Pulling up the Tarnished Anchor:

The End of Silver as a Global Unit of Account

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1 Introduction

When does a global monetary system of fixed exchange rates end? One way of characterizing the demise of a fixed-exchange rate systems is when countries sever their formal links to it, such as when the U.S. closed the gold window in 1971 and abandoned the Bretton Woods System, or when England, in 1914, embargoed gold exports and imposed exchange controls, thus marking the end of the classical gold standard era. For other historical monetary systems, such as bimetallism, the transition can be slower, and the answer as to "when a system ended" is not always obvious. Individual countries can jettison an existing arrangement by changing the formal rules that bind them to the existing system, but that does not necessarily imply that the unit of account, serving as an anchor for prices, ceases to function globally. Even the exit of economically or systemically important countries from a metallic, fixed-exchange rate regime may not end the precious metal's role as a numeraire and unit of account.

To shed new light on the relationship between price dynamics and global monetary transitions, we focus on the demonetization of silver in the nineteenth century. We connect a well-studied feature of metallic standards, their use as global price anchors, with a key historical change in the global monetary system - the abandonment of the use of silver as a global unit of account. Silver's historical role as a price anchor ended as countries overwhelmingly left silver and bimetallic and standards in the latter half of the century in favor of gold; however, the shift to gold was not instantaneous, permitting us to examine how silver's declining use as a unit of account affected global prices.

We develop a dynamic, general equilibrium model of the global economy that can be used to consider the gradual transition away from silver as a price anchor. Because both gold and silver were used to back currencies, the global monetary system is modeled as bimetallic, with both precious metals having monetary and non-monetary roles in the economy. Because the abandonment of silver as a unit of account occurred gradually, we calibrate the model using data on global commodity prices to understand the price dynamics and transition away from silver. The calibration delivers several interesting and novel results. First, the model shows that silver ceased functioning as a global price anchor in the mid-1890s, nearly two decades after many important countries abandoned bimetallism. That is, the price of silver was highly correlated with agricultural prices through the mid-1890s, but not so thereafter. Furthermore, our model matches the increased volatility of commodity prices observed after the mid- 1890s, suggesting that silver played an important role in anchoring price expectations in the global economy. The timing of the end of silver's role as a price anchor is consistent with several historical facts: many countries continued to use silver as a unit of account in the 1870s and 1880s (i.e., Japan, Russia, the United States, Brazil, Mexico, Spain, China) and global political forces in the 1890s conspired against silver as a price anchor (e.g., India's suspension of the free coinage of silver in 1893 and the defeat of William Jennings Bryan and the "silverites" in the U.S. presidential election of 1896).

2 Demonetization of Silver and Global Price Dynamics

The use of silver as a global price anchor gradually ended in the late nineteenth century as countries switched from paper, silver, and bimetallic standards to monometallic gold standards, either de jure or de facto (Eichengreen 1996, Meissner 2005). Until its demise, countries used silver, often in conjunction with gold, to fix exchange rates. Under bimetallism, both gold and silver as numeraires, with all other goods priced relative to the mint par ratio of gold to silver. A country that legally permitted bimetallism was often effectively on either a gold or silver monometallic standard, depending on how the mint price of gold to the mint price of silver compared to the world price ratio of the two metals (Officer 2008).[1]

Previous studies have attempted to model or estimate the demise of bimetallism by focusing on changes in the longstanding global mint ratio of 15.5 silver ounces to one gold ounce. The standard story postulates that Germany's decision to demonetize silver led many countries to drop silver and switch over to gold (Friedman and Schwartz 1963, Gallarotti 1994). Flandreau (1996), however, argues that France, being the largest bimetallic country in the world, was the marginal player in the bimetallic system that kept the silver-gold price ratio at 15.5:1 and that Germany's decision did not spell the end of bimetallism.[2] Rather,

[1] However, gold and silver appear to have circulated simultaneously in France from 1852-72 (Flandreau 1996).

[2] Friedman (1990) argues that the United States made a big mistake when it demonetized silver in 1873, referring to this episode as Crime of 1873. He argues that this legislation destabilized prices. Velde (2002) employs a general equilibrium model to test Friedman's hypothesis that the United States could have maintained the silver-gold price ratio at 15.5:1. He finds that the United States could have kept the silver-gold price ratio fixed up until the mid-1890s. Oppers (1996), on the other hand, finds that much of the deflation of the 1870s and 1880s could have been avoided if France and Belgium continued to freely coin silver.

they suggest that France's 1873 decision to limit the coinage of silver violated bimetallism, and led to the eventual rise of the international gold standard and to a floating silver-gold price ratio. Building on their work, Meissner (2013) argues that bimetallism would have been unsustainable after 1875. On the other hand, Morys (2007) suggests that bimetallism may have unofficially ended even earlier, suggesting that that the large gold discoveries in California and Australia in the 1850s increased the supply of gold and made the emergence of the classical gold standard imminent by the end of the 1860s.

Even though bimetallism reached its unofficial end by at least the early 1870s as countries "scrambled for gold," scholars have noted that that many countries, including Austria-Hungary, Brazil, China, Japan, Mexico, Russia and Spain, continued to back money with silver well after that date (Flandreau 2004, Bordo and Kydland 1995). China and Spain remained on a silver standard for the entire period, while the other countries gradually transitioned to gold.[3] Hence, silver's monetary importance persisted well after the unofficial end of bimetallism. In relation to pinpointing bimetallism's demise, however, comparatively little is known about how the gradual demonetization of silver and the transition to a new global monetary system influenced price dynamics. For an individual country, once legal backing of money with silver ended (either de jure or de facto), prices were free to fluctuate relative to that metal; it no longer served as a numeraire. Commodity prices, however, were largely set in competitive, global markets, suggesting that correlations between precious metals and commodity prices could persist even after legal backing ends for a single country. On the other hand, eventual widespread demonetization of silver at some point likely ended silver's role as a price anchor. And as that took place, were commodity prices cast adrift or did widespread gold standard adoption stabilize price movements?

Scholars have also noted that fixed exchange rate regimes seem to have delivered relatively stable long-run prices over the period 1870-1914, with their path characterized by a shallow U shape with the nadir occurring in the 1890s. General price deflation from the 1870s until the mid-1880s is typically attributed to a rise in the demand for monetary gold as countries moved towards the gold standard as well as economic growth. The trend in prices reversed in the mid-1890s following the discovery of gold in South Africa and the invention of the Sinai process that lowered the price of gold extraction.

[3]Austria-Hungary became a de facto member of the gold standard in 1892, Brazil and Mexico joined in 1905 and 1906, while Japan and Russia became members of the gold club in 1897.

3 Model

Consider a simple dynamic model of the international bimetallic monetary system in which time is indexed by $t \in \mathbb{N}$. There are overlapping generations of agents who maximize their expected utility over consumption, and we introduce money demand through a cash-in-advance constraint.

All generations of agents have a constant-elasticity-of-substitution (CES) utility function. There are four goods in the economy—silver, gold, an agricultural commodity, and a metallic commodity that is different from gold and silver—and those goods must be purchased with monetary gold or silver. At the start of each period t, a new agent is born and then chooses how much of her income to spend on each consumption good. Each newborn agent inherits from the previous generation a complete equity share in a representative firm that produces the four goods in the economy, and it is this equity holding that she derives her income from. At the end of each period t, a newborn agent consumes each of the goods she has purchased and realizes her utility before dying and being replaced by a newborn agent at the start of period $t+1$. Note that agents do not have bequest motives in this setup. The assumption that there are overlapping generations is a simplification of the model that allows us to solve the model and closely focus on the relationship between the prices of the four goods in the economy.[4] In this setup, then, the utility of an agent born in period t is given by

$$u(Y_{ct}, Y_{at}, Y_{st}, Y_{gt}) = \left(\mu_c^{\frac{1}{\theta}} Y_{ct}^{\frac{\theta-1}{\theta}} + \mu_a^{\frac{1}{\theta}} Y_{at}^{\frac{\theta-1}{\theta}} + \mu_s^{\frac{1}{\theta}} Y_{st}^{\frac{\theta-1}{\theta}} + (1 - \mu_c - \mu_a - \mu_s) Y_{gt}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}, \quad (3.1)$$

where Y_{ct} is consumption of the metallic commodity in period t, Y_{at} is consumption of the agricultural commodity in period t, Y_{st} is consumption of silver in period t, Y_{gt} is consumption of gold in period t, $\theta > 0$ is a constant that measures the elasticity of substitution between goods, and μ_c, μ_a, μ_s are positive constants satisfying $\mu_c + \mu_a + \mu_s < 1$. These constants capture agents' preferences for each good relative to the others.

The total quantities of gold, silver, the agricultural commodity, and the metallic commodity produced by the representative firm in period t are given by G_t, S_t, A_t, and C_t, respectively. We set gold as the numeraire so that the price of gold is normalized to one in all periods and then denote the prices of silver, the agricultural commodity, and the metallic

[4] This simple model can be extended to include infinitely-lived agents and intertemporal consumption smoothing. However, such an extension distracts from the key focus of this paper, namely the relationship between various commodity prices and silver prices.

commodity in period t by p_{st}, p_{at}, and p_{ct}, respectively. In equilibrium, these prices adjust so that the goods markets all clear.

The cash-in-advance constraint in this model states that an agent born in period t must hold enough money in the form of gold and silver in period t to purchase all the goods she consumes in that period. If in period t we respectively denote the quantity of monetary silver and gold in the economy by M_{st} and M_{gt}, then it follows that this constraint is given by

$$M_{gt} + M_{st}p_{st} \geq Y_{st}p_{st} + Y_{at}p_{at} + Y_{ct}p_{ct} + Y_{gt}. \tag{3.2}$$

Note that $M_{st}p_{st}$ is equal to the quantity of monetary silver in terms of the numeraire gold. Although equation (3.2) describes the total quantity of money in the economy, it is still necessary to specify the ratio of monetary gold to monetary silver. We assume that

$$\alpha M_{gt} = M_{st}, \tag{3.3}$$

so that $\alpha > 0$ is a constant that measures the proportion of the world money supply that is denominated in silver relative to gold. A value of α that is close to zero implies that the economy primarily uses gold as its money, while a very large value of α implies that the economy primarily uses silver as its money.

It is worth clarifying the timing of events in this setup. At that start of each period t, the first thing to occur is the realization of the firm's production of each of the four goods. After observing these realizations, the firm sets the prices of those goods and this in turn describes the value of the firm's output and hence the value of the newborn agent's equity this period. The representative agent is then extended monetary credit equal to the value of the firm's output and she then uses that money to purchase her desired quantity of each good. Finally, the firm uses the income it receives from these purchases to pay back the agent's creditor on the agent's behalf.[5] Note that this implies that the cash-in-advance constraint given by equation (3.2) above holds with equality.

[5] One alternative specification of our setup features an infinitely-lived representative agent and labor income, just like in the simplest model of Rotemberg (1987). In that model, the only source of income is labor income and there are no financial assets traded. Such a specification generates predictions that are practically identical to those in this setup. Another alternative is the model of Flandreau (1996), which also generates the same results as in this setup but does so largely by ignoring the issues related to production and firm ownership.

Theorem 3.1. *In each period t, the equilibrium quantities $Y_{st}, Y_{at}, Y_{ct}, Y_{gt}, M_{st}, M_{gt}$ and prices p_{st}, p_{at}, and p_{ct} are given by the solution to the system of equations*

$$M_{gt} + M_{st}p_{st} = Y_{ct}p_{ct} + Y_{at}p_{at} + Y_{st}p_{st} + Y_{gt}, \tag{3.4}$$

$$Y_{ct}p_{ct}^{\theta}(1-\mu_c) = \left(Y_{at}p_{at}^{\theta} + Y_{st}p_{st}^{\theta} + Y_{gt}\right)\mu_c, \tag{3.5}$$

$$Y_{at}p_{at}^{\theta}(1-\mu_a) = \left(Y_{ct}p_{ct}^{\theta} + Y_{st}p_{st}^{\theta} + Y_{gt}\right)\mu_a, \tag{3.6}$$

$$Y_{st}p_{st}^{\theta}(1-\mu_s) = \left(Y_{ct}p_{ct}^{\theta} + Y_{at}p_{at}^{\theta} + Y_{gt}\right)\mu_s, \tag{3.7}$$

$$C_t = Y_{ct}, \tag{3.8}$$

$$A_t = Y_{at}, \tag{3.9}$$

$$S_t = Y_{st} + M_{st}, \tag{3.10}$$

$$G_t = Y_{gt} + M_{gt}, \tag{3.11}$$

$$\alpha M_{gt} = M_{st}. \tag{3.12}$$

Equations (3.5)-(3.7) represent an agent's demand for the metallic commodity, the agricultural commodity, and silver in period t, while equations (3.8)-(3.11) represent the market-clearing conditions for the metallic commodity, agricultural commodity, silver, and gold markets in period t. Equation (3.4) simply follows from the cash in advance constraint equation (3.2). The demand for each good by an agent born in period t is easily derived and depends only on the relative prices and endowments of each good. This is a standard property of CES utility.

In general, the system of equations from Theorem 3.1 cannot be solved analytically in closed form. However, we are able to simulate this economy and generate correlations between the equilibrium prices p_{ct}, p_{at}, and p_{st}. These simulated correlations can then be compared with the actual correlations observed in the data. All that remains to fully specify the model, then, is some structure on the random shocks that affect the representative firm's production of the metallic commodity, the agricultural commodity, silver, and gold. We assume that the quantities of each good produced evolve according to logarithmic AR-1 processes, so that for all $t \geq 1$,

$$\log S_t = \bar{S} + \rho_S \log S_{t-1} + \tilde{s}_t \tag{3.13}$$

$$\log G_t = \bar{G} + \rho_G \log G_{t-1} + \tilde{g}_t \tag{3.14}$$

$$\log A_t = \bar{A} + \rho_A \log A_{t-1} + \tilde{a}_t \tag{3.15}$$

$$\log C_t = \bar{C} + \rho_C \log C_{t-1} + \tilde{c}_t, \tag{3.16}$$

where $\bar{S}, \bar{G}, \bar{A}, \bar{C} \in \mathbb{R}$ and $\rho_S, \rho_G, \rho_A, \rho_C$ are constants with values between zero and one, and the shocks $\tilde{s}_t, \tilde{g}_t, \tilde{a}_t$, and $\tilde{c}_t$ are all i.i.d. over time with $\tilde{s}_t \sim \mathrm{N}(0, \sigma_s^2)$, $\tilde{g}_t \sim \mathrm{N}(0, \sigma_g^2)$, $\tilde{a}_t \sim \mathrm{N}(0, \sigma_a^2)$, and $\tilde{c}_t \sim \mathrm{N}(0, \sigma_c^2)$. Note that this specification does not preclude some correlation across shocks to production.

There are a number of alternative specifications of this setup that generate similar predictions. Much like the cash-in-advance constraint from equation (3.2) above (with equality), however, practically all of these alternatives generate a positive relationship between money and nominal output in the spirit of a new-Keynesian aggregate demand curve.[6] The goal of this paper is to investigate the joint dynamics of the prices of silver, the agricultural commodity, and the metallic commodity in a bimetallic system, and this is largely determined by the positive relationship between money and output that is common among these setups. For this reason, we construct a relatively parsimonious model with the understanding that this model avoids complexities that distract from our main focus even though the model can be extended to include many of these complexities without affecting the main results.

[6]Two notable examples are Svensson (1985), who develops a model in which there is intertemporal consumption smoothing and money must be acquired before the representative agent knows how much to spend on consumption, and Woodford (2003), who presents a model in which a central bank adjusts the money supply to target a particular value for nominal GDP. In both cases, real money balances and output are positively linked.

4 Simulations

The most important contribution of the model described above is to demonstrate how the relationship between the price of silver and the price of agricultural commodities and other metallic commodities significantly changes as silver stops being used as a global unit of account. We also show that this changed relationship across various commodity prices matches what we observe in the data when we compare the period 1870-1896 with the period 1897-1913.[7]

In order to demonstrate this prediction of the model, we wish to perform simulations using several different calibrations of the model. For each of these simulations, we set the elasticity of substitution between goods $\theta = 0.5$. This value of θ represents relatively low substitutability across goods, an assumption that is supported by the fact that agricultural commodities, metallic commodities, silver, and gold are unlikely to be highly substitutable in the real world.[8] We also set the values of μ_c, μ_a, and μ_s equal to 0.4, 0.4, and 0.1, respectively.

For the constant α, which measures the proportion of the world money supply that is denominated in silver relative to gold, we consider several values that are greater than zero to represent scenarios in which silver is acting as a global unit of account. The results of the simulations are insensitive to the different values of $\alpha > 0$ that we choose. We then compare these results to a simulation with $\alpha = 0$, a calibration that represents a scenario in which silver is no longer acting as a global unit of account.

The values for the parameters that determine the behavior of the random shocks that affect the representative firm's production of the metallic commodity, the agricultural commodity, silver, and gold as given by equations (3.13)-(3.16) are chosen to match the observed time-series statistics in the price data for the period 1897-1913. More specifically, we choose $\alpha = 0$ and then calibrate the parameters from equations (3.13)-(3.16) so that the simulations match the time-series moments in the price data for 1897-1913.

The moments of the time-series data for the periods 1870-1896 and 1897-1913 are presented in Tables 1-3 below. The results for 1,000 simulations that follow the procedure described above are presented in Tables 4-5 below.

[7]The data for silver and various commodity prices is from Blattman, Hwang, and Williamson (2004).

[8]Note that an alternative specification might set θ equal to 1 so that each agent has Cobb-Douglas utility as in Flandreau (1996). This alternative specification does not qualitatively change the results of the model.

5 Conclusion

We examine the demise of the global bimetallic monetary system in the late nineteenth century. Previous studies have often focused on the role that different countries played in destabilizing the silver-gold price ratio of 15.5:1 during the 1870s. Instead, we focus on silver's continued influence on global commodity prices and the precious metal's role as a global unit account for the remainder of the classical gold standard period given that it took decades for many countries to switch to a monometallic gold standard. We develop a dynamic general equilibrium to examine silver's influence on global commodity prices from 1870-1913. Our evidence suggests that silver remained highly correlated with agricultural and metals prices until the mid 1890s. Furthermore, the model matches the increased volatility of commodity prices following silver's diminished role in global commodity price determination. With the defeat of the "silverites" and William Jennings Bryan's bid for President of the United States in 1896 and India's demonetization of silver in 1893, silver ceased to exert a significant influence on global prices and appears to have lost its role as a global unit of account.

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Good	Time Period	Standard Deviation (%)	Autocorrelation
Silver	1870-1896	0.224	0.990
Silver	1897-1913	0.086	0.961
Copper	1870-1896	0.311	0.987
Copper	1897-1913	0.188	0.946
Tin	1870-1896	0.350	0.975
Tin	1897-1913	0.290	0.980
Cotton	1870-1896	0.347	0.972
Cotton	1897-1913	0.254	0.969
Corn	1870-1896	0.278	0.909
Corn	1897-1913	0.276	0.954
Wheat	1870-1896	0.232	0.916
Wheat	1897-1913	0.190	0.895

Table 1: Data.

	Silver	Copper	Tin	Cotton	Corn
Copper	0.833	1.000			
Tin	0.662	0.755	1.000		
Cotton	0.864	0.865	0.819	1.000	
Corn	0.419	0.170	0.272	0.317	1.000
Wheat	0.749	0.715	0.457	0.600	0.321

Table 2: Data correlations 1870-1896.

	Silver	Copper	Tin	Cotton	Corn
Copper	0.726	1.000			
Tin	0.168	0.329	1.000		
Cotton	-0.177	-0.081	0.631	1.000	
Corn	-0.379	-0.185	0.560	0.547	1.000
Wheat	-0.327	-0.348	0.249	0.412	0.510

Table 3: Data correlations 1897-1913.

Good	Value of α	Standard Deviation (%)	Autocorrelation
Silver	0	0.136	0.938
Silver	0.25	0.266	0.946
Silver	0.5	0.241	0.944
Silver	0.75	0.234	0.941
Silver	1.0	0.234	0.941
Silver	1.5	0.227	0.941
Silver	2.0	0.225	0.945
Metallic Commodity	0	0.214	0.965
Metallic Commodity	0.25	0.330	0.968
Metallic Commodity	0.5	0.313	0.970
Metallic Commodity	0.75	0.312	0.969
Metallic Commodity	1.0	0.320	0.968
Metallic Commodity	1.5	0.312	0.970
Metallic Commodity	2.0	0.280	0.967
Agricultural Commodity	0	0.211	0.922
Agricultural Commodity	0.25	0.390	0.925
Agricultural Commodity	0.5	0.363	0.924
Agricultural Commodity	0.75	0.353	0.918
Agricultural Commodity	1.0	0.349	0.918
Agricultural Commodity	1.5	0.353	0.926
Agricultural Commodity	2.0	0.355	0.925

Table 4: 1,000 simulations.

Value of α		Silver	Metallic Commodity
0	Metallic Commodity	0.317	1.000
0	Agricultural Commodity	-0.184	-0.047
0.25	Metallic Commodity	0.785	1.000
0.25	Agricultural Commodity	0.900	0.571
0.5	Metallic Commodity	0.786	1.000
0.5	Agricultural Commodity	0.883	0.546
0.75	Metallic Commodity	0.767	1.000
0.75	Agricultural Commodity	0.881	0.543
1.0	Metallic Commodity	0.772	1.000
1.0	Agricultural Commodity	0.877	0.524
1.5	Metallic Commodity	0.757	1.000
1.5	Agricultural Commodity	0.882	0.535
2.0	Metallic Commodity	0.709	1.000
2.0	Agricultural Commodity	0.886	0.480

Table 5: 1,000 simulations.