Credit risk transfer and bank insolvency risk[*]

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— PRELIMINARY DRAFT, COMMENTS WELCOME —

Abstract The theoretical framework in this paper shows that some transactions to transfer portfolio credit risk to external investors increase the default risk of banks. This is in particular likely if a bank sells the senior tranche and retains a sufficiently large first-loss position. The results do not rely on banks increasing leverage after the risk transfer, nor on banks taking on new risks, although these could aggravate the effect. High leverage and concentrated business models increase the vulnerability to the mechanism. The literature on credit risk transfers and information asymmetries generally tends to advocate the retention of "information-sensitive" first-loss positions. The present study shows that, under certain conditions, such an approach may harm financial stability, and thus calls for further reflection on the structure of securitisation transactions and portfolio insurance.

Keywords: Credit risk transfer, default risk, financial stability, portfolio insurance, risk management.

JEL Classification Numbers: G21, G28, G32.

1 Introduction

This paper studies the direct impact of credit risk transfer to remove credit risk from the balance sheet of financial institutions on their failure probabilities. Such credit risk transfer can take place, for example, by means of purchasing credit porfolio insurance, or by means of selling tranches in a securitized loan portfolio to third-party investors. This paper focuses on deriving some general results that help to evaluate whether a credit risk transfer to third-party investors reduces or increases the failure probability of a financial institution.

The possibility of higher insolvency risk as a consequence of removing credit risk from the balance sheet may sound counterintuitive, because such credit risk transfer reduces the "overall uncertainty" of the institution's cash flow. However, the reduction in "overall uncertainty" of a bank's cash flow does not necessarily imply a smaller probability of failure for that institution. The reason is straightforward. Credit risk transfer to third-party investors may improve the banks' position if the risk materializes. However, since such protection is not for free, ex post, the bank will be worse off in all other states of the world. If credit risk transfer isolates risks from the balance sheet that materialize only after a bank would fail with certainty, or if the risk materializes only if the bank survives with certainty, such protection will not provide benefits in terms of avoiding bank failures. The present paper shows that holds true for some common credit risk transfer strategies.

Interest expenses on bank debt may drop in response to an overall reduction in the uncertainty of a bank's cash flows. Such a reduction in debt servicing costs can attenuate the potentially harmful direct impact of credit risk transfer on the probability of failure. One may suspect that such an indirect benefit could outweigh the harmful direct impact if the bank's funding cost were to fall sufficiently. However, our results show this does not hold true if credit risk transfer is fairly priced and if depositors, despite of deposit insurance, fully appreciate the reduction in the risk of the bank's cash flow. The intuitive reason is that the depositors still share in the burden of the cost of credit risk protection if the bank fails. Therefore, the potential drop in interest expenses will not completely reflect cost of the

credit risk protection.

Our results show that strategies where banks reduce exposures, but retain a sufficiently large first-loss exposure are in particular susceptible to a potential increase in default risk. Despite of this adverse impact on bank default risk, such credit risk transfer strategies often receive some sort of favorable treatment. One example is the regulation on credit risk retention, which provide a more favorable regulatory treatment of securitization transactions in which banks retain the equity tranches. Another example is the government facilitating or providing forms of credit risk insurance if banks retain a first loss position. A third example is the case where capital requirements provide capital relief on an insured portfolio, if an institution insures losses beyond a certain amount. In the latter case, the credit risk transfer may even have a twofold harmful impact on the bank solvency by not only increasing default risk because of the direct impact the credit risk transfer, but also by increasing default risk as a consequence of smaller capital buffers in response to a lower requirement. In each of these examples, a poorly calibrated credit risk transfer can increase the probability of bank failures.

Whether credit risk transfer increases bank default risk depends on a several parameters. One parameter is the leverage of a bank. Banks operating with higher leverage need to meet a higher threshold return on their assets to be able to meet their debt obligations. Designing a credit risk transfer that helps the bank to meet a higher threshold can be more challenging. Therefore, the weaker the capital position of a bank, the more likely it is that credit risk transfer increases the default risk of a bank.

A second parameter is the thickness of the junior and senior positions. In particular, it is important how the thickness of the junior and senior positions compares to the threshold return that a banks needs to meet to be able to meet its debt obligations. For example, if a specialized bank transfers the risk of a senior position in a loan portfolio, and the guaranteed return on the portfolio after the credit risk is less than the payoff promised to the bank's depositors, then it is very likely that the credit risk transfer increases the default risk of the

bank.

A third parameter is the degree of concentration of the banks' business model. The direction of the impact is particularly clear for banks with more concentrated business models. For diversified institutions, the direction of the impact of a credit risk transfer can be less clear. The reason is that banks may reap benefits from diversification, as they receive income from other sources, as well as suffer additional losses, when risks from other exposures materialize. Such variation in income from other sources blurs the answer to the question on how the thickness of the junior and senior positions compares to the minimum return on the insured portfolio necessary to repay the bank's depositors.

The results in this paper do not rely on a particular form of the distribution function of the payoffs on loan portfolios. This is an attractive feature, because the risk profiles of the retained tranches can be highly dependent on distributional assumptions. However, as a consequence, the results have less to say about the direction of the impact if the insured risk represents only a small portion of the balance sheet. In those cases, proving the beneficial impact of credit risk transfer may require additional analysis, which can be based on simulations.

2   Related literature

2.1   Theoretical

The information asymmetries at the heart of credit risk transfers have been a flourishing topic of theoretical banking research. Starting with the seminal paper of Pennacchi (1988), one of the main objectives is the design of transactions to adequately deal with the incentive problems following from information asymmetries; see, e.g., Gorton and Pennacchi (1990, 1995), Boot and Thakor (1993), Riddiough (1997), DeMarzo and Duffie (1999) and DeMarzo (2005). A general recommendation is to construct and sell relatively 'information insensitive' tranches, i.e., securities whose payoffs are relatively unaffected by private information.

Usually these are the most senior tranches, although this need not necessarily be the case; see, e.g., Chiesa (2008). The importance of studying the effects of securitization transactions on the stability of banks is generally recognized; see, e.g., Pennacchi (1988, p. 393). Nevertheless, the direct impact of credit risk transfer received limited attention in the theoretical literature.

The design of retention regulation is the subject of a growing literature. In Greenbaum and Thakor (1987), the optimum level of risk retained by the bank allows for the truthful revelation of the credit risk. Guo and Wu (2014) point out that mandatory retention requirements may undermine the possibility to signal the quality of underlying assets by voluntary risk retention. Kiff and Kisser (2014) suggest that, without retention requirements, ineffective retention schedules may prevail if banks have incentives to economize on capital. They suggest regulators could opt to impose equity tranche retention to better align incentives. Fender and Mitchell (2009) and Cerasi and Rochet (2014) prove the retention of mezzanine tranches to be more effective in aligning incentives in certain cases. The reason is that losses as a result of (for example) adverse macro-economic conditions may entirely wipe out the equity tranches. Pagès (2013) explores how optimal retention schemes can be implemented in potentially more cost-effective ways. In general, these studies recommend a flexible design of retention regulation in recognition of the variation in risk characteristics of securitized assets and the differences in economic conditions.

The present paper focuses on the direct impact of the securitization transaction, but does not discuss the indirect impact as a consequence of changes to the risk management practices and investment decisions at banks. Other theoretical studies discuss this indirect channel without elaborating on how the structure of the transaction affects the direct impact of credit risk transfer on the insolvency probability of banks. For example, Wagner and Marsh (2006) show that credit risk transfer improves the risk-return trade-off for banks because it facilitates diversification of their investments. As a consequence, banks increase the level of leverage. Hence, the "overall" impact could be more or less stability. Similarly, Wagner (2007) argues

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that enhanced liquidity of bank assets as a consequence of credit risk transfer techniques makes banks safer. However, as banks optimally increase their exposure in response to the enhanced liquidity, the overall impact on stability may be negative. In the model of Allen and Carletti (2006), credit risk transfer from the banking to the insurance sector can be beneficial because of improved risk sharing, but may also increase the potential of contagion because insurers liquidate safe long-term assets if adverse shocks materialize resulting in mark-to-market losses at banks. Another example is the study of Shin (2009), which emphasizes that securitization, as a form of credit risk transfer, facilitates a system-wide credit expansion, which could promote financial instability through lower lending standards.

Our paper is related to Van Oordt (2014), who focuses on the question whether risk sharing among different financial institutions through the exchange of tranches in securitisations enhances financial stability. In such a setup, all credit risk remains in the financial sector. Securitisation may then both stabilize or destabilize the financial sector depending on the design of the tranches. These results complement the results of Shaffer (1994) and Wagner (2010), who show that linear risk sharing within the financial system increases joint failure risk. However, in contrast to the current study, Van Oordt (2014) does not discuss the effects of credit risk transfer on the stability of the financial sector if credit risk is transferred to third-party investors.

2.2   Empirical

Empirical studies tend to focus more on the default risk of the underlying than on the insolvency risk of the issuer. Several studies that discuss the empirical relation between risk of institutions and credit risk transfer are discussed below. However, it should be kept in mind that those studies generally do not disentangle the direct impact of credit risk transfer on insolvency risk and the aforementioned indirect impact as a consequence of changes to risk management practices and investment decisions at banks.

Cebenoyan and Strahan (2004) document that banks that actively buy and sell loans on the secondary markets tend to have lower risk-weighted capital ratios. Similarly, Casu et al. (2013) observe that securitizing banks have lower capital ratios on average. However, they do not find evidence of a significant impact on the performance of first-time securitizers when compared to a matched sample of banks that do not securitize. Le et al. (2016) document securitizing banks to be more risky using several accounting-based measures, such as the ratio of non-performing loans, the loan loss allowance and charge-offs.[1] Sarkisyan and Casu (2013) assess the impact of securitisation on the Z-scores of issuing bank holding companies in the US. They document lower Z-scores if issuing banks retain larger interests, which suggests a positive relation with bank failure risk, which is in line with the expected theoretical relation.

Franke and Krahnen (2007) document evidence that post-announcement systematic risk, as measured market betas, is higher for banks using collateralized debt obligations to transfer risks to market participants. Similarly, Uhde and Michalak (2010) report that securitization tends to increase systematic risk of banks. Trapp and Weiß (2016) document higher measures of downside tail risk, such as Marginal Expected Shortfall and ΔCoVaR for banks involved in securitization.

Interestingly, Lockwood et al. (1996) observe that the relation depends on the capital position of the securitizing banks. They document that systematic risk, as measured by market betas, does increase upon announcement of securitisation transactions for banks with weaker capital positions, while a reduction in risk is observed for banks with stronger capital positions. Such a bifurcate relation is consistent with our theoretical result that whether credit risk transfer increases or decreases insolvency risk crucially depends on how prices and the retained interest relate to the level of capital ratio of a bank. For example, a strategy of selling senior claims on a credit portfolio is harmful in terms of insolvency risk if the retained first-loss position is relatively thick compared to the capital ratio.

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[1]Simulation results of Chang and Chen (2016) suggest that default risk is negatively related to credit risk transfer if the bank acts as a protection buyer, and positively if it acts as a protection seller.

3 Model Setup

We consider a one-period model with a bank holding a unit investment financed with deposits, d, and equity, $1 - d$. An overview of the balance sheet is provided in Figure 1. At the start of the period, the bank holds investments ω in asset X and $(1 - \omega)$ in asset Y. Asset X is the loan pool for which the firm considers transferring the credit risk to third-party investors. Asset Y represents the pool with all other investments of the firm. The assets generate end-of-period cash flows denoted by x and y. The originator operates in a perfect information world. The only uncertainty is the end-of-period realization (x, y). The density of the continuous joint distribution function is denoted as $\phi(x, y)$ with full support $[0, \overline{x}] \times [\underline{y}, \overline{y}]$.

At the start of the period, the firm transfers credit risk of fraction $r \in [0, 1]$ of its position in asset X using credit risk transfer strategy (i, k). Credit risk transfer can take three different forms, i.e., the form of transferring the risk of a *senior* position, the risk of a *junior* (first-loss) position, or a *full* risk transfer, which are denoted as $i \in \{s, j, f\}$, respectively. For $i = j$, the firm exchanges the end-of-period cash flow $s^s(x, k) = \min\{x, k\}$ with third-party investors against a unit price $p(k)$. If $i = s$, the firm exchanges the cash flow $s^j(x, k) = \max\{x - k, 0\}$ against a unit price $q(k)$. If $i = f$, third-party investors obtain both cash flows for a price $p(k) + q(k)$.[2] Proceeds are invested against the risk-free rate, which we normalize to zero. Hence, the end-of-period cash flow of the firm, $v_{i,k}(x, y, r)$, equals
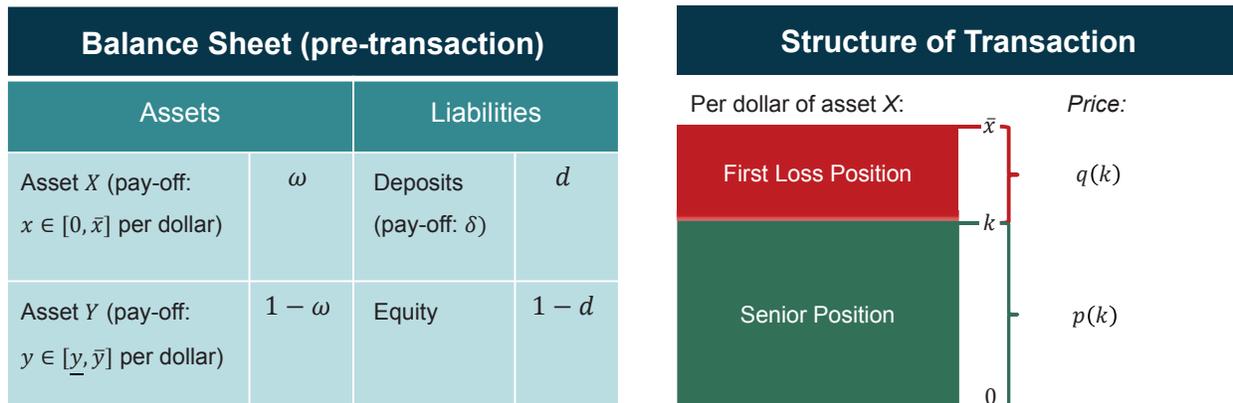
$$v_{i,k}(x, y, r) = \omega x + (1 - \omega)y +$$
$$+ r\omega \left[\mathbf{1}_{i \in \{s,f\}} \left(p(k) - s^s(x, k) \right) + \mathbf{1}_{i \in \{j,f\}} \left(q(k) - s^j(x, k) \right) \right]. \quad (1)$$

This setup is sufficiently flexible to cover various forms of credit risk transfer. One

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[2]The level of $p(k) + q(k)$ is independent of threshold k if the law of one price holds true, since $s^s(x, k) + s^j(x, k) = x$ for any k. If the law of one price does not hold true, then the issuing firm could optimally choose k to maximize the proceeds from the credit risk transfer. For our results it is irrelevant whether the law of one price holds true.

Figure 1: Balance sheet and credit risk transfer in the model

Balance Sheet (pre-transaction)			
Assets		Liabilities	
Asset X (pay-off: $x \in [0, \bar{x}]$ per dollar)	ω	Deposits (pay-off: δ)	d
Asset Y (pay-off: $y \in [\underline{y}, \bar{y}]$ per dollar)	$1 - \omega$	Equity	$1 - d$

Structure of Transaction

Per dollar of asset X:

First Loss Position — $q(k)$

Senior Position — $p(k)$

Note: The figure summarizes the model information on the bank's balance sheet before the credit risk transfer and the potential structure of the transaction.

interpretation is securitization, where the banks sells senior and/or junior tranches in the securitized loan portfolio X. A second interpretation is where the bank buys credit portfolio insurance for X. If the bank retains the junior tranche, it would be comparable to insuring the returns on X with a deductible $\bar{x} - k$ against a premium $k - p(k)$. The transfer of the risk in the junior position could be considered as insuring the returns on X against a premium $\bar{x} - k - q(k)$ with a insurance coverage limit of $\bar{x} - k$. A third interpretation is that the bank uses bank credit derivatives to hedge portfolio risk.

Insolvency occurs if the end-of-period cash flow $v_{i,k}(x, y, r)$ is insufficient to repay the depositors the amount $\delta_{i,k}(r)$, where the $\delta_{i,k}(r)$ represents the principal amount and interest owed to depositors. The level of this solvency threshold in the absence of credit risk transfer is denoted as $\delta_{i,k}(0) = \delta$ for any (i, k). In other words, the probability of insolvency with securitisation strategy i equals

$$\pi_{i,k}(r) = \Pr\left[v_{i,k}(x, y, r) < \delta_{i,k}(r)\right]. \tag{2}$$

Our intention is to assess how different credit risk transfer strategies affect the level of $\pi_{i,k}(r)$.

With the threshold return $x_i^*(y, r)$ denoting the minimum level of x such that

$v_{i,k}(x, y, r) \geq \delta_{i,k}(r)$, the probability of insolvency can be obtained from evaluating the double integral

$$\pi_{i,k}(r) = \int_0^\infty \int_0^{x_{i,k}^*(y,r)} \phi(x, y) \mathrm{d}x \mathrm{d}y. \tag{3}$$

Comparing the levels of $\pi_{i,k}(r)$ and $\pi_{i,k}(0)$ reveals whether credit risk transfer strategy (i, k) is beneficial or harmful in terms of insolvency risk of the originator, where $\pi_{i,k}(0)$ can be obtained from evaluating (3) with

$$x_{i,k}^*(y, 0) = \delta + \frac{1 - \omega}{\omega}(\delta - y). \tag{4}$$

One important challenge is that evaluating (3) requires an accurate approximation of the return distribution $\phi(x, y)$. However, it is well-known that the return distributions of junior and senior positions in credit portfolios strongly depend on the dependence structure among loan defaults and the likelihood of tail events; see, e.g., Duffie et al. (2009) and Gennaioli et al. (2012). Hence, one may suspect that the conclusions may strongly depend on the underlying assumptions regarding the specification of $\phi(x, y)$. In other words, results based on evaluating (3) using simulations or an analytical solution may be subject to debate. Instead, we will focus on deriving results that apply to any continuous $\phi(x, y)$ with full support $[0, \overline{x}] \times [\underline{y}, \overline{y}]$.

4   Results

Not in every situation, it is necessary to specify $\phi(x, y)$ to evaluate the effect of a credit risk transfer on insolvency risk. The reason is that one can directly compare the levels of the solvency threshold returns $x_{i,k}^*(y, r)$ and $x_{i,k}^*(y, 0)$. If $x_{i,k}^*(y, r) \leq x_{i,k}^*(y, 0)$ for every $y \in [\underline{y}, \overline{y}]$, then it must hold true that $\pi_{i,k}(r) \leq \pi_{i,k}(0)$: The credit risk transfer reduces insolvency risk. Similarly, if $x_{i,k}^*(y, r) \geq x_{i,k}^*(y, 0)$ for every $y \in [\underline{y}, \overline{y}]$, then it must hold true that $\pi_{i,k}(r) \geq \pi_{i,k}(0)$: The credit risk transfer increases insolvency risk. Finally, if

$x^*_{i,k}(y, r) < x^*_{i,k}(y, 0)$ for some levels of $y \in [\underline{y}, \overline{y}]$, and $x^*_{i,k}(y, r) \geq x^*_{i,k}(y, 0)$ for some other levels of $y \in [\underline{y}, \overline{y}]$, then the direction of the impact of the credit risk transfer on $\pi_{i,k}$ will be indeterminate without further specifying $\phi(x, y)$.

4.1 Exogenous Funding Costs

We start our analysis from the point of view of a specialized bank ($\omega = 1$) with risk-insensitive debt for illustrative purposes. This stacks the cards in favour of our approach, because the level of $x^*_{i,k}(y, r)$ does not depend on y in this case.[3] The following proposition summarizes the effect of a credit risk transfer on the insolvency risk of a specialized firm.

Proposition 1 *Credit risk transfer reduces the probability of insolvency of a specialized bank with risk-insensitive debt if*

1. *$p(k) \geq \delta$ in case of transferring the risk in a senior position;*

2. *$p(k) + q(k) \geq \delta$ in case of a full risk transfer;*

3. *$k + q(k) \geq \delta$ in case of transferring the risk in a junior position.*

Otherwise, credit risk transfer strictly increases the probability of insolvency.

Proof. See Appendix. ∎

Why can a reduction in the "overall risk" increase the insolvency risk of the firm? In the case of a full risk transfer, the reason is obvious. In the context of a specialized bank, selling both tranches boils down to the liquidation of the firm. If the liquidation value of X is insufficient to repay creditors, i.e., if $p(k) + q(k) < \delta$, selling both tranches effectively eliminates the chance of survival when a high return materializes. The credit risk transfer is the inverse of "gambling for resurrection."

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[3]If $\omega = 1$, then we have $\partial x^*_{i,k}(y, r)/\partial y = 0$ and $\partial x^*_{i,k}(y, 0)/\partial y = 0$ by construction. As a consequence, it is impossible to have $x^*_{i,k}(y, r) < x^*_{i,k}(y, 0)$ for some levels of $y \in [\underline{y}, \overline{y}]$ and $x^*_{i,k}(y, r) \geq x^*_{i,k}(y, 0)$ for some other levels of $y \in [\underline{y}, \overline{y}]$. Hence, our approach always provides an answer in this case.

If the firm retains the risk in the senior position, the reason is similar, but the constraint is weaker. It will always be possible to design some credit risk transfer such that $\delta \leq k < \bar{x}$, which implies $\delta < k + q(k)$, as long as the best possible outcome exceeds the insolvency threshold, i.e., $\bar{x} > \delta$. Hence, even if the liquidation value of X is insufficient to repay creditors, then it will still be possible to reduce the probability of insolvency of the bank by transferring some of the upward potential. However, transferring risk in a junior position increases the insolvency probability if $k + q(k) < \delta$. In this case, the proceeds from the transfer will be insufficient to repay depositors, even if the retained senior position yields the full pay-off k.

The strongest constraint in Proposition 1 to determine whether the removal of credit risk has a beneficial effect, is the one regarding the case where the bank retains the first-loss position. However, the reason why insolvency risk may increase in this case is more subtle. Proposition 1 implies that, unless the retained first-loss position is sufficiently thin, transferring the risk in the senior position must increase the probability of insolvency. This is because of the following reason. Transferring credit risk in a senior position can be considered as protecting the bank against realizations of the (tail) events where $x < k$. This protection comes at a premium, $k - p(k)$. However, the end-of-period cash flow of a protected bank will be insufficient to achieve solvency if the insured risk materializes under the condition in the proposition, i.e., $p(k) < \delta$. Hence, insolvencies are not avoided. On the contrary, the transfer increases the insolvency risk of the firm, because the cost of the credit risk protection raises the minimum return necessary to remain solvent above the original level.

Proposition 1 shows that whether a credit risk transfer is beneficial or harmful in terms of insolvency risk depends on whether the insolvency threshold of the specialized firms, δ, is sufficiently low. Parameter δ is positively related to the leverage of a bank. It follows that a certain credit risk transfer may be beneficial in terms of insolvency risk for a well-capitalized issuer, while the same transaction could be harmful for an issuer with a more fragile financial structure, i.e., a higher δ. The more leveraged a bank is, the more likely it is that a credit

risk transfer will be harmful in terms of insolvency risk.

Note that the qualitative results in Proposition 1 are independent of the scale of the securitisation transaction: The direction of the effect is independent of r. Of course, this does not apply to the magnitude of the effect, which will increase in the scale of the transaction.

In a similar fashion, the following lemma describes how credit risk transfer affects the insolvency risk of a bank that also invests in a portfolio of other assets, Y, for a given realization y.

Lemma 1 *Given some realization y, the credit risk transfer reduces the probability of insolvency of a bank with risk-insensitive debt if*

1. *$p(k) \geq x_{i,k}^*(y, 0)$ in case of transferring the risk in a senior position;*

2. *$p(k) + q(k) \geq x_{i,k}^*(y, 0)$ in case of a full risk transfer;*

3. *$k + q(k) \geq x_{i,k}^*(y, 0)$ in case of transferring the risk in a junior position.*

Otherwise, credit risk transfer strictly increases the probability of insolvency.

The mechanics behind this lemma are exactly the same as those described above, with the only difference that the ex-post realization of y will change the threshold that determines whether a credit risk transfer reduces or increases the bank's insolvency risk. Hence, uncertainty regarding the ex-post return on the bank's other investments leads to uncertainty regarding the question whether a credit risk transfer reduces bank insolvency risk.

Lemma 1 exposes a potentially poisonous interaction between the benefits of diversification and the impact of credit risk transfers. Suppose that a credit risk transfer would reduce bank insolvency risk if the performance of Y is in accordance with expectations. Then we have that this credit risk transfer will certainly reduce bank insolvency risk if y is higher than expected, because it is associated with a lower threshold for beneficial credit risk transfers $x_{i,k}^*(y, 0)$ in Lemma 1; see Eq. (4). However, the same credit risk transfer strategy may turn out to increase bank insolvency risk in case of a lower than expected y, because a lower y

result in a higher threshold $x^*_{i,k}(y,0)$. Hence, the credit risk transfer will be beneficial when the firm also benefits from profits due to diversification in Y, while the credit risk transfer may turn out to be harmful when the bank is in poor conditions as diversification in Y results in losses. This interaction between the benefits of the two different risk management strategies, i.e., diversification and credit risk protection, may pose a challenge for risk managers in practice.

The portfolio of other assets, Y, could solely contain a risk-free asset (i.e., if $y = \underline{y} = \overline{y}$). If Y is a risk-free asset, then Lemma 1 will directly answer the question as to whether the credit risk transfer reduces or increases bank insolvency risk. However, with variation in the bank's return on the investment in Y, the uncertainty regarding y will blur the answer to the question on how the thickness of the junior and senior positions compares to the minimum return on X necessary to pay off the depositors. Nevertheless, in this case, one can still be certain about the direction of the impact if Lemma 1 provides the same answer for any $y \in [\underline{y}, \overline{y}]$. Therefore, the larger the risk in terms of $(\overline{y} - \underline{y})$ and the degree of diversification in terms of $(1 - \omega)$, the less likely it is that it will be possible to determine the effect of a credit risk transfer strategy on bank default risk without explicit assumptions regarding $\phi(x,y)$.

This is shown in the following propositions. The first proposition gives the condition under which a credit risk transfer strategy is beneficial in terms of insolvency risk.

Proposition 2a *Credit risk transfer strictly reduces the insolvency probability of a firm with risk-insensitive debt if*

1. *$p(k) > \overline{\delta}$ in case of transferring the risk in a senior position;*

2. *$p(k) + q(k) > \overline{\delta}$ in case of a full risk transfer;*

3. *$k + q(k) > \overline{\delta}$ in case of transferring the risk in a junior position,*

where $\overline{\delta} = \delta + \frac{1-\omega}{\omega}(\delta - \underline{y})$.

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Proof. See Appendix. ∎

The proposition replaces the δ in Proposition 1 by $\overline{\delta}$. The $\overline{\delta}$ depends on the original insolvency threshold and a layer of uncertainty which is an increasing function of $\delta - \underline{y}$. The magnitude of $\delta - \underline{y}$ is the maximum shortfall in the return on Y relative to the firm's overall insolvency threshold. The larger the magnitude of this downside risk, the less likely it is that one can be certain that a credit risk transfer reduces the insolvency probability. For lower realizations of y, the firm needs a higher realization of x to stay solvent. In other words, diversification raises the threshold that determines whether credit risk transfer strategies are beneficial in terms of insolvency risk for lower values of y. If the magnitude of the downside risk is sufficiently large, then there will exist contingencies in which the credit risk transfer will be harmful in terms of insolvency risk, which implies that the default risk of the bank will depend on the functional form of $\phi(x, y)$.

Similarly, the following proposition gives the condition under which a securitisation strategy is harmful in terms of insolvency risk.

Proposition 2b *Credit risk transfer strictly increases the insolvency probability of a firm with risk-insensitive debt if*

1. *$p(k) < \underline{\delta}$ in case of transferring the risk in a senior position;*

2. *$p(k) + q(k) < \underline{\delta}$ in case of a full risk transfer;*

3. *$k + q(k) < \underline{\delta}$ in case of transferring the risk in a junior position,*

where $\underline{\delta} = \delta - \frac{1-\omega}{\omega}(\overline{y} - \delta)$.

Proof. See Appendix. ∎

The proposition replaces the δ in Proposition 1 by $\underline{\delta}$, which is a decreasing function of $\overline{y} - \delta$. The magnitude of $\overline{y} - \delta$ reflects the upward potential of the return on investment Y relative to the overall insolvency threshold. If the maximum realization of y is too large, then there will exist some contingencies in which the credit risk transfer will be beneficial in

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terms of insolvency risk, which implies that the increase in insolvency risk is not guaranteed for every $\phi(x, y)$.

In summary, a higher degree of diversification limits the number of cases in which it will be possible to discern whether a credit risk transfer increases or reduces bank default risk without making assumptions on $\phi(x, y)$. In the context of bank loan portfolios, diversification will be more restrictive regarding conclusions on a reduction in default risk than on an increase in default risk, because the magnitude of $\overline{y} - \delta$ in Proposition 2b can be considered small relative to $\delta - \underline{y}$ in Proposition 2a. The reason is that, in the context of loan portfolios, $\overline{y}$ will be close to one since it is directly linked to the gross return charged to borrowers, while δ will be close to one due to bank leverage. By contrast, $\underline{y}$ will generally be far from one because it depends on the loss in the worst case scenario.
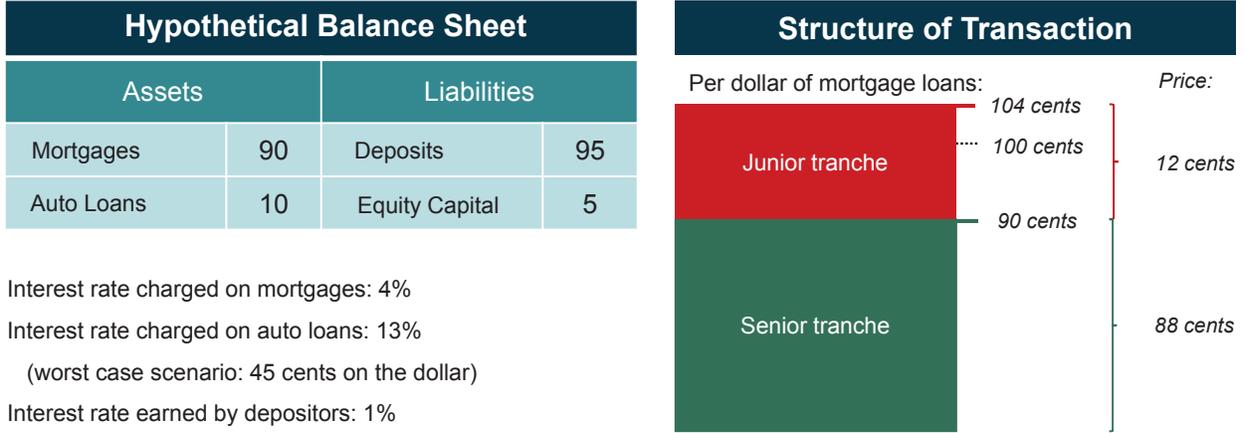
4.2   Illustration

Before continuing to risk-sensitive bank debt, it will be useful to provide an illustration on the results so far (parameter values have been chosen such that it doesn't matter for the conclusions whether we consider risk-sensitive or risk-insensitive bank debt).

Consider a hypothetical bank specialized in mortgage loans for house purchases and auto loans. The balance sheet of the bank is provided in Figure 2. The bank charges its clients, on average, an interest rate of 4 per cent on mortgage loans, and an interest rate of 13 per cent on auto loans. In the worst case scenario, the portfolio of auto loans will return 45 cents on the dollar. Moreover, depositors earn on average an interest rate of 1 percent on their balances. Based on this information, we have that the parameters values $\omega = 0.9$, $d = 0.95$, $\overline{x} = 1.04$, $\underline{y} = 0.45$, $\overline{y} = 1.13$, $\delta = 0.9595$, and $\omega = 0.9$.

The bank considers transferring the risk for the pool of mortgage loans to reduce the risk on the balance sheet. Figure 2 also shows the potential structure of the securitized loan pool, with the threshold between the senior and the junior tranche set at 90 cents on the dollar. Market analysis suggests that, for each dollar of mortgages in the pool, the securitized senior

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Figure 2: Hypothetical bank balance sheet and securitization transaction

Hypothetical Balance Sheet			
Assets		Liabilities	
Mortgages	90	Deposits	95
Auto Loans	10	Equity Capital	5

Interest rate charged on mortgages: 4%

Interest rate charged on auto loans: 13%

  (worst case scenario: 45 cents on the dollar)

Interest rate earned by depositors: 1%

Structure of Transaction

Per dollar of mortgage loans: *Price:*

Junior tranche — 104 cents / 100 cents / 90 cents   *12 cents*

Senior tranche   *88 cents*

Note: This securitization transaction illustrates the results in Propositions 2a–3b. The direct impact of selling a proportion of the senior tranche is an increase in the default risk of the bank, while the direct impact of selling a proportion of the equity tranche is a reduction in default risk. Parameter values have been chosen such that it doesn't matter whether the interest rate earned by depositors is fixed (Propositions 2a and 2b), or whether it changes in response to the securitization transaction as implied by Eq. (5) (Propositions 3a and 3b).

position could be sold to third-party investors for 88 cents, while the first-loss position could be sold for 12 cents (respectively, an effective interest rate of approximately 2.3 per cent and 16.7 per cent). This information gives the parameter values $k = 0.9$, $p(0.9) = 0.88$ and $q(0.9) = 0.12$.

Is selling a proportion of the tranches in the securitization beneficial for the bank from a default risk point of view? From Proposition 2a, we have $\bar{\delta} \approx 1.016$, which is smaller than $k + q(k) = 1.02$, and hence, selling the first-loss position while retaining the senior position will certainly reduce the bank's probability of default. On the other hand, from Proposition 2b, we have $\underline{\delta} \approx 0.941$, which is larger than $p(k) = 0.88$, and therefore, selling the senior position while retaining the first-loss position will certainly increase the bank's probability of default. Interestingly, the first transaction, which increases default risk, is in general not compliant with risk retention regulation that has been introduced in several jurisdictions, while the second transaction, which increases default risk, generally is. We will return to this issue at a later point in the paper.

4.3 Endogenous Funding Costs

So far, the analysis relies on assuming that bank's funding cost does not respond to the credit risk transfer. Note, however, that the potentially harmful direct impact of credit risk transfer on the probability of failure could be attenuated by a reduction in the rate of interest on bank deposits in response to credit risk transfer. In this section, we discuss the results under the assumption that the rate of interest on bank deposits is risk-sensitive.

Endogenous funding costs are modelled as follows. Without accounting for default costs, the amount of the principal plus interest at the end of the period, $\delta_{i,k}(r)$, is implicitly defined as

$$d = \int_0^\infty \int_0^\infty \phi^Q(x,y) \min\{v_{i,k}(x,y,r), \delta_{i,k}(r)\} \mathrm{d}x \mathrm{d}y, \tag{5}$$

where $\phi^Q(x,y)$ reflects the risk-neutral probability density function of the depositors. The form in Eq. (5) is flexible to allow for subjectivity in perceived probabilities in terms of divergence in opinions à la Chan and Kanatas (1985) or various types of risk preferences, since it is not assumed that $\phi^Q(x,y)$ reflects the true $\phi(x,y)$. Moreover, for the purpose of the discussion, it will also be convenient to define the subjective, or risk-neutral, probability of default from the perspective of the bank's depositors as

$$\pi_{i,k}^Q(r) = \int_0^\infty \int_0^{x_{i,k}^*(y,r)} \phi^Q(x,y) \mathrm{d}x \mathrm{d}y. \tag{6}$$

The general line of the proof of the propositions with endogenous bank funding costs is as follows. The credit risk transfer strategy must increase or decrease the bank's probability of default in (3) for any continuous probability density function $\phi(x,y)$ with full support $[0,\overline{x}] \times [\underline{y}, \overline{y}]$, if the derivative

$$\frac{\partial x_{i,k}^*(y,r)}{\partial r} \lessgtr 0 \tag{7}$$

for every $(y,r) \in [\underline{y}, \overline{y}] \times [0,r]$. A difficult factor in showing under which conditions this holds true is that the derivative in (7) is also a function of $\partial \delta_{i,k}(r)/\partial r$, which is typically nonzero if

the bank's funding cost responds to changes in the risk of the investments held by the bank. The proof relies on deriving the level of $\partial \delta_{i,k}(r)/\partial r$ from Eq. (5) with implicit derivation.

In case of deriving the condition for the credit risk transfer for the senior tranche, it will be necessary to also assume that the risk premium required by depositors when putting money in a hypothetical bank investing solely in the senior position is at least smaller or equal to the risk premium required by third-party investors or the insurer, i.e.,

$$k - p(k) \geq k - p^Q(k) = k - \int_0^\infty \int_0^\infty \phi^Q(x, y) \min\{x, k\} \mathrm{d}x \mathrm{d}y. \tag{8}$$

Note that this condition is even satisfied if depositors, despite of deposit insurance, fully appreciate the reduction in the risk of the bank's cash flow in lowering the interest rate that they require, and if credit risk transfer is fairly priced (i.e., providers of credit risk protection price risk using the same $\phi^Q(x, y)$).[4]

The reason why the proof requires an assumption such as the one in (8) is straightforward. After transferring the credit risk in the senior position, the bank bears the cost of credit risk protection. With risk-sensitive debt, the interest expenses of the bank may drop in response to the credit risk transfer. The condition in (8) ensures that the interest expenses of the bank do not drop more than is justified by the reduction in asset risk, although they are allowed to drop with less.

With these preparations, the following proposition shows that the conditions under which the credit risk transfer increase the risk of insolvency change, but not by much.

Proposition 3a *The credit risk transfer strictly increases the insolvency probability of a bank with risk-sensitive debt if*

1. *$k < \underline{\delta}$ in case of transferring the risk in a senior position and the assumption in (8) holds true;*

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[4]Explicit or implicit guarantees on bank debt, such as deposit insurance, reduce the level of $k - p^Q(k)$. On the other hand, subsidies on securitization or credit portfolio insurance programs could impact the level of the risk premium $k - p(k)$.

2. $p(k) + q(k) + (\delta - d) < \underline{\delta}$ *in case of a full risk transfer;*

3. $k + q(k) + (\delta - d) < \underline{\delta}$ *in case of transferring the risk in a junior position,*

where $\underline{\delta} = \delta - \frac{1-\omega}{\omega}(\overline{y} - \delta)$.

Proof. See Appendix. ∎

With risk-sensitive debt, the conditions for the full risk transfer and the transfer of the risk in the junior position also include and adjustment for the risk premium required by depositors in the absence of credit risk transfer, i.e., $\delta - d$. The magnitude of this component can be illustrated with an example: A specialized bank which is financed with 10 per cent equity and a risk premium required by depositors of 2 per cent corresponds to parameter values of $\underline{\delta} = 0.918$ and $\delta - d = 0.018$. This implies that, with risk-insensitive debt, a full credit risk transfer increases the insolvency probability, if the bank receives less than 91.8 cents on the dollar, while with risk-sensitive debt, the threshold is reduced to 90 cents on the dollar. In other words, the relevant threshold depends not only on the amount of debt, but also on the risk premium promised to depositors.

Moreover, with risk-sensitive debt, transferring the risk in the senior position can still increase the default risk of the firm. The underlying reason is that the interest expense will not drop by the full amount of the cost of the insurance, even if the credit risk transfer is fairly priced (i.e., using the same $\phi^Q(x, y)$), and if depositors, despite of deposit insurance, fully appreciate the reduction in the risk of the bank's cash flow in lowering interest required on bank deposits. The reason is that the depositors still end up paying for the cost of credit risk protection if the bank fails, which occurs with a perceived probability of $\pi_{s,k}^Q(r)$. Therefore, the funding cost of the bank does not drop by as much as the cost of the credit risk protection, resulting, overall, in a higher threshold return necessary to avoid bank failures.

Similarly, the conditions under which credit risk transfer reduce the insolvency probability do not change much, as is shown in the following proposition.

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Proposition 3b *The credit risk transfer strictly reduces the insolvency probability of a bank with risk-sensitive debt if*

1. $p(k) > \overline{\delta}$ *in case of transferring the risk in a senior position;*

2. $p(k) + q(k) + (\delta - d) > \overline{\delta}$ *in case of a full risk transfer;*

3. $k + q(k) > \overline{\delta}$ *in case of transferring the risk in a junior position,*

where $\overline{\delta} = \delta + \frac{1-\omega}{\omega}(\delta - \underline{y})$.

Finally, it may be useful to stress that the statements on the reductions and increases in the bank's default probabilities in the propositions apply to both the actual and the perceived failure probabilities, i.e., $\pi_{i,k}(r)$ and $\pi_{i,k}^{Q}(r)$. The reason is that the propositions do not rely on the precise functional form of $\phi(x, y)$. This implies that the direction of the impact will be correct, even if the perceived default probability is far from accurate.

5 Discussion

5.1 Adverse Selection

Adverse selection in the context of credit risk transfer generally refers to the case where the lender uses private information to select lower quality loans into the underlying asset pool of the securitisation. Our framework is sufficiently broad to allow for adverse selection. Asset X may represent the pool with a selection of low-quality borrowers, while Y represents the pool of loans to high-quality borrowers. Of course, insurance providers may require a higher premium, or third-party investors may require a higher yield on tranches in securitisations, if they anticipate adverse selection. Hence, adverse selection may potentially reduce the level of $p(k)$ and $q(k)$. From our propositions, such a discount makes it more likely that the conditions for which credit risk transfers increase the insolvency probability are binding. Therefore, anticipation of adverse selection can be a factor that contributes to an increase in bank insolvency risk as a consequence of a credit risk transfer.

5.2 Moral Hazard

With moral hazard, the end-of-period cash flow from the asset pool (after deducting the operating cost) may depend on the structure of the credit risk transfer. For example, after transferring the risk in the senior position, the originator could save on the operating cost of the asset pool by a reduction in monitoring. Theoretically, such an action may result in higher losses in states corresponding to adverse tail events and lower costs in the other states of the world; see, e.g., Chiesa (2008). Dependence between the return distribution (net of the operating cost) and the structure of the credit risk transfer is a violation of the assumptions in the model. The essential difference with adverse selection is that, once asset pool X is constructed, the return distribution is not affected by the credit risk transfer strategy. With moral hazard, the return distribution (net of the operating cost) does change depending on the structure of the credit risk transfer. Therefore, the propositions do not generally hold true with moral hazard. However, in the special case that moral hazard results in a cost saving by the bank without affecting the risk distribution of the retained interest, then the propositions can still be applied by simply adding the bank's cost saving to the purchase price paid by the third-party investors.

6 Concluding Remark

After the financial crisis, authorities in several jurisdictions implemented forms of mandatory credit risk retention for banks engaging in credit risk transfer. For example, credit risk retention regulation has been implemented in the EU (Regulation No 575/2013, Article 405) and the US (CFR Title 17, Chapter II, Part 246).[5] Inspired by the literature on information asymmetries in credit risk transfers, one of the main objectives has been to better align the incentives of banks and investors.

Credit risk retention regulation usually requires exposure to a certain fraction of the

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[5]In 2014, the Canadian Securities Administrators (CSA) decided not to propose the introduction of mandatory credit risk retention in Canada.

unhedged credit risk after the credit risk transfer, where the institution has the choice of different forms of credit risk retention. Broadly speaking, under current regulation, the choice boils down to either retaining an exposure of not less than 5 per cent to the credit risk in all positions ("vertical" interest), or retaining an exposure to a first-loss position of not less than 5 per cent ("horizontal residual" interest).[6] Moreover, in some jurisdictions, including the US, a combination of vertical and horizontal residual interests is also allowed.

Although aligning incentives between banks and investors is important, credit risk retention regulation in its current form may advocate transactions whose direct impact is to increase bank insolvency risk. For example, in the illustration discussed in Subsection 4.2, transferring the credit risk in the first-loss position while retaining the risk in the senior position would reduce bank insolvency risk, but this transaction would typically not comply with the aforementioned credit risk retention regulation. In contrast, the transaction in which the bank transfers the risk in the senior position and retains the risk in the first-loss position does comply with credit risk retention regulation, but the direct impact of this transaction is to increase bank insolvency risk.

Whether a transaction in which the bank retains the first-loss position increases insolvency risk depends on how the retained position compares to the unweighted capital ratio of the bank. To see this, let the unweighted capital ratio ("leverage ratio") be denoted as $l = 1 - d$, and let the size of the economic interest in the form of a retained first loss position be denoted as $f = 1 - p(k)$. Let f^* denote the mandatory minimum retained interest if the bank retains a first loss position, such that $f \geq f^*$. Then using this notation in, for example, the condition in Proposition 1, gives that any credit risk transfer of the senior position complying with the regulation will increase insolvency risk of the bank if
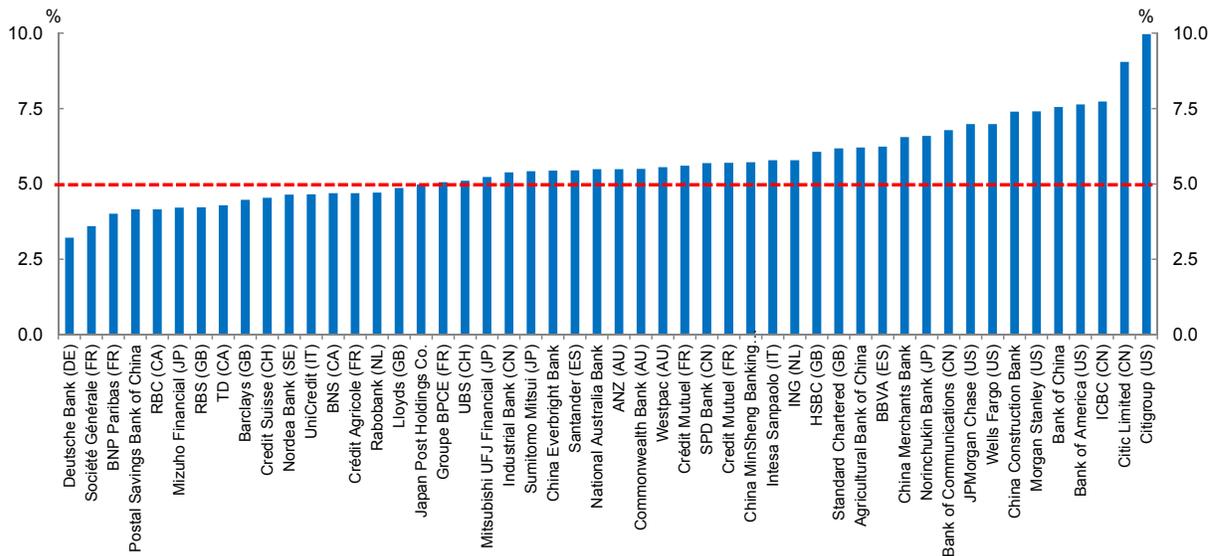
$$f^* > l - (\delta - d). \tag{9}$$

With $\delta - d \geq 0$, we have that the condition in (9) is a binding constraint if $f^* > l$, which

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[6] For a precise description of the options, we refer to the regulatory texts.

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Figure 3: Commercial Banks' Tangible Common Equity, per cents of Total Assets

Note: The figure illustrates the condition in Eq. (9) for the 50 largest commercial banks by total assets. The red dashed line corresponds to the minimum mandatory retained interest of 5 per cent, i.e., f^*. The blue bars correspond to the unweighted capital ratios, i.e., l. With $\delta - d \geq 0$, the condition in (9) is binding for more than a third of the banks. Source: Fitch Connect. The numbers refer to end of fiscal 2016 for most banks.

depends on the level of the unweighted capital ratio of a bank.

Figure 3 illustrates this condition by reporting the level of tangible common equity as a ratio of total assets for the 50 largest commercial banks around the globe. About a third of these banks operate with an unweighted capital ratio smaller than 5 per cent, which suggests that the condition in (9), with a current $f^* = 0.05$, is far from hypothetical in the current banking landscape. Moreover, some have suggested to increase the level of f^* going forward, which corresponds to an upward shift of the horizontal dashed line in Figure 3. For example, lawmakers in the EU have proposed that the European Banking Authority should review the level of the risk retention rate between 5 per cent and 20 per cent every two years; see the report of the Committee on Economic and Monetary Affairs (2016).

Without an increase in the levels of banks' capital ratios, risk retention requirements may move banks further into the quadrant of retaining thick first-loss positions and relatively thin capital buffers. The current analysis shows that such a combination could turn out to be

a toxic cocktail in times of financial headwinds. Of course, increasing banks' capital ratios may be one way to address this issue, but another – and perhaps politically more feasible – solution is to limit the option of complying with risk retention regulation through the retention of first-loss positions for those banks with low unweighted capital ratios.

Appendix: Proofs

A.1 Proposition 1

Proposition 1 follows from Propositions 2a and 2b with $\omega = 1$.

A.2 Propositions 2a and 2b

From (2), we have

$$\pi_{i,k}(0) = \Pr\left[\omega x < \delta - (1 - \omega)y\right]; \tag{10}$$

$$= \Pr\left[\omega x < \delta - (1 - \omega)y, x \leq k\right] + \Pr\left[\omega x < \delta - (1 - \omega)y, x > k\right]; \tag{11}$$

$$:= A_0 + B_0. \tag{12}$$

A.2.1 Transfer risk in the senior position

From (1) and (2), we have for $i = s$

$$\pi_{s,k}(r) = \Pr\left[\omega\left(x + rp(k) - r\min\{x, k\}\right) < \delta - (1 - \omega)y\right]; \tag{13}$$

$$= \Pr\left[\omega x < \delta - (1 - \omega)y + \frac{r}{1 - r}\left(\delta - (1 - \omega)y - \omega p(k)\right), x \leq k\right]$$

$$+ \Pr\left[\omega x < \delta - (1 - \omega)y + \omega r(k - p(k)), x > k\right]; \tag{14}$$

$$:= A_s + B_s. \tag{15}$$

From a comparison of (11) and (14) follows $A_s > A_0$ if $\delta - (1-\omega)y - \omega p(k) > 0 \forall y \in [\underline{y}, \overline{y}]$, i.e., if $p(k) < \delta - \frac{1-\omega}{\omega}(y - \delta) \forall y \in [\underline{y}, \overline{y}]$, which is implied by $p(k) < \delta - \frac{1-\omega}{\omega}(\overline{y} - \delta)$. Further, from a comparison of (11) and (14) follows $B_s \geq B_0$, since $k - p(k) > 0$. Hence, $p(k) < \delta - \frac{1-\omega}{\omega}(\overline{y} - \delta)$ implies $A_s + B_s > A_0 + B_0$, or, $\pi_{s,k}(r) > \pi_{i,k}(0) \forall r \in (0, 1]$.

From a comparison of (11) and (14) follows $A_s < A_0$ if $\delta - (1-\omega)y - \omega p(k) < 0 \forall y \in [\underline{y}, \overline{y}]$, i.e., if $p(k) > \delta + \frac{1-\omega}{\omega}(\delta - y) \forall y \in [\underline{y}, \overline{y}]$, which is implied by $p(k) > \delta + \frac{1-\omega}{\omega}(\delta - \underline{y})$. Further,

from (14), with this condition we have

$$B_s = \Pr\left[\omega k < \omega x < \delta - (1-\omega)y + \omega r(k - p(k))\right];$$

$$\leq \Pr\left[\omega k < \omega x < \delta - (1-\omega)\underline{y} + \omega r(k - \delta - \frac{1-\omega}{\omega}(\delta - \underline{y}))\right];$$

$$= \Pr\left[k < \frac{x - rk}{1 - r} < \delta + \frac{1-\omega}{\omega}(\delta - \underline{y})\right]. \tag{16}$$

Since $k > p(k)$, with this condition we also have $k > \delta + \frac{1-\omega}{\omega}(\delta - \underline{y})$. Using this inequality in (14) gives

$$B_s \leq \Pr\left[\delta + \frac{1-\omega}{\omega}(\delta - \underline{y}) < \frac{x - rk}{1 - r} < \delta + \frac{1-\omega}{\omega}(\delta - \underline{y})\right];$$

$$= 0.$$

Following a similar argument, $B_0 = 0$ if $k > \delta + \frac{1-\omega}{\omega}(\delta - \underline{y})$. Hence, $p(k) > \delta + \frac{1-\omega}{\omega}(\delta - \underline{y})$ implies $A_s < A_0$ and $B_s = B_0 = 0$, or, $\pi_{s,k}(r) < \pi_{i,k}(0)\forall r \in (0, 1]$.

A.2.2   Full risk transfer

From (1) and (2) follows the insolvency probability for $i = f$ as

$$\pi_{f,k}(r) = \Pr\left[\omega x < \delta - (1-\omega)y + \frac{r}{1-r}(\delta - (1-\omega)y - \omega(p(k) + q(k)))\right]. \tag{17}$$

From a comparison of (10) and (17) follows $\pi_{f,k}(r) < \pi_{i,k}(0)\forall r \in (0, 1]$ if $\delta - (1-\omega)y - \omega(p(k) + q(k)) < 0\forall y \in [\underline{y}, \overline{y}]$, i.e., if $p(k) + q(k) < \delta + \frac{1-\omega}{\omega}(\delta - y)\forall y \in [\underline{y}, \overline{y}]$, which is implied by $p(k) + q(k) > \delta + \frac{1-\omega}{\omega}(\delta - \underline{y})$.

From a comparison of (10) and (17) follows $\pi_{f,k}(r) > \pi_{i,k}(0)\forall r \in (0, 1]$ if $\delta - (1-\omega)y - \omega(p(k) + q(k)) > 0\forall y \in [\underline{y}, \overline{y}]$, i.e., if $p(k) + q(k) < \delta - \frac{1-\omega}{\omega}(y - \delta)\forall y \in [\underline{y}, \overline{y}]$, which is implied by $p(k) + q(k) < \delta - \frac{1-\omega}{\omega}(\overline{y} - \delta)$.

A.2.3 Transfer risk in the junior position

From (1) and (2) follows the insolvency probability for $i = j$ as

$$\pi_{j,k}(r) = \Pr\left[\omega\left(x - r\max\{x - k, 0\} + rq(k)\right) < \delta - (1 - \omega)y\right]; \tag{18}$$

$$= \Pr\left[\omega\left(x + rq(k)\right) < \delta - (1 - \omega)y, x \leq k\right]$$

$$+ \Pr\left[\omega x < \delta - (1 - \omega)y + \frac{r}{1 - r}\left(\delta - (1 - \omega)y - \omega(k + q(k))\right), x > k\right]; \tag{19}$$

$$:= A_j + B_j. \tag{20}$$

From a comparison of (11) and (19) that $A_j \leq A_0$, since $x \leq x + rq(k)$. Further, from a comparison of (11) and (19) follows that $B_j < B_0$ if $\delta - (1 - \omega)y - \omega(k + q(k)) < 0 \forall y \in [\underline{y}, \overline{y}]$, i.e., if $k + q(k) > \delta + \frac{1-\omega}{\omega}(\delta - y) \forall y \in [\underline{y}, \overline{y}]$, which is implied by $k + q(k) > \delta + \frac{1-\omega}{\omega}(\delta - \underline{y})$. Hence, this condition implies $A_j + B_j < A_0 + B_0$, or, $\pi_{j,k}(r) < \pi_{i,k}(0) \forall r \in (0, 1]$.

Since $x \leq x + rq(k)$, it follows from a comparison of (11) and (19) that $A_j = A_0$ if $\omega\left(x + rq(k)\right) < \delta - (1 - \omega)y \forall (x, y) \in [0, k] \times [\underline{y}, \overline{y}]$, which is implied by $\omega\left(k + rq(k)\right) < \delta - (1 - \omega)\overline{y}$, i.e., $k + rq(k) < \delta + \frac{1-\omega}{\omega}(\delta - \overline{y})$. Further, from a comparison of (11) and (19) follows that $B_j > B_0$ if $\delta - (1 - \omega)y - \omega(k + q(k)) > 0 \forall y \in [\underline{y}, \overline{y}]$, i.e., if $k + q(k) < \delta - \frac{1-\omega}{\omega}(y - \delta) \forall y \in [\underline{y}, \overline{y}]$, which is implied by $k + q(k) < \delta - \frac{1-\omega}{\omega}(\overline{y} - \delta)$. This condition for $B_j > B_0$ implies the (weaker) condition for $A_j = A_0$, since $k + rq(k) < k + q(k)$. Hence, $k + q(k) < \delta - \frac{1-\omega}{\omega}(\overline{y} - \delta)$ implies $A_j + B_j > A_0 + B_0$, or, $\pi_{j,k}(r) > \pi_{i,k}(0) \forall r \in (0, 1]$.

A.3 Propositions 3a and 3b

A.3.1 Full risk transfer

For the strategy with the full risk transfer, we have

$$x_{f,k}^*(y, r) = \frac{\delta_{f,k}(r) - (1 - \omega)y - r\omega\left(p(k) + q(k)\right)}{(1 - r)\omega}. \tag{21}$$

From this follows

$$\frac{\partial x_{f,k}^*(y,r)}{\partial r} = \frac{(1-r)\frac{\partial \delta_{f,k}(r)}{\partial r} + \delta_{f,k}(r) - (1-\omega)y - \omega\left(p(k) + q(k)\right)}{(1-r)^2\omega}. \tag{22}$$

To obtain $\partial \delta_{f,k}(r)/\partial r$, we write (5) for the full risk transfer as

$$d = \int_0^\infty \int_{x_{f,k}^*(y,r)}^\infty \phi^Q(x,y)\delta_{i,k}(r)\mathrm{d}x\mathrm{d}y +$$

$$\int_0^\infty \int_0^{x_{f,k}^*(y,r)} \phi^Q(x,y)\left(\omega\left[x + r\left(p(k) + q(k)\right)\right] + (1-\omega)y\right)\mathrm{d}x\mathrm{d}y. \tag{23}$$

Taking the implicit derivative of (23) to r gives

$$0 = \int_0^\infty \int_{x_{f,k}^*(y,r)}^\infty \phi^Q(x,y)\frac{\partial \delta_{f,k}(r)}{\partial r}\mathrm{d}x\mathrm{d}y +$$

$$\omega \int_0^\infty \int_0^{x_{f,k}^*(y,r)} \phi^Q(x,y)\left[\left(p(k) + q(k)\right) - x\right]\mathrm{d}x\mathrm{d}y, \tag{24}$$

where we use the fact that $v_{f,k}(x_{f,k}^*(y,r),y,r) = \delta_{i,k}(r)$. Rewriting Eq. (24) using the notation for $\pi_{i,k}^Q(r)$ in (6) gives

$$\frac{\partial \delta_{f,k}(r)}{\partial r} = -\frac{\omega \pi_{f,k}^Q(r)\left(p(k) + q(k) - \delta_{f,k}(r)\right) + \omega \int_0^\infty \int_0^{x_{f,k}^*(y,r)} \phi^Q(x,y)(\delta_{f,k}(r) - x)\mathrm{d}x\mathrm{d}y}{1 - \pi_{f,k}^Q(r)}. \tag{25}$$

Moreover, rewriting (23) gives

$$(\delta_{f,k}(r) - d) = \omega(1-r)\int_0^\infty \int_0^{x_{f,k}^*(y,r)} \phi^Q(x,y)(\delta_{f,k}(r) - x)\mathrm{d}x\mathrm{d}y$$

$$+ \omega r \int_0^\infty \int_0^{x_{f,k}^*(y,r)} \phi^Q(x,y)(\delta_{f,k}(r) - (p(k) + q(k)))\mathrm{d}x\mathrm{d}y$$

$$+ (1-\omega)\int_0^\infty \int_0^{x_{f,k}^*(y,r)} \phi^Q(x,y)(\delta_{f,k}(r) - y)\mathrm{d}x\mathrm{d}y \tag{26}$$

Using (26) to rewrite Eq. (24) gives

$$\frac{\partial \delta_{f,k}(r)}{\partial r} = -\frac{(\delta_{f,k}(r) - d) - \int_0^\infty \int_0^{x_{f,k}^*(y,r)} \phi^Q(x,y)\left[\delta_{f,k}(r) - \omega\left(p(k) + q(k)\right) - (1-\omega)y\right] \mathrm{d}x\mathrm{d}y}{\left(1 - \pi_{f,k}^Q(r)\right)(1-r)}.$$

(27)

Combining the specification of $\partial \delta_{f,k}(r)/\partial r$ in (27) and that of $x_{f,k}^*(y,r)$ in (22), while using the definition of $\pi_{i,k}^Q(r)$ in (6), gives the final equation for the derivative of the threshold return with respect to changes in the fraction of asset X involved in the credit risk transfer

$$\begin{aligned}
\frac{\partial x_{f,k}^*(y,r)}{\partial r} = &\left[\left(1 - \pi_{f,k}^Q(r)\right)(d - \omega(p(k) + q(k)) - (1-\omega)y)\right.\\
&\left.+ \int_0^\infty \int_0^{x_{f,k}^*(y,r)} \phi^Q(x,y)\left(d - \omega(p(k) + q(k)) - (1-\omega)y\right)\mathrm{d}x\mathrm{d}y\right]\Bigg/\\
&\left[(1-r)^2\omega\left(1 - \pi_{f,k}^Q(r)\right)\right].
\end{aligned}$$

(28)

The denominator of Eq. (28) is positive for any $r \in [0,1]$. Moreover, the numerator will be positive if $p(k) + q(k) > d + \frac{1-\omega}{\omega}(d - \underline{y})$, because it implies $d - \omega(p(k) + q(k)) - (1-\omega)y > 0$ for any $y \in [\underline{y}, \overline{y}]$. Similarly, the numerator will be negative if $p(k) + q(k) < d + \frac{1-\omega}{\omega}(d - \overline{y})$, because it implies $d - \omega(p(k) + q(k)) - (1-\omega)y < 0$ for any $y \in [\underline{y}, \overline{y}]$. This proofs the statements regarding the full risk transfer in Propositions 3a and 3b.

A.3.2 Transfer risk in the junior position

For the strategy transferring the risk in the first-loss position, we have in Eq. (1) that

$$x_{j,k}^*(y,r) = \begin{cases} \dfrac{\delta_{j,k}(r) - (1-\omega)y - r\omega\left(k + q(k)\right)}{(1-r)\omega} & \text{if } y < y_{j,k}^*(r); \\[4mm] \dfrac{\delta_{j,k}(r) - (1-\omega)y - r\omega q(k)}{\omega} & \text{if } y \geq y_{j,k}^*(r), \end{cases}$$

(29)

where $y_{j,k}^*(r)$ is the (unique) solution of $x_{j,k}^*(y_{j,k}^*(r), r) = k$, i.e.,

$$y_{j,k}^*(r) = \frac{\delta_{j,k}(r) - \omega\left(rq(k) + k\right)}{1 - \omega}. \tag{30}$$

From (29), it follows that

$$\frac{\partial x_{j,k}^*(y,r)}{\partial r} = \begin{cases} \dfrac{(1-r)\frac{\partial \delta_{j,k}(r)}{\partial r} + \delta_{j,k}(r) - \omega\left(k + q(k)\right) - (1-\omega)y}{(1-r)^2\omega} & \text{if } y < y_{j,k}^*(r); \\[2ex] \dfrac{\frac{\partial \delta_{j,k}^*(r)}{\partial r} - \omega q(k)}{\omega} & \text{if } y \geq y_{j,k}^*(r). \end{cases} \tag{31}$$

To obtain the level of $\partial \delta_{j,k}(r)/\partial r$ in Eq. (31), we rewrite Eq. (5) as

$$\delta_{j,k}(r) - d = \int_0^{y_{j,k}^*(r)} \int_0^k \phi^Q(x,y) \left[\delta_{j,k}(r) - \omega\left(x + rq(k)\right) - (1-\omega)y\right] dxdy$$
$$+ \int_0^{y_{j,k}^*(r)} \int_k^{x_{j,k}^*(y,r)} \phi^Q(x,y) \left[\delta_{j,k}(r) - \omega\left((1-r)x + r\left(k + q(k)\right)\right) - (1-\omega)y\right] dxdy$$
$$+ \int_{y_{j,k}^*(r)}^{\infty} \int_0^{x_{j,k}^*(y,r)} \phi^Q(x,y) \left[\delta_{j,k}(r) - \omega\left(x + rq(k)\right) - (1-\omega)y\right] dxdy. \tag{32}$$

Taking the implicit derivative of Eq. (32), while using $x_{j,k}^*(y_{j,k}^*(r), r) = k$ and $v_{j,k}(x_{j,k}^*(y,r), y, r) = \delta_{j,k}(r)$, gives

$$\frac{\partial \delta_{j,k}(r)}{\partial r} = \int_0^{y_{j,k}^*(r)} \int_0^k \phi^Q(x,y) \left[\frac{\partial \delta_{j,k}(r)}{\partial r} - \omega q(k)\right] dxdy$$
$$+ \int_0^{y_{j,k}^*(r)} \int_k^{x_{j,k}^*(y,r)} \phi^Q(x,y) \left[\frac{\partial \delta_{j,k}(r)}{\partial r} - \omega q(k) + \omega\left(x - k\right)\right] dxdy$$
$$+ \int_{y_{j,k}^*(r)}^{\infty} \int_0^{x_{j,k}^*(y,r)} \phi^Q(x,y) \left[\frac{\partial \delta_{j,k}(r)}{\partial r} - \omega q(k)\right] dxdy. \tag{33}$$

Rewriting Eq. (33), while using the definition of $\pi_{j,k}^Q(r)$ in Eq. (6), gives

$$\frac{\partial \delta_{j,k}(r)}{\partial r} = -\frac{\omega q(k)\pi_{j,k}^Q(r) - \omega \int_0^{y_{j,k}^*(r)} \int_k^{x_{j,k}^*(y,r)} \phi^Q(x,y)\left(x - k\right) dxdy}{1 - \pi_{j,k}^Q(r)}. \tag{34}$$

Moreover, rewriting Eq. (32) with the definition of $\pi_{j,k}^{Q}(r)$ gives

$$
\left(1 - \pi_{j,k}^{Q}(r)\right)\delta_{j,k}(r) = \left(1 - \pi_{j,k}^{Q}(r)\right)d
$$
$$
+ \int_{0}^{y_{j,k}^{*}(r)}\int_{0}^{k}\phi^{Q}(x,y)\left[d - \omega\left(x + rq(k)\right) - (1-\omega)y\right]\mathrm{d}x\mathrm{d}y
$$
$$
+ \int_{0}^{y_{j,k}^{*}(r)}\int_{k}^{x_{j,k}^{*}(y,r)}\phi^{Q}(x,y)\left[d - \omega\left((1-r)x + r(k + q(k))\right) - (1-\omega)y\right]\mathrm{d}x\mathrm{d}y
$$
$$
+ \int_{y_{j,k}^{*}(r)}^{\infty}\int_{0}^{x_{j,k}^{*}(y,r)}\phi^{Q}(x,y)\left[d - \omega\left(x + rq(k)\right) - (1-\omega)y\right]\mathrm{d}x\mathrm{d}y. \quad (35)
$$

Then, using Eqs. (34) and (35) in Eq. (31) gives, for $y < y_{j,k}^{*}(r)$,

$$
\frac{\partial x_{j,k}^{*}(y,r)}{\partial r} = \left[\left(1 - \pi_{j,k}^{Q}(r)\right)\left[d - (1-\omega)y - \omega\left(k + q(k)\right)\right]\right.
$$
$$
+ \int_{0}^{y_{j,k}^{*}(r)}\int_{0}^{k}\phi^{Q}(x,y)\left[d - \omega\left(x + q(k)\right) - (1-\omega)y\right]\mathrm{d}x\mathrm{d}y
$$
$$
+ \int_{0}^{y_{j,k}^{*}(r)}\int_{k}^{x_{j,k}^{*}(y,r)}\phi^{Q}(x,y)\left[d - \omega\left(k + q(k)\right) - (1-\omega)y\right]\mathrm{d}x\mathrm{d}y
$$
$$
+ \left.\int_{y_{j,k}^{*}(r)}^{\infty}\int_{0}^{x_{j,k}^{*}(y,r)}\phi^{Q}(x,y)\left[d - \omega\left(x + q(k)\right) - (1-\omega)y\right]\mathrm{d}x\mathrm{d}y\right]\Bigg/
$$
$$
\left[\left(1 - \pi_{j,k}^{Q}(r)\right)\omega(1-r)^{2}\right]. \quad (36)
$$

The denominator of Eq. (36) is positive for any $r \in [0,1]$. Moreover, the numerator will be positive if $k + q(k) < d - \frac{1-\omega}{\omega}(\overline{y} - d)$, because it implies $d - (1-\omega)y - \omega(k + q(k)) > 0$ for any $y \in [\underline{y}, \overline{y}]$, and it implies $d - (1-\omega)y - \omega(x + q(k)) > 0$ for any $(x,y) \in [0,k] \times [\underline{y}, \overline{y}]$. Finally, $y < y_{j,k}^{*}(r)$ is implied by $k + q(k) < d - \frac{1-\omega}{\omega}(\overline{y} - d)$, because, from Eq. (30), $y < y_{j,k}^{*}(r)$ requires the weaker condition $k + rq(k) < \delta_{j,k}(r) - \frac{1-\omega}{\omega}(\overline{y} - \delta_{j,k}(r))$ (this condition is weaker because $\delta_{j,k}(r) \geq d$ and $0 \leq r \leq 1$). This proofs the statement regarding the increase in insolvency risk in Proposition 3a.

Moreover, rewriting Eq. (33) gives,

$$\frac{\partial \delta_{j,k}(y,r)}{\partial r} = \left[\int_0^{y_{j,k}^*(r)} \int_k^{x_{j,k}^*(y,r)} \phi^Q(x,y)\left[x - \omega\left(k + q(k)\right)\right] \mathrm{d}x\mathrm{d}y \right.$$

$$- \omega q(k) \int_0^{y_{j,k}^*(r)} \int_0^k \phi^Q(x,y)\mathrm{d}x\mathrm{d}y$$

$$\left. - \omega q(k) \int_{y_{j,k}^*(r)}^{\infty} \int_0^{x_{j,k}^*(y,r)} \phi^Q(x,y)\mathrm{d}x\mathrm{d}y \right] \Big/$$

$$\left[1 - \pi_{j,k}^Q(r) \right]. \tag{37}$$

The denominator of Eq. (37) is positive for any $r \in [0,1]$. Moreover, it can easily be verified in Eq. (29) that, if $k+q(k) > \delta_{j,k}(r) + \frac{1-\omega}{\omega}(\delta_{j,k}(r) - \underline{y})$, $x_{j,k}^*(y,r) < k+q(k)$ for any $y \in [\underline{y}, \overline{y}]$. This implies that both parts of the numerator in Eq. (37) are negative. As a consequence, $\partial \delta_{j,k}(r)/\partial r < 0$ if $k+q(k) > \delta_{j,k}(r) + \frac{1-\omega}{\omega}(\delta_{j,k}(r) - \underline{y})$. Finally, with $\partial \delta_{j,k}(r)/\partial r < 0$, it is easily verified in Eq. (31) that $\partial x_{j,k}^*(y,r)/\partial r < 0$ for any $y \in [\underline{y}, \overline{y}]$. This proof the statement regarding reduction in the probability of insolvency with the transfer of the risk in the senior position in Proposition 3b.

A.3.3 Transfer risk in the senior position

For the strategy transferring the risk in the senior position, we obtain from Eq. (1) that

$$x_{s,k}^*(y,r) = \begin{cases} \dfrac{\delta_{s,k}(r) - (1-\omega)y - r\omega p(k)}{\omega} & \text{if } y < y_{s,k}^*(r); \\[3mm] \dfrac{\delta_{s,k}(r) - (1-\omega)y - r\omega\left(k - p(k)\right)}{\omega(1-r)} & \text{if } y \geq y_{s,k}^*(r), \end{cases} \tag{38}$$

where $y_{s,k}^*(r)$ is the (unique) solution of $x_{s,k}^*(y_{s,k}^*(r),r) = k$, i.e.,

$$y_{s,k}^*(r) = \frac{\delta_{s,k}(r) - \omega\left(rp(k) + (1-r)k\right)}{1-\omega}. \tag{39}$$

From (38), it follows that

$$\frac{\partial x^*_{s,k}(y,r)}{\partial r} = \begin{cases} \dfrac{\frac{\partial \delta_{s,k}(r)}{\partial r} - \omega\left(k - p(k)\right)}{\omega} & \text{if } y < y^*_{s,k}(r); \\[2em] \dfrac{(1-r)\frac{\partial \delta_{s,k}(r)}{\partial r} + \delta_{s,k}(r) - \omega p(k) - (1-\omega)y}{(1-r)^2 \omega} & \text{if } y \geq y^*_{s,k}(r). \end{cases} \tag{40}$$

To obtain the level of $\partial \delta_{s,k}(r)/\partial r$ in Eq. (40), we rewrite Eq. (5) as

$$\begin{aligned} \delta_{s,k}(r) - d &= \int_0^{y^*_{s,k}(r)} \int_0^k \phi^Q(x,y) \left[\delta_{s,k}(r) - \omega\left((1-r)x + rp(k)\right) - (1-\omega)y\right] \mathrm{d}x \mathrm{d}y \\ &+ \int_0^{y^*_{s,k}(r)} \int_k^{x^*_{s,k}(y,r)} \phi^Q(x,y) \left[\delta_{s,k}(r) - \omega\left(x - r\left(k - p(k)\right)\right) - (1-\omega)y\right] \mathrm{d}x \mathrm{d}y \\ &+ \int_{y^*_{s,k}(r)}^{\infty} \int_0^{x^*_{s,k}(y,r)} \phi^Q(x,y) \left[\delta_{s,k}(r) - \omega\left((1-r)x + rp(k)\right) - (1-\omega)y\right] \mathrm{d}x \mathrm{d}y. \end{aligned} \tag{41}$$

Taking the implicit derivative of Eq. (41), while using $x^*_{s,k}(y^*_{s,k}(r), r) = k$ and $v_{s,k}(x^*_{s,k}(y,r), y, r) = \delta_{s,k}(r)$, gives

$$\begin{aligned} \frac{\partial \delta_{s,k}(r)}{\partial r} &= \int_0^{y^*_{s,k}(r)} \int_0^k \phi^Q(x,y) \left[\frac{\partial \delta_{s,k}(r)}{\partial r} - \omega\left(p(k) - x\right)\right] \mathrm{d}x \mathrm{d}y \\ &+ \int_0^{y^*_{s,k}(r)} \int_k^{x^*_{s,k}(y,r)} \phi^Q(x,y) \left[\frac{\partial \delta_{s,k}(r)}{\partial r} - \omega\left(p(k) - k\right)\right] \mathrm{d}x \mathrm{d}y \\ &+ \int_{y^*_{s,k}(r)}^{\infty} \int_0^{x^*_{s,k}(y,r)} \phi^Q(x,y) \left[\frac{\partial \delta_{s,k}(r)}{\partial r} - \omega\left(p(k) - x\right)\right] \mathrm{d}x \mathrm{d}y. \end{aligned} \tag{42}$$

Rewriting Eq. (42), while using the definition of $\pi^Q_{s,k}(r)$ in Eq. (6), gives

$$\begin{aligned} \frac{\partial \delta_{s,k}(r)}{\partial r} &= \Bigg[\omega \pi^Q_{s,k}(r)\left(k - p(k)\right) - \omega \int_0^{y^*_{s,k}(r)} \int_0^k \phi^Q(x,y)\left(k - x\right) \mathrm{d}x \mathrm{d}y \\ &- \omega \int_{y^*_{s,k}(r)}^{\infty} \int_0^{x^*_{s,k}(y,r)} \phi^Q(x,y)\left(k - x\right) \mathrm{d}x \mathrm{d}y \Bigg] \Bigg/ \\ &\left[1 - \pi^Q_{s,k}(r)\right]. \end{aligned} \tag{43}$$

Then, using Eqs. (41) and (42) in Eq. (40) gives, for $y \geq y_{s,k}^*(r)$,

$$
\frac{\partial x_{s,k}^*(r)}{\partial r} = \left[\left(1 - \pi_{s,k}^Q(r)\right) [d - \omega p(k) - (1 - \omega)y] \right.
$$

$$
+ \int_0^{y_{s,k}^*(r)} \int_0^k \phi^Q(x,y) [d - \omega p(k) - (1 - \omega)y] \, dx dy
$$

$$
+ \int_0^{y_{s,k}^*(r)} \int_k^{x_{s,k}^*(r)} \phi^Q(x,y) [d - \omega (x + k - p(k)) - (1 - \omega)y] \, dx dy
$$

$$
+ \left. \int_{y_{s,k}^*(r)}^{\infty} \int_0^{x_{s,k}^*(y,r)} \phi^Q(x,y) [d - \omega p(k) - (1 - \omega)y] \, dx dy \right] \Big/
$$

$$
\left[\omega \left(1 - \pi_{s,k}^Q(r)\right) (1 - r)^2 \right]. \tag{44}
$$

The numerator of Eq. (44) is positive for any $r \in [0, 1]$. It can easily be verified that each of the parts in the numerator is negative if

$$
p(k) > d + \frac{1 - \omega}{\omega}(d - \underline{y}), \tag{45}
$$

since this implies $d < \omega p(k) - (1 - \omega)y$ for any $y \in [\underline{y}, \overline{y}]$ (note that $x + k - p(k) > p(k)$ for any $x \geq k$). However, Eq. (44) only holds true for $y \geq y_{s,k}^*(r)$, which is implied by

$$
(1 - r)k + rp(k) > \delta_{s,k}(r) + \frac{1 - \omega}{\omega}(\delta_{s,k}(r) - \underline{y}). \tag{46}
$$

The conditions in (45) and (46) jointly hold true if $p(k) > \delta_{s,k}(r) + \frac{1-\omega}{\omega}(\delta_{s,k}(r) - \underline{y})$, since $k > p(k)$ and $d \leq \delta_{s,k}(r)$. This proofs the statement regarding the reduction in the probability of insolvency as a consequence of the transfer of the risk in the senior position in Proposition 3b.

Moreover, using Eq. (43) in Eq. (40) gives, for $y < y^*_{s,k}(r)$,

$$\frac{\partial x^*_{s,k}(r)}{\partial r} = \left[(k - p(k)) - \int_0^{y^*_{s,k}(r)} \int_0^k \phi^Q(x,y)\,(k-x)\,\mathrm{d}x\mathrm{d}y \right.$$
$$\left. - \int_{y^*_{s,k}(r)}^{\infty} \int_0^{x^*_{s,k}(y,r)} \phi^Q(x,y)\,(k-x)\,\mathrm{d}x\mathrm{d}y \right] \Bigg/$$
$$\left[1 - \pi^Q_{s,k}(r) \right]. \tag{47}$$

With the definition of the pricing of the risk premium by depositors in (8), we have from (44) that, for $y \geq y^*_{s,k}(r)$,

$$\frac{\partial x^*_{s,k}(r)}{\partial r} = \frac{(k - p(k)) - \left(k - p^Q(k)\right) + \int_{y^*_{s,k}(r)}^{\infty} \int_{x^*_{s,k}(y,r)}^{k} \phi^Q(x,y)\,(k-x)\,\mathrm{d}x\mathrm{d}y}{1 - \pi^Q_{s,k}(r)}. \tag{48}$$

The denominator of this derivative is positive, as well as the double integral in the numerator. As a consequence, Eq. (48) must be positive if $(k - p(k)) \geq \left(k - p^Q(k)\right)$. Hence, $\partial x^*_{s,k}(r)/\partial r > 0$ for $y \geq y^*_{s,k}(r)$, where the latter is implied by $\delta_{s,k}(r) + \frac{1-\omega}{\omega}(\overline{y} - \delta_{s,k}(r)) > (1 - r)k + p(k)$. Since $k \geq (1 - r)k + p(k)$ for any $r \in [0, 1]$, this proofs the statement regarding the increase in insolvency risk of the transfer of the risk in the senior position in Proposition 3a.

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