Credit Expansion and Credit Misallocation

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Abstract

This paper studies the effectiveness of market-based monetary policy in creating economic stimulus. We show that if the central bank injects too much liquidity into a two-sector economy, overheating can build up in the sector with lower financing friction, crowding out the demand for liquidity in the sector with higher friction. The crowding-out occurs in a self-reinforcing spiral because of feedback between the supply of liquidity and the demand for liquidity. This limit to market-based monetary policy derives from misaligned lending incentives between the central bank and financial intermediaries. As a result, monetary policy, in relying on the credit market to allocate liquidity across the economy, could actually distort the credit market.

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1 Introduction

During and after the severe financial crisis of 2007-2009, central banks around the world adopted traditional as well as unconventional credit easing policies to inject liquidity into their banking systems in an attempt to save the economy from recession. The liquidity injections came in various forms in different countries.[1] However, one common aim of the credit easing policies was to ensure the banking systems would have sufficient liquidity to extend loans to the economy. Economic stimulus with liquidity injections, however, does not always work. Very often, liquidity is markedly unevenly distributed; overheating builds up in some sectors while other sectors make little recovery.[2] What happened in recent years in China, now the world's second largest economy, is an illustrative case in point.

Facing a sharp decline in the external demand for exports and the danger of plunges in economic growth, the Chinese government implemented an economic stimulus package unprecedented in history, notably through some very aggressive credit expansion policies. The official figure shows that a total amount of RMB 7.37 trillion (around USD 1.08 trillion) of bank credit was injected into China's economy in the first half of 2009, up 200% on the same period a year earlier (when the global financial crisis was not yet in full swing).[3] One immediate and pronounced phenomenon following the liquidity injections was the surge in house prices, with a 50% increase within one year in many cities.[4] Asset prices were also climbing in other asset classes, like commodities. Ironically, in the name of stimulating the real economy, small and medium-sized businesses in China had been experiencing even more difficult times in financing and obtaining corporate liquidity following the economic stimulus. The underground real interest rate surged to 30% in 2010 in some regions. Most small and medium-sized firms with little collateral had been virtually left out of the bank

[1] In the U.S., the Federal Reserve adopted an unconventional policy of credit easing through a combination of lending to financial institutions, providing liquidity directly to key credit markets, and purchases of long-term securities (Bernanke (2009)). Central banks in Europe and Japan used similar 'quantitative easing' policies, while many emerging economies undertook aggressive credit expansion.

[2] In our paper, as shown later, overheating can be defined as the situation in which an increase, rather than a decrease, in the interest rate coincides with the increase in the supply of liquidity.

[3] Data is from the statistics and analysis department of the People's Bank of China, China's central bank (http://www.pbc.gov.cn/publish/english/963/index.html).

[4] The data are from China Real Estate Index System (CREIS).

credit market.[5] The *People's Daily*, China's leading official newspaper, wrote:

"Massive funds pulled out the real sector and flowed into the real estate sector, *crowding out* the real economy."[6]

The experience of China was not unique. Some researchers contend that the financial integration but without the necessary financial deepening in the EU's peripheral countries since the late 1990s had led to the massive foreign credit, mainly through domestic banks, pouring in the real estate sector in these countries, which in turn was a root cause of the subsequent European crisis (see, e.g., Reis (2014)).[7] Notably, the pre-crisis asset bubble in the U.S., which was the origin of the 2007-2009 crisis, was also rooted in an environment of massive liquidity injections and credit expansion (see, e.g., the recent evidence by Chakraboty, Goldstein and MacKinlay (2013)).[8]

In this paper, we study the effectiveness of credit easing policy, and provide one perspective for understanding its observed limitations. Our paper demonstrates the relationship between liquidity injections, asset prices and the *distribution* of liquidity in the economy, and shows the economic consequences of this distribution for aggregate economic performance.

We build a stylized model, which features an economy with two sectors with different degrees of financial frictions, labeled Sector 1 and Sector 2. Concretely, the two sectors differ in firm asset specificity: firms in Sector 1 have higher asset specificity than firms in Sector 2. For example, Sector 1 could be the real sector in the economy while Sector 2 could be the financial sector (e.g., housing, commodities, equities, etc.). In general, assets tend to be more specific (in operation) across firms for real business (Williamson (1985, 1986)), while assets in the financial sector tend to be more homogeneous. Rigorously, however, the model is about any two sectors with differences in asset specificity.

[5] See, e.g., the Citibank-CCER report of "Financing and growth of small and medium-sized businesses in China" (http://www.ccstock.cn/stock/jinrongjigou/2011-07-21/A516377.html, the link is only available in Chinese).

[6] http://news.xinhuanet.com/house/2011-12/09/c_122399353.htm, the link is only available in Chinese.

[7] Some other countries like Thailand had a similar experience (see, e.g., Reinhart and Kaminsky (1999)).

[8] In fact, in response to the bursting of the Internet bubble in early 2000, the Federal Reserve Bank had adopted a policy of unprecedented credit easing, with the Federal funds (effective) rate having been historically low at below 2.5% for a prolonged period between 2001 and 2005. The trend of low Federal funds rates started from the early 1990s. (http://www.federalreserve.gov/releases/h15/data.htm). See also related evidence in Gilchrist and Zakrajsek (2013) and Krishnamurthy and Vissing-Jorgensen (2011).

The model has three dates. At the initial date, the economy is hit by an unexpected negative (aggregate) liquidity shock; firms with assets in place need to make a liquidity investment to enable their project to deliver cash flow at the intermediate date. If commercial banks increase their lending to the corporate sector (i.e., provide liquidity support to firms), firms are able to make the investment and will thus realize their cash flow; this will also increase the aggregate available cash in the industry.[9] However, the friction of unverifiability of cash flow means that bank lending needs to be secured by collateral. Lending then crucially depends on the collateral value of a firm's asset, which is the asset re-sale value in the secondary market at the intermediate date. The re-sale value in turn depends on the aggregate available cash in the industry at the intermediate date (as in Shleifer and Vishny (1992)) as well as the expected fundamental value of the asset at the final date. When the central bank injects liquidity into the commercial banking system, a feedback loop can form between bank lending, asset prices and collateral values.

We show that the strength of feedback, however, is asymmetric across sectors. This is because the response of the asset price to liquidity injections is asymmetric across sectors. In a secondary asset market, buyer firms can not only use their own cash when buying but can also borrow from the sellers by pledging their assets as collateral (i.e., *asset leverage*). Hence, the asset price in a secondary market reflects the level of asset leverage buyers can access. For Sector 2, with lower asset specificity, buyer firms are able to leverage more when buying (i.e., higher asset leverage), considering that their lower asset specificity raises debt capacity more (Williamson (1988)).[10] Therefore, the asset price in Sector 2 responds more strongly to liquidity injections than that in Sector 1 does.

This asymmetry across sectors creates a 'crowding-out' effect. If too much liquidity is injected into the economy, the asset price and thus the collateral value in Sector 2 can increase so fast that it leads to a rise in the real interest rate in the economy. This is because when the collateral value increases, firms' collateral constraints are relaxed; more firms thus qualify to borrow and compete for loans, pushing up the interest rate. With the collateral value in Sector 1 not responding much, the higher interest rate reduces the ability of firms to borrow in that sector, *crowding out* the liquidity entering Sector 1. The crowding-out manifests in a self-reinforcing spiral as more liquidity

[9] The term 'firm' is used in a broad sense, encompassing individual investors. It is possible that an individual rather than a corporation invests in the asset (financial) market, e.g., to buy houses.

[10] See the empirical evidence by Adrian and Shin (2010) and Kalemli-Ozcan et al. (2011), among others.

flowing into Sector 2 pushes up the interest rate, and the increased interest rate leads to *additional* liquidity flowing out of Sector 1 (into Sector 2), pushing up the interest rate further, and so on. In short, too much liquidity injected actually *reduces* the liquidity entering Sector 1. If, on the other hand, too little liquidity is injected, Sector 1 of course cannot obtain much liquidity. We show that there exists an optimal level of liquidity injection for the central bank.

One would expect that more (less) liquidity leads to a decrease (increase) in interest rates. The Japanese experience during the 1980s, however, is a stark example of the tightening policy accompanying a (slight) decrease in real interest rates. In fact, the tightening policy in Japan in the latter 1980s was followed by a fall in asset prices and thus a reduction in the collateral values of firm assets; the reduction in the creditworthiness of Japanese corporations at least in part contributed to lower demand for credit, decreasing interest rates (see, e.g., Bernanke and Gertler (1995)).[11] What happened recently in China can be regarded as the same sort of problem the Japanese faced, but in the opposite direction. That is, the massive liquidity injections and credit expansion in China created overheating in some sectors (e.g., the real estate sector) which led to the effective demand for credit shooting up, in turn causing real interest rates to rise. This potentially generated a downward 'crowding-out' spiral across sectors, as China's media reported.

The model has two empirical implications. The first is a cross-sectional implication. The model implies that for a country with a poorer contracting institution (see, e.g., LaPorta, Lopez-de-Silanes, Shleifer and Vishny (1997, 1998), Djankov, Hart, McLiesh and Shleifer (2008)), crowding-out across sectors is more likely. The second is a time-series implication. At times of greater uncertainty about economic prospects, crowding-out is more likely to occur in response to liquidity injections.

Related literature. Our paper highlights the effect of financial frictions on the allocation and distribution of liquidity in the economy. The financial friction in our model is the unverifiability of cash flows combined with collateral constraints (see, e.g., Hart and Moore (1994, 1998)). In the two-sector economy setting, we show that liquidity not only tends to move to the sector with lower friction (i.e., the *allocation effect*) but also that the sector with lower friction can crowd out the other sector in attracting liquidity (i.e., the *crowding-out effect*). In the literature on fiscal policy,

[11] Bernanke and Gertler (1995) write: "the crash of Japanese land and equity values in the latter 1980s was the result (at least in part) of monetary tightening; ... [T]his collapse in asset values reduced the creditworthiness of many Japanese corporations and banks...". See also Benmelech and Bergman (2012).

the crowding-out effect means that an increase in government spending can lead to a reduction in private investment, because more spending can increase interest rates due to increased borrowing (see, e.g., Blanchard (2008)). Our paper demonstrates the crowding-out effect across two (private) sectors in the context of liquidity injections. The mechanism of the crowding-out effect in our model is different, through the feedback between liquidity injections (credit expansion), collateral values, and interest rates.[12]

In the literature on financial frictions (see, e.g., the seminal work by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997)),[13] Benmelech and Bergman (2012) built a novel framework for studying the interplay between financing frictions, liquidity, and collateral values. The authors show that the credit easing policy sometimes does not work because additional liquidity injections do not raise firm asset collateral values and thus credit traps can form. Additional liquidity injections in their model are ineffective but harmless to the economy. Our paper contributes to this literature in two ways. First, we study the *distribution* of liquidity in an economy, and demonstrate the danger of *excessive* liquidity injections (beyond inflation): excessive liquidity actually hurts aggregate economic performance because it causes misallocation of liquidity in the economy. Second, we provide a new micro-foundation for the effect of liquidity injections on asset prices. We believe that the new micro-foundation is more robust, complementing Benmelech and Bergman's work.

The work by Shleifer and Vishny (1992) and Geanakoplos (2010) helps explain why asset prices in financial markets do not only depend on asset fundamentals but also on the aggregate liquidity in the economy. Shleifer and Vishny (1992) adopt an industry equilibrium approach with asset specificity. Geanakoplos (2010) as well as Simsek (2012) uses a general-equilibrium framework with heterogeneous beliefs.[14] In this literature, aggregate liquidity in the economy is exogenously given.

[12] In the macroeconomics literature on bubbles, Tirole (1985) and Tirole and Farhi (2012) show that bubbles in unproductive assets can crowd out investments in unrelated real assets. In contrast, in our paper, overheating occurs in the productive investment of a less frictioned sector, crowding out investment in a sector with higher friction. Our paper studies the effect of different levels of liquidity injections in a two-sectors economy with financial contracting frictions.

[13] Brunnermeier, Eisenbach and Sannikov (2013) provide a recent excellent survey.

[14] A strand of finance literature uses a general equilibrium framework to study the interplay between liquidity, leverage, and asset prices (e.g., Holmstrom and Tirole (1997) and Acharya and Viswanathan (2011)). Simsek (2012) studies asset pricing under collateral constraints with belief disagreements as in Geanakoplos (2010).

In our paper, we show the effect of policy (liquidity injections) on the aggregate liquidity, and the asymmetric response of asset prices across sectors to the change in aggregate liquidity.

Our paper is related to the literature that links finance and macroeconomics. Shleifer and Vishny (2010a,b) present a theory justifying the credit easing policies of governments. In their model, distressed asset prices lead to banks' incentives to speculate, at the expense of funding new real investments, justifying ex post government intervention. Diamond and Rajan (2006) study money policy in a banking framework and explore the connection between money, banks and aggregate credit. Allen and Gale (2000) study the consequences of credit expansion and highlight the problem of risk shifting of borrowers and its asset price implications. Acharya and Naqvi (2012) show that abundant liquidity can cause moral hazard problems of loan officers inside banks, inducing excessive credit volume and having asset price implications. Compared with the above work, our paper focuses on the interplay between two sectors in the economy that have different degrees of financial friction. The mechanism generating the asset price change is heterogeneous beliefs with asset specificity, instead of moral hazard problems of risk shifting.[15]

The paper is organized as follows. In Section 2, we present the model and the equilibria, and analyze the welfare implications of the model. In Section 3, we discuss empirical implications of the model. Section 4 concludes.

2 Model

In this section, we present the model and the equilibria.

2.1 Setup

Consider an economy with two sectors, labeled Sector 1 and Sector 2. The two sectors differ in firm asset specificity, which we will elaborate on. Each sector consists of a continuum of self-employed firm-households of measure one.[16] For ease of exposition, we do not distinguish between the two sectors at this stage; so for now we can regard that there is only one sector. Later, we will model

[15] Our paper is related to the growing literature on unconventional monetary (credit) policies (see, e.g., Reis (2009), Gertler and Kiyotaki (2010), Gertler and Karadi (2011)). As Gertler and Kiyotaki (2010) write, "Since these policies are relatively new, much of the existing literature is silent about them."

[16] See Mendoza (2010) for the setup of self-employed firm-households.

the interplay between the two sectors in detail. In the economy, there is also a set of commercial banks that supply capital to firms, and a central bank. The model has three dates: T_0, T_1 and T_2. There is no time discount or the time between dates is short.

2.1.1 Firms

Each firm has an asset *in place* at T_0 - an identical investment project across all firms. Firms undertook their project before T_0, which is expected to generate a constant cash flow C at T_1 and a random cash flow $\widetilde{x}$ at T_2, where $\widetilde{x}$ has one of two realizations, $\widetilde{x} \in \{u, d\}$, and $u > d > 0$.

Only a part of the cash flow of the project is contractible. More specifically, the cash flow C is uncontractible while a part of the cash flow $\widetilde{x}$ is contractible. The contractible part of $\widetilde{x}$ is a constant amount X, where $0 \leq X \leq d$; the remaining part $\widetilde{x} - X$ is uncontractible. As is standard in the incomplete contracting literature (e.g., Hart and Moore (1998)), the interpretation is the following.

The project's cash flow is unverifiable. In the event that the owner of a project defaults at T_2, outside investors (i.e., debt-holders) obtain and exercise the control right over the asset; outside investors can only realize a cash flow X when they seize and operate the asset at T_2 due to asset specificity. That is, the term X measures asset specificity; the lower X, the higher the asset specificity. Alternatively (and intuitively), we can think that the payoff of the project at T_2 has two components: the cash flow $\widetilde{x} - X$ and the liquidation or salvage value of the project's (fixed) asset, X; while the cash flow is unverifiable, the project's (fixed) asset can be contracted as collateral, and outside investors can realize its liquidation (salvage) value. In this case, the term X equivalently measures firm asset collateralizability at T_2.[17] Williamson (1998) stresses the link between asset specificity, the liquidation value of assets, and debt capacity. He argues that assets with low specificity have high liquidation values, which raise debt capacity.[18] As shown later, asset specificity, the term X, determines *asset leverage* in the secondary asset market.

[17] Firm asset collateralizability at T_1 is different, which is measured by P, as shown later.

[18] See Benmelech (2009) and Benmelech and Bergman (2009, 2011) for evidence.

2.1.2 Liquidity shock

The economy (firms) suffers an *unexpected* (aggregate) liquidity shock at T_0 (as in the business cycle literature, e.g., Kiyotaki and Moore (1997)). That is, a firm has to invest an additional amount I at T_0 to enable its project to deliver the cash flow C, where $I < C$; otherwise its project delivers zero cash flow at T_1.

Firms differ in their level of internal capital at T_0. Suppose the amount of internal capital of a firm at T_0 is A, which means that the firm needs an amount of external capital, $B \equiv I - A$, to be able to make its liquidity investment. We assume that B has a probability distribution (pdf), $f(B)$, across firms, within the support $[0, I]$. Let $F(\cdot)$ denote the cumulative distribution function (cdf) of $f(\cdot)$. Clearly, giving the distribution of B is equivalent to giving the distribution of A.[19]

Faced with limited internal capital, firms seek to raise external capital by borrowing from commercial banks.[20] The borrowing (debt) is short-term, that is, a firm needs to repay its debt at T_1. We will show that long-term debt with maturity T_2 is not optimal or infeasible. Firms that do not make the liquidity investment can *deposit* their spare internal capital with commercial banks at T_0.

It is common knowledge that firms will have diverging (heterogeneous) beliefs at T_1. For simplicity, we assume that there are two types of beliefs at T_1: high beliefs and low beliefs. For the high (respectively low) beliefs, the probability of realizing u of $\widetilde{x}$ is θ_H (respectively θ_L), where $\theta_H > \theta_L$. That is, high beliefs correspond to $\Pr[\widetilde{x} = u] = \theta_H$ and low beliefs to $\Pr[\widetilde{x} = u] = \theta_L$. Ex ante, before T_1, the probability of developing high beliefs is π. We also denote the true probability of realizing u of $\widetilde{x}$ as θ.

It is realistic to model diverging (heterogeneous) beliefs among firms. In fact, in an economic recession, agents are often quite uncertain about economic prospects and have diverging views.

There is a secondary asset market at T_1, where firms with heterogeneous beliefs trade their

[19] For simplicity and without loss of generality, we do not explicitly model firms' investment and financing decisions prior to T_0, which is not the focus of the paper.

[20] Rajan (1992) provides justifications for debt financing.

assets.[21] As in Geanakoplos (2010), Miller (1977) and Harrison and Kreps (1978), short-selling is not allowed for the secondary market. In reality, short-selling is either impossible or with constraints.

2.1.3 Commercial banks

There is a large number of commercial banks that make loans to firms. Each individual commercial bank is price-taking. That is, the lending by any one bank does not affect the market-wide interest rate, at which firms can borrow. Denote the net interest rate of bank loans by r. As the cash flow C of a firm's project is not contractible, the only means to force a firm to repay is to contract the firm's asset (project) as collateral. If the firm does not repay, the bank can threaten to liquidate the firm's project to sell in the secondary market at T_1. We denote by P the market price of the asset (project) in the secondary market. If a firm has the full bargaining power in renegotiating with its bank, then a firm will never be able to commit to repay more than P at T_1 (e.g., Hart and Moore (1994)). Therefore, the collateral value of a firm's asset at T_1 is P.[22]

Both P and r will be endogenized.

2.1.4 Central bank

After the economy suffers the systemic liquidity shock, the central bank chooses an amount of liquidity, Q, to inject into the commercial banking system at T_0, where $Q \in \left[0, \overline{Q}\right]$; $\overline{Q}$ is the maximum amount of liquidity the central bank can inject, which reflects the government's constraint in economic stimulus. We interpret liquidity as loanable funds. Essentially, as in the literature on unconventional monetary policies,[23] we have abstracted away the institution and assumed that the central bank can directly determine the amount of loanable funds in the commercial banking system. This simplification is to capture the fact that the central bank can use various policy tools to influence bank credit available to the economy, for example, the policies of direct lending to

[21] We assume that firm projects are not "mature" enough at T_0 and thus firm assets cannot be traded at T_0 due to the inalienability of human capital (Hart and Moore (1994)).

[22] We will prove that $P > X$, so the long-term debt with maturity at T_2 is not optimal or feasible for some firms since they can raise less external financing by using long-term debt than by using short-term debt. This is in the spirit of Hart and Moore (1994) on the optimal debt maturity choice. Also, if the support of B is assumed to be $[X, I]$, long-term debt becomes infeasible for all firms.

[23] See, e.g., Reis (2009), Gertler and Kiyotaki (2010), Gertler and Karadi (2011) and Benmelech and Bergman (2012).

financial institutions, equity injections to increase bank capital, and so on.[24] We can also interpret liquidity injections in our model as a *credit expansion* policy, as many emerging market economies have exercised in response to financial crises. In fact, our paper aims to study consequences of excessive liquidity injections *beyond inflation*, by focusing on the effect of financial contracting frictions on the distribution of liquidity.[25] For simplicity and without loss of generality, we assume that each commercial bank obtains a fixed amount of liquidity, aggregating to Q.

The liquidity injection Q is essentially 'outside liquidity' to the private sector in the spirit of Kiyotaki and Moore (2002) while the liquidity ultimately from the private sector itself (i.e., the bank deposits by non-investing firms) is 'inside liquidity'. As shown later, in our model the liquidity injection Q gets fully repaid by the private sector (i.e., investing firms) at T_1. In this sense, we might also interpret the liquidity injection Q as a fiscal policy (such as a credit expansion policy); for example, the government borrows the Q amount of *real goods* from foreign countries to support the domestic economy, and the Q amount of borrowing is fully repaid later (with the source of the output of the domestic private sector).

We will model two alternative objective functions of the central bank, which deliver qualitatively equivalent results. The first is for the central bank to maximize the number of firms in Sector 1 that can make the liquidity investment. This can be justified by the government caring more about outcomes such as employment (e.g., small and medium-sized businesses) than purely firms' profits. If Sector 1, relative to Sector 2, is disproportionately more important in these aspects, the government may make Sector 1 its first priority in economic stimulus. The alternative objective function of the central bank is to maximize the total surplus of the economy (both sectors).

Figure 2 summarizes the main setup of the model.

[24] It is also worth noting that all quantities in our model are in 'real' and not 'nominal' terms. Basically, we abstract away the nominal side of the economy (e.g., inflation) and focus on the real side.

[25] Gertler and Kiyotaki (2010) write: "From the standpoint of the Federal Reserve, these "credit" policies represent a significant break from tradition. In the post war era, the Fed scrupulously avoided any exposure to private sector credit risk. However, in the current crisis the central bank has acted to offset the disruption of intermediation by making imperfectly secured loans to financial institutions and by lending directly to high grade non-financial borrowers. In addition, the fiscal authority acting in conjunction with the central bank injected equity into the major banks with the objective of improving credit flows. ... Since these policies are relatively new, much of the existing literature is silent about them."

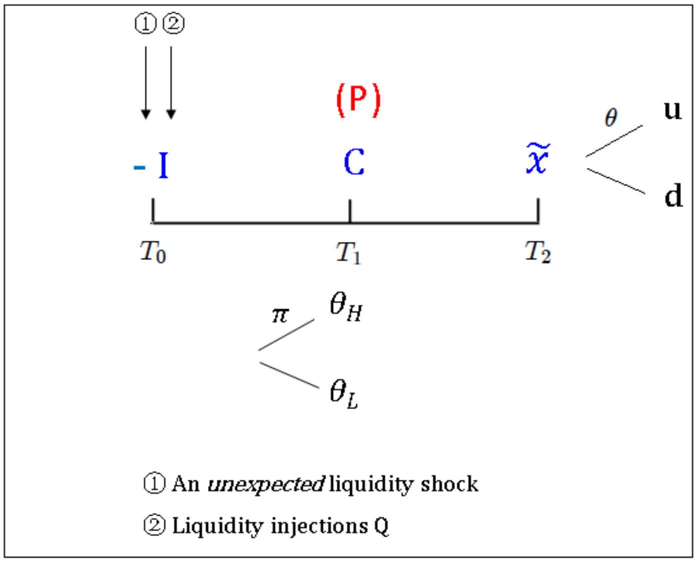


Figure 2: Main setup of the model

2.2 One-sector economy equilibrium

In this subsection, we solve for the equilibrium of the one-sector economy. The one-sector equilibrium highlights a new micro-foundation for the effect of liquidity injections on asset prices, complementing Benmelech and Bergman's (2012) work.

We first state the equilibrium concept.

One-sector economy equilibrium An equilibrium of the one-sector economy consists of the following four elements:

(i) Firms optimize their investment and borrowing choices at T_0 given the interest rate r;

(ii) Commercial banks optimize their lending decisions at T_0 given the collateral value of firm asset, P, and the market interest rate r;

(iii) The bank credit market clears at T_0. That is, the aggregate supply of bank credit is equal to the total demand of credit from firms;

(iv) The secondary asset market clears at T_1. That is, there is an asset market equilibrium at T_1, where the equilibrium asset price is P.

2.2.1 Solving for the equilibrium

We examine equilibrium elements (ii), (iii) and (iv) in order, and finally check element (i).

First, we consider the decisions of commercial banks at T_0. If commercial banks *rationally anticipate* that the collateral value of a firm's asset is P at T_1, and given the interest rate r, they would grant a loan to a firm with a maximum amount $\frac{P}{1+r}$.[26] Hence, the marginal firm that can undertake the liquidity investment, denoted B^*, is

$$B^* = \frac{P}{1+r}. \tag{1}$$

We will verify, by considering their participation conditions, that firms $B \in [0, B^*]$ undertake the liquidity investment while firms $B \in (B^*, I]$ do not. Basically, B^* measures *corporate leverage* in a sector.[27]

Second, the credit market must clear such that the total supply of funds should be equal to the total demand for funds at T_0. The supply of funds is from commercial banks, which have two sources of funding: the liquidity injection Q and the deposits from the non-investing firms $\int_{B^*}^{I}(I-B)f(B)dB$. The total demand of funds (by the investing firms) is $\int_0^{B^*} Bf(B)dB$. Thus, we have

$$\int_{B^*}^{I}(I-B)f(B)dB + Q = \int_0^{B^*} Bf(B)dB. \tag{2}$$

Adding $\int_0^{B^*}(I-B)f(B)dB$ to both sides of this equation, equation (2) can be equivalently rewritten as

$$\int_0^{I}(I-B)f(B)dB + Q = I \cdot F(B^*). \tag{2'}$$

[26] Considering that each commercial bank has a fixed amount of funding to lend, and given the market interest rate r, an individual commercial bank has no incentive to use an interest rate different from r. In fact, if it charges an interest rate lower than r, its profits become less. On the other hand, if it charges an interest rate higher than r, it loses all its customers. So it is optimal for the individual commercial bank to use the market interest rate r as well. Essentially, the commercial banks behave competitively and are price-takers.

[27] For corporate leverage, in our model the asset price and the collateral value are the same, which are P. We could use a more complicated setup in which the collateral value is positively correlated with the asset price, but not the same.

Equation (2') has an intuitive interpretation. Each project requires an amount I of investment and thus the total amount of investment in the economy is $I \cdot F(B^*)$, which is financed by the liquidity injection Q and the aggregate inside liquidity, the internal capital of all firms.

Third, solving for the market equilibrium of the secondary asset market at T_1, we obtain the equilibrium asset price P. Firms have different beliefs at T_1. The asset valuation under high beliefs, denoted $E^H(\widetilde{x})$, is $E^H(\widetilde{x}) = u \cdot \theta_H + d \cdot (1 - \theta_H)$. Likewise, the asset valuation under low beliefs, denoted $E^L(\widetilde{x})$, is $E^L(\widetilde{x}) = u \cdot \theta_L + d \cdot (1 - \theta_L)$. Clearly, $E^H(\widetilde{x}) > E^L(\widetilde{x})$. The difference in valuations among firms motivates them to trade. Also, as in Shleifer and Vishny (1992), only industry participants, *who have previous periods of experience in managing assets*, can operate the assets to generate cash flows at T_2. Thus, buyers in the secondary market are firms with high beliefs. Buyers can not only use their own funds but also use leverage when buying. Therefore, the asset price depends on the total liquidity that buyers can access.

Based on the above analysis, we have the asset price, P, in the secondary asset market at T_1:

$$P = \begin{cases} E^H(\widetilde{x}) & \text{if } \Gamma(B^*, r) > E^H(\widetilde{x}) \\ \Gamma(B^*, r) & \text{if } \Gamma(B^*, r) \in [E^L(\widetilde{x}), E^H(\widetilde{x})], \\ E^L(\widetilde{x}) & \text{if } \Gamma(B^*, r) < E^L(\widetilde{x}) \end{cases} \tag{3}$$

where

$$\Gamma(B^*, r) = \frac{\pi \left\{ \int_0^{B^*} [C - B(1+r)] f(B) dB + \int_{B^*}^{I} (1+r)(I-B) f(B) dB \right\} + X}{1 - \pi}.$$

The key to understanding the asset price is expression $\Gamma(B^*, r)$, which is in the spirit of Geanakoplos (2010). The asset price reflects not only the asset's expected future fundamental value at T_2 but also the current liquidity that buyers can access at T_1. The current liquidity at T_1 available to buyers has two components. First, a firm that made the liquidity investment needs to repay its bank loans resulting in its net liquidity of $C - B(1+r)$; a firm that did not make the investment can withdraw its deposits from banks resulting in its net liquidity of $(1+r)(I-B)$. As all the firms have the experience in managing the assets before T_1, buyers are of total measure π. Thus, the aggregate internal funds of buyers at T_1 are $\pi \left\{ \int_0^{B^*} [C - B(1+r)] f(B) dB + \int_{B^*}^{I} (1+r)(I-B) f(B) dB \right\}$.[28]

[28] In our model, in equilibrium every investing firm has sufficient cash to repay its debt at T_1 even if it is the marginal firm that borrows the highest amount. That is, there is no default.

Second, a buyer uses his own asset plus his purchased assets as collateral for borrowing, in which case he can borrow an amount X against each asset.[29] In sum, the aggregate liquidity available to buyers includes the aggregate internal funds of buyers (i.e., the first term of the numerator) and the aggregate liquidity borrowed against the assets in the economy as collateral (i.e., the second term of the numerator). The denominator is the quantity of assets put up for sale.

Moreover, the asset price is truncated by upper and lower bounds, reflecting its dependence on the asset's expected future fundamental value at T_2. If the asset price calculated in Γ is higher than $E^H(\widetilde{x})$, this means the total available liquidity is excessive. Thus, in equilibrium, the asset price is $E^H(\widetilde{x})$, at which the firms with high beliefs are indifferent between buying and not, some of whom do not participate in buying, and the total liquidity used to buy is less than $\pi\left\{\int_0^{B^*}[C-B(1+r)]f(B)\,dB+\int_{B^*}^{I}(1+r)(I-B)f(B)\,dB\right\}+X$. At the other extreme, if the asset price calculated in Γ is lower than $E^L(\widetilde{x})$, this means that there is too little total available liquidity. Thus, the equilibrium asset price is $E^L(\widetilde{x})$, at which the firms with low beliefs are indifferent between selling and not, some of whom do not participate in selling, and the total quantity of assets to sell is less than $1-\pi$.

In what follows, we denote the pricing function of (3) as $P=p(\widetilde{x},B^*,C,X,r)$.

Finally, we check the firms' participation condition at T_0. We find the condition under which a firm is willing to make its liquidity investment. Given the interest rate r, if a firm with internal capital A borrows an amount $B=I-A$ to invest in its project, its payoff is $C-B(1+r)$. Alternatively, the firm can deposit its internal capital in commercial banks and realize a payoff of $A(1+r)$. Thus, the firm is willing to invest if and only if

$$C-B(1+r)\geq A(1+r)\quad\Leftrightarrow\quad C-\ I(1+r)\geq 0. \tag{4}$$

Note that inequality (4) does not depend on A, which means either all firms or none are willing to make the liquidity investment. For simplicity, we focus on the set of equilibria in which inequality (4) is satisfied, that is where all firms are willing to invest. In this case, whether a firm *actually* makes the liquidity investment is completely determined by whether it can satisfy the borrowing constraint, $B\leq B^*$ (defined in (1)). In other words, all firms *want* to invest but whether they can actually invest depends on whether they *can* obtain loans.

[29] We can focus on the equilibrium that the sellers' funds are big enough to satisfy the buyers' borrowing.

Based on the above analysis, we have Proposition 1.

Proposition 1 *The equilibrium of the one-sector economy is characterized by a triplet* $\{B^*, P, r\}$*, which, given* Q*, solves the system of equations (1) to (3), and satisfies condition (4).*

To summarize, the analysis above captures the endogenous feedback loop under liquidity injections between bank lending, the asset price, and the collateral value, illustrated in Figure 3. Our paper provides a new micro-foundation for the endogenous feedback loop.

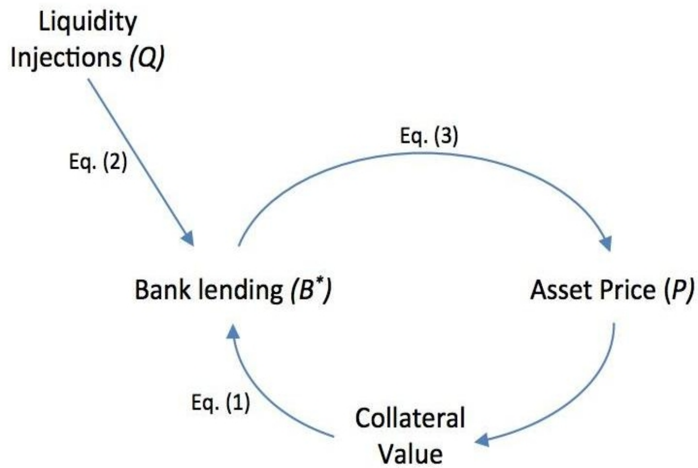


Figure 3: Feedback loop

2.2.2 Characterizing the equilibrium

Exogenous collateral prices Before we proceed to characterizing the equilibrium, it is instructive to analyze how liquidity injections or credit expansion policies work if the collateral value (the asset price) is exogenously given and constant. By equation (2), we have that B^* is increasing in Q (in fact, $\frac{dB^*}{dQ} = \frac{1}{I \cdot f(B^*)} > 0$), that is, the more liquidity injected, the more firms are being financed. Notably, equation (2), which determines the number of firms obtaining financing, is independent of the collateral value (the asset price) P. If the collateral value is exogenously given and constant, then by equation (1) the interest rate r goes down when liquidity injections, Q, increase. In short, the mechanism can be summarized as follows: the additional supply of liquidity lowers the interest rate; for a given collateral value, more firms are able to obtain financing, which clears the market. From the above analysis, a low collateral value *per se* may not prevent firms from obtaining financing under liquidity injections, because an even lower interest rate can channel lending to firms.[30]

In our model, the collateral value (the asset price) is endogenous to liquidity injections and the equilibrium interest rate may be non-monotonic in liquidity injections. We are interested in the answers to the following comparative static questions: What is the effect of liquidity injections on the equilibrium asset price and the equilibrium interest rate (i.e., the functions $P(Q)$ and $r(Q)$)? What is the role of asset specificity in the effect?

First, we examine how liquidity injections impact the asset price P. Intuitively, liquidity injections enable more firms to make their liquidity investment at T_0 and hence increase the liquidity in the industry at T_1, which in turn may raise the equilibrium asset price.

Formally, we can prove that Γ in (3) is an increasing function of Q. In fact, from (2), B^* is increasing in Q. We can also obtain that the *total* derivative $\frac{d\Gamma}{dB^*}$ is positive if $C - I(1+r) > 0$. Therefore, under this condition, Γ is certainly increasing in B^* and thus in Q.

Lemma 1 *The equilibrium price* Γ *is increasing in* Q *if* $C - I(1+r) > 0$ *in equilibrium.*

Proof: See the Appendix.

The intuition for Lemma 1 is the following. Liquidity injections Q enable more firms, which are otherwise unable, to make the liquidity investment. Suppose in the economy there is one more firm switching from non-investing to investing at T_0. Given r, this increases the liquidity in the sector at T_1 by an amount $C - I(1+r)$, which corresponds to the NPV of the marginal investment in (4).[31] More liquidity in the sector (industry) at T_1 increases the asset price, in the spirit of Shleifer and Vishny (1992). We show that the condition in Lemma 1, which is also condition (4) in Proposition 1, holds under general parameter values and hence Γ is increasing in Q.

Crucially, we need to consider the lower bound of P in (3). The asset specificity plays an important role here. If X is low, the buyers cannot leverage much when buying. Hence, the asset

[30] Benmelech and Bergman (2012) and others study credit traps, where the interest rate may hit zero bound. Our paper has a different focus.

[31] There is a further general equilibrium effect because r changes. We prove that the overall effect is positive when $C - I(1+r) > 0$. See the proof of Lemma 1 in the Appendix.

price is low. It is possible that X is so low that the asset price is trapped at the lower bound $E^L(\widetilde{x})$ no matter what Q $(\in [0, \overline{Q}])$ is; that is, P does not change with Q.

We have Proposition 2.

Proposition 2 *If $X \leq \underline{X}$, where $\underline{X}$ is a (positive) cutoff, the equilibrium asset price $P(Q)$ is a constant, equal to $E^L(\widetilde{x})$, no matter what the size of the liquidity injection Q $(\in [0, \overline{Q}])$ is. If $X > \underline{X}$, $P(Q)$ is (weakly) increasing in Q under general parameter values.*[32]

Proof: See the Appendix.

In Proposition 2, asset specificity plays an important role in determining the asset price. The reason is that asset specificity determines the leverage level of buyers (i.e., the amount of margin financing buyers can access) and thus influences the total liquidity available for purchases and thus the asset price. Proposition 2 gives the cleanest case for liquidity injections having little impact on the asset price.[33] This happens when the financing friction is sufficiently high (X sufficiently low).[34]

Margin levels can differ substantially across sectors. For example, the margin level for purchasing financial assets including real estate can be significant, while the margin level for purchasing real assets like equipment or machines can be negligible. On the eve of the 2007-2009 financial crisis, the haircut in trading mortgage-backed security (MBS) often amounted to less than 10%, or the margin level in terms of the debt-to-asset ratio equated to above 90%; the margin level for trading stocks is typically around 50% (see Gorton and Metrick (2010) and Adrian and Shin (2010) for evidence on margins). Different margin levels across industries clearly give cross-sectional asset price implications.[35]

[32] For our purpose, we focus on the set of equilibria in which Γ is lower than and not binding at $E^H(\widetilde{x})$. This can be achieved by assuming that θ_H and thus $E^H(\widetilde{x})$ are sufficiently big, *ceteris paribus*.

[33] For the benefit of a clean analysis and for our purpose, we have divided X into two regions in the analysis: $X \leq \underline{X}$ and $X > \underline{X}$. The merit of the cleanness can be further seen later when we discuss Figures 4a and 4b. The results of the paper however hold generally (for some $\overline{Q}$).

[34] In this sense, buyers' internal funds and external funds are complementary in purchasing assets; more internal funds push up the asset price only when the external funds are above a threshold.

[35] See, e.g., Garleanu and Pedersen (2011) and Frazzini and Pedersen (2011) for evidence on margin-based asset pricing.

That the asset price can be pegged to the valuation under low beliefs has empirical support. In fact, in an economic downturn, agents are often quite uncertain about economic prospects. In particular, in the aggregate, agents appear to be acting as if under "Knightian uncertainty" (i.e., agents use a worst-case for the uncertain probabilities in valuations, see, e.g., Caballero and Krishnamurthy (2008)), and asset prices are depressed and exhibit features of "flight to quality" phenomena. Those times may correspond to π being low in our model, in which case margin levels can be crucial in determining how asset prices respond to liquidity injections.

In light of the earlier discussion, a low collateral value *per se* may not prevent firms from obtaining financing under liquidity injections, as an even lower interest rate can channel lending to firms. What matters in our model is that liquidity injections may have *asymmetric* impacts on collateral prices across the two sectors (with different Xs), which can generate the crowding-out effect.

Next, we examine the response of the equilibrium interest rate to the liquidity injection. On the one hand, liquidity injections translate into additional supply of loans. On the other hand, liquidity injections may raise the asset price and thus the collateral value (for bank loans); the higher collateral value means that more firms can borrow and *compete* for loans. That is, the effective demand for liquidity increases. As a result of the simultaneous increase in both the supply of and the demand for liquidity, the equilibrium interest rate can either go up or down.

Formally, from the optimal lending condition (1), we have the equilibrium interest rate as $r = \frac{P(Q)}{B^*(Q)} - 1$, where $P(Q)$ has the properties of Proposition 2 and $B^*(Q)$ is uniquely determined by market clearing (2). We also have that $B^*(Q)$ is increasing in Q. Therefore, if $P(Q)$ is constant, then r is certainly decreasing in Q. If $P(Q)$ is increasing in Q, it is ambiguous whether r is decreasing or increasing in Q. Indeed, we prove that r can go in either direction depending on whether $P(Q)$ increases faster or slower than $B^*(Q)$. Also, we have the following result: when Q is very low (close to 0), r is generally decreasing in Q. To see the intuition, we consider the equilibrium interest rate when $Q = 0$; in this case, all bank loans are 'inside liquidity' and no cash flow at T_1 is used to repay the interest on 'outside liquidity' Q, and hence the asset price relative to the threshold B^* (that is, $\frac{P(Q)}{B^*(Q)}|_{Q=0}$) is high; when Q increases away from zero, a part of the cash flow at T_1 starts to be repaid as the interest on 'outside liquidity', and hence $\frac{P(Q)}{B^*(Q)}$ decreases.

Proposition 3 summarizes the relationship between r and Q.

Proposition 3 *If the financing friction is high, $X \leq \underline{X}$, the equilibrium interest rate $r(Q)$ is strictly decreasing in Q ($\in [0, \overline{Q}]$). For lower financing frictions, $X > \underline{X}$, under some distribution $f(B)$ and some parameters, $r(Q)$ decreases first and then increases in Q, that is, there exists a minimum $r(Q)$, denoted $r_{\min}$, for an interior $Q \in (0, \overline{Q})$.*

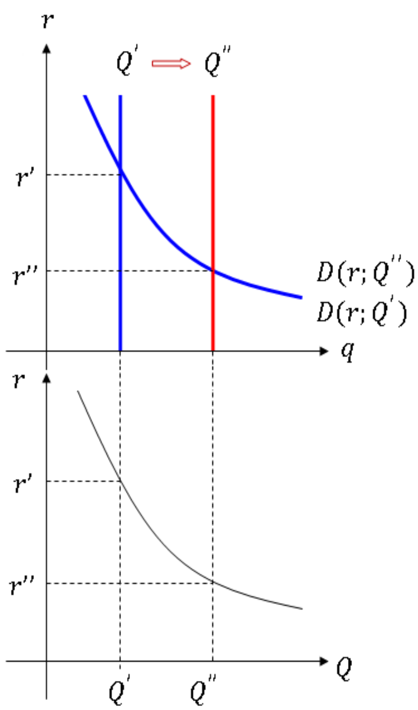
Proof: See the Appendix.

The interest rate equilibrates the (effective) demand for liquidity, captured by the collateral value (asset price) P, and the supply of liquidity, captured by the threshold B^*, through $1 + r = \frac{P(Q)}{B^*(Q)}$. Proposition 3 delineates two important cases. First, if the asset price is constant, then the interest rate certainly decreases in liquidity injections because the supply of loans increases with liquidity injections. Second, the asset price may increase very fast, faster than B^*. In this case, the interest rate increases with liquidity injections. In fact, the latter case happens when the density $f(B)$ is thick in some region of B. Note that $\frac{dB^*}{dQ} = \frac{1}{I \cdot f(B^*)}$. If the density of $f(B^*)$ is high, B^* increases very slowly in Q, not as fast as $P(Q)$ does, hence the interest rate increases.

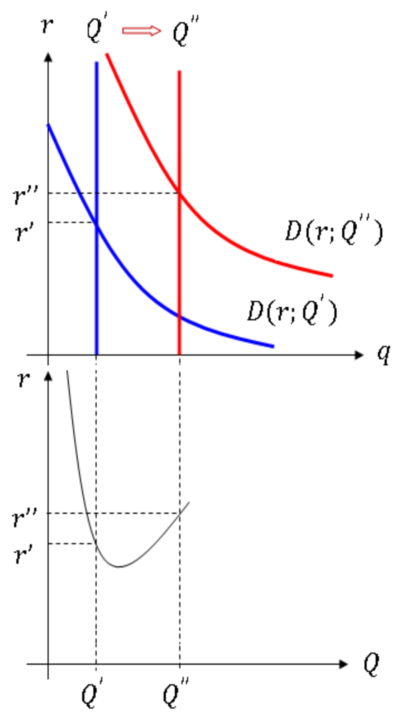
Remark We can explain the above mechanisms in the framework of the demand and supply equilibrium. When more liquidity is injected, which is on the supply side, the interest rate typically falls. However, in the model, the supply may also influence the demand. Additional liquidity injected may push up the asset price and thus the collateral value; hence the *effective* demand for liquidity also increases. When the increase in the effective demand is greater than the increase in supply, the interest rate goes up rather than down.

Figures 4a and 4b illustrate the equilibrium interest rate under the demand and supply equilibrium. The effective demand of liquidity is a decreasing function of the interest rate (for a given collateral value), i.e., the higher the interest rate, the lower the demand. Thus, the additional supply of liquidity typically causes the equilibrium interest rate to fall, which is the case of Figure 4a. However, in general equilibrium, the supply may also change the effective demand, i.e., the effective demand curve shifts upward because the collateral value increases. The demand curve may move upward very fast, and hence the equilibrium interest rate is non-monotonic in liquidity supply, which is the case of Figure 4b. Formally, in our model, the aggregate supply of liquidity is

$Q + \int_{B^*}^{I} (I - B) f(B) dB$ and the aggregate demand is $\int_0^{B^*} B f(B) dB$. By (2'), we can equivalently rewrite the supply as $S(Q) = Q + \int_0^{I} (I - B) f(B) dB$ and the demand as $D(r; Q) = I \cdot F\left(\frac{P(Q)}{1+r}\right)$, where both supply and demand are functions of Q. In Figure 4a, $P(Q)$ is a constant while in Figure 4b, $P(Q)$ is increasing in Q.



(a) When $X \leq \underline{X}$



(b) When $X > \underline{X}$

Figure 4: Liquidity market equilibrium

Figure 5 depicts the response of the interest rate to liquidity injections for the one-sector economy under two cases of Proposition 3.

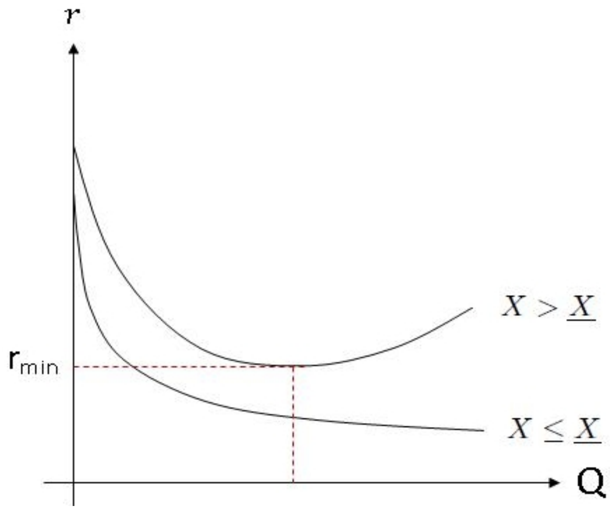


Figure 5: The response of the interest rate to liquidity injections for the one-sector economy

2.3 Two-sector economy

In this subsection, we analyze the economy with two sectors: Sector 1 and Sector 2. We study the distribution of liquidity in the economy and the distribution's effect on aggregate economic performance. We first solve for the two-sector equilibrium to highlight the crowding-out effect, and then consider the optimal decision of the central bank and analyze the welfare implications of the model.

2.3.1 Two-sector equilibrium: the crowding-out effect

In order to highlight the key mechanism underlying the crowding-out effect, at this stage we assume that the two sectors differ only in their asset specificity. That is, we assume that the cash flow of the project, the internal capital distribution of firms, and all other aspects are identical across the two sectors. The only difference is that the two sectors have different Xs, the contractible part of the cash flow at T_2. Sector 1 has a higher friction, a lower X denoted X_1, where $X_1 \leq \underline{X}$, while Sector 2 has a lower friction, a higher X denoted X_2, where $X_2 > \underline{X}$.[36] Note that X determines *asset*

[36] For cleanness, we consider that $X_2 > \underline{X} \geq X_1$. When $X_2 > X_1 \geq \underline{X}$, the result in this subsection (i.e., the unique optimal level of liquidity injection as will be shown in Propositions 5 and 7) still holds. However, a stricter condition is required, that is, the maximum amount of allowed liquidity injection $\overline{Q}$ cannot be too large. The reason is that when $X_2 > X_1 > \underline{X}$, the interest rates in both sectors are increasing in the liquidity inflow when the liquidity inflow is sufficiently high. Hence, the crowding-out effect occurs only when $\overline{Q}$ is not very large.

leverage while B^* measures *corporate leverage* in a sector; asset leverage (at T_1) impacts corporate leverage (at T_0).

The central bank can decide the aggregate liquidity to inject into the economy but cannot control the allocation of the liquidity across the two sectors in the economy, which is determined by commercial banks based on market forces. Market forces mean that: 1) commercial banks make loans only based on firms' *repaying ability* or collateral values (no matter which sector firms are from) and 2) there is a single interest rate in the economy (for both sectors). Also, the central bank cannot control the (real) interest rate directly. In fact, central banks generally control only the overnight interest rate but cannot control long-term real interest rates – the ultimate interest rates that matter for real businesses (see Blinder (1998, p. 70)); also, in our model, one r may map into two Qs (i.e., not a one-to-one mapping).

We solve for the two-sector economy equilibrium: for a given Q, how the liquidity is allocated and distributed across the two sectors. The equilibrium is given by the following equation system and condition:

$$r = \frac{P_i}{B_i^*} - 1 \tag{5a}$$

$$\int_{B_i^*}^{I} (I - B)\, f(B)\, dB + Q_i = \int_0^{B_i^*} B f(B)\, dB \tag{5b}$$

$$P_i = p(\widetilde{x}, B_i^*, C, X_i, r) \tag{5c}$$

$$C - \; I(1 + r) \geq 0 \tag{5d}$$

$$Q = \sum_i Q_i, \tag{5e}$$

where $i = 1$ and 2.

In (5a)-(5e), the variables with superscript i denote the variables for sector i, where $i = 1$ and 2. In particular, Q_i is defined as the *net* amount of outside-sector liquidity entering sector i, or the *net* liquidity inflow into sector i. Equations (5a), (5b) and (5c), and condition (5d) correspond, in order, to the respective equations (1), (2) and (3), and condition (4) of the one-sector economy. The three equations ((5a) through (5c)) and condition (5d) give the equilibrium *within* each sector. The interaction *between* the two sectors is given by (5a) and (5e). First, equation (5a) states that there is a single interest rate, in equilibrium, for the two sectors. Second, equation (5e) reflects that the aggregate outside-sector liquidity of the two sectors must equal Q.

In the two-sector economy, there is an integrated bank credit market at T_0, through which capital can flow across sectors. That is, a non-investing firm in one sector saves its spare internal capital in commercial banks and can actually become an ultimate liquidity provider to investing firms in the other sector. In fact, in (5a)-(5e), even when $Q = 0$, Q_i might not be equal to zero, in which case, measuring the liquidity flow across the two sectors. In contrast, the secondary asset markets for Sectors 1 and 2 at T_1 are segmented. That is, only industrial participants active in a particular sector can buy assets from their peers in the same sector.

Under general conditions, the equation system and condition of (5a)-(5e) have a unique solution and hence there is a unique equilibrium. We have Proposition 4.

Proposition 4 *The (market) equilibrium of the two-sector economy is characterized by a set* $\{B_i^*, P_i, Q_i, r\}$ *where* $i = 1$ *and* 2*, which, given* Q*, solves the system of equations (5a), (5b), (5c) and (5e), and satisfies condition (5d). If the conditions in Proposition 3 are satisfied, there is a unique equilibrium for the two-sector economy.*

Proof: See the Appendix.

We can think that there are two steps in solving for the two-sector equilibrium in (5a)-(5e). First, given Q_i for each sector, solve for the equilibrium within each sector, that is, solve for the triplet $\{B_i^*, P_i, r_i\}$. In particular, we obtain the function $r_i(Q_i)$. Second, by considering the link between the two sectors, (5a) and (5e), we can work out Q_i (for $i = 1$ and 2) for a given Q. That is, by considering $r_1(Q_1) = r_2(Q_2) = r$ and $Q_1 + Q_2 = Q$, we obtain the unique Q_1 and Q_2, and r.

We conduct the comparative static analysis on $Q_1(Q)$, i.e., how liquidity injections, Q, determine the net liquidity entering Sector 1. We find a unique Q that maximizes Q_1.

Proposition 5 *If the conditions in Proposition 3 are satisfied, there is a unique* Q*, denoted by* Q^**, that maximizes* Q_1 *(or* B_1^**).*

Proof: See the Appendix.

Figure 6 depicts the unique Q^*. In the figure, $Q^* = Q_1 + Q_2$. If the liquidity injection exceeds this level, all additional liquidity goes to Sector 2 and, further, some liquidity in Sector 1 is actually *squeezed out* due to the increased interest rate. Therefore, overall, the liquidity in Sector 2 increases while that in Sector 1 decreases. In fact, if Q increases above Q^*, Sector 1 cannot attract more liquidity as there is otherwise no interest rate at which the market across the two sectors can clear; thus Sector 2 must attract more liquidity, which will certainly increase the interest rate as we go along the increasing portion of the liquidity curve of Sector 2.

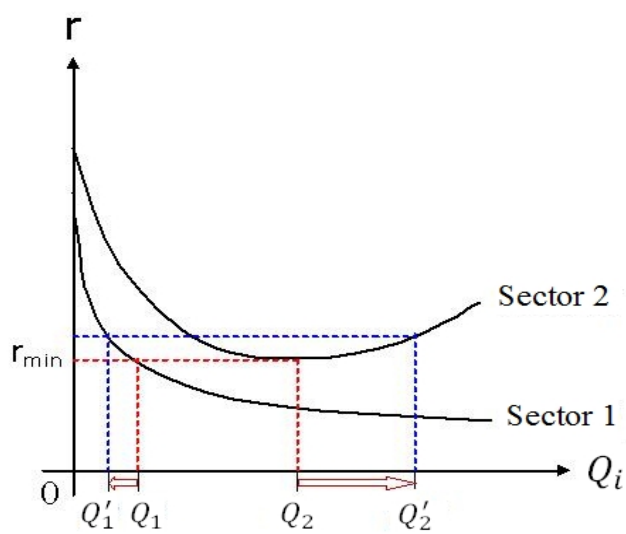


Figure 6: Crowding-out effect in the two-sector economy equilibrium

It is important to examine the process by which the new equilibrium is reached when some additional amount of liquidity (beyond Q^*) is injected. In Figure 6, in which an additional amount of liquidity $\Delta Q = (Q_1' + Q_2') - (Q_1 + Q_2)$ is injected, we examine how Q_1 reaches Q_1'. Suppose initially all additional liquidity, ΔQ, enters Sector 2; this would push up the interest rate; the higher interest rate would *squeeze* some liquidity out of Sector 1, which means that more liquidity would flow into Sector 2, pushing up the interest rate further, and so on in a spiral. Essentially, there is a spiral in reaching the new equilibrium. From the perspective of Sector 2, the spiral results in a multiplier $\frac{Q_2' - Q_2}{\Delta Q} > 1$; that is, although the (aggregate) additional amount of liquidity injection is ΔQ, the increment of liquidity in Sector 2 is actually more than ΔQ; the leap is at the expense of liquidity flows to Sector 1, causing a liquidity drain out of Sector 1. In short, one sector enters a liquidity-asset price 'inflationary' cycle while the other enters a 'deflationary' cycle, and the two cycles reinforce each other.

In sum, we can divide the support of liquidity injection, Q, into two regions: $Q \leq Q^*$ and $Q > Q^*$. In the region of $Q \leq Q^*$, the liquidity in both sectors increases with liquidity injections. In the region of $Q > Q^*$, additional liquidity injected increases the liquidity in one sector but reduces that in the other, that is, the 'crowding-out' effect occurs. The 'crowding-out' occurs in a spiral. We have Proposition 6.

Proposition 6 *When $Q \leq Q^*$, there is an 'allocation' effect, i.e., liquidity in both sectors increases with liquidity injections but Sector 1 obtains less liquidity than Sector 2. When $Q > Q^*$, there is a 'crowding-out' effect, i.e., more liquidity injected increases the liquidity in Sector 2 but reduces it in Sector 1; the crowding-out occurs in a self-reinforcing spiral.*

Proof: See the Appendix.

Although the investments in the two sectors have the same surplus (i.e., $C - I$), the liquidity is unevenly distributed across the two sectors. In particular, if too much liquidity is injected into the economy, Sector 1 can actually suffer a liquidity outflow. As long as the collateral value (the asset price) in one sector increases faster than that in the other sector, there is the possibility of crowding out.

2.3.2 The central bank's decisions and the welfare implications

We model two alternative objective functions of the central bank, which deliver qualitatively equivalent results. That is, there is an optimal level of liquidity injection for the central bank and the liquidity injection should not be too high. This result is robust under either objective function. The first objective function is that the central bank targets to maximize the number of firms in Sector 1 that can undertake the liquidity investment. This may be because the government cares more about outcomes such as employment (e.g., small and medium-sized businesses) than purely firms' profits. If Sector 1, relative to Sector 2, is disproportionately more important in these aspects, the government may make Sector 1 its first priority in economic stimulus. In this case, there exists an optimal level of liquidity injection for the central bank, as shown in Proposition 5.[37]

Second, we assume that the government is to maximize the total surplus in terms of cash flows of liquidity investments in the economy. To make the problem interesting and relevant, we assume that the two sectors differ in both their cash flow at T_0 and their asset specificity while holding all other aspects same. Specifically, we assume that $C_1 > C_2$ and $X_1 < X_2$, where C_i and X_i are, respectively, the cash flow at T_1 and the asset specificity for sector i, and $i = 1$ and 2. That is, Sector 1 has a higher cash flow and higher asset specificity.

The total surplus of liquidity investments in the economy is then

$$W = \int_0^{B_1^*} (C_1 - I) f(B)\, dB + \int_0^{B_2^*} (C_2 - I) f(B)\, dB.$$

The total surplus W is, in the end, divided among investing firms (profits), non-investing firms (interest on deposits) and the government (interest on Q) (see the proof of Proposition 7 in the Appendix). We make one remark. In general, it is difficult to conduct welfare analysis on models with heterogeneous beliefs (see, e.g., Brunnermeier, Simsek and Xiong (2012)). However, this is not the case with our model. In our model, we assume that the cash flow $\widetilde{x}$ at T_2 (over which firms develop heterogeneous beliefs) and the distribution of beliefs (i.e., the probability π) are identical across the two sectors. More importantly, the asset, which generates cash flow $\widetilde{x}$, is *in place* at T_0; liquidity injections are irrelevant to it. Liquidity injections only impact the liquidity investment I and the cash flow C subsequently. Hence, we can conduct the welfare analysis of liquidity injections by calculating how many firms undertake the liquidity investment to realize the positive surplus $C - I$, which is how W is calculated in our model.

[37] In terms of modelling, we can assume that the liquidity investment for firms in Sector 1 generates not only cash flow C but also some non-pecuniary payoffs.

The central bank's optimization problem is:

$$\max_{Q} \sum_{i} \int_{0}^{B_i^*} (C_i - I) f(B)\, dB \tag{Program 1}$$

$$\text{s.t.} \quad r = \frac{P_i}{B_i^*} - 1 \tag{6a}$$

$$\int_{B_i^*}^{I} (I - B) f(B)\, dB + Q_i = \int_{0}^{B_i^*} B f(B)\, dB \tag{6b}$$

$$P_i = p(\widetilde{x}, B_i^*, C_i, X_i, r) \tag{6c}$$

$$C_i - I(1+r) \geq 0 \tag{6d}$$

$$Q = \sum_{i} Q_i, \tag{6e}$$

where $i = 1$ and 2.

The difference between (5a)-(5e) and Program 1 is that the former gives an equilibrium for a given Q while the latter requires the central bank to choose a Q to maximize the objective function subject to the two-sector market equilibrium.

Solving Program 1, we have the following result.

Proposition 7 *Under certain conditions, Program 1 has a unique interior optimal Q ($\in (0, \overline{Q})$). That is, the central bank has a unique optimal level of liquidity injection to maximize the total surplus of liquidity investments in the economy.*

Proof: See the Appendix.

The intuition behind Proposition 7 is as follows. If the liquidity injection Q increases beyond a critical level, in equilibrium this causes the number of firms in Sector 1 that undertake the liquidity investment to decrease while increasing the number of investing firms in Sector 2 (i.e., the crowding-out effect). However, the surplus generated per firm in Sector 1 is greater than that in Sector 2 because $C_1 > C_2$. These are two opposing forces. Under certain conditions, the total loss of surplus in Sector 1 can be higher than the total gain of surplus in Sector 2. Therefore, there exists an optimal level of liquidity injection. Note that by (3), when π is small the cash flow C is less important than X in determining the asset price.

Under either objective function of the central bank, we draw the same conclusion that the level of liquidity injection should not be too high. In fact, the key underlying mechanism for the optimal liquidity injection under either objective function is the crowding-out effect.

Program 1 also corresponds to the *constrained second best equilibrium* of the two-sector economy. In this equilibrium, the social planner (the government) decides the optimal Q by taking account of the incentives of the private sector (i.e., the constraints of (6a)-(6e)). If liquidity injections exceed the constrained optimal level, not only are investments in Sector 1 crowded out but also the total surplus in terms of cash flows in the economy is reduced. In this sense, excessive liquidity injections lead to a *misallocation of liquidity* in the economy: lower-surplus projects obtain more liquidity while higher-surplus projects *lose* liquidity such that the total surplus in the economy is reduced. This gives the welfare (efficiency) implication of the model.

Corollary 1 *If liquidity injections exceed the constrained optimal level given by Program 1, there is a misallocation of liquidity in the economy.*

3 Empirical implications

We have shown that if too much liquidity is injected into the economy, there is the possibility of crowding-out across sectors. In this section, we intend to understand under what circumstances crowding-out is more likely to occur and what its implication is for the optimal liquidity injection. We derive two implications, a cross-sectional one and one in the time series.

We first analyze the cross-sectional implication. Recall that the term X measures the contractible part of the project payoff, which depends on the project's asset specificity. We expect that the cross-country difference in payoff contractibility is small for industries with low asset specificity while the cross-country difference is larger for industries with higher asset specificity. For example, the leverage (margin) level in the real estate sector may not vary much across countries while the leverage level in the real sector (e.g., trading machines and equipment) can differ significantly across countries. This is similar to the phenomenon that the degree of external finance dependence in industries with higher asset tangibility is similar across countries while that in industries with lower asset tangibility varies largely across countries. In fact, for a country with poorer contracting institutions, creditors can realize a lower fraction of the investment payoff in case of default if asset tangibility is low (see, e.g., La Porta et al. (1997, 1998), Djankov et al. (2008), Almeida and Campello (2007)). This can be due to an inefficiency of contract enforcement or high transaction costs in financial markets in liquidating collateral. Specifically, holding X_2 (the contractible part of the project payoff for Sector 2 with lower asset specificity) similar for all countries, we expect X_1 (the contractible part of the project payoff for Sector 1 with higher asset specificity) to be lower for a country with poorer contacting institutions.

By conducting a comparative statics analysis on two countries with a similar X_2 but different levels of X_1, we have the following prediction.

Implication 1 (Cross-sectional implication) For a country with poorer contracting institutions, the crowding-out occurs under a wider range of Q and the optimal level of liquidity injection is lower.

Proof: See the Appendix.

As for the time-series implication, our main interest is in the uncertainty about economic prospects. Specifically, consider a mean-preserving spread of θ_L and θ_H, that is, hold the average of θ_L and θ_H constant while increasing their distance, $\theta_H - \theta_L$. Then, the distance $\theta_H - \theta_L$ measures the degree of divergence in investors' opinions. We can characterize the uncertainty about economic prospects by the degree of divergence in opinions. The higher the degree of divergence, the higher the uncertainty about economic prospects. We have Implication 2.

Implication 2 (Time-series implication) At a time of higher uncertainty about economic prospects, the crowding-out occurs under a wider range of Q and the optimal level of liquidity injection is lower.

Proof: See the Appendix.

The intuition behind Implication 2 is as follows. We consider a pair (θ'_L,θ'_H), which is a mean-preserving spread of the pair (θ_L,θ_H), that is, $\theta'_L < \theta_L < \theta_H < \theta'_H$. Under the mean-preserving spread, the lower bound of the asset price decreases, that is, $E^{\theta'_L}(\widetilde{x}) < E^{\theta_L}(\widetilde{x})$. For Sector 1, if its original asset price binds at the lower bound $E^{\theta_L}(\widetilde{x})$, the divergence of opinion reduces the asset price. In contrast, for Sector 2, under the divergence of opinion, the asset price remains unchanged for the range of high Q. Therefore, the gap in asset prices between the two sectors widens. As a result, the optimal level of liquidity injection is lower. Hence, for a given Q, the crowding-out occurs more likely under higher uncertainty.

4 Conclusion

The paper studies why economic stimulus by way of liquidity injections (credit easing) may not work. We highlight the friction of imperfect financial contracting. We show that there is feedback between liquidity injections and firm asset collateral values in an economy. This feedback, however, is asymmetric across sectors. The sector with lower friction has stronger feedback. This asymmetry creates a crowding-out effect. For the sector with lower friction, the collateral value responds strongly to liquidity injections, which can push up the interest rate; the increased interest rate reduces the liquidity in the sector with higher friction as the collateral value in that sector responds feebly. Therefore, even if the collateral value in one sector increases with liquidity injections, as long as the increase is not as fast as that in the other sector, there is the possibility of crowding out, which holds back economic recovery.

Our paper has two empirical implications. The model implies that crowding-out is more likely in a country with poorer contracting institutions and at a time of greater uncertainty about economic prospects. Authorities should be more cautious about the crowding-out effect in these situations.

The paper highlights limitations of (unconventional) monetary policy in stimulating economic growth. While financial intermediaries have an (informational) advantage/expertise in allocating capital to firms, the market frictions also mean that implementation of monetary policy through financial intermediaries is not perfect. The paper implies that monetary policy in conjunction with fiscal policy to target some specific sectors/industries might have better effects in economic stimulus. Regulation on the leverage level in more speculative industries when other industries are in distress might be helpful in reducing the crowding-out effect.

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6 Appendix

A numerical example:

We provide a numerical example for the crowding-out effect, illustrating the existence of relevant parameters in the propositions. That is, the numerical example is to highlight the qualitative (rather than quantitative) aspect of the model.

We choose parameter values as simple as possible. We set the parameter values for the project as $I = 1$, $C = 2.2$, $\pi = 0.4$, $E^H(\widetilde{x}) = 1.3668$, $E^L(\widetilde{x}) = 1.0334$. Note that we do not need to specify exactly u, d, θ_H and θ_L, any combination of which that satisfies that $u > d > 0$, $0 \leq X_1 < X_2 \leq d$, $u \cdot \theta_H + d \cdot (1 - \theta_H) = 1.3668$ and $u \cdot \theta_L + d \cdot (1 - \theta_L) = 1.0334$ works. The firm distribution is $f(B) = \log \frac{1}{1-B}$ for $B \in [0, 1)$. As the maximum total amount of liquidity that any one sector can demand is $Q^{\max} = \int_0^1 Bf(B)dB = 0.75$, we set $\overline{Q} = 2Q^{\max} = 1.5$.

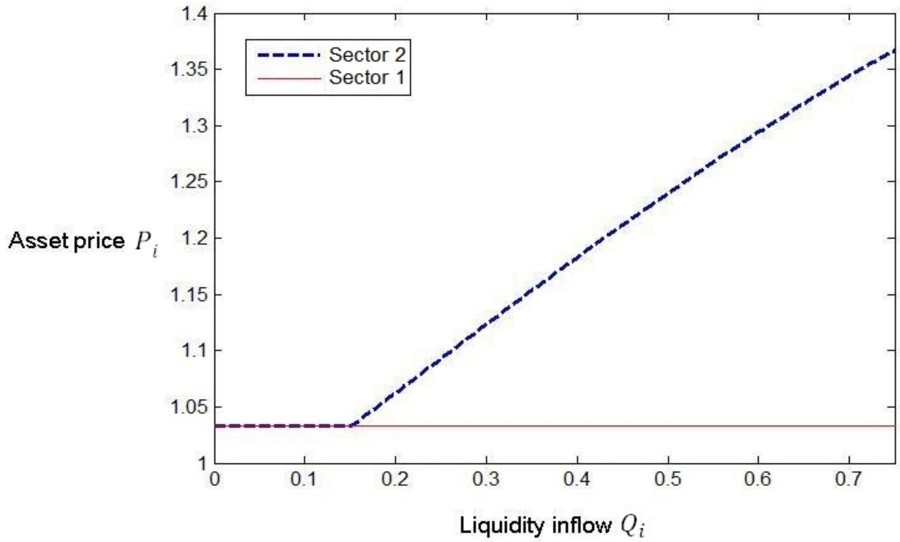
Given that $E^L(\widetilde{x}) = 1.0334$, we can calculate the threshold $\underline{X}$ in Proposition 1, which is $\underline{X} = 0.0501$. We set $X_1 = 0.05$ and $X_2 = 0.35$, where $X_1 < \underline{X} < X_2$.

For Sector 1, we can work out that the asset price is $P_1 = 1.0334$ for any $Q_1 \in [0, 0.75]$. The asset price is trapped at $E^L(\widetilde{x}) = 1.0334$. The interest rate $r_1(Q_1)$ is a strictly decreasing function of liquidity inflow Q_1.

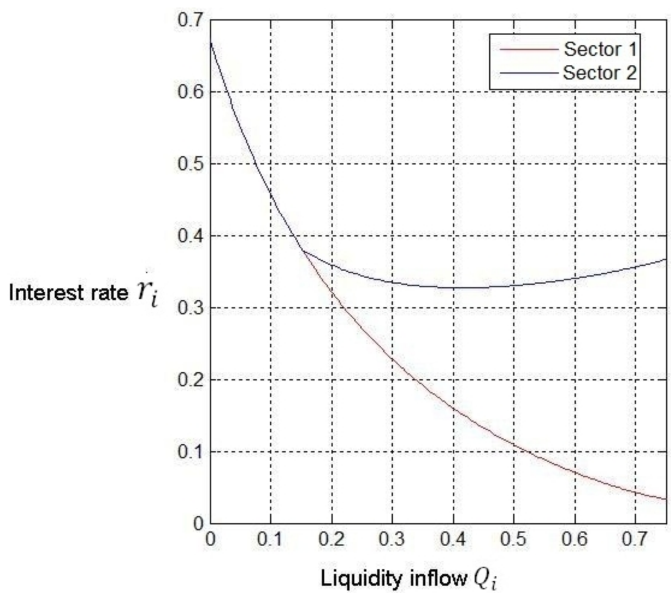
For Sector 2, the asset price $P_2(Q_2)$ is (weakly) increasing in Q_2. When Q_2 is small, liquidity injections are not sufficient to push the asset price above $E^L(\widetilde{x})$ and the asset price is $P_2 = 1.0334$; when Q_2 is big enough, $P_2(Q_2)$ is strictly increasing in Q_2, with the maximum asset price being $P_2(Q_2 = 0.75) = 1.3667$ (which is lower than $E^H(\widetilde{x}) = 1.3668$). As for the equilibrium interest rate $r_2(Q_2)$, it is non-monotonic and 'U'-shaped, with the minimum interest rate being $\mathrm{r}_{\min} = 0.3270$.

The optimal amount of liquidity injection to maximize liquidity investments in Sector 1 is $Q^* = 0.6113$, at which the distribution of the liquidity injection across the two sectors is $Q_1^* = 0.4163$ and $Q_2^* = 0.1950$, respectively.

Figure A1(a) shows the asset price response in each sector to its (net) liquidity inflow Q_i and Figure A1(b) depicts the interest rate response in each sector.



(a) Asset prices responses to liquidity inflow



(b) Interest rate responses to liquidity inflow

Figure A1: A numerical example

Proof of Lemma 1:

By using the credit market clearing condition (2), the expression of $\Gamma(B^*, r)$ in (3) can be rewritten as

$$\Gamma(B^*, r) = \frac{\pi[C \cdot F(B^*) - Q(1+r)] + X}{1-\pi}.$$

By substituting (1) into this equation, r can be eliminated. That is,

$$\Gamma\left(B^{*}\right)=\frac{\pi C\cdot F\left(B^{*}\right)+X}{(1-\pi)+\pi\frac{Q}{B^{*}}}.$$

From (2), B^{*} is completely and uniquely determined by Q. Hence, Γ can be expressed in terms of Q:

$$\Gamma\left(Q\right)=\frac{\pi C\cdot F\left(B^{*}\left(Q\right)\right)+X}{(1-\pi)+\pi\frac{Q}{B^{*}(Q)}}.$$

Now we derive the total derivative $\frac{d\Gamma}{dQ}$.

From (2), we can work out

$$\frac{dB^{*}}{dQ}=\frac{1}{If\left(B^{*}\right)}. \tag{A1}$$

From the expression $\Gamma(B^{*},r)$ in (3), we have the total derivative:

$$\begin{aligned}\frac{d\Gamma}{dB^{*}}&=\frac{\partial\Gamma}{\partial B^{*}}+\frac{dr}{dB^{*}}\frac{\partial\Gamma}{\partial r}\\&=\frac{\partial\Gamma}{\partial B^{*}}+\frac{d}{dB^{*}}\left(\frac{P}{B^{*}}-1\right)\frac{\partial\Gamma}{\partial r}\\&=\frac{\partial\Gamma}{\partial B^{*}}+\left(\frac{1}{B^{*}}\frac{dP}{dB^{*}}-\frac{P}{B^{*2}}\right)\frac{\partial\Gamma}{\partial r},\end{aligned} \tag{A2}$$

where the equality in the second line uses the optimal lending condition (1). Given the interior region of (3) in which $P=\Gamma$, we can rewrite equation (A2) as

$$\frac{d\Gamma}{dB^{*}}=\frac{\partial\Gamma}{\partial B^{*}}+\left(\frac{1}{B^{*}}\frac{d\Gamma}{dB^{*}}-\frac{\Gamma}{B^{*2}}\right)\frac{\partial\Gamma}{\partial r}. \tag{A3}$$

Noticing that the term $\frac{d\Gamma}{dB^{*}}$ appears on both sides of equation (A3), we solve for this term

$$\frac{d\Gamma}{dB^{*}}=\frac{\frac{\partial\Gamma}{\partial B^{*}}-\frac{\Gamma}{B^{*2}}\frac{\partial\Gamma}{\partial r}}{1-\frac{1}{B^{*}}\frac{\partial\Gamma}{\partial r}}. \tag{A4}$$

Also, from the expression of $\Gamma(B^{*},r)$ in (3), we have

$$\begin{aligned}\frac{\partial\Gamma}{\partial B^{*}}&=\frac{\pi}{1-\pi}\left[C-I\left(1+r\right)\right]f\left(B^{*}\right),\\\frac{\partial\Gamma}{\partial r}&=\frac{\pi\left\{\int_{0}^{B^{*}}-Bf\left(B\right)dB+\int_{B^{*}}^{I}\left(I-B\right)f\left(B\right)dB\right\}}{1-\pi}=-\frac{\pi}{1-\pi}Q,\end{aligned}$$

where the second equality follows from the credit market clearing condition (2).

Therefore, we can use these expressions to simplify expression (A4):

$$\frac{d\Gamma}{dB^{*}}=\frac{\left[C-I\left(1+r\right)\right]f\left(B^{*}\right)+\left(1+r\right)\frac{Q}{B^{*}}}{\frac{1-\pi}{\pi}+\frac{Q}{B^{*}}}. \tag{A5}$$

Overall, we have

$$\frac{d\Gamma}{dQ} = \frac{dB^*}{dQ}\frac{d\Gamma}{dB^*} = \frac{\left[C - I\left(1+r\right)\right] f\left(B^*\right) + \left(1+r\right)\frac{Q}{B^*}}{If\left(B^*\right)\left(\frac{1-\pi}{\pi} + \frac{Q}{B^*}\right)}. \tag{A6}$$

By (A6), we conclude that a sufficient condition for $\frac{d\Gamma}{dQ} > 0$ is $C - I(1+r) > 0$.

In fact, if $\frac{dr}{dB^*}$ is positive, we immediately conclude that Γ must increase in Q because $\Gamma = B^*\left(1+r\right)$ by (1) and B^* is increasing in Q. If $\frac{dr}{dB^*}$ is negative, clearly $\left[C - I\left(1+r\right)\right] f\left(B^*\right) - \frac{dr}{dB^*}Q > 0$, meaning $\frac{d\Gamma}{dQ} > 0$. In short, if $C - I(1+r)$ is positive and close to zero, $\frac{dr}{dB^*}$ must be negative; if $\frac{dr}{dB^*}$ is positive, $C - I\left(1+r\right)$ must be far above zero and exceed the effect of $\frac{dr}{dB^*}$.

Proof of Proposition 2:

For ease of exposition, we reiterate the equilibrium asset price here. That is

$$P = \begin{cases} E^H\left(\widetilde{x}\right) & \text{if} \quad \Gamma\left(B^*, r\right) > E^H\left(\widetilde{x}\right) \\ \Gamma\left(B^*, r\right) & \text{if} \quad \Gamma\left(B^*, r\right) \in \left[E^L\left(\widetilde{x}\right), E^H\left(\widetilde{x}\right)\right], \\ E^L\left(\widetilde{x}\right) & \text{if} \quad \Gamma\left(B^*, r\right) < E^L\left(\widetilde{x}\right) \end{cases}$$

where

$$\Gamma\left(B^*, r\right) = \frac{\pi\left[\int_0^{B^*}\left[C - B\left(1+r\right)\right] f\left(B\right) dB + \int_{B^*}^{I}\left(1+r\right)\left(I - B\right) f\left(B\right) dB\right] + X}{1-\pi}.$$

From Lemma 1, we know that $\Gamma(Q)$ is an increasing function of Q if $C - I(1+r) > 0$. We show that the condition $C - I(1+r) > 0$ holds under general parameter values. In fact, by $1+r = \frac{\Gamma(B^*, r)}{B^*}$, we have $C - I(1+r) = C - I\frac{\Gamma(B^*, r)}{B^*}$, where B^* is completely determined by Q and r is endogenous. *Ceteris paribus*, if π is sufficiently low, Γ is low and thus $C - I(1+r) > 0$. The numerical example above in the Appendix is one case.

We consider the lower bound of the asset price, $E^L(\widetilde{x})$. For our purpose, we choose the parameters to make sure that the upper bound of the asset price is not binding (i.e., the asset price calculated in $\Gamma(Q)$ is always below $E^H(\widetilde{x})$). We define two cutoffs, $\underline{X}$ and $\overline{X}$, and divide X into three ranges: $X < \underline{X}$, $X > \overline{X}$, and $\underline{X} \leq X \leq \overline{X}$.

The first range of X is the case in which the asset price is trapped at the lower bound no matter what Q ($\in [0, \overline{Q}]$) is. That is, even if $Q = \overline{Q}$, the asset price calculated in Γ is still (weakly) below $E^L(\widetilde{x})$. Therefore, $\underline{X}$ satisfies $E^L(\widetilde{x}) = \Gamma|_{Q=\overline{Q},\ X=\underline{X}}$.

The third range of X is the case in which the asset price is above the lower bound for the whole region of $Q \in [0, \overline{Q}]$. That is, even if $Q = 0$, the asset price calculated in Γ is still (weakly) above $E^L(\widetilde{x})$. Therefore, $\overline{X}$ satisfies $E^L(\widetilde{x}) = \Gamma|_{Q=0,\ X=\overline{X}}$.

In the second range of $\underline{X} \leq X \leq \overline{X}$, the asset price is the constant $E^L(\widetilde{x})$ when Q is low and then increases in Q when Q is higher.

Proof of Proposition 3:

When $X \leq \underline{X}$, the asset price P is constant in Q. Also considering $\frac{dB^*}{dQ} > 0$, we have that when $X \leq \underline{X}$, the equilibrium interest rate $r(Q)$ is strictly decreasing in Q ($\in [0, \overline{Q}]$).

We consider the equilibrium interest rate when $X > \underline{X}$. By $r = \frac{\Gamma}{B^*} - 1$, we have

$$\frac{dr}{dB^*} = \frac{1}{B^*}\frac{d\Gamma}{dB^*} - \frac{\Gamma}{B^{*2}}. \tag{A7}$$

Plugging (A5) into (A7), we have

$$\frac{dr}{dB^*} = \frac{\left[C - I\left(1+r\right)\right] f\left(B^*\right) B^* - \frac{1-\pi}{\pi}\Gamma}{\left(\frac{1-\pi}{\pi}B^* + Q\right) B^*}. \tag{A8}$$

By $1 + r = \frac{P(Q)}{B^*(Q)}$, considering $P = \Gamma$, we have

$$\begin{aligned}\frac{dr}{dQ} &= \frac{dB^*}{dQ}\frac{dr}{dB^*} \\ &= \frac{1}{If\left(B^*\right)}\frac{\left[C - I\left(1+r\right)\right] f\left(B^*\right) B^* - \frac{1-\pi}{\pi}\Gamma}{\left(\frac{1-\pi}{\pi}B^* + Q\right) B^*} \\ &= \frac{\left[C - I\left(1+r\right)\right] B^* - \frac{1-\pi}{\pi}\frac{\Gamma}{f(B^*)}}{\left(\frac{1-\pi}{\pi}B^* + Q\right) B^* I}.\end{aligned} \tag{A9}$$

Hence, $\frac{dr}{dQ} > 0$ if and only if $f(B^*) > \frac{1-\pi}{\pi}\left[\frac{C}{1+r} - I\right]^{-1}$ by noting that $\frac{\Gamma}{B^*} = 1+r$. In equilibrium, we guarantee that $C > I(1+r)$, so the right-hand side of this condition is bounded. When $f(B^*)$ is sufficiently high (respectively low), $\frac{dr}{dQ}$ is positive (respectively negative). The numerical example above in the Appendix illustrates these.

We also consider the special case of $\frac{P(Q)}{B^*(Q)}|_{Q=0}$. In this case, B^* solves $\int_{B^*}^{I}(I-B)\,f(B)\,dB = \int_0^{B^*} Bf(B)\,dB$ by noting that B^* has a unique solution based on (2'), and $\Gamma = \frac{\pi C \cdot F(B^*)+X}{1-\pi}$. Hence, r is determined.

To summarize, when $X > \underline{X}$, we can choose some function $f(B)$ (i.e., $f(B)$ is sufficiently high in some region of B) and some $\overline{Q}$ such that r decreases first and then increases in Q within the interval $Q \in [0, \overline{Q}]$.

Proof of Proposition 4:

First, given a Q_i for each sector, solve for the equilibrium within each sector, that is, solve for the triplet $\{B_i^*, P_i, r_i\}$. In particular, we obtain the function $r_i(Q_i)$. If conditions in Proposition 3 are satisfied, $r_1(Q_1)$ is a decreasing function and $r_2(Q_2)$ is a 'U'-shaped function. Second, by considering the link between the two sectors, (5a) and (5e), we can work out Q_i (for $i = 1$ and 2) for a given Q. That is, by considering $r_1(Q_1) = r_2(Q_2) = r$ and $Q_1 + Q_2 = Q$, we obtain the unique Q_1 and Q_2, and r. In fact, by aggregating $r_1(Q_1)$ and $r_2(Q_2)$, we can obtain an 'aggregate' function $r(Q)$. That is, for a given r, we find the corresponding Q_1 which solves $r_1(Q_1) = r$ and Q_2 which solves $r_2(Q_2) = r$, and then aggregate Q_1 and Q_2 as $Q = Q_1 + Q_2$. Under generally chosen parameters, the 'aggregate' function $r(Q)$ is a 'U'-shaped function. The numerical example above in the Appendix is one case. Thus, for a given Q, we have unique Q_1, Q_2 and r.

Proof of Proposition 5:

From the proof of Proposition 4, we have that the 'aggregate' function $r(Q)$ is a 'U'-shaped function (i.e., decreasing first and then increasing). By Proposition 3, $r_1(Q_1)$ is a decreasing function. Therefore, to maximize Q_1, we need to choose a Q to minimize $r(Q)$. Clearly, there is a unique Q that minimizes r and thus maximizes Q_1.

Proof of Proposition 6:

Considering that $r_1(Q_1)$ is a decreasing function and $r_2(Q_2)$ is a 'U'-shaped function, we find a unique Q_1^* that solves $r_1(Q_1^*) = r_{\min}$ and a unique Q_2^* that solves $r_2(Q_2^*) = r_{\min}$. We define $Q^* = Q_1^* + Q_2^*$. By $r_1(Q_1) = r_2(Q_2) = r$ and $Q_1 + Q_2 = Q$, we have that Q_1 is increasing in Q when $Q < Q^*$ and decreasing in Q when $Q > Q^*$, and that Q_2 is increasing in Q.

Proof of Proposition 7:

The conditions to guarantee that Program 1 has a unique interior optimal Q ($\in (0, \overline{Q})$) are that π is sufficiently small and $f(B)$ is sufficiently high in some region of B.

First, we show that W, the total surplus of liquidity investments in the economy, is divided among investing firms, non-investing firms, and the government. For simplicity, we first consider the one-sector economy.

$$
\begin{aligned}
W &= \int_0^{B^*} (C-I) f(B)\, dB \\
&= \underbrace{\int_0^{B^*} C f(B)\, dB}_{\text{Cash flow generated at } T_1} - \underbrace{\int_0^{B^*} I f(B)\, dB}_{\text{Investment cost at } T_0} \\
&= \left\{ \underbrace{\underbrace{\int_0^{B^*} [C - B(1+r)] f(B)\, dB}_{\text{Investing firms}} + \underbrace{(1+r)\int_{B^*}^{I} (I-B) f(B) dB}_{\text{Non-investing firms}} + \underbrace{(1+r)Q}_{\text{The government}}}_{\text{Cash flow generated at } T_1} \right\} \\
&\quad - \left\{ \underbrace{\underbrace{\int_0^{B^*} (I-B) f(B) dB}_{\text{Internal funds of investment firms}} + \underbrace{\int_{B^*}^{I} (I-B) f(B) dB}_{\text{Deposits of non-investing firms}} + \underbrace{Q}_{\text{Liquidity injections of the government}}}_{\text{Investment cost at } T_0} \right\} \\
&= \underbrace{\int_0^{B^*} [C - (I-B) - (1+r) B] f(B)\, dB}_{\text{Surplus for investing firms}} + \underbrace{r\int_{B^*}^{I} (I-B) f(B) dB}_{\text{Surplus for non-investing firms}} + \underbrace{rQ}_{\text{Surplus for the government}}
\end{aligned}
$$

In the above, when we decompose the first term and the second term in the second line (into the third line and the fourth line, respectively), we have used the results of (2) and (2'). For the two-sector economy, it is easy to show:

$$
\begin{aligned}
W &= \sum_i \int_0^{B_i^*} (C_i - I) f(B)\, dB \\
&= \underbrace{\sum_i \int_0^{B_i^*} [C_i - (I-B) - (1+r)B] f(B)\, dB}_{\text{Surplus for investing firms}} + \underbrace{\sum_i r \int_{B_i^*}^{I} (I-B) f(B) dB}_{\text{Surplus for non-investing firms}} + \underbrace{rQ}_{\text{Surplus for the government}}
\end{aligned}
$$

Second, we prove that when Q is sufficiently high, $\frac{dW}{dQ} < 0$. We have that $\frac{dW}{dQ} = \frac{dr}{dQ}\frac{dW}{dr}$. Because $\frac{dr}{dQ} > 0$, the sign of $\frac{dW}{dQ}$ is the same as the sign of $\frac{dW}{dr}$. We calculate $\frac{dW}{dr}$:

$$
\begin{aligned}
\frac{dW}{dr} &= \frac{d\sum_i \int_0^{B_i^*} (C^i - I) f(B)\, dB}{dr} \\
&= \frac{dB_1^*}{dr}(C_1 - I) f(B_1^*) + \frac{dB_2^*}{dr}(C_2 - I) f(B_2^*) \\
&= \frac{dQ_1}{dr}\frac{dB_1^*}{dQ_1}(C_1 - I) f(B_1^*) + \frac{dQ_2}{dr}\frac{dB_2^*}{dQ_2}(C_2 - I) f(B_2^*) \\
&= \frac{dQ_1}{dr}\frac{(C_1 - I) f(B_1^*)}{I f(B_1^*)} + \frac{dQ_2}{dr}\frac{(C_2 - I) f(B_2^*)}{I f(B_2^*)} \\
&= \frac{dQ_1}{dr}\frac{C_1 - I}{I} + \frac{dQ_2}{dr}\frac{C_2 - I}{I}.
\end{aligned} \tag{A10}
$$

In the third line above, we have used the result of (A1).

For Sector 1, the asset price is binding at $E^L(\widetilde{x})$, denoted by $\underline{V}$. By (1), we have $r = \frac{\underline{V}}{B_1^*} - 1$. So, $\frac{dr}{dB_1^*} = -\frac{\underline{V}}{(B_1^*)^2}$. Thus,

$$
\begin{aligned}
\frac{dQ_1}{dr} &= \frac{dB_1^*}{dr}\frac{dQ_1}{dB_1^*} \\
&= -\frac{(B_1^*)^2}{\underline{V}}(I f(B_1^*)) < 0.
\end{aligned} \tag{A11}
$$

For Sector 2, the asset price is not binding at $E^L(\widetilde{x})$. By (A9), we have

$$
\frac{dQ_2}{dr} = \frac{\left(\frac{1-\pi}{\pi} B_2^* + Q_2\right) B_2^* I}{[C_2 - I(1+r)] B_2^* - \frac{1-\pi}{\pi}\frac{\Gamma_2}{f\left(B_2^*\right)}}. \tag{A12}
$$

When $f(B_2^*)$ is sufficiently high, $\frac{dQ_2}{dr}$ is positive. Note that B_1^* and consequently $\frac{dQ_1}{dr}$ do not depend on C_1. Therefore, for given $\frac{dQ_1}{dr}$, $\frac{dQ_2}{dr}$ and C_2, we can find a sufficiently big C_1 such that $\frac{dQ_1}{dr}\frac{C_1-I}{I} + \frac{dQ_2}{dr}\frac{C_2-I}{I} < 0$. So $\frac{dW}{dr} < 0$.

We need that the asset price in Sector 1 is binding at $E^L(\widetilde{x})$ and that in Sector 2 is not. Note that $C_1 > C_2$ and $X_1 < X_2$. When π is chosen to be sufficiently small, the first term in the numerator of (3) which involves C has a limited role in determining the asset price while the second term which is about X becomes crucial. Hence, when the gap between X_1 and X_2 is sufficiently big, the result is obtained.

A numerical example is used to illustrate the existence of relevant parameters. We continue the previous simulation exercise. Let $I = 1$, $\pi = 0.185$, $E^H(\widetilde{x}) = 1.2$, and $E^L(\widetilde{x}) = 1.1$. The firm distribution is $f(B) = \log \frac{1}{1-B}$ for $B \in [0, 1)$. As the maximum total amount of liquidity that any one sector can demand is $Q^{\max} = \int_0^1 Bf(B)dB = 0.75$, we set $\overline{Q} = 2Q^{\max} = 1.5$. We choose $C_1 = 5.6 > C_2 = 4$ and $X_1 = 0.01 < X_2 = 0.4$. We can find that the optimal level of liquidity injection to maximize the total surplus of the economy is $Q = 1.18$, which lies within $(0, \overline{Q})$, where $\overline{Q} = 1.5$.

Proof of Implication 1:

We consider two countries, Countries North (N) and South (S), with the same X_2 but different levels of X_1. Country S has a lower level of X_1 than Country N does. First, to show a clean result, we consider the extreme case that for Country N $X_1^N = X_2^N > \underline{X}$. For Country S, $X_1^S < \underline{X} < X_2^S$ and the asset price in Sector 1 binds at the constant $E^L(\widetilde{x})$. So, in Country N the two sectors are identical and thus the liquidity injection Q is always equally distributed between the two sectors and there is no crowding-out effect; hence the optimal liquidity injection should be $\overline{Q}$. In contrast, in Country S, there is a crowding-out effect as shown in the main model, where the optimal level of liquidity injection is lower than $\overline{Q}$. Therefore, the optimal level of liquidity injection is lower for Country S than for Country N. This also means that the range of Q in which crowding-out occurs is wider for Country S than for Country N. Second, we consider the general case that for Country N $X_1^N < X_2^N$. Clearly, when the difference between X_1^N and X_2^N is small enough, the two interest rate-liquidity injection curves shown in Figure A1(b) are very close to each other, so the optimal level of liquidity injection for Country N is lower than but close to $\overline{Q}$. Hence, the result under the extreme case $X_1^N = X_2^N$ applies to the general case (qualitatively).

Proof of Implication 2:

We prove that under a mean-preserving spread, the optimal level of liquidity injection is lower. We consider a pair (θ'_L,θ'_H), which is a mean-preserving spread of the pair (θ_L,θ_H), that is, $\theta'_L < \theta_L < \theta_H < \theta'_H$. Under the mean-preserving spread, the lower bound of the asset price decreases, that is $E^{\theta'_L}(\widetilde{x}) < E^{\theta_L}(\widetilde{x})$. For Sector 1, if its original asset price binds at the lower bound $E^{\theta_L}(\widetilde{x})$, the divergence in opinions lowers the asset price. In contrast, for Sector 2, under the divergence of opinions, the asset price remains unchanged for the range of high Q. Figure A2(a) shows the asset price response in each sector to liquidity injections under the original set of beliefs (the left) and under the new set of beliefs (the right).

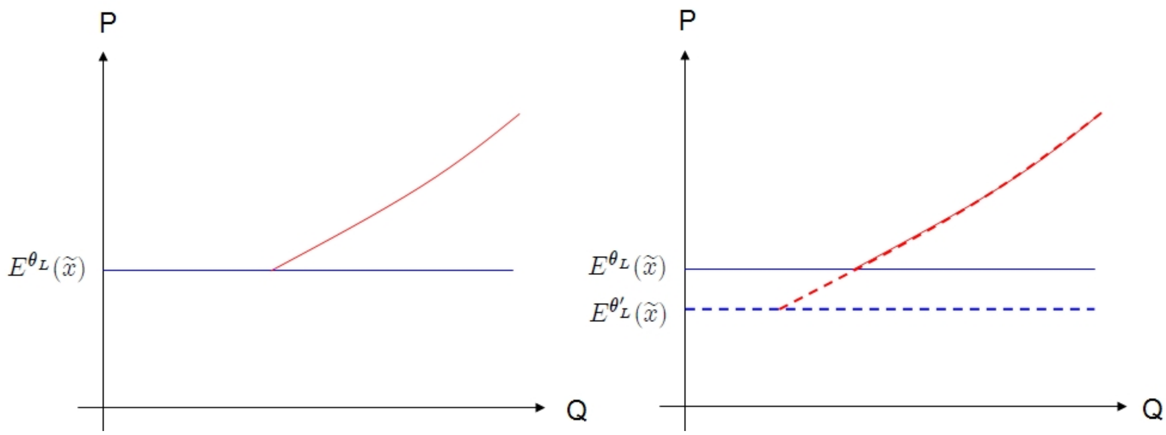


Figure A2(a): The asset price response to liquidity injections under the original set of beliefs (the left) and under the new set of beliefs (the right)

Because of the asset price changes, the interest rate-liquidity injection curve for Sector 1 shifts down. The interest rate-liquidity injection curve for Sector 2 remain unchanged for the range of high Q. Figure A2(b) shows the interest rate response in each sector to liquidity injections under the original set of beliefs (the left) and under the new set of beliefs (the right).

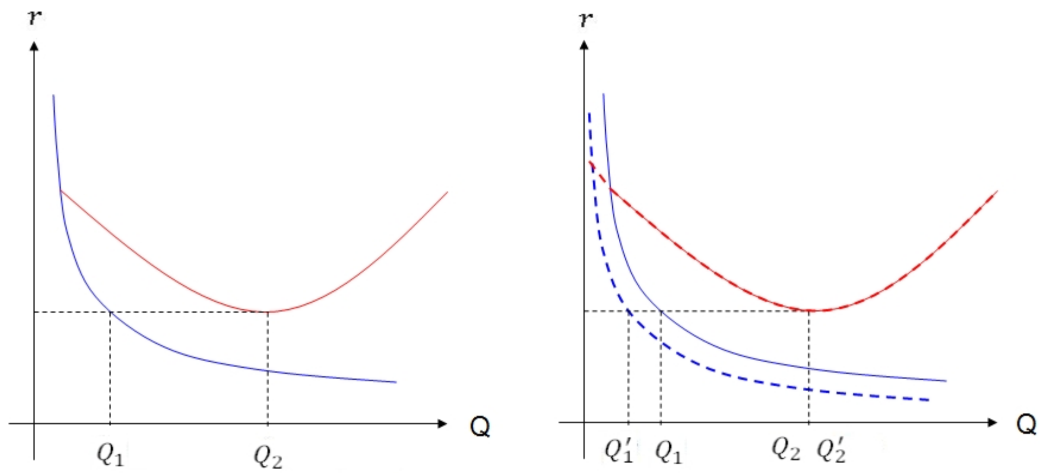


Figure A2(b): The interest rate response to liquidity injections under the original set of beliefs (the left) and under the new set of beliefs (the right)

From Figure A2(b), we can see that the optimal level of liquidity injection under the original set of beliefs is Q_1+Q_2 and under the new set of beliefs is $Q_1'+Q_2'$. Clearly, $Q_1'+Q_2' < Q_1+Q_2$. Because crowding-out occurs for $Q > Q_1 + Q_2$ under the original set of beliefs and for $Q > Q_1' + Q_2'$ under the new set of beliefs, this means that the range of Q in which crowding-out occurs is wider under the new set of beliefs than under the original set of beliefs. That is, for a given Q, crowding-out is more likely under the new set of beliefs than under the original set of beliefs.