

# Optimal inflation in a model of inside money

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## Optimal inflation

Inflation produced by lump-sum transfers can be optimal in models of outside money

- see Levine 1990, Kehoe et al 1992, Green and Zhou 2005, Molico 2006, Deviatov 2006
- the transfers improve extensive margins in a way that more than offsets their harmful effect on intensive margins

This paper: inflation can be optimal in a model of inside money—essentially, in Cavalcanti and Wallace 1999

## Inflation in Cavalcanti and Wallace (1999)

- Inflation occurs if more inside money is issued at each date than is redeemed (retired)
- Even with individual money holdings in  $\{0, 1\}$ , inflation allows the post-trade distribution of money holdings to differ from the pre-trade distribution
- Using a representative-agent notion of welfare, we show in some numerical examples that some inflation is optimal

## Interpretation of *inside money*

trade-credit instruments

- issued by monitored people when they buy from nonmonitored people
- used by nonmonitored people in trade among themselves
- redeemed (accepted) by *all* monitored people when they sell to non-monitored people

## Interpretation of *inflation* with $\{0, 1\}$ money holdings

With divisible money, a standard normalization:

- holds the stock of money fixed
- represents inflation by a proportional tax on money holdings

Our approach: a probabilistic version of such a tax:

- a person who ends a period with a unit of money loses it with some probability

## The background environment (Trejos-Wright 1995)

- discrete time
- unit measure of infinitely-lived people who maximize expected discounted utility
- period utility is  $u(\cdot) - c(\cdot)$ , where  $y^* = \arg \max[u(y) - c(y)] > 0$
- production is perishable
- pairwise meetings at random
  - prob of being producer or consumer =  $1/K$ , where  $K \geq 2$
  - prob of no meeting =  $1 - (2/K)$

## Monitoring (Cavalcanti-Wallace 1999)

Initial and permanent split of people into two groups

- fraction  $\alpha$  are  $m$  people: perfectly monitored
- fraction  $1 - \alpha$  are  $n$  people: anonymous (not monitored at all) and can hide money
- $\alpha$  is exogenous (society's monitoring capacity)

## Money (only durable object)

- inside money
  - issuer-specific and perfectly recognizable (no counterfeiting)
  - issued only by  $m$  people and the planner
- individual money holdings in  $\{0, 1\}$



## Comments on the model

- We see money-transactions and credit-transactions
- To get both, need some monitoring, but not perfect monitoring
- The above model is an extreme way to get both:
  - model analogue of money-transaction: production by  $n$  person
  - model analogue of credit-transaction: production by  $m$  person
- In the above model, allowing inside money raises welfare

## Symmetric and steady-state allocations

- Allocation: initial distributions of money holdings, trades, transfers
- Steady states: everything is constant
- Symmetry:
  - all people in the same situation take the same action (could be a lottery)
  - all monies issued by  $m$  people who have not defected and any money issued by the planner are treated as perfect substitutes

## Weakly implementable allocations and the optimum problem

Implementable allocations: immune to

- individual defection and cooperative defection of any pair in a meeting
- Only punishment: an  $m$  agent  $\rightarrow$   $n$  agent (and cannot issue money)

Optimum problem: choose an implementable, symmetric, steady-state allocation to maximize ex ante expected utility before assignment of

- monitored status
- initial money holdings

## Features of our examples

- If  $\alpha = 1$ , then first-best is implementable; i.e., impose

$$\frac{u(y^*)}{c(y^*)} \geq 1 + K(1 - \beta)/\beta. \quad (1)$$

- If  $\alpha = 0$ , then paying interest on money would be good; i.e, impose

$$\frac{u(y^*)}{c(y^*)} < 1 + \frac{K(1 - \beta)/\beta}{1 - \theta}, \quad (2)$$

when  $\theta$  (fraction with money) = 1/2.

- For given  $u$ ,  $c$ , and  $K$ , let  $\beta^*$  be such that (1) holds at equality and let  $\beta^{**}$  be such that (2) holds at equality. We set  $\beta \approx (\beta^* + \beta^{**})/2$ .

## Examples

$$u(y) = 1 - e^{-10y}, c(y) = y, K = 3$$

Implies

$$y^* = \ln(10)/10 \approx .23$$

and

$$\beta^* \approx 0.51 \text{ and } \beta^{**} \approx 0.67 \text{ and, hence, } \beta = .59$$

$$\alpha \in \{1/4, 1/2, 3/4\}$$

Table 1. Aggregates			
	$\alpha = 1/4$	$\alpha = 1/2$	$\alpha = 3/4$
ex ante welfare*	.233	.326	.431
pre-meeting welfare, $m$	.380	.432	.488
pre-meeting welfare, $n$ without money	.100	.113	.133
pre-meeting welfare, $n$ with money	.358	.401	.458
pre-meeting fraction of $n$ with money**	.299	.371	.398
inflation rate	.082	.104	.114

\* Welfare is relative to first-best welfare:  $[u(y^*) - c(y^*)]/[K(1 - \beta)]$ .

\*\*  $m$  people hold no money.

Table 2. Output in meetings ( $y/y^*$ )			
(producer)(consumer)	$\alpha = 1/4$	$\alpha = 1/2$	$\alpha = 3/4$
$(n0)(n1)$	.606	.663	.739
$(n0)(m)$	.606	.663	.739
$(m)(n0)$	.296	.141	.107
$(m)(n1)$	.717	.818	.911
$(m)(m)$	.717	.818	.911

- Money trades: consumer surrenders one unit in rows 1, 2, and 4
- inflow to  $n$  (row 2)  $>$  outflow from  $n$  (row 4)  $\Rightarrow$  inflation
- Binding producer IR constraint in rows 1, 2, 4, 5.
- $n$  consumer faces a lower average price than an  $n$  producer

## Robustness of the optimality of inflation

- almost certainly generic
- could also hold with a larger set of individual money holdings:  $\{0, 1, 2, \dots, B\}$
- will not hold with a degenerate distribution of money holdings

Why did it take so long to get this result????