

From time-varying macro-dynamics to time-varying estimates of DSGE parameters*

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Abstract

This paper estimates a 7 variable TVP-VAR on US data using kernel methods. We identify monetary policy shocks using sign restrictions for each period in our sample. We then fit the Smets-Wouters (2007) model to these impulse responses, tracing out evolutions in the structural DSGE parameters over time. Parameters defining nominal rigidities move a lot. Some real side ones move a lot (investment adjustment costs) and some are flat (eg the discount rate). Monetary policy parameters change, but not as much as is evident in other studies of the Great Moderation and its causes, contrary to the ‘indeterminacy’ theory of the Great Inflation.

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1 Introduction

1.1 What we do and find

This paper estimates a time-varying parameter VAR in the 7 US variables used to estimate the widely cited DSGE model of [Smets and Wouters \(2007a\)](#) [Hereafter SW] (closely related to the model of [Christiano et al. \(2005\)](#)). We then use sign-restrictions (following [Uhlig \(2005\)](#), [Canova and Nicolo \(2002\)](#) and others) to identify monetary policy shocks, and compute the impulse response of our observables to those shocks *for each observation in our sample period*. Finally, we use a minimum-distance estimator (see, for example, [Theodoridis \(2011\)](#)) to fit SW model to these impulse response functions, once again, *for each observation in our sample period*. We therefore map how the estimated reduced form macroeconomic dynamics translate into implied shifts in the structural DSGE parameters. In effect, we are taking the exercise that [Christiano et al. \(2005\)](#) did for their fixed-coefficient VAR-estimating a DSGE model by fitting the IRF to a monetary policy shock - but doing it for the hypothetical VARs implied by each value in the sequence of estimated VAR parameters (and using sign restrictions for identification, rather than zero restrictions as in [Christiano et al. \(2005\)](#)).

We uncover very substantial fluctuations in the DSGE parameters that best fit the evolving impulse response functions. For example: the probability that prices and wages are *not* reset varies between 0.6 and 0.85 for prices, and between 0.6 and 0.9 for wages. These values translate in to changes in the implied frequency of prices that are large relative to the range of values reported in micro studies. There are also significant changes in the coefficient that wage and price setters load onto past inflation when indexing between resets: from 0.45 to 0.7 in the case of wages, and from 0.25-0.7 in the case of prices. We compute that there is an increase in implied values for h , which encodes habits in consumption, from around 0.75 to 0.85. The investment adjustment costs parameter rises precipitously in the boom years before 2000 and falls just as quickly thereafter. Interestingly a parameter that is probably most securely micro-founded, the discount rate, is estimated to be pretty constant. We also detect variations in the parameters defining the behaviour of monetary policy. The standard deviation of monetary policy shocks varies between 0.18 and 0.1; the persistence in these shocks varies between 0.2 and 0.45. The coefficients in the policy rule are, in absolute terms, more stable than some of the others, but they still each vary by as much as 0.2 from their maximum to minimum values. There is no indication that there was ever a period where monetary policy was insufficiently responsive to inflation such that there was indeterminacy, in contrast to some other work on the Great Moderation. Our parameter estimates also seem to conflict with characterisations of monetary policy as being universally less responsive to inflation before Volcker than after him, a feature that emerges in some previous work, e.g. [Clarida et al. \(2000\)](#).

1.2 Connections to prior literature

Our paper is at the same time derivative of and hopefully contributes to several strands of literature in empirical macro. We make these connections below, distinguishing between the methodological literature on characterising and detecting structural change, and the substantive literature on generating and explaining change in VAR and DSGE parameters.

1.2.1 Methodological literature on characterising structural change

The first line of work we want to emphasise is methodological. This paper is the latest in a series of papers we have written seeking to comment on the industry-standard method for estimating stochastic time varying parameter VAR models, offering another complimentary tool in the applied macro armoury.¹ That industry standard was set by [Cogley and Sargent \(2005\)](#), [Cogley et al. \(2010\)](#) and others that followed them in many papers, including, for example: [Benati and Surico \(2008\)](#), [Clarida et al. \(2000\)](#), [Benati and Mumtaz \(2007\)](#) and [Mumtaz and Surico \(2009\)](#). This estimates the sequence of VAR parameters and volatilities by using the Gibbs Sampler to characterise the joint posterior density. Our alternative is to use a kernel estimator. This paper, which applies the method to a 7 variable VAR, is part of an effort to underscore the practical advantages of our method, namely, that it is quick and can handle large dimensions. It takes fractions of a second or seconds to run, rather than hours or days. Also, the industry standard method, when combined with the desire to impose the condition that the VAR parameter sequences imply instantaneous VARs are stationary, often breaks down due to a failure to find enough satisfactory draws. Our method does not have this problem, since we compute point estimates directly, rather than indirectly via characterising the likelihood using MCMC methods. We can inspect whether the stationarity condition is satisfied *ex post*. As an aside, in our previous work we stressed that this kernel estimator has good theoretical properties too: it is consistent and asymptotically normal. So even if the DGP is as the industry standard stochastic time varying coefficient model would have it, it may pay to use a kernel estimator. So far as we know, comparable results are not available for the Gibbs-Sampler based method, though, as we and no doubt many other practitioners know, they appear to work well in realistic monte carlos.

Of course, our offer of a tool to complement the method used by Cogley, Sargent and others should be seen in the context of the larger literature spanning other methods for describing structural change. This embraces at least three literatures, including i) the literature on smooth, deterministic change, exemplified by [Priestley \(1965\)](#), [Dahlhaus \(1996\)](#) and [Robinson \(1991\)](#); ii) the literature on estimating VARs with parameters that follow a Markov process [see, for example, [Sims and Zha \(2006\)](#)]; iii) the literature on identifying infrequent and abrupt, non-parametric structural change, including, for example [Chow \(1960\)](#), [Brown et al. \(1974\)](#) and [Ploberger and Kramer \(1992\)](#).

1.2.2 Substantive literature on detecting changes in DSGE parameters

The second line of work which is relevant for this paper is substantive, and concerns the findings in our paper. The broad umbrella for this work is that it asks whether DSGE parameters are time-varying or not, or, to put it more pejoratively, and borrowing the title of [Fernandez-Villaverde and Rubio-Ramrez \(2008\)](#), asks “how structural are structural parameters?”

Mapping from changes in the reduced form VAR to DSGE parameters There are 3 variants of this kind of work. Variant A, in which our paper sits, first estimates reduced form time-variation and then maps that into, or seeks to interpret this as caused by, time variation in DSGE

¹These papers span: [Kapetanios and Yates \(2011\)](#), which reworked the analysis of evolving inflation persistence in [Cogley and Sargent \(2005\)](#) using kernel methods; [Giraitis et al. \(2011\)](#) which derives the theoretical results on consistency and asymptotic normality of the kernel estimator for an AR model where the coefficients follow a bounded random walk; [Giraitis et al. \(2012\)](#) which extends these results to the case of a VAR with stochastic volatility.

parameters. The closest paper in this vein to ours in execution is [Hofmann et al. \(2010\)](#). They estimate a 4 variable TVP-VAR using the industry standard Gibbs-Sampling algorithm and identify technology and demand shocks using sign restrictions. They then take three snapshots of the implied estimated impulse responses (at the beginning, middle and end of their sample) and to these they fit a New Keynesian model with sticky prices, sticky wages (indexation in both) and habits in consumption. The model could be described as a Smets-Wouters model without capital formation. Their three point estimates show changes in DSGE parameters that are larger than those we uncover (see Table 1 of their paper). For example: median estimates of the price indexation parameter are 0.15 for 1960, 0.8 for 1974 and 0.17 for 2000. And for wages the analogous figures are 0.3, 0.91 and 0.17.

The main point of departure from this paper for us is the use of the kernel estimator to generate the sequence of reduced form VAR coefficients. As a consequence this allows us to estimate a larger, 7-variable VAR on an (updated) [Smets and Wouters \(2007a\)](#) dataset. The hope is that by using more data we can improve identification. Our paper also differs on a few details. First, we allow all the parameters of the SW model to move around, whereas [Hofmann et al. \(2010\)](#) fix some of theirs at calibrated values. (In particular, they fix the discount rate, the elasticity of labour supply, and the mark-ups in product and labour markets. Our results provide more support for fixing the discount rate than the elasticity of labour supply, which does show some movement across the sample.) Second, we chose to fit the DSGE model to an identified monetary policy shock.² We make this choice on two grounds. First, we expect that the response of real variables to a monetary shock should be the acid test of the extent of nominal rigidities. (If the classical dichotomy held, and prices were flexible, real variables would be invariant to a monetary policy shock). Second, we want to preserve a parallel with the original paper by CEE, who fitted a DSGE model to the impulse response to a monetary policy shock recovered from a fixed coefficient VAR.

Several other papers adopt this same general approach of making connections between connection between time-varying reduced form VAR dynamics and changes in structural DSGE parameters. [Cogley and Sargent \(2005\)](#) build a 3 variable time-varying coefficient VAR to characterise shifts in macroeconomic dynamics. And later, in their joint work with Primiceri, they seek to find structural explanations via a small DSGE model.³ In a similar vein, [Sargent and Surico \(2011\)](#) interpret shifts in the money growth - inflation correlation as accountable for by changes in the monetary policy rule. And in [Cogley et al. \(2012\)](#) changes in the correlation between nominal interest rates and inflation, and inflation persistence are associated with shifts in the degree of indexation of firms' prices, shifts which come about because of changes in the monetary regime. [Gali and Gambetti \(2009\)](#) estimate a TVP-VAR involving labour productivity and hours work, and uncover changes in the impulse response to identified technology shocks. The fixed-coefficient literature to which these two papers address themselves was an argument about key parameters of the DSGE model that should be taken as the DGP. If hours work did fall, as Gali's striking 1999 paper found⁴, following a technology shock, then this suggested either that technology shocks were not major contributors to the business cycle (indicating a small value for the parameter governing the variance of these shocks) or, for example, that prices were sticky (whereupon the conventional result in the flex price RBC model that hours rise after a technology shock is overturned). The TVP-VAR results are therefore to be interpreted as

²In order gain extra precision in the identification of the monetary policy shock, we also identify technology, labour supply and aggregate demand shocks. For an exposition of how precision in identifying one shock is improved by identifying other shocks, see [Paustian \(2007\)](#)

³[Cogley et al. \(2010\)](#).

⁴See: [Gali \(1999\)](#).

alluding to potential changes in, eg, the volatility of technology shocks and the degree of stickiness in prices.

Estimating time-varying DSGE parameters directly Other researchers have avoided first estimating changes in reduced form VAR coefficients and sought to estimate changes in DSGE parameters directly. Within this category of papers, there are two tactics. One is to embed time-variation into the DSGE model itself. Another is simply to estimate the DSGE model on different samples.

[Fernandez-Villaverde and Rubio-Ramrez \(2008\)](#) build a DSGE model that includes stochastic processes for policy rule and price/wage stickiness parameters, processes over which agents in the model form rational expectations. They find abundant evidence of time variation. In one sense, our exercise is more limited and less constructive: we don't offer a DSGE model with full-blown time variation in place of the fixed coefficient variety, a model that would be a potential replacement for the fixed-coefficient DSGE model. Instead we undertake the narrower task of shedding light on whether the benchmark fixed coefficient variety is deficient or not, specifically asking whether the coefficients in the fixed coefficient DSGE model are really fixed. In another our paper offers something new relative to [Fernandez-Villaverde and Rubio-Ramrez \(2008\)](#). Freed from the computational burden of computing expectations over the time-variation in the DSGE parameters, we can look for time-variation in *all* of the model's (19) parameters at the same time. By contrast, [Fernandez-Villaverde and Rubio-Ramrez \(2008\)](#) allow only one parameter at a time to move.⁵ Note too that [Fernandez-Villaverde and Rubio-Ramrez \(2008\)](#) use full information methods. We resort to partial information methods, partly to preserve the parallel with the CEE work in the fixed coefficient context, and partly noting the costs and benefits of the two approaches (the benefit of our approach being the insurance against model misspecification, the cost being that partial information methods aggravate identification problems).

[Cogley and Sbordone \(2008\)](#) is another example of a DSGE model that embodies explicit time variation. They estimate a VAR with a time-varying trend inflation rate, imposing on the VAR cross equation restrictions implied by a version of the New Keynesian model linearised aroundtime-varying trend. This time-variation allows the model to explain inflation well despite having no "backward-lookingness" in the form of indexation. This paper therefore makes an intimate connection between a TVP-VAR and a DSGE model related to the one considered here, re-interpreting the previous result that the data need indexation in the Philips Curve as reflecting the fact that the model omits time varying trend inflation.

[Justiniano and Primiceri \(2008\)](#) estimate a DSGE model with time-varying volatilities of the structural shocks in the model, and interpret the Great Moderation through the lens of this model. [Born and Pfaifer \(2011\)](#) likewise estimate time-variation in volatilities, with a particular focus on the changing volatility of monetary and fiscal policies.

The second tactic, as described above, is to estimate DSGE models on different samples. [Smets and Wouters \(2007b\)](#) estimate their model over two sub-samples of US data and conclude that (variances of shocks aside), structural DSGE parameters are stable. [Benati \(2008\)](#) estimates a small New Keynesian model on various subsamples corresponding to different monetary regimes. He finds that the indexation parameter, corresponding to inflation persistence in the Philips Curve, varies substantially between monetary regimes, and therefore adduces that the reduced form property of

⁵This was confirmed to us in email correspondence with Fernandez-Villaverde.

inflation persistence derives, ultimately, not from indexation, but from the behaviour of monetary policy. Canova and Sala (2009) estimates the simplest New Keynesian model on rolling samples using full information Bayesian methods. He finds evidence that policy and private sector parameters change, and also instability in the variance of the shocks. Canova and Ferroni (2011) conduct a similar exercise using the Smets and Wouters (2007a) model, augmented to allow for real balances to affect consumption and for money growth to enter the policy rule. Giacomini and Rossi (2009) report rolling regression estimates of the [SW2003] model in the course of developing a KLIC based method of conducting rolling comparisons of the performance of competing models. Castelnuevo (2012) estimates on US data a rolling-sample version of the model of Andrs et al. (2006) (a New Keynesian model without capital, but with habits, indexation, and costs of adjusting portfolios to bring in a role for money).⁶

The subsample and rolling-regression literature here is connected with our paper not just in terms of its substantive focus (DSGE parameter change) but also methodologically. Appropriately specified kernel functions can produce either rolling regressions or subsample estimates. Kernel estimators next these alternatives. For example, a kernel function that equal-weights all observations within a window, and zero-weights those outside it is a rolling-regression. Which, then, would be most appropriate? A richer kernel, or those that produce rolling regressions? The answer to this depends on the kind of structural change one is trying to model, which, unfortunately, is not known ex ante. Loosely speaking, one might say that provided the structural change is sufficiently gradual, richer (eg normal) kernels will be optimal, rather than rolling-regression kernels. Note that flat, rolling-regression kernels are not optimal in the case that we consider, where change in the DSGE parameters is derived from persistent, stochastic evolution of the reduced-form VAR parameters.

1.2.3 Diagnosing the causes of the Great Moderation

One of the focal points of the literature on structural change in macroeconomic dynamics has been to try to diagnose the causes of the Great Moderation - crudely, the phenomena including the rise and fall of the mean, variance and persistence of inflation and the fall in the volatility of output. Many of the papers mentioned above seek to quantify the contribution of shock volatilities versus other things, and those factors directly attributable to policy and those not. Two prominent, early fact-finding papers were McConnell and Perez-Quiros (2000) and Stock and Watson (2002). A survey of some of the subsequent literature is Velde (2004)

Our paper makes a limited contribution to this literature in so far as it quantifies changes in the systematic component of monetary policy. As already trailed, we tend to find that there is not such a dramatic difference between pre and post-Volcker as sometimes reported in other papers. We cannot say much more than this because of the way we estimated the DSGE model. Recall that we are fitting it to the impulse response to a monetary policy shock, and not taking a stand on, and therefore not producing estimates of changes in the rest of the stochastic structure of the model. So we are not

⁶Also worth mentioning is the literature on detecting instabilities in DSGE model parameters. An example is Inoue and Rossi (2011). They develop and test an algorithm for recursively identifying sets of stable and unstable parameters in a DSGE model. They test for joint stability of all parameters, and, if this rejects, eliminate the parameter with the lowest individual p-value (corresponding to a test that this individual parameter is stable), and re-test for joint stability, proceeding like this until that test does not reject. This identifies a set of stable parameters. They apply this to a New Keynesian model, and find widespread evidence of parameter instability, including changes in parameters defining nominal rigidities, habits and monetary policy parameters.

able to quantify the contribution of ‘good luck’ properly.

2 Kernel estimates of a 7 variable TVP-VAR

2.1 The kernel estimator, what it is and why we use it

For simplicity, suppose we have a univariate time series process generated thus:

$$\begin{aligned} y_t &= \rho_t y_{t-1} + h_t^{1/2} \varepsilon_t \\ \rho_t &= \rho_{t-1} + u_t \\ \ln h_t &= \ln h_{t-1} + z_t \end{aligned}$$

Where we assume in addition that ρ_t is bounded between 0 and 1.

Cogley and Sargent (2005) set the industry standard method for estimating this process using the Gibbs Sampler. The joint posterior for $\rho = \{\rho_1, \rho_2, \dots\}, \sigma_u^2, \sigma_z^2, h = \{h_1, h_2, \dots\}$ is factored into conditionally independent blocks which allow numerical characterisation of the posterior. We instead estimate ρ (or rather its multivariate counterpart in in our 7 variable VAR) using the kernel estimator;

$$\hat{\rho}_{n,t} := \frac{\sum_{k=1}^n K\left(\frac{t-k}{H}\right) y_k y_{k-1}}{\sum_{k=1}^n K\left(\frac{t-k}{H}\right) y_{k-1}^2}$$

Where K is some kernel (in our case the normal kernel), H defines the bandwidth of the kernel (and is chosen optimally). If K was simply a vector of 1s, this would simply give the full sample OLS estimate for all periods. K can equivalently be specified so that it delivers a rolling window OLS estimate.

In two previous papers we have established the consistency of this estimator. In Giraitis et al. (2011) we work through the univariate model above, but with time-invariant volatilities, (ie $h_t = h, \forall t$). In Giraitis et al. (2012) we extend these results to the multivariate case with stochastic volatility. In both cases, we require the ρ_t ’s (or the equivalent multivariate object, as appropriate) to be bounded within an interval such that the instantaneous VAR at any t is stationary. This requirement is implemented in most of the studies exploring changing macroeconomic dynamics using the Gibbs Sampler based methods, on the grounds that a VAR that at any point violates this notion of instantaneous stationarity is not economically meaningful. In our case we also require this to establish consistency and asymptotic normality. The consistency result is not available for the classical version of the Gibbs Sampling method to our knowledge. This may not be so interesting for all practitioners, for two reasons: i) most (though not all) papers are Bayesian, though many stress the use of uninformative priors and are using the method for its numerical machinery rather than philosophical stance on statistics; ii) in our experience the Gibbs Sampler does well in Monte Carlo tests and this is probably widely known and understood.

However two practical advantages of the kernel method may, we think, be compelling. And actually both advantages combine to make the application presented here possible. The first is that the method takes seconds to run rather than the minutes, hours or days with the competitor Gibbs Sampling based

method. Second, the kernel method works with large dimensions. The Gibbs Sampling method seems to break down in large dimensions (we guess for 5 or more) on account of it being either time consuming to the point of being impractical, or impossible to find sequences of draws of the ρ 's that satisfy the boundary conditions, especially in common applications to macro time series which embody lots of persistence. Why do we not suffer this problem? Our method delivers classical point estimates (and standard errors) directly, and not via first characterising the likelihood/posterior. So if these point estimates do satisfy the boundary condition (and in the applications we have presented before, and in this paper, they do) the researcher is done. If not, there is the option (that we do not explore ourselves) of combining the classical estimates with prior information about the boundary conditions: there is a long tradition of Bayesian nonparametric econometrics to draw on.

For the purposes of this paper - looking for structural change in the parameters of a medium scale DSGE model, we are driven to use the kernel method as the only method that will handle a large enough dimension of observables. Of course, in principle, although SW (and others) have estimated this model on the 7 observables articulated by the model, we could look for structural change using a smaller vector of time series, small enough to be handled by the Gibbs-Sampling method. We could estimate a 4 variable reduced form TVP-VAR, identify monetary policy shocks using just those 4 observables, and fit our DSGE model each period to the impulse response function to those shocks. But we know that partial information methods like the one we are using aggravate identification problems that are already an issue with DSGE models, and extra observables are likely to be very useful indeed in pinning down the DSGE parameters.

2.2 The data

As advertised earlier, we use the 7 variable quarterly dataset for the US compiled by SW, comprising: quarterly growth in GDP, CPI inflation, hours worked, quarterly growth in investment, quarterly growth in consumption, quarterly growth in real wages and the Fed Funds rate. The dataset in the 2007 AER depository is updated to 2010Q2.

2.3 Identification and computation of the impulse response functions

We identify four shocks. We are going to fit the DSGE model to the impulse response function to a monetary policy shock. We consider this to be the key moment for estimating parameters defining nominal frictions, which we suspect - not least because of the previous work by [Fernandez-Villaverde and Rubio-Ramrez \(2008\)](#) and [Hofmann et al. \(2010\)](#) to be the least microfounded of all of the DSGE parameters. Plus, this preserves a parallel with the CEE paper in 2005 which fit a fixed coefficient variety of a similar DSGE model to the IRF to a monetary policy shock. In contrast to CEE, we will identify this shock using sign restrictions, following [Uhlig \(2005\)](#), [Canova and Nicolo \(2002\)](#) and others.

Recapping on this method briefly, and following the exposition in [Canova \(2007\)](#): the output of the kernel estimation procedure is a set of reduced form VARs that we can write thus:

$$Y_t = \hat{A}_t Y_{t-1} + Z_t, \hat{\Sigma}_t = Z_t Z_t'$$

The estimated variance-covariance matrix of the reduced form VAR residuals, $\hat{\Sigma}$, (suppressing the t notation momentarily) can be decomposed thus:

$$\hat{\Sigma} = PVP' = \tilde{P}\tilde{P}' = \tilde{P}HH'\tilde{P}'$$

Where P collects the eigenvectors, V is a diagonal matrix of eigenvalues, $\tilde{P} = PV^{1/2}$ and H satisfies $HH' = I$. We interpret $H(\omega)$, $0 \leq \omega \leq 2\pi$ as a matrix that rotates P by the angle ω . We search through the space of H 's by choosing random values for ω until we collect 100 for which $\tilde{P}H$ satisfies the requisite sign restrictions.

We identify four shocks in total, including, in addition to the monetary policy shock: a technology shock, a labour supply shock, and a demand shock. We identify these additional shocks to increase the precision with which the monetary policy shock is identified, noting the results in [Paustian \(2007\)](#).

The relevant sign restrictions are encoded in the table below:

	Δc	Δi	Δy	h	π	$\Delta(w/p)$	i
mon pol	-	-	-		-		+
technology		+	+		-		
labour supply			+	+	-	-	
demand	+		+		+	+	+

In words: a monetary policy shock is taken to be a shock that, if interest rates rise, induces inflation, output, investment and consumption to fall, with the responses on hours worked and the real wage left free. This scheme follows previous work by [Uhlig \(2005\)](#) and others. Technology shocks are identified such that an improvement in technology causes inflation to fall, output and investment to rise, with the response of interest rates, hours worked and consumption left free. (Leaving the hours worked responses free is compelling here given the prior controversy about that response, and the ease with which this sign can be reversed by appropriate parameterisation of a DSGE model). An outward shock to labour supply is identified as one that leads to hours worked increasing at the same time as the real wage falls, and output increases, with other responses left free. This scheme bears some resemblance to the one used by [Chang and Schorfheide \(2003\)](#): they took an outward shift in labour supply to be something that reduced labour productivity: this is consistent with us assuming that real wages fall. A demand shock is one that induces positive comovement in interest rates, inflation, output and consumption, with the responses of hours and investment left free.

In estimating and identifying the impulse response to the monetary policy (and other) shocks there are two sources of uncertainty. The first is the sampling variability experienced in estimating the reduced form VAR parameters, and the variance-covariance of the VAR residuals. The second is the uncertainty manifest by the fact that many rotations of the variance covariance of the reduced form residuals will satisfy the sign restrictions above. We report only uncertainty generated from this second rotational source. This is for expositional clarity, but we might note in addition that with increasingly long samples the sampling uncertainty would of course disappear, while the rotational uncertainty would not, so, in a sense, the latter is the dominant source of variability.

2.4 Estimated IRFs to a monetary policy shock over our sample

Our estimated impulse responses - holding the variance of the reduced form shocks constant - are reported in Figures 1 and 2 below. Figure 1 and 2 plots 3d charts of the IRFs through time. For each period, and for each horizon, we are reporting the median across impulse responses that satisfy the sign restrictions. (For the moment we defer the task of addressing the well known comment by Fry and Pagan (2007) on this practice for future work).

The broad picture spits out what our identification procedure demanded: a contractionary policy shock that therefore causes the short rate to rise reduces inflation, investment, consumption and output. The sign on real wages and hours worked were left free, and note that these both fall. Hours and inflation show signs of a hump-shaped response (the peak response not being on impact, but later). All charts show pronounced time-variation in the IRFs. For example, there is a clear increase in the magnitude of the effect of the policy shock on some real variables in the early 2000s, relative to periods just prior to that.

Figure 3 depicts the IRFs in a slightly different way. We calculate the cumulated sum of these responses and, for each IRF, and for each time period, we plot four lines, the first, fourth, eighth and twelfth period responses. If the responses are defined as R_i , then the cumulated responses are defined as:

$$\begin{aligned} CR_1 &= R_1 \\ CR_4 &= \sum_{i=1}^4 R_i \\ CR_8 &= \sum_{i=1}^8 R_i \\ CR_{12} &= \sum_{i=1}^{12} R_i \end{aligned}$$

Loosely speaking these lines could be viewed as the integral of the response at each horizon. If $CR_8 = CR_{12}$ then the variable has returned back to its steady-state in period $Q = 8$ and it stays there. If $CR_8 < 0$ ($CR_8 > 0$) and $CR_{12} > CR_8$ ($CR_{12} < CR_8$) then the series has returned to its steady state and moves to the opposite direction.

Figure 3 brings out more starkly some of the more detailed time-variation occurring in the IRFs to the monetary policy shock. As the chart shows, the cumulative impact at all horizons and for most responses shows quite a bit of variation. To take a few examples: if we look at the charts for consumption, output, hours [“labour”], we can see that for some periods the turquoise line is below the red line (implying that the monetary policy shock is still propagating and being amplified out to 12 quarters) yet at other times the reverse is true (implying that the shock has started to dissipate by 12 quarters). Another example: for most charts, the cumulative impulse at all horizons varies a lot. The impact of the shock on the growth in investment out to 12 quarters varies between -0.2 and -1.2; the impact on output at 12 quarters varies between -0.2 and 0. The impact on real wages out to 4 quarters varies from -0.1 to 0.2.

A necessary condition for us to get time variation in the implied DSGE estimates is that these impulse responses to a monetary shock vary: this condition is clearly satisfied in our case, so we proceed

therefore to the step of mapping from the IRFs to the DSGE parameter estimates.

3 Mapping from the kernel estimates of the TVP-VAR to structural DSGE estimates

Mapping from the IRFs to a monetary policy shock to our structural DSGE estimates requires us i) to specify an IRF fitting procedure and ii) to specify a DSGE model. This section explains these two briefly.

3.1 The Smets-Wouters (2007) model

We fit the widely used SW model to these IRFs. This model is well known. A schematic description is as follows: the model features optimising consumers and firms; a government setting fiscal policy; and a monetary authority. There are habits in consumption and investment adjustment costs. Firms experience sticky prices, and labour suppliers sticky wages, following [Calvo \(1983\)](#). In periods when prices and wages are not set optimally, they are indexed to the most recent observed inflation/wage inflation rate (respectively) using a linear rule. Monetary policy is assumed to follow an interest rate rule that has the policy rate responding to its own lag, inflation, the level and the change in the output gap (Note: there is no time-varying inflation target). The model defines laws of motion for the same observeables estimated in the VAR, namely: consumption, investment, output, inflation, the one period interest rate, hours worked and real wages. In the appendix we sketch these laws of motion to make the paper somewhat more self contained, and to establish the meaning of our notation.

3.2 A minimum distance estimation procedure

Collect together the parameters of the DSGE model into the vector θ . Denote the impulse responses to a monetary policy shock of the VAR and the model by \hat{R}_{VAR} and $R_{DSGE}(\theta)$ respectively. Then the minimum distance estimator of θ , $\hat{\theta}_{MDE}$ minimises the distance between the two, ie is given by:

$$\hat{\theta}^{MDE} = \arg \min \left\| \hat{R}_{VAR} - R_{DSGE}(\theta) \right\|$$

Where R is defined so that all horizons are weighted equally and $\| \cdot \|$ denotes the Euclidean norm.

Note that we conduct this minimisation i) for every time period in the sample, and ii) for every rotation of the reduced form residual covariance matrix that satisfies our sign restrictions, so in our results section we plot $\hat{\theta}_{j,t}^{MDE}$ where t denotes the time period, and runs from 1955Q1 to 2010Q2 and j denotes the particular rotation and runs from 1 to N , where we routinely set $N=100$.

We use Matlab's FMINUNC function to find the minimum, which is a gradient based method. To ensure robustness, for each $R_{i,j}$ (ie for each rotation that satisfies the sign restrictions) and each sample period we try 100 starting values to initialise the search for the minimum.

This method is a partial information method. We are fitting to only a limited number of the infinitely many moments implied by the model. It has been pointed out by others (see, for example, [Canova and](#)

Sala (2009)) that such methods aggravate already worrisome identification problems in the estimation of DSGE models. However, the method has the advantage in that it provides some insurance against model misspecification. The full information estimates will give us consistent estimates only in the event that the DSGE model is the true data generating process. And if we worry that the DSGE model might be sufficiently far away from this DGP then an estimator of the sort used here is useful and justified.

3.3 Results: time series for implied DSGE parameters

Our benchmark estimation results are presented in Figures 4 and 5. The charts plot the median and 16%-84% confidence sets that result from fitting the entire set of rotations of the reduced form shocks that satisfy the sign restrictions. Marked as a blue diamond are the SW estimates produced from their full information Bayesian Maximum Likelihood procedure, which we report as a comparison. These SW estimates very often are different from the average of our “sub-sample” estimates. This is to be expected: our estimates differ not only because they are sub-sample, but because SW used full information/Bayesian estimates, with informative priors, and ours are partial information, classical estimates.

We report on the parameter estimates in several blocks; nominal rigidities; real side of the model; monetary and fiscal policy.

Nominal rigidities. We estimate very pronounced changes in the parameters defining nominal wage and price rigidity. ξ_p , the “Calvo parameter” for prices, which encodes the probability of not re-setting prices, is estimated to be about 0.72 in 1955, falls steadily to a low point of 0.6 in 1985 (a period which, roughly speaking, captures the ‘Great Inflation’), and then rises sharply to 0.83, by 2005 (a period which brackets the ‘Great Moderation’), before falling back sharply to 0.7 by 2010.⁷ There is strong circumstantial evidence, as also pointed out by Fernandez-Villaverde and Rubio-Ramrez (2008) and Hofmann et al. (2010), that this parameter is a reduced form for some underlying state-dependent model of prices in which the frequency of price changes is inversely related to inflation itself. The equivalent parameter for wages, ξ_w follows a very similar path indeed, as we would expect if this speculation about the underlying state-dependent pricing model is correct, since wage inflation has followed a similar path to price inflation, but note that despite the pronounced time-variation in both parameters we find that the probability of not re-setting is always lower for prices than wages.⁸

The indexation parameter is perhaps the most controversial aspect of the DSGE model: micro evidence on prices strongly suggests that there is no indexation; yet indexation in prices and wages greatly improves the fit of the DSGE model to macro time series. i_p records the coefficient in the one argument linear rule that firms use to multiply with last period’s inflation to index prices. We estimate that this begins in 1955 at 0.4, rising to almost 0.7 in 1985 or so, before falling to a low point of around 0.25 in 2005, and then rising sharply again to 0.5 by 2010. This parameter is very closely correlated with the estimated path for the Calvo price parameter ξ_p , for reasons which are obviously not interpretable through the lens of the time-invariant DSGE model of nominal rigidities. This parameter varies a lot with monetary regimes, as Benati (2008) showed, but our estimates reveal that there is also

⁷In terms of the price average duration the last two numbers imply that this varies from, approximately, 2 to 8 quarters, respectively (red diamonds in Figure 4).

⁸Mapping the minimum and the maximum value of ξ_w to wages average duration we realise that wages were reset every 2 and 10 quarters, respectively (red diamonds in Figure 4).

clearly a lot of instability *within* institutionally-defined regimes. For example, it is not the case that indexation-induced persistence is greater pre than post-Volcker. Estimated indexation varies a lot in both sub-regimes (Note too that we are separately estimating the contribution of monetary policy to inflation persistence here, and will report on the policy block of parameters below). The equivalent parameter for wages, i_w follows a very similar path, but fluctuating in a slightly narrower range (0.4-0.7, as opposed to 0.25-0.7 for prices).

These parameter fluctuations echo those found in [Fernandez-Villaverde and Rubio-Ramrez \(2008\)](#) and [Hofmann et al. \(2010\)](#). Relative to the latter, which is the closest paper to ours in execution, we find slightly smaller fluctuations in the parameters defining nominal rigidity. There are still quite a few differences between their method and ours to account for the mildly contrasting results: we use kernel methods to estimate the reduced form VAR, they use the random coefficients model; we use a 7 variable VAR and they use 4 variables; we allow all parameters to vary over time, they fix many at calibrated values; our identification scheme differs from their in some details; and we fit only to a monetary policy shock.

Our results emphasise that more research may be needed to refine the nominal rigidities in the canonical DSGE model, echoing many previous papers. It is well known that the details of optimal monetary policy depend a lot on the nature of nominal rigidities. Examples being: the stickier are wages relative to prices, the more weight the authorities should place on nominal wage stabilisation relative to price stabilisation ([Erceg et al. \(2000\)](#)); the presence of indexation implies the authorities should stabilise a quasi-difference of inflation involving the indexation parameter itself ([Woodford \(2003\)](#)). Finding such a large amount of variation in the nominal rigidity parameters is disquieting since they are important for optimal policy.

Real economy parameters. There are several points worth noting here. First, on h , the parameter that encodes habits in consumption. This parameter is estimated at about 0.77 in 1955 and fluctuates between this value and about 0.83 until the early 2000s, when it jumps from a trough of 0.75 to 0.88. These changes are smaller than for the indexation parameters that are engineered to generate persistence on the nominal side, but in terms of half-lives of consumption these are still pretty large changes. Consumption ends up more backward-looking and less sensitive to real interest rate changes than it begins the sample. The inverse intertemporal elasticity of substitution (σ_c) fluctuates quite a bit, showing a particularly large fall from a peak in 1970 of about 1.25 to a trough of about 0.8 in 1980. The inverse Frisch elasticity of labour supply (σ_L) shows a marked fall between 1985 when it peaks around 5.25, to the early 2000s when it troughs at around 2. The parameter governing the costs of adjusting investment (Φ) is pretty flat for most of the sample, but then shows a large rise from a trough of around 2 in 1990 to a peak of about 5 in 2005, before falling back sharply to 2 again. The greater this parameter, the more detached is investment from the traditional cost of finance manifest in Tobin's Q. This suggests that the DSGE model had a hard time to explain the boom investment during the 1990s, and the subsequent 'post Y2K' bust in the 2000s. Interestingly, the discount rate, β , is found to be relatively constant. We draw comfort from this. There is a wide body of evidence, macro/finance and micro/experimental, that the discount rate is close to but less than 1. So this is probably the most micro-founded parameter of all in the DSGE model. Stepping back: where the DSGE model is most micro-founded, we find the parameter estimates to be flat; where it is least well-microfounded, we find the parameter to be quite variable. We take our flat β to strengthen the case for interpreting our results as indicating something useful about the DSGE model's failings. If everything

were moving, even parameters that were relatively solidly evidenced outside the model, we could be more sceptical that this estimated variation was just noise, or indicative of poor identification. Note that the flat β confirms ex post that the data would in some sense support the fixing of β in Hofmann et al. (2010).

Policy parameters. Monetary policy is assumed to have been characterised by an interest rate rule such that the interest rate responds to its own lag, a term in the inflation rate, the output gap and the change in the output gap (sometimes known as the ‘speed limit’). The responsiveness of interest rates to inflation, r_π , is the least precisely estimated of the monetary policy parameters, and moves around the least relative to the confidence sets that we compute, staying within a range of 1.7 to 1.9. On the face of it our estimates fail to confirm previous work that argues that the Great Inflation was caused by a breach of the Taylor principle. We effectively discard all DSGE parameter constellations that imply indeterminacy by penalising these heavily in the minimum distance procedure. But the profile of the estimates of the monetary policy and other parameters suggests that this constraint is never binding: if it were we would expect the minimisation to drive one of the parameters somewhere in the model to a boundary. We discard separately (even amongst otherwise stationary solutions) candidate parameter sets in which r_π is less than 1, but this coefficient is never driven to that boundary in estimation.⁹

The coefficient on lagged interest rates, ρ , fluctuates between 0.75 and 0.95, which we take to be quite large movements, especially relative to the small confidence sets we estimated. Interestingly, the coefficient on lagged interest rates rises at the point when the indexation parameters for prices and wages rise. It has been remarked in previous work [kydland et al ier] that interest rate inertia can generate reduced form inflation persistence and therefore *substitute* for persistence imparted by indexation in price and wage setting. But our estimates tell a story of policy induced and nominal rigidity-induced persistence going hand in hand, acting as *complements*. The coefficients on the level and change in the output gap also vary somewhat, between 0.04 and 0.14 for the level term and 0.15 and 0.3 for the speed limit term. The most pronounced movement in the monetary policy parameters occurs in the early 2000s, a time when the Fed was keeping rates as low as possible to avoid the threat of deflation. Our estimates pick this up as being associated with a rise in interest rate inertia and the responsiveness to the speed limit, and a fall in the response to the level of the output gap.

Finally, the volatility of the estimated monetary policy shock process falls steadily through the sample period, from about 0.18 in 1955 to about 0.12 by 2010. We are used to reading that this volatility is markedly higher pre than post Volcker, but our graphs tell a slightly different story. True enough, this volatility is, on average, higher pre than post-Volcker. But its fall starts in 1970, and is actually complete by 1985 when the volatility troughs. Thereafter, for the bulk of the Volcker regime, it rises, until Greenspan arrives.

Taken together, the variation we detect in policy parameters is considerably less than that uncovered in much of the other work whose focus was understanding the causes of the Great Inflation and its subsequent moderation. At face value this suggests less of a role for monetary policy in explaining changes in macro dynamics than other accounts which take different approaches. Having noted that, we don’t want to push this interpretation too hard, as it requires making the leap of faith that the

⁹We appreciate that a value >1 is not going to be necessary or sufficient to guarantee stability, but it is nevertheless interesting that our estimation generates values always >1 , when others, using estimates of the interest rate reaction function only, or structural models, have found differently.

world really was characterised by not only separate regimes but also separate DSGE economies each period.

3.4 Sensitivity: time varying Σ

It has been common for researchers estimating TVP-VARS using the Gibbs Sampling algorithm and the random coefficients model to allow for time varying volatilities. Especially following the comments by Sims on the first Cogley-Sargent work in this vein [Sims ()] where he worried that the apparent changes in the reduced form VAR dynamics had been amplified by imposing that the VAR had time-invariant volatilities. In this section, we comment on whether the results described above survive us allowing the volatilities to be time varying. On the whole, the results survive. We still find substantial instability in the DSGE parameter estimates, and the paths of the estimates largely follow those derived under the assumption of fixed volatilities. These findings are evident from Figures 6 and 7, which plot the median for the time-invariant volatilities against the confidence sets for those estimated assuming time-varying volatilities.

4 Conclusions

In this paper, we have estimated a time-varying parameter VAR on the 7 variable data set SW used to estimate their medium scale DSGE model. This is currently impractical using the widely known Gibbs-Sampling algorithms to estimate stochastic time-varying coefficient models, given the need to impose that at each point in time the implied VAR satisfies stationarity conditions. So we have deployed a kernel estimator, explained in prior work, that is known to deliver consistent estimates of the VAR parameters, and has no problem handling large dimension systems (and indeed spits out results in seconds). Armed with this estimated TVP-VAR, we have identified monetary policy shocks using sign restrictions, and computed the impulse response to those shocks implied by each of the instantaneous VARs that the TVP-VAR produces, for each observation period in our data set. We have then fit the SW model to these impulse response functions using a minimum distance procedure. In doing this, we are, roughly speaking, replicating what CEE did for their model, (similar in design to SW), for impulse responses to a monetary policy shock that were recovered from a fixed-coefficient VAR. We therefore map the time-varying evolution in reduced form dynamics into time-variation in the structural parameters of the DSGE model.

We find that many of the parameters of the model are subject to substantial time-variation. Those that move most are those that define the nominal rigidities in the model: the Calvo and indexation parameters for prices and wages. These have long been the focus of criticism from outside the DSGE community (see, for example, Chari et al. (2009)), and recognised by those practising within it to be work in progress. But other parameters appear to move a lot too. For example, the parameter that defines the cost of adjusting investment. Smaller, but still economically meaningful movements are detected in the parameter that encodes habits in consumption, and those define the behaviour of the monetary policy authorities.

It is hardly surprising that the changing reduced form VAR parameters translate into changes in the DSGE parameters. Small samples are noisy. And we know that the DSGE model itself suffers from

identification problems. We should surely expect these estimates to move a round quite a bit over time. So is there really anything to be read into these estimates? We believe they deserve attention for a few reasons. First, the movements are smooth, relatively low frequency and, in the case of the parameters defining nominal rigidities, correlate in a systematic way with inflation, which, as others have noted, hints at an underlying state-dependent model for pricing. These movements look less like those one would expect if the underlying cause was small-sample noise. Second, the parameters that move most are those that are accepted to be the least well micro-founded, and whose justification relies most on being necessary to fit the (fixed-coefficient) macro time series dynamics: those defining nominal rigidities and investment adjustment costs. Those that move least - for example the discount rate, are those for which there is a wide body of prior evidence that this is an invariant behavioural parameter.

A final feature of our results that may be of interest is the relatively small movements in the parameters that define monetary policy compared to those estimated by papers in the Great Moderation literature. For example, taken literally, we find no support for the argument that there have been periods where interest rates do not satisfy the “Taylor Principle”, and that the Great Inflation was therefore the result of a period where policy had led to indeterminacy. We would not want to push this interpretation too far though, since it is stretching it somewhat to characterise central bank policy as implementing an entirely different but parsimonious rule for rates each period in the sample.

These results confirm finding in two other similar papers, particularly [Fernandez-Villaverde and Rubio-Ramrez \(2008\)](#) and [Hofmann et al. \(2010\)](#). Those papers, as well as recording changes in monetary policy behaviour, put the spotlight on the nominal rigidities in the DSGE model by recording that estimates of those parameters moved over time, and in ways correlated with inflation. We have found this too. Although, as we have emphasised already, our work also suggests that other parameters on the real side seem to vary substantially too.

A Appendix

A.1 Charts

Figure 1: IRFs Constant Σ_v : A

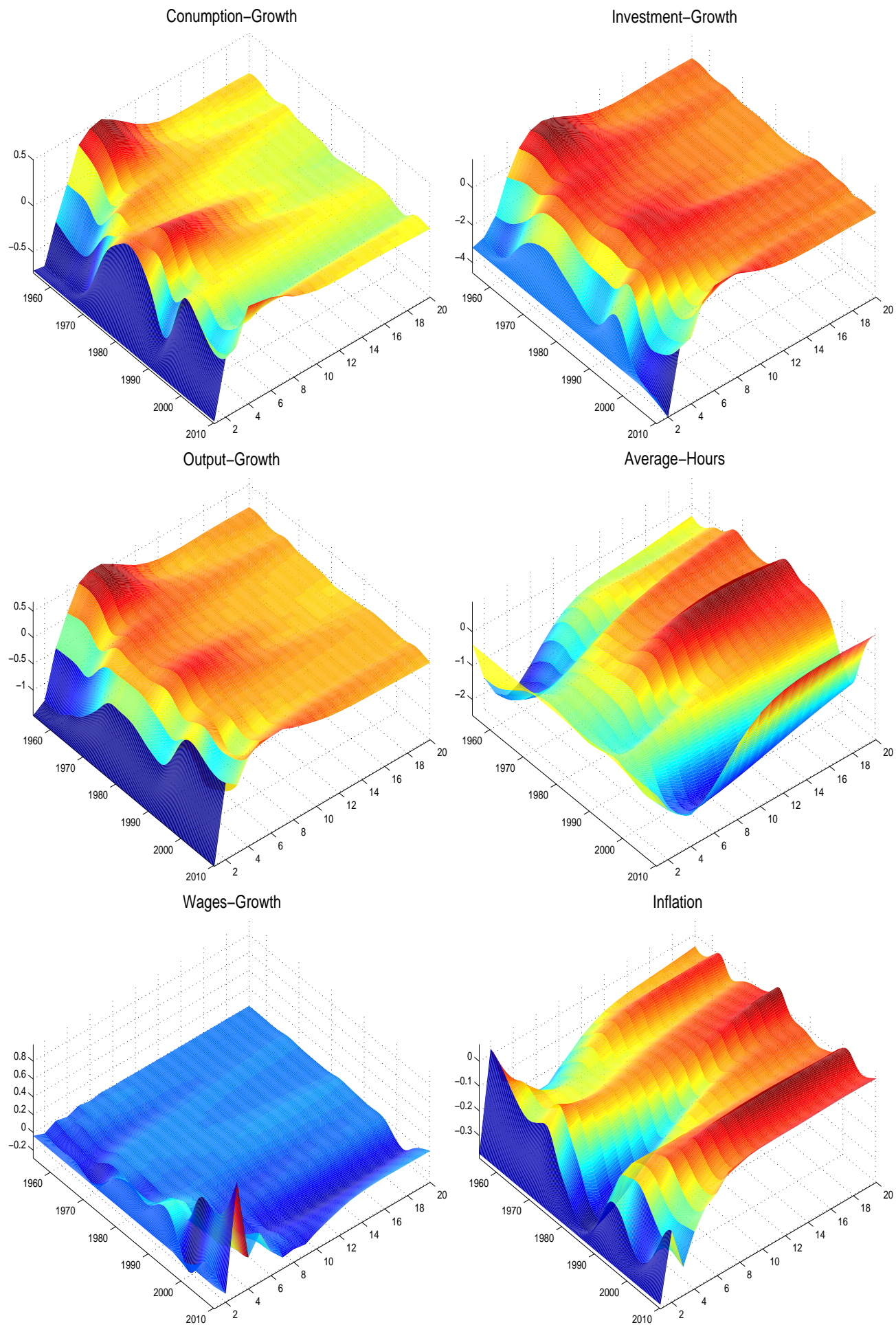


Figure 2: IRFs Constant Σ_v : B

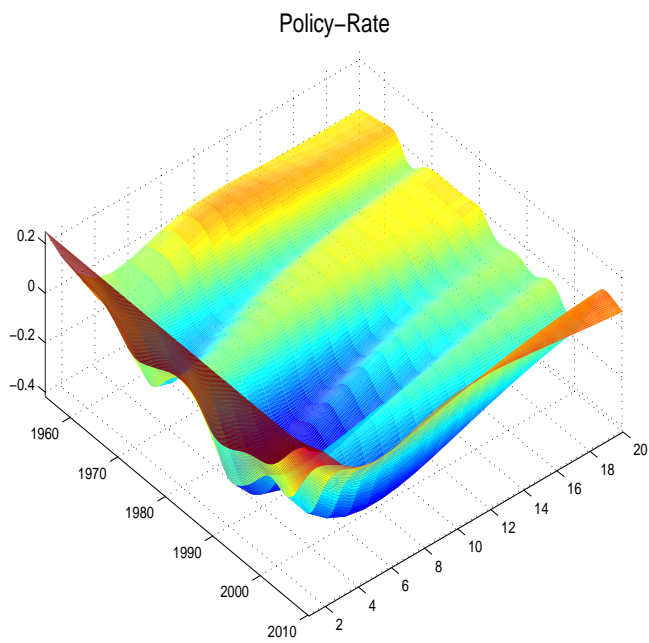


Figure 3: IRFs Summary Constant Σ_v

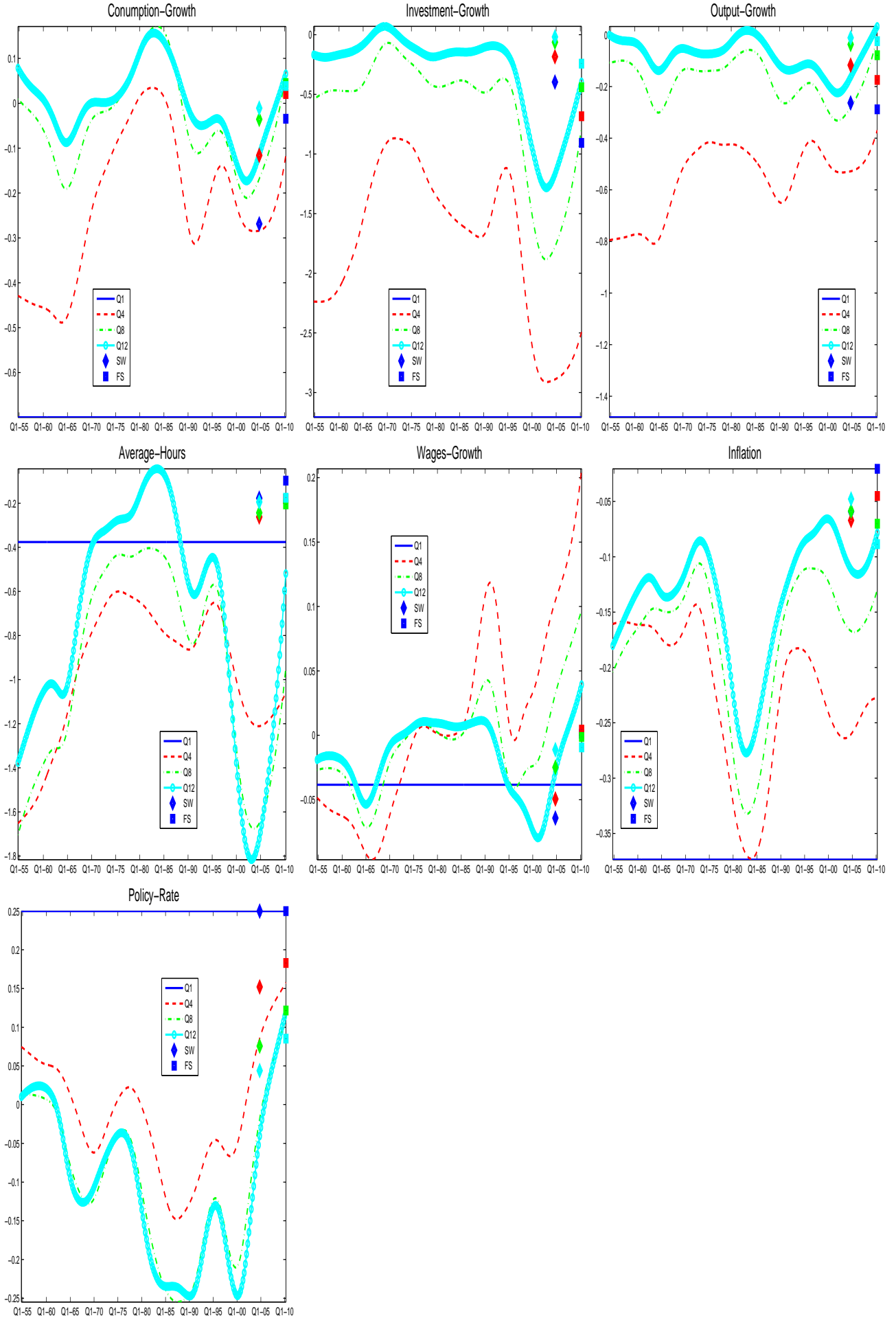


Figure 4: Structural Parameters Constant Σ_v : A

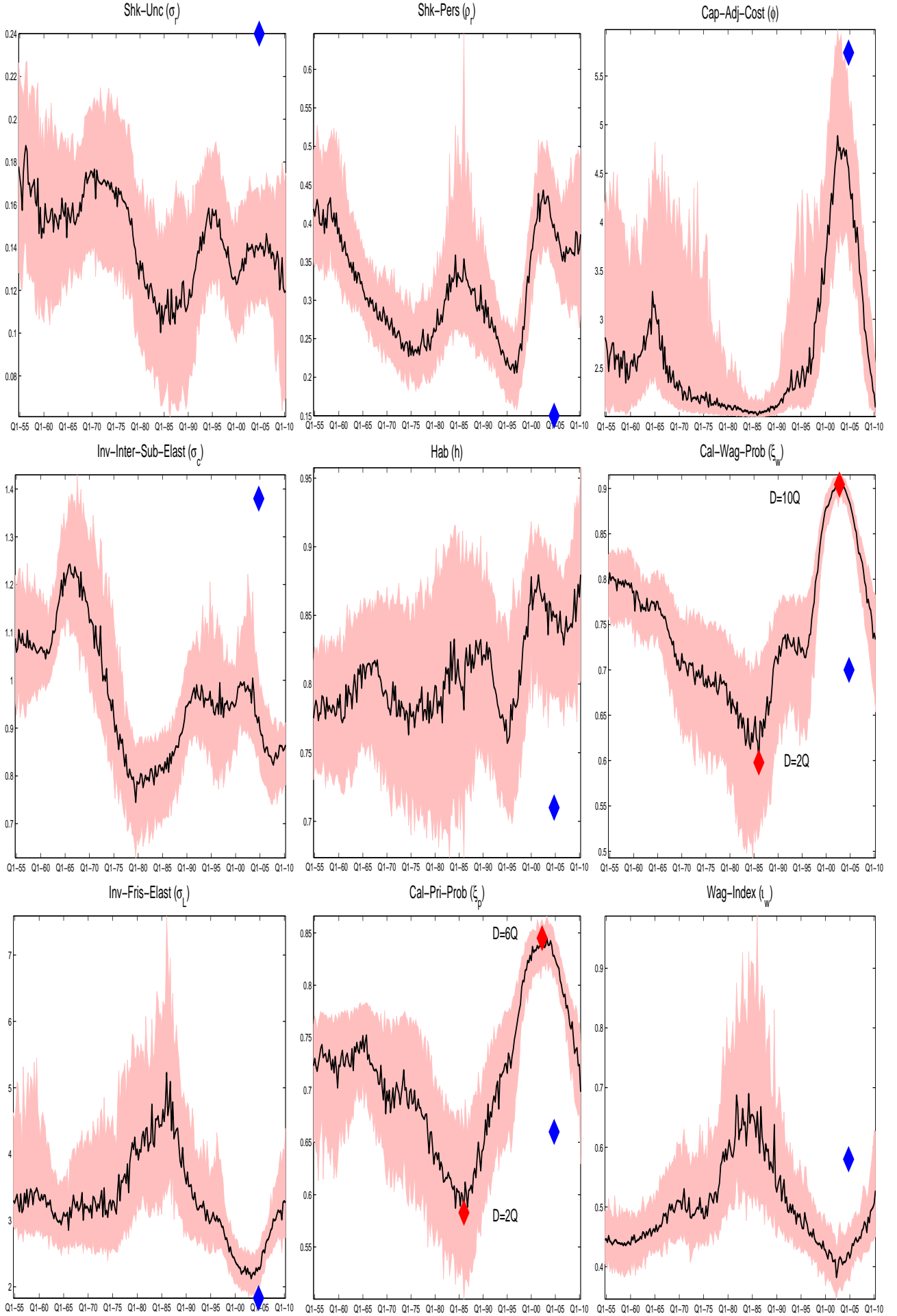


Figure 5: Structural Parameters Constant Σ_v : B

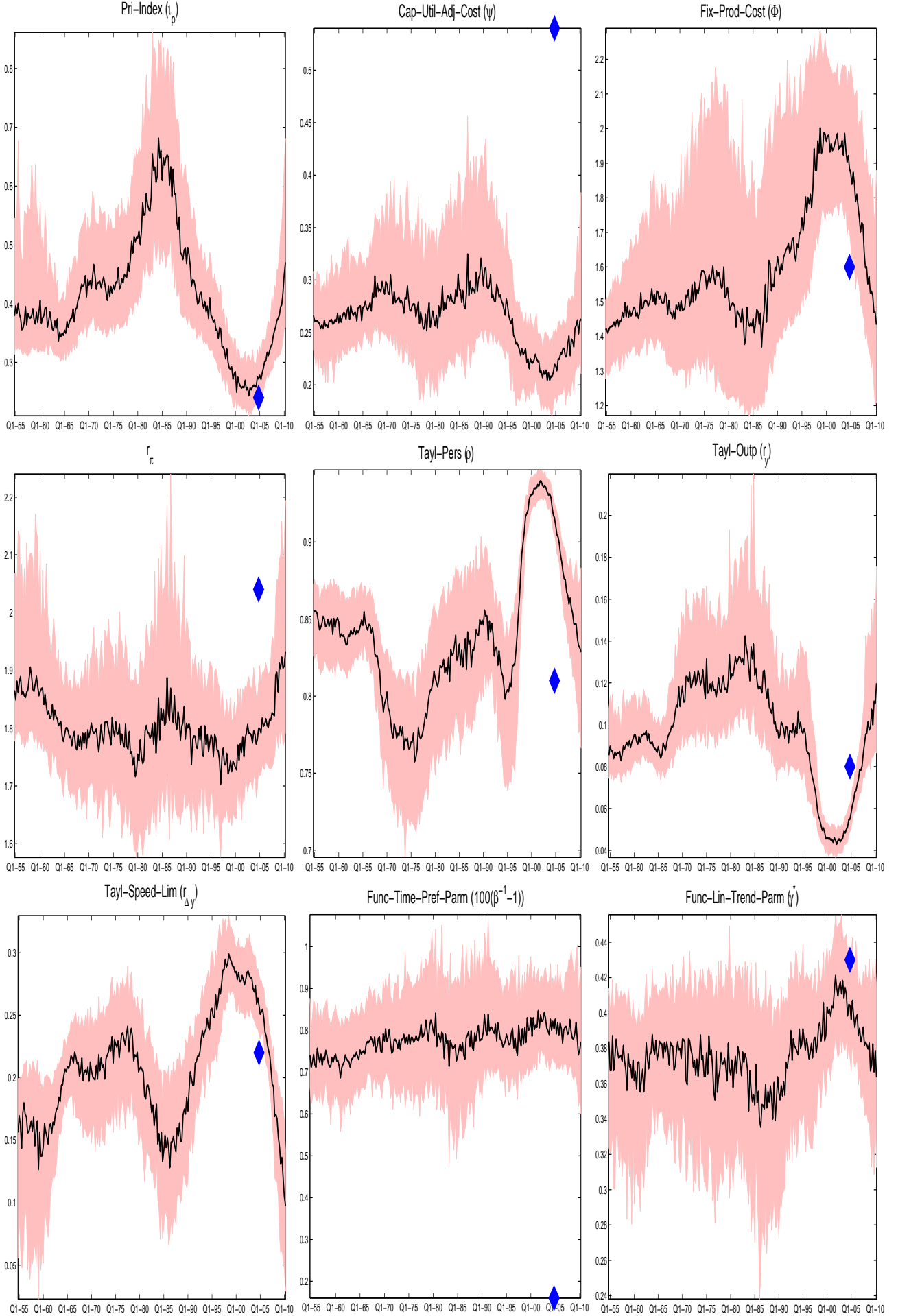


Figure 6: Structural Parameters Time-Varying Σ_v : A

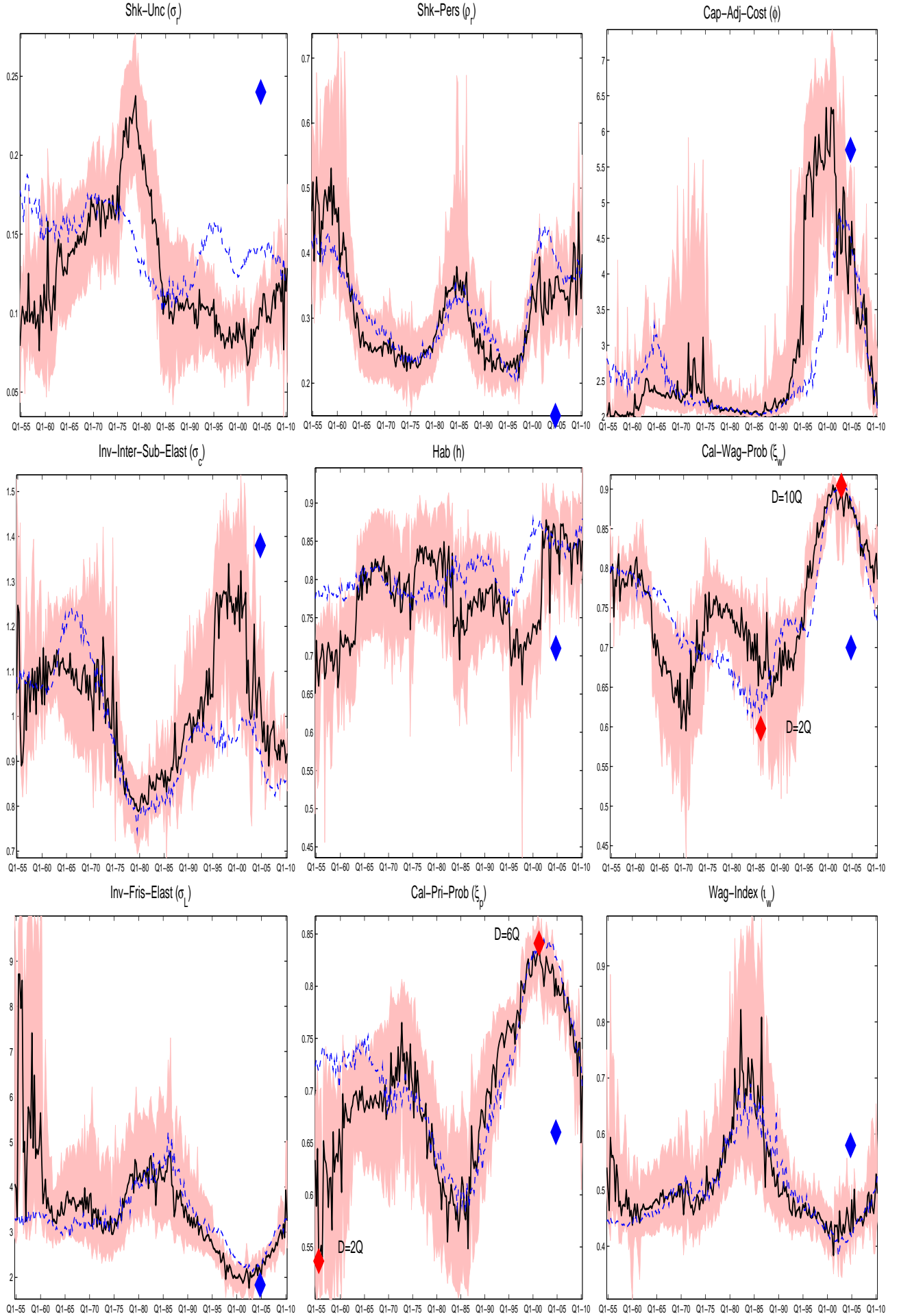


Figure 7: Structural Parameters Time-Varying Σ_v : B

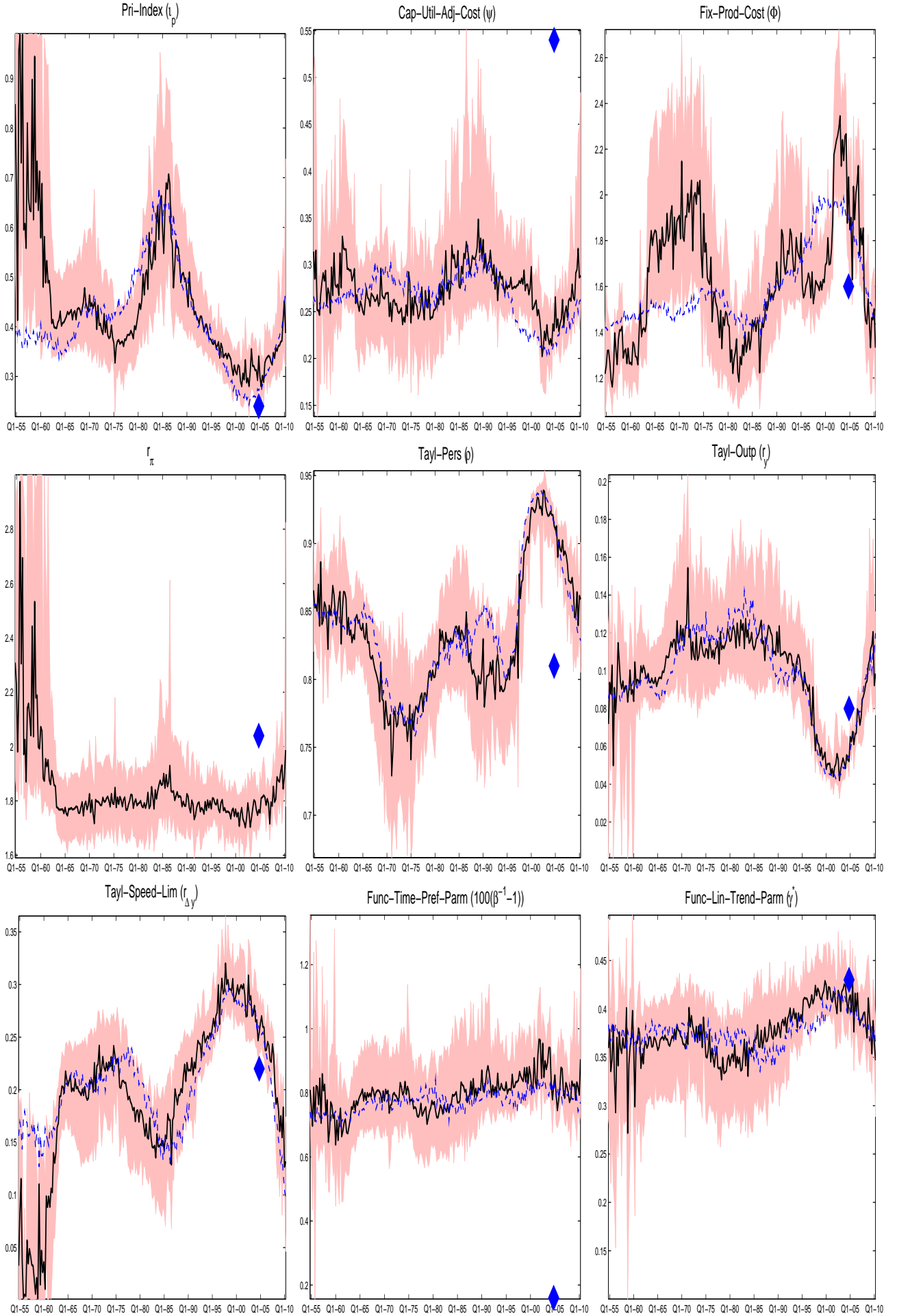


Figure 8: IRFs Time-Varying Σ_v : A

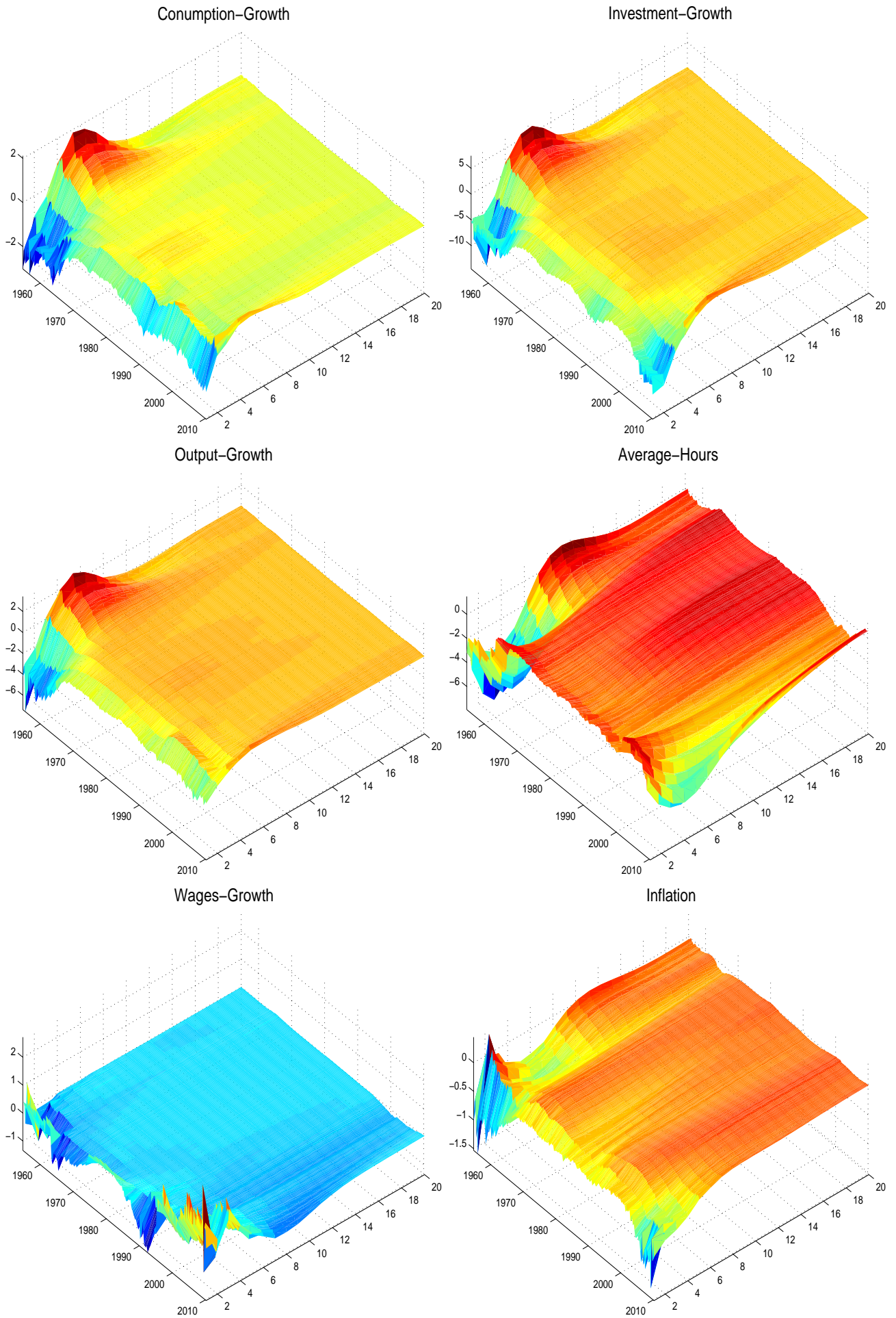


Figure 9: IRFs Time-Varying Σ_v : B

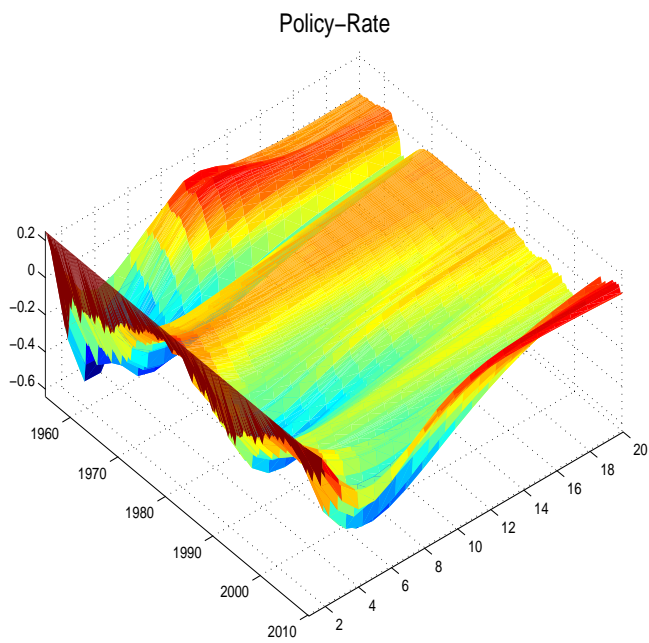
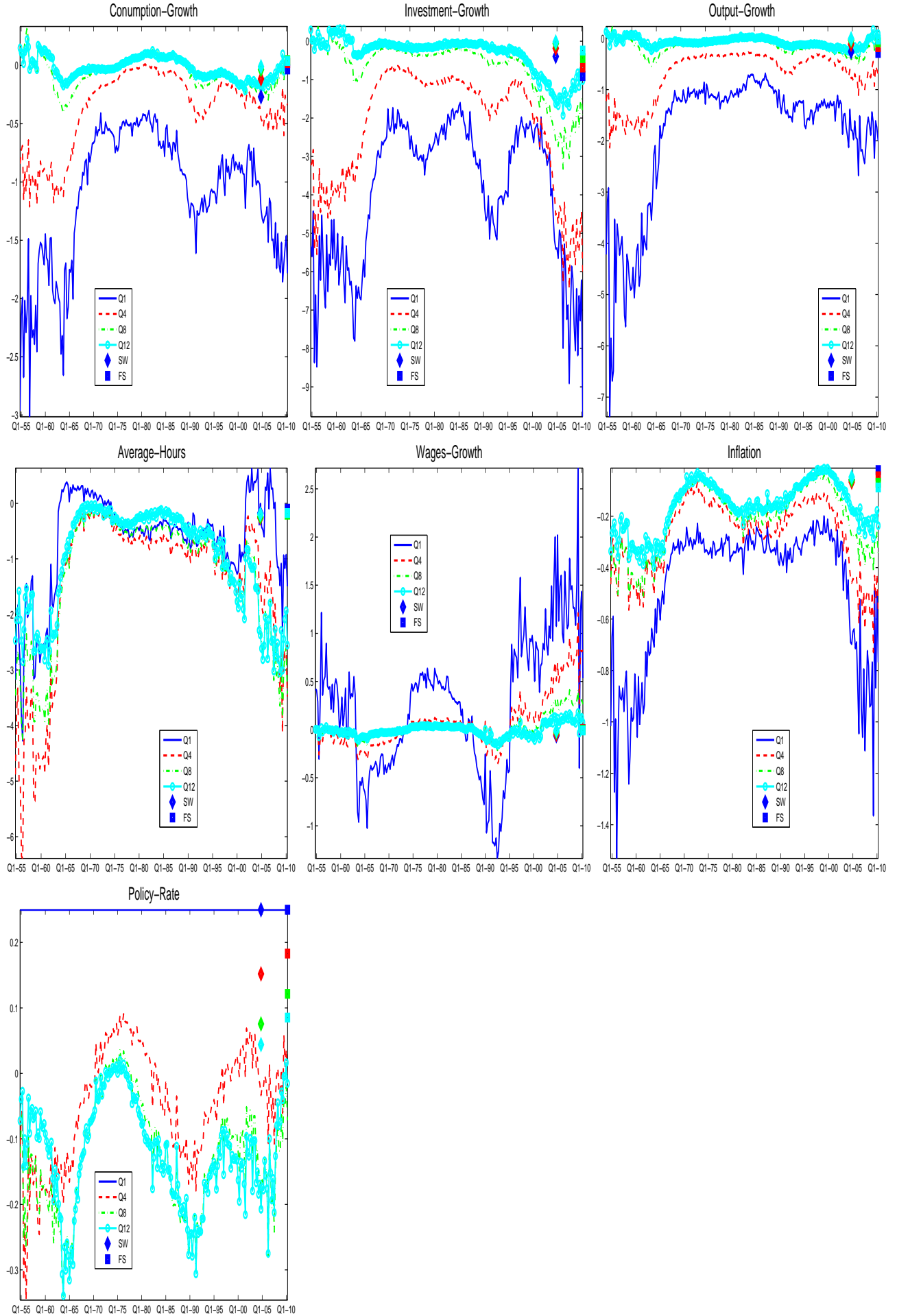


Figure 10: IRFs Summary Time-Varying Σ_v



A.2 The linearized, Smets-Wouters model

To make the paper self-contained, this subsection briefly discusses some of the key linearized equilibrium conditions of [Smets and Wouters \(2007a\)](#) model. Readers who are interested in the agents' decision problems are recommended to consult the references mentioned above directly. All the variables are expressed as log deviations from their steady-state values, \mathbb{E}_t denotes expectation formed at time t , '−' denotes the steady state values and all the shocks (η_t^i) are assumed to be normally distributed with zero mean and unit standard deviation.

The demand side of the economy consists of consumption (c_t), investment (i_t), capital utilisation (z_t) and government spending ($\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \sigma_g \eta_t^g$) which is assumed to be exogenous. The market clearing condition is given by

$$y_t = c_y c_t + i_y i_t + z_y z_t + \varepsilon_t^g \quad (\text{A.1})$$

where y_t denotes the total output and Table (1) provides a full description of the model's parameters. The consumption Euler equation is given by

$$c_t = \frac{\lambda/\gamma}{1 + \lambda/\gamma} c_{t-1} + \left(1 - \frac{\lambda/\gamma}{1 + \lambda/\gamma}\right) \mathbb{E}_t c_{t+1} + \frac{(\sigma_C - 1) (\bar{W}^h \bar{L} / \bar{C})}{\sigma_C (1 + \lambda/\gamma)} (l_t - \mathbb{E}_t l_{t+1}) - \frac{1 - \lambda/\gamma}{\sigma_C (1 + \lambda/\gamma)} (r_t - \mathbb{E}_t \pi_{t+1}) \quad (\text{A.2})$$

where l_t is the hours worked, r_t is the nominal interest rate and π_t is the rate of inflation. If the degree of habits is zero ($\lambda = 0$), equation (A.2) reduces to the standard forward looking consumption Euler equation. The linearised investment equation is given by

$$i_t = \frac{1}{1 + \beta \gamma^{1-\sigma_C}} i_{t-1} + \left(1 - \frac{1}{1 + \beta \gamma^{1-\sigma_C}}\right) \mathbb{E}_t i_{t+1} + \frac{1}{(1 + \beta \gamma^{1-\sigma_C}) \gamma^2 \varphi} q_t \quad (\text{A.3})$$

where i_t denotes the investment and q_t is the real value of existing capital stock (Tobin's Q). The sensitivity of investment to real value of the existing capital stock depends on the parameter φ (see, ?). The corresponding arbitrage equation for the value of capital is given by

$$q_t = \beta \gamma^{-\sigma_C} (1 - \delta) \mathbb{E}_t q_{t+1} + (1 - \beta \gamma^{-\sigma_C} (1 - \delta)) \mathbb{E}_t r_{t+1}^k - (r_t - \mathbb{E}_t \pi_{t+1}) \quad (\text{A.4})$$

where $r_t^k = -(k_t - l_t) + w_t$ denotes the real rental rate of capital which is negatively related to the capital-labour ratio and positively to the real wage.

On the supply side of the economy, the aggregate production function is define as

$$y_t = \phi_p (\alpha k_t^s + (1 - \alpha) l_t) \quad (\text{A.5})$$

where k_t^s represents capital services which is a linear function of lagged installed capital (k_{t-1}) and the degree of capital utilisation, $k_t^s = k_{t-1} + z_t$. Capital utilization, on the other hand, is proportional to the real rental rate of capital, $z_t = \frac{1-\psi}{\psi} r_t^k$. The accumulation process of installed capital is simply described as

$$k_t = \frac{1 - \delta}{\gamma} k_{t-1} + \frac{\gamma - 1 + \delta}{\gamma} i_t \quad (\text{A.6})$$

Monopolistic competition within the production sector and Calvo-pricing constraints gives the following New-Keynesian Phillips curve for inflation

$$\begin{aligned}\pi_t = & \frac{i_p}{1 + \beta\gamma^{1-\sigma_C}i_p}\pi_{t-1} + \frac{\beta\gamma^{1-\sigma_C}}{1 + \beta\gamma^{1-\sigma_C}i_p}\mathbb{E}_t\pi_{t+1} \\ & - \frac{1}{(1 + \beta\gamma^{1-\sigma_C}i_p)} \frac{(1 - \beta\gamma^{1-\sigma_C}\xi_p)(1 - \xi_p)}{(\xi_p((\phi_p - 1)\varepsilon_p + 1))}\mu_t^p + \varepsilon_t^p\end{aligned}\quad (\text{A.7})$$

where $\mu_t^p = \alpha(k_t^s - l_t) - w_t$ is the marginal cost of production and $\varepsilon_t^p = \rho_p\varepsilon_{t-1}^p + \sigma_p\eta_t^p - \mu_p\sigma_p\eta_{t-1}^p$ is the price mark-up price shock which is assumed to be an ARMA(1,1) process. Monopolistic competition in the labour market also gives rise to a similar wage New-Keynesian Phillips curve

$$\begin{aligned}w_t = & \frac{1}{1 + \beta\gamma^{1-\sigma_C}}w_{t-1} + \frac{\beta\gamma^{1-\sigma_C}}{1 + \beta\gamma^{1-\sigma_C}}(\mathbb{E}_tw_{t+1} + \mathbb{E}_t\pi_{t+1}) - \frac{1 + \beta\gamma^{1-\sigma_C}i_w}{1 + \beta\gamma^{1-\sigma_C}}\pi_t \\ & + \frac{i_w}{1 + \beta\gamma^{1-\sigma_C}}\pi_{t-1} - \frac{1}{1 + \beta\gamma^{1-\sigma_C}} \frac{(1 - \beta\gamma^{1-\sigma_C}\xi_w)(1 - \xi_w)}{(\xi_w((\phi_w - 1)\varepsilon_w + 1))}\mu_t^w + \varepsilon_t^w\end{aligned}\quad (\text{A.8})$$

where $\mu_t^w = w_t - \left(\sigma_l l_t + \frac{1}{1-\lambda}(c_t - \lambda c_{t-1})\right)$ is the households' marginal benefit of supplying an extra unit of labour service and the wage mark-up shock $\varepsilon_t^w = \rho_w\varepsilon_{t-1}^w + \sigma_w\eta_t^w - \mu_w\sigma_w\eta_{t-1}^w$ is also assumed to be an ARMA(1,1) process.

Finally, the monetary policy maker is assumed to set the nominal interest rate according to the following Taylor-type rule

$$r_t = \rho r_{t-1} + (1 - \rho)[r_\pi\pi_t + r_y(y_t - y_t^p)] + r_{\Delta y}[(y_t - y_t^p) + (y_{t-1} - y_{t-1}^p)] + \varepsilon_t^r \quad (\text{A.9})$$

where y_t^p is the flexible price level of output and $\varepsilon_t^r = \rho_r\varepsilon_{t-1}^r + \sigma_r\eta_t^r$ is the monetary policy shock.¹⁰

¹⁰The flexible price level of output is defined as the level of output that would prevail under flexible prices and wages in the absence of the two mark-up shocks.

A.3 Tables

Table 1: Parameter descriptions and estimated values from ?

Symbols	Description	M_0
γ	Steady State Growth Rate	1.00
π	Steady State Inflation	1.00
Φ	Fixed Cost	1.50
S''	Steady State Capital Adjustment Cost Elasticity	5.74
α	Capital Production Share	0.19
σ	Intertemporal Substitution	1.38
h	Habit Persistence	0.71
ξ_w	Wages Calvo Parameter	0.70
σ_l	Labour Supply Elasticity	1.83
ξ_p	Prices Calvo Parameter	0.66
i_w	Wage Indexation	0.58
i_p	Price Indexation	0.24
z	Capital Utilisation Adjustment Cost	0.27
ϕ_π	Taylor Inflation Parameter	2.04
ϕ_r	Taylor Inertia Parameter	0.81
ϕ_y	Taylor Output Gap Parameter	0.08
ϕ_{dy}	Taylor Output Gap Change Parameter	0.22
ρ_g	Government Spending Shock Persistence	0.97
ρ_{ms}	Policy Shock Persistence	0.15
ρ_p	Price Mark-up Shock Persistence	0.89
ρ_w	Wage Mark-up Shock Persistence	0.96
ma_p	Price Mark-up MA Term	0.69
ma_w	Wage Mark-up MA Term	0.84
σ_g	Government Spending Shock Uncertainty	0.53
σ_{ms}	Policy Shock Uncertainty	0.24
σ_p	Price Mark-up Shock Uncertainty	0.14
σ_w	Wage Mark-up Shock Uncertainty	0.24

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