# Credit Markets, Limited Commitment, and Government Debt

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March 21, 2011

**Abstract**

## 1 Introduction

In order to understand recent events in credit markets and their implications for aggregate economic activity, it is necessary that we understand underlying credit market frictions and attempt to sort out which frictions, if any, are important in explaining observations. One set of credit market frictions is captured in the costly state verification models of Williamson (1987), Bernanke and Gertler (1989), and Bernanke, Gertler, and Gilchrist (1999). Under some circumstances, the costly state verification friction can amplify aggregate disturbances, with this amplification mechanism sometimes called a "financial accelerator" (Bernanke, Gertler and Gilchrist 1999). Another important friction works through the value of collateral in Kiyotaki and Moore (2008) and Kocherlakota (2008). In these models, asset "bubbles" are a good thing in that the collapse of an asset bubble can tighten liquidity constraints and reduce aggregate investment and output.

This paper explores inefficiencies that can arise in the context of a limited commitment friction, and explores the potential role of government debt in overcoming these inefficiencies. A standard textbook story is that, in a frictionless world, Ricardian equivalence holds, so that the size of the government debt and the timing of taxation is irrelevant. Then, as the typical story goes, suppose that there is a credit market "imperfection," for example there is a gap between the rates of interest at which consumers can lend and borrow, respectively, and the government can borrow at a lower rate than can private sector agents. Then government debt matters: effectively the government can run a welfare-improving credit program by using the tax system.

But what happens if we are more explicit about the underlying private-information or limited-commitment frictions that give rise to the credit market imperfection? Then, what might matter is whether the government has better information, or faces less severe incentive constraints, perhaps because it is better-able to collect on its debts than private sector lenders. However, we also know, for example from work by Lagos and Rocheteau (2008) and Rocheteau (2009), for example, that safe government debt can serve an important role in transactions when private information limits the role of private liquid assets in exchange. This insight that is closely related to (if not identical) to the insight we get from modern monetary theory on the role for money (e.g. Kocherlakota 1998). A goal of this paper is to transfer that insight to the study of exchange using credit, under limited commitment, and to expand on that idea.

In limited commitment models, such as Kehoe and Levine (1993) and Sanches and Williamson (2008), there typically exist multiple equilibria, which arise for much the same reason that models of money exhibit multiple equilibria. For example, in overlapping generations models of money (e.g. Wallace 1980), turnpike models (Townsend 1980), or search models of money (Lagos and Wright 2005), there typically exists an efficient equilibrium with valued money (given the appropriate monetary policy rule), an inefficient equilibrium where money is not valued, and many other inefficient equilibria with valued money. An equilibrium exists where money is not valued since each individual anticipates that other individuals will not accept money in the future, so he or she does not accept it either. Similarly, for example in Sanches and Williamson (2008), under circumstances where there is perfect recordkeeping and therefore no role for monetary exchange, there exist two steady state credit equilibria with limited commitment. In the efficient equilibrium, lending is supported in equilibrium because it is suboptimal for any borrower to default. If default were to occur (out of equilibrium), then the borrower would give up any prospect of future borrowing and be relegated to autarky forever. However, in the inefficient equilibrium there is no borrowing, as an individual lender correctly anticipates that no one will lend in the future, so that a would-be borrower would lose nothing from defaulting and is therefore not credit-worthy.

In the model constructed here, I explore that possibility of asymmetric equilibria where there is a set of creditworthy borrowers who always repay their debts, and another set of borrowers who will always default if anyone chooses to lend to them. The model is set up so that these two sets of borrowers are sometimes indistinguishable. There is potentially an adverse selection friction in the credit market, and the possibility of default in equilibrium.

In this context, government debt may sometimes improve matters, and sometimes it will not. In equilibria where there are global punishments, more government debt cannot improve the equilibrium allocation. In this case, severe private punishments discipline creditors to the extent that the government has no advantage. Perhaps more realistically though, we can consider other equilibria, with individual punishments. In this case, default will ban an agent from future credit arrangements (where he or she can be identified), but an individual's default does not affect anyone else. Now there is a clear advantage for the government. In some cases, effectively displacing private credit with government debt can yield a welfare improvement, though the introduction of government debt may not stand in for severe punishments. However, the fact that an agent who defaults has to incur the cost of acquiring government debt in order to trade acts to discipline borrowers.

In equilibria where some fraction of borrowers always defaults, consumption tends to fall and incentive constraints to tighten as the fraction of defaulters in the population increases. In this context, government debt can work to effectively solve the adverse selection problem in the credit market, and in so doing it relaxes incentive constraints. Just as in symmetric equilibria, government debt can sometimes replicate the equilibrium allocation with global punishments, but in general this does not happen.

# 2 The Model

Time is indexed by $t = 1, 2, 3, ...$, and each period consists of two subperiods, in which trade occurs, respectively, in a centralized market ($CM$) and a decentralized market ($DM$). There is a continuum of agents with mass 2, half of whom are buyers, with the other half being sellers. Each buyer has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t [-H_t + u(x_t)] \tag{1}$$

where $H_t$ is labor supply during the $CM$, $x_t$ is consumption in the $DM$, and $0 < \beta < 1$. Assume that $u(\cdot)$ is strictly concave, strictly increasing, and twice continuously differentiable with $u(0) = 0$, $u'(0) = \infty$, and define $x^*$ to be the solution to $u'(x^*) = 1$, and $x^{**}$ the solution to $x^{**} = \beta u(x^{**})$. A seller has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t (X_t - h_t), \tag{2}$$

where $X_t$ is consumption in the $CM$, and $h_t$ is labor supply in the $DM$. Buyers can produce only in the $CM$, and sellers produce only in the $DM$. When productive, an agent has access to a technology which permits the production of one unit of the perishable consumption good for each unit of labor input.

The government can tax buyers lump-sum in the $CM$, and can issue one-period government bonds. In the $CM$, agents first meet in a centralized location, where debts from the previous $DM$ are settled, taxes are paid to the government, and the government makes the payoffs on the government bonds issued in the previous period. Then, in the latter part of the $CM$, government bonds are sold on a Walrasian market in which exchange is anonymous. A key assumption is limited commitment, i.e. all exchange is voluntary. In particular, buyers cannot be forced to pay their taxes.

During the $DM$, each buyer is randomly matched with a seller. A fraction $\rho$ of $DM$ meetings are *limited-information meetings*, where the seller does not have access to the buyer's history. Even though there is limited information in this sense, the interaction between the buyer and seller in the meeting will be publicly recorded. The remaining fraction $1-\rho$ of $DM$ meetings are *full-information meetings*, where the seller has access to the public record and the interaction between buyer and seller is recorded.

# 3 Symmetric Stationary Equilibria with Global Punishments

First, we will analyze equilibria that are symmetric and stationary, in that each buyer and each seller receive the same allocation, and consume the same amount in each period. Further, these equilibria will be supported by off-equilibrium-path global punishments, in which all agents are punished for the bad behavior of any agent. We will first examine equilibria without government debt, and then introduce government debt to show what difference this makes.

## 3.1 No Government Debt

In a $DM$ meeting, we will assume that the buyer makes a take-it-or-leave-it offer to the seller. Let $v$ denote the continuation value (constant for all $t$) for a buyer at the end of the $CM$, with $\hat{v}$ denoting the punishment continuation value. Then, $v$ is determined by

$$v = \max_{l}[u(l) - l + \beta v] \tag{3}$$

subject to

$$l \leq \beta(v - \hat{v}). \tag{4}$$

Here, $l$ is the quantity of goods received by the buyer from the seller (i.e. the loan quantity) during the $DM$ and the buyer promises to repay $l$ goods in the following $CM$ so as to make the seller indifferent to accepting the contract offer. Inequality (4) is an incentive constraint which states that, given limited commitment, the buyer must have the incentive to repay the loan during the $CM$ rather than facing punishment, represented by the continuation value $\hat{v}$.

We assume that no one can be forced to work, so that the worst possible punishment is $\hat{v} = 0$, i.e. perpetual autarky. Recall that $\hat{v} = 0$ is accomplished off-equilibrium with global punishments. If any buyer defaults then this triggers global autarky. Note that global autarky is also an equilibrium, since if $\hat{v} = 0$ then $v = l = 0$ solves the problem above. Reformulating the problem, equilibrium $v$ solves

$$v = \psi(v),$$

where

$$\psi(v) = u(\beta v), \text{ for } 0 \leq v \leq \frac{x^*}{\beta},$$

$$\psi(v) = u(x^*) - x^* + \beta v, \text{ for } v \geq \frac{x^*}{\beta}.$$

There are then two stationary symmetric equilibria. The first is $v = l = 0$, which we have already discussed. In the second equilibrium, either (i) $x^* \leq x^{**}$ and $v = \frac{u(x^*)-x^*}{1-\beta}$ with $l = x^*$, or (ii) $x^* > x^{**}$ and $v = \frac{l}{\beta} = \frac{x^{**}}{\beta}$. The first equilibrium is inefficient, but the second is efficient, in either case (i) or case (ii). The incentive constraint does not bind in the efficient equilibrium in case (i), but it binds in case (ii).

Note that, in the equilibrium where $v > 0$, efficient trade is supported in spite of the fact that the seller does not observe the buyer's history in a limited-information meeting during the $DM$. If a buyer defaults on any loan contract, whether the loan was received in a limited-information or full-information meeting, this will trigger global autarky, so that no loans are made on the off-equilibrium path.

## 3.2 Government Debt

Now, suppose that the government issues $B$ units of government bonds each period in the $CM$. Each bond is a promise to pay one unit of goods in the next $CM$, and these promises sell at the price $q$. Further, each buyer incurs a tax $\tau$ during the $CM$ to pay the net interest on the government's debt. Now, if the equilibrium is one where buyers hold government debt and borrow from sellers in the $DM$, the continuation value $v$ is determined by

$$v = \max_{l,b} \{-qb + u(l + \beta b) - l - \beta\tau + \beta v\} \tag{5}$$

subject to

$$l + \beta\tau \leq \beta (v - \hat{v}), \tag{6}$$

where $l$ is the quantity borrowed during the $DM$, and $b$ denotes the quantity of bonds acquired by the buyer in the $CM$. As well, in equilibrium, the demand for government debt is equal to the supply,

$$b = B, \tag{7}$$

and the government budget constraint holds, or

$$\tau = B(1 - q). \tag{8}$$

First, if the constraint (6) does not bind, then in equilibrium $l+\beta B = x^*$, $q = \beta$, and $\tau = B(1 - \beta)$, so from (5), we have

$$v = \frac{-\beta(1-\beta)B + u(x^*) - x^*}{1-\beta}, \tag{9}$$

and then the incentive constraint (6) holds if and only if $x^* \leq x^{**}$, which is the same condition we obtained without government debt, and the equilibrium allocation is identical, so $B$ is irrelevant in this case.

Next, consider the case where the incentive constraint (6) binds. Then, from (5)-(8) we get

$$v = -qB + u(\beta v + \beta B q), \tag{10}$$

$$q = \beta u'(\beta v + \beta Bq) \tag{11}$$

Then, from (10) there are two solutions for $v + qB$, i.e.

$$v + qB = \frac{x^{**}}{\beta},$$

or $v + qB = 0$. Then, from (11), if $v + qB = \frac{x^{**}}{\beta}$ then $q = \beta u'(x^{**})$, and if $v + qB = 0$, then $q = \infty$. Here, we need to check that the incentive constraint is binding, which it always is in the equilibrium where $v + qB = 0$, but in the other equilibrium, we require $x^* > x^{**}$. Again, $B$ is irrelevant for the equilibrium, and we get exactly the same equilibria as without government debt.

So far we have looked at the case where there is always some lending in equilibrium. This requires, from above, that $B < \frac{\min(x^*,x^{**})}{\beta}$. Suppose that this condition does not hold. First, suppose that there is efficient exchange in the $DM$, so that $x^*$ is consumed by buyers, and buyers need not trade all their bonds in the $DM$, so that $q = \beta$. Here, we can show that we can sustain the equilibrium with no lending where $x^*$ is consumed in the $DM$ and the incentive constraint does not bind, if and only if

$$\frac{x^*}{\beta} \leq B \leq \frac{u(x^*) - x^*}{1-\beta},$$

and a necessary condition for this equilibrium to exist for some $B$ is that $x^* \leq x^{**}$. It is also straightforward to show that the only value of $B$ that will support an equilibrium where the incentive constraint binds and there is no lending is the case where $B = \frac{x^{**}}{\beta}$, in which case a necessary condition for existence is $x^* > x^{**}$. This then is just a special case of the equilibrium considered above with government debt in which lending goes to zero, and we know this is identical to the equilibrium with $B = 0$.

Thus, in these equilibria, the issue of government debt accomplishes nothing. Effectively, the government faces the same limited commitment problem in collecting taxes as do private sector sellers in collecting on their debts. Therefore, the issue of government debt cannot allow for a superior equilibrium allocation.

# 4 Symmetric Equilibria with Individual Punishment

Now, suppose that punishments are carried out, off equilibrium, only at the individual level. The key implication of this is that a buyer who defaults on his or her private or public liabilities may still be able to consume in limited information meetings in the $DM$, whereas with global punishments this is not possible. Thus, we are just examining some different equilibria, but ones which are in some sense more realistic.

What changes here is that we do not necessarily have $\hat{v} = 0$. When a buyer defaults, he or she will be able to acquire consumption goods in limited information meetings in the $DM$, so long as he or she mimics the equilibrium behavior of buyers. Mimicing equilibrium behavior requires that the defaulter acquire the same quantity of bonds, $b$, to offer in exchange, receiving the same loan quantity $l$ in limited information meetings as do buyers in equilibrium. A defaulting buyer will of course not be able to trade in full information meetings in the $DM$. It may also be the case that the best a buyer can do in the event of default is autarky. Therefore, in the case where buyers trade away all their bonds in the $DM$, we have

$$\hat{v} = \max\left[0, \frac{-qb + \rho u(l + \beta b) + (1-\rho)\beta b}{1-\beta}\right] \tag{12}$$

## 4.1 Equilibria with no Government Debt

In this case it will always be optimal for a buyer who defaults to mimic the behavior of other buyers on the off-equilibrium path. In the case where there is efficient trade in the $DM$, we have

$$v = \frac{u(x^*) - x^*}{1-\beta}, \tag{13}$$

$$\hat{v} = \frac{\rho u(x^*)}{1-\beta}, \tag{14}$$

and the incentive constraint (4) gives

$$x^* \leq \beta(1-\rho)u(x^*), \tag{15}$$

so if (15) holds, then this equilibrium exists.

Now, suppose that trade is not efficient in the $DM$, i.e. the incentive constraint (4) binds. Then from (3), (4), and (12), we can solve for the quantity of goods $x$ consumed by the buyer in the $DM$, i.e. $x$ solves

$$x = \beta(1-\rho)u(x),$$

and we require that the solution satisfy $x < x^*$, in order that the incentive constraint not bind, so this equilibrium exists if and only if

$$x^* > \beta(1-\rho)u(x^*). \tag{16}$$

It will be useful to define $x_1$ as the solution to

$$x_1 = \beta(1-\rho)u(x_1),$$

so that the incentive constraint does no bind if and only if $x_1 \geq x^*$, and otherwise the incentive constraint binds.

## 4.2 Equilibria with Government Debt

Now, suppose that $B > 0$. To focus on whether or not the issue of government debt can improve matters, we will construct equilibria in which government debt is just sufficiently large to crowd out private credit. First, suppose that government debt supports an equilibrium where trade in the $DM$ is efficient, i.e. $\beta B = x^*$ and $q = \beta$. Then,

$$v = \frac{u(x^*) - x^*(2-\beta)}{1-\beta},$$

$$\hat{v} = \frac{\rho\left[u(x^*) - x^*\right]}{1-\beta}$$

Then, the incentive constraint (6) holds if and only if

$$x^* \leq \frac{\beta(1-\rho)u(x^*)}{1-\rho\beta} \tag{17}$$

So inequality (17) is necessary and sufficient for this equilibrium to exist. Now, let $x_2$ be the solution to

$$x_2 = \frac{\beta(1-\rho)u(x_2)}{1-\rho\beta},$$

and so we can write (17) as

$$x_2 \geq x^*$$

Next, consider equilibria with the incentive constraint (6) binding. Here, first construct an equilibrium where a buyer who defaults, off-equilibrium, mimics the behavior of non-defaulters. Then, from (5), (6), and (12), and given $q = \beta u'(x)$, where $x$ is the quantity of goods exchanged in $DM$ meetings, the following equation solves for $x$

$$x\left[1 - \beta u'(x)\right] = \beta(1-\rho)[u(x) - x], \tag{18}$$

and it is easy to show that there is a unique solution to (18) with $x > 0$. For this to be an equilibrium, we require that two conditions be satisfied. First $x < x^*$, or from (18),

$$x^* > \frac{\beta(1-\rho)u(x^*)}{1-\rho\beta},$$

or $x_2 < x^*$. Second $\hat{v} \geq 0$, or from (12), and using (18), the solution to (18) must satisfy

$$u'(x) \leq \frac{\rho + \beta(1-\rho)}{\beta} \tag{19}$$

so from (18), the solution to (18) will satisfy (19) if and only if

$$x^{***} \leq x^{**},$$

where $x^{***}$ solves

$$u'(x^{***}) = \frac{\rho + \beta(1-\rho)}{\beta},$$

and $x^{**}$ solves

$$x^{**} = \beta u(x^{**}). \tag{20}$$

Finally, consider an equilibrium where the incentive constraint (6) binds, but a buyer who defaults does not mimic the behavior of other buyers on the off-equiliibrum path, so $\hat{v} = 0$. Then, solving for $x$ from from (5), (6), and (8), with $q = \beta u'(x)$ and $\hat{v} = 0$ gives

$$x = x^{**}, \tag{21}$$

so again there are two conditions that must hold for this to be an equilibrium, the first, required for the incentive constraint to bind, is

$$x^{**} < x^*,$$

and the other is that a buyer who defaults not want to mimic other buyers on the off-equilibrium path, i.e. from (12) and (21),

$$u'(x^{**}) \geq \frac{\rho + \beta(1-\rho)}{\beta}$$

or

$$x^{**} \leq x^{***}$$

## 4.3 Welfare

If we add utilities across agents in any of the equilibria we are studying here, we will obtain a welfare measure

$$W = u(x) - x,$$

where $x$ is the quantity of goods exchanged in $DM$ meetings and $u(x) - x$ is total surplus in a $DM$ meeting. In any of the equilibria under study we have $x \leq x^*$, so $W$ is increasing $x$ over the relevant range. As a result, if we want to compare welfare given two alternative equilibrium allocations, we need only determine $x$ is in the the two equilibria and compare.

Now, from the previous two subsections, $x_2 > x_1$ for all parameter values, so if $x_1 \geq x^*$, so there is efficient trade ($x = x^*$) in the $DM$ when $B = 0$, there is also efficient trade in the equilibrium with government debt, as $x_2 > x^*$. Thus, if $x_1 \geq x^*$, i.e. if (15) holds, the economy with all trade accomplished with government debt fares no worse than the pure private-credit economy.

Next, it is possible to have $x_2 \geq x^* > x_1$ for some parameter values, in which case the incentive constraint binds in the private-credit economy ($x < x^*$), but $DM$ exchange is efficient with government debt. Thus, in this case, government debt increases welfare. Then, in the case where $x^* > x_2 \geq x^{***}$, incentive constraints bind in both the private-credit and government debt equilibria. However, in the government debt equilibrium, we have $x^{**} \leq x < x^*$, and in the private-credit equilibrium $x < x^{**}$, so welfare is higher with government debt. Similarly, in the case $x^{***} \geq x_2$, we have $x = x^{**}$, so $x$ is smaller in the equilibrium with private credit, and welfare is also higher with government debt.

In conclusion, government debt improves welfare, principally by relaxing the incentive constraint. With government debt, a buyer who defaults needs to bear the cost of acquiring government debt in order to consume in the event of default, which lowers the payoff from defaulting. At the extreme, the cost of acquiring debt is so high that a defaulter lives in autarky forever. However, in a private-credit equilibrium, an agent can default and consume on the off-equilibrium path without working, which makes the incentive constraint tighter.

# 5 Asymmetric Equilibria with Individual Punishment and Equilibrium Default

We will now consider equilibria where agents behave asymmetrically, with some buyers defaulting in equilibrium. These are equilibria where a fraction $\alpha$ of buyers (the *good buyers*) never defaults, but a fraction $1-\alpha$ (*bad buyers*) will default on their debts if anyone chooses to lend to them. Now, the continuation value $v$ for a good buyers, in the case where all bonds are sold during the $DM$ (the relevant case for the results we consider), is given by

$$v = \max_{l_1,l_2,b} \left\{ -qb + \rho u(l_1 + \beta b) + (1-\rho)u(l_2 + \beta b) - \rho \frac{l_1}{\alpha} - (1-\rho)l_2 - \beta\tau + \beta v \right\} \tag{22}$$

subject to

$$\frac{l_1}{\alpha} + \beta\tau \leq \beta \left( v - \hat{v} \right), \tag{23}$$

$$l_2 + \beta\tau \leq \beta \left( v - \hat{v} \right) \tag{24}$$

In (22)-(24), the good buyer takes out a loan $l_1$ in a limited information meeting, and a loan $l_2$ in a full information meeting, and sells bonds $b$ in each type of meeting. In order to receive consumption goods from the seller in a limited-information meeting, a bad buyer must mimic the behavior of a good buyer. Thus, there is a pooling equilibrium in which the good borrower promises to make a payment $\frac{l_1}{\alpha\beta}$ on a loan from a seller so as to compensate the seller for defaults by bad buyers. Thus, limited-information loans carry a default premium. In full information meetings, bad buyers will not receive loans. Constraints (23) and (24) are the incentive constraints that must hold for a good buyer following a limited-information meeting and a full-information meeting, respectively.

A bad buyer can consume the same quantity as a good buyer in a limited information meeting in the decentralized market if he or she mimics the behavior of a good buyer. In any event, a bad buyer cannot trade in full-information meetings in the decentralized market, and always defaults on his or her loans. As in the previous section, a bad buyer always has the option of choosing autarky, i.e. his or her continuation utility $\hat{v}$ must exceed zero, so we get

$$\hat{v} = \max\left[0, \frac{-qb + \rho u(l_1 + \beta b) + (1-\rho)\beta b}{1-\beta}\right], \tag{25}$$

and note here that the bad buyer must purchase the same quantity of government bonds $b$ as a good buyer, in order to mimic the good buyer's behavior in decentralized trading.

Further, taxes on good buyers finance the net interest on government bonds, or

$$\tau = \frac{B(1-q)}{\alpha}. \tag{26}$$

and the bond market clears, which implies

$$b = B, \tag{27}$$

if bad buyers mimic good buyers, or

$$\alpha b = B, \tag{28}$$

if bad buyers choose autarky.

## 5.1 Asymmetric Equilibria Without Government Bonds

First, suppose that $B = 0$. In this case, either both incentive constraints (23) and (24) bind, constraint (23) binds and (24) does not, (23) does not bind and (24) does, or neither constraint binds. We will consider each case in turn.

### 5.1.1 Both Incentive Constraints Bind

Since $b = B = 0$, and given that (23) and (24) bind, from (22) - (25) we get

$$v = \rho u[\alpha\beta(v-\hat{v})] + (1-\rho)u[\beta(v-\hat{v})] + \beta\hat{v}, \tag{29}$$

$$\hat{v} = \rho u[\alpha\beta(v-\hat{v})] + \beta\hat{v}. \tag{30}$$

Then, subtracting (30) from (29), we get

$$v - \hat{v} = (1-\rho)u[\beta(v-\hat{v})], \tag{31}$$

which solves for $v - \hat{v}$. There are two solutions, one with $v - \hat{v} = 0$, which is an equilibrium, and one with $v - \hat{v} > 0$. This latter solution is an equilibrium if and only if the incentive constraints (23) and (24) indeed bind. That is, from (23) and (24), we require $v - \hat{v} \leq \frac{\hat{x}}{\beta\alpha}$ and $v - \hat{v} \leq \frac{x^*}{\beta}$, where $\hat{x}$ solves

$$u'(\hat{x}) = \frac{1}{\alpha}.$$

Thus, from (31), the equilibrium with $v - \hat{v} > 0$ exists if and only if

$$\frac{\hat{x}}{\alpha} \geq \beta(1-\rho)u\left(\frac{\hat{x}}{\alpha}\right), \tag{32}$$

and

$$x^* \geq \beta(1-\rho)u\left(x^*\right). \tag{33}$$

Thus, letting $x_L$ denote consumption by buyers in limited information meetings in the $DM$, and $x_F$ consumption in full-information meetings, we have $x_L = \alpha x_1$ and $x_F = x_1$. Recall that $x_1$ is consumption in the symmetric private-credit equilibrium with a binding incentive constraint, which is just the special case where $\alpha = 1$.

#### 5.1.2 Limited Information Incentive Constraint Binds, Other Incentive Constraint Does Not

Now, consider the case where (23) binds, but (24) does not. Then, from (22) - (25) we get

$$v = \rho u[\alpha\beta(v-\hat{v})] + (1-\rho)[u(x^*) - x^*] + \beta(1-\rho)v + \beta\rho\hat{v}, \tag{34}$$

$$\hat{v} = \rho u[\alpha\beta(v-\hat{v})] + \beta\hat{v}. \tag{35}$$

Then, subtracting (35) from (34) and solving for $v - \hat{v}$, we obtain

$$v - \hat{v} = \frac{(1-\rho)[u(x^*) - x^*]}{1-\beta(1-\rho)} \tag{36}$$

As in the previous subsection, we then have to check that (23) in fact binds, and (24) does not, or $v - \hat{v} \leq \frac{\hat{x}}{\beta\alpha}$ and $v - \hat{v} \geq \frac{x^*}{\beta}$, which from (36) gives us the necessary and sufficient conditions for existence of this equilibrium,

$$\frac{\hat{x}}{\alpha}\left[1-\beta(1-\rho)\right] + x^*\beta(1-\rho) \geq \beta(1-\rho)u(x^*), \tag{37}$$

$$x^* \leq \beta(1-\rho)u(x^*). \tag{38}$$

Then, from (37) and (38), a necessary condition for existence of this equilibrium is

$$x^* \leq \frac{\hat{x}}{\alpha}$$

From (36), note that

$$x_L = \frac{\alpha\beta(1-\rho)[u(x^*) - x^*]}{1-\beta(1-\rho)},$$

and of course $x_F = x^*$.

### 5.1.3 Limited Information Incentive Constraint Does Not Bind, Other Incentive Constraint Does

Next, we analyze the case where (23) does not bind, but (24) does. Then, from (22) - (25) we get

$$v = \rho u(\hat{x}) + (1-\rho)u[\beta(v-\hat{v})] - \rho\frac{\hat{x}}{\alpha} + \beta\rho v + \beta(1-\rho)\hat{v}, \tag{39}$$

$$\hat{v} = \rho u(\hat{x}) + \beta\hat{v}. \tag{40}$$

Then, subtracting (40) from (41) we get

$$v - \hat{v} = \frac{(1-\rho)u[\beta(v-\hat{v})] - \rho\frac{\hat{x}}{\alpha}}{1-\beta\rho}, \tag{41}$$

which solves for $v-\hat{v}$. Now, there may be no solutions to (41) with $v-\hat{v}>0$, there is a knife-edge case where there is one solution, and there can be two solutions. A solution to (41) with $v-\hat{v}>0$ exists if and only if

$$\tilde{x}\,(1-\beta\rho) + \rho\beta\frac{\hat{x}}{\alpha} \leq \beta(1-\rho)u(\tilde{x}), \tag{42}$$

where $\tilde{x}$ solves

$$u'(\tilde{x}) = \frac{1-\beta\rho}{\beta(1-\rho)}.$$

Then, if solutions to (41) exist, it must be the case that (23) does not bind, and (24) does, or $v-\hat{v} \geq \frac{\hat{x}}{\beta\alpha}$ and $v-\hat{v} \leq \frac{x^*}{\beta}$. This implies that a necessary condition for existence of this equilibrium is

$$\frac{\hat{x}}{\alpha} \leq x^*$$

From (41), $x_F$ solves

$$x_F = \frac{\beta(1-\rho)u(x_F) - \beta\rho\frac{\hat{x}}{\alpha}}{1-\beta\rho},$$

and of course we have $x_L = x^*$.

### 5.1.4 Neither Incentive Constraint Binds

If (23) and (24) do not bind, then from (22) - (25) we get

$$v = \rho u(\hat{x}) + (1-\rho)u(x^*) - \rho\frac{\hat{x}}{\alpha} - (1-\rho)x^* + \beta v,$$

$$\hat{v} = \rho u(\hat{x}) + \beta\hat{v},$$

and so, solving for $v-\hat{v}$, we get

$$v - \hat{v} = \frac{(1-\rho)u(x^*) - \rho\frac{\hat{x}}{\alpha} - (1-\rho)x^*}{1-\beta}. \tag{43}$$

For this to be an equilibrium requires that (23) and (24) not bind, or $v-\hat{v} \geq \frac{\hat{x}}{\beta\alpha}$ and $v - \hat{v} \geq \frac{x^*}{\beta}$, respectively, which gives, using (43),

$$[1-\beta(1-\rho)]\frac{\hat{x}}{\alpha} + \beta(1-\rho)x^* \leq \beta(1-\rho)u(x^*),$$

$$\beta\rho\frac{\hat{x}}{\alpha} + (1-\beta\rho)x^* \leq \beta(1-\rho)u(x^*)$$

## 5.2 Asymmetric Equilibria with Government Bonds

In asymmetric equilibria it is difficult to characterize all equilibria with a positive supply of government bonds, principally due to the fact that the price of bonds is endogenous, and this price will enter the incentive constraints. However, we can say a lot about the effects of government intervention in the bond market by considering equilibria where the government intervenes to the extent that it drives out activity in the credit market. We will first consider such equilibria where incentive constraints bind, and then look at the case where they do not.

### 5.2.1 Incentive Constraints Do Not Bind

First, suppose that government debt supports an equilibrium where trade in the decentralized market is efficient, i.e. $\beta B = x^*$ and $q = \beta$. Then, from (22)-(28), we get

$$v = -x^* + u(x^*) - \frac{x^*(1-\beta)}{\alpha} + \beta v,$$
$$\hat{v} = -x^* + \rho u(x^*) + (1-\rho)x^* + \beta\hat{v},$$

so we can solve for $v - \hat{v}$ to get

$$v - \hat{v} = \frac{1}{1-\beta}\left\{(1-\rho)u(x^*) - x^*\frac{[1-\beta+\alpha(1-\rho)]}{\alpha}\right\}$$

Then, we have to check that the incentive constraint does not bind in equilibrium, i.e. $v - \hat{v} \geq \frac{x^*(1-\beta)}{\alpha\beta}$ or

$$x^* \leq \frac{\alpha\beta(1-\rho)u(x^*)}{1-\beta[1-\alpha(1-\rho)]},$$

which is the necessary and sufficient condition for this equilibrium to exist.

### 5.2.2 Incentive Constraints Bind

Next, consider the case where the government sets $B$ so that incentive constraints bind, there is no private lending, and all bonds are sold by good buyers in the decentralized market. Also, suppose that it is optimal for bad buyers and defaulting good buyers to mimic the behavior of good buyers, i.e. $\hat{v} \geq 0$. Then, from (22)-(28), we get

$$v = -qB + u(\beta B) - \frac{\beta B(1-q)}{\alpha} + \beta v, \tag{44}$$

$$\hat{v} = -qB + \rho u(\beta B) + (1-\rho)\beta B + \beta \hat{v}, \tag{45}$$

$$\frac{\beta B(1-q)}{\alpha} = \beta(v - \hat{v}), \tag{46}$$

$$q = \beta u'(\beta B). \tag{47}$$

Then, from (44)-(47), and since $x = \beta B$, we obtain

$$x[1 - \beta u'(x)] = \alpha\beta(1-\rho)[u(x) - x], \tag{48}$$

which solves for $x$. It is straightforward to show that a unique solution to (48) always exists, and this solution is an equilibrium if and only if the solution satisfies $x < x^*$, or

$$x^* > \frac{\alpha\beta(1-\rho)u(x^*)}{1 - \beta[1 - \alpha(1-\rho)]} \tag{49}$$

and $\hat{v} \geq 0$, or

$$-xu'(x) + \rho u(x) + (1-\rho)x \geq 0 \tag{50}$$

Further, from (48) we can write (50) as

$$u'(x) \leq \frac{\alpha\beta(1-\rho) + \rho}{\beta[\alpha(1-\rho) + \rho]} \tag{51}$$

Then, letting $x_3$ denote the solution to

$$u'(x_3) = \frac{\alpha\beta(1-\rho) + \rho}{\beta[\alpha(1-\rho) + \rho]}, \tag{52}$$

using (48) we can write (51) as

$$x_3 \geq \tilde{x} \tag{53}$$

where $\tilde{x}$ solves

$$\tilde{x} = \beta[\alpha(1-\rho) + \rho]u(\tilde{x})$$

Next, construct an equilibrium with a binding incentive constraint where bad buyers and good buyers who default do not mimic good buyers. In this case $\hat{v} = 0$ and since only good buyers hold bonds, good buyers do not have to bear the tax payments that pay off bonds traded by defaulting agents. Therefore, $v$ is determined just as in the symmetric equilibrium (the equilibrium with $\alpha = 1$) and so $x = x^{**}$. For this to be an equilibrium, we require

$$x^{**} < x^*,$$

and that it not be in the interest of a bad buyer or defaulting good buyer to mimic good buyers, or

$$u'(x^{**}) \geq \rho + \beta(1-\rho),$$

i.e. $x^{**} \leq x^{***}$.

# 6 An Example

Things get somewhat complicated in analyzing asymmetric equilibria with default. For now, I'll use an example to illustrate some of the main points. Assume that $u(x) = 2x^{\frac{1}{2}}$. Then, $x^* = 1$, and $x^{**} = 4\beta^2$. First, if we consider symmetric equilibria with global punishments, then incentive constraints do not bind if and only if $\beta \geq \frac{1}{2}$, in which case $x = 1$. If $\beta < \frac{1}{2}$, then incentive constraints bind and $x = 4\beta^2 < 1$. The introduction of government bonds in this case cannot improve the equilibrium allocation.

Next, in a symmetric equilibrium with individual punishments, and with no government bonds issued, incentive constraints do not bind if and only if $\beta \geq \frac{1}{2(1-\rho)}$, in which case $x = 1$, but if $\beta < \frac{1}{2(1-\rho)}$ then incentive constraints bind and $x = 4\beta^2(1-\rho)^2$. Therefore, as compared to the case with global punishments, consumption and welfare are lower. Now, with government bonds, incentive constraints do not bind if and only if $\beta \geq \frac{1}{2-\rho}$, and in this case $x = 1$. But if $\beta \leq \frac{1-2\rho}{2(1-\rho)}$ then incentive constraints bind and

$$x = \frac{\beta^2 \left[1 + 2(1-\rho)\right]^2}{\left[1 + \beta(1-\rho)\right]^2}.$$

However, if $\frac{1-2\rho}{2(1-\rho)} \leq \beta < \frac{1}{2}$, then $x = 4\beta^2$. Here, note that we can do better, and never do worse, by replacing private credit arrangements with trade in safe government bonds. Note, however that, even with government bonds in the system, welfare is typically not as high as in the case with global punishments, i.e. fiscal policy cannot substitute perfectly for the severest possible punishment for default. But note that, if $\frac{1-2\rho}{2(1-\rho)} \leq \beta < \frac{1}{2}$, then government debt can replicate the global punishment equilibrium. Further, there is also a region of the parameter space, where $\frac{1-2\rho}{2(1-\rho)} < \beta < \frac{1}{2-\rho}$ where the introduction of government bonds cannot support an equilibrium that drives out private credit. In this case, it is possible that government bonds can improve welfare, but will coexist at the optimum with private credit.

Finally, in an asymmetric equilibrium with equilibrium default, consider equilibria where incentive constraints do not bind. In that case, consumption by buyers in limited information meetings, is $x_L = \alpha^2$, and in full-information meetings it is $x_F = 1$. This equilibrium exists if and only if

$$\beta \geq \frac{1}{(1-\rho)(1+\alpha)}$$

There may be an equilibrium where the incentive constraint binds in full-information meetings but not in limited-information meetings, but that is a little difficult to analyze. However, an equilibrium where both incentive constraints bind is not so hard. In that case, the equilibrium exists if and only if

$$\beta \leq \frac{\alpha^{\frac{1}{2}}}{2(1-\rho)}$$

and in this equilibrium,

$$x_L = 4\alpha\beta^2(1-\rho)^2;\; x_F = 4\beta^2(1-\rho)^2.$$

Now, consider asymmetric equilibria with government bonds, issued in sufficient quantities to drive out private credit. First, if neither incentive constraint binds, then $x_L = x_F = 1$, and this equilibrium exists if and only if

$$\beta \geq \frac{1}{1+\alpha(1-\rho)}.$$

Next, in an equilibrium where incentive constraints bind and defaulting buyers receive strictly positive continuation utility,

$$x_L = x_F = \frac{\beta^2\left[1+2\alpha\beta(1-\rho)\right]^2}{\left[1+\alpha\beta(1-\rho)\right]^2},$$

and this equilibrium exists if and only if

$$\beta < \frac{1}{1+\alpha(1-\rho)}$$

and

$$\beta \leq \frac{1-2\rho}{2\alpha(1-\rho)}$$

Then, in an equilibrium with government bonds where both incentive constraints bind, and defaulting buyers prefer autarky, we have

$$x_L = x_F = 4\beta^2,$$

and this equilibrium exists if and only if

$$\beta < \frac{1}{2},$$

and

$$\beta \geq \frac{1-2\rho}{2\alpha(1-\rho)}$$

Note that higher $\alpha$ tends to tighten incentive constraints and reduce consumption. In general, a larger fraction of buyers who default is a drag on everyone. However, government debt again tends to mitigate the problem, relaxing incentive constraints and increasing consumption. However, even with government debt in the system, default still tends to tighten the incentive constraints and make agents worse off than they would otherwise be, say with a higher $\alpha$ in equilibrium.

# 7 References


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