The Effect of Mortgage Payment Reduction on Default: Evidence from the Home Affordable Refinance Program

Jun Zhu

Jared Janowiak

Lu Ji

Kadiri Karamon

Douglas McManus

January 2014

Abstract: This paper evaluates the effect of payment reduction on mortgage default within the context of the Home Affordable Refinance Program (HARP). We find that mortgage default is sensitive to payment reduction across univariate, duration, and hazard modeling approaches. A relative risk Cox model of default with time-varying covariates estimates that a 10% reduction in mortgage payment is associated with about a 10 to 11% reduction in monthly default hazard for loans. This finding is robust to the inclusion of empirically important mortgage risk drivers (such as current LTV and FICO score) as well as controlling for selection effects based on observables. A theorem is developed that allows for interpreting monthly default hazard estimates from the perspective of cumulative default.

Author contacts: jzhu@urban.org, jared_janowiak@freddiemac.com, lu_ji@freddiemac.com, kadiri_karamon@freddiemac.com, and douglas_mcmanus@freddiemac.com.

1. **Introduction**

Housing default played a central role in the 2008 financial crisis and subsequent great recession. Policy interventions to prevent default have focused on reducing payments for borrowers with negative equity. A key question for policymakers and lenders in designing interventions for homeowners with negative equity is understanding the effect of payment reduction on default. Investors who manage portfolios of mortgages with variable payments, such as adjustable rate mortgages (ARMs), also need to understand this relationship for valuation purposes. Some research on this topic has been conducted, primarily by examining the effects of payment changes as a result of interest rate movements for ARM loans. A lack of any sort of resetting features for fixed-rate loans has precluded analysis for this market segment, which makes up the single largest share of outstanding residential mortgages. This paper addresses this gap by using Freddie Mac's data from the U.S. Treasury Home Affordable Refinance Program (HARP). The HARP program offers refinancing options to borrowers with loans guaranteed by Freddie Mac or Fannie Mae who may otherwise be unable to get traditional refinancing due to deterioration in the value of their home; many of these borrowers are able to lower their payments as a result. This unique data allows for the analysis of the effect of payment reduction on default among fixed-rate borrowers with little or negative equity.

The HARP program began in April 2009. For a loan to be eligible to refinance with Freddie Mac under HARP, it must meet several criteria[1]. First, the loan must have been sold to Freddie Mac or Fannie Mae prior to June 2009. Second, the borrower must have made all mortgage payments for the twelve months prior to the date of application for the refinance. Third, it must have a current loan-to-value (LTV) ratio above 80% and up to 125%. When refinancing under the HARP program, borrowers are able to choose different mortgage product types. For example, a borrower with a fixed-rate 30-year mortgage could switch to a mortgage with a shorter term (e.g. 15 years) or to an ARM product. Such changes may influence the default probability of the loan,

[1] The HARP Program was expanded for loans funded on are after January 1, 2012. This expansion, referred to as "HARP 2", relaxed some of the eligibility requirements. First, the upper limit of 125% LTV was removed. Second, the delinquency requirement is slightly relaxed, allowing borrowers who missed at most a single payment 7 to 12 months prior to the HARP application date to participate. The available window of payment performance following refinancing for these loans is too short for estimation and therefore they are excluded from this research. Note the eligibility criteria for loans to refinance with Fannie Mae under the HARP program differ slightly.

presenting a selection problem and consequently making it difficult to isolate the effect of payment reduction. In order to control for this selection, we restrict our sample to the most common mortgage choice observed in our available data — borrowers with fixed-rate 30-year mortgages refinancing into fixed-rate 30-year mortgages[2].

Within this data we find that cumulative default rates are significantly lower for borrowers who receive relatively larger reductions in payment as a result of refinancing. In a hazard modeling framework, we estimate that default is roughly unit elastic with respect to payment reduction; a 10% reduction in payment corresponds to about a 10% to 11% reduction in the monthly default hazard for fixed-rate 30-year loans. This result is consistent with some recent studies of payment reduction in ARM loans (e.g., Fuster and Willen (2012), Tracy and Wright (2012)) and extends this area of research to a set of mortgages with potentially greater relevance to policymakers.

The rest of the paper is organized as follows. First, the relevant literature on mortgage default and payment reduction is reviewed. Second, the dataset is described in more detail and summary statistics are given. Third, the empirical hazard model is described and parameter estimates are reported, followed by a discussion of these results and a conclusion.

2. Literature Review

This research touches on several strands of literature. First, it builds on earlier work (Zhu (2012)) that evaluates the impact of the HARP program on mortgage default by estimating an average treatment effect of HARP, comparing HARP refinances to similar HARP-eligible loans that did not participate in the program. Zhu (2012) finds a strong treatment effect in terms of lower default rates among loans participating in the HARP program, and this finding is robust to several approaches to accounting for selection effects related to program participation. This paper extends this work by focusing within the HARP program to understand the differential effect of payment reduction.

[2] 30-year term loans that refinance into shorter term loans tend to have very small default rates empirically. This phenomenon is consistent with the dual-trigger hypothesis of mortgage default, in that the choice to shorten the term likely indicates a liquidity position strong enough to absorb a payment increase.

Two related empirical papers examine the effect of payment reduction on mortgage default. Fuster and Willen (2012) examine the effect of reduced payment for interest-only (IO) ARM mortgages due to declines in interest rates, implementing a competing risk model of default and prepayment. One limitation of their research is that it focuses on a narrow segment of the market, ARM IO loans within the private-label securities. Tracy and Wright (2012) take a similar approach to ours in examining payment changes for ARM loans due to downward interest rate adjustments within the GSE loan population. In contrast to these studies, our research considers loans in the broad fixed-rate conforming (GSE) marketplace.

This research also nests into the broader literature of mortgage default. The classical theory of mortgage default is an option-based approach wherein the option to default (a "put") is exercised by a borrower if the equity in the property (e.g., the value of the property less principal owed) falls below some threshold (see for example, Foster and Van Order (1984); Kau, Keenan and Kim (1994); and Vandell (1995)) for an extensive review of this literature). Another vein of research extends this model by combining the "in-the-money" put option of default with liquidity constraints, commonly referred to as the "dual trigger" model (Elmer and Seelig (1999); more recently Campbell and Cocco (2011)). Research from the recent mortgage crisis has focused on the dual trigger model, finding evidence that both liquidity and equity factors drive mortgage default (Elul et al. (2010)). This research takes a dual trigger approach by modeling the default decision controlling for the property equity of the borrower as well as liquidity via payment reduction.

Finally, this research makes a technical contribution through the adjustment for selection in hazard models using inverse probability weighting. Inverse probability weighting (IPW) is a common technique for adjusting for sample selection in the context of linear regression models (for example, see Wooldridge 2010). Of concern in our research is whether or not the estimated effect of payment reduction on default is biased due to observing only certain types of borrowers that choose to participate in the HARP program. If a representative sample of observations not receiving the treatment (e.g. not participating in the HARP program) is available, the IPW methodology is to first estimate a logistic model of selection based on observables. The second step is to estimate the model of interest, weighting observations by the inverse of their predicted probabilities from the first stage logistic model. Pan and Schaubel (2008) extend the IPW methodology to hazard models and show that weighting observations by these inverse probabilities in the partial information likelihood function of a hazard model leads to asymptotically unbiased estimates. As far as the authors know, this is the first paper in economics to use this methodology.

3. Data and Summary Statistics

We use Freddie Mac loan-level data for our empirical analysis. This data contains detailed information about loan and borrower characteristics of HARP refinances funded by Freddie Mac from the inception of the program in April 2009 through December 2011. Monthly loan performance is observed on these loans from origination of the loan through September 2013. The sample is restricted to loans where the previous mortgage and subsequent HARP refinance both are fixed-rate loans with a 30-year term. This restriction makes the analysis of payment reduction (the central aim of this research) clearer by avoiding issues around changes in product and term. The data includes traits of the HARP refinance mortgage itself as well as the previous loan. Table 1 reports summary statistics for variables of interest. The data consists of a random subsample of 64,810 HARP refinances with 2.06 million loan-month records. The naming convention in this analysis is that variables prefixed "pre" correspond to the mortgage prior to refinancing, and "post" correspond to the HARP refinance mortgage. For example, "Post FICO" refers to the FICO credit score used to originate the HARP refinance, while "Pre FICO" refers to the credit score used to originate the prior loan that eventually was refinanced in HARP.

[Insert Table 1]

First, note "Post LTV" is on average higher than "Pre LTV", with a mean of 95% compared to 79%. Loans in this sample have experienced deterioration in home values. Second, observe that the note rates on average have declined as a result of refinancing through the HARP program. Average note rates declined from 6.17% to 4.97%. Since the sample is restricted to borrowers refinancing from a fixed-rate 30-year product into another fixed-rate 30-year product, this decline in note rate leads to a reduction in monthly principal and interest payments. This is seen in comparing "Post Payment" to "Pre Payment", which on average declines by $211 per month from $1,410 to $1,198. Measured in percentage terms, the average reduction in monthly principal and interest (P&I) is about 15%.

Finally, note that the FICO credit score of these borrowers has tended to rise from the time of origination of the original mortgage to the time of HARP refinancing. This reflects the HARP requirement that while these borrowers must have high LTV ratios, they must have been current on their mortgage payments for the previous 12 months.

[Insert Figure 1]

Figure 1 shows a histogram of the percent payment reduction as a result of HARP refinancing. Payment reduction in this research is defined in percent terms as

$$Payment\ Reduction = 100 * \frac{Pre\ Payment - Post\ Payment}{Pre\ Payment}$$

where payments are defined as principal and interest (P&I). The distribution of payment reduction is centered at a median of 15% with a first quartile of 12% and a and third quartile of 19%. The concentration of this distribution suggests that results from this dataset may not be appropriate to extrapolate very large payment reductions.

Tables 2 reports average default rates cross-sectionally across the loans in the sample by the percentage of payment reduction. The default event is defined as the loan being cumulatively delinquent for three or more months at any time following the HARP refinance (referred to as “ever D90+”). The sample has an average performance history of 32 months, with a maximum possible history of 50 months. There is a clear negative relationship between payment reduction and default among HARP loans, with default rates for loans receiving 20% or greater payment reduction being less than half that of loans with reductions between 0% and 10%. These univariate statistics support the view that greater reductions in payment are associated with greater reductions in default.

[Insert Figure 2]

The univariate statistics reported in Table 2 do not condition on other factors normally associated with default, nor do they take into account potential differences in the ages of loans; such differences could bias univariate summaries. To correct for the differences in duration of loans prior to default, Figure 2 reports Kaplan-Meier default estimates for loans in the sample stratified by the amount of payment reduction. The qualitative pattern for HARP loans displayed in Table 2 persists in Figure 2; greater levels of payment reduction are associated with lower cumulative default rates when controlling for the duration of the loan prior to default. Inspecting Figure 2 can give a rough approximation of the effect of payment reduction not controlling for other factors. For instance, at the 46 month horizon, the Ever D90+ rate for loans with a 0% to 10% payment reduction is 6.4%, and 5.9% for loans with a 10% to 15% payment reduction. Taking the midpoints of these intervals as 5% and 12.5% suggests a 7.5% reduction in payment implies a (6.4% - 5.9%)/6.4% = 7.8% reduction in default rate, or a 10% reduction in payment corresponds to about a 10% reduction in the monthly default hazard; this result is fairly similar to those from models reported below that control for other variables known to influence default behavior.

4. **Empirical Model Specification**

The summary statistics reported in the last section fail to take into account the influence of other covariates on mortgage default besides payment reduction. The empirical approach taken to correct for this is to model the monthly default hazard of a loan using a Cox relative risk model[3]. In this framework, the instantaneous probability of default is described by a hazard function:

$$h(t) = \lim_{\tau \to 0} \frac{P(t \leq T < t + \tau | T \geq t)}{\tau}$$

This hazard function for loan *i* at time *t* is modeled as

$$h(t; x_{it}) = h_0(t) \exp[\beta' x_{it}]$$

[3] Note that in many instances the term "Cox model" is used to describe a proportional hazards model. However, in the case where time-varying covariates are included, the proportional hazard property no longer holds. This paper adopts the convention of referring to this as a relative risk model, as in Kalbfleisch and Prentice (2002).

where x_{it} is a vector of (possibly time-varying) covariates and $h_0(t)$ is an arbitrary baseline hazard function. This general class of models is used in studying time-to-failure when data is right censored. In this context, the failure event is mortgage default. For the remainder of this study, default is defined as the first time a loan becomes more than three months' delinquent (abbreviated D90+).

This modeling approach is similar to that taken in other recent work on mortgage default (see, for example, Haughwout, Okah, and Tracy (2010)). Besides defaulting, mortgages can also terminate by prepayment. The model expressed above only directly models default and treats prepayment as a censored observation. The motivation for this, as opposed to a richer model capturing default and prepayment behavior (say, by a competing hazards framework) is that the likelihood of a HARP loan terminating through prepayment (e.g. refinancing) within this sample is quite low. The program is designed to provide refinance opportunities to borrowers who otherwise would not meet underwriting criteria necessary to refinance via traditional market channels.

5. **Empirical Estimation Results**

[Insert Table 3]

Table 3 reports coefficient estimates from Cox relative risk regression of default estimates of various specifications for the HARP sample. All reported coefficients are in the hazard ratio form $exp(\hat{\beta})$. Specifications (1) and (2) in Table 3 are the main models of interest, containing payment reduction and other control variables of interest. In specification (1), controlling for state and vintage effects using fixed effects, the estimated hazard ratio of payment reduction is 0.896. A convenient way to interpret hazard ratios for continuous variables is to subtract 1.0 from the estimate and multiply by 100. The result (-10.4) is the semi-elasticity of the monthly default hazard with respect to payment reduction. Though this entire analysis is focused on measuring the elasticity of default with respect to payment reduction, we choose to scale the units of payment reduction for all regression analyses by a factor of 10 (e.g., 1.0 corresponds to 10% payment reduction, 2.0 corresponds to 20% payment reduction, etc) to be able to frame results as relative to a 10% payment change. The authors feel this is a more intuitive framing of the issue than estimating a purely marginal elasticity.

Therefore, a 10% increase in the payment reduction, holding all other variables in the model constant, is associated with a 10.4% reduction in the monthly default hazard, with a 95% confidence interval of 1.0% to 18.2%. Similarly, in specification (2) controlling for state and vintage effects with strata (where the nonparametric baseline hazard is allowed to vary by state and vintage), a 10% payment reduction corresponds to a 10.2% reduction in monthly default hazard.

Current LTV (CLTV) is a time-varying covariate in the model. In each period *t* following the origination of the HARP refinance, an estimate of the current property value it is calculated using the Freddie Mac Weighted Repeat Sales Index (WRSI) house price index. CLTV is defined as the current unpaid loan balance divided by this estimate of property value. Inclusion of this variable allows the model to capture movement in house prices following origination and the equity position of the borrower. Its hazard ratio estimate in specification (1) is 1.027. Using the interpretation suggested above implies a one percentage-point increase in the CLTV in month *t* raises the hazard of default in that month by 2.7%. The CLTV estimate is also statistically significant at the 99% level. The hazard ratio for CLTV is relatively stable across specifications.

Two additional time-varying covariates are included to control for changing macroeconomic conditions. The first is 2-Year HPA (measured at the zip-code level with the Freddie Mac Weighted Repeat Sales Index). This is the change in house prices (as measured by the Freddie Mac WRSI house price index) over the previous two years; specifically, in month *t*, the annualized percentage change in house prices between months *t*-24 and *t*. Its estimate is statistically significant across each specification, suggesting recent house price momentum influences default even if CLTV is already included as a covariate. It has the expected negative relationship, with a hazard ratio of less than unity implying greater recent house price appreciation is associate with lower levels of default. The unemployment rate in percent, reported at the county level by the Bureau of Labor Statistics, is included to capture local labor market conditions though not statistically significant in any of the specifications. This variable may be too coarse to be strongly correlated with an individual household's income and liquidity.

Two FICO-related variables are included in the analysis. Post FICO is the FICO score at the time of origination of the HARP refinance. Delta FICO is the difference in FICO scores between the time of origination of the HARP refinance and the origination of the previous loan. Post FICO is statistically significant at the 99% level across all specifications with a hazard ratio of between 0.991 and 0.992. A one unit increase in Post FICO is associated with a 0.8% to 0.9% decrease in the monthly default hazard. Delta FICO is not statistically significant, suggesting that the migration of the borrower's credit quality over the life of the previous loan has no influence on mortgage default when controlling for the FICO score at the time of the HARP origination.

The overall picture emerging from the results in specifications (1) and (2) in Table 3 is that FICO, CLTV, and payment reduction are statistically significant determinants of default at the 1% level. Information related to the previous loan is generally not statistically significant in this analysis. A natural question is whether or not there is an interaction effect between Post FICO score or Post LTV and payment reduction. Borrowers' default sensitivity to payment reduction may also depend on these other two measures known to influence default. Tables 4a and 4b present model estimates similar to those presented in Table 3, but segmented by New LTV and New FICO, respectively. Table 4a suggests that borrowers with relatively higher LTV ratios are relatively more sensitive to payment reduction, more than twice more sensitive than for those borrowers with some equity in their property. Table 4b suggests that default sensitivity to payment reduction does not vary much depending upon the borrower's FICO score at the time of refinancing.

This evidence of interaction between payment reduction and LTV has important policy implications. It suggests that payment reduction has a significant effect on default both for borrowers with equity and those underwater. Additionally, borrowers who are underwater are far more sensitive to such payment reductions. Similarly, the lack of significant interaction between payment reduction and FICO score suggests that both borrowers of varying credit quality are responsive to payment reduction (in terms of default) in roughly similar ways. It is important to note that estimates for the other model parameters are generally stable compared to the specification without these interactions.

6. **Robustness Check – Selection into HARP Program**

One concern with the results reported in Tables 3, 4a, and 4b is that there is a selection effect in terms of which borrowers choose to refinance within the HARP program. If borrowers who choose to participate are motivated to stay in their homes and therefore less likely to default, the estimate of the effect of payment reduction may be biased. To control for this, an inverse probability weighting approach is taken. Pan and Schaubel (2008) extend inverse probability weighting via a logistic selection equation to hazard models, and we follow their approach here. First, a logistic model of HARP participation is estimated on a sample of HARP and eligible non-HARP loans. Predicted probabilities of HARP participation are taken from this model and used as inverse weights in a hazard model of default.

The sample used for the selection model consists of all of the HARP loans in our existing sample, as well as a random sample of 743,725 loans that are observed to be eligible for HARP participation but do not enter the program. This stacked sample is stratified based on vintage (for HARP loans, the year of HARP refinancing; for eligible loans that do not HARP, the year of observed eligibility). A separate logistic model of HARP participation is estimated for each year with FICO score, splined CLTV, splined UPB, and servicer and state fixed effects. Table 5a reports the logit parameter estimates for the continuous variables from this model. Table 5b reports type III analysis for the model, both for the coefficients listed in Table 4a as well as the categorical state and servicer fixed effects.

First, note that the servicer fixed effect variable has the largest Type III SS for each regression vintage. The particular servicer for a loan has a stronger influence on participation within the HARP program than the other variables included in the model. The next most influential variable in terms of Type III SS is loan balance (UPB). The coefficient on UPB in Table 5a is positive, indicating larger balance loans are more likely to participate in the HARP program. A spline is included with a knot point at $200,000[4]. This spline term is generally negative which acts to lower the importance of UPB above $200,000, but it is still the case that increases in UPB are associated with higher rates of HARP refinancing, even above $200,000. This makes intuitive sense; larger balance loans will have proportionally large monthly P&I payments. Participation in HARP will lead to a large savings in absolute dollar terms for such large balance borrowers.

[4] The spline basis function used is UPB_Spline = max{UPB - $200,000,0}. The UPB slope moves from 1.2 for balances below $200,000 to 1.2 – 0.92 = 0.28 for balances above $200,000.

CLTV, measured here as the LTV ratio at the time of HARP refinancing (for HARP loans) or HARP eligibility (for eligible loans that do not HARP), is the next most influential variable in terms of Type III SS. A spline variable is included at 100% LTV. Loans having a CLTV of less than 100% at the time of observed eligibility are associated with a lower likelihood of participation in HARP, while loans having CLTV of 100% or more are more likely to participate. Qualitatively, borrowers who are not underwater are relatively less likely to participate in HARP.

Finally, note that pre-FICO (a seasoned measurement observed at origination of the eligible non-HARP loan, or at the time of origination of the pre-HARP loan) score is not a strong driver of HARP participation. Since we do not observe updated FICO credit scores for the HARP eligible sample, we use the origination FICO on these loans.

With predicted probabilities of HARP participation from the models presented in Tables 5a and 5b, we re-estimate the default hazard models presented in Table 3 using the inverse probability weighting. The results are displayed in Table 6. First, note that the hazard ratios of payment reduction in specifications (1) and (2) are very similar between the results with and without the inverse probability weights. A slightly stronger effect of payment reduction is estimated when using these weights to control for selection based on observables, with a 10% reduction in payment being associated with a 11% reduction in monthly default hazard (in the case of fixed effects) or 11.9% (in the case of baseline hazard stratification). The other covariate weighted parameter estimates in Table 6 are relatively stable in comparison with the unweighted estimates in Table 3. This suggests that whatever selection process there is associated with borrowers choosing to participate in HARP based on observable factors, it does not strongly affect the default sensitivity of borrowers with respect to FICO, LTV, local area unemployment, or recent HPA experience.

Tables 7a and 7b present model estimates segmented by LTV and FICO of the new HARP loan to analyze interaction between these variables and payment reduction. These are analogous to those results presented in Table 4, only now weighting observations using their inverse probability weights from the first-stage logistic regression. The same qualitative trend in Table 4a is present in Table 7a – borrowers with higher LTV ratios at the time of HARP refinancing display greater sensitivity to payment reduction when compared to borrowers with lower LTV ratios. The evidence from Table 7b is less clear. While the unweighted estimates in Table 4b suggest that the payment reduction effect is relatively consistent across different FICO score segments, the weighted estimates in Table 7b are inconclusive in regards to the lowest category (FICO ≤ 700). Nevertheless, the estimates for the two other categories of FICO borrowers (700 to 750 and 750 and above) are relatively similar and in line with those obtained from a model without using the inverse probability weights. On balance, there is not sufficient evidence to assert that the sensitivity of borrowers to payment reduction varies across FICO.

7. **Selection within HARP Program**

Another possible concern regarding the validity of the estimates presented here is that there may be selection issues within the HARP program. For example, if borrowers were able to in some way influence the degree of payment reduction they receive from HARP refinancing, then motivation or ability to do so may be correlated with their propensity to default. Borrowers with less than 20% equity are generally unable to refinance in the existing marketplace, and as such the HARP program is the only opportunity available for such refinancing (without paying down principal to sufficiently reduce the LTV ratio to 80% or below). Though the HARP program technically allows for borrowers to pursue refinancing with servicers who are not their existing servicer, this option is generally not taken; more than 80% of borrowers refinance with their existing servicer or lender. Consequently, they are likely price takers with respect to the note rate on the new HARP refinance, with little ability to influence the rate on the new loan (and therefore the degree of payment reduction they receive). Based on this feature, the authors contend there is little opportunity for selection issues regarding the degree of payment reduction within the sample of HARP refinances.

8. **Discussion**

The payment reduction elasticity of monthly default hazard described in the last section is interpreted as a 10% payment reduction corresponds to a 10% to 11% reduction in default hazard. In comparing this result with other findings, recall that payment reduction here is defined as percentage reduction in the principal and interest payment. There would be a stronger effect if one were to apply this point estimate to a program that targets a broader payment measure such as principal, interest, taxes, insurance, and association fees (PITIA).

This result is of a similar magnitude to those computed in other recent research on the effects of payment reduction on default. Fuster and Willen (2012) find that cutting borrower payments in half lowers the monthly default hazard by approximately two-thirds; assuming linearity, this is very similar to the findings presented here, suggesting a 10% payment reduction corresponds to about a 13% reduction in default hazard. They focus on interest-only (IO) mortgage loans and examine payment changes within the IO period of the loan. While technically they examined changes in the interest payment only, it is analogous to the P&I payment of a fixed-rate mortgage in the sense that it is the portion of a mortgage payment excluding taxes, insurance, and association fees. To this extent, we believe a direct comparison of our point estimates with theirs is appropriate. Tracy and Wright (2012) computed a 22.5% reduction in monthly default hazard from a 10% payment reduction for borrowers with current LTV above 80% (16.6% reduction for borrowers with current LTV at or below 80%). It is important to note that the payment changes observed by Tracy and Wright (2012) are materially smaller than those observed in this sample or in Fuster and Willen (2012), and their larger magnitude estimates may be partially due to the smaller incremental payment changes observed in their data.

Modeling monthly default hazard is a parsimonious econometric approach to take in that typical mortgage performance datasets provide monthly information of the payment history of the loan. However, at times it may be of interest to understand the effect of payment reduction on the cumulative default probability of the loan (say, over its lifetime). A theorem developed in this paper (see appendix for details) derives a relationship between the elasticity of monthly default hazard and the elasticity of cumulative default. It finds that if the monthly default hazard elasticity of a variable is θ, then the cumulative default probability elasticity of that variable is between $\theta[1\text{-}F]$ and θ, where F is the estimated level of cumulative default. For example, the estimated monthly default hazard elasticity of payment reduction in this research is approximately -1.0 to -1.2. If the view of lifetime default probability of a particular loan (prior to payment reduction) were that it has a 10% probability of default, then applying this estimate would suggest that a 10% payment reduction would lead to a reduction in lifetime default probability of between 9% and 12%. This result is a useful guide for interpreting estimates more globally. It is intuitive that the sensitivity of lifetime default should not exceed that of the average monthly hazard rate. To our knowledge this is the first analytic approach taken to link monthly hazard estimates to the concept of cumulative or lifetime default.

9. Conclusion

In this paper, we examined the effect of reducing mortgage payments for borrowers on default within the HARP program. It was shown that there are significantly lower default rates for loans receiving larger reductions in payment: a reduction in payment by 10% reduces expected defaults by 10-11%. This result holds even when taking into account the duration dimension of loan default in a model which controls for other variables known to strongly influence mortgage default. We find results that are qualitatively similar to other recent work analyzing ARM resets and payment reduction, and add to this literature by using unique data from HARP refinances. The estimated effect of payment reduction is similar for borrowers of differing FICO credit scores, and is relatively stronger for those with negative equity. An inverse probability weighting approach is taken to control for potential selection bias based on observables, and does not significantly alter our estimates. Finally, we develop a theorem that links monthly hazard estimates to cumulative default.

10. Appendix: Linking Monthly Default Hazard to Lifetime Default

A primary goal of this paper is to estimate the default elasticity of payment reduction, where default is characterized as a monthly hazard rate. Because mortgage performance data typically is observed at a monthly frequency, it is convenient to develop monthly default models. However, it is frequently of interest to understand what a view of lifetime default is, or in this case, what the effect of payment reduction means on the probability of ever defaulting at some point in life of a loan. The following theorem characterizes the relationship between these two elasticities. The elasticity of lifetime (or cumulative) default is bounded above by the monthly hazard elasticity, and below by a multiple of the monthly hazard that depends upon the cumulative default level.

Theorem: Assume a monthly default hazard function $h(t,p)$ exists, is differentiable with respect to p, and integrable with respect to t where t is time and p is some other variable of interest that is not dependent on t. If the elasticity of monthly default with respect to p, $\eta_h(t,p) = \frac{\partial h(t,p)}{\partial p}\frac{p}{h(t,p)} = \theta(p)$, is not a function of t then the elasticity of cumulative default with respect to p is bounded above by $\theta(p)$and below by $\theta(p)$[1-F] where F is the cumulative default level.

Proof: First, the definition of a cumulative distribution function of default given a hazard function that depends upon t and some other variable p is

$$F(t,p) = 1 - \exp\left[-\int_0^t h(s,p)ds\right]$$

The elasticity of lifetime default with respect to p is

$$\eta_F(t,p) = \frac{\partial F(t,p)}{\partial p}\frac{p}{F(t,p)} = \exp\left[-\int_0^t h(s,p)ds\right]\left[\frac{\partial}{\partial p}\int_0^t h(s,p)ds\right]\frac{p}{F(t,p)} = p\frac{1-F(t,p)}{F(t,p)}\left[\frac{\partial}{\partial p}\int_0^t h(s,p)ds\right]$$

Assuming $h(t,p)$ is differentiable with respect to p, we can apply Lebintz's Rule[5] to differentiate under the integral sign

$$\eta_F(t,p) = p\frac{1-F(t,p)}{F(t,p)}\left[\frac{\partial}{\partial p}\int_0^t h(s,p)ds\right] = p\frac{1-F(t,p)}{F(t,p)}\left[\int_0^t \frac{\partial h(s,p)}{\partial p}ds\right]$$

The definition of elasticity with respect to p can be rearranged to give

$$\frac{\partial h(s,p)}{\partial p} = \eta_h(t,p)\frac{h(t,p)}{p} = \frac{\theta(p)}{p}h(t,p)$$

and substituted into the previous expression to give

$$\eta_F(t,p) = \theta(p)\frac{1-F(t,p)}{F(t,p)}\left[\int_0^t h(s,p)ds\right] = \theta(p)\frac{1-F(t,p)}{F(t,p)}ln\left(\frac{1}{1-F(t,p)}\right)$$

[5] See Casella and Berger (2002), p. 69 for a statement of the theorem.

The assumption that the elasticity of monthly default hazard does not depend upon *t* is necessary for this simplification (e.g. the factoring of $\theta(p)$ out of the integral expression). It can be shown this expression is monotonically decreasing in *F* over the unit interval and L'Hospital's rule gives that it is bounded above by $\theta(p)$ and below by 0. A stricter lower bound can be established by inspecting the infinite Taylor (Mercator) series representation of the natural logarithm

$$\ln[1 - F(t,p)] = -F(t,p) - \left(\frac{F(t,p)^2}{2} + \frac{F(t,p)^3}{3} + \frac{F(t,p)^4}{4} + \cdots\right)$$

This expression will be more negative than $-F(t,p)$ for $0 < F(t,p) < 1$. Re-arranging $\ln[1 - F(t,p)] < -F(t,p)$ gives $1 < \frac{1}{F(t,p)} ln\left(\frac{1}{1-F(t,p)}\right)$, providing the lower bound of $\theta(p)$[1-*F*] for $\eta_F(t,p)$. ■

References

Campbell, T., and Dietrich, J. K. (1983). The Determinants of Default on Conventional Residential Mortgages, Journal of Finance, 48(5), 1569-1581.

Campbell, J., and Cocco J. (2011). A Model of Mortgage Default, NBER Working Papers 17516, National Bureau of Economic Research, Inc.

Casella, G., and Berger, R. L. (2002). Statistical Inference (Second Edition). Pacific Grove, CA: Duxbury/Thomson Learning.

Cordell, L., Dynan, K., Lehnert, A., Liang, N., and Mauskopf, E. (2009). Designing Loan Modifications to Address the Mortgage Crisis and the Making Home Affordable Program. Federal Reserve Board, Finance and Economic Discussion Series, 2009-43.

Demyanyk, Y., Koijen,R., and Van Hemert, O. (2011). Determinants and Consequences of Mortgage Default. Working paper.

Elmer, P., and Seelig, S. (1999). Insolvency, Trigger Events, and Consumer Risk Posture in the Theory of Single-Family Mortgage Default. FDIC Working Paper 98-3.

Elul, R., Souleles, N., Chomsisengphet, S., Glennon, D., and Hunt, R. (2010). What `Triggers' Mortgage Default? Federal Reserve Bank of Philadelphia Working Paper No. 10-13.

Fuster, A., and Willen, P. (2012). Payment Size, Negative Equity, and Mortgage Default. FRB of New York Staff Report No. 582. Available at SSRN: http://ssrn.com/abstract=2186626.

Gerardi K., and Li, W. (2010). Literature Review of Foreclosure Prevention Efforts. Working Paper.

Gerardi, K., Shapiro, A., and Willen, P. (2008). Subprime Outcomes: Risky Mortgages, Homeownership Experiences, and Foreclosures. Federal Reserve Bank of Boston working paper.

Haughwout, A. Okah, E., and Tracy, J. (2009). Second Chances: Subprime Mortgage Modification and Re-Default. FRB of New York Staff Report No. 417. Available at SSRN: http://ssrn.com/abstract=1527870.

Kau, J., Keenan, D., and Kim, T. (1994). Default Probabilities for Mortgages. Journal of Urban Economics, 35, 278-296.

Pan, Q. and D. Schaubel (2008). Proportional Hazards Models Based on Biased Samples and Estimated Selection Probabilities. The Canadian Journal of Statistics / La Revue Canadienne de Statistique, 36(1), 111-127.

Tracy, J. and Wright, J. (2012). Payment Changes and Default Risk: The Impact of Refinancing on Expected Credit Losses. FRB of New York Staff Report No. 562. Available at SSRN: http://ssrn.com/abstract=2080261 or http://dx.doi.org/10.2139/ssrn.2080261

Vandell, K. (1995). How Ruthless is Mortgage Default? A Review and Synthesis of the Evidence. Journal of Housing Research, 6 (2), 245-264.

Wooldrigdge, J. (2010). Econometric Analysis Of Cross Section And Panel Data. London: The MIT Press.

Zhu, J. (2012). Refinance and Mortgage Default: An Empirical Analysis of the HARP's Impact on Default Rates. Available at SSRN: http://ssrn.com/abstract=2184514.

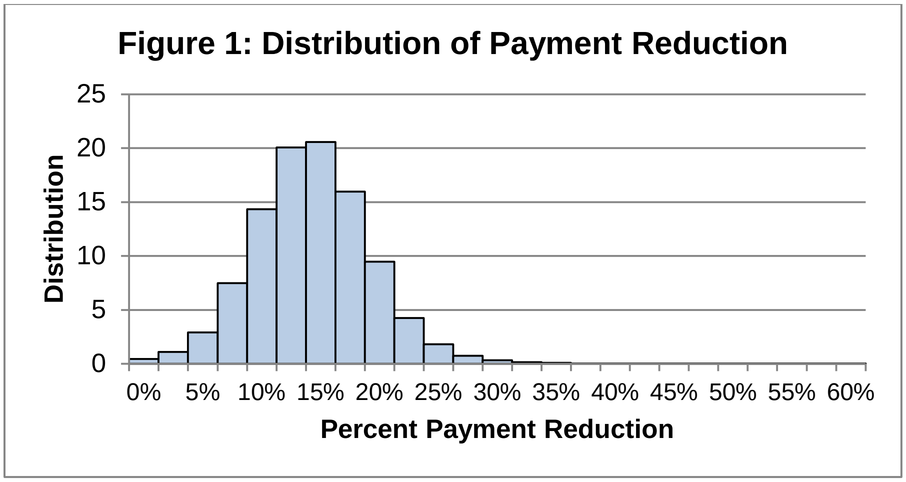

Figure 1 reports the empirical histogram of payment reduction in the sample of HARP refinances. Payment reduction is defined as the percentage change in payment as a result of HARP refinances. The sample is restricted to loans where payment reduction is nonnegative (e.g. payment does not go up). The sample consists of 64,810 randomly selected Freddie Mac HARP refinances of 30-year fixed-rate mortgages into 30-year fixed-rate mortgages.

Figure 2: HARP 1: Kaplan-Meier Failure Estimates by Payment Reduction

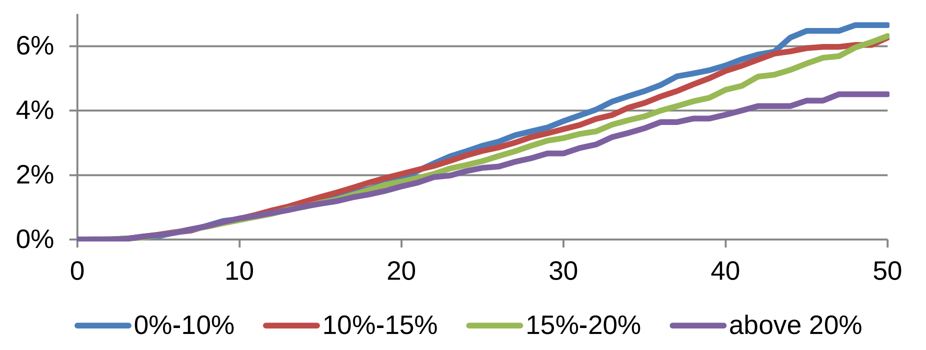

Figure 2 reports Kaplan-Meier default curves for the sample stratified by payment reduction (the percent change in payment as a result of HARP refinancing). The Y-axis measures cumulative defaults and the X-axis is the loan age in months. The sample consists of 64,810 randomly selected Freddie Mac HARP refinances of 30-year fixed-rate mortgages into 30-year fixed-rate mortgages, with of 2.06 million loan-month records.

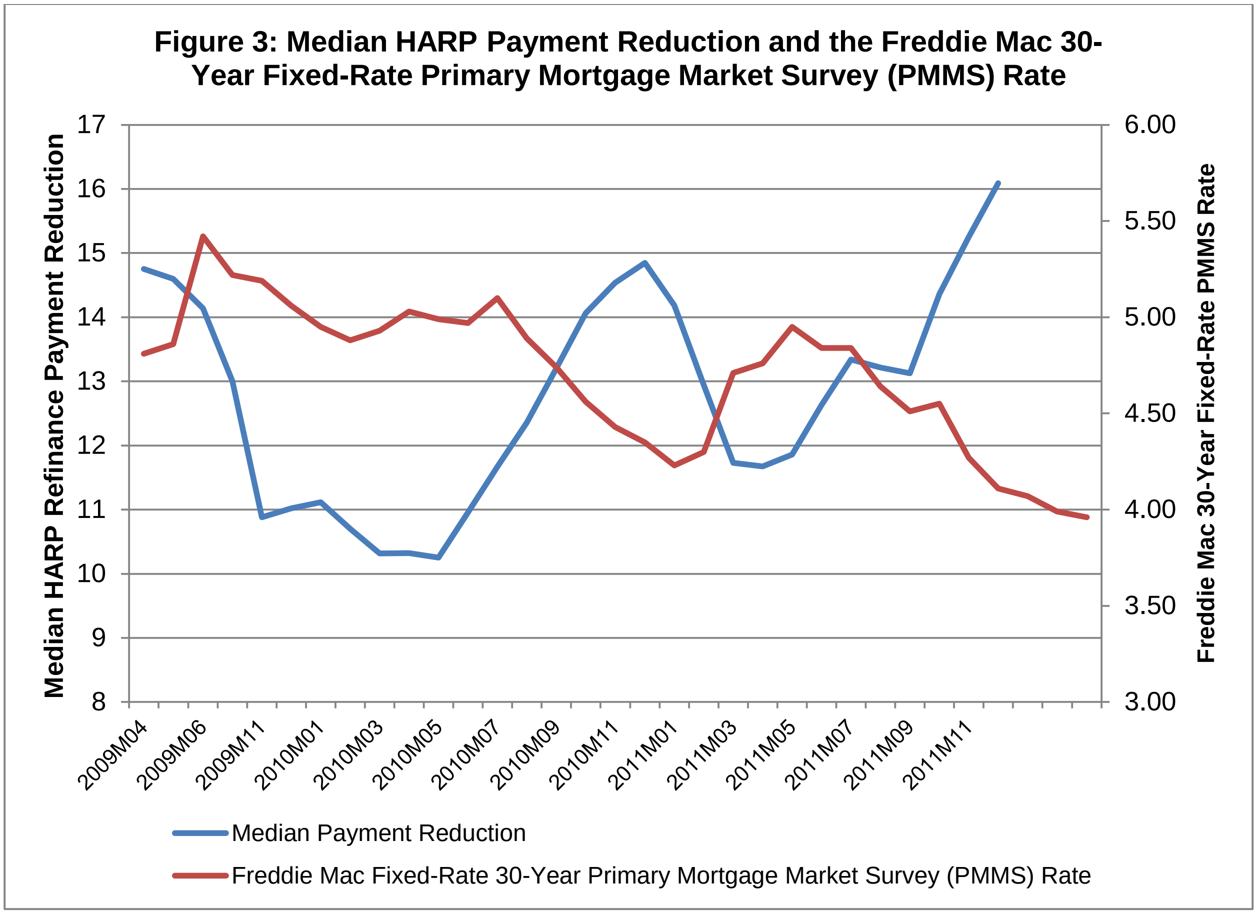
Figure 3: Median HARP Payment Reduction and the Freddie Mac 30-Year Fixed-Rate Primary Mortgage Market Survey (PMMS) Rate
Median HARP Refinance Payment Reduction
Freddie Mac 30-Year Fixed-Rate PMMS Rate
17
16
15
14
13
12
11
10
9
8
6.00
5.50
5.00
4.50
4.00
3.50
3.00
2009M04
2009M06
2009M11
2010M01
2010M03
2010M05
2010M07
2010M09
2010M11
2011M01
2011M03
2011M05
2011M07
2011M09
2011M11
Median Payment Reduction
Freddie Mac Fixed-Rate 30-Year Primary Mortgage Market Survey (PMMS) Rate

Blah

Table 1: Summary Statistics

	Mean	S.D.	P25	P50	P75
Post FICO	735	55	703	746	777
Pre FICO	729	50	695	737	769
Post Note Rate	4.968%	0.423%	4.625%	4.990%	5.250%
Pre Note Rate	6.166%	0.532%	5.875%	6.250%	6.500%
Post LTV	95%	11%	87%	93%	102%
Pre LTV	79%	10%	75%	80%	80%
Post UPB	$223,266	$92,830	$149,400	$210,966	$288,110
Pre UPB	$232,716	$95,904	$156,000	$220,000	$300,000
Post P&I	$1,198	$500	$803	$1,131	$1,543
Pre P&I	$1,410	$569	$960	$1,336	$1,808
Payment Reduction	15%	6%	12%	15%	19%
CLTV	90%	14%	81%	89%	98%
2-Year HPA	-5.9%	4.3%	-8.1%	-5.3%	-3.0%

Table 1 reports summary statistics a Freddie Mac sample of 64,810 randomly selected Freddie Mac HARP refinances of 30-year fixed-rate mortgages into 30-year fixed-rate mortgages. CLTV and 2-Year HPA are time-varying and statistics are reported over the sample of 2.06 million loan-month records.

Table 2: HARP Average Ever D90+ Rates

Payment Reduction	Average D90+ Rate
0-10%	3.90%
10%-15%	3.80%
15%-20%	3.00%
above 20%	2.00%

Table 2 reports cumulative default rates within the sample of 64,810 HARP loans stratified by payment reduction (percentage change in payment as a result of HARP refinancing). The average number of months of observed performance history is 32 months, with a maximum possible window of 50 months.

Table 3: Hazard Model Results

	(1)	(2)
New FICO	0.991***	0.991***
Delta FICO	1.001	1
CLTV	1.027***	1.027***
HPA Growth	0.984***	0.983***
Unemployment Rate	1.008	1.01
Payment Reduction	0.896**	0.898**
State	FE	Strata
Vintage	FE	Strata
-2 Log L (Intercept Only)	52,808	37,579
-2 Log L (Int + Covariates)	51,430	36,352

Table 3 reports estimates from a Cox relative risk hazard model with time to default as the dependent variable (default is defined as the first time a loan is ever 3 months or greater past due). Current LTV (CLTV), 2-Year HPA, and unemployment rate are time-varying covariates, all other variables are static observed at the time of loan origination. Hazard ratios of parameter estimates $\exp(\widehat{\beta})$ are reported. The sample consists of 64,810 randomly selected Freddie Mac HARP refinances of 30-year fixed-rate mortgages into 30-year fixed-rate mortgages, with 2.06 million loan-month records. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 4a: Hazard Model Results Stratified by New LTV

	(1)	(2)	(3)	(4)	(5)	(6)
	(80,95]	(95,110]	(110,125]	(80,95]	(95,110]	(110,125]
New FICO	0.99***	0.991***	0.994***	0.99***	0.991***	0.994***
Delta FICO	1.00	1.00	1.003**	1.000	1	1.003**
CLTV	1.034***	1.03***	1.024***	1.036***	1.032***	1.027***
HPA Growth	0.979***	0.991	0.993	0.979	0.994	0.994
Unemployment Rate	1.014	0.991	1.008	1.014	0.993	1.006
Payment Reduction	0.923**	0.921**	0.727***	0.926**	0.922**	0.741**
State	FE	FE	FE	Strata	Strata	Strata
Vintage	FE	FE	FE	Strata	Strata	Strata
-2 Log L (Intercept Only)	24,867	17,360	5,582	17,649	11,599	3,597
-2 Log L (Int + Covariates)	24,152	16,908	5,482	17,009	11,231	3,540

Table 4b: Hazard Model Results Stratified by Post FICO

	(1)	(2)	(3)	(4)	(5)	(6)
	≤700	(700,750]	>750	≤700	(700,750]	>750
New FICO	0.994***	0.992***	0.99***	0.994***	0.991***	0.99***
Delta FICO	0.999**	1.002*	1.004***	0.999**	1.002*	1.004***
CLTV	1.018***	1.029***	1.041***	1.019***	1.029***	1.041***
HPA Growth	0.993	0.977**	0.971***	0.998	0.974**	0.965***
Unemployment Rate	1.01	1.018	0.992	1.013	1.022	0.986
Payment Reduction	0.91**	0.881**	0.904**	0.915***	0.883***	0.896***
State	FE	FE	FE	Strata	Strata	Strata
Vintage	FE	FE	FE	Strata	Strata	Strata
-2 Log L (Intercept Only)	21,521	13,730	11,601	14,572	8,983	7,884
-2 Log L (Int + Covariates)	21,263	13,523	11,322	14,366	8,851	7,673

Tables 4a and 4b report estimates from a Cox relative risk hazard model with time to default as the dependent variable (default is defined as the first time a loan is ever 3 months or greater past due). Current LTV (CLTV), 2-Year HPA, and unemployment rate are time-varying covariates, all other variables are static observed at the time of loan origination. Hazard ratios of parameter estimates $\exp(\widehat{\beta})$ are reported. The sample consists of 64,810 randomly selected Freddie Mac HARP refinances of 30-year fixed-rate mortgages into 30-year fixed-rate mortgages, with 2.06 million loan-month records. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 5a: Logit Model Parameter Estimates from Selection Model

	2009	2010	2011
CLTV	-0.057***	-0.059***	-0.028***
CLTV (Spline 100)	0.105***	0.098***	0.08***
UPB	1.20***	1.30***	1.20***
UPB Spline $200k	-0.92***	-0.98***	-0.89***
FICO	-0.0001*	0.0011***	0.0009***
-2 Log L (Int Only)	655,747	1,232,730	928,274
-2 Log L (Int + Covariates)	557,148	1,046,005	804,838

Table 5b: Type III Analysis (Wald χ^2)

	2009	2010	2011
CLTV	9,924	21,714	3,281
CLTV (Spline 100)	8,697	15,563	8,756
UPB	13,509	29,581	20,764
UPB Spline $200k	4,073	8,687	5,499
FICO	3	409	208
Servicer Fixed Effects	32,754	56,589	50,562
State Fixed Effects	6,466	10,570	3,116

Tables 5a and 5b report estimates from a logistic model with participation in HARP as a binary dependent variable. Current LTV (CLTV) is defined as the origination LTV for HARP loans in the sample, and for eligible non-HARP loans is estimated at the time of observed eligibility using the Freddie Mac Weighted Repeat Sales Model (WRSI) house price index. FICO credit score is observed at the time of refinance for HARP loans, and at the time of origination for eligible non-HARP loans. For HARP loans, UPB is the origination unpaid balance at the time of refinance; for eligible non-HARP loans it is the current balance at the time of observed eligibility. The sample consists of 64,810 randomly selected Freddie Mac HARP refinances and 743,725 randomly selected eligible non-HARP loans. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 6: Hazard Model Results (IPW)

	(1)	(2)
New FICO	0.991***	0.991***
Delta FICO	1.001*	1
CLTV	1.027***	1.027***
HPA Growth	0.991	0.983
Unemployment Rate	0.989	1.01
Payment Reduction	0.881***	0.898***
State	FE	Strata
Vintage	FE	Strata
-2 Log L (Intercept Only)	35,035	37,579
-2 Log L (Int + Covariates)	33,880	36,352

Table 6 reports estimates from a Cox relative risk hazard model with time to default as the dependent variable (default is defined as the first time a loan is ever 3 months or greater past due). Current LTV (CLTV), 2-Year HPA, and unemployment rate are time-varying covariates, all other variables are static observed at the time of loan origination. Hazard ratios of parameter estimates $\exp(\widehat{\beta})$ are reported. The sample consists of 64,810 randomly selected Freddie Mac HARP refinances of 30-year fixed-rate mortgages into 30-year fixed-rate mortgages, with 2.06 million loan-month records. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 7a: Hazard Model Results Segmented by New LTV (IPW)

	(1)	(2)	(3)	(4)	(5)	(6)
	(80,95]	(95,110]	(110,125]	(80,95]	(95,110]	(110,125]
New FICO	0.99***	0.99***	0.993***	0.99***	0.991***	0.993***
Delta FICO	1	1.001	1.003***	1	1.001	1.003
CLTV	1.028***	1.034***	1.037***	1.034***	1.041***	1.046***
HPA Growth	0.982***	1.02**	1.01	0.979**	1.016	1.022
Unemployment Rate	1.011	0.964*	0.963	1.005	0.975	0.976
Payment Reduction	0.894*	0.908**	0.66***	0.871**	0.951	0.666***
State	FE	FE	FE	Strata	Strata	Strata
Vintage	FE	FE	FE	Strata	Strata	Strata
-2 Log L (Intercept Only)	23,268	16,231	4,646	17,035	10,641	2,874
-2 Log L (Int + Covariates)	22,572	15,710	4,509	16,440	10,240	2,788

Table 7b: Hazard Model Results Segmented by Post FICO (IPW)

	(1)	(2)	(3)	(4)	(5)	(6)
	≤700	(700,750]	>750	≤700	(700,750]	>750
New FICO	0.993***	0.99***	0.988***	0.993***	0.991***	0.991***
Delta FICO	1	0.998*	1.005***	1	0.999	1.005
CLTV	1.019***	1.032***	1.036***	1.019***	1.03***	1.038***
HPA Growth	0.999	0.989	0.999	0.995	0.985	0.984
Unemployment Rate	0.999	1.014	0.946	1	0.996	0.958
Payment Reduction	0.974	0.801***	0.812**	0.975	0.835**	0.787***
State	FE	FE	FE	Strata	Strata	Strata
Vintage	FE	FE	FE	Strata	Strata	Strata
-2 Log L (Intercept Only)	19,776	12,468	9,523	13,461	8,319	6,829
-2 Log L (Int + Covariates)	19,497	12,231	9,323	13,270	8,201	6,682