A MODEL OF COMMODITY MONEY WITH MINTING AND MELTING

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Motivation

Commodity money systems based on metals begin with something ***divisible,*** with value conferred by ***alternative uses.***

One might think these features would make a commodity money system easy for monetary authorities to implement, and for economic theory to understand, but this is not the case.

Historically, minting technology has meant coining metals in ***discrete amounts.***

In addition, environments had

- ***large costs*** of adjusting the structure of weights among coins—both technological, and imposed by the monetary authority

- sizable ***carrying costs,*** verification costs

- ***information problems*** associated with decentralized production and exchange, and limited record-keeping and enforcement

These features meant alternative designs of the commodity money system likely had big ***welfare effects*** and ***distributional effects.***

There is a set of important questions facing the theory of commodity money:

- ***Minting/melting:*** Relationship between metals' value in alternative uses, the supply of metal coins for use in transactions, and legally-set weights of coins
- ***Changing world supply of metals***
- ***Debasement/seigniorage:*** How should a monetary authority optimally tax commodity money?
- ***Bimetallism:*** Is a bimetallic system based on a legal ratio between coins of the two metals stable, or a knife-edge system in which changes in the market ratio drive one metal entirely into its alternative use?
- ***Gresham's law:*** Does good money drive out bad?
- ***Denomination structure:*** Should exchange rates between coins of different weights/metals be set? If yes, what should the composition of denominations in the money supply be? If no, what should the distribution of coins of different sizes be?

This paper uses modern monetary theory and computational methods to improve our understanding of commodity money systems.

It is part of a larger research program to which Warren has contributed numerous papers, with Anji and with others, on a number of these questions.

In an earlier paper (2011), Redish and Weber studied another random matching model, with both silver and gold coins. That paper explored the welfare and distributional effects of a shortage of small coins.

However, that paper did not tackle the question of how to model the ***opportunity cost*** of commodity money—how to model the commodity's alternative use. In that paper, coins yielded a flow of dividends.

This present paper allows silver to be held as ***jewelry***—which yields utility, but cannot be used in transactions—or as ***coins***—which make random matches between consumers and producers potentially productive, but don't yield utility, and in fact have a carrying cost.

By endogenizing the quantity of money, this paper represents a significant step forward in this research program.

Features of the model

One metal, in fixed supply.

Monetary authority determines the metal content of coins, and the number of different sizes of coins.

<u>Model</u>

First sub-period

- **Consumer/producer shock** is realized, **i.i.d.** across agents and over time (**1/2** of each)
- <u>Fraction</u> of agents are randomly **bilaterally matched**

Second sub-period

- Agents can change the mix of their coins and jewelry through **minting/melting**

During a period, the agent ***transitions*** to a new coin/jewelry portfolio, as a function of his ***portfolio coming in,*** realization of his ***type shock,*** and the ***portfolio of the agent with whom he is matched*** (if matched)***.*** The agent's ***minting/melting decision*** is conditioned on the outcome of the bilateral match.

Preferences: $$\underbrace{u(c)}_{\text{utility of consumption}} - \underbrace{q}_{\text{disutility of production}} + \underbrace{\mu(b_1 j)}_{\text{utility of jewelry}} - \underbrace{\gamma(s_1 + s_2)}_{\text{carrying costs}}$$

Agent's portfolio: $$y = \{s_1, s_2, j\}$$

- $\mathbf{s_1}$ small coins ($\mathbf{b_1}$ ounces of silver)
- $\mathbf{s_2}$ large coins **($\mathbf{b_2 = \eta b_1}$,** where $\eta = 2, 3, 4, \ldots$)
- **j** jewelry (in units of the small coin)

Question: Does the large coin need to be an integer multiple of the small coin?

Commodity money has an alternative use (there is an opportunity cost of tying up the commodity in money form).

Coin holdings of both consumer and producer in a match are <u>observable</u>.

Jewelry cannot be used for payment because its metal content cannot be ascertained.

Assumptions on the environment rule out credit.

TIOLI offer made by potential consumer: $(\mathbf{q}, \mathbf{p_1}, \mathbf{p_2})$

$\mathbf{q}$ = quantity of the perishable good to be produced for the consumer

$\mathbf{p_1}$ = number of small coins offered (if $\mathbf{p_1} < \mathbf{0}$, the producer is asked to make change)

$\mathbf{p_2}$ = number of large coins offered

Could it ever be optimal for agents to exchange coins without producing/consuming, as a way of altering their portfolio, without minting/melting and paying seigniorage?

Value functions in steady state equilibrium:

Bellman equation at the start of the second sub-period:

$$\mathbf{v(y) = \max_{(z_1, z_2)} \{ \beta\, w(s_1 + z_1, s_2 + z_2, j - z_1 - \eta z_2) - S(z_1, z_2, j) \}}$$

- $\mathbf{z_1, z_2}$ are numbers of small and large coins minted **(+)** or melted **(–)**
- paid only on minting, not melting
- jewelry is given up as seigniorage:

$$\mathbf{S(z_1, z_2, j) = \max\{ \mu(b_1 j) - \mu[b_1 j - b_1 \underbrace{\sigma(z_1 + \eta z_2)}_{\textit{seigniorage}}], 0 \}}$$

Bellman equation at the start of the first sub-period:

$$\mathbf{w(y)} = \theta \underbrace{\sum_{\tilde{\mathbf{y}}} \pi(\tilde{\mathbf{y}}) \max_{\mathbf{q},\, \mathbf{p_1}, \mathbf{p_2}} [\mathbf{u(q)} + \mathbf{v(s_1 - p_1, s_2 - p_2)}]}_{\textbf{buyers}}$$

$$+ \underbrace{(\mathbf{1} - \theta)\mathbf{v(y)}}_{\substack{\textbf{sellers and} \\ \textbf{unmatched agents}}} + \underbrace{\mu(\mathbf{b_1\, j})}_{\substack{\textbf{utility from} \\ \textbf{jewelry}}} - \underbrace{\gamma(\mathbf{s_1 + s_2})}_{\textbf{carrying costs}}$$

- $\pi(\mathbf{y})$ is the fraction of agents with **y** at the start of the first sub-period.
- **θ =** fraction of agents who are buyers in a bilateral match
- γ = utility cost of holding a coin

Asset holdings:

$\lambda^{b}(\mathbf{k_1, k_2; y, \tilde{y}})$ = probability a buyer leaves with $\mathbf{k_1 = s_1 - p_1}$ and $\mathbf{k_2 = s_2 - p_2}$

$\lambda^{s}(\mathbf{k_1, k_2; y, \tilde{y}})$ = same for seller

Asset distributions:

Going into the <u>second</u> sub-period: $\pi'(\mathbf{k_1, k_2, j})$

Going into the <u>first</u> sub-period: $\pi(\mathbf{k_1, k_2, h})$, where $\mathbf{h = j - z_1 - \eta z_2}$ after minting/melting

Asset holdings satisfy: $\sum_{\mathbf{y}} \pi(\mathbf{y}) = \sum_{\mathbf{y}} \pi'(\mathbf{k_1, k_2, h}) = \mathbf{1}$

Market-clearing (stock of silver is held)

Results

$\mathbf{u(q) = q^{1/4}}$

$\mathbf{\mu(b_1 j) = .05(b_1 j)^{1/2}}$ **utility from jewelry**

$\mathbf{\beta = .9}$

$\mathbf{\gamma = .001}$ **carrying cost of a coin**

Various values

$\mathbf{b_1}$	**ounces of silver in a small coin**
$\mathbf{\eta}$	$\mathbf{b_2 = \eta b_1}$
$\mathbf{\theta}$	**fraction of agents who are buyers in a bilateral match**
$\mathbf{m}$	**per capita amount of silver in the economy**

Welfare criterion: ***Ex ante welfare***

Single silver coin

The optimal coin size reflects the tradeoff between ***more handling costs*** (smaller coin) and ***lower probability of a successful match.***

Only producers with small silver holding are willing to produce a lot of output.

Distributional effect of a larger coin size

The fraction of agents who are made better off by a change in coin size may be below 50%, even though ex ante welfare is higher.

With a larger coin, a larger fraction of agents don't have any coins. Fewer trading opportunities. More silver held as jewelry.

Two coins

Provided coin sizes are chosen optimally, two coins result in higher ex ante welfare than one coin.

Introducing a second coin does not necessarily increase ex ante welfare. Yet agents may mint them, given the opportunity.

Would agents vote to prohibit the second denomination?

Historical applications

Urbanization increases trading opportunities. In the model, an increase in θ causes the optimal coin size to increase, but then to steadily decrease.

When the opportunity to trade arises infrequently, agents want most of their silver in the form of jewelry. A small coin allows this, while at the same time facilitating trade.

As trade becomes more frequent, agents are willing to hold more of their silver as coins, so the carrying cost per coin is important, and agents want a larger coin. As the frequency of trade continues to increase, the added flexibility of a smaller coin becomes more valuable, and optimal size decreases.

> Is it known what happens to the ratio of silver in coinage to silver in other uses?

The timing of the industrialization of London and Venice, and the introduction of a larger coin, are consistent with this.

> What happens to the distribution of welfare as trading opportunities become more frequent (urbanization)? Does the distribution become more equal, and is it known whether that is consistent with historical experience?

A few comments

Political economy potential of this research program. Distributional effects of changes in the number and sizes of coins can be explored.

How should we think about a change in the distribution of welfare in an infinitely-lived agent model in which, given enough elapsed time, every agent spends the same duration of time with each possible portfolio?

It would be interesting to study **international flows of the commodity.** In an ex ante sense, would agents favor a balance of payments deficit in order to build up the stock of the commodity? A model with an endogenous total supply of the commodity would allow us to study inflation.

It would be interesting to incorporate **physical capital,** so that the impact of shifts in the value of the alternative use could be studied.

Conclusion

A model capable of studying the distributional and welfare effects of the structure of denominations or coin sizes is central to all of the fundamental questions of commodity money listed earlier.

Such a model must be based on decentralized monetary exchange, which accommodates heterogeneity of transactions and wealth.

In this paper, and in the research program about commodity money that both Anji and Warren have contributed to more generally, Anji and Warren study an important set of questions thoroughly and carefully, in an elegant model that is structured to capture key aspects of the technology and information frictions.

The scholarship on which this paper rests is abundantly evident in the modeling decisions, questions posed, and how the model is interpreted in order to shed light on historical experiences.