Rational blinders: is it possible to regulate banks using their internal risk models?[*]

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Abstract

The regulation of banks uses their internal risk estimates to compute finer capital requirements. The underlying risk models have been blamed for their optimism, which I link to a hidden information problem between a regulator and a bank better informed about risk models. I first show that low incentives to use cautious models can seriously cripple current reforms: a regulatory tightening to compensate for "model risk" (switching to Basel III/increasing a floor on capital ratios) leads to a contraction of credit, which favors a wider adoption of optimistic models, increases risk and can make counter-cyclical buffers pro-cyclical. Second, giving proper incentives is difficult: as model uncertainty relates to tail risk, over-optimism is typically revealed only when it is too late to impose a penalty. The framework allows to derive predictions on the use of internal models and to compare different regulatory solutions. More broadly, this paper shows how incentives impact the development of applied economic models.

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1   Introduction

Many examples during the recent crisis revealed that many risk models used by financial institutions were unable to take into account extreme risks. Dowd *et al.* (2008) illustrate the extent to which some older models used in practice were flawed: "25-sigmas events" happening several times in a row in August 2007 were supposed to occur once in every 10^{135} years! Surprisingly, new generations of more robust risk models were already available *before* that time. It is crucial for regulatory purposes to understand why agents can choose internal models too optimistic regarding extreme risks, and how this can be avoided.

The regulation of banks relies heavily on internal models to compute risk-sensitive capital requirements[1]. Danielsson (2008), Rochet (2010) or Eichengreen (2011) argue that this gives banks incentives to use optimistic models to increase leverage. A recent study by Barclays Equity Research shows that investors share this concern[2]. More than half of the investors surveyed do not trust risk weightings, 80% think the way the banks' risk models work is a significant driver of major differences between European banks' risk weightings, and even more think model discretion should be removed (Samuels, Harrison, and Rajkotia (2012)).

This paper offers a tractable analytical framework to reflect on these issues. I consider financial intermediaries (or banks) with limited liability, competing both to attract investors and to lend to final borrowers. Banks have some freedom to choose an internal model, report a risk estimate to the regulator, who chooses a capital constraint depending on the report. The capital constraint thus depends on the model chosen.

In the first part of the paper I analyze the regulation in its current and near future form, where banks face few penalties for reporting over-optimistic models for some types of risk. The regulatory response to fears of over-optimism has been to tighten capital requirements and set aside provisions for "model risk". But model risk arises when wrong models are chosen because of unbiased mistakes, not bad incentives. I show to what extents things can go wrong if "hidden model" is mistaken for model risk: a regulatory tightening, e.g.

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[1] *"Because the most accurate information regarding risks is likely to reside within a bank's own internal risk measurement and management systems, supervisors should utilize this information to the extent possible."* (FED Task Force on Internal Credit Risk Models (1998)).

[2] For other examples see "Investors lose faith in risk measures" by B. Masters ""Whale" makes a big splash on risk models" by P. Triana, both in the *Financial Times*, 24.05.12 and 28.05.12.

switching to Basel III, can *increase* the risk that a bank defaults (Proposition 2). When the regulation tightens, the supply of bank loans decreases and the interest rate on loans goes up. Due to this macroeconomic effect, using optimistic models to bypass the regulation becomes more profitable, and the wider adoption of over-optimistic models can lead to an increase in the average risk of banks. In the U.S., the rules recently proposed by the regulatory agencies[3] would make a bank's capital ratio the lower of an internally measured ratio and a standardized measure, which thus acts as a floor. Increasing this floor has the same unwanted effects. As a corollary, counter-cyclical capital ratios can turn out to be pro-cyclical. The analysis also delivers new testable predictions about the choice of risk models.

Regulators are currently looking for ways to restore investors' confidence in risk weightings, but they cannot substitute a "naive" regulatory tightening for a solution to a hidden information problem. In the second part of the paper, I study a backtesting mechanism (as the one used for market risk models) where penalties punish banks using optimistic models when unlikely levels of losses occur. I give sufficient conditions for implementing the first-best outcome (Proposition 3), and study a difficulty specific to these internal models: they typically differ in their predictions about extreme levels of losses, thus revelation of information relies on penalties after high losses. But if the regulation is very sensitive to the model reported, a bank with a very optimistic model is allowed a leverage so high that it may already be in default when it should be punished. I give conditions under which this prevents the regulator from implementing the first-best (Proposition 4), in which case a trade-off appears between the cost and the risk-sensitiveness of the regulation, so that an incentive-compatible regulation may make little use of the information conveyed by internal models.

The framework developed in this paper is very flexible and allows for a number of extensions, which I make available in a separate Internet Appendix[4]. I discuss in particular the possibilities to rely more on the ex ante auditing of internal models or on a benchmarking mechanism, which are both possible avenues for future regulatory reforms.

Background. Let me illustrate the issue with the following example. To compute the capital requirement k to be held for a corporate/sovereign or bank exposure, a bank under

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[3]See http://www.federalreserve.gov/newsevents/press/bcreg/20120607a.htm.
[4]Available at http://sites.google.com/site/jecolliardengl/RB-Appendix.pdf.

the Basel framework can opt for the "advanced-internal ratings based approach", in which case k is computed as follows:

$$k = \left(LGD \times \mathcal{N} \left(\frac{\mathcal{N}^{-1}(PD)}{\sqrt{1-R}} + \sqrt{\frac{R}{1-R}} \times \mathcal{N}^{-1}(0.999) \right) - PD \times LGD \right) \frac{1 + (M - 2.5)b}{1 - 1.5b}$$

(1)

where PD is the probability of default, LGD the loss given default, R a correlation coefficient, M the effective maturity, b a maturity adjustment coefficient, and $\mathcal{N}$ the cdf of a Gaussian distribution. The *risk-weighted asset* will be obtained by multiplying k by $12.5 \times EAD$, where EAD is the exposure at default. With the regulator's approval, the bank can use its own internal model to compute PD, LGD and EAD (PD also enters the formulas defining b and R). These parameters can be computed by the bank in several ways, but a large bank would typically use a Jarrow-Turnbull model. Tarashev (2008) performs an interesting exercise by comparing the regulatory capital obtained with different academic models. On a sample of BBB-rated bonds for instance, a bank choosing the most pessimistic model would have to keep 2.8% regulatory capital, compared to 2.3% with the most optimistic one, i.e. 18% less (Table 5 in Tarashev (2008))! This difference is only due to the different structures of these models; there is even more scope for gaming in their implementation.

Such differences are likely to give rise to a strategic selection of models. The first part of the paper gives some empirical implications about this selection (Implications 1 and 2). An exogenous increase of demand for banks' loans should cause more banks to use optimistic models, as proxied by the proportion of assets for which banks choose to adopt the "internal ratings based approach" of Basel instead of the "standardized" approach. Increasing costs (e.g. through regulation) in a "shadow" banking sector competing with the regulated sector has the same effect. Since the implementation of Basel III leads to heavier capital charges, internal models may also be used to mitigate the additional costs of complying with the regulation. An implication of the second part is that good candidate models to bypass the regulation are those which are proven to be over-optimistic only when the situation is so bad that the regulator cannot be too harsh on banks, by fear of deepening an economic downturn.

The analysis fits the regulation of credit risk well, but section 4 in particular could also apply to market risk or operational risk. Credit risk models are extremely difficult to backtest

due to their time horizon (typically one-year) and the scarcity of available data. The formula in equation 1 is supposed to lead in the end to enough capital to cover losses with probability 0.999. If the regulator only looked at "violations" (as for market risk), reporting a risk ten times too low would go unnoticed for a hundred years on average. Advanced tests can be used of course, such as the one proposed by Lopez and Saidenberg (2000), but the problem of insufficient data cannot be completely bypassed, see also Kupiec (2002). Thus the regulator can hardly backtest credit risk models, while checking their methodology ex-ante is also a difficult task (see Jackson and Perraudin (2000)).

Finally, the hidden information problem I consider here also turns up in other forms of "regulation". A rating agency estimating the creditworthiness of a firm or the risk of a pool of loans similarly has to rely partly on internal models chosen by an agent. Inside a firm, the models a team uses to manage risk may also determine the compensation its members receive[5]. With slight amendments, the model I develop here can be used to analyze this larger class of problems.

Related literature.   There is a huge literature on the regulation of banks and on the Basel framework in particular. Kim and Santomero (1988) and Rochet (1992) show that risk-based capital requirements are necessary to control banks' risks without inducing inefficient asset allocations. The use of VaR for market risk and internal ratings for credit risk has been seen as a way to implement such capital requirements. Dangl and Lehar (2004) for instance have analyzed the risk-taking of a bank under VaR based regulation. Several papers like Danielsson, Shin, and Zigrand (2004), Heid (2007) or Kashyap and Stein (2004) have criticized the pro-cyclical equilibrium effects of using risk-based capital requirements. The first part of this paper also focuses on equilibrium effects but without the assumption that risk-based requirements stem from a correct representation of risk. This part is linked to models of competition between leveraged banks, for instance Herring and Vankudre (1987), Matutes and Vives (2000), Bolt and Tieman (2004). Closely related are also recent papers studying the choice between Basel's "standardized" and the "internal ratings based" approaches:

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[5]A recent example of improper risk valuation driven by employees' incentives is presented in "Deutsche Bank: Show of strength or a fiction?", by T. Braithwaite, M. Mackenzie and K. Scannell, *Financial Times*, 05.12.12.

Ruthenberg and Landskroner (2008) and Antao and Lacerda (2011) (empirical), and Hakenes and Schnabel (2011) (theory).

Carey and Hrycay (2001) estimate the extent to which portfolio managers can use different methodologies to game the regulation, and conclude on the necessity to monitor the use of credit risk models. Jacobson, Linde, and Roszbach (2006) show on a sample of Swedish banks that banks using different methodologies leads to different estimates of economic capital. They also suggest that "*given the fact that many supervisors will have an informational disadvantage [...], internal models are likely to become instrumental in banks' search for lower regulatory capital buffers*". Feess and Hege (2011) are to my knowledge the only authors to study this problem theoretically, but they focus on the choice between using internal models or not, not on the choice of one model rather than another.

Several papers have considered the possibility of biased models for the regulation of *market risk*. An interesting difference with credit risk models is that market risk is evaluated on a *daily* basis, typically by the value at risk at the 99% level, so that after 100 days it is already possible to detect blatant over-optimism. Incentives not to use optimistic market risk models are provided in the Basel framework, as studied theoretically by Lucas (2001) and Cuoco and Liu (2006). Empirical studies by Berkowitz and O'Brien (2002), Perignon, Deng, and Wang (2008) and Perignon and Smith (2010) show that VaRs reported for market risk were actually *too conservative*, implying that the penalty for under-reporting the VaR is probably too high. If banks respond to incentives to choose pessimistic market risk models, they probably also respond to incentives to choose optimistic credit risk models.

The second part focusing on how the regulator could elicit the revelation of the true model echoes Chan, Greenbaum, and Thakor (1992), Freixas and Rochet (1998) and Giammarino, Lewis, and Sappington (1993) on fairly priced deposit insurance. In these papers, information is revealed by using risk-based insurance premia and capital ratios, a possibility I do not consider here as insurance premia are not risk-based in most countries, or in too crude a way to realistically assume that they could give correct incentives. I discuss the links with these papers further in section 4.2. Another difference is that in my paper the hidden information is about "models", which typically give similar predictions except for high levels of losses, which adds an interesting difficulty to the design of the regulation.

This paper contributes to yet another strand of the literature, concerned with "markets for models/theories". Banks in this paper are on the demand side of such a market. Examples include Hong, Stein, and Yu (2007), who study agents relying on partial models and shifting from one to the other depending on their observations, and Cogley, Colacito, and Sargent (2007) who study rational learning of macroeconomic models with a feedback of learning on economic variables. Few papers look at situations where the demand for models is not directly derived from their predictive power only. Exceptions include Millo and MacKenzie (2009) who study the usefulness of simple risk management models for internal communication, and Ghosh and Masson (1994), who suggest that governments could pretend to believe in false economic models so as to gain bargaining power against other countries.

The remainder of the paper is organized as follows: section 2 develops the general framework; section 3 derives empirical implications from a stylized representation of the current regulation; section 4 asks whether the regulator could use backtesting to reveal the true model and discusses policy implications; section 5 presents other applications of the framework.

2 Framework

Agents and assets. To study how market prices depend on the models chosen by intermediaries and how these prices determine the incentives to choose a given model, I introduce three types of agents:

-*Borrowers* need to finance risky projects. Their demand for loans $D(r_L)$ is a function of the gross interest rate r_L. D is decreasing, $D(1) = +\infty$ and $\lim_{+\infty} D(r_L) = 0$. $r_L(L)$ is the inverse demand function. A random proportion t of borrowers will default, according to a distribution defined below. A defaulting loan yields 0 (failure of the borrower's project)[6].

-*Investors* can invest their large initial wealth W in a safe asset yielding the exogenous riskless rate r_0 with certainty or lend to intermediaries at a rate r_D, but not directly to borrowers.

-*Intermediaries* can lend to borrowers, invest in the safe asset, and borrow M from investors at r_D. They initially own K (equity) and are protected by limited liability. Finally, I assume

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[6]Default is assumed to be independent of the interest rate and the amount lent. Relaxing this assumption makes the analysis more cumbersome (adverse selection) without altering the main results.

that a debt contract between an investor and an intermediary cannot be made contingent on the intermediary's subsequent choice of leverage or assets.

There is a continuum $[0, 1]$ of each type of agents, all risk-neutral and price-takers on a perfect competitive market. Finally, a benevolent *regulator* can set limits to intermediaries' leverage and aims at maximizing social welfare. Throughout the paper a female pronoun refers to the regulator, and a male pronoun to an intermediary. Fig. 1 sums up the market structure.[7]

[Insert Fig. 1 here.]

Model uncertainty. The proportion t of defaulting borrowers follows a distribution $F(t, \sigma)$ with support over $[0, 1]$, interpreted as the correct risk model in this economy. Two assumptions are needed to study the strategic choice of risk models: the plausibility of different models and asymmetric information between the bank and the regulator:

M1: Let $\{F(., \sigma), \sigma \in [\sigma_{min}, \sigma_{max}]\}$ be a family of cumulatives over $[0, 1]$, parameterized by σ, twice-continuously differentiable in both arguments. Denote $f(., \sigma)$ the corresponding pdfs. The family of distributions has the monotone likelihood ratio property:

$$\forall t_0, t_1, \sigma_0, \sigma_1 \text{ with } t_1 \geq t_0, \ \sigma_1 \geq \sigma_0, \ \frac{f(t_1, \sigma_1)}{f(t_1, \sigma_0)} \geq \frac{f(t_0, \sigma_1)}{f(t_0, \sigma_0)}$$

M2: A given σ is randomly selected in $[\sigma_{min}, \sigma_{max}]$ according to some pdf. $\Psi(.)$, density $\psi(.)$. Intermediaries observe σ before they take any decision, but σ remains hidden to the regulator. t, the proportion of defaulting loans, is drawn from $F(., \sigma)$.

M1 means there exists a rich set of different plausible models indexed by σ, which can be interpreted as one model with different parameterizations, or models from different families. Moreover, models with a low σ give risk estimates unambiguously more optimistic than models with a high σ. Finally, **M2** means that intermediaries know the true model σ while the regulator does not, thus an extreme form of asymmetric information.

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[7]All figures are in the Appendix A.6, the notations used are summed up in A.1.

2.1 The intermediary's program

Taking the model chosen and prices as given, I first derive the demand for funds and the supply of credit by an intermediary. Take r_0, r_D, r_L as given with $r_L \geq r_D \geq r_0$. It never pays off for an intermediary to borrow $M > 0$ and invest at r_0 since investors ask for $r_D \geq r_0$. Thus we have either $L = M + K$ with M possibly zero, or $L = M = 0$ (intermediaries invest their equity in the safe asset)[8].

Due to limited liability, an intermediary's realized profit if he lends L and a proportion t of borrowers do not repay can be written as $\max\left(0, r_L(1-t)L - r_D M\right)$. The intermediary cannot repay his debt if there have been too many defaults in his portfolio, that is if:

$$t > 1 - \frac{r_D}{r_L}\left(1 - \frac{K}{L}\right) = \theta(L) \tag{2}$$

$\theta(L)$ is the *maximum proportion of sustainable losses*, that an intermediary can bear without defaulting. It will be easier in many proofs to work with θ instead of L. Inverting equation 2, L is determined by θ as:

$$L(\theta) = \frac{r_D K}{r_D - r_L(1 - \theta)} \tag{3}$$

Thus, denoting $\pi(\theta, \sigma)$ the intermediary's expected profit if he chooses $L > 0$, we have

$$\pi(\theta, \sigma) = \int_0^\theta (r_L(1-t)L(\theta) - r_D(L(\theta) - K))f(t, \sigma)dt \tag{4}$$

$$= r_L L(\theta) \times s(\theta, \sigma) \tag{5}$$

$$\text{with } s(\theta, \sigma) = F(\theta, \sigma)\mathbb{E}_\sigma(\theta - t | t \leq \theta)$$

Profit is thus the product of two terms. $r_L L(\theta)$ are the revenues if all borrowers repay their debt. The second term is the probability that the bank survives, times expected repayments above the level enabling the bank to survive. $s(\theta, \sigma)$ is the proportion of expected "surplus" repayments, with the convention that it is 0 if the intermediary defaults. Finally the operator $\mathbb{E}_\sigma$ denotes an expectation according to the distribution $f(., \sigma)$.

Notice that the bank is supposed to compute expected profit according to the true model

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[8]It is possible to have non-zero reserves by assuming random deposit withdrawals. This would not affect the main results, except that for some parameters the regulation would not be binding.

σ, not to the model reported to the regulator. Several interpretations of this assumption are possible, the most simple being that the bank takes into account that the model he uses is biased. Section 5.2 develops formally another argument: since the bank defaults for losses above θ, forecasting mistakes made about such losses are privately irrelevant. Model uncertainty for credit risk is typically about high levels of default, whether the bank uses a correct or biased model may thus not change the computation of its profit.

The intermediary will either invest all equity in the riskless asset and not borrow, or maximize $\pi(\theta(L), \sigma)$ in L, taking prices as given. As is detailed in the next subsection, he also faces a regulatory constraint on the ratio K/L, which has to be larger than some α, and the intermediary's program can be written as:

$$\max_L \pi(\theta(L), \sigma) \text{ s.t. } L \leq K/\alpha \tag{6}$$

An increase in L expands the scale of operation, bringing more profit for a given proportion of expected surplus repayments. But this proportion itself is increasing in θ and thus decreasing in L: less leverage means a lower probability of default and less debt to repay. Profit is decreasing and then increasing in L as on Fig. 2. Thus only three choices make sense: (i) investing K in loans without borrowing ($L = K$), (ii) investing K in the safe asset without borrowing ($L = 0$), (iii) borrowing until the constraint binds and investing everything in loans ($K/L = \alpha$).

Lemma 1. *Let (M^*, L^*) be the profit-maximizing choice of the intermediary. There exists $\underline{r_L}$ such that:*
-If $r_L \geq \underline{r_L}$ then $L^* = K/\alpha, M^* = K(1-\alpha)/\alpha$.
-If $r_L < \underline{r_L}$ then $M^* = 0$. $L^* = K$ if $r_L \geq r_0/\mathbb{E}_\sigma(1-t)$, and $L^* = 0$ otherwise.

The value of $\underline{r_L}$ and the proof are in Appendix A.2. The intermediary uses the maximum leverage allowed by the regulation if r_L is high enough to compensate for the high risk of defaulting, and otherwise doesn't borrow but invests in loans or in the safe asset.

2.2 Regulation under complete information

I first analyze the optimal capital ratio the regulator can set if she also knows σ. It will be convenient to define the rate r_L at which investors would break even if they could lend directly to borrowers:

$$r_L(L) = r_L^e = \frac{r_0}{\mathbb{E}_\sigma(1-t)} \tag{7}$$

When intermediation is necessary and for a given level of capital, reaching a supply of loans of $D(r_L^e)$ may require a high leverage, and thus the possibility that an intermediary defaults. I take as given that the regulation is motivated by limiting losses to a public deposit insurance fund. For simplicity I consider only complete deposit insurance with a fixed premium[9]. Then $r_D = r_0$ since loans to intermediaries are riskless, and investors' welfare is constant. The regulator sets capital requirements, taking into account the surplus of borrowers, the profit of intermediaries, and the cost of repaying losses to depositors. I assume a deadweight cost $c > 0$ from taxation.

Under complete information, the regulator can use "model-sensitive" capital constraints $K/L \geq \alpha(\sigma)$. It is convenient to translate this constraint on the capital ratio into a constraint on maximum sustainable losses:

$$K/L \geq \alpha(\sigma) \Leftrightarrow \theta \geq 1 - (r_D/r_L)(1 - \alpha(\sigma)) = \theta(\sigma)$$

The constraint faced by the intermediary is just $\theta \geq \theta(\sigma)$, the regulator asks the intermediary to have enough capital to bear at least $\theta(\sigma)$ losses in his portfolio. As shown in the previous subsection, this constraint will be binding for a high enough interest rate r_L. We thus have the following objective function for the regulator, to maximize in θ for a given σ:

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\begin{aligned}
V(\theta, \sigma) &= r_0 W + \int_0^\theta (r_L L(1-t) - r_0(L-K)) f(t,\sigma) dt + \mathbb{E}_\sigma(1-t)\left(\int_0^L r_L(u)du - r_L(L)L\right) \\
&\quad - (1+c)\int_\theta^1 (r_0(L-K) - (1-t)r_L L) f(t,\sigma) dt \\
&= \underbrace{r_0(W + K - L)}_{\text{Safe asset}} + \underbrace{\mathbb{E}_\sigma(1-t)\int_0^L r_L(u)du}_{\text{Surplus from loans}} - \underbrace{c\int_\theta^1 (r_0(L-K) - (1-t)r_L L) f(t,\sigma) dt}_{\text{Deadweight costs}} \tag{8}
\end{aligned}
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[9]If investors are not fully insured, know the true risk and can fully monitor the bank, then in equilibrium $r_L = r_L^e$ but intermediaries can still choose over-optimistic models, see section 5.3. Notice that in the few countries where the premium is risk sensitive, it actually gives further incentives to get optimistic risk weights.

Both r_L and L depend on the θ chosen by the regulator. To keep things simple, I assume demand to be so elastic that the effect of θ on r_L is negligible. Otherwise increasing θ could lead to an increase in r_L, and the optimal θ could be increasing in σ only by parts.

Lemma 2 (First-best). *For a given level of capital K[10]:*

-If $D(r_L^e) \geq K$ the first-best is to set $\theta^* = 1$ ($L = K$) and let intermediaries invest in loans up to the point where $r_L = r_L^e$.

-If $D(r_L^e) < K$, c is high enough and demand $D(.)$ is elastic enough, then $V(\theta, \sigma)$ is concave and the optimal regulatory threshold $\theta^*(\sigma)$ is increasing and satisfies:

$$r_L(\sigma) = \frac{r_0}{\mathbb{E}_\sigma(1-t)} + \frac{c}{\mathbb{E}_\sigma(1-t)}(1 - F(\theta^*(\sigma), \sigma)) \underbrace{(r_0 - r_L(\sigma)\mathbb{E}_\sigma(1-t|t > \theta^*(\sigma)))}_{\geq 0} \quad (9)$$

The case for a model-based regulation here is straightforward: when the true model is more pessimistic (higher σ), there is less surplus to gain by expanding credit and more risks of default for a given level of θ, hence the regulator wants to restrict leverage more. Note that with the first-best regulation the regulator implements a capital requirement that involves under-investment ($r_L > r_L^e$) to decrease the costs to taxpayers.

2.3 Numerical example

Consider the following example, to be kept for illustration throughout the paper. The proportion of defaults follows a Beta distribution with parameters $a = 3.5$, $b = 31.5$ but many values are possible for b, uniformly distributed from $b = 13$ up to $b = 50$, which is the most optimistic model. Take $\sigma = 1/b$ so that **M1** holds.

In practice, regulators aim at capping the probability that each intermediary defaults, typically 0.1% in the Basel framework. Assume this is an approximate solution to equation 9. Since an intermediary defaults with probability $1 - F(\theta, \sigma)$ when the true model is σ,

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[10]The result depends on the assumption that K is fixed in the short-run, otherwise it is optimal for the regulator to impose 0 leverage. As shown in the Internet Appendix **B.1**, the results are qualitatively unchanged if there is some informational cost of levying capital (Myers and Majluf (1984)). This is consistent with the spirit of the Basel framework, although recent papers like Admati *et al.* (2011) argue that a more ambitious regulatory reform should aim at decreasing the cost of capital, thus allowing for higher capital ratios.

$\theta^*(\sigma)$ has to satisfy $F(\theta^*(\sigma), \sigma) = 1 - p$ to ensure a default probability lower than p. For easier visualization I assume a $p = 0.05$ probability to default, unlike in Basel. Plotting the CDFs we can easily see $\theta^*(1/31.5)$ and $\theta^*(1/50)$ on Fig. 2 (left).

Fig. 2 (right) plots the profit $\pi(\theta(L), 1/31.5)$ as a function of L, with $r_L = 1.1, r_D = 1, K = 1$ and when defaults follow the "true" Beta distribution. Investing K in loans is less profitable than investing in the safe asset in this example. However, investing in loans with a high enough leverage is even more profitable, as the bank exploits the government's guarantee on its debt. If the regulator knows the true model and imposes $L \leq L(\theta^*(1/31.5))$, the bank prefers not to use any leverage. But if she falsely believes that $\sigma = 1/50$ and imposes $L \leq L(\theta^*(1/50))$, the bank chooses maximum leverage. This illustrates the adverse selection problem: in this extreme example, if the intermediary is successful at convincing the regulator that $\sigma = 1/50$ he will increase his expected profit by 10% and default with a 25% probability, five times higher than the regulator's objective.

[Insert Fig. 2 here.]

3 Model choice and market equilibrium

3.1 Equilibrium

I now study how banks choose their risk models in equilibrium, in a stylized representation of the current regulatory structure.

-**T=0** the regulator specifies a rule linking any model σ to a capital ratio $\alpha(\sigma)$, and the requirements that an internal model has to satisfy. These requirements define a set of models accepted by the regulator. For simplicity assume all models with a positive probability to be the true model are accepted, so that this set is the interval $[\sigma_{min}, \sigma_{max}]$.

-**T=1** σ is drawn from the distribution $\Psi(.)$. Each intermediary observes σ, remains unleveraged or reports a model $\sigma' \in [\sigma_{min}, \sigma_{max}]$ at a vanishingly small cost $\zeta > 0$. [11].

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[11]The assumption that different intermediaries know the same σ is not key to the main results but allows to easily define a competitive equilibrium, see the Internet Appendix **B.3**. ζ corresponds to the cost of developing an internal model, and implies that such a model will be used only if it is a source of profit above the riskless rate.

-**T=2** An intermediary who has reported a model σ' chooses a supply of loans and a demand for deposits maximizing profit, such that $K/L \geq \alpha(\sigma')$ and $L \leq M + K$, taking prices as given. r_L, r_D, M and L are simultaneously determined by competitive equilibrium conditions.

-**T=3** a proportion t of borrowers default, where t is drawn from the distribution $F(., \sigma)$.

The capital ratio $\alpha(\sigma)$ links a bank's model to capital requirements, exactly as in equation 1: a model σ determines LGD and PD, which in turn determine R and capital requirements. $\alpha(.)$ can incorporate additional measures of the regulator, such as floors, extra safety margins and so on. The model can also accommodate a number of measures taken to ensure models are not too biased: comparison with "industry standards", required assumptions of the model, reasonable performance of the model on historical data... all enter the definition of the interval $[\sigma_{min}, \sigma_{max}]$. But then a bank is free to choose among all models that can get the regulator's approval. As a consequence, the situation is equivalent to a "delegation game" (Holmstrom (1977) and Alonso and Matouschek (2008)) in which banks are offered a set of attainable leverage ratios from which they can choose[12]. This assumption fits the letter of the Basel Accords, except the requirement that the bank has used the model for internal purposes for several years before it can be used for regulatory purposes (as shown in section 5.2, this is unlikely to make a big difference).

Intermediaries' choice at $T=1$ and $T=2$. Solving the model backwards, I first define formally the equilibrium of the subgame starting at $T=1$ when a given σ is realized:

Definition 1 (Equilibrium with choice of a risk model)**.** *For an increasing $\alpha(.)$ and a given realization of σ, an equilibrium is a 5-uple $(r_L, r_D, \mu_l, \mu_r, \mu_s)$ and a function $h : (\sigma_{min}, \sigma_{max}] \to [0,1]$ where a proportion $h(\sigma')$ of intermediaries choose σ', μ_l choose σ_{min} and $K/L = \alpha(\sigma_{min})$, μ_r choose $K = L$, μ_s choose $L = 0$ and invest K in the safe asset, and:*

-Each intermediary's choice given his model, r_L and r_D is a solution to the intermediary's program of Lemma 1, the supply of loans by intermediaries is equal to $D(r_L)$ and funds borrowed by intermediaries equal funds supplied by investors at an interest rate r_D.

-Investors are indifferent between lending to intermediaries and buying the safe asset.

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[12]A difference with these papers being that here the agents are interacting with each other.

-No intermediary wants to choose a different σ' or change his investment strategy.

This definition simply enlarges the concept of competitive equilibrium by requiring that no intermediary wants to choose a different model. Since investors are fully insured, it must be the case by condition 2 that $r_D = r_0$. This equality implies that if $r_L \geq r_L^e$ it always pays to borrow at least a little, whereas if $r_L < r_L^e$ an unleveraged intermediary prefers the safe asset to loans, so in both cases $\mu_r = 0$. We know from Lemma 1 that, depending on r_L, either an intermediary uses no leverage at all or his capital constraint is binding. Then for any $\sigma \in (\sigma_{min}, \sigma_{max}]$ we have $h(\sigma) = 0$ and only two strategies may be used: not borrowing and investing in the safe asset (proportion μ_s of intermediaries), or choosing the most optimistic model and using maximum leverage (proportion μ_l). Thus only the minimum of the function $\alpha(.)$, $\alpha(\sigma_{min})$, will matter. I denote $\bar{\alpha}$ this minimum.

Proposition 1. *For given $\bar{\alpha}$ and σ, starting at $T = 1$ there exists a unique equilibrium, in which $\mu_l(\bar{\alpha}, \sigma)$ intermediaries choose the most optimistic model.*
μ_l increases if the demand function shifts from D to $D' \geq D$. μ_l decreases in σ.

Since the equilibrium is unique, I denote $r_L(\bar{\alpha}, \sigma)$ the equilibrium interest rate on loans, and $p_d(\bar{\alpha}, \sigma)$ the expected proportion of defaulting intermediaries in equilibrium, where:

$$p_d(\bar{\alpha}, \sigma) = \mu_l(\bar{\alpha}, \sigma) \left(1 - F \left(1 - \frac{r_0(1 - \bar{\alpha})}{r_L(\bar{\alpha}, \sigma)}, \sigma \right) \right) \tag{10}$$

Corollary 1. *When $\mu_l(\bar{\alpha}, \sigma) < 1$, p_d increases if the demand function shifts from D to $D' \geq D$.*

See Appendix A.4 for the proof. This proposition illustrates the role of demand in giving incentives to choose a model: when (i) all intermediaries use an optimistic model, they are able to use a high leverage and the supply of loans is high, which lowers the interest rate on loans. This is an equilibrium if and only if the interest rate is not so low as to make it more profitable not to borrow, that is if demand is high enough. Conversely, if (ii) few intermediaries use leverage, the supply of loans is low and the interest rate high. To have an equilibrium the interest rate must be low so that it doesn't pay to use even the most optimistic model, thus demand has to be low. An increase in demand then leads to a wider adoption of optimistic models and a higher risk in the banking sector.

The regulator's choice at $T = 0$. The regulator anticipates that intermediaries will choose either $L = 0$ or σ_{min}. Banks always choose the same internal model, if they use one, thus the regulator may just as well choose the function $\alpha(.)$ constant and equal to $\bar{\alpha}$. The equilibrium levels of r_L and μ_l depend both on $\bar{\alpha}$ and σ and the regulator's objective is:

$$\max_{\bar{\alpha}} \int_{\sigma_{min}}^{\sigma_{max}} (\gamma(\bar{\alpha}, \sigma) - c\mu_l(\bar{\alpha}, \sigma)\delta(\bar{\alpha}, \sigma))\psi(\sigma)d\sigma \tag{11}$$

$$\gamma(\bar{\alpha}, \sigma) = r_0(W + K - (K/\bar{\alpha})) + \mathbb{E}_\sigma(1 - t)\int_0^{K/\bar{\alpha}} r_L(u)du$$

$$\delta(\bar{\alpha}, \sigma) = \int_{1 - \frac{r_0(1-\bar{\alpha})}{r_L(\bar{\alpha}, \sigma)}}^1 (r_0((K/\bar{\alpha}) - K) - (1 - t)r_L(\bar{\alpha}, \sigma)K/\bar{\alpha})f(t, \sigma)dt$$

The regulator has to select a single $\bar{\alpha}$ to solve the trade-off between the expected surplus $\gamma(\bar{\alpha}, \sigma)$ and the expected deadweight losses from taxation $c\mu_l(\bar{\alpha}, \sigma)\delta(\bar{\alpha}, \sigma)$, in expectation over all realizations of σ. A necessary condition for an interior solution is:

$$E_\Psi\left(\mathbb{E}_\sigma(1 - t)r_L(\bar{\alpha}, \sigma)\right) = r_0 - \frac{\bar{\alpha}^2}{K}c \times E_\Psi\left(\frac{d}{d\bar{\alpha}}(\mu_l(\bar{\alpha}, \sigma)\delta(\bar{\alpha}, \sigma))\right)$$

This condition is similar to equation 9: the expected interest rate on loans implemented by the regulator is distorted from the average (over σ) break-even interest rate due to the cost of public funds. But now the regulator needs to take into account that the proportion of intermediaries who will adopt optimistic models is affected by $\bar{\alpha}$ and σ. The latter effect is in the right direction: when σ increases and risk is more severe, we know from Proposition 1 that μ_l decreases so that less intermediaries are at risk. The flexibility given to intermediaries thus has a positive effect. However it also limits the extent to which the regulator can control risk:

Proposition 2 (Counterproductive tightening). *For a low enough elasticity of the demand for loans, tightening capital requirements increases* $\mu_l(\bar{\alpha}, \sigma)$. *If* $\mu_l(\bar{\alpha}, \sigma)$ *is low enough,* $p_d(\bar{\alpha}, \sigma)$ *increases.*

Proof: the equilibrium is defined by μ_l and r_L satisfying the following two equations:

$$\mu_l K = D(r_L)\bar{\alpha} \tag{12}$$

$$r_0\bar{\alpha} = r_L s\left(1 - \frac{r_0}{r_L}(1 - \bar{\alpha}), \sigma\right) \tag{13}$$

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Equation 13 defines r_L as the interest rate such that an intermediary is indifferent between choosing $L = K/\bar{\alpha}$ and $L = 0$. When $\bar{\alpha}$ increases there are two effects: r_L increases so that $D(r_L)$ decreases, and for a given r_L the product $D(r_L)\bar{\alpha}$ increases. If demand is rigid enough the first effect is negligible, so $D(r_L)\bar{\alpha}$ increases in equation 12 and μ_l has to increase. $p_d(\bar{\alpha}, \sigma)$ is the product of μ_l and the probability that an intermediary with maximum leverage fails (equation 10). The effect of increasing $\bar{\alpha}$ on the second term is negative: each intermediary has a lower leverage and the interest rate on loans increases. But when μ_l is small enough this negative effect is smaller than the positive impact of $\bar{\alpha}$ through the increase of μ_l. ∎

Intuitively, there are three effects when the regulator increases $\bar{\alpha}$. First, an intermediary already using the most optimistic model has less leverage than before, which decreases losses to taxpayers. When the true model is quite pessimistic and few intermediaries use the optimistic model, this effect is small. Second, choosing the most optimistic model is less profitable because it allows less leverage. Third, since intermediaries have a tighter capital constraint, the supply of loans decreases and the interest rate r_L goes up. This increase makes it *more* profitable to use the most optimistic model. When demand is rigid enough, the third effect is stronger than the second, so an increase in $\bar{\alpha}$ leads more intermediaries to adopt the most optimistic model, which in turn increases risk. Thus a naive tightening of the regulation can counter-intuitively increase risk, precisely in those states of the world where the true model is quite pessimistic and risk is already high, as on Fig. 3.

3.2 Empirical and policy implications: market and regulation

Even when investors are fully insured, the market still gives a counterweight to incentives to use optimistic models: when more banks adopt optimistic models and use a high leverage, the interest rate on loans goes down and increasing leverage is less profitable. Some banks then choose not to adopt over-optimistic models and to remain unleveraged, which can be thought of as banks sticking to more traditional activities, for which model uncertainty and risk are low. A wide adoption of over-optimistic models is possible only if there is a high demand for loans due to, say, a boom in a given sector, a housing bubble or a long period of accommodating monetary policy.

Proposition 2 shows that market and regulation are partial substitutes in limiting the use of over-optimistic models. A tighter regulation restricts leverage and loan supply, increases the interest rate on loans, and thus incentives to use optimistic models. Tightening the regulation can thus be counterproductive. In particular, the strong increase of capital requirements with the transition to Basel III gives incentives to develop optimistic models so as to minimize the impact of increased capital requirements[13]. This is not just a "Peltzman effect", the propensity of agents to behave less cautiously when they feel safer: the effect here comes from the substitutability between market incentives and regulatory incentives.

Examples. Before turning to empirical implications, I illustrate the main results using the same example as in section 2 and a demand for loans equal to $D(r_L) = \frac{\eta}{(r_L-1)}, \eta = 1$. On Fig. 3 I plot for different choices of $\bar{\alpha}$ by the regulator the expectation over σ of the welfare, the volume of loans, the proportion of defaulting intermediaries, and the number of intermediaries with optimistic models. Tightening the regulation leads more intermediaries to adopt the most optimistic model in this example and, as a result, for low levels of $\bar{\alpha}$ the default probability increases when regulation tightens.

The optimal $\bar{\alpha}$ for the regulator can be identified on the figure and is close to 8.2%. Assume now that the regulator selects this value of $\bar{\alpha}$; Fig. 4 shows the same variables, but for the different realizations of σ. As expected from Proposition 1, when the true risk parameter is higher less intermediaries try to bypass the regulation.

Finally, for the same optimal $\bar{\alpha}$ and the median value of σ, I plot the same variables on Fig. 5, letting η vary between 0.05 and 5. As expected from Proposition 1, an increase in demand leads more intermediaries to adopt an optimistic model, and risk rises accordingly. When demand becomes so high that all intermediaries already use maximum leverage, an increase in demand only leads to a higher interest rate and thus less risk.

[Insert Fig. 3, 4 and 5 here.]

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[13]According to several commentators, this is exactly how many banks have reacted. See for instance "Banks turn to financial alchemy in search for capital" by T. Braithwaite, *Financial Times*, 24.10.11, or "Fears rise over banks' capital tinkering" by B. Masters, P. Jenkins and M. Johnson, *Financial Times*, 13.11.2011.

Empirical predictions. The theoretical framework gives new predictions about the use of internal risk models by regulated financial institutions. Getting data about what models are used in different institutions is challenging, but an available proxy is the percentage of their assets for which the regulatory risk weights are computed using an internal model. Under the null assumption that the development and adoption of new models is unaffected by economic incentives, this percentage should not be correlated with changes in regulatory or market conditions. This paper on the contrary predicts the following:

Empirical implication 1. *A more intensive use of internal models to compute risk weights should be caused by:*

-1. A positive shock on the demand for loans (μ_l is increasing in η).

-2. A negative shock on the riskiness of borrowers (μ_l is decreasing in σ).

-3. A regulatory tightening, if the demand for loans is not too elastic (Proposition 2).

Points 1 and 2 gives a cross-country implication: the imposition of the same regulatory floor on risk weights in different countries should lead to different choices of models, in a way that flattens the average default probability across countries.

A concern with the regulation of banks is that a regulatory tightening will be neutralized by transfers of assets from the regulated banks to the shadow banking sector, or other un-regulated entities. This can be introduced parsimoniously in the model by assuming there is a supply of loans $S(r_L, \sigma, c)$ by the shadow banking sector, where c is some measure of lending costs in this sector, so that $\partial S / \partial r_L \geq 0$, $\partial S / \partial \sigma \leq 0$ and $\partial S / \partial c \leq 0$. If the r_L solving $D(r_L) = S(r_L, \sigma, c)$ is such that $r_0 \bar{\alpha} < r_L s (1 - (r_0 / r_L \bar{\alpha})(1 - \bar{\alpha}), \sigma)$ then in equilibrium the regulated sector is active, and section 3.1 can easily be adapted with a demand "net of supply by the shadow banks" $\bar{D}(r_L, \sigma, c) = D(r_L) - S(r_L, \sigma, c)$ instead of $D(r_L)$. Of particular interest here are equilibria with $0 < \mu_l < 1$. r_L is still determined by equation 13, is independent of c and increasing in $\bar{\alpha}$. Then μ_L is determined by:

$$\mu_l \frac{K}{\bar{\alpha}} = D(r_L) - S(r_L, \sigma, c) \tag{14}$$

The derivatives $\partial r_L / \partial \bar{\alpha} \geq 0$ and $\partial S / \partial c \leq 0$ immediately give us the following:

Empirical implication 2. *-1. A regulatory tightening causes an increase in the supply of loans by the shadow banking sector, $(\partial S/\partial r_L) \times (\partial r_L/\partial \bar{\alpha}) \geq 0$.*

-2. A negative shock on the shadow banks' supply (higher funding costs, more regulation...) causes more intermediaries to adopt over-optimistic models (μ_l increases when c increases).

Policy implications.   The model has important implications for current policy debates:

-1. When banks can choose over-optimistic models, tightening the regulation can increase the risk of default in the banking sector, in particular if interest rates on loans react strongly to the drop in supply. A typical example is the transition to Basel III. Several regulators are also currently considering the imposition of floors on certain risk weights (see for instance the "Collins amendment" in the next update of the U.S. regulation), the $\bar{\alpha}$ chosen by the regulator in the model can also be interpreted as such a floor. A prediction of the model is that higher floors will lead to the selection of more optimistic models, and possibly to an increase in the total risk of the banking sector.

-2. Counter-cyclical capital ratios can have a pro-cyclical effect on the default risks in the banking sector. This is a consequence of point 1: although in the model optimal capital requirements should vary over the cycle (as η and the distribution Ψ change), increasing $\bar{\alpha}$ when risks are higher can actually increase risk.

-3. The presence of a shadow banking sector reduces the effect described in Proposition 2, but does not cancel it. This is shown by differentiating equation 14, which gives:

$$\frac{\partial \mu_l}{\partial \bar{\alpha}} = \left(\frac{1}{K} D(r_L) + \frac{\bar{\alpha}}{K} \times \frac{\partial r_L}{\partial \bar{\alpha}} D'(r_L) \right) - \left(\frac{1}{K} S(r_L, \sigma, c) - \frac{\bar{\alpha}}{K} D(r_L) \frac{\partial r_L}{\partial \bar{\alpha}} \frac{\partial S(r_L, \sigma, c)}{\partial r_L} \right)$$

The first term in brackets is the effect without the supply by shadow banks, as in Proposition 2. The second term is negative and thus reduces this effect, but can be made very small if for instance $\bar{\alpha}$ and $S(r_L, \sigma, c)$ are small.

-4. Regulatory changes implying a decrease in supply by the shadow banking sector[14] lead to more adoption of optimistic risk models in the regulated sector, and increase the probability that a regulated bank defaults.

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[14]See for instance the "Initial Integrated Set of Recommendations to Strengthen Oversight and Regulation of Shadow Banking" published by the FSB on 19.11.12.

The message from these different points is that the possibility to bypass the regulation by using more optimistic models is not a secondary problem requiring a fix. On the contrary, it is a serious loophole that can completely undo important regulatory reforms.

Discussion. A natural question is of course why the regulator would let banks so much freedom. In practice the regulator defines a certain number of requirements and backtests the model, which must also have been used by the bank for two years before it can serve the computation of regulatory capital. This prevents banks from using models totally off the mark, but not from using models slightly over-optimistic. Backtesting for example does not often lead to the rejection of a credit risk model given the low power of the tests.

I see four reasons why regulators may have postponed dealing with the hidden information problem: (i) when Basel II was put in place, internal models were already in use and had no reason to be biased, hence regulators thought they could rely on them but neglected incentives to tweak the models in the future. (ii) The regulator can consider that the priority is to give incentives to use quantitative models to increase transparency, and that the market will penalize banks using unrealistic models. (iii) Banks' defaults affect non-national investors, so that regulators use their discretion in allowing more or less optimistic models to favor national banks, which is exactly what the Basel framework was supposed to avoid (Rochet (2010)). (iv) The next section shows that giving incentives to use the correct model is a difficult task, the regulator may prefer to deal with this problem by using more cautious capital requirements. But this first part and in particular Proposition 2 show that adding capital requirements to cover "model risk" is not sufficient, and can actually be counter-productive. It is therefore necessary for the regulation to take the hidden information problem seriously and adopt a mechanism giving incentives to use the correct model.

4 Optimal regulation with hidden model

To focus on information problems and abstract from market equilibrium effects, I assume from now on that r_L is given, or equivalently that demand is very elastic. Moreover I assume $r_L > r_0/E_{\sigma_{max}}(1-t)$ so that any capital requirement will be binding, even if the true model

is the most pessimistic one. As a consequence, when she sets $\alpha(\sigma)$ the regulator equivalently sets a constraint of the form $\theta \geq \theta(\sigma)$, which will be easier to work with.

Assume the regulator wants to implement a leverage constraint dependant on the intermediary's type and expressed as $\theta = \theta(\sigma)$, maximizing under incentive compatibility constraints:

$$\mathbb{E}_\psi(V(\theta(\sigma), \sigma)) - \text{Expected costs of the regulation}$$

where V is the social welfare function studied in section 2.2. Absent costs, the regulator would implement the first-best $\theta^*(.)$. There are three constraints for the regulator: (i) incentive compatibility (IC) - a bank must be better off telling the truth about the model; (ii) limited liability (LL) - the regulator cannot tax more than what the intermediary has earned, in particular it is impossible to "punish" a defaulting intermediary; (iii) giving the agent more than a type-dependent outside option (IR) (Jullien (2000)) - he can choose not to borrow at all or opt for the "standardized approach", not use any internal model and earn a profit that will depend on the true state of the economy. I assume that a bank can opt out of the mechanism and then get $\bar{\pi}(\sigma)$ if σ is the true parameter, with $\bar{\pi}' \leq 0$.

In this section I study a backtesting mechanism, used for market risk models, and show a specific difficulty of using such a natural mechanism for credit risk models. The framework is flexible enough to study alternative mechanisms that could be used in practice, some of them are developed in the Internet Appendix.

4.1 On the difficulty of backtesting internal models ex post

Sufficient conditions to reach the first-best. Consider a mechanism where an intermediary observes the true model σ, announces some parameter σ' and faces the constraint $\theta \geq \theta(\sigma')$. Given the assumption $r_L > r_0/E_{\sigma_{max}}(1-t)$, the intermediary chooses $\theta = \theta(\sigma')$, then suffers some level of defaults t in his portfolio and finally pays a transfer $T(\sigma', t)$ if $t \leq \theta(\sigma')$. It will be useful to denote $u(\theta, t)$ the profit before transfers of an intermediary choosing θ when t defaults realize:

$$u(\theta, t) = r_L L(\theta)(1-t) - r_0(L(\theta) - K) \tag{15}$$

The regulator's program is the following:

$$\max_{\theta(.),T(.,.)} \mathbb{E}_\psi(V(\theta(\sigma),\sigma) + cE_\sigma(T(\sigma,u))) \text{ with:} \tag{16}$$

$$\forall \sigma, \sigma', \ \pi(\theta(\sigma),\sigma) - \mathbb{E}_\sigma(T(\sigma,t)) \ \geq \pi(\theta(\sigma'),\sigma) - \mathbb{E}_\sigma(T(\sigma',t)) \quad (IC)$$

$$\forall \sigma, \ \pi(\theta(\sigma),\sigma) - \mathbb{E}_\sigma(T(\sigma,u)) \qquad\qquad \geq \bar{\pi}(\sigma) \qquad\qquad (IR)$$

$$\forall \sigma, t, \ u(\theta(\sigma),t) \qquad\qquad\qquad \geq T(\sigma,t) \qquad\qquad (LL)$$

The spirit of such a regulation is easy to understand: the regulator offers a profile of transfers $T(\sigma,t)$ such that an intermediary reporting σ is heavily taxed if the realized level of defaults is relatively unlikely, given the model announced. The backtesting mechanism used for market risk models belongs to this class of mechanisms.

Consider a simpler example with only two types σ_1, σ_2, $\sigma_2 > \sigma_1$, two possible realizations of defaults $\underline{t}, \bar{t}$, $\bar{t} > \underline{t}$, and $\Pr(t = \underline{t}|\sigma_i) = p_i$, $p_1 > p_2$. To satisfy (IC) and bind (IR) the regulator gives $\bar{\pi}(\sigma_1)/p_1$ to a type reporting σ_1 if $\underline{t}$ realizes, 0 otherwise, and $\bar{\pi}(\sigma_2)$ to type σ_2 irrespective of the realization. (IC) for type σ_2 gives $\bar{\pi}(\sigma_2)/\bar{\pi}(\sigma_1) \geq p_2/p_1$. It is impossible to reach the first-best if the outside option of type σ_1 is much higher than that of type σ_2 and the likelihood ratios of the two states under both models are not different enough. Put differently, if profit decreases quickly in σ it has to be the case that the different models give very different predictions, otherwise a rent has to be left to the regulated. Fig. 6 gives an example where $p_1 = 0.5$, $p_2 = 0.25$ and $\bar{\pi}(\sigma_1) = 1$. $\bar{\pi}(\sigma_2) = 0.8$ in the first graph and 0.4 in the second. The condition $\bar{\pi}(\sigma_2)/\bar{\pi}(\sigma_1) \geq p_2/p_1$ ensures that the two lines cross at a point with a positive payoff after $\bar{t}$, which will give contracts satisfying $(IC),(LL)$ and binding (IR) for both types. The following proposition generalizes this idea to a continuum of types:

Proposition 3. *If, in addition to **M1**, $F(.,.)$ is log-concave in its second argument and $\bar{\pi}$ log-convex, then with $\theta(.) = \theta^*(.)$ the menu of transfers $T(.,.)$ defined below satisfies (IC)*

and (LL) and is such that (IR) is binding for every σ.

$$T(\sigma, t) = \begin{cases} \max(0, u(\theta^*(\sigma), t)) & \text{if } t > a(\sigma) \\ u(\theta^*(\sigma), t) - \frac{\bar{\pi}(\sigma)}{F(a(\sigma), \sigma)} & \text{if } t \le a(\sigma) \end{cases}$$

with $a(\sigma)$ increasing and such that:

$$\frac{F_2'(a(\sigma), \sigma)}{F(a(\sigma), \sigma)} = \frac{\bar{\pi}'(\sigma)}{\bar{\pi}(\sigma)}$$

[Insert Fig. 6 here.]

With the proposed menu, an intermediary reporting model σ gets $\frac{\bar{\pi}(\sigma)}{F(a(\sigma), \sigma)}$ if the realized level of defaults is less than $a(\sigma)$, and zero otherwise. By definition such a mechanism satisfies the limited liability condition. Moreover, if he reports truthfully the intermediary gets exactly $\bar{\pi}(\sigma)$ in expectation, thus the mechanism binds condition (IR). We only have to find $a(\sigma)$ such that (IC) holds for all types. Under **M1** we can induce truthful revelation with an increasing $a(.)$: intermediaries announcing a low σ get a high payoff but only if the level of defaults is under a low threshold (which will be crossed only with a small probability if their report is truthful), intermediaries announcing a higher σ get a lower payoff but more often. The two other assumptions ensure that this particular mechanism satisfies (IR) and (IC). Since $F(., .)$ is decreasing in its second argument and $\bar{\pi}$ is decreasing, the log-concavity of $F(., .)$ in σ expresses the idea that the different distributions do not give too similar predictions as σ increases, and the log-convexity of $\bar{\pi}$ implies that the outside option does not decrease too quickly as σ increases. The intuition is the same as in the binary example above. See Appendix A.5 for the full proof.

Fig. 7 gives an example, the parameters are the same as in the example of section 3.2 and r_L is equal to 1.15. On the left panel I plot the expected payoff an intermediary gets if the true parameter is σ and he reports σ', for all values of σ' and different values of σ. By construction of the mechanism the maximum payoff is obtained for $\sigma' = \sigma$. On the right panel I show how this is achieved by plotting the payoff an intermediary gets when he reports the true σ and t defaults realize.

[Insert Fig. 7 here.]

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The logic behind this result is simple: if risk is low, the intermediary is ready to pay high penalties if high default levels realize, because this event is unlikely. In principle, observing the level of defaults ex-post gives a powerful tool to punish the users of over-optimistic models. A limitation is that the intermediary's outside option should not be too sensitive to his type. If this outside option is the profit of a bank under Basel's standardized approach, an implication is that a more risk-sensitive standardized approach can make the revelation of models in the advanced internal ratings based approach more difficult.

A negative result. The optimal menu of penalties may include transfers for levels of default above those at which a bank defaults itself. Due to limited liability, the regulator cannot impose penalties for high levels of default, it may thus be necessary to *subsidize* defaulting banks who had announced very pessimistic parameters. This is the case with the mechanism of Proposition 3 when $a(\sigma) > \theta^*(\sigma)$.

This happens in particular when models are difficult to backtest. For low levels of risk there is a lot of historical data to calibrate different models, such that they tend to deliver similar predictions, while for extreme levels data is much more sparse. This is at the same time the reason why the regulator would like to use the bank's expertise, and why it is difficult to punish overoptimism. I model this situation in a stylized way by assuming the different models are perfectly equivalent up to a given level of defaults:

Definition 2. *Models $\sigma \in [\sigma_{min}, \sigma_{max}]$ are **distinguishable only above** $\hat{t}$ if for any $(\sigma, \sigma') \in [\sigma_{min}, \sigma_{max}]^2$ and for any $t < \hat{t}$ we have $f(t, \sigma) = f(t, \sigma')$.*

Take any two models σ, σ' with $\sigma < \sigma'$ such that $\theta(\sigma) \leq \theta(\sigma')$. Assume σ is so low that the regulator wants to implement $\theta(\sigma) < \hat{t}$, in which case a bank using model σ will default for levels of losses that give no information on which is the true model. I prove that in this case it will be necessary to subsidize a bank using model σ after it defaults. By contradiction, assume it is not the case. Then for any $t \geq \theta(\sigma)$ we have $u(\theta(\sigma), t) = T(\sigma, t) = 0$. To bind the constraint (IR) for type σ we thus need:

$$\int_0^{\theta(\sigma)} [u(\theta(\sigma), t) - T(\sigma, t)] f(t, \sigma) dt = \bar{\pi}(\sigma)$$

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A bank of type σ' gets $\bar{\pi}(\sigma')$ for reporting truthfully, while reporting σ gives:

$$\int_0^1 [u(\theta(\sigma),t) - T(\sigma,t)]f(t,\sigma')dt = \int_0^{\theta(\sigma)} [u(\theta(\sigma),t) - T(\sigma,t)]f(t,\sigma)dt = \bar{\pi}(\sigma)$$

where the second term is implied by $f(t,\sigma) = f(t,\sigma')$ for $t \leq \theta(\sigma) \leq \hat{t}$. Since $\sigma < \sigma'$ we have $\bar{\pi}(\sigma) > \bar{\pi}(\sigma')$, which violates (IC) for type σ', a contradiction. It is thus necessary to have $T(\sigma,t) < 0$ at least for some $t > \hat{t}$: to have (IC) and bind (IR), it must be the case that a type with a low σ gets a positive payoff for some realizations of t that have a higher probability when the true model is σ than when it is a more pessimistic model.

It is politically unfeasible for the regulator to commit to a mechanism where taxpayers' money is used to subsidize defaulting banks. Then the reasoning above shows that she has to use transfers such that for any σ with $\theta(\sigma) < \hat{t}$, an intermediary truthfully reporting σ' must get at least $\bar{\pi}(\sigma)$. Hence the following proposition:

Proposition 4. *If models $\sigma \in [\sigma_{min}, \sigma_{max}]$ are distinguishable only above $\hat{t}$ and $\theta(\sigma_{min}) \leq \hat{t}$:*
-To satisfy (IC), (LL) and bind (IR), the regulator must commit to subsidizing defaulting banks in some states of the world: $\forall \sigma$ s.t. $\theta(\sigma) < \hat{t}$, $\exists t > \hat{t}$ $T(\sigma,t) < 0$.
-If unable to commit, the regulator has to set $T(.,.)$ such that each type gets at least $\bar{\pi}(\sigma_{min})$, the highest reservation value, in order to get truthful revelation by all types.

Remark 1. *If the regulator is unable to commit to subsidizing an intermediary after he defaults, if $\theta^*(\sigma_{min}) < \hat{t}$ there is a trade-off between extracting the intermediaries' surplus and how model-sensitive the regulation can be.*

When it is impossible to "punish" the use of optimistic models, the only possibility is to give a "bonus" for the use of pessimistic models. This "bonus" can be very costly here since all intermediaries have to get the reservation value of the most optimistic type. To avoid these costs, the only solution is to increase the capital requirements of the most optimistic types such that there is no $\theta(\sigma)$ below $\hat{t}$. This means reducing the model-sensitiveness of the regulation compared to the first-best solution, more so when backtesting is more difficult ($\hat{t}$ is higher).

4.2 Discussion and policy implications

This section concludes on a rather negative note. In principle the regulator could use the observation of the realized level of defaults to detect over-optimistic models, as is done for market risk. Credit risk however is likely to be different: models typically yield different predictions for tail values only, and when high levels of default are reached it is likely that the institution will already be at risk. To preserve incentive compatibility, capital requirements have to be increased, so that the bank can still be punished ex-post in states of the world where over-optimism can be detected. This limits how sensitive to the intermediary's model the regulatory constraint can be. But if the regulation has to be less reactive to the intermediary's report, using internal models for regulatory purposes is also less useful. Notice that for market risk the regulator wants regulatory capital to cover losses during 99% of trading days, against (implicitly) 99.9% of *years* for credit risk. In the terms of the model θ^* is larger in the latter case; it is tempting to consider market risk as the case where backtesting can work ($\theta^* > \hat{t}$) and credit risk as the case where it cannot ($\theta^* < \hat{t}$).

The regulator may have more options in a dynamic environment where losses appear over time, as opposed to this static framework. Dewatripont and Tirole (1994), Decamps, Rochet, and Roger (2004) or Rochet (2010) actually advocate abandoning the regulatory use of internal models and having a regulator prompt to intervene when simple capital ratios hit certain thresholds. A similar problem may appear however: if large "tail" losses follow a sequence of moderate losses, a lot of information can be gained by dynamically monitoring the bank's assets. However if large losses appear so suddenly that it is already too late to take corrective action, the situation corresponds to the case $\theta^*(\sigma) > \hat{t}$ above.

Models of adverse selection in a banking context similar to the menu of transfers discussed above have been used to study deposit insurance premia. In Chan, Greenbaum, and Thakor (1992), low-risk (resp. high-risk) banks self-select a menu with a low insurance premium and a high capital requirement (resp. a high insurance premium and a low capital requirement). A similar mechanism could theoretically be used in our context. A problem however is that the demand for loans is inelastic in the aforementioned paper, so that the regulator does not take into account that higher capital ratios and/or higher insurance premia will imply fewer loans, a concern that seems currently extremely important. Increasing capital requirements

when risk is low would decrease the amount of loans in the economy precisely when they have a higher social value. It is in principle possible to do better by using transfers to banks depending on the model they reported and on the level of defaults that realizes, which gives information about whether the model used is realistic or not. In Giammarino, Lewis, and Sappington (1993), the problem studied is different: the regulator is able to check the quality of the bank's assets at no cost through auditing, but a risk-based regulation may give the bank incentives not to spend enough effort to increase the quality of its assets. But internal risk models today are used precisely because it is assumed that the bank has more information about its assets than the regulator can acquire through simple auditing procedures. Hence the main problem seems to be adverse selection, not moral hazard, although it is interesting to introduce the moral hazard element into the picture (see the Internet Appendix **B.5**).

Solving the asymmetric information problem studied in section 3 is hard because the regulator wants a safe intermediary to have a high leverage. This partly explains why the problems underlined in section 3 actually take place, and reinforces the warning issued in that section. Different options exist for a second-best regulation, depending on the tools that the regulator has. The Internet Appendix develops several of them, in particular a mechanism based auditing internal models ex ante. The second-best regulation involves a trade-off: the goal is to obtain finely risk-sensitive capital requirements, but then capital requirements will also be *model*-sensitive. More sensitivity to the model used gives more incentives to misreport, which increases regulatory costs.

5 Other applications of the framework

5.1 Gradual adoption of new models

It is certainly not always the case that risk models are deliberately chosen to bypass regulatory constraints. More plausibly, there is a process in which new models are developed, with a competitive advantage for more "useful" models. Either their users tend to favor them, or their "suppliers", often specialized firms, realize that models both plausible and not too pessimistic attract more customers. The equilibrium of Proposition 1 is the outcome of such a process. Note that this process can also take place in a world where banks do not know

better than the regulator which model is the correct one.

Imagine that at the beginning $1 - \mu_{S,0}$ banks use all available models in the same proportions, $\mu_{S,0}$ invest in the safe asset only. In each subsequent period, each bank can choose a new risk model. They cannot compute precisely which model is the best to use, and tend to adopt models which seem widely used and profitable: with $n_{i,t}$ the number of banks using strategy i in period t, $\pi(i,t)$ the profit made by a bank adopting this strategy in t, and $\bar{\pi}_t$ the average profit in t, assume:

$$\forall i, \forall t \geq 0, \ n_{i,t+1} = \frac{\pi(i,t)}{\bar{\pi}_t} n_t$$

In these "replicator dynamics" the total number of banks stays constant, a more profitable model is more adopted, and if the process converges to some distribution of strategies and market prices then they form an equilibrium in the sense of Definition 1.

In the framework of section 3.1 the choices of different banks are strategic substitutes: incentives to choose the most optimistic model are higher when r_L is higher, and r_L is higher when less banks choose the most optimistic model. This has interesting dynamic implications if we simulate the process. I take the same parameters for the distribution of defaults and demand as in section 3.2, $\mu_{S,0} = 90\%$, and there are 1000 models giving values of θ between $\theta(\sigma_{min})$ and 1. Fig. 8 shows the evolution over 10 periods of the proportion of intermediaries choosing the most optimistic model, and the distribution of intermediaries over the different models available in period 10. 24% of intermediaries choose the most optimistic model, and 72% invest in the safe asset only.

[Insert Fig. 8 here.]

At the beginning, most intermediaries do not invest in the risky asset, hence r_L is high and larger than r_L^e. It is extremely profitable to use the most optimistic models, and many intermediaries switch to them. The supply of loans increases, r_L decreases and in period 2 already the adoption of optimistic models is smaller. r_L continues to shrink gradually, but as it does it becomes profitable to use low levels of leverage, and we obtain a process in two waves: forerunners rush to very optimistic models and r_L drops, then these first models are gradually abandoned and replaced by more conservative ones. In the end the

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process converges to an asymmetric situation with banks using either maximum leverage or no leverage at all, corresponding to the equilibrium of section 3.1.

Empirical implication 3. *When new models become available, or the regulatory use of internal models is allowed for new assets, the adoption of optimistic models should be quick at the beginning, and then slow down as it becomes less profitable to invest in these assets.*

5.2 The cost of bad forecasts

I have made the assumption that an intermediary can choose an over-optimistic model with no more cost than if he chose a more realistic one. In a more general framework it could be costly, or even impossible, to choose an optimistic model to bypass regulation and at the same time allocate between the several types of borrowers as the true model advises. This would add a countervailing force giving incentives to stay closer to the true model.

For this countervailing force to be of any importance, optimistic models must be too optimistic *for default levels below the default point*. If the model just underestimates the probability of extreme events and the intermediary defaults for events less extreme, the forecasting mistake is privately irrelevant. Assume the true model is σ and the intermediary reports σ', r_L is multiplied if the level of realized defaults is t by $\epsilon(|f(t,\sigma') - f(t,\sigma)|)$, with $\epsilon \geq 0$ a decreasing function and $\epsilon(0) = 1$. In words, the return on each loan (conditional on repayment) is discounted, and the discount is higher when the probability of the realized default level was more badly forecast. Equation 5 can be rewritten as:

$$\tilde{\pi}(L,\sigma',\sigma) = \int_0^{\theta^C(L,\sigma')} \left(r_L L(1-t)\epsilon(|f(t,\sigma') - f(t,\sigma)|) - r_D(L-K) \right) f(t,\sigma)dt$$

$$\text{where } \theta^C(L,\sigma') \quad s.t. \quad r_L L(1 - \theta^C(L,\sigma'))\epsilon(|f(t,\sigma') - f(t,\sigma)|) - r_D(L-K) = 0$$

Assume the most optimistic model σ_{min} and the true model σ are distinguishable only in the tail above $\theta^C(L,\sigma_{min})$. Then $\theta^C(L,\sigma_{min})$ and the bank's profit are the same as without costs for forecast errors since below $\theta^C(L,\sigma_{min})$ the optimistic model's predictions are correct.

Costs associated to forecasting errors do not give incentives to choose correct models if some models are available which are both optimistic regarding the probability of extreme events, and realistic everywhere else, which is precisely the fact highlighted in Proposition 4.

5.3 Other extensions

The model allows for many other extensions. Some are detailed in the Internet Appendix.

It is possible to take into account additional countervailing forces to over-optimism, with random deposit withdrawals, risk aversion or a charter-value effect for instance. The main economic mechanisms would not be affected. The only assumption needed in this paper is that these forces are not strong enough to avoid biased models, which is what investors believe (Samuels, Harrison, and Rajkotia (2012)).

On top of the adverse selection problem considered here, moral hazard may arise if banks choose in which assets to invest before reporting risk estimates. As suggested by Carey and Hrycay (2001): "*investments might be focused in relatively high-risk loans that a scoring model fails to identify as high-risk, leading to an increase in actual portfolio risk but no increase in the banks estimated capital allocations*". Section **B.5** of the Internet Appendix sketches an extension where banks focus on complex assets for which there is model uncertainty, even when they are on average riskier than simple assets.

My model's assumptions match the case of banks relying on retail funding. They can be adapted to wholesale funding, deposits being replaced by uninsured debt. If the banks' lenders know the true model they charge higher rates to banks adopting optimistic models and in equilibrium $r_L = r_L^e$. Regulation then relies on market discipline: the regulator makes sure investors are given quantitative estimates of risk, and that the methodology used is clear enough for them to detect over-optimistic models. This is optimal if the regulator cares only about protecting investors. But if a bank's default has some externalities, then risk is too high. Another interpretation is that the possibility to choose risk measures enables intermediaries and their creditors to bypass the regulation and reach the level of leverage maximizing their joint profit. An application is the case of an originator of securitized products (intermediary) facing investors legally prevented from investing in low-grade assets. Optimistic risk measures can be used to label products as "investment grade" and sell them to regulated investors, who are aware that ratings are unrealistic but want to bypass the regulation (see Pagano and Volpin (2009) and Bolton, Freixas, and Shapiro (2012)).

Other mechanisms to reveal the banks' private information can be analyzed. A possible solution is auditing: the regulator inspects internal models and looks for suspicious "tweaks".

Since auditing is costly, the regulator audits just enough to prevent banks from misreporting, and takes into account that a more risk-sensitive regulation gives more incentives to misreport. Section **B.2** shows that the second-best regulation is less risk-sensitive than the first-best, due to two effects: banks knowing risk is high have less incentives to misreport when allowed a higher leverage, and when pretending risk is low would give them a lower leverage. For high auditing costs, these effects make the second-best regulation much less risk-sensitive, so that using internal models unnecessarily complicated. This extension is close to Prescott (2004). In his paper the amount a bank invests is fixed. As a result, the bank with the highest risk has more incentives to misreport and second-best capital ratios are above the first-best, but not necessarily less risk-sensitive.

Another solution is to use reports from different banks, or benchmarking. I show in **B.3** that the first-best satisfies Maskin monotonicity (Maskin (1999)) and how to adapt the canonical mechanism for Nash implementation[15]. However, the assumption of section 3 that all banks have the same information is meant as a simplifying assumption allowing to consider a representative bank (see footnote 11). I show Proposition 1 still obtains in a model where banks have different monitoring technologies, so that their models are not directly comparable and they have different σ. Clearly a bank's information about internal models has both an idiosyncratic and a common component, studying mechanisms along the lines of Cremer and McLean (1988) would be interesting for future research.

In section 3.1, it seems that the regulator has a simple solution: if all banks report σ' and market prices are inconsistent with the report, she could infer that banks have lied and use this information to choose θ. However this reaction will be anticipated by market participants in equilibrium, which can make market signals much less informative or even destroy the existence of an equilibrium (see Bond, Goldstein, and Prescott (2010)). In **B.4**, I show that the regulator can indeed use a bank's market value to learn σ if its shares are priced by investors who know the true model. This is not a very compelling case, because with perfectly informed shareholders it would not be necessary to have a Basel type regulation, as high capital ratios would not be that costly. If instead the regulator uses the market price of junior debt, no equilibrium exists when model uncertainty is too high.

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[15]Which is not trivial as there is an interaction between transfers and limited liability as in section 4.

6   Conclusion

"Model-based" regulation exploits banks' better information about their own risks to compute capital ratios. This information however is private, and financial intermediaries cannot be expected to develop unbiased models if they face incentives to do otherwise. The process of elaboration/adoption of new models may be biased towards more "profitable" models.

This paper gives new empirical implications on how the choice of internal models reacts to market and regulatory changes. A regulation failing to give proper incentives to develop cautious models can cripple other regulatory changes, in particular a regulatory tightening can lead to the wide adoption of over-optimistic risk models and increase risk.

This problem can only be addressed by giving incentives to use the best possible models. A regulation relying more on internal risk measures makes it harder for the regulator to reveal an intermediary's true model, for two reasons. First, it gives more incentives to use slightly over-optimistic models as it will enable the intermediary to increase leverage. Second, if intermediaries are allowed to use more leverage it is more likely that they will default for high levels of losses. Since these high levels are the ones that enable the regulator to identify optimistic models, it becomes more difficult to punish over-optimistic intermediaries.

The current trend in regulation is towards less use of internal models, at the cost of a more distortive regulation that does not react finely to a bank's risks. A more ambitious avenue would be to keep using internal models to make the regulation more efficient, and give the regulators more tools to ensure the figures reported are unbiased.

There are other instances in which a strategic use of models can take place. The regulation of insurance companies in the Solvency II framework is comparable. Internal models are also used to measure the performance of employees and desks, to convey information from one hierarchic level to another, to rating agencies, shareholders... Sibbertsen, Stahl, and Luedtke (2008) cite as evidence of model risk a report according to which some investors "*tend to apply an across-the-board discount of about 20% to the published numbers*". This discount cannot stem from honest and random mistakes about the true model, much rather from the suspicion that models are deliberately chosen to bias the reported information. Hence the relevant framework here may not be model risk, but "hidden model".

A   Appendix - Figures and proofs

A.1   Notations

r_L	gross interest rate on loans to borrowers.
r_D	gross interest rate on loans to intermediaries.
r_0	risk-free rate, normalized to 1.
r_L^e	first-best interest rate on loans, equals $r_0/(1 - \mathbb{E}(t))$.
$D(.)$	demand for loans by final borrowers.
$r_L(.)$	inverse demand function for loans.
L	amount lent by an intermediary/the representative intermediary.
M	amount borrowed by an intermediary.
K	capital owned by an intermediary.
W	investors' wealth.
t	random proportion of defaulting loans.
$f(.,\sigma), F(.,\sigma)$	family of pdf and cdf, parameterized by σ, modeling the proportion of defaulting loans.
$\psi(.), \Psi(.)$	pdf and cdf from which the true σ is drawn.
θ	default point, maximum proportion of defaults an intermediary can suffer in his portfolio.
$s(\theta, \sigma)$	expected proportion of surplus repayments in an intermediary's portfolio.
$\pi(\theta, \sigma)$	expected profit of an intermediary.
$V(\theta, \sigma)$	social welfare.
$\alpha(\sigma)$	minimum capital ratio required from an intermediary reporting model σ.
$\bar{\alpha}$	minimum of the function $\alpha(.)$.
$\theta(\sigma)$	minimum θ allowed by the regulation if the intermediary reports model σ.
η	parameter of the demand function used in simulations.
μ_l, μ_r, μ_s	proportions of intermediaries with max. leverage / investing K in loans / K in the safe asset.
p_d	expected number of defaulting intermediaries in equilibrium.

A.2   Proof of Lemma 1

For a given constraint $K/L \geq \alpha$, the intermediary's program if he invests in loans is:

$$\max_{\theta} r_L L(\theta) s(\theta, \sigma), \text{ s.t. } \theta \geq 1 - (r_D/r_L)(1 - \alpha) = \underline{\theta}$$

Notice in particular that $\alpha \leq 1$ and $r_L \geq r_D$ implies that $\underline{\theta} \geq 0$ and $r_D - r_L(1 - \theta) \geq 0$. It is easy to compute that $s'(\theta, \sigma) = F(\theta, \sigma)$. Then we have

$$\pi_1'(\theta, \sigma) = \frac{r_L L(\theta)(F(\theta, \sigma)(r_D - r_L(1 - \theta)) - r_L s(\theta, \sigma))}{r_D - r_L(1 - \theta)} \tag{17}$$

Denoting $G(\theta) = F(\theta, \sigma)(r_D - r_L(1 - \theta)) - r_L s(\theta, \sigma)$, we have $G'(\theta) = f(\theta, \sigma)(r_D - r_L(1 - \theta))$ as both terms $r_L F(\theta, \sigma)$ cancel out, thus G' is always positive. This implies that $\pi(\theta, \sigma)$ is either decreasing and then increasing in θ, always increasing or always decreasing. Finally, we have $\pi_1'(1, \sigma) = \frac{r_L K}{r_D}(r_D - r_L(1 - \mathbb{E}(t)))$, $\lim_{\theta \to \underline{\theta}} \pi_1'(\theta, \sigma) = -\infty$. Using equation 7 defining r_L^e:

-If $r_L \geq (r_D/r_0)r_L^e$, then $\pi_1'(1, \sigma) \leq 0$ and π_1' is negative for every θ, thus if he invests in loans the intermediary chooses $\theta = \underline{\theta}$. Notice that we also have $r_L(1 - \mathbb{E}(t)) \geq r_0$, hence the intermediary prefers investing in loans to investing in the safe asset.

-If $(r_D/r_0)r_L^e > r_L > r_L^e$ then the intermediary chooses $\theta = \underline{\theta}$ or $L = K$, since profit is either first decreasing and then increasing in θ, or always increasing. Direct comparison shows that he will choose $\theta = \underline{\theta}$ if and only if

$$r_L \geq r_D \left(\frac{1 - \mathbb{E}(t) - s(\underline{\theta}, \sigma)}{(1 - \underline{\theta})(1 - \mathbb{E}(t))} \right) = r_1(r_D, \underline{\theta})$$

-If $(r_D/r_0)r_L^e > r_L^e > r_L$ profit is decreasing and then increasing in θ, but investing K in the safe asset yields more than in loans. The intermediary chooses $\theta = \underline{\theta}$ over $L = 0$ if and only if

$$r_L \geq \frac{r_0 r_D}{r_D s(\underline{\theta}, \sigma) + r_0(1 - \underline{\theta})} = r_2(r_D, \underline{\theta})$$

-If $(r_D/r_0)r_L^e > r_L^e = r_L$ the previous condition applies, except that the intermediary is indifferent when he doesn't borrow between investing in the safe asset or in loans.

We have to compare the different thresholds for r_L. First, we have:

$$
\begin{aligned}
r_1(r_D, \underline{\theta}) \quad &> (r_D/r_0)r_L^e \\
\Leftrightarrow \quad \underline{\theta} \quad &> \mathbb{E}(t) + s(\underline{\theta}, \sigma) \\
\Leftrightarrow \quad \underline{\theta} \quad &> \int_0^1 t f(t, \sigma) dt + \underline{\theta} F(\underline{\theta}, \sigma) - \int_0^{\underline{\theta}} t f(t, \sigma) dt \\
\Leftrightarrow \quad \underline{\theta}(1 - F(\underline{\theta}, \sigma)) \quad &> \int_{\underline{\theta}}^1 t f(t, \sigma) dt \Leftrightarrow \underline{\theta} > \mathbb{E}(t | t \geq \underline{\theta})
\end{aligned}
$$

The last inequality is false. Developing and rearranging $r_1(r_D, \underline{\theta})$ and $r_2(r_D, \underline{\theta})$, we get:

$$
\begin{aligned}
r_1(r_D, \underline{\theta}) > r_2(r_D, \underline{\theta}) \quad &\Leftrightarrow \quad r_1(r_D, \underline{\theta}) > r_L^e \Leftrightarrow r_2(r_D, \underline{\theta}) > r_L^e \\
&\Leftrightarrow \quad r_D > \frac{r_0(1 - \underline{\theta})}{1 - \mathbb{E}(t) - s(\underline{\theta}, \sigma)}
\end{aligned}
\tag{18}
$$

This last inequality may be true or false depending on r_D. Thus we have two cases to consider and

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the conditions above prove the following:

If $r_D > (r_0(1 - \theta))/(1 - \mathbb{E}(t) - s(\underline{\theta}, \sigma))$: when $r_L < r_L^e$ the intermediary chooses $r_L^* = 0$, when $r_1(r_D, \underline{\theta}) > r_L \geq r_L^e$ he chooses $L^* = K$, when $r_L > r_1(r_D, \underline{\theta})$ he chooses $\theta^* = \underline{\theta}$. Now if $r_D \leq (r_0(1 - \theta))/(1 - \mathbb{E}(t) - s(\underline{\theta}, \sigma))$: if $r_L < r_2(r_D, \underline{\theta})$ the intermediary chooses $L^* = 0$, if $r_L \geq r_2(r_D, \underline{\theta})$ he chooses $\theta^* = \underline{\theta}$. This implies the proposition where $\underline{r_L} = \max(r_1(r_D, \underline{\theta}), r_2(r_D, \underline{\theta}))$.

A.3   Proof of Lemma 2

When demand is close to perfectly elastic r_L does not depend on α, such that choosing α is equivalent to choosing $\theta = 1 - \frac{r_0}{r_L}(1 - \alpha)$. Moreover we can write:

$$L(\theta) = \frac{r_0 K}{r_0 - r_L(1 - \theta)}$$

L is obviously decreasing in θ. The first-order condition in θ gives us:

$$
\begin{aligned}
V_1'(\theta, \sigma) &= L'(\theta)\left(r_L \mathbb{E}_\sigma(1 - t) - r_0 - c\int_\theta^1 (r_0 - r_L(1 - t))f(t, \sigma)dt\right) \\
&= L'(\theta)\left(r_L \mathbb{E}_\sigma(1 - t) - r_0 - c(1 - F(\theta, \sigma))(r_0 - r_L \mathbb{E}_\sigma(1 - t|t > \theta))\right) = 0
\end{aligned}
$$

Assumption **M1** implies that a distribution with a higher σ dominates a distribution with a lower one in the sense of first-order stochastic dominance. As a result when σ increases $\mathbb{E}_\sigma(1 - t)$ and $\mathbb{E}_\sigma(1 - t|t > \theta)$ decrease, $(1 - F(\theta, \sigma))$ increases such that, since $L'(\theta) \leq 0$, $V_1'(\theta, \sigma)$ increases. Hence $V_{1,2}''(\theta, \sigma) \geq 0$. We can then compute:

$$V_{11}''(\theta, \sigma) = L''(\theta)(r_L \mathbb{E}_\sigma(1 - t) - r_0) - cL''(\theta)\int_\theta^1 (r_0 - r_L(1 - t))f(t, \sigma)dt + cL'(\theta)(r_0 - r_L(1 - \theta))$$

L'' is positive, thus the first term may be positive since in general the regulator will allow less leverage than what would lead to $r_L = r_L^e$. When the regulator reduces the leverage further the supply of loans decreases at a declining speed, hence welfare losses due to credit restriction increase more slowly, which gives some convexity in θ to V. When costs are high enough however this effect is compensated by the two other terms: by definition of θ we have $r_0 - r_L(1 - t) \geq 0$ for $t \geq \theta$, such that they are negative. Hence if c is high enough V_{11}'' is negative and $V_{1,2}''(\theta, \sigma)$ positive, so that for every σ there is a unique maximum of V for $\theta = \theta^*(\sigma)$, with θ^* increasing.

A.4   Proof of Proposition 1 and Corollary 1

When $r_D = r_0$, inequality 18 is equivalent for any θ to $\theta > \mathbb{E}(t) + s(\theta)$, which was proven to be wrong in A.3. Thus we have $r_L^e \geq r_2(r_0, \theta(\sigma_{min})) \geq r_1(r_0, \theta(\sigma_{min}))$, an intermediary will choose either $L = 0$ or $\theta = \theta(\sigma_{min})$. In equilibrium a proportion μ_l of intermediaries thus choose model

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σ_{min} and $L = K/\bar{\alpha}$, and the others invest only in the safe asset. If the interest rate on loans is r_L, the supply of loans must equal the demand:

$$\mu_l \frac{K}{\bar{\alpha}} = D(r_L) \tag{19}$$

It is impossible to have $\mu_l = 0$ in equilibrium since this would require an infinite interest rate r_L, at which supply would be positive. Hence there are two possibilities: if $0 < \mu_l < 1$ it must be the case that intermediaries are indifferent between investing only in the safe asset and choosing maximum leverage, in which case r_L will be equal to $r_2(r_0, \theta(\sigma_{min}))$. This condition can be rewritten as:

$$r_0\alpha = r_L s\left(1 - \frac{r_0(1-\alpha)}{r_L}, \sigma\right) \tag{20}$$

The second possibility is to have $\mu_l = 1$, in which case the left-hand side of equation 20 has to be lower than the right-hand side, or equal.

Start with equation 20. The right-hand side is increasing in r_L, for $r_L \to +\infty$ it goes to infinity, and for $r_L = r_0(1 - \alpha)$ it is equal to zero. Hence there is a unique value r_L^* for which there is equality. Then by using equation 19 we can compute $\mu_l^* = D(r_L^*)(\bar{\alpha}/K)$. If we find $\mu_l^* \leq 1$ we have an equilibrium. Moreover it is unique: if we choose a higher μ_l then r_L has to be lower, in which case the right-hand side in equation 20 becomes strictly lower than the left-hand side, which cannot be the case in equilibrium. If instead we find $\mu_l^* > 1$, it implies that $\mu_l = 1$ is an equilibrium, and r_L is determined by $K/\bar{\alpha} = D(r_L)$. Again this equilibrium is unique: if we decrease μ then r_L will increase, and the right-hand side in equation 19 will become even greater.

Assume now that we start the same reasoning with a higher σ. Due to assumption **M1**, $s(., \sigma)$ is decreasing in σ, to have an equality in equation 20 we need a higher r_L, which will give a lower $D(r_L)$ in equation 20, and hence a lower equilibrium μ_l. If we start with a σ such that $\mu_l = 1$ in equilibrium, an increase in σ will make it less interesting to invest in loans, which will either let μ_l unchanged or induce a switch from the case $\mu_l = 1$ to the case $\mu_l < 1$.

Finally, assume we do the same reasoning with a higher demand function D'. This does not affect the determination of r_L^*, but since $D'(r_L^*) > D(r_L^*)$, μ_l will be higher.

The corollary follows from equation 10 defining $p_d(\bar{\alpha}, \sigma)$ as the product of $\mu_l(\bar{\alpha}, \sigma)$ and the default probability of a bank with maximum leverage. When $\mu_l(\bar{\alpha}, \sigma) < 1$, increasing demand leaves $r_L(\bar{\alpha}, \sigma)$ unchanged, the first term μ_l increases and the second term is unchanged, hence the product increases.

A.5 Proof of Proposition 3

Define $U(\sigma', \sigma)$ the expected profit of an intermediary reporting σ' when the true parameter is σ:

$$U(\sigma', \sigma) = F(a(\sigma'), \sigma) \frac{\bar{\pi}(\sigma')}{F(a(\sigma'), \sigma')}$$

$$U_1'(\sigma', \sigma) = \frac{(\bar{\pi}'(\sigma')F(a(\sigma'), \sigma) + a'(\sigma')f(a(\sigma'), \sigma)\bar{\pi}(\sigma'))F(a(\sigma'), \sigma')}{F(a(\sigma'), \sigma')^2}$$
$$- \frac{(a'(\sigma')f(a(\sigma'), \sigma') + F_2'(a(\sigma'), \sigma'))\bar{\pi}(\sigma')F(a(\sigma'), \sigma)}{F(a(\sigma'), \sigma')^2} \tag{21}$$

Incentive compatibility requires for every σ:

$$U_1'(\sigma, \sigma) = 0 \Leftrightarrow \frac{F_2'(a(\sigma), \sigma)}{F(a(\sigma), \sigma)} = \frac{\bar{\pi}'(\sigma)}{\bar{\pi}(\sigma)} \tag{22}$$

Notice first that the left-hand side is increasing in a (MLRP), and under the assumptions of the proposition the left-hand side is decreasing in σ and the right-hand side increasing. This ensures that if there exists a solution $a(.)$ it is increasing in σ. Is it always possible to find such an $a(\sigma)$? Since $\bar{\pi}$ is the payoff of an intermediary not using any leverage or allowed some default point θ independent of σ, it can be written as $\bar{\pi}(\sigma) = r_L K L(\theta)s(\theta, \sigma)$, with $\theta = 1$ if no leverage is allowed. $s(\theta, \sigma)$ can be rewritten as $s(\theta, \sigma) = \int_0^\theta F(t, \sigma)dt$. Thus we can rewrite equation 22 as:

$$\frac{F_2'(a(\sigma), \sigma)}{F(a(\sigma), \sigma)} = \frac{\int_0^\theta F_2'(t, \sigma)dt}{\int_0^\theta F(t, \sigma)dt}$$

$a(\sigma)$ can take values between 0 and 1, the right-hand side is negative. We have $F_2'(1, \sigma)/F(1, \sigma) = 0$. If $\lim_{a \to 0} \frac{F_2'(a, \sigma)}{F(a, \sigma)} = -\infty$ there exists a value a to satisfy the required equality. If this limit is a negative number $-k$, then for every t in $[0, 1]$ we have $F_2'(t, \sigma) \geq -kF(t, \sigma)$. Integrating we find:

$$\frac{\int_0^\theta F_2'(t, \sigma)dt}{\int_0^\theta F(t, \sigma)dt} \geq -k$$

hence for a given σ the right-hand side is always lower than $\frac{F_2'(1, \sigma)}{F(1, \sigma)}$ and greater than $\lim_{a \to 0} \frac{F_2'(a, \sigma)}{F(a, \sigma)}$. Since $\frac{F_2'(a, \sigma)}{F(a, \sigma)}$ is increasing in a there is always a unique a satisfying the inequality for a given σ, and hence there always exists a unique solution $a(.)$ increasing and satisfying the first-order condition.

We now have to show that the second-order condition is met. We can use equation 22 to replace $F_2'(a(\sigma'), \sigma')$ in equation 21. After some rearrangements this gives us:

$$U_1'(\sigma', \sigma) \geq 0 \Leftrightarrow \frac{f(a(\sigma'), \sigma)}{f(a(\sigma'), \sigma')} \geq \frac{F(a(\sigma'), \sigma)}{F(a(\sigma'), \sigma')}$$

by the monotone likelihood ratio property we thus have $U_1'(\sigma', \sigma) \geq 0 \Leftrightarrow \sigma' \leq \sigma$, hence truthfully reporting σ globally maximizes $U(., \sigma)$.
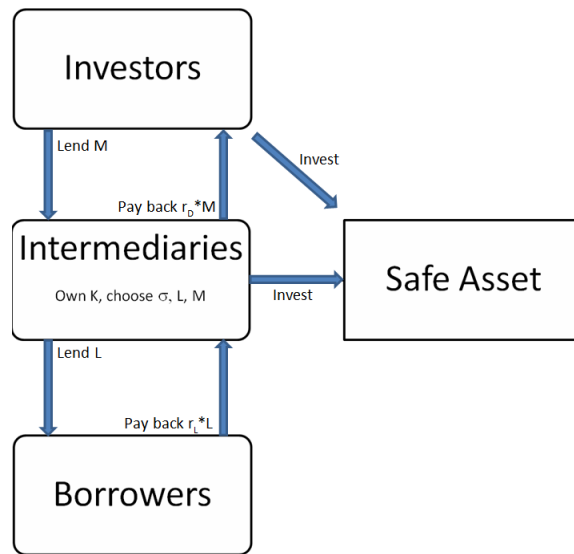
A.6 Figures
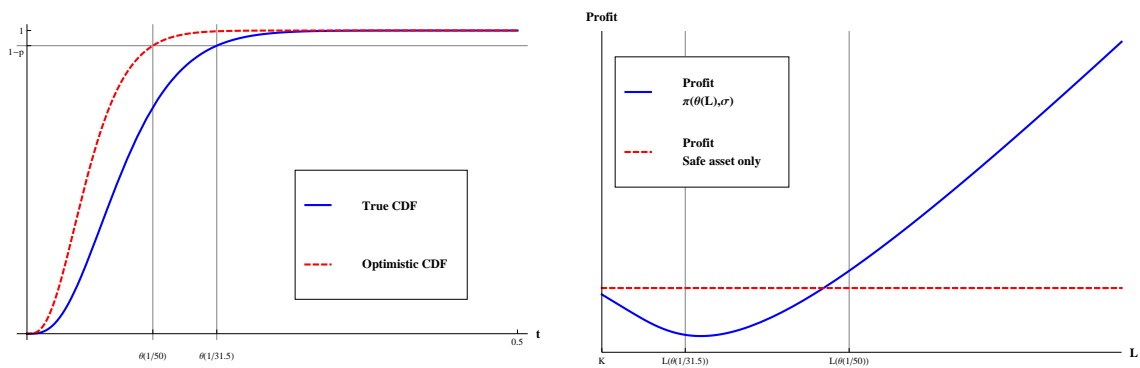

Figure 1: Market structure.

Figure 2: Cumulatives and minimum default points (left), and profit as a function of loans (right).
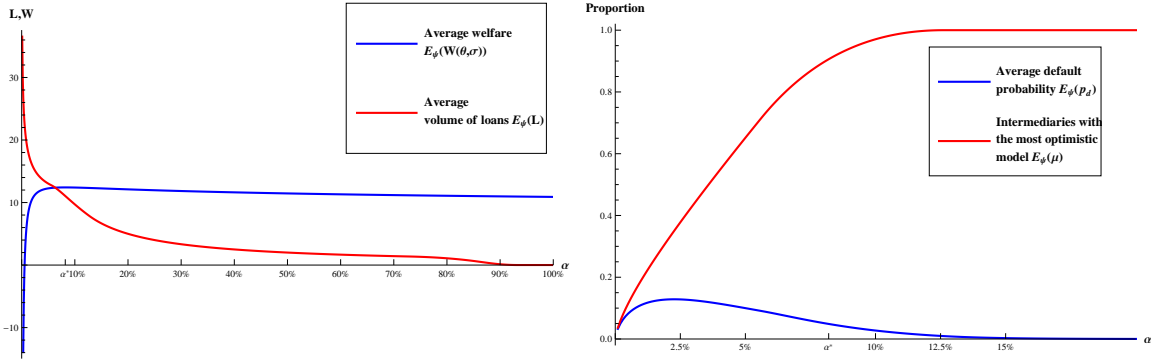
Figure 3: Expected welfare, volume of loans, intermediaries using the most optimistic model and default probability as $\bar{\alpha}$ increases.
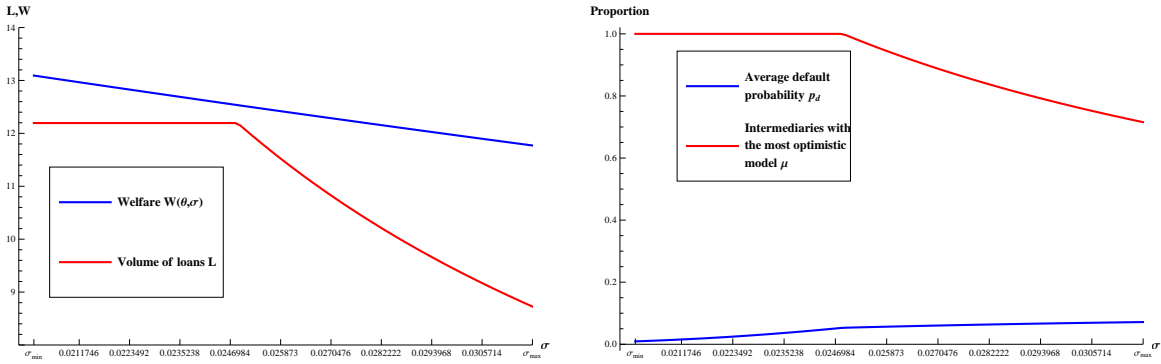

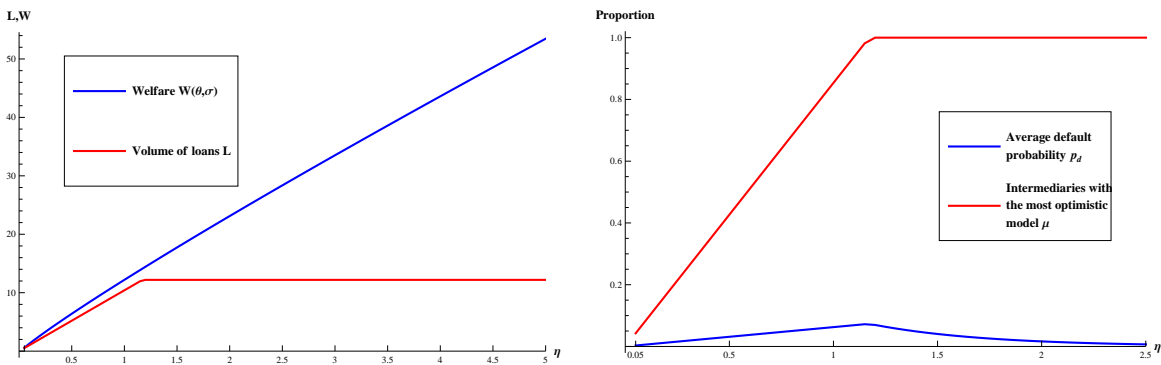Figure 4: Welfare, volume of loans, intermediaries using the most optimistic model and default probability as σ increases.

Figure 5: Welfare, volume of loans, intermediaries using the most optimistic model and default probability as η increases.
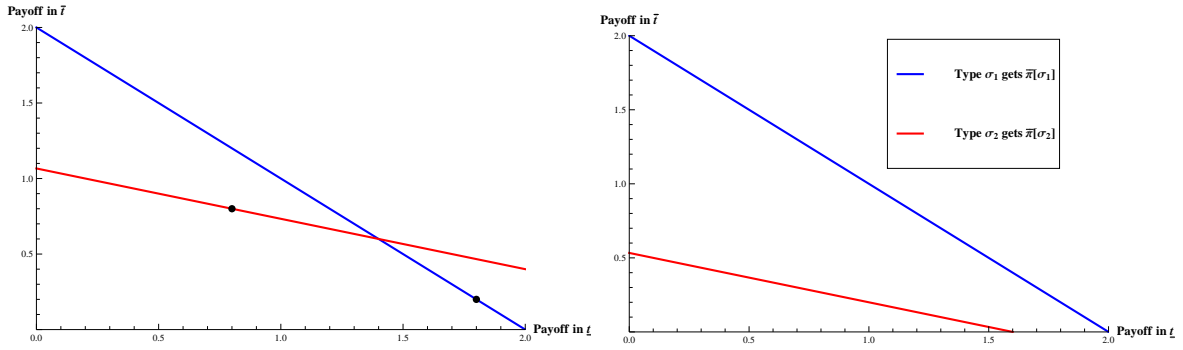
Figure 6: Example of separation when $\bar{\pi}(\sigma_2)/\bar{\pi}(\sigma_1) > p_2/p_1$, and non separation otherwise.
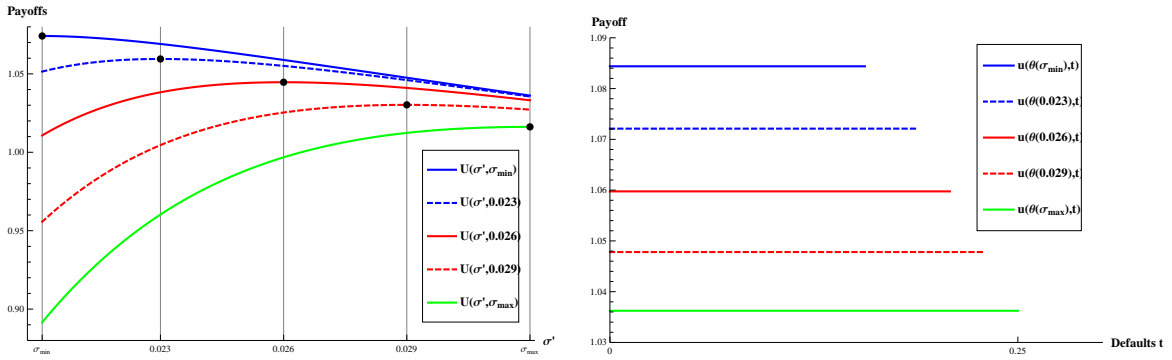

Figure 7: Expected payoff for a given σ to report σ' (left), and payoff from reporting the truth depending on the level of defaults (right).
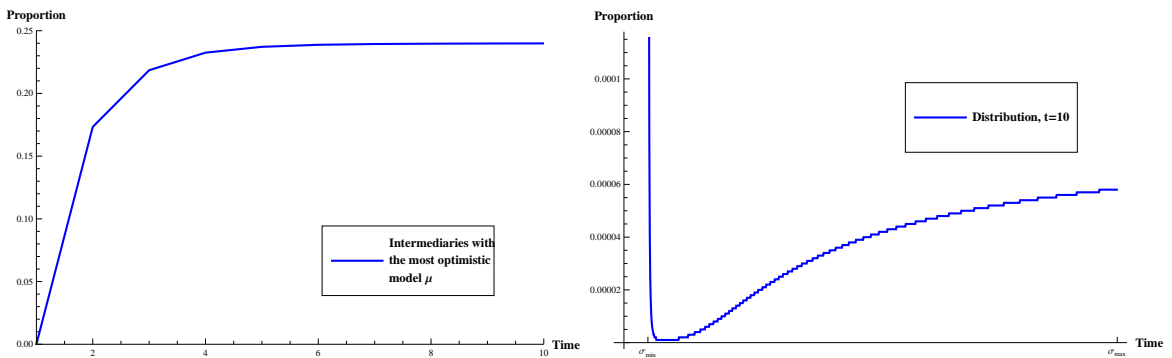

Figure 8: Distribution of intermediaries using the different models under incomplete regulation after 10 periods, and use of the most optimistic model over time.

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