

# Idea Flows, Economic Growth, and Trade

Fernando Alvarez    Francisco Buera

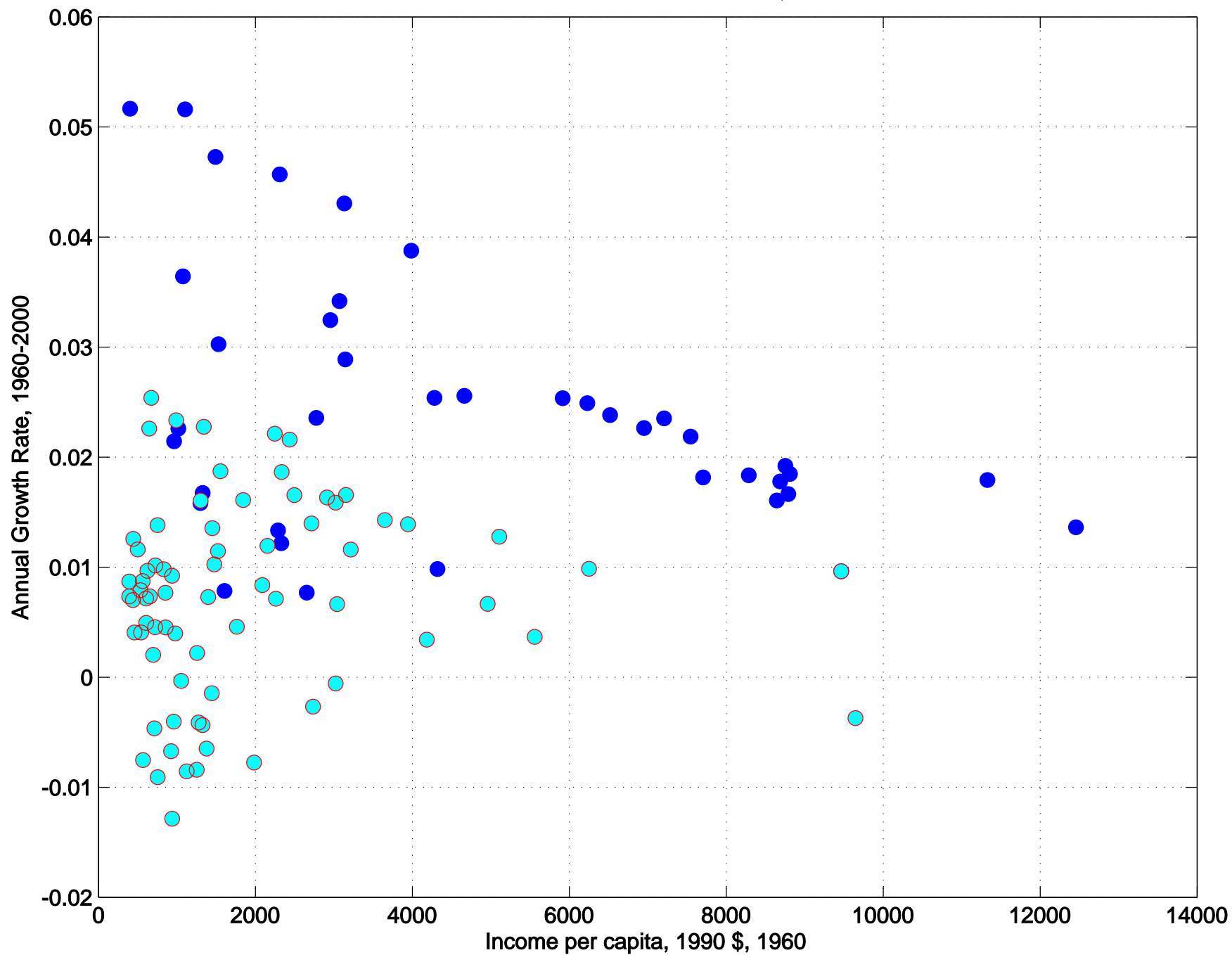
Robert E. Lucas, Jr.

Atlanta Fed Conference Honoring Warren Weber

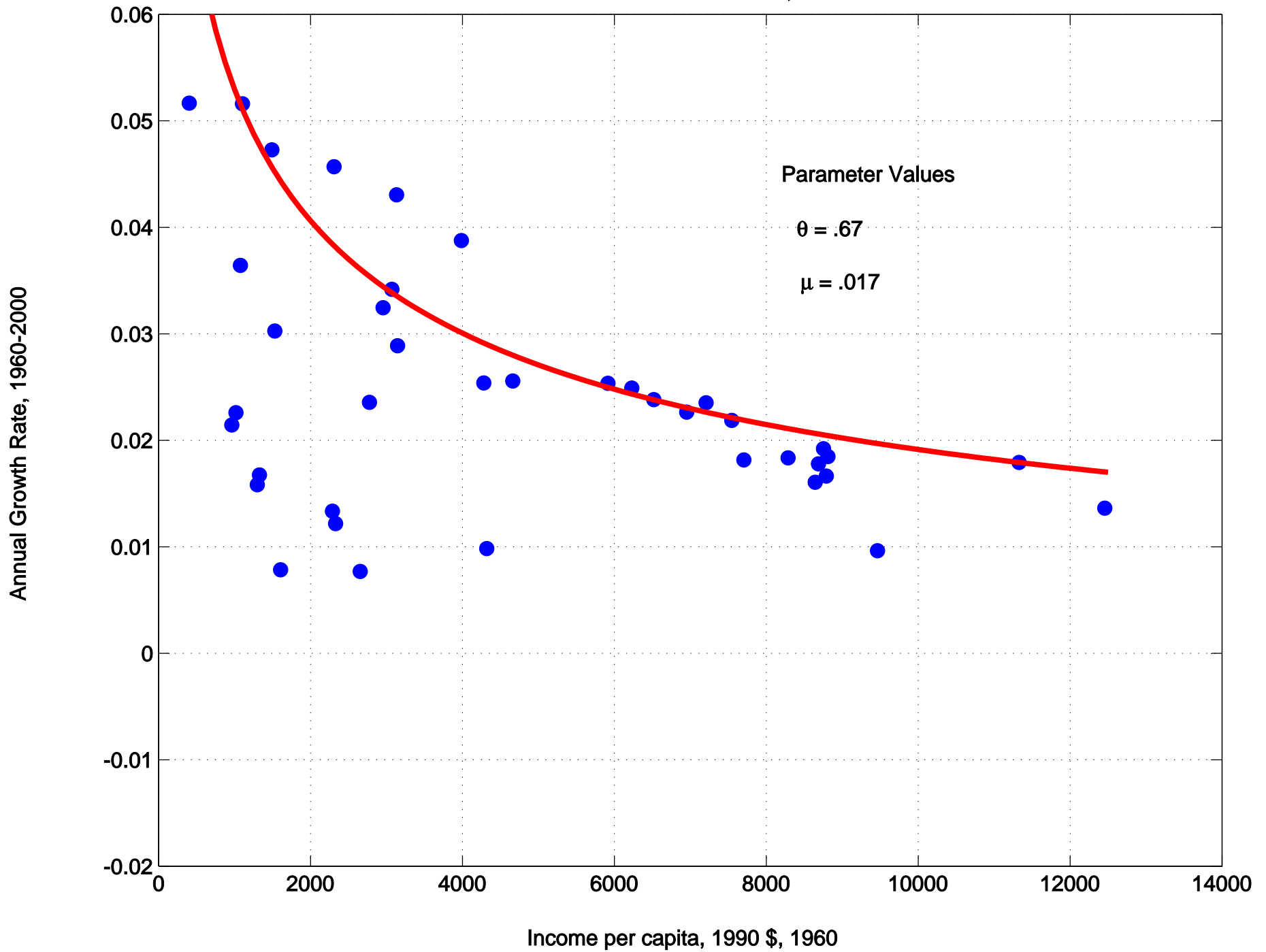
February 17, 2012

- Economists have long believed that international trade serves as a vehicle for the diffusion of technology—of ideas
- Much evidence of important growth effects of openness, trade
- Example is Lucas (2009) replication of Sachs/Warner (1995)
- Next slides, based on Maddison data, 1960 - 2000

INCOME LEVELS AND GROWTH RATES, 112 COUNTRIES



# INCOME LEVELS AND GROWTH RATES, 39 OPEN ECONOMIES



- What we do not have is good understanding of how this trade/diffusion effect works
- What is the process that links trade policy to diffusion, growth?
- What are the key parameters of this process?
- What evidence do we have on their magnitudes?
- Seek a framework that can help make progress on these questions

- We develop an endogenous growth model with many countries that explicitly connects trade and trade policy to sustained growth rates and transition dynamics
- Model is built on work of Eaton, Kortum in two ways:
  - (2002) static theory of technology-based trade, adapted in Alvarez, Lucas (2007)
  - vision of technology diffusion proposed in Kortum (1997), Eaton and Kortum (1999), adapted in Alvarez, Buera, Lucas (2008), Lucas (2009)
- View technology as distribution of productivity-related knowledge held by heterogeneous, individual people, firms, countries

- Construct a model of  $n$  country world, engaged in continuously balanced trade
- Many goods, many people in each: details later
- State variables are  $F_1, F_2, \dots, F_n$  : right cdfs of “cost” in  $\mathbb{R}_+$   
$$F_i(z) = \Pr\{\text{randomly chosen good has cost } \geq z \text{ if produced in } i\}$$
- Densities  $f_i = -\partial F_i(z, t)/\partial z$

(In E/K, A/L  $F_i$ 's are Weibull RVs. Not so here.)

- Constant trade costs matrix  $K = [\kappa_{ij}] : \kappa_{ij} =$  units of goods arriving in  $i$  per unit shipped from  $j$
- Populations  $L = (L_1, \dots, L_n)$
- Given  $K, L$  and technology profile  $F$ , can solve for static trade equilibrium, including wage rates  $w = (w_1, \dots, w_n)$
- Theory also gives us equilibrium distributions of the costs of producers in  $j$  who sell in country  $i$ , all  $i, j$ .
- From these, can calculate right cdfs  $G_1, G_2, \dots, G_n$  where

$$G_i(z, t) = \Pr\{\text{seller active in } i \text{ at } t \text{ has cost } \geq z\}$$



- We will use these to motivate a law of motion for  $F_1, F_2, \dots, F_n$  of the form

$$\frac{\partial \log(F_i(z, t))}{\partial t} = \alpha_i \log(G_i(z, t)) \quad (*)$$

- Trade theory tells us how  $F, K$  determine  $G$ ; gives autonomous system
- Simulatable model of world trade, economic growth
- Law of motion (\*) is main new idea of this paper

# PLAN OF TALK

- 1 Technology Diffusion, Closed Economy
- 2 Technology Diffusion,  $n$ -Country World
- 3 Numerical Illustrations
- 4 Conclusions

# 1 Diffusion in Closed Economy

- Consumers have identical preferences over  $[0, 1]$  continuum of goods

$$C = \left[ \int_0^1 c(s)^{1-1/\eta} ds \right]^{\eta/(\eta-1)}$$

- Good  $s$  is produced with a linear, labor-only technology

$$y(s) = \frac{l(s)}{z(s)}$$

- $l(s)$  is labor input—1 unit per person— and  $z(s)$  is cost (labor requirement)

- Exploit symmetry of utility function
- Re-label goods by their costs  $z$  and write time  $t$  utility as

$$C(t) = \left[ \int_{\mathbb{R}_+} c(z)^{1-1/\eta} f(z, t) dz \right]^{\eta/(\eta-1)}$$

- Here  $f(\cdot, t)$  is the density of costs.
- Use  $F(z, t)$  for the **right** cdf of cost, so density is  $f(z, t) = -\partial F(z, t)/\partial z$

- In competitive equilibrium, price of good  $z$  is  $p(z) = wz$
- Ideal price index for the economy at date  $t$  is

$$p(t) = \left[ \int_{\mathbb{R}_+} p(z)^{1-1/\eta} f(z, t) dz \right]^{\eta/(\eta-1)}$$

- Real per capita GDP  $y(t)$  is real wage  $w/p(t)$  :

$$y(t) = \left[ \int_{\mathbb{R}_+} z^{1-1/\eta} f(z, t) dz \right]^{-\eta/(\eta-1)}$$

- Now need to describe evolution of  $f, F$

- Model technological diffusion as process of search involving *technology managers*
- One manager per good, each identified with current cost  $z$
- Technology management requires no time, earns no private return
- Manager of good  $z$  operates the CRS, zero profit production described earlier
- Or others imitate him and do so: Who cares?

- Each manager  $z$  meets others at given rate  $\alpha$  per unit of time
- Each meeting a random draw from the population  $f, F$  of managers of all goods
- When he meets another with cost  $z' < z$  he adopts  $z'$  for his own good
- Motivate law of motion for  $F$  as

$$\begin{aligned}
 F(z, t + \Delta) &= \Pr\{\text{my cost} > z \text{ at } t + \Delta\} \\
 &= \Pr\{\text{my cost} > z \text{ at } t\} \times \Pr\{\text{no lower draw in } (t, t + \Delta)\} \\
 &= F(z, t)F(z, t)^{\alpha\Delta}.
 \end{aligned}$$

- Continuous draws, not Poisson arrivals

- Let  $\Delta \rightarrow 0$  to obtain:

$$\frac{1}{F(z, t)} \frac{\partial F(z, t)}{\partial t} = \alpha \log(F(z, t)) \quad (\text{DE})$$

- Write out general solution:

$$\log(F(z, t)) = \log(F(z, 0))e^{\alpha t}$$

where  $F(z, 0)$  is any given initial distribution (right cdf)

- Clear that model implies GDP growth of some kind.
- Easy to compute. How to interpret results, characterize possibilities?



- For empirical reasons, interest is in sustained growth of economies that either grow at a constant rate or will do so asymptotically
- Central construct is *balanced growth path* (BGP): right cdf  $\Phi(z)$  (density  $\phi = -\Phi'(z)$ ) and a growth rate  $\nu > 0$  such that  $F(z, t) = \Phi(e^{\nu t} z)$  and

$$\log(\Phi(e^{\nu t} z)) = \log(\Phi(z)) e^{\alpha t}$$

- On BGP

$$f(z, t) = -\frac{\partial F(z, t)}{\partial z} = \phi(e^{\nu t} z) e^{\nu t}$$

- Then real GDP path is

$$\begin{aligned} y(t) &= \left[ \int_{\mathbb{R}_+} z^{1-1/\eta} \phi(e^{vt} z) e^{vt} dz \right]^{-\eta/(\eta-1)} \\ &= e^{vt} \left[ \int_{\mathbb{R}_+} x^{1-1/\eta} \phi(x) dx \right]^{-\eta/(\eta-1)} \end{aligned}$$

- We show that BGP takes Weibull form

$$\Phi(z) = \exp(-\lambda z^\theta)$$

for some pair  $\lambda, \theta > 0$  and  $\nu = \alpha\theta$

- Note that Weibull RV is just exponential RV raised to power  $\theta$

- Also show that if initial distribution  $F(z, 0)$  satisfies

$$\lim_{z \rightarrow 0} \frac{f(z, 0)z}{1 - F(z, 0)} = \frac{1}{\theta}$$

for some  $\theta > 0$  and

$$\lim_{z \rightarrow 0} \frac{\log [F(z^\theta, 0)]}{z} = -\lambda$$

for some  $\lambda > 0$  then

$$\lim_{t \rightarrow \infty} \log [F(e^{-\alpha\theta t} z, t)] = -\lambda z^{1/\theta} \quad \text{for all } z > 0$$

- Parameter  $\theta$  measures mass near  $z = 0$
- Costs are headed to zero so long run behavior determined by “left tail”
- High  $\theta$  value means more low cost ideas waiting to be discovered
- Inverse of cost is productivity so high  $\theta$  means thick “right tail” of productivity distribution, high growth rate
- Power Law

## 2 Diffusion in World Economy

- An  $n$  country world. Populations  $L_1, \dots, L_n$
- Each country in autarky exactly as discussed
- Now open all of them to Eaton-Kortum trade in all goods
- Rename each good by its cost profile  $z = (z_1, \dots, z_n)$

$$C_i(t) = \left[ \int_{\mathbb{R}_+^n} c_i(z)^{1-1/\eta} f(z, t) dz \right]^{\eta/(\eta-1)}$$

where  $f(z, t) = \prod_{i=1}^n f_i(z_i, t)$  is joint density

- Given fixed trade costs  $K$ , technologies  $F = (F_1, \dots, F_n)$ , solve for balanced growth equilibrium wages  $w = (w_1, \dots, w_n)$
- See E/K (2002), A/L (2007), this paper for details
- Turn to dynamics. Technology managers native to  $i$  now draw ideas from all managers, foreign and domestic, whose goods are currently being sold in  $i$
- Searching managers include all managers in  $i$ , good and bad
- They meet managers from all  $j$ , but only those good enough to sell goods in  $i$

- Want to replace autarky law of motion

$$\frac{\partial \log(F_i(z, t))}{\partial t} = \alpha_i \log(F_i(z, t))$$

with

$$\frac{\partial \log(F_i(z, t))}{\partial t} = \alpha_i \log(G_i(z, t)) \quad (*)$$

where

$$G_i(z, t) = \Pr\{\text{seller active in } i \text{ at } t \text{ and has cost } \geq z\}$$

- Theory tells us who these sellers are, given  $K$ ,  $F$ , and  $w$

- Distributions  $G_i$  stochastically dominate  $F_i$
- Statement of familiar static gains from trade
- Also key to dynamic gains:  $G_i$  provides people in  $i$  a better intellectual environment than  $F_i$  does
- Formula is

$$G_i(z, t) = \sum_{j=1}^n \int_z^{\infty} f(z_j, t) \prod_{k \neq j} F_k \left[ \frac{w_j(t) \kappa_{ik}}{w_k(t) \kappa_{ij}} z_j \right] dz_j$$



- Can show that if (i) some  $F_i$  consistent with sustained growth under autarchy, and (ii) trade is possible between any pair, then for all  $i$

$$\lim_{z \rightarrow 0} \frac{z g_i(x)}{1 - G_i(x)} = \frac{1}{\theta} \quad \text{where} \quad \theta = \max_i \theta_i$$

- That is  $G'_i$ 's inherit common, fattest left tail from  $F'_i$ 's
- Can also show that all countries have common BGP growth rate  $\nu = \theta \sum_{i=1}^n \alpha_i$  where  $\theta = \max_i \theta_i$
- Scale economy? Yes. And note that trade costs not in the formula
- Does not take much trade for the **really** good ideas to get around: think of Marco Polo and pasta

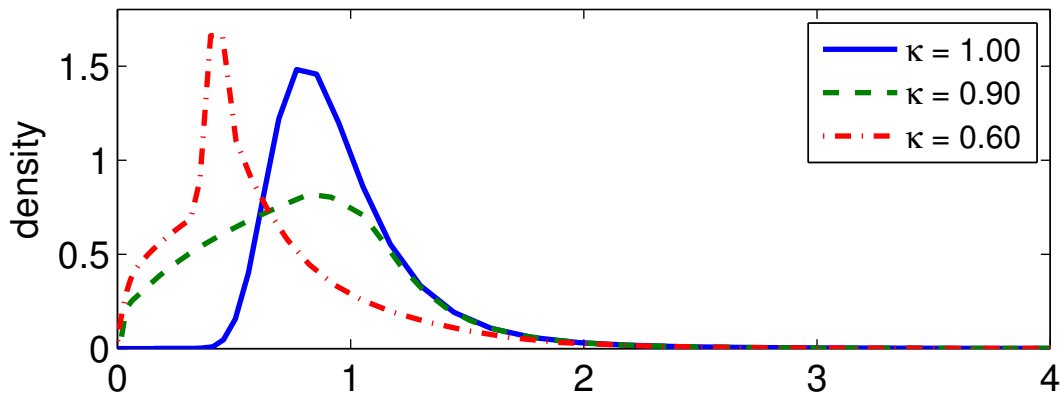
### 3 Numerical Illustrations

- Begin with world of  $n$  identical countries; symmetric trade cost  $\kappa$
- Already know a lot from general theory:
  - Relative wages identical: set  $w = 1$ , all countries
  - common BGP growth rate:  $\nu = \theta \sum_{i=1}^n \alpha_i$ , where  $\theta = \max_i \theta_i$
  - $\nu$  independent of  $\kappa$  value

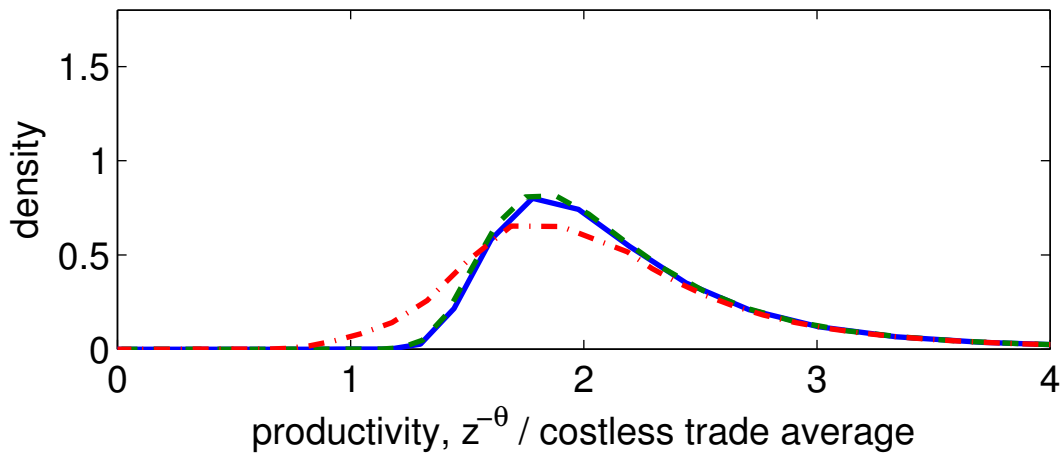
- Like to know more:
  - what do the distributions  $F$  and  $G$  look like on a BGP?
  - how do trade volumes and GDP levels vary with trade costs  $\kappa$  and substitution elasticities  $\eta$ ?
- Consider world of identical economies,  $n = 50$ ,  $\theta = 0.2$ ,  $\alpha = .002$ ,  $\nu = 0.02$

- Next figure describes the distributions  $F$  and  $G$
- Have plotted distributions of productivities,  $1/z$ , rather than costs  $z$
- $x$  -axis on both panels is BGP productivity relative to mean of 1 for  $\kappa = 1$  (costless trade) world
- Top panel shows productivity densities of each country ( $F$ ) at different  $\kappa$  levels. Bottom panel shows densities of sellers' productivities ( $G$ ) at different  $\kappa$  levels
- Both are Frechet distributions in right tail: note common tails on each panel

Potential Producers from a Country



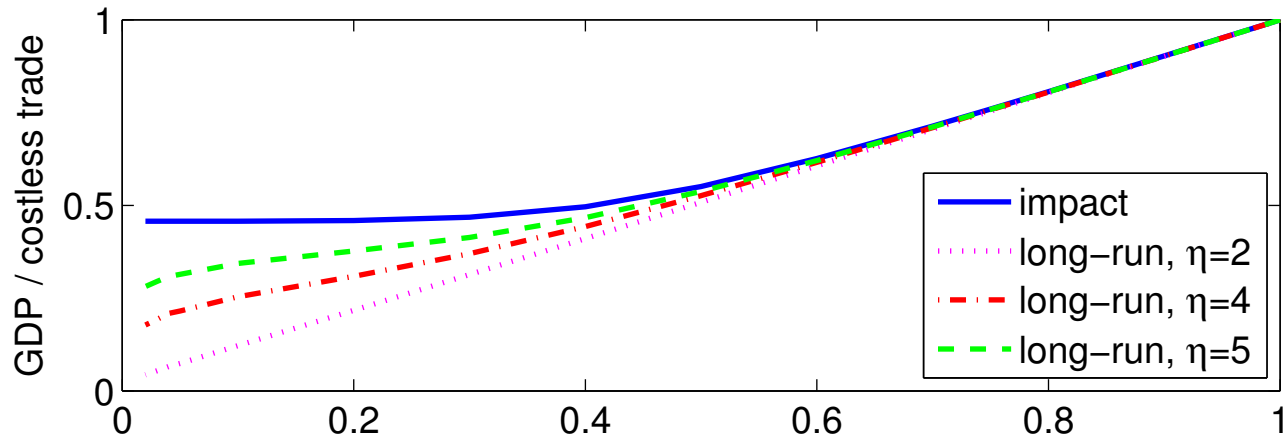
Sellers to a Country



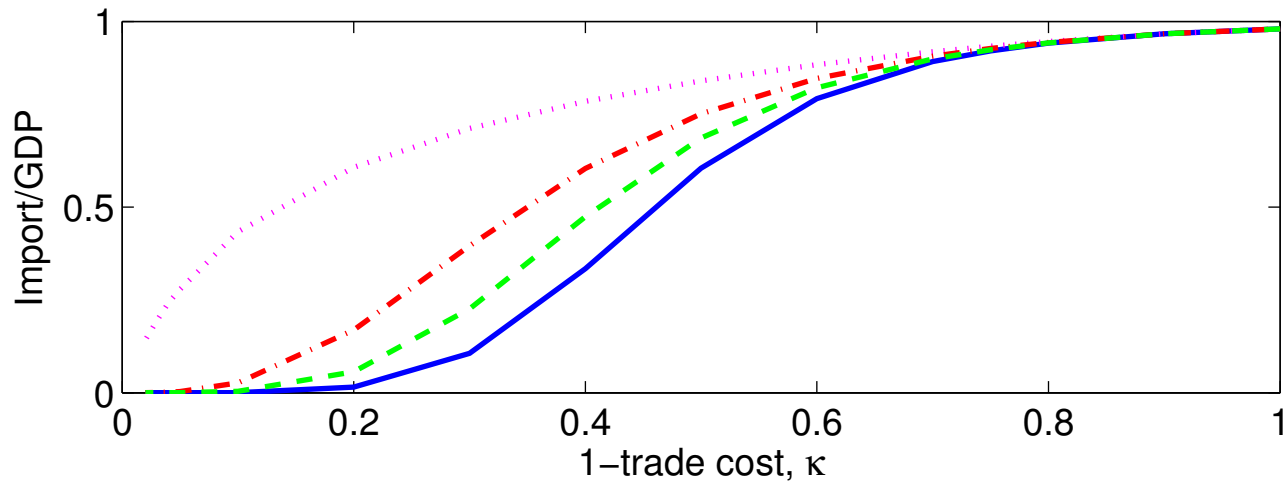
- Next figure also describes the symmetric world economy: the effects of changes in trade costs on real incomes and trade shares
- The x-axis shows trade costs, varying from autarky for all ( $\kappa = 0$ ) to costless trade ( $\kappa = 1$ ).
- Top panel plots per capita gdp levels, relative to gdp with costless trade
- Bottom panel plots trade volumes, relative to the costless trade case

- Solid blue lines show impact, static trade effects
- Other three curves on top panel are real gdp levels along the BGP
- Levels are shown for three values of  $\eta$ : elasticity of substitution
- Three curves on bottom panel are trade share levels along the BGP for three  $\eta$  values

Per-capita Income



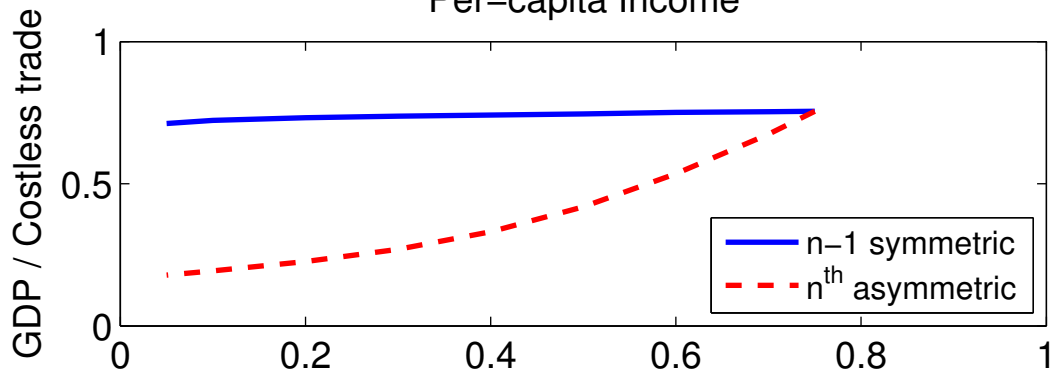
Trade Share



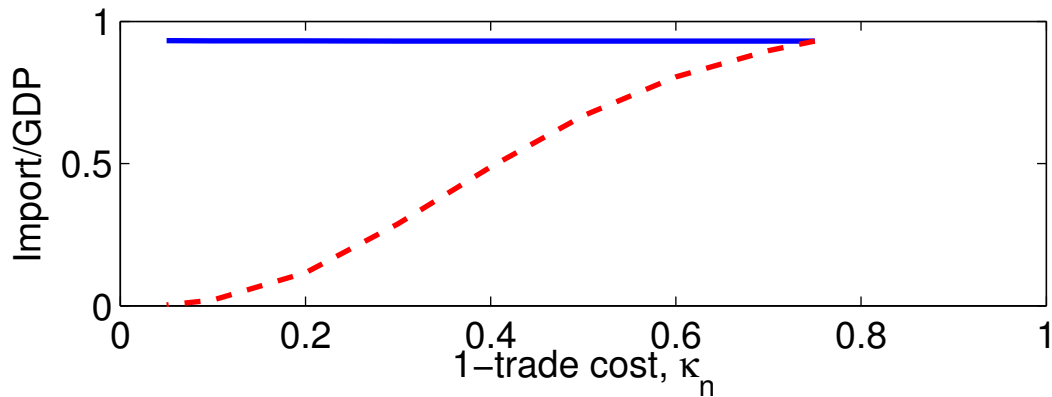


- Next figures describe world with  $n$  countries:
  - $n - 1$  identical (as above), common trade costs  $\kappa = .75$
  - one small, open, larger trade cost  $\kappa_n$  applied to all imports
- First figure shows BGP income levels and trade shares—relative to costless trade benchmark—for different trade cost levels of deviant

Per-capita Income

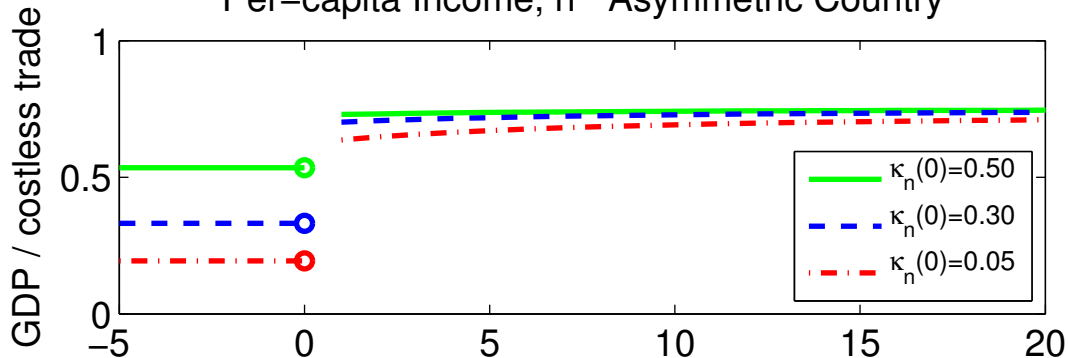


Trade Share

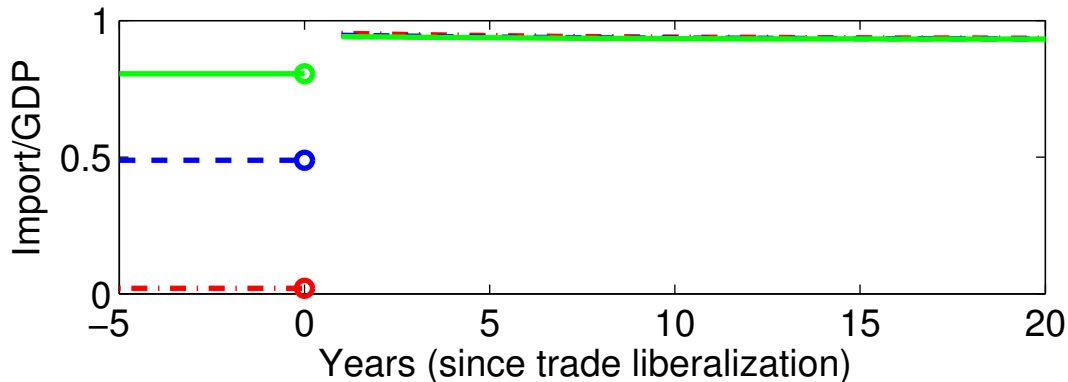


- Last figure shows time series of income and trade shares of the deviant country
- Deviant begins with higher trade costs, poorer economy
- At  $t = 0$ , deviant adopts trade cost  $\kappa = .75$  of other countries
- Immediate jump in income, trade share shown: static trade effect
- Slow convergence to common BGP also shown

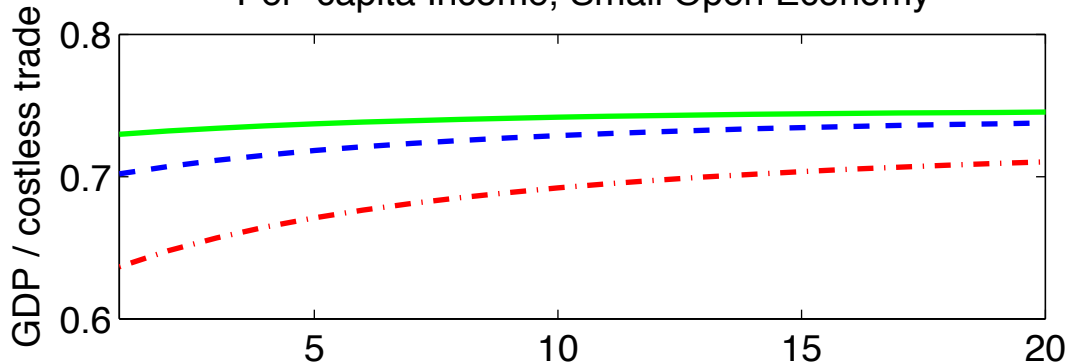
Per-capita Income,  $n^{\text{th}}$  Asymmetric Country



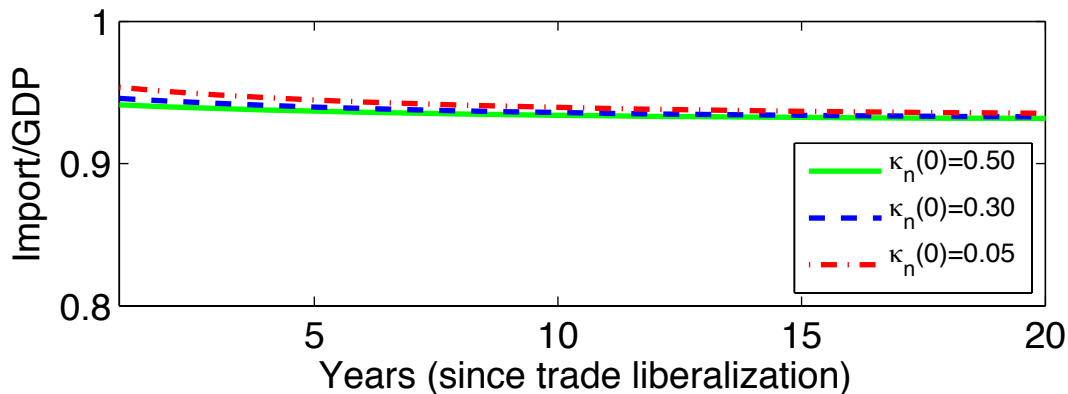
Volume of Trade,  $n^{\text{th}}$  Asymmetric Country



Per-capita Income, Small Open Economy



Volume of Trade, Small Open Economy



### 3 Conclusions

- Basic model general enough to support realistic calibration, policy simulations: see e.g. Alvarez/Lucas (2007)
- Our immediate goal here more modest: to understand the operating characteristics of a new, combined model of trade and growth
- General structure shares features of von Neumann (1937) or Parente/Prescott (1994): long run growth rate common to all; different policies induce different income *levels*
- Model makes operational distinction between static effects of trade policies and dynamic effects via trade related technology diffusion