The Risky Capital of Emerging Markets

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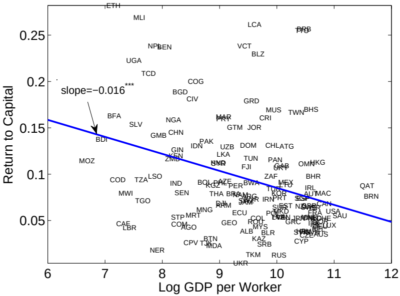
Lucas Paradox Revisited



Lucas Paradox: Why don't returns to capital equalize?

Our answer: Capital in poorer countries is riskier.

$\Rightarrow$ Investor demands higher average returns there.

We show: differences in risk **quantitatively** account for differences in returns.

What we do...

First pass: workhorse CCAPM... works qualitatively; not quantitatively (paper)

- Given $cov(r_j, \Delta c_{US})$ from data, need $\gamma \approx 900$ to fit observed return diffs.

  $\Rightarrow$ Lucas Paradox: just another asset pricing puzzle! Details

Towards a resolution:

Key Building Blocks

1. Aguiar and Gopinath (2007): shocks to trend growth key in poor/emerging market BC's +

2. Bansal and Yaron (2004): implications of these shocks for asset prices/returns, i.e., compensation for "long-run risks"

What we find...

Investments in poor countries are more exposed to global long-run risk
⇒ US investor demands higher average returns.

1. Risk accounts for 60-70% of return diff. btwn US & set of poorest cntrs
2. Results robust to different levels of disaggregation; grouping of countries
3. LRR key; SRR implies (tiny) *negative* risk premia in poor countries

Measuring Returns to Capital for US Investor

Environment: J regions;

2 sectors: K is freely traded—P_I, C is not (builds on Hsieh and Klenow, 2007);

Representative US agent consumes, considers buying K in US, investing it in j.

Motivation: Poor countries import capital goods from rich.

(see Eaton and Kortum, 2001, Burstein et al., 2011, Mutreja et al., 2012)

Return from potential investment in j for US investor:

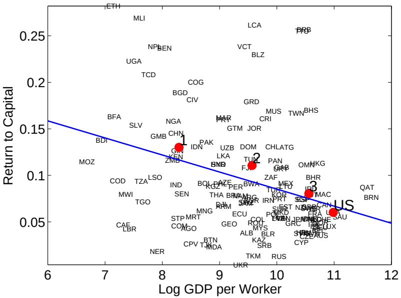
$$R_t^j = \underbrace{\underbrace{\alpha \frac{P_{Y,j,t} Y_{j,t}}{K_{j,t}}}_{D_{j,t} \text{ in } C_{US,t}} \frac{1}{P_{I,t}}}_{\text{dividend yield, D/P}} + \underbrace{(1-\delta_{j,t+1}) \frac{P_{I,t+1}}{P_{I,t}}}_{\text{capital gains, } \Delta P}$$

$P_I, P_{Y,j}$ are prices of K, Y relative to US price of C (Gomme et al., 2011).

Returns Revisited: "Bundles" of countries

Assume common $\alpha = 0.3$. ▸ Robustness ▸ Open

144 countries bundled according to mean output per worker over period.



Data:

- PWT 8.0, 1950-2009: $K_{j,t}, \delta_{j,t}, P_{Y,US,2005} Y_{j,t}$—adjust by $\dfrac{\frac{P_{Y,US,t}}{P_{Y,US,2005}}}{P_{C,US,t}}$
- BEA 1950-2009:
  - $P_{I,US}$—price index of equipment + structures
  - $P_{Y,US}$—price index of output
  - $P_{C,US}$—price index of non-durables + services

Model

Endowment economy in spirit of Bansal and Yaron (2004):

- EZ US investor consumes, gets dividends from inv'ts in US + abroad

  1. Two sources of risk:
     - Small but persistent global component in $\Delta c^{(*)}$ and $\Delta d^{(*)}$ (*=foreign)
     - Small but persistent $*$-specific component; orthogonal to global component
  2. US investor prices exposure to global component
  3. Key challenge in calibration: disentangle global from *-specific shocks

Model With Global and Local Shocks: US Processes

Representative US-based investor endowed with c, d, d^*

$$\begin{aligned}\Delta c_{t+1} &= \mu_c + x_t + \eta_{t+1} \\ x_{t+1} &= \rho x_t + e_{t+1} \\ \Delta d_{t+1} &= \mu_d + \phi x_t + \pi \eta_{t+1} + \mu_{t+1}\end{aligned}$$

$$\eta_{t+1} \sim N\left(0, \sigma_\eta\right), e_{t+1} \sim N\left(0, \sigma_e\right), \mu_{t+1} \sim N\left(0, \sigma_\mu\right)$$

US variables:

- $\mu_c(\mu_d)$: unconditional mean growth rate of consumption (dividend)
- η: transitory shock to consumption growth
- ϕ: exposure of dividend growth to trend shock in consumption growth
- π: exposure of dividend growth to transitory shock in consumption growth
- μ: transitory shock to dividend growth

Model With Global and Local Shocks: Foreign Processes

$$\begin{aligned} \Delta c^*_{t+1} &= \mu^*_c + \xi^* x_t + x^*_t + \pi^*_c \eta_{t+1} + \eta^*_{t+1} \\ x^*_{t+1} &= \rho^* x^*_t + e^*_{t+1} \\ \Delta d^*_{t+1} &= \mu^*_d + \tilde{\phi}^* \left(\xi^* x_t + x^*_t\right) + \pi^* \eta_{t+1} + \pi^*_d \mu_{t+1} + \pi^*_{cd} \eta^*_{t+1} + \mu^*_{t+1} \end{aligned}$$

$$\eta^*_{t+1} \sim N\left(0, \sigma_{\eta^*}\right), e^*_{t+1} \sim N\left(0, \sigma_{e^*}\right), \mu^*_{t+1} \sim N\left(0, \tilde{\sigma}_{\mu^*}\right)$$

- $\mu^*_c(\mu^*_d)$: unconditional mean growth rate of consumption (dividend)
- ξ^*: exposure of cons. growth to trend shock in US cons. growth
- π^*_c: exposure of cons. growth to transitory shock in US cons. growth
- η^*: transitory shock to consumption growth
- $\tilde{\phi}^*$: exposure of dividend growth to local trend shock
- π^*: exposure of dividend growth to transitory shock in US cons. growth
- π^*_d: exposure of dividend growth to transitory shock in US dividend growth
- π^*_{cd}: exposure of dividend growth to transitory shock in cons. growth
- μ^*: transitory shock to dividend growth

Model: US Investor's Preferences

Representative US-based investor's preferences

$$V_t = \left[(1-\beta) C_t^{\frac{\psi-1}{\psi}} + \beta \nu_t \left(V_{t+1} \right)^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}}$$

$$\nu_t \left(V_{t+1} \right) = \left[E_t \left(V_{t+1}^{1-\gamma} \right) \right]^{\frac{1}{1-\gamma}}$$

- $\nu_t \left(V_{t+1} \right)$ is certainty equivalent function
- ψ is IES; $\gamma \geq 0$ is risk aversion coefficient; CRRA special case iff $\gamma = 1/\psi$

Risk premia for US and foreign assets:

$$\mathbb{E}\left[R_t^e\right] = \underbrace{\left(\phi - \frac{1}{\psi}\right)\left(\gamma - \frac{1}{\psi}\right) \frac{\kappa_{m,1}}{1-\kappa_{m,1}\rho} \frac{\kappa_1}{1-\kappa_1 \rho} \sigma_e^2}_{\text{long-run risk}} + \underbrace{\gamma \pi \sigma_\eta^2}_{\text{short-run risk}}$$

$$\mathbb{E}\left[R_t^{e*}\right] = \underbrace{\left(\phi^* - \frac{1}{\psi}\right)\left(\gamma - \frac{1}{\psi}\right) \frac{\kappa_{m,1}^*}{1-\kappa_{m,1}^*\rho} \frac{\kappa_1}{1-\kappa_1 \rho} \sigma_e^2}_{\text{long-run risk}} + \underbrace{\gamma \pi^* \sigma_\eta^2}_{\text{short-run risk}}$$

where $\phi^* \equiv \tilde{\phi}^* \xi^*$ and κ's are endogenous objects. SDF

Model: Calibration of US Parameters

$$\begin{aligned}\Delta c_{t+1} &= \mu_c + x_t + \eta_{t+1} \\ x_{t+1} &= \rho x_t + e_{t+1} \\ \Delta d_{t+1} &= \mu_d + \phi x_t + \pi \eta_{t+1} + \mu_{t+1}\end{aligned}$$

$\gamma = 10, \psi = 1.5, \beta = 0.99$; $\rho = 0.93$ (Ferson, 2013; Bansal, Kiku, Yaron, 2012)

$$\begin{aligned}\mathbb{E}\left[\Delta c_t\right] &= \mu_c \\ \mathrm{cov}\left(\Delta c_t, \Delta c_{t+1}\right) &= \rho \frac{\sigma_e^2}{1-\rho^2} \\ \mathrm{var}(\Delta c_t) &= \frac{\sigma_e^2}{1-\rho^2} + \sigma_\eta^2 \\ \mathbb{E}\left[\Delta d_t\right] &= \mu_d \\ \sqrt{\frac{\mathrm{cov}\left(\Delta d_{t+1}, \Delta d_t\right)}{\mathrm{cov}\left(\Delta c_{t+1}, \Delta c_t\right)}} &= \phi \\ \mathrm{cov}\left(\Delta d_t, \Delta c_t\right) &= \phi \frac{\sigma_e^2}{1-\rho^2} + \pi \sigma_\eta^2 \\ \mathrm{var}(\Delta d_t) &= \phi^2 \frac{\sigma_e^2}{1-\rho^2} + \pi^2 \sigma_\eta^2 + \sigma_\mu^2\end{aligned}$$

Model: Calibration of Foreign Parameters

Rewrite the foreign dividend process:

$$\Delta d^*_{t+1} \quad = \quad \mu^*_d + \phi^* x_t + \tilde{\phi}^* x^*_t + \pi^* \eta_{t+1} + \pi^*_d \mu_{t+1} + \pi^*_{cd} \eta^*_{t+1} + \mu^*_{t+1}$$

Iterative procedure to identify ϕ^* from:

$$\begin{aligned} \text{cov}\left(r^*_{m,t}, r_{m,t}\right) \quad = \quad & \frac{1}{\psi^2}\frac{\sigma_e^2}{1-\rho^2} + \pi^*\pi\sigma_\eta^2 + \pi^*_d\sigma_\mu^2 + \\ & \frac{\kappa_{m,1}}{1-\kappa_{m,1}\rho}\frac{\kappa^*_{m,1}}{1-\kappa^*_{m,1}\rho}\left(1-\frac{1}{\psi}\right)\left(\phi^*-\frac{1}{\psi}\right)\sigma_e^2 \end{aligned}$$

$+$ moment conditions for remaining parameters.

For intuition, consider following moment:

$$\text{cov}\left(\Delta d^*_t, \Delta d_t\right) \quad = \quad \phi\phi^*\frac{\sigma_e^2}{1-\rho^2} + \pi^*\pi\sigma_\eta^2 + \pi^*_d\sigma_\mu^2$$

- Both dividend growth and returns comove due to common transitory and persistent shocks, but return comovement is more sensitive to latter.
- Persistent shock $\Rightarrow \Delta$ asset prices due to change in future growth prospect $\Rightarrow$ cov $\left(r^*_{m,t}, r_{m,t}\right)$ relative to cov $\left(\Delta d^*_t, \Delta d_t\right)$ is larger.

▸ Solution ▸ Alternative Calibration ▸ Other Parameters

Data: Moments

US Moments						
Consumption	$\mathbb{E}[\Delta c_t]$	$\text{cov}(\Delta c_{t+1}, \Delta c_t)$	$\text{std}(\Delta c_t)$			
	0.019	0.00024	0.022			
Dividends	$\mathbb{E}[\Delta d_t]$	$\sqrt{\frac{\text{cov}(\Delta d_{t+1}, \Delta d_t)}{\text{cov}(\Delta c_{t+1}, \Delta c_t)}}$	$\text{cov}(\Delta d_t, \Delta c_t)$	$\text{std}(\Delta d_t)$		
	-0.006	2.19	0.00018	0.026		
Foreign Moments						
Portfolio	$\mathbb{E}[\Delta d_t^*]$	$\text{cov}\left(\Delta d_{t+1}^*, \Delta d_t^*\right)$	$\text{cov}(\Delta d_t^*, \Delta c_t)$	$\text{std}(\Delta d_t^*)$	$\text{cov}(\Delta d_t^*, \Delta d_t)$	$\text{cov}(r_t^*, r_t)$
1	-0.017	0.00122	0.00011	0.083	0.00032	0.00109
2	-0.015	0.00156	0.00011	0.074	0.00033	0.00100
3	-0.011	0.00075	0.00015	0.063	0.00040	0.00085

- US Consumption TS: 1929-2009, Portfolio TS: 1950-2009
- Portfolio moment computed as mean of country-specific moments
- $\uparrow \text{cov}(\Delta d_t^*, \Delta d_t) + \downarrow \text{cov}(r^*, r)$ wrt income $\Rightarrow \uparrow \pi_d^* + \downarrow \phi^*$

Question: What are the implications for returns to capital?

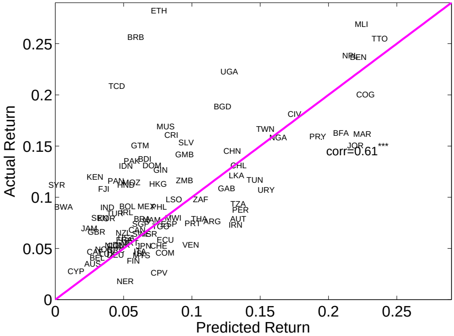
Parameters

Predicted VS Actual Returns: 3, 5, 10 Bundles

3 Portfolios			5 Portfolios			10 Portfolios		
	$\widehat{r}$	r		$\widehat{r}$	r		$\widehat{r}$	r
1	10.55	13.02	1	11.02	14.39	1	12.38	16.40
2	9.30	11.07	2	10.80	10.89	2	9.65	12.38
3	7.18	8.05	3	8.75	11.40	3	9.95	10.28
US	5.94	6.02	4	8.30	10.07	4	11.31	11.38
			5	6.28	6.75	5	7.47	10.65
			US	5.94	6.02	6	10.02	12.00
						7	8.06	9.76
						8	8.50	10.40
						9	6.60	7.63
						10	6.04	6.00
						US	5.94	6.02
Average:	8.24	9.54		8.52	9.92		8.72	10.26
Spread: 1-US	4.61	7.00		5.08	8.36		6.44	10.38
Percent of actual	66			61			62	
corr($\widehat{r}$, r)	1.00			0.92			0.91	

Model delivers 61-66% of spread. ▸ Rebalanced Portfolios

Predicted VS Actual Returns by Country



- 96 countries in PWT in or prior to 1961: corr($\hat{r}, r$) is 0.61
- predicted semi-elasticity of returns wrt income is 55% of actual: -0.023 (model) vs. -0.013 (data)

Decomposition: Long VS Short Run Risk

	Actual	Predicted						
Portfolio	r	$\widehat{r}$		$\widehat{r}^f$		$\widehat{r}^e_{sr}$		$\widehat{r}^e_{lr}$
1	13.02	10.55	=	1.29	+	−0.28	+	9.54
2	11.07	9.30	=	1.29	+	−0.18	+	8.20
3	8.05	7.18	=	1.29	+	0.08	+	5.82
US	6.02	5.94	=	1.29	+	0.22	+	4.44

- High returns mainly driven by long-run risk
- Short-run risk compensation in low/middle-income countries

Conclusion

Can differences in risk **quantitatively** account for differences in returns?

Yes!

Risk (measured properly) accounts for 60-70% of difference in returns between poorest countries and US.

Key implication: Despite large return differentials, observed capital allocation is not so distant from that predicted by theory.

Future work should investigate sources of differences in long-run risk.

Model: Calibration of Foreign Parameters

Remaining moments on dividends similar to those for US.

Let $\rho^* = \rho$, $\sigma^2_{\mu^*} \equiv {\pi^*_{cd}}^2 \sigma^2_{\eta^*} + \tilde{\sigma}^2_{\mu^*}$.

Guess ϕ^*, then use $\text{cov}\left(\Delta d_t^*, \Delta d_t\right)$ and following moment conditions:

$$
\begin{aligned}
\mathbb{E}\left[\Delta d_t^*\right] &= \mu_d^* \\
\text{cov}\left(\Delta d_t^*, \Delta c_t\right) &= \phi^* \frac{\sigma_e^2}{1-\rho^2} + \pi^* \sigma_\eta^2 \\
\text{cov}\left(\Delta d_{t+1}^*, \Delta d_t^*\right) &= \left(\tilde{\phi}^* \sigma_{e^*}\right)^2 \frac{\rho^*}{1-{\rho^*}^2} + \left(\phi^* \sigma_e\right)^2 \frac{\rho}{1-\rho^2} \\
\text{var}\left(\Delta d_t^*\right) &= \left(\phi^*\right)^2 \frac{\sigma_e^2}{1-\rho^2} + \frac{\left(\tilde{\phi}^* \sigma_{e^*}\right)^2}{1-\rho^2} + {\pi^*}^2 \sigma_\eta^2 + {\pi_d^*}^2 \sigma_\mu^2 + \sigma_{\mu^*}^2
\end{aligned}
$$

Verify ϕ^* using $\text{cov}\left(r_{m,t}^*, r_{m,t}\right)$ above.

Notable difference:

$\text{cov}\left(\Delta c_{t+1}^*, \Delta c_t^*\right)$ not needed b/c $\tilde{\phi}^*$ and σ_{e^*} need not be separately identified.

Back

Alternative Calibration Strategies and Biases

Alternative moment for ϕ^*

$$\phi^*_{\text{biased}} = \sqrt{\frac{\text{cov}\left(\Delta d^*_{t+1}, \Delta d^*_t\right)}{\text{cov}\left(\Delta c_{t+1}, \Delta c_t\right)}} = \phi^* \sqrt{1 + \frac{1}{\xi^{*2}} \frac{\sigma^2_{e^*}}{\sigma^2_e}}$$

- larger country-specific trend shock $\sigma^2_{e^*}$ $\Rightarrow$ larger bias
- $\sigma^2_{e^*} = 0$ yields nearly identical results in our calibration
- $\pi^*_d = 0$ reverses returns; it ignores comovement due to transitory shocks

			$\sigma_{e^*} = 0$		
Portfolio	Actual	Benchmark	Autocovariance	Baseline	$\pi^*_d = 0$
1	13.02	10.55	11.06	10.56	8.76
2	11.07	9.30	12.05	9.33	9.28
3	8.05	7.18	9.59	7.19	10.47

▸ Calibration

Calibrated Model: Parameters

Preferences:	$\gamma = 10$	$\psi = 1.5$	$\beta = 0.99$			
Consumption:	$\rho = 0.93$	$\mu_c = 0.019$	$\sigma_e = 0.006$	$\sigma_\eta = 0.02$		
Portfolio	μ_d	ϕ	π	π_d	σ_μ	$\tilde{\phi}^* \sigma_{e^*}$
1	-0.017	5.14	-1.24	-0.16	0.074	0.005
2	-0.015	4.23	-0.81	-0.00	0.061	0.011
3	-0.011	2.87	0.34	0.27	0.056	0.008
US	-0.006	2.19	0.98	-	0.020	-

Moments

Lucas Paradox: Just Another Asset Pricing Puzzle

With CRRA (γ) preferences:

$$\mathbb{E}[r_{jt}^e] \approx \gamma cov(r_{jt}, \Delta c_t)$$

- $r_{jt}^e \equiv r_{jt} - r_{ft}$ is excess (net) return on portfolio j over 3-month t-bill at t
- Δc_t is US (ND+S) consumption growth during 1950-2009

				$\widehat{r^e}$	
Portfolio	r^e	$\text{cov}(r, \Delta c)$	$\gamma =$	10	889
1	11.80	0.00016		0.16	14.06
2	9.85	0.00009		0.09	7.93
3	6.83	0.00004		0.04	3.92
US	4.80	0.00004		0.04	3.23
Spread: 1-US	7.00	0.00012		0.12	10.83

γ falls as granularity increases; at country level it is very high, $\approx$ 500. Back

Predicted VS Actual Returns—Annually Rebalanced Portfolios

3 Portfolios			5 Portfolios			10 Portfolios		
	$\widehat{r}$	r		$\widehat{r}$	r		$\widehat{r}$	r
1	10.56	12.10	1	11.03	13.00	1	11.44	14.91
2	8.74	10.23	2	10.06	10.83	2	10.59	10.92
3	7.04	7.57	3	8.14	10.18	3	9.77	10.49
US	5.94	6.02	4	7.47	8.49	4	10.36	11.17
			5	7.18	7.26	5	8.25	10.51
			US	5.94	6.02	6	8.05	9.88
						7	8.33	8.96
						8	6.62	8.01
						9	7.28	6.87
						10	7.09	7.64
						US	5.94	6.02
Average:	8.07	8.98		8.31	9.30		8.52	9.58
Spread: 1-US	4.61	6.08		5.09	6.98		5.50	8.89
Percent of actual	76			73			62	
corr($\widehat{r}$, r)	1.00			0.96			0.91	

▸ Results

Details: SDF

Key asset-pricing equation for any asset:

$$\mathbb{E}_t[M_{t+1} R_{t+1}] = 1,$$

M_{t+1} is US investor's SDF.

In logs,

$$m_{t+1} = \theta \log \beta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1)\, r_{ct+1}$$

where $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$
and r_{ct+1} is return on asset that pays consumption as dividend.

Solving the Model

Standard approach; approximations to make exercise computationally feasible

Note that $z_t = \log\left(\frac{P_t}{D_t}\right)$ is enough to characterize returns

- For each asset $k = \mathit{home}, *$, and asset that pays off aggregate consumption

To solve:

- Conjecture: $z_t^k = A_0^k + A_1^k x_t$
- Approximate log returns: $r_{m,t+1}^k = \kappa_0^k + \kappa_1^k z_{t+1}^k + \Delta d_{t+1}^k - z_t^k$
  where κ's depend on $\overline{z}^k$
- Restrictions from Euler equation gives excess return (as in text) and rf rate
- Solve numerically for $\overline{z}^k = A_0^k\left(\overline{z}^k\right)$ and for $A_1^k\left(\overline{z}^k\right)$

▸ Model

Alternative Measurement Approaches

1. Caselli and Feyrer (2007): country-specific P_I, P_C, P_Y from PWT.
2. Country-time specific α's + non-reproducible capital adjustment from WDI

	All Years				1996			
Portfolio	Baseline	Country prices	Country α's	Country prices & α's	Baseline	Country prices	Country α's	Country prices & α's
1	13.02***	12.00***	13.22***	13.63***	5.38**	3.32	8.24**	7.27
	(0.76)	(0.66)	(0.76)	(1.05)	(.78)	(2.81)	(1.25)	(2.00)
2	11.07***	10.53***	13.15***	13.23***	5.21	5.89	8.12*	7.51
	(0.61)	(0.62)	(0.75)	(0.68)	(0.99)	(1.73)	(1.29)	(1.42)
3	8.05***	9.36**	9.17***	11.39***	3.91	10.03*	6.47	14.09***
	(0.45)	(0.33)	(0.49)	(0.44)	(0.85)	(1.48)	(1.29)	(1.22)
US	6.02	8.22	6.20	9.39	3.64	7.23	5.52	9.55
	(0.35)	(0.31)	(0.39)	(0.50)	-	-	-	-

Notes: Table reports the returns to capital across portfolios under a number of measurement approaches. Baseline uses US prices from BEA. Country prices uses country-specific P_Y, P_I, P_C from PWT. Country α's uses country-year α from PWT and subtracts from α the share of payments to non-reproducible capital from WDI, dropping the countries that have negative α for at least one year. Country prices and α's uses country prices and country-year α as described above. Baseline and Country prices cover years from 1950 to 2008. Country α's and Country prices and α's cover years from 1970 to 2008. The portfolios include only countries for which data are available. Standard errors are reported in parentheses. Asterisks denote significance of difference from US values: ***: difference significant at 99%, **: 95%, and *: 90%.

▸ Main

Returns for Countries With Open Capital Accounts

Countries with open capital accounts obey link btwn GDP and returns. Back

	Measure of Openness		
Portfolio	Chinn, Ito	Quinn	Grilli, Milesi-Ferretti
1	10.38***	12.39***	11.48***
	(0.66)	(0.67)	(0.59)
2	8.74***	11.27***	10.28***
	(0.61)	(0.62)	(0.86)
3	5.61	6.66	7.57**
	(0.50)	(0.51)	(0.63)
US	5.22	6.06	5.85
	(0.38)	(0.38)	(0.52)

Notes: Table reports the returns to capital across portfolios for economies that are characterized as open according to three indices: Chinn/Ito, Quinn, and Grilli/Milesi-Ferretti, respectively. Chinn/Ito and Quinn openness cutoff is median value in sample. Grilli/Milesi-Ferretti openness indicator is unity. Standard errors are reported in parentheses. Asterisks denote significance of difference from US values: ***: difference significant at 99%, **: 95%, and *: 90%.

Supplementary Results

Note:

- A's depend on US parameters only
- $\sigma^2_{\mu^*}$, $\tilde{\phi}^*\sigma^2_{e^*}$ and calibrated parameters are sufficient to compute $\kappa^*_{m,1}$

$$
\begin{aligned}
A^*_{m,0} &= \frac{\theta \log \beta - \gamma\mu + (\theta - 1)\left(\kappa_0 + A_0\left(\kappa_1 - 1\right)\right) + \mu^*_d + \kappa^*_{m,0} + \frac{1}{2}\left(\frac{\kappa^*_{m,1}}{1-\kappa^*_{m,1}\rho^*}\right)^2 (\tilde{\phi}^*\sigma^*_e)^2}{1 - \kappa^*_{m,1}} \\
&+ \frac{\frac{1}{2}\left(\pi^* - \gamma\right)^2 \sigma^2_\eta + \frac{1}{2}\left((\theta - 1)\kappa_1 A_1 + \frac{\kappa^*_{m,1}}{1-\kappa^*_{m,1}\rho}\left(\phi^* - \frac{1}{\psi}\right)\right)^2 \sigma^2_e + \frac{1}{2}\pi^{*2}_d \sigma^2_\mu + \frac{1}{2}\sigma^2_{\mu^*}}{1 - \kappa^*_{m,1}}
\end{aligned}
$$

▸ Solution