simeq \sum_{t=1}^T \ln\left(\frac{1}{40,000}\sum_{i=1}^{40,000} w_t^i\right) \tag{B.2}$$

B.3   MCMC Algorithm

The random walk Metropolis-Hastings algorithm used for estimation works as follows:

- Step 1. Compute the posterior log-likelihood for 500 draws from the priors. Call the draw with the highest posterior log-likelihood value θ^*.

- Step 2. Starting from θ^*, generate a MCMC chain using the following random-walk proposal density
$$\theta_{j+1}^{prop} = \theta_j^{prop} + c\mathcal{N}(0,\Lambda), \quad j = 1, ..., 100,000$$
where Λ is the covariance matrix of 500 draws from the priors and $c > 0$ is a tuning parameter set to determine the acceptance ratio.

- Step 3. Compute the acceptance ratio $\varphi = \min\left\{\frac{p(\theta_{j+1}^{prop}|v^T)}{p(\theta_j|v^T)}, 1\right\}$. Given a draw u from the standard uniform distribution. Then $\theta_{j+1} = \theta_{j+1}^{prop}$ if $u < \varphi$ and $\theta_{j+1} = \theta_j$ otherwise. Repeat for $j = 1, ..., 10,000$.

- Step 4. Update the random walk proposal density in the following way. Update Λ to be the covariance matrix from the previous draws $\{\theta_j\}_1^{10,000}$. Update θ^* to be the mean of previous draws $\{\theta_j\}_1^{10,000}$. Starting from the new θ^*, proceed through steps 2 and 3 for 38,000 draws from the new MCMC chain.

We burn the first 15,000 draws from the final MCMC chain and thin every 25 draws.

C   Log-Linearized Model Equations

The log-linearized system of equations for the variant of the model without default are:

$$\hat{c}_t - \frac{1}{1+h}E_t\hat{c}_{t+1} + \frac{1-h}{1+h}\hat{R}_t = \frac{h}{1+h}\hat{c}_{t-1}$$

$$\frac{1}{1-h}\hat{c}_t + \frac{n}{1-n}\hat{n}_t - \hat{A}_t + \frac{\tau}{1-\tau}\hat{\tau}_t = \frac{h}{1-h}\hat{c}_{t-1}$$

$$\frac{c}{y}\hat{c}_t + \frac{g}{y}\hat{g}_t = \hat{A}_t + \hat{n}_t$$

$$\frac{b}{y}\hat{b}_t - \frac{g}{y}\hat{g}_t - \frac{z}{y}\hat{z}_t + \tau(\hat{\tau}_t + \hat{A}_t + \hat{n}_t) = R * \frac{b}{y}(\hat{R}_{t-1} + \hat{b}_{t-1})$$

$$\hat{g}_t = (1-\rho^g)\hat{g}_{t-1} - \gamma^{g,L}(1-\rho^g)b_{t-1} + \sigma_{g,p}\epsilon_t^g, \quad \epsilon_t^g \sim N(0,1)$$

$$\hat{\tau}_t = (1-\rho^\tau)\hat{\tau}_{t-1} + \gamma^{\tau,L}(1-\rho^\tau)b_{t-1} + \sigma_{\tau,p}\epsilon_t^\tau, \quad \epsilon_t^\tau \sim N(0,1)$$

$$\hat{z}_t = (1-\rho^z)\hat{z}_{t-1} + \sigma_{z,p}\epsilon_t^z, \quad \epsilon_t^z \sim N(0,1)$$

$$\hat{A}_t = (1-\rho^a)\hat{A}_{t-1} + \sigma_{a,p}\epsilon_t^a, \quad \epsilon_t^z \sim N(0,1)$$

Table 1: 10-yr Nominal Interest Rate Spread (against Germany)

	2009	2010	2011
Greece	2.35	9.1	19.2
Portugal	0.77	3.62	11.15
Italy	0.87	1.69	4.88
Spain	0.67	2.47	3.6

Table 2: Calibration and Priors

Calibration

	Italy	Greece
β	0.99	0.99
$\bar{n}$	0.75	0.75
$\bar{g}/\bar{y}$	0.196	0.181
$\bar{b}/\bar{y}$	1.081*4	1.095*4
τ	0.413	0.333

Priors

	Function	Italy	Greece
h	Beta	0.5	0.5
		(0.2)	(0.2)
$\tilde{b}^*$	Uniform	1.6	1.6
		(0.013)	(0.013)
$\gamma^{\tau,L}$	Gamma	0.5	0.5
		(0.2)	(0.2)
$\gamma^{g,L}$	Gamma	0.4	1.1
		(0.2)	(0.3)
ρ^a	Beta	0.8	0.8
		(0.1)	(0.1)
ρ^g	Beta	0.5	0.5
		(0.2)	(0.2)
ρ^τ	Beta	0.5	0.5
		(0.2)	(0.2)
ρ^z	Beta	0.5	0.5
		(0.2)	(0.2)
$\sigma_{a,p}$	Gamma	0.005	0.005
		(0.003)	(0.003)
$\sigma_{g,p}$	Gamma	0.005	0.005
		(0.003)	(0.003)
$\sigma_{\tau,p}$	Gamma	0.005	0.005
		(0.003)	(0.003)
$\sigma_{z,p}$	Gamma	0.2	0.2
		(0.1)	(0.1)

Table 3: Greece Estimates.

	Prior		Posterior: $\delta^A = 0.3$		Posterior: $\delta^A = 0.2$		Posterior: Linear	
	mean	[5, 95]	median	[5, 95]	median	[5, 95]	median	[5, 95]
h	0.5	[0.17, 0.83]	0.08	[0.02, 0.16]	0.12	[0.04, 0.26]	0.12	[0.03, 0.27]
$\tilde{b}$	1.6	[1.42, 1.78]	1.5	[1.43, 1.54]	1.48	[1.44, 1.53]	-	-
$\gamma^{\tau,L}$	0.4	[0.14, 0.78]	0.3	[0.2, 0.44]	0.2	[0.14, 0.3]	0.23	[0.11, 0.38]
$\gamma^{g,L}$	1.1	[0.66, 1.64]	1.26	[1.16, 1.44]	1.31	[0.97, 1.58]	1.09	[0.67, 1.65]
ρ^a	0.8	[0.61, 0.94]	0.92	[0.90, 0.93]	0.94	[0.93, 0.96]	0.92	[0.90, 0.94]
ρ^g	0.8	[0.61, 0.94]	0.92	[0.87, 0.95]	0.90	[0.87, 0.94]	0.92	[0.84, 0.97]
ρ^z	0.5	[0.17, 0.83]	0.5	[0.43, 0.62]	0.62	[0.35, 0.70]	0.57	[0.33, 0.79]
ρ^τ	0.5	[0.17, 0.83]	0.67	[0.61, 0.74]	0.71	[0.65, 0.78]	0.64	[0.53, 0.73]
$\sigma_{a,p}$	0.01	[0.003, 0.02]	0.018	[0.014, 0.021]	0.019	[0.014, 0.027]	0.017	[0.014, 0.022]
$\sigma_{g,p}$	0.02	[0.003, 0.05]	0.04	[0.034, 0.043]	0.04	[0.037, 0.046]	0.040	[0.033, 0.049]
$\sigma_{z,p}$	0.5	[0.35, 0.68]	0.44	[0.38, 0.49]	0.52	[0.48, 0.65]	0.51	[0.40, 0.65]
$\sigma_{\tau,p}$	0.01	[0.003, 0.02]	0.013	[0.010, 0.016]	0.012	[0.010, 0.015]	0.013	[0.009, 0.017]

Table 4: Smoothed estimates of measurement error.

Greece

		b_t	g_t	T_t	y_t	R_t
Nonlinear $\delta^A = 0.3$	mean absolute value	0.0038	0.0029	0.0070	0.0077	0.0003
	relative standard deviation	0.097	0.070	0.255	0.225	0.118
Nonlinear $\delta^A = 0.2$	mean absolute value	0.0048	0.0033	0.0069	0.0082	0.0003
	relative standard deviation	0.136	0.081	0.235	0.233	0.129
Linear	mean absolute value	0.0039	0.0034	0.0061	0.0082	0.0003
	relative standard deviation	0.099	0.081	0.191	0.239	0.093

Table 5: Model-implied $\tilde{s}^{max}$ and estimated $\tilde{s}^*$

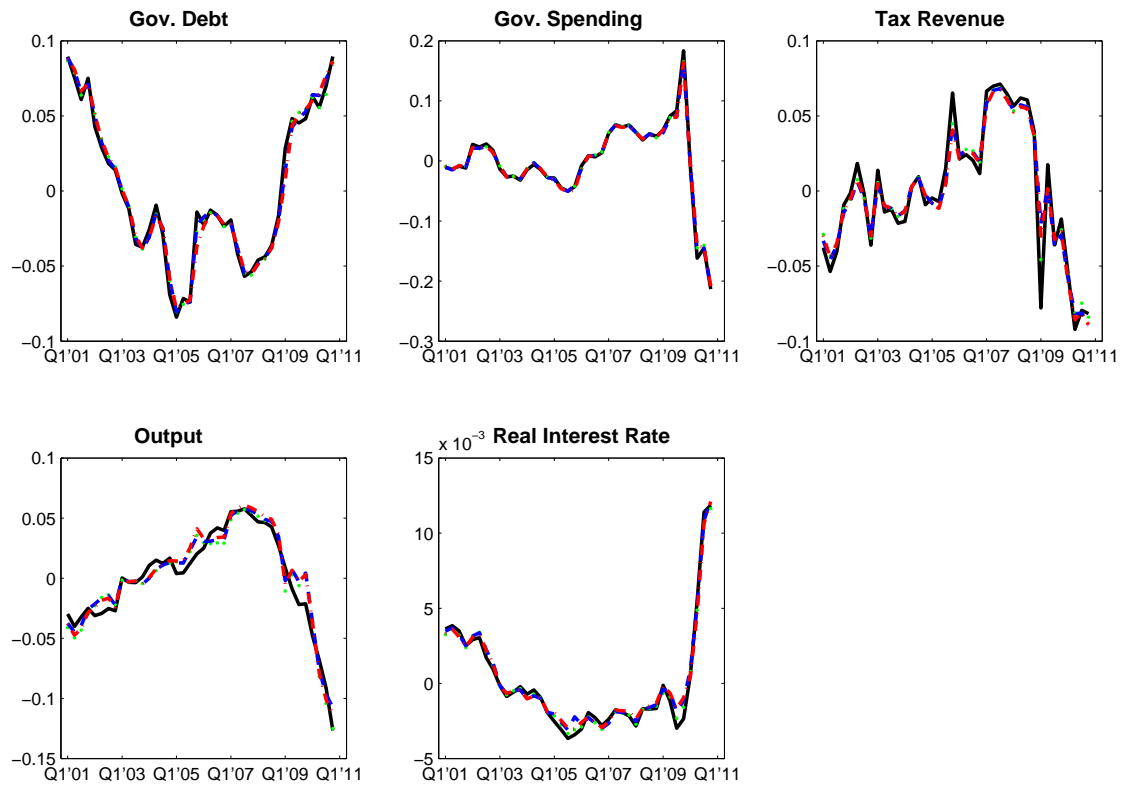|	Greece $\delta^A = 0.3$		Greece $\delta^A = 0.2$	
	median	[5, 95]	median	[5, 95]
$\tilde{s}^{max}$	4.91	[4.78, 5.07]	4.97	[4.84, 5.09]
$\tilde{s}^*$	1.47	[1.43, 1.52]	1.49	[1.43, 1.53]
β^{pol}	0.30	[0.29, 0.31]	0.30	[0.29, 0.31]

Figure 1: Fitted values for various estimations for Greece. Black, solid lines: data. Blue, dashed lines: Nonlinear model with $\delta^A = 0.3$. Red, dotted-dashed lines: Nonlinear model with $\delta^A = 0.2$. Green, dotted lines: Linear model.
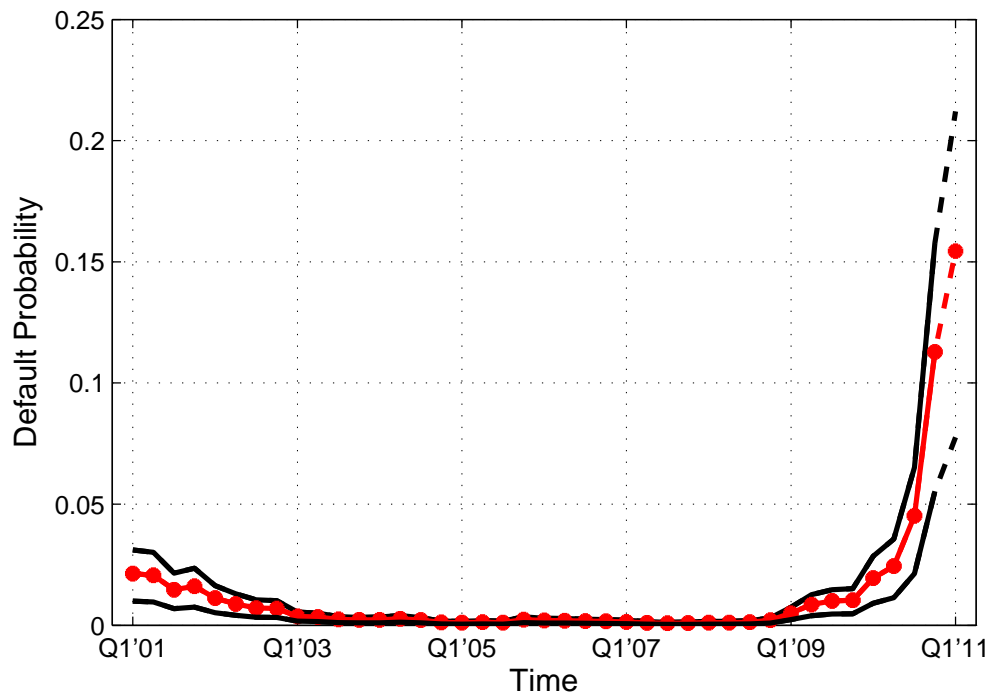
Figure 2: Model-implied sovereign default probabilities for Greece. Solid lines denote the median and 90% confidence interval probabilities for in-sample debt-to-GDP ratios. Dashed lines denote the median and 90% confidence interval probabilities for out-of-sample debt-to-GDP ratios.
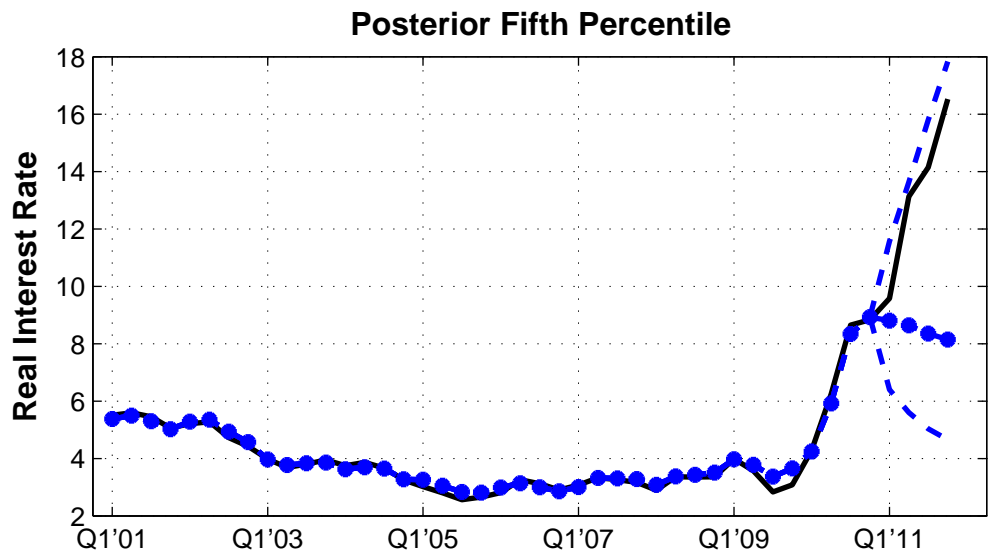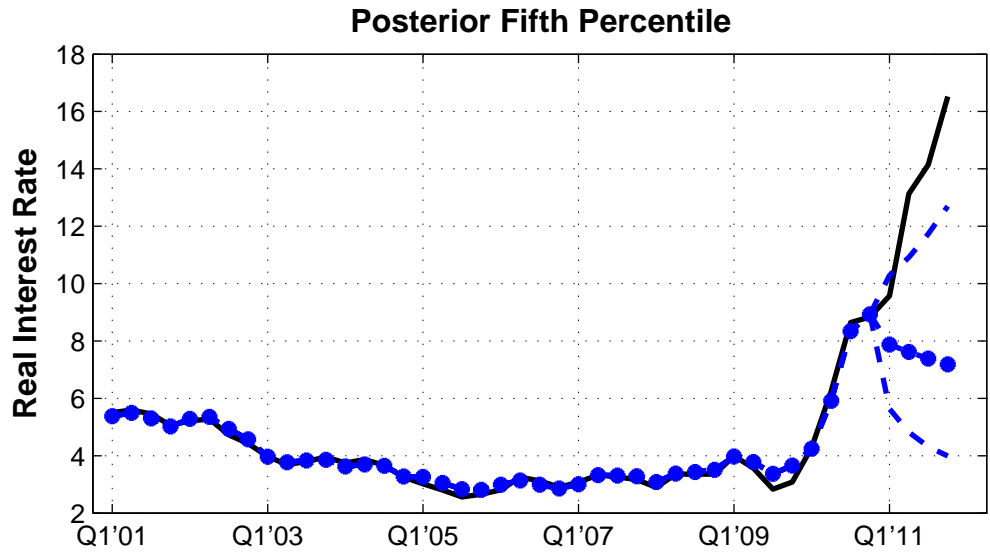
Figure 3: Data (black, solid line) versus fitted and forecast values (blue, dotted and dashed lines) for the Greek interest rate. The median (blue, dotted line) and 90% interval (blue, dashed lines) of model-implied interest rate forecasts for 2011 are calculated based on the posterior median parameter estimates (top panel) and 5 percentile parameter estimates (bottom panel).