A Model of Commodity Money with Minting and Melting

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Any study of the money supply [of medieval Europe] needs to take account not only of the total face value of the currency, but also of the metals and **denominations** *of which it is composed.*

(Mayhew 2004)

What were these denominations?

- 800-1200 A.D. most European states issued only one coin type - a penny containing $\sim$ 1.7 gms fine silver

Two major changes to European monetary systems:

- Debasement of the penny - to a varying extent across mints
  - In England - in 1160, still $\sim$ 1.4 gms
  - In Venice - in 1160, $\sim$ 0.10 gms
- Introduction of a larger coin - at different times across mints
  - In Venice - grosso 1194: 2.18 gms (26d)
  - in England - groat 1351: 4.66 gms (4d)

What drove the changes?

- Conventional view - debasement:
  - debasements were revenue generators
  - debasements created more units of money so facilitated more exchange
- Conventional view - larger coins:
  - large coins were needed to pay urban workers
- Our view
  - changes in coin types were consistent with welfare increasing responses to change in the economic environment

What drove the changes?

- we build a random matching model to assess these views
- the paper extends existing search models:
  - to allow for multiple coins
  - to allow for an endogenous quantity of money

Preview of results

We find that:

- the size of a coin affects social welfare
- the size of a coin has distributional consequences
- the frequency of trade affects the optimal coin size
- the stock of monetary metal affects optimal coin size
- permitting minting of two types of coin *may* raise social welfare

Preview of results

We use these results to reconsider the motives for coinage changes:

- debasement may have been a response to urbanization rather than (only) generating revenue or making 'more' units of the medium of exchange
- large coins may have been a response to silver discoveries rather than a response to urbanization

Outline of talk

- Model
- Results
- Apply model to historical choices of denomination
- Conclude/further research

Environment

- Time discrete and infinite
- One nonstorable, perfectly divisible consumption good
- One storable metal (silver) in **fixed supply** (m)

Environment

- Silver can be held as coins or jewelry (bullion)
- Silver coins are indivisible, but can be minted or melted
- Silver coin contains b_1 ounces of silver
  - possible second silver coin contains $b_2 = \eta b_1$ ounces of silver

Environment

- Agents hold

  s_1 small silver coins
  s_2 large silver coins
  j units silver jewelry

$\Rrightarrow$ Only coins can be used in trade

$\Rrightarrow$ Only jewelry yields utility (similar to Velde-Weber)

Agents

- $[0, 1]$ continuum, infinitely-lived
- Preferences:

$$u(c) - q + \mu(b_1 j) - \gamma(s_1 + s_2)$$

$$u(0) = 0, u' > 0, u'(0) = \infty, u'' < 0$$

γ utility cost of holding a coin

- Maximize expected discounted (β) lifetime utility
- θ prob of a being a buyer or seller in a single coincidence match

Trade

- Each period has two subperiods
  1. First subperiod: decentralized trade in bilateral matches
     - Preference assumption rules out double coincidence matches
     - past trading histories private (no monitoring or commitment technology) - rules out gift-giving equilibrium
     - agents are anonymous - rules out credit
  2. Second subperiod: agents can alter coin/jewelry portfolio by minting or melting
     - Can change how metal stocks held – no change in quantity of silver

Choices

1st sub period

- Single coincidence matches: potential consumer makes TIOLI offer (q, p_1, p_2)
- Buyer 'sees' seller's portfolio

2nd sub period

- Agents make portfolio adjustment after trade (z_1, z_2)
- z_i is the amount of coins minted (melted if negative)

Model: Value functions

- Expected value of holding $y_t = (s_{1t}, s_{2t}, j_t)$ beginning second subperiod

$$
\begin{aligned}
v_t(y_t) \quad = \quad & \max_{z_{1t}, z_{2t}} \{\beta w_{t+1}(s_{1t} + z_{1t}, s_{2t} + z_{2t}, j_t^s - z_{1t} - \eta z_{2t}) \\
& -S(z_{1t}, z_{2t}; j_t)\}
\end{aligned}
$$

$S(z_{1t}, z_{2t}; j_t)$ is seigniorage

Model: Value functions

- Expected value of holding y_t beginning of first subperiod

$$
\begin{aligned}
w_t(y_t) &= \theta \sum_{\tilde{y}_t} \pi_t(\tilde{y}_t) \max_{\Lambda} [u(q_t) + v_t(s_{1t} - p_{1t}, s_{2t} - p_{2t}, j_t)] \\
&\quad + (1-\theta) v_t(y_t) + \mu(b_1 j_t) - \gamma(s_{1t} + s_{2t})
\end{aligned}
$$

where:

- $\Lambda =$ set of all feasible TIOLI offers
- $\pi_t(y_t) =$ fraction of agents with y_t beginning first subperiod
- $\tilde{y}$ denotes seller portfolios

Model: Equilibrium

- Steady state symmetric equilibrium:

  Value functions w, v; asset holdings π; and quantities p_1, p_2, z_1, z_2, q that satisfy

  1. Bellman equations
  2. asset transitions
  3. market clearing

Results

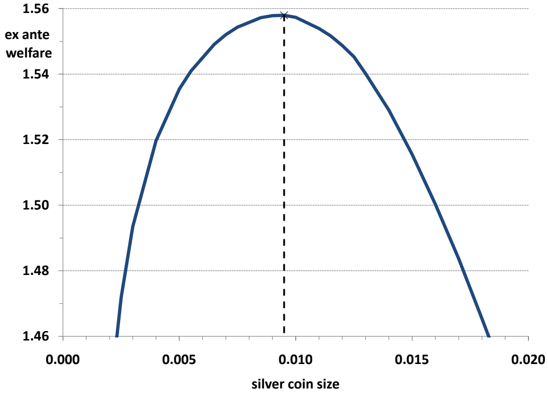
- Numerical – analytic results not possible
- Assume:

$$\beta = 0.9$$
$$\sigma = 0.04$$
$$\gamma = 0.001$$
$$u(q) = q^{1/4}$$
$$\mu(b_1 j) = 0.05(b_1 j)^{1/2}$$

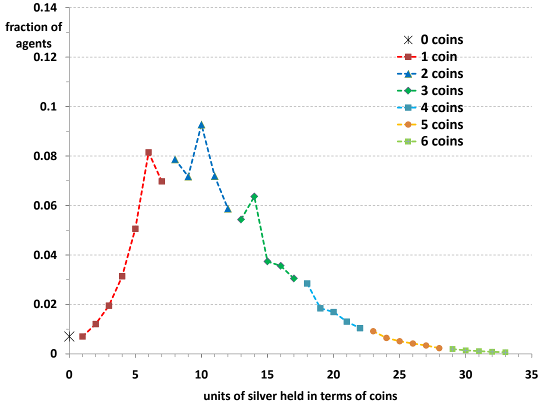
- Base case:

$$\theta = \frac{1}{3}$$
$$m = 0.1$$

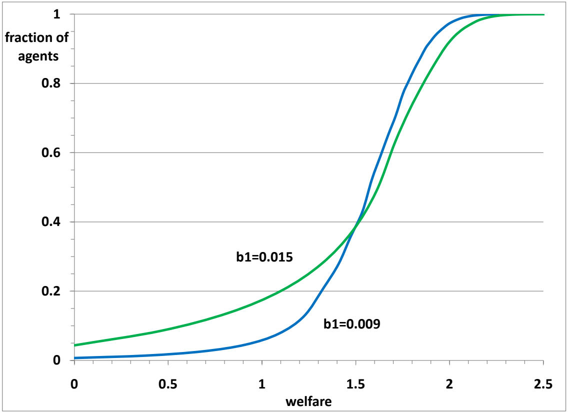
Single coin: Welfare effect of changing coin size



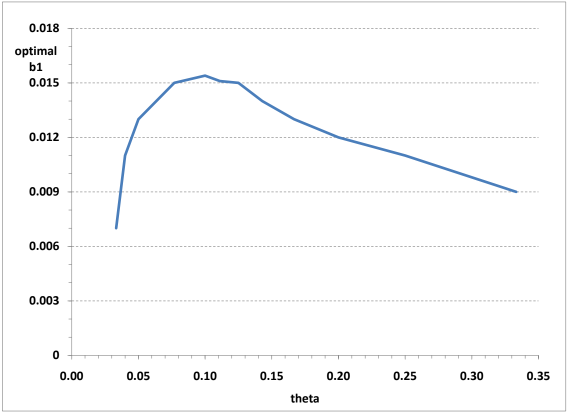
Single coin: Distribution of coin and jewelry holdings



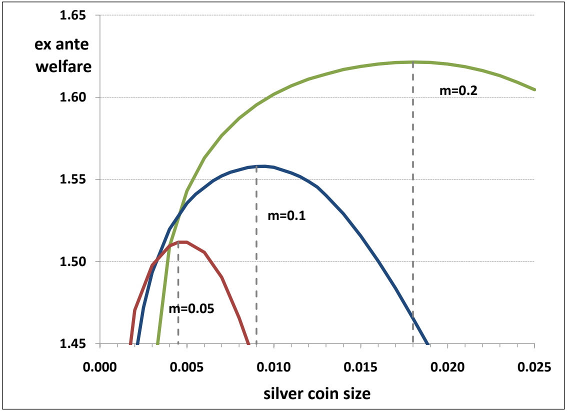
Distribution of welfare



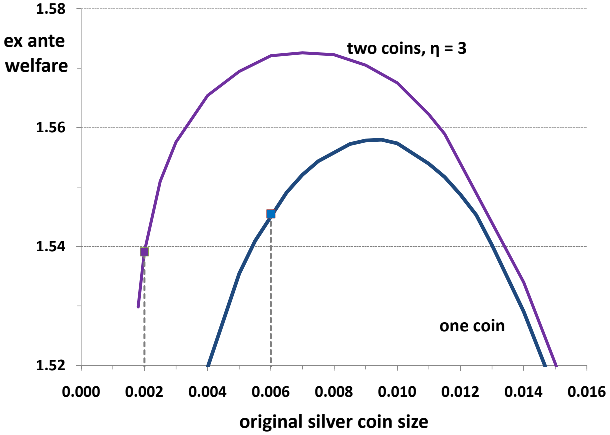
Optimal coin size depends on trading frequency



Optimal coin size depends on quantity of metal



Adding a second coin type *may* increase welfare



Shift to smaller coins

- Pennies in 800 A.D. were $\sim$ 1.7 gms of fine silver
- By 1160
  - In England still $\sim$ 1.4 gms
  - In Venice $\sim$ 0.10 gms

Motives for smaller coins

- The model suggests that optimal coin size depends on trading frequency
- Venice urbanized earlier and much more than England
  - Venice urbanized from 1000 AD
  - English market towns grew especially after 1250
- This difference in debasement policy is consistent with a social welfare maximizing response to urbanization

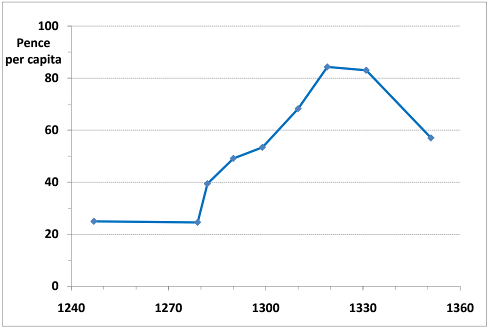
Introduction of grossi and groats

- In 1194 Venice introduced large silver coins
  - grossi weighing 2.18 gms of 96.5% fine silver
  - contained the same fine silver as about 26 denari
- Not until 1351 did the English produce large silver coins
  - groats weighing 4.66 gms of 92.5% fine silver
  - contained the same fine silver as 4 pence.

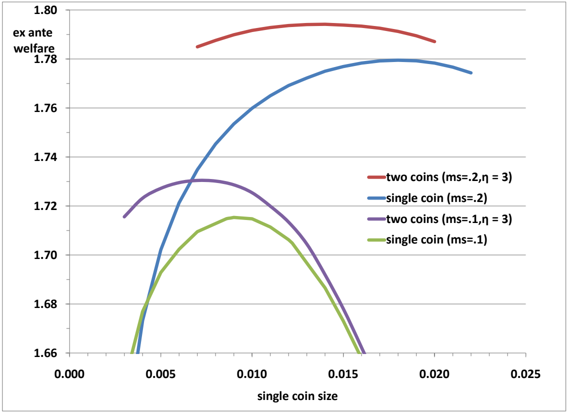
Silver flows

- The model suggests that larger stocks of silver imply larger optimal coin size
- The late 12th century saw large increases in silver
    - 1160-1320 known for the large amounts of silver mined
    - Flows (from Saxony) went first to Venice
    - in England inflows came later
- The introduction of grossi and groats may have been motivated by the larger silver stocks

Money stock in England



Two coins with varying metal stocks



Conclusion

- Coin size/type affects welfare in the economy
- Debasement of the penny is consistent with a monetary policy that valued social welfare
- Silver inflows in the 13th century give a rationale for increasing coin sizes

Next direction - outstanding issues

- Why debase rather than introduce a second (smaller) coin?
- Build a model where agents benefit from a large gold coin