Tokenomics: Dynamic Adoption and Valuation[*]

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Abstract

We provide a dynamic asset-pricing model of (crypto-)tokens on (blockchain-based) platforms, and highlight their roles on endogenous user adoption. Tokens intermediate transactions on decentralized networks, and their trading creates an inter-temporal complementarity among users, generating a feedback loop between token valuation and platform adoption. Consequently, tokens capitalize future platform growth, accelerate adoption, and reduce user-base volatility. Equilibrium token price increases non-linearly in platform productivity, user heterogeneity, and endogenous network size. The model also produces explosive growth of user base after an initial period of dormant adoption, accompanied by a run-up of token price volatility. We further discuss how our framework can be used to discuss cryptocurrency supply, token competition, and pricing assets under network externality.

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1   Introduction

Blockchain-based cryptocurrencies and tokens have taken the world by storm. According to CoinMarketCap.com, the entire cryptocurrency market capitalization has also grown from around US$20 billion to around US$600 billion over last year, with active trading and uses; virtually unknown a year ago, initial coin offerings (ICOs) have also attracted more attention than the conventional IPOs, raising 3.5 billion in more than 200 deals in 2017 alone, according to CoinSchedule. In order to draw a line between reckless speculation and financial innovation, and understand how tokens should be regulated, it is important to first understand how cryptocurrencies or tokens (henceforth generically referred to as "token") derive value and the roles they play in the development and adoption of the virtual economy.

To this end, we develop the first dynamic model of a virtual economy with endogenous user adoption and native tokens that facilitate transactions and business operations. We anchor token valuation on the fundamental productivity of the (blockchain-based) platform, and demonstrate how tokens derive value as an exchangeable asset with limited supply that users hold to derive utility available solely on the platform. We then pin down the dynamics of token price as the solution to a second-order ODE. We are also the first to highlight two important roles of tokens in business development (fundraising included). First, because the expected price appreciation makes it an attractive to hold tokens, early users can capitalize future growth of the platform, leading to accelerated adoption. Second, the expected price appreciation diminishes as the platform technology matures and more users adopt, which moderates the volatility of user base due to productivity shocks through endogenous token price changes.

Specifically, we consider a continuous-time economy with a continuum of agents who differ in their needs to conduct transactions on the blockchain. We broadly interpret transaction as including not only typical money transfer (e.g., on the Bitcoin blockchain) but also signing smart contracts (e.g., on the Ethereum blockchain). Accordingly, we model agents' gain from blockchain transaction as a flow utility that depends on agent-specific transaction needs, the size of blockchain community, and the current productivity of blockchain platform ("productivity" broadly interpreted) that loads on exogenous shocks. Very importantly, the larger the community is, the more surplus can be realized through trades among agents on the blockchain (i.e., higher flow utility of tokens). Exogenous shocks to productivity can be

broadly interpreted as shocks to the general usefulness of the platform, technological changes, or regulatory shocks such as the ban on cryptocurrency trading by several governments.

In our model, agents make a two-step decision: (1) whether to incur a participation cost to meet potential trade counterparties (i.e., to join the community); (2) how many tokens to hold, which depends on both blockchain trade surplus ("transaction motive") and the expected future token price ("investment motive"). A key innovation of our model is that not only does one user's adoption exhibit externality on others, but the investment motive also introduces an inter-temporal complementarity in user base. We model productivity as a geometric Brownian motion, thus exogenous shocks have permanent impact on the level of productivity. A positive shock to platform productivity directly increases the user base today due to the higher flow utility of holding tokens. Furthermore, agents now expect more users to join the community in the future, which leads to a stronger future demand for tokens and thus token price appreciation. This investment motive creates a stronger demand for tokens today and greater adoption. Our model highlights this indirect feedback effect that arises because token price reflects agents' expectation of future popularity of the platform.

We characterize the non-degenerate Markov equilibrium with platform productivity as the state variable, and derives the token valuation as the solution to a second-order ordinary differential equation. Akin to many equilibrium models that feature interaction between financial markets and the real economy, the financial side of our model is the endogenous price of tokens, whereas the real side is the size of user base that determines the benefits (the utility flow) of agents who trade on the blockchain. Token price affects user adoption through the expected price appreciation, while user base affects token price through its impact on flow utility and agents' token demand. This mutual feedback naturally triggers a question: how a platform with embedded tokens differs from one without?

We therefore compare the endogenous S-curve of adoption of a platform with embedded tokens to one without tokens (where agents use dollars or other media of exchange). Both platforms have exactly the same process of productivity growth. We find that without tokens, user adoption is below the first-best level which entails full adoption as long as the platform productivity is above an initial threshold. Tokens can improve welfare because the user base grows faster and reaches full adoption faster. This result derives from that token price reflects agents' expectation of future popularity of the platform, thus the investment

motive induces more agents to join the platform. That said, a caveat against tokens is that non-fundamental driven expected price appreciation can lead to over-adoption in the early stage of the platform, not to mention that tokens can accelerate the demise of a bad platform whose productivity drifts downward (as agents forecast a smaller user base in the future, they shun away from holding tokens with expected price depreciation). In sum, embedding tokens on a platform front-loads the prospect of platform, so depending on the expectation of fundamentals (i.e., productivity in our model), it can accelerate adoption or precipitate abandonment.

Furthermore, we show that introducing tokens can reduce user base volatility, making it less sensitive to productivity shocks. The key driver is again the agents' investment motive—their decision to participate depends on their expectation of future token price appreciation. Consider a negative shock that reduces the flow utility, and thus user adoption. This direct negative effect is mitigated by an indirect effect through token price: A lower adoption now means more agents can be brought onto the platform in future. Agents' expected stronger token price appreciation therefore induces them to adopt and hold tokens. Similarly, a positive productivity shock increases adoption by increasing the flow utility. However, as the pool of potential newcomers shrinks, the expected token price appreciation declines, discouraging agents from joining the platform and holding tokens.

Having clarified the roles of tokens analytically, we fully solve the Markovian equilibrium, and calibrate our model to existing data. Our quantitative exercise shows that the mechanism in our model indeed induces a form of inter-temporal complementarity in user base, which leads to tokens accelerating adoption and reducing user base volatility. The exercise also helps understand several empirical patterns in token price: the endogenous user adoption can generate run-ups in token valuation and volatility.

Finally, we extend our model along several dimensions. Specifically, we demonstrate how endogenizing productivity growth can further strengthen the economic channels we highlight, how time-varying systematic risk of tokens can produce a sharp rise and fall of token price under rational expectations, and how our model can be used to analyze cryptocurrency competition and the design of state-contingent token supply.

Overall, our model sheds light on the pricing of cryptocurrencies and tokens in peer-to-

peer networks that include but is not restricted to permissioned and permissionless blockchains.[1] Many digital currencies and tokens, payment-focused or not, have been introduced and associated with platforms and virtual economies: Linden dollar for the game Second Life, WoW Gold for the game World of Warcraft, Facebook Credits, Q-coins for Tencent QQ, Amazon coins, to name a few.[2] Our model offers a pricing formula and reveals how introducing native tokens benefits users and accelerates adoption. While the Blockchain technology certainly gives platforms unprecedented flexibility and commitment power in introducing native tokens and designing their attributes, our model potentially applies to other trusted platforms or virtual systems such as email protocols and online social network, and adds new insights to asset pricing and macro models with network externality.

Literature Review. Our paper foremost contributes to the emerging literature on blockchains and cryptocurrencies. Among early studies, Cong and He (2018) examine informational issues in generating decentralized consensus, with implications on industrial organization; Biais, Bisiere, Bouvard, and Casamatta (2017) and Saleh (2017) analyze the mining or minting games through Proof-of-Work and Proof-of-Stake; Easley, O'Hara, and Basu (2017), Huberman, Leshno, and Moallemi (2017), and Cong, He, and Li (2018) study the miners' market in terms of compensation, organization, and micro-structure; Harvey (2016) briefly surveys the mechanics and applications of crypto-finance; Yermack (2017) and Cao, Cong, and Yang (2018) evaluates the potential impacts of the technology on corporate governance and financial reporting.

To be clear, the concepts of digital currency and distributed ledger have been separately developed earlier and Nakamoto's innovation lies in combining them to enable large-scale application (Narayanan and Clark (2017)): embedding a native currency into a blockchain system helps incentivize record-keepers (e.g., miners in protocols using proof-of-work) of decentralized consensus, which in turn prevents double-spending (Nakamoto (2008)). Our paper emphasizes the impact of introducing native tokens on the incentives of users to adopt.

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[1]To be precise, the networks in question should be viewed as complete networks with economy of scale, which is different from incomplete networks that many recent studies focus on (e.g., Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012)).

[2]Even before the heated debate on cryptocurrencies, economists, and commentators were already raising questions such as "Could a gigantic nonsovereign like Facebook someday launch a real currency to compete with the dollar, euro, yen and the like?" (Yglesias (2012)). Gans and Halaburda (2015) provides an insightful introduction on how payment systems and platforms are related.

Specifically, we focus on the valuation of cryptocurrencies and tokens under endogenous user adoption in a *dynamic* framework that highlights inter-temporal feedback effects. In contrast, other models in the literature are static. For example, Gans and Halaburda (2015) is among the earliest studies on platform-specific virtual currencies and users' network effects. Ciaian, Rajcaniova, and Kancs (2016) test quantity theory of money using Bitcoin data assuming exogenous user demand for Bitcoins. Fernández-Villaverde and Sanches (2016) and Gandal and Halaburda (2014) consider the competition among alternative cryptocurrencies. Pagnotta and Buraschi (2018) studies the pricing of Bitcoins under exogenous network and adoption. Closer to our paper is Athey, Parashkevov, Sarukkai, and Xia (2016) that emphasizes agents' dynamic learning on a binary technology quality and decision to use bitcoins for money transfer, but does not model blockchain productivity and user-base externality.

Several contemporaneous models analyze cryptocurrencies in the context of initial coin offerings (ICOs). Li and Mann (2018) demonstrate that staged coin offerings mitigate coordination issues; Sockin and Xiong (2018) study how households first purchase an indivisible cryptocurrency which serves as membership certificate that enables them to match and trade in a second period; Catalini and Gans (2018) study entrepreneurs' discretionary pricing to ensure the value of crypto-tokens issued to fund start-ups; Chod and Lyandres (2018) discuss the risk diversification benefit of ICOs; most recently, Bakos and Halaburda (2018) compare the adoption acceleration benefit of tokens we highlight with traditional user subsidy through VC capital.

We differ in our emphasis of the role of crypto-tokens as media of exchange in decentralized virtual economy, and their effects on endogenous user adoption.[3] We differ also in allowing agent heterogeneity and divisible holdings of tokens, and uniquely pining down the non-degenerate dynamics of token valuation and user adoption.[4] Unlike many studies focusing on permissionless blockchains maintained by decentralized miners, our framework does not rely on the specific mechanism for consensus generation, and consequently applies equally to permissioned blockchains or platforms owned by trusted third parties with net-

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[3]Also related are discussions on the design of cryptocurrencies/tokens and platforms, such as Gans and Halaburda (2015), Halaburda and Sarvary (2016), Chiu and Wong (2015), and Chiu and Koeppl (2017). Our framework can directly tie the protocol-based design on supplies to valuation and adoption, enabling us to evaluate various design objectives.

[4]Bakos and Halaburda (2018) also discuss user adoption, but in a setting without technological uncertainty or user heterogeneity. Like Sockin and Xiong (2018), the model features a two-period set-up in which tokens serve as platform membership.

work effect of user adoption.

We organize the remainder of the article as follows. Section 2 sets up the model and characterizes the dynamic equilibrium. Section 3 performs quantitative analysis and discusses model implications. Section 4 contains extensions and further discussions. Section 5 provides further institutional background on crypto-currency and crypto-tokens. Section 6 concludes.

2   A Dynamic Model of Adoption and Valuation

Many features in our model are motivated by stylized facts about existing platforms, blockchains, and crypto-tokens. We refer to interested readers to Section 5 for a brief introduction and the institutional background.

2.1   Setup

Consider a continuous-time economy with a unit measure of agents. Generic goods serve as numeraire ("dollar"). We work under the risk-neutral measure, and discuss agents' risk aversion and the physical measure for calibration and risk premium in Section 3.1.

Our model of cryptocurrency or crypto-tokens (generically referred to as "token") starts with an exogenous process of the productivity of blockchain platform for peer-to-peer transactions, a geometric Brownian motion

$$\frac{dA_t}{A_t} = \mu^A dt + \sigma^A dZ_t^A. \tag{1}$$

A_t represents the quality or usefulness of blockchain platform. We focus on the case of a promising yet risky platform, i.e., $\mu^A > 0$ and $\sigma^A > 0$. While pure technological shocks in cryptography or consensus algorithms obviously affect A_t, systematic shifts in user preferences, regulatory changes, creative platform use, and complementary innovations can all play a role.[5] We later discuss in Section 4.1 how A_t can also depend on the user base.

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[5]For example, the effectiveness of the blockchain technology – provision of decentralized consensus – is

Agent $i \in [0,1]$ obtains a flow of utility or "trade surplus" by holding certain medium of exchange—typically native notkens—for peer-to-peer transactions on this platform. Let $x_{i,t}$ denote the value of holdings in terms of the numeraire, the trade surplus in reduced-form is,[6]

$$x_{i,t}^{1-\alpha} \left(N_t A_t e^{u_i} \right)^\alpha dt. \tag{2}$$

The medium of exchange can be the numeraire itself (i.e., dollar) or the embedded native token. In the latter case,

$$x_{i,t} = P_t k_{i,t}, \tag{3}$$

where P_t is the unit price of token in terms of the numeraire and $k_{i,t}$ is the units of token. The trade surplus depends on the common productivity A_t, the idiosyncratic productivity u_i, and noticeably the user base N_t—the total measure of agents that decide to join the blockchain network (i.e., $x_{i,t} > 0$). Introducing N_t into the trade surplus captures the network externality among users, consistent with the evidence on social interaction and market participation in Hong, Kubik, and Stein (2004). For example, it reflects the ease to find trading or contracting counterparties in a large community. Alternatively, we could introduce total volume, which would not affect the economic channels we focus on—endogenous network effect, both contemporaneous and inter-temporal.

User type u_i captures the heterogeneity in agents' needs to transact on the blockchain. The interpretation depends how we understand the blockchain trade surplus. For example, if the trade surplus comes from a payment blockchain (e.g., Bitcoin and Ripple), a high value of u_i reflects agent i's urge to conduct a transaction on the platform, be it an international remittance or a purchase of illegal drugs; if the trade surplus arises from smart contracting

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affected by the protocol design and participants' behavior. Biais, Bisiere, Bouvard, and Casamatta (2017) study the stability of consensus, while Cong and He (2018) relate the quality of blockchain platform to miner/keeper activities. On the regulatory side, the Bitcoin platform became popular as a venue to transfer capital overseas in Greece during the country's financial distress in 2015. Interests rose quickly amidst fears of capital controls (Lee and Martin (2018)). On the other extreme of regulatory impact, in 2017 and 2018, China and Korea have introduced various measures to limit cryptocurrency trading and usages, which are widely considered as a negative shock to the Bitcoin platform.

[6]The multiplicative specification is for analytical convenience. Note that trivially, zero adoption is always an equilibrium. Our focus is therefore on the non-degenerate equilibrium with positive adoption. Alternatively, we can rule out the degenerate equilibrium using a different functional form of trade surplus: $\left(P_t k_{i,t} \right)^{1-\alpha} \left(A_t e^{u_i} \right)^\alpha dt + \left(P_t k_{i,t} \right)^{1-\alpha} N_t^\alpha dt$, i.e., with N_t entering the surplus in an additive form. Under this specification, there are always participating agents whose u_i is high enough. We can also show that our results are qualitatively robust to alternative specifications that feature decreasing total return i.e., $\left(x_{i,t} \right)^{1-\alpha-\gamma} \left(N_t A_t e^{u_i} \right)^\alpha$ with $\gamma > 0$. Results are available upon request.

for business operations that are only possible on the blockchain (e.g., Ethereum), u_i then reflects the productivity of such entrepreneurial projects; if the trade surplus derives from decentralized computation (e.g., Dfinity) or data storage (e.g., Filecoin), u_i reflects the need for secure and fast access to computing power and data. Such heterogeneity renders the determination of user base non-trivial. Let $G(u)$ denote the cumulative probability distribution of u_i in the cross section (and $g(u)$ the density).[7] For simplicity, we require $g(u)$ to be continuously differentiable over its support $[\underline{U}, \overline{U}]$, where $\underline{U} < 0$ and $\overline{U} > 0$ could be negative infinity and infinity. In the case of Normal distribution, $g(u) = \sqrt{\frac{1}{2\pi\theta^2}} e^{-\frac{u^2}{2\theta^2}}$.

To join the platform and obtain this trade surplus, an agent incurs a flow cost ϕdt, which can be cognitive in nature. For example, transacting on the platform takes effort and attention. That said, agents can easily abstain from participating in the ecosystem any time and save the cost, reflecting the reality that joining or leaving an online platform is rather frictionless. Agents with very high u_i finds it profitable to join the platform, while agents with very low u_i does not. For these reasons and modeling convenience, we assume $\phi > 0$, although one could alternatively allow negative idiosyncratic preference to endogenize the adoption threshold.

Finally, we assume that agents do not face financial constraints and do not default, so they may borrow or lend, as much as they like, at the risk-free rate rdt. Abstracting away financial frictions distinguishes and highlights our theoretical contributions.

2.2 Token-based Equilibrium

In what follows, we focus on the joint dynamics of token valuation and user adoption on platforms requiring native tokens as the medium of exchange. Later to highlight the role of tokens on user adoption, we compare the equilibrium to an equilibrium without native tokens.

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[7]We note that time variation in agents' type can be accommodated. For example, we can set let $u_{i,t}$ follow a Ornstein-Uhlenbeck process

$$du_{i,t} = -\mu^U u_{i,t} dt + \sigma^U dZ_{i,t}^U,$$

where $Z_{i,t}^U$ is an idiosyncratic, standard Brownian motion, and μ^U and σ^U are common among agents. By setting the initial cross-sectional distribution to be the stationary distribution of the corresponding Fokker–Planck equation $g(u) = \sqrt{\frac{1}{2\pi\theta^2}} e^{-\frac{u^2}{2\theta^2}}$, where $\theta = \frac{\sigma^U}{\sqrt{2\mu^U}}$ captures the effective heterogeneity of agents (the ratio of user heterogeneity scaled by the rate of mean reversion), we obtain exactly the same aggregate dynamics (e.g., token price and adoption) as the one with time-invariant type, provided $dZ_{i,t}^U$ is perfectly insured among agents.

We also remind the readers that the flow utility can only be realized on the platform. This is consistent with the fact that most blockchain platforms serve unique purposes: Bitcoin facilitates anonymous and international transactions; Filecoin allows P2P storage-sharing; Ethereum enables smart contracting. We discuss platform competition in Section 4.3.

Agents' objective function. Let $y_{i,t}$ denote agent i's *cumulative* profits from blockchain activities. Agent i then maximizes life-time utility under the risk-neutral measure,

$$\mathbb{E}\left[\int_{t=0}^{\infty} e^{-rt} dy_{i,t}\right]. \tag{4}$$

In addition to the typical "convenience yield" associated with currencies and commodities, providing services on blockchain platforms and conducting business through smart contracts typically requires holding or locking up certain amounts of native tokens. Technical limitations of decentralized ledgers also necessitate token holding. We provide institutional details and examples in Section 5.

In line with these considerations, agents must hold tokens for at least an instant to derive utility flow. This holding period is important because it exposes users to token price fluctuation over dt, so that users of the platform care not only the surplus from conducting trade with peer users but also the future token price, which in turn depends on further user base.

In equilibrium, agents take as given the equilibrium price dynamics, which we conjecture to be a diffusion process,

$$dP_t = P_t \mu_t^P dt + P_t \sigma_t^P dZ_t^A. \tag{5}$$

We confirm the conjecture once we clear the token market and define the Markov equilibrium. Throughout the paper, we use capital letters for aggregate and price variables that individuals take as given, and lower-case letters for individual-level variables.

Individual optimization. Without financial constraint, agents can borrow and lend at the risk free rate. Their maximization problems reduces to maximizing profit flow at each t, i.e.,

$$dy_{i,t} = \max \left\{ 0, \max_{k_{i,t}>0} \left[\underbrace{(P_t k_{i,t})^{1-\alpha} (N_t A_t e^{u_i})^{\alpha} dt}_{\text{blockchain trade surplus}} + \underbrace{k_{i,t} \mathbb{E}_t [dP_t]}_{\text{price change}} - \underbrace{\phi dt}_{\text{participation cost}} - \underbrace{P_t k_{i,t} r dt}_{\text{financing cost}} \right] \right\},$$

$$(6)$$

where the outer "max" operator reflects agent i's freedom to leave the platform and obtain zero profit, and the inner "max" operator reflects agent i's choice of optimal operation scale. We apply the conditional expectation operator, $\mathbb{E}_t[\cdot]$ to token price change, by referring to the the risk-neutrality in the life-time utility (Equation (4)), and using the law of iterated expectation. Under the conjecture of price dynamics,

$$\mathbb{E}_t[dP_t] = P_t \mu_t^P dt. \tag{7}$$

Conditional on joining the platform, agent i chooses the optimal token holdings, $k_{i,t}^*$, by the first order condition,

$$(1-\alpha) \left(\frac{N_t A_t e^{u_i}}{P_t k_{i,t}^*} \right)^{\alpha} + \mu_t^P = r, \tag{8}$$

that is the sum of *marginal* production on the platform and expected token price change equals r. Rearranging the equation, we have the following lemma for optimal token holdings.

Lemma 1 (Optimal Token Holdings). *At time t, agent i holds $k_{i,t}^*$ units of tokens if she participates, where*

$$k_{i,t}^* = \frac{N_t A_t e^{u_i}}{P_t} \left(\frac{1-\alpha}{r - \mu_t^P} \right)^{\frac{1}{\alpha}}. \tag{9}$$

It has the following properties: (1) $k_{i,t}^*$ increases in N_t. (2) $k_{i,t}^*$ decreases in token price P_t. (3) $k_{i,t}^*$ increases in A_t and u_i. (4) $k_{i,t}^*$ increases in the expected token price change, μ_t^P.

Agents hold more token when the common productivity or their agent-specific transaction need is high, and also when the community is larger because it is easier to conduct trades in the ecosystem. Equation (9) also reflects an investment motive to hold tokens, that is $k_{i,t}$ increases in the expected token price appreciation, μ_t^P.

Substituting $k_{i,t}^*$ into the profit function, we solve the maximized profits (conditional on

agent i joining the platform),

$$N_t A_t e^{u_i} \alpha \left(\frac{1 - \alpha}{r - \mu_t^P} \right)^{\frac{1-\alpha}{\alpha}} - \phi. \tag{10}$$

Apparently, agent i chooses to hold tokens (i.e., join the platform) if the expression is non-negative. Taking logarithm of it, we solve $\underline{u}_t$, the user participation threshold.

Lemma 2 (**Adoption Threshold**). *Agent i joins the token-based platform if $u_i \geq \underline{u}_t$, where*

$$\underline{u}_t \triangleq \underline{u}\left(N_t; A_t, \mu_t^P\right) = -\ln\left(N_t\right) + \ln\left(\frac{\phi}{A_t \alpha}\right) - \left(\frac{1 - \alpha}{\alpha}\right) \ln\left(\frac{1 - \alpha}{r - \mu_t^P}\right). \tag{11}$$

$\underline{u}_t$ *decreases in A_t, N_t, and μ_t^P.*

$\underline{u}_t$ is decreasing in A_t because a more productive platform attracts more users. $\underline{u}_t$ also decreases in N_t because individuals' profits from platform activities increases in the current size of user base. We aggregate individuals' adoption decision to obtain the size of user base:

$$N_t = 1 - G\left(\underline{u}_t\right). \tag{12}$$

Interestingly, adoption threshold decreases if agents expect token price to increase more (i.e., higher μ_t^P), as the next proposition reveals.

Proposition 1 (**Endogenous User Base**). $\exists \underline{A}$ *such that given μ_t^P and $A_t > \underline{A}$, there exists a non-degenerate solution to Equations (11) and (12). If $G(\cdot)$ and $g(\cdot)$ have increasing hazard rate, the non-degenerate solution is unique.*[8] *Importantly, N_t is increasing in μ_t^P.*

Given the common productivity A_t and agents' expected price appreciation μ_t^P, we note that zero adoption is always a solution: the trade surplus is zero when $N_t = 0$, and the total token return is less than r under the risk-neutral measure, so agents refrain from holding tokens. one may suspect that there are multiple values of N_t that satisfy Equations (11) and (12). To get the intuition for existence and uniqueness, consider the properties of a *response* function $R\left(n; A_t, \mu_t^P\right)$ that maps a hypothetical value of N_t, say n, to the measure

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[8]Increasing hazard rate means $\frac{g(u)}{1-G(u)}$ is increasing in u, which is equivalent to $1-G(u)$ being log-concave. This is a standard assumption in the mechanism design literature to avoid the technically complicated "ironing" of virtual values.
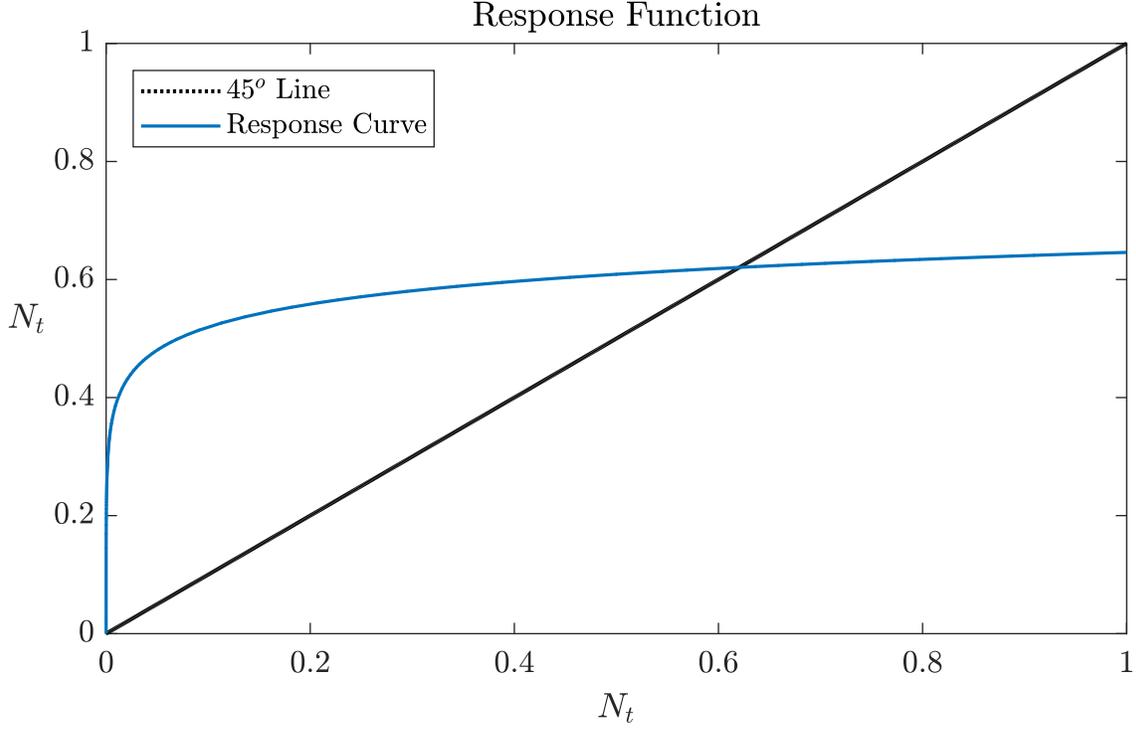
Figure 1: Determining User Base.

of agents who choose to join the community after knowing $N_t = n$. As depicted in Figure 1, the response curve originates from zero. If $A_t > \underline{A}$, then the response function is above the 45° line at some $n \in (0,1)$. But we know $R\left(1; A_t, \mu_t^P\right) \leq 1$, therefore the response curve crosses the 45° line at least once in the range of $(0,1]$. We show in the appendix that the monotone hazard rate implies the $g(u)$ is well-behaved that the response curve crosses the 45° line exactly once, which pins down the unique N_t. Importantly, a higher μ_t^P shifts the response curve upwards, resulting in a bigger N_t of user adoption.

Token Pricing Formula. We now provide the token pricing formula under endogenous adoption, which nests several valuation ingredients salient among practitioners. Define the *participants' aggregate transaction need* as

$$S_t := \int_{\underline{u}_t}^{\overline{U}} e^u g\left(u\right) du, \tag{13}$$

where $g\left(u\right)$ is then the density function of u_i. S_t is the integral of e^{u_i} of participating agents.

We consider a fixed supply of tokens, M.[9] The market clearing condition is

$$M = \int_{u_i \geq \underline{u}_t} k_{i,t} di, \tag{14}$$

Substituting in agents' optimal token holdings, we have the following proposition.

Proposition 2 (**Token Pricing Formula**). *The token-market clearing condition offers a token price formula:*

$$P_t = \frac{N_t S_t A_t}{M} \left(\frac{1 - \alpha}{r - \mu_t^P} \right)^{\frac{1}{\alpha}}. \tag{15}$$

The token price increases in N_t, the size of blockchain user base – the larger the ecosystem is, the higher trade surplus individual participants can realize by holding tokens, and stronger the token demand. Our model contributes to the literature of asset pricing by providing a theoretical foundation for the commonly used valuation-to-user base ratio in the technology industry, especially popular for valuing firms and platforms whose customer base feed on endogenous network effects. Here, the P-N ratio increases in the blockchain productivity, expected price appreciation, and network participants' aggregate transaction need, while decreases in token supply M. It is worth emphasizing that the asset we price is a blockchain token, not equity stakes of firms. The formula reflects certain observations by practitioners, such as incorporating DAA (daily active addresses) and NVT Ratio (market cap to daily transaction volume) in token valuation framework, but instead of heuristically aggregating such inputs into a pricing formula, we solve an equilibrium with *both* endogenous token pricing and adoption.[10]

Solving the Markov Equilibrium. The token pricing formula suggests that there exists a Markov equilibrium with A_t being the only aggregate state variable.

Definition 1 (**Equilibrium**). *For any initial value of A_0, the distribution of idiosyncratic productivity u_i given by the density function $g(u)$, and any endowments of token holdings*

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[9]This is consistent with many ICOs that fix the supply of tokens. Because resources for business operations on-chain are all discussed in real terms, we can simply normalize M to one due to money neutrality.

[10]See, for example, *Today's Crypto Asset Valuation Frameworks* by Ashley Lannquist at Blockchain at Berkeley and Haas FinTech.

among the agents, $\{k_{i,0}, i \in [0, 1]\}$, such that

$$M = \int_{i \in [0,1]} k_{i,0} di,$$

a Markov equilibrium with state variable A_t is described by the stochastic processes of agents' choices and token price on the filtered probability space generated by Brownian motion $\{Z_t^A, t \geq 0\}$ under the risk-neutral measure, such that

(1) Agents know and take as given the process of token price;

(2) Agents optimally choose consumption, and token and off-blockchain investments;

(3) Token price adjusts to clear the token market as in Proposition 2;

(4) All variables are functions of A_t that follows an autonomous law of motion given by Equation (1) that maps any path of shocks $\{Z_s, s \geq t\}$ to the current state A_t.

This conjecture of Markov equilibrium with state variable A_t is consistent with the equilibrium conditions. For example, by Itô's lemma, μ_t^P is equal to $\left(\frac{dP_t/P_t}{dA_t/A_t}\right)\mu^A + \frac{1}{2}\frac{d^2P_t/P_t}{dA_t^2/A_t^2}\left(\sigma^A\right)^2$, which is a univariate function of A_t in the Markov equilibrium. From Proposition 1, we can solve $\underline{u}_t$ and N_t, which only depends on A_t and μ_t^P, and thus, are also univariate functions of A_t. Hence, all the endogenous aggregate variables only depend on the state variable A_t.

So far, we have shown that once the token pricing function $P(A_t)$ is known, we can solve for μ_t^P using Itô's lemma, and then, the optimal token holdings, $k_{i,t}^*$ using Equation (9). From Proposition 1, we solve for the user base, N_t, and the lower bound of participants' idiosyncratic productivity, $\underline{u}_t$. Substituting these variables into the token pricing formula (Equation (15)), we have the right-hand side depends only on A_t, $P(A_t)$, and the first and second derivatives of $P(A_t)$ through μ_t^P. Therefore, rearranging the token pricing formula, we have a second-order ordinary differential equation ("ODE") for $P(A_t)$:

$$\frac{\left(\sigma^A\right)^2}{2}\frac{d^2P_t/P_t}{dA_t^2/A_t^2} + \mu^A\left(\frac{dP_t/P_t}{dA_t/A_t}\right) + (1-\alpha)\left(\frac{N_t S_t A_t}{M P_t}\right)^\alpha - r = 0. \qquad (16)$$

By imposing proper boundary conditions, we can solve for $P(A_t)$.[11] The first is

$$\lim_{A_t \to 0} P(A_t) = 0, \tag{17}$$

so that when the platform is not productive any more, token price collapses to zero , ruling out pure speculation ($\mu_P \to r$) when $A_t \to 0$. The second involves that as A_t increases, token price after full adoption ($N_t = 1$) behaves according to

$$\overline{P}(A_t) = \frac{\overline{S} A_t}{M} \left(\frac{1 - \alpha}{r - \mu^A} \right)^{\frac{1}{\alpha}}, \tag{18}$$

where recall that the constant $\overline{S} \equiv \int_{\underline{U}}^{\overline{U}} e^u g(u)\, du$ is the participants' aggregate need for trade. Note that (18) solves the (2) when $N_t \geq 1$, and is essentially the "Gordon Growth Formula" in our setting. For any finite $\underline{U}$, we require value matching and smooth pasting to rule out arbitrage. In other words, A^* and $P(A^*)$ are endogenously determined by

$$P(A^*) = \overline{P}(A^*) \quad and \quad P'(A^*) = \overline{P}'(A^*) \tag{19}$$

When $\underline{U} = -\infty$, we simply explore the asymptotic behavior of the economy. When A_t approaches infinity, N_t approaches one and $\underline{u}_t$ approaches $-\infty$, then the natural *asymptote* of $P(A_t)$ is:

$$\lim_{A_t \to +\infty} \overline{P}(A_t) - P(A_t) = 0, \tag{20}$$

The asymptote not only satisfies the token pricing formula for $A_t \to \infty$, but also has P_t purely driven by A_t in the sense that with full adoption ($N = 1$), $\sigma_t^P = \sigma^A$ (i.e., no excess volatility).

The lower boundary (17) determines the level of price in the region of interest ($N_t < 1$), while the upper boundary (19) or (20) specifies the long-term dynamics of price. Together, they rule out the degenerate equilibrium of $P(A_t) = 0$ for any $A_t > 0$.[12] Similar to Brun-

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[11]The existence of ODE requires a unique mapping from A_t, $P(A_t)$, and $P'(A_t)$ to $P''(A_t)$. The question is: given A_t and P_t, can we uniquely solve μ_t^P using the token pricing formula? Since $\underline{u}_t$ decreases in μ_t^P (and N_t increases in μ_t^P), the right-hand side of the token market clearing condition increases in μ_t^P, so we can uniquely pin down μ_t^P given A_t and P_t.

[12]Ruling out such a degenerate equilibrium is non-trivial because the dollar value of tokens, i.e., $P_t k_{i,t}$, enters into the trade surplus, so when $P_t = 0$, the "cash flow" (i.e., trade surplus) of token is zero. However, given Equation (20), agents expect token price to be positive in the far future where A_t is sufficiently large.

nermeier and Sannikov (2014), we characterize the equilibrium that is Markov in the state variable A_t.

Proposition 3 (**Markov Equilibrium**). *Given the boundary conditions described in (17) and (19), there exists a unique Markov equilibrium with A_t as the state variable for any finite lower bound $\underline{U}$ of user type.*[13] *Equation (9) solves participants' optimal token holdings $k_{i,t}^*$ given the token price function $P(A_t)$, and Proposition 1 solves the user base N_t and the threshold of adoption $\underline{u}_t$. The token price is then the unique solution to the second-order ordinary differential equation (15) in Proposition 2.*

In the appendix we show that our ODE (15) satisfies all the regularity conditions for us to apply an argument similar in spirit to that in the Picard-Lindelof Theorem for initial value problems, in order to obtain uniqueness for our boundary value problem (Jackson (1968)). The existence of a unique equilibrium distinguishes our paper from studies such as Sockin and Xiong (2018) that focus on equilibrium multiplicity, and allows us to highlight dynamics of adoption and valuation.

Figure 2 summarizes the key economic mechanism that follows from all the results thus far, where the blue, black, and red arrows show respectively the user-base externality, the transaction motive of token holdings, and the investment motive of token holdings.

2.3 The Impact of Tokens on Adoption

We next investigate the role of tokens in users' dynamic adoption. To proceed, let us consider an alternative setup where agents can conduct businesses and enjoy the trade surplus on the same blockchain platform without holding tokens, but use dollar, the numeraire, as

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Hence reasoning backward instant by instant, token price stays positive so that arbitrage opportunities (i.e., a jump of token price from zero to positive within an instant of dt) do not exist. More rigorously, given Equation (20) and $\overline{P}(A_t) > 0$, for a sufficiently small ϵ, there exists a sufficiently large $\overline{A}(\epsilon)$ such that $P(A_t)$ is within the ϵ-neighborhood of $\overline{P}(A_t)$ (and thus, positive) for any $A_t > \overline{A}(\epsilon)$. We can also rule out the equilibrium of $P_t = 0$ by setting the trade surplus as a function of $k_{i,t}$ instead of $P_t k_{i,t}$, and our results are qualitatively robust under this alternative setup.

[13]We note that the second-order ODE (15) is non-linear, $\underline{U}$ being finite is a *sufficient* condition for the boundary conditions (17) and (20) to guarantee a unique solution (Jackson (1968)). When $\underline{U} = -\infty$, the solution is likely still unique, but the proof is beyond this paper. Moreover, taking finite $\underline{U}$ is not an issue for all practical purposes and quantitative analysis, because any numerical procedure has to approach full adoption at finite values of A_t, and we do not think there are agents who are infinitely averse towards the platform in real life.
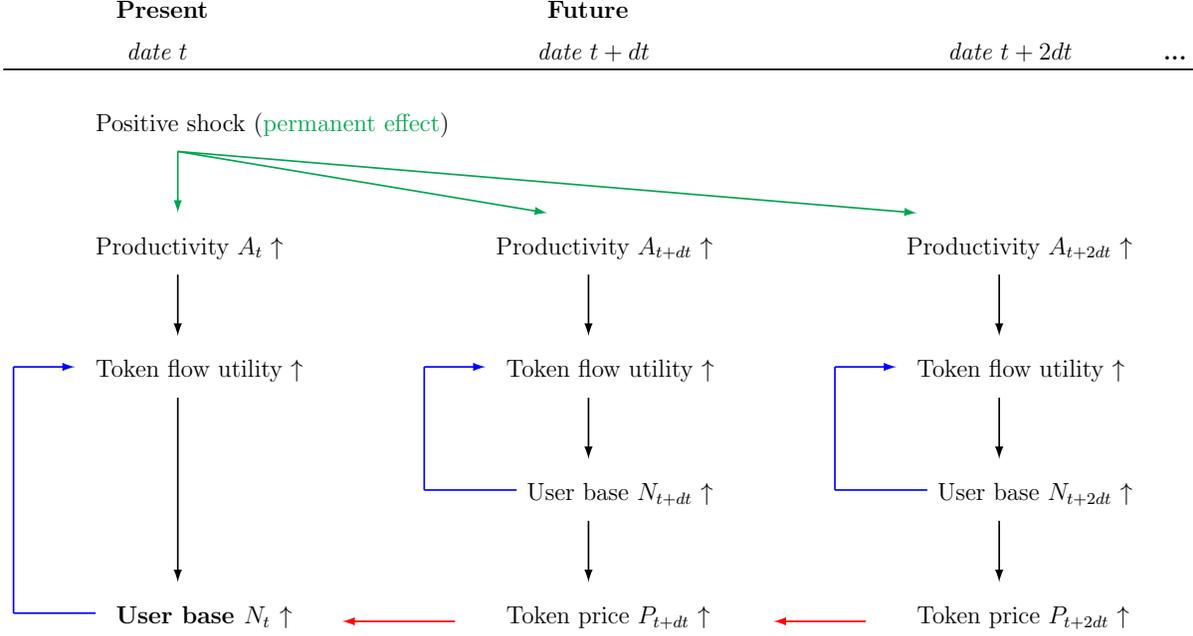
Figure 2: **The Economic Mechanism in a Nutshell.** The green arrows point to the increases of the current and future (expected) levels of productivity A, which lead to higher flow utilities of tokens, and in turn, larger user bases N as highlighted by the black arrows. The blue arrows show that increases in user base result in even higher flow utility due to the contemporaneous network externality. Finally, more users push up the token prices P in future dates, which feed into a current expectation of token price appreciation, leading to greater current adoption.

the medium of exchange. Here, the only difference from the token-based equilibrium is that agents' profits are not exposed to the token price fluctuation:

$$dy_{i,t} = \max\left\{0, \max_{x_{i,t}>0}\left[\underbrace{(x_{i,t})^{1-\alpha}\left(N_t A_t e^{u_i}\right)^\alpha dt}_{\text{blockchain trade surplus}} - \underbrace{\phi dt}_{\text{participation cost}} - \underbrace{x_{i,t} r dt}_{\text{financing cost}}\right]\right\}. \quad (21)$$

Conditional on joining the platform (i.e., $x_{i,t} > 0$), the first order condition for $x_{i,t}$ is

$$(1-\alpha)\left(\frac{N_t A_t e^{u_i}}{x_{i,t}}\right)^\alpha = r. \quad (22)$$

Rearranging the equation, we have the following lemma on optimal operation scale.

Lemma 3 (**Optimal Scale without Token**). *At time t, agent i's optimal level of wealth held on the platform without token is given by*

$$x_{i,t}^* = N_t A_t e^{u_i}\left(\frac{1-\alpha}{r}\right)^{\frac{1}{\alpha}} \quad (23)$$

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which increases in N_t, A_t, and u_i.

Substituting $x_{i,t}^*$ into the profit function, we have the maximized profit (conditional on joining the platform) equal to

$$N_t A_t e^{u_i} \alpha \left(\frac{1-\alpha}{r} \right)^{\frac{1-\alpha}{\alpha}} - \phi. \tag{24}$$

An agent joins the platform only if the maximized profits are positive. Taking logarithm of Equation (24), we have the following lemma of adoption threshold.

Lemma 4 (Adoption Threshold without Token). *Agent i joins the platform without token if $u_i \geq \underline{u}_t^{NT}$, where*

$$\underline{u}_t^{NT} = -\ln(N_t) + \ln\left(\frac{\phi}{A_t \alpha} \right) - \left(\frac{1-\alpha}{\alpha} \right) \ln\left(\frac{1-\alpha}{r} \right), \tag{25}$$

and the superscript "NT" is for "no tokens". $\underline{u}_t^{NT}$ decreases in A_t and N_t.

We can aggregate individuals' adoption decision,

$$N_t^{NT} = 1 - G\left(\underline{u}_t^{NT} \right). \tag{26}$$

Equations (25) and (26) jointly determine $\underline{u}_t^{NT}$ and N_t^{NT}.

Token Acceleration of Adoption. We next prove an important result on how introducing tokens can accelerate adoption.

Proposition 4 (Adoption Level).

When $\mu_A > 0$, N_t^{NT} is always smaller than N_t for $t > 0$, where N_t is given in Proposition 1; when $\mu_A < 0$, the result is the opposite.

In other words, users adopt more with tokens when they believe the platform productivity is improving. We note that having no tokens is equivalent to setting μ_t^P to zero in Proposition 1. Introducing tokens adds an investment motive to agents' adoption decision through the expectation of token appreciation, which is in turn driven by the growth of A_t. This channel speeds up user-base expansion. We can also entertain the flip side of this effect: when A_t

is expected to deteriorate (e.g., due to $\mu^A < 0$), the associated token depreciation ($\mu_t^P < 0$) accelerates the collapse of user base and the demise of the platform. Our quantitative analysis focuses on the case where $\mu^A > 0$.

Note that in the system without token, transactions are settled on dollars, and we simplify the analysis by assuming that the price of dollar in goods is fixed at one. In reality, the value of dollar declines over time due to inflation, which is likely to strengthen the token effect by adding a dollar depreciation term in Equation (25). It is true that non-blockchain-based transferable participation rights may serve a similar function in accelerating adoption, but it is the significant reduction in transaction cost, reliance on trusted third parties, and the growth in crypto-awareness and popularity brought forth by the blockchain technology that catalyzed the qualitative change in practice.

Token Reduction of User Base Volatility. Without native tokens, agents' decision to participate is purely driven by the current level of blockchain productivity A_t and their idiosyncratic transaction needs u_i. Therefore, the user base N_t varies only with A_t, and its volatility is tied to the exogenous volatility of blockchain productivity. Introducing tokens also changes the volatility of N_t through the fluctuation of the expected price change, because now, agents' decision to participate also depends on μ_t^P.

To derive the dynamics of N_t, we first conjecture that N_t follows a diffusion process in equilibrium

$$dN_t = \mu_t^N dt + \sigma_t^N dZ_t^A. \tag{27}$$

Strictly speaking, N_t follows a reflected (or "regulated") diffusion process that is bounded below at zero and bounded above at one, so we study the interior behavior of N_t. In the appendix, we solve the volatility of N_t for the case without token and the one with token. The following proposition summarizes the results.

Proposition 5 (**User Base Volatility**). *In an economy without tokens, the diffusion of dN_t is*

$$\sigma_t^N = \left(\frac{g\left(\underline{u}_t^{NT}\right)}{1 - g\left(\underline{u}_t^{NT}\right)/N_t^{NT}} \right) \sigma^A. \tag{28}$$

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In an economy with tokens, the diffusion of dN_t is

$$\sigma_t^N = \left(\frac{g\left(\underline{u}_t\right)}{1 - g\left(\underline{u}_t\right)/N_t} \right) \left[\sigma^A + \left(\frac{1-\alpha}{\alpha} \right) \left(\frac{\sigma_t^{\mu^P}}{r - \mu_t^P} \right) \right], \tag{29}$$

where, $\sigma_t^{\mu^P}$ is the diffusion of μ_t^P in its law of motion,

$$d\mu_t^P = \mu_t^{\mu^P} dt + \sigma_t^{\mu^P} dZ_t^A. \tag{30}$$

Comparing Equations (28) and (29), we see that introducing tokens could alter the volatility dynamics of user base through the fluctuation of expected token price change, i.e., $\sigma_t^{\mu^P}$. A priori, having a native token may either amplify or dampen the shock effect on the the user base, depending on the sign of $\sigma_t^{\mu^P}$. By Itô lemma, $\sigma_t^{\mu^P} = \frac{d\mu_t^P}{dA_t}\sigma^A A_t$, so the sign of $\sigma_t^{\mu^P}$ depends on whether μ_t^P increases or decreases in A_t.

We note that μ_t^P weakly decreases in A_t (and thus, $\sigma_t^{\mu^P} < 0$), precisely because of the endogenous user adoption. Consider an increase in A_t, which corresponds to an increase in N_t, reducing potential newcomers to join the community in future. Recall that token price appreciation is driven by the future increase in both A_t and N_t, so when there is less potential for N_t to grow, the expected token price appreciation, i.e., μ_t^P, declines. Because μ_t^P decreasing in A_t, introducing token can *reduce* the conditional volatility of user base. That said, because $\underline{u}_t^{NT}$ and $\underline{u}_t$ could differ in general, there could be regions where the conditional volatility of user base is higher when token is introduced. In our calibration later, we find that the region of intermediate adoption whereby tokens reduce user base volatility is significant.

Given the roles of the tokens, entrepreneurs may want to introduce them in a platform. For example, suppose the platform can collect a fee of ϕ from the users, greater adoption would increase the revenue of the platform. One way to kill two birds with one stone is to issue tokens to early investors through ICOs which brings in capital for developing the platform as well. Then through retaining some tokens, the early investors and entrepreneurs can also enjoy the token price appreciation. Our on-going work and Bakos and Halaburda (2018) explore such strategic considerations of the entrepreneurs and platforms designers.

2.4 Tokenized, Tokenless, and First-best Economy

Given the roles of the tokens, entrepreneurs may want to introduce them in a platform. For example, suppose the platform can collect a fee of ϕ from the users, greater adoption would increase the revenue of the platform. One way to kill two birds with one stone is to issue tokens to early investors through ICOs which brings in capital for developing the platform as well. Then through retaining some tokens, the early investors and entrepreneurs can also enjoy the token price appreciation. Our model thus provide one rationale for introducing tokens in startup platforms. Our on-going work explore such strategic considerations of the entrepreneurs and platforms designers.

While tokens may benefit entrepreneurs who care about user adoption, are they always welfare improving? To answer this question, let us consider a planner's problem. Since there is a buyer and a seller to any trading of media of exchange, what matters for welfare is the amount of resources allocated to the blockchain platform. Given a user base N_t, the socially optimal amount of capital an adopted user i allocates onto the blockchain is

$$x_{i,t}^* = N_t A_t e^{u_i} \left(\frac{1-\alpha}{r} \right)^{\frac{1}{\alpha}}. \tag{31}$$

Let $\mathcal{U}_t$ denote the set of user base with measure N_t. Then the total welfare flow (if positive) with user base N_t is

$$\int_{i \in \mathcal{U}_t} \left[\alpha N_t A_t e^{u_i} \left(\frac{1-\alpha}{r} \right)^{\frac{1-\alpha}{\alpha}} - \phi \right] di = N_t \left[\alpha \left(\frac{1-\alpha}{r} \right)^{\frac{1-\alpha}{\alpha}} A_t \int_{i \in \mathcal{U}_t} e^{u_i} di - \phi \right] \tag{32}$$

To maximize the total welfare flow, $\mathcal{U}_t$ should include the agents with higher u_i and therefore should follow a threshold cutoff, i.e., $\mathcal{U}_t(u) = \{i : u_i \geq u\}$ for some $u \geq \underline{U}$. Therefore, we should set $N_t = 1$ to maximize (31); if the optimized (31) is negative, then zero adoption would be optimal. The switching from zero adoption to full adoption happens at

$$\underline{A}^{FB} = \phi \left[\alpha \left(\frac{1-\alpha}{r} \right)^{\frac{1-\alpha}{\alpha}} \int_{u=\underline{U}}^{\overline{U}} e^u g(u) du \right]^{-1} \tag{33}$$

When $\int_{u=\underline{U}}^{\overline{U}} e^u g(u) du < \infty$, welfare maximization has a bang-bang solution, requiring full-

adoption if $A \geq \underline{A}^{FB}$ and zero adoption otherwise. It is straight forward to check that $\underline{A}^{FB} \leq \underline{A}^{Tokenless}$, where $\underline{A}^{Tokenless}$ is the $\underline{A}$ in Proposition (1) with $\mu^P = 0$. The adoption acceleration of token can potentially improve welfare by bring the outcome closer to first-best. That said, if μ^P is too big, it is possible $\underline{A}(\mu^P) < \underline{A}^{FB}$: when agents are counting on the appreciation of token price, they could over-adopt relative to the first-best outcome. Therefore, when the platform productivity is smaller than $\underline{A}^{FB}$, adoption acceleration is welfare-destroying.

3   Quantitative Analysis

3.1   The Physical Measure and Calibration

Risk Aversion and SDF. So far we have worked under the risk-neutral measure. To relate our model to data and to understand the returns of cryptocurrencies and crypto-tokens, we introduce agents' risk aversion and discuss price and adoption dynamics under the physical measure. To this end, we assume that agents' risk preference is given by a stochastic discount factor ("SDF") Λ satisfying

$$\frac{d\Lambda_t}{\Lambda_t} = -r dt - \eta d\hat{Z}_t^\Lambda, \tag{34}$$

where r is the risk-free rate and η is the price of risk for systematic shock $\hat{Z}_t^\Lambda$ under the physical measure.[14] Let dZ_t^A denote the SDF shock under the risk-neutral measure. Using the Girsanov Theorem, we have

$$dZ_t^\Lambda = d\hat{Z}^\Lambda + \eta dt. \tag{35}$$

Throughout this paper, we use "ˆ" indicate the physical measure. A unique SDF gives the price of Arrow-Debreu securities in each state of the world (a complete market).

Let ρ denote the instantaneous correlation between the SDF shock and the blockchain productivity shock. The usefulness of a particular platform evolves with the economy, as agents discover new ways to utilize the technology, which in turn depends on the progress of complementary technologies. As aforementioned, macro and regulatory events affect the

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[14]Chen (2010) shows that the SDF in the form of Equation (34) can be generated from a consumption-based asset pricing model.

usage of a blockchain platform. The crypto beta from ρ is priced under the physical measure, generating a link between token price fluctuation and expected return.

Under the physical measure, A_t has the following law of motion, $dA_t = \hat{\mu}^A A_t dt + \sigma^A A_t d\hat{Z}_t^A$, where using Girsanov Theorem, we know that $\hat{\mu}^A$ is equal to $\mu^A + \eta\rho\sigma^A$, and $d\hat{Z}_t^A$ is the Brownian productivity shock under the physical measure, given by $d\hat{Z}_t^A = dZ_t^A - \eta\rho dt$.

Calibration. We now calibrate our model under the physical measure, and analyze the equilibrium outcome quantitatively. Our calibration is guided by the growth of token price and blockchain user base in the period from July 2010 and April 2018. In the model, since we fix the supply of tokens at M, the variation of token price P_t drives that of market capitalization (i.e., $P_t M$). We map the dynamics of P_t to that of the *aggregate* market capitalization of 16 major cryptocurrencies.[15] Since we study a representative token economy, we choose to focus on the aggregate token valuation that averages out idiosyncratic movements due to specificalities of token protocols and highlights the common feature, which is decentralized ledger or computing platform powered by the blockchain technology.

Accordingly, we collect the number of active user addresses for major cryptocurrencies, and map the aggregate number to N_t in our model. Since the beginning month of our sample is unlikely to be the initial date of blockchain application to peer-to-peer platform, we choose to map the maximum number of active addresses, which was achieved in December 2017, to $N_t = 0.5$ in our model, and scale the number of active addresses in other months. As a result, we focus on the model performances in the states where $N_t \in [\underline{N}, 0.5]$, i.e., the early stage of adoption. $\underline{N}$ will be explained later together with Figure 4. We take a Normal distribution for u^i with $g(u) = \sqrt{\frac{1}{2\pi\theta^2}} e^{-\frac{u^2}{2\theta^2}}$, and adjust parameters so that the model generates the following patterns in data: (1) the growth of N_t over time; (2) the evolution of the growth *rate* of N_t; (3) the co-movement between P_t and N_t; (4) the dynamics of user base volatility. We later juxtapose our model-generated results along these dimensions together with the data.

The key parameters for the equilibrium dynamics of N_t and P_t are μ^A, σ^A, α, and θ. First,

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[15]We include all cryptocurrencies with complete market cap and active address information on bitinfocharts.com, namely, AUR (Auroracoin), BCH (Bitcoin Cash), BLK (BlackCoin), BTC (Bitcoin), BTG (Bitcoin Gold), DASH (Dashcoin), DOGE (DOGEcoin), ETC (Ethereum Classic), ETH (Ethereum), FTC (Feathercoin), LTC (Litecoin), NMC (Namecoin), NVC (Novacoin), PPC (Peercoin), RDD (Reddcoin), VTC (Vertcoin). They represent more than 2/3 of the entire crypto market.

σ^A and μ^A determines the time scale of this economy *under the physical measure*, i.e. how fast N_t and P_t grows, by pinning down the growth rate of A_t. The instantaneous expected growth rate of A_t is $\hat{\mu}^A = \mu^A + \eta\rho\sigma^A$. η is set to 1, roughly in line with the maximum Sharpe ratio given by the efficient frontier of U.S. stock market. ρ is set to 1, a conservative value relative to the beta of technology sector (Pástor and Veronesi (2009)).

σ^A is set to 200%, which contributes most to A_t's growth rate under the physical measure and the growth of N_t. As previously discussed, our interpretation of A_t emphasizes the general usefulness of blockchain platform rather than narrowly defined technological progress. The type of activities on blockchain depends on the progress of complementary and competing technologies, government regulations, and critically, users' creativity and perception of the technology, suggesting a fast yet volatile growth of A_t. Given this annual growth rate of A_t, N_t the sample span of approximately eight years implies the growth of N_t from 0.0001 (i.e., $\underline{N}$) to 0.5, which corresponds to the growth of normalized number of addresses in data.

We set one unit of time to be one year, and $r = 5\%$. Because P_t converges to a multiple of A_t, μ_t^P converges to μ^A. Under the risk-neutral measure, the expected token price change has to be lower than r (otherwise agents invest as much as they can in tokens), so we set μ^A equal to 2%. The gap between μ^A and r determines how widely μ_t^P varies, which will be explained in detail with Figure 5, so we use the volatility of percentage change of user addresses to discipline the choice of this parameter. Specifically, we map the data moment to average σ_t^N/N_t in the states where $N_t \in [\underline{N}, 0.5]$.

While σ^A pins down the growth rate of N_t, θ is responsible for the curvature of its growth path. In data, user-base growth rate rises over time, which will be shown to be qualitatively consistent with the model dynamics (i.e., a S-shaped development of N_t). To quantitatively match this pattern, we set $\theta = 10/\sqrt{2}$ (i.e., a cross-section variance of 50 for u_i).

Parameter α governs the co-movement between N_t and P_t. We can think of token as an asset that pays a flow dividend (i.e., the blockchain trade surplus). α governs the decreasing return to N_t in the user-surplus flow. Therefore, a higher value of α increases the co-movement between P_t and N_t through a cash flow channel. We set α to 0.3 so that the model generates the joint dynamics of P_t and N_t in the states where $N_t \in [\underline{N}, 0.5]$.

The remaining parameters do not affect much the equilibrium dynamics. Recall that ϕ is the cost of joining the blockchain community, measured in goods. We set ϕ equal to 1

Table 1: Calibration

Parameter	Value	Model	Data
		Panel A: Key Parameters	
(1) σ^A	200%	Growth rate of N_t	Growth of user address
(2) θ	$10/\sqrt{2}$	Curvature of N_t growth	Curvature of user-address curve
(3) α	0.3	Comovement: N_t & P_t	User address & crypto market cap
(4) μ^A	2%	$\frac{\sigma_t^N}{N_t}$, vol. of N_t % change	Vol. of user-address % change
		Panel B: Other Parameters	
(5) ϕ	1	Scaling effect on A_t	
(6) M	1 billion	Monetary Neutrality	
(7) ρ	1	Shock correlation: SDF & A_t	
(8) r	5%	Risk-free rate	
(9) η	1	Price of risk	

as a reference point for other parameters. We set M, the supply of tokens, to 10 billion. Our model features monetary neutrality, that is the equilibrium outcome stays the same, for instance, if M is doubled and P_t halved.

3.2 Dynamic Adoption and Token Valuation

Token Price. The token price is solved as a function of blockchain productivity A_t. The left panel of Figure 3 plots dP_t/dA_t against $\ln(A_t)$. The curve starts at $A_t = 1e - 21$ (i.e., $\ln(A_t) = -48.35$), a number that we choose to be close to zero, i.e., the left boundary. The curve ends at $A_t = 1e8$ (i.e., $\ln(A_t) = 18.42$), a sufficiently large number such that beyond this point, $P(A_t)$ and its asymptote $\overline{P}(A_t)$ become indistinguishable, as shown in the left panel of 3 (the convergence of derivative). Another message from the left panel is that over time, token price becomes increasingly sensitive to the variation in A_t. When the user base is small, token price is less responsive to the growth of A_t, because A_t is multiplied by N_t when entering into the blockchain trade surplus. As A_t grows and N_t approaches 1, P_t becomes more sensitive to A_t.

The right panel of Figure 3 plots the logarithm of token price against the size of user base, both being functions of A_t in the Markov equilibrium. This graph is particularly interesting because it directly links token price to various stages of adoption in equilibrium. On a logarithm scale, token price increases fast with adoption in the early stage, and then
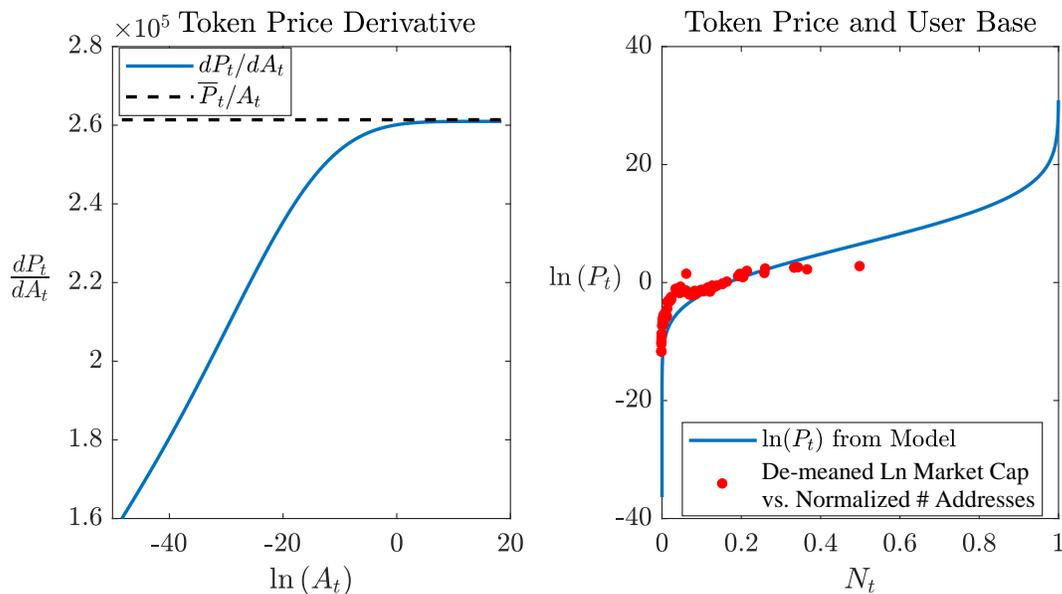
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Figure 3: Dependence of Token Price on Blockchain Productivity and User Base.

gradually rises with user base, but when the system has accumulated a critical mass of users, the increase of token price speeds up and converges to its long-run path. More generally, token price could have exponential growth with respect to the user base, an important implication for trading cryptocurrencies. The comovement of P_t and N_t matches well the pattern in data, for which we only observe the early stage of adoption. Next, we will explore more in detail the dynamics of user adoption.

User Base. The solid line in Figure 4 plots the user base N_t against the logarithm of A_t. The curve exhibits S-shaped development. When the blockchain technology is not so efficient, the growth of user base in response to technological progress is small. But as N_t increases, the growth of user base feeds on itself – the more agents join the ecosystem, the higher surplus it is from trading on the blockchain. As a result, the growth of N_t speeds up in the interim range of blockchain productivity. User adoption eventually slows down when the pool of newcomers get exhausted. This model does not feature population growth, on the basis that population growth relative to changes in A_t is small.

Both the growth rate and curvature of N_t over time match well the pattern in data. As previously discussed, we map the highest number of user addresses (December 2017) to $N_t = 0.5$, and record its corresponding value of $\ln(A_t)$ in our model. We scale the number of
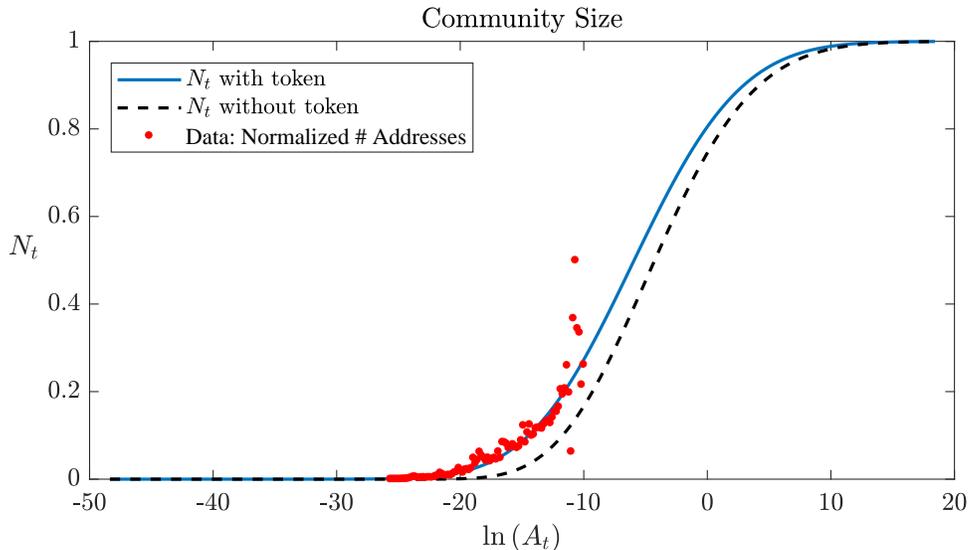
Figure 4: Dependence of User Base on Blockchain Productivity

addresses in other months by that of December 2017. With December 2017 as the reference point, we calculate the corresponding value of $\ln(A_t)$ for each observation by applying the annualized growth rate of 202% to the value of $\ln(A_t)$ in December 2017. The leftmost state that maps to the data has $N_t = \underline{N} = 0.0001$.

Figure 4 also compares the user adoption with and without tokens or blockchain native tokens. The former strictly dominates the latter. The two eventually converge to one as A_t grows. Next, we explain in detail such comparison.

3.3 The Impact of Introducing Tokens on Adoption

When token is introduced as the required medium of exchange on a platform, its market price reflects agents' anticipation of future technological progress and user adoption, which translates into the expected token price appreciation. Tokens therefore accelerates adoption because agents join the community not only to enjoy the trade surplus but also the return from rising token price.

The Adoption Acceleration Effect. Comparing users' adoption decision in Proposition 1 and 4, the only difference is that without tokens, the price appreciation term, i.e., μ_t^P, drops out in the lower bound of idiosyncratic productivity. Therefore, in states where $\mu_t^P > 0$, a blockchain system with tokens has a larger community. The intuition is simply that agents

hold tokens to enjoy not only the trade surplus ("utility purchase") uniquely available on-chain, but also the token price appreciation ("investment purchase"). In a system without tokens, the investment-driven demand is shut down.

Intuitively, if A_t is expected to grow fast (for example, due to a larger and positive μ^A), μ_t^P tends to be positive. Therefore, by introducing a native token that is required to be the medium of exchange, a blockchain system can capitalize future productivity growth in token price, and thereby accelerate adoption. This is the growth effect of introducing tokens in promising platforms (large and positive μ^A).

In essence, token enables a feedback loop reflecting the inter-temporal complementarity of user base. By capitalizing future productivity growth and popularity (i.e., large N), token induces faster adoption in the early stage. As the user base is *expected* to expand fast, token price is expected to appreciate because the flow surplus is expected to be higher for all participants and the demand for tokens expected to be higher. The expected price appreciation feeds into a stronger investment-driven demand for tokens.

We note that a predetermined token supply schedule is important. If token supply can arbitrarily increase ex post, then the expected token price appreciation is delinked from the technological progress. Predeterminancy or commitment can only be credibly achieved through the decentralized consensus mechanism empowered by the blockchain technology. In contrast, traditional monetary policy has commitment problem – monetary authority cannot commit not to supply more money when its currency value is relatively high.

The Volatility Reduction Effect. As we discussed following Proposition 5, introducing tokens also changes the volatility of N_t through the fluctuation of the expected price change, because now, agents' decision to participate also depends on μ_t^P. Now we numerically analyze how introducing tokens *reduces* the volatility of user base.

The left panel of Figure 5 plots σ_t^N, and compares the cases with and without token across different stages of adoption. Apparently, introducing token reduces the volatility of N_t. Both curves starts at zero and ends at zero, consistent with the S-shaped development in Figure 4 where both curves starts flat and ends flat. This volatility reduction effect is more prominent in the early stage of development when A_t and N_t are low. Note that σ_t^N can be slightly higher when token is introduced because the first brackets in Equations (28) and (29) can differ due to the difference between $\underline{u}_t^{NT}$ and $\underline{u}_t$ even for the same value of N_t.
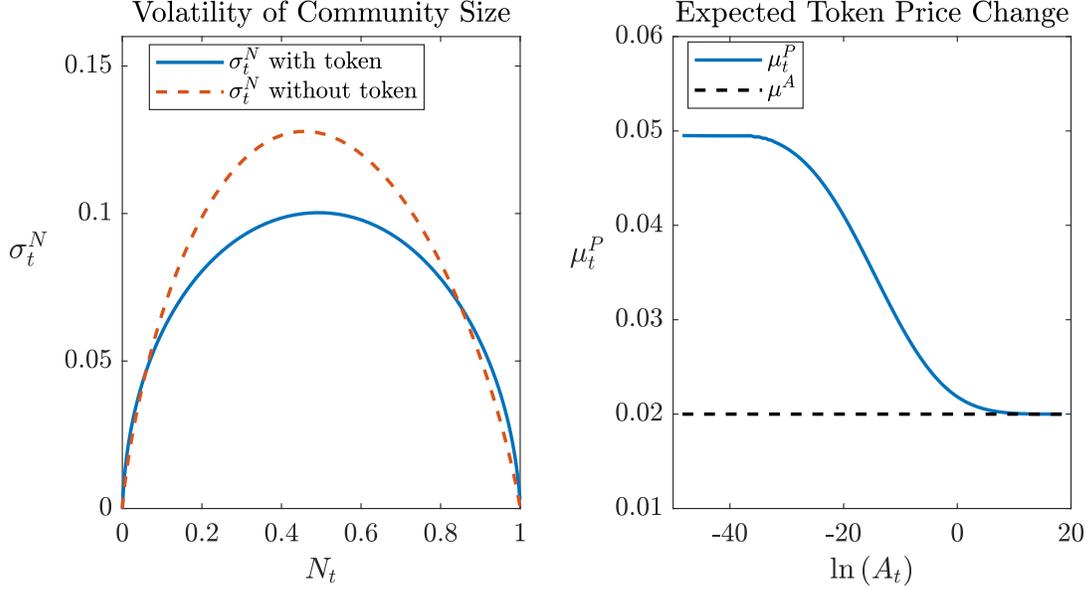
Figure 5: Volatility of User Base.

The right panel of Figure 5 plots μ_t^P against $\ln(A_t)$, showing their negative relation that causes $\sigma_t^{\mu^P} < 0$, which generates the volatility reduction effect. The intuition behind was explained following Proposition 5. When A_t is low and N_t is low, token price is expected to increase fast reflecting both the future growth of A_t and N_t. As A_t and N_t grow, the pool of newcomers is getting exhausted and there is less potential for N_t to growth. As a result, the expected token appreciation declines, and agents are less willing to join the community and hold tokens. Consider a positive shock to A_t (i.e., $dA_t > 0$), this negative impact on N_t through the expected token appreciation counteracts the direct positive impact on N_t through a higher trade surplus, making N_t less responsive to dA_t than the case without tokens. Similarly, following a negative shock to A_t (i.e., $dA_t < 0$), the trade surplus declines, but μ_t^P increases, which induces more agents to hold tokens than the case without tokens.

The figure also implies a negative time-series correlation between N_t and μ_t^P, which is to be distinguished from the partial equilibrium comparative statics in Proposition 1. As more and more people adopt, the investment motive declines because the contemporaneous adopter demand dominates and is priced in.
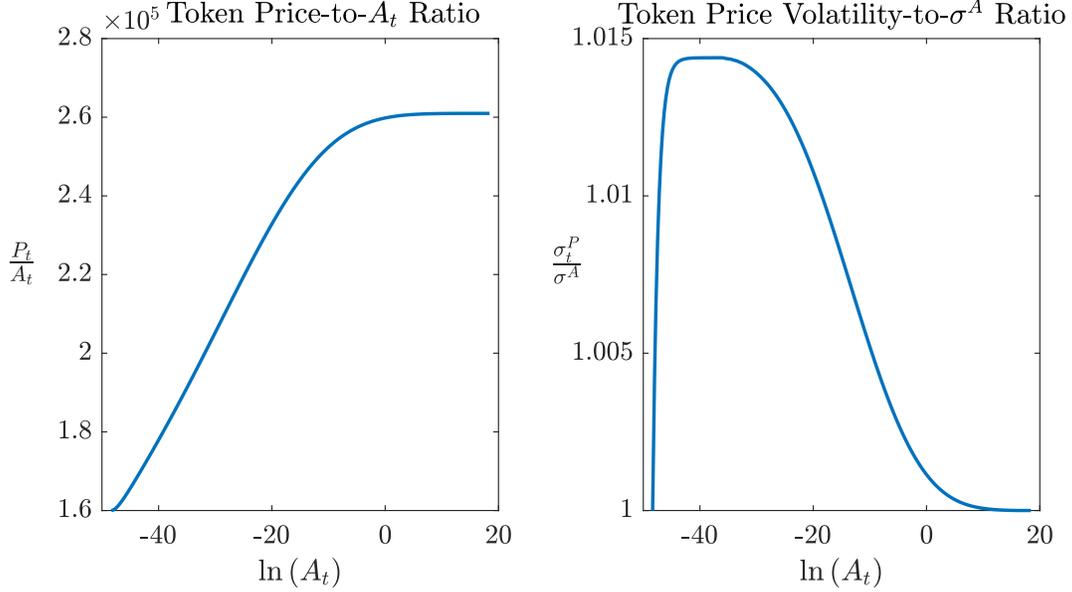
Figure 6: Token Price-to-Blockchain Productivity Ratio and Volatility.

3.4   The Impact of Endogenous Adoption on Token Price

The Run-up of Token Price Volatility. The dynamics of user adoption in turn affects the volatility of token price. When $\phi = 0$, agents' decision to participate becomes irrelevant. Every agent participates, so $N_t = 1$, and token price is given by Equation (18) and the ratio of P_t to A_t is a constant. Therefore, the diffusion of token price, i.e., σ_t^P, is equal to σ^A.

A key theme of this paper is the endogenous dynamics of user adoption. When $\phi > 0$, the ratio of P_t to A_t depends on the $\underline{u}_t$, the threshold value of agent-specific needs for blockchain transactions, above which an agent participates. The variation of N_t feeds into P_t/A_t, and therefore, amplifies the volatility of token price beyond σ^A, the level of volatility when the issue of user adoption is irrelevant.

The left panel of Figure 6 plots P_t/A_t. As shown in Equation (15), this ratio follows closely the dynamics of N_t shown in Figure 4, but is steeper in the early stage (i.e., low A_t region). The right panel of Figure 6 plots the ratio of token price volatility to σ^A, which eventually converges to 1 as N_t approaches one and P_t approaches its asymptote. At its height, endogenous user adoption amplifies token price volatility (or instantaneous standard deviation, to be precise) by 1.4%, but the excess volatility eventually declines with further adoption. We remind the reader that the result obtains under the premise that token price is

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purely driven by the fundamental productivity and there is no token/platform competition as described in Section 4.3. The large volatility observed in practice is likely due to platform competition and non-fundamental-based speculations.

What is interesting is the qualitative implications: as a new platform is gradually adopted, one may observe dormant token price variation, followed by a volatility run-up before the eventual stabilization. A key result of this paper is this the mutual effect between N_t and P_t in both growth and volatility. Especially in the early stage of adoption, user participation amplifies the growth and volatility of token price, while at the same time, is being affected by agents' expectation of future token price appreciation and the volatility of such expectation.

Risk and Return under the Physical Measure The expected token price appreciation under the physical measure is

$$\hat{\mu}_t^P = \mu_t^P + \eta \rho \sigma_t^P. \tag{36}$$

The covariance between token price change and SDF shock, i.e., $\rho \sigma_t^P$, is priced at η. If the shock to blockchain productivity is orthogonal to SDF shock ($\rho = 0$), then $\hat{\mu}_t^P = \mu_t^P$. Next, we graphically illustrate how risk premium varies across different stages of platform development. Figure 7 plots the token risk premium, i.e., $\hat{\mu}_t^P - r$, under the physical measure against $\ln(A_t)$. As A_t increases, the risk-neutral drift of token price (μ_t^P) declines, while the volatility (σ_t^P) follows a hump-shaped curve, which dominates the dynamics of risk premium.

Our risk premium of 200% is higher than the average annual return to the cryptocurrency portfolio in our sample (27%). The main reason is the decline of cryptocurrency market value in 2018. In the next section, we discuss an extension of the model that features time-varying token beta, which may generate such a decline and a risk premium more in line with data.

4 Discussion and Extension

4.1 Endogenous Growth: from User Base to Productivity

In this paper, our primary focus is on the endogenous and joint dynamics of token price and user base. Through token price that reflects agents expectation, the popularity of a platform in the future increases the present user base. Our analysis thus far has taken the blockchain productivity process as exogenous. In reality, many token and cryptocurrency
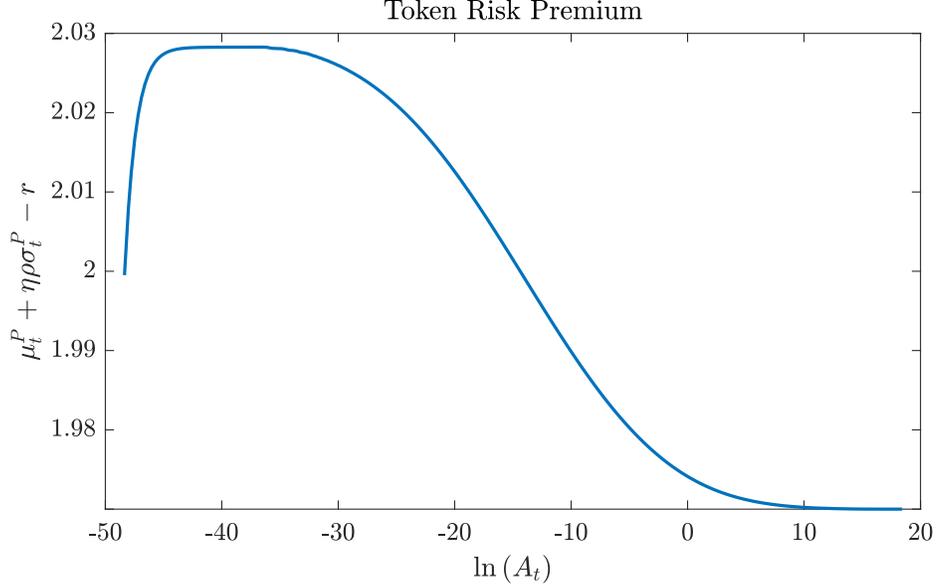
Figure 7: Expected Capital Gain under the Physical Measure.

applications feature an endogenous response of platform productivity to the variation of user base.

A defining feature of blockchain technology is the provision of consensus on decentralized ledgers. In a "proof-of-stake" system, the consensus is more robust when the user base is large and dispersed because no single party is likely to hold a majority of stake; in a "proof-of-work" system, more miners potentially deliver faster and more reliable confirmation of transactions, and miners' participation in turn depends on the size of user base through the associated media coverage (attention in general), transaction fees, and token price. More broadly, A_t represents the general usefulness of the platform. When more users participate, more types of activities can be done on the blockchain. Moreover, a greater user base potentially directs greater resources and research into the blockchain community, accelerating the technological progress.

The endogeneity of blockchain productivity and its dependence on the user base highlight the decentralized nature of this new technology. To reflect this fact and discuss its theoretical implications related to the growth and volatility amplification effects, we modify the process of A_t as follows:

$$\frac{dA_t}{A_t} = \left(\mu_0^A + \mu_1^A N_t\right) dt + \sigma^A dZ_t. \tag{37}$$

By inspection of Equation (2), the definition of trade surplus, it seems that A_t and N_t are

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not separately identified from the perspective of individual users, because either of the two is simply part of the marginal productivity. However, this argument ignores the fact that by feeding N_t into the process of A_t, the growth rate of A_t is no longer i.i.d.

Consider the case where $d\mu^A(N_t)/dN_t > 0$. A higher level of N_t now induces faster growth of A_t, which leads to a higher level of N_t in the future. Similarly, a lower current level of N_t translates into a downward shifting of the path of N_t going forward. In other words, the endogenous growth of A_t induces *persistence* in N_t. In our benchmark setting, N_t is reset every instant, depending on the exogenous level of A_t. Yet, here path dependence arises, which tends to amplify both the growth and unconditional volatility of N_t by accumulating and propagating shocks to A_t. A formal analysis of this extension is certainly important in the light of improving quantitative performances of the model.

Another way to achieve such path dependence is to assume that agents' decision to join the community or quit incurs an adjustment cost, so N_t becomes the other aggregate state variable, just as in macroeconomic models where capital stock becomes an aggregate state variable when investment is subject to adjustment cost. However, this specification does not capture the endogenous growth of A_t.

4.2   New Economy, Token Beta, and "Bubble"

So far, we have fixed the correlation between SDF shock and shock to A_t as a constant. Yet as a blockchain platform or the general technology gains popularity, eases, its token is becoming a systematic asset. Pástor and Veronesi (2009) emphasize that the beta of new technology tends to increases as it becomes mainstream and well adopted. Here, we allow the correlation between SDF and A_t to depend on N_t, and study its implications on token price. Specifically, we decompose the technological shock into two components under the physical measure,

$$d\hat{Z}_t = \rho(N_t) d\hat{Z}_t^\Lambda + \sqrt{1 - \rho(N_t)^2} d\hat{Z}_t^I, \tag{38}$$

where the standard Brownian shock, $d\hat{Z}^I$, is independent from the SDF shock, $d\hat{Z}_t^\Lambda$. Therefore, the correlation between technological shock and SDF shock is $\rho(N_t)$, where we assume $d\rho/dN_t > 0$, that is the blockchain productivity shock becomes increasingly systematic as

the user base grows. Under the risk-neutral measure, we have

$$\frac{dA_t}{A_t} = \left[\hat{\mu}^A - \eta\rho\left(N_t\right)\sigma^A\right]dt + \sigma^A dZ_t, \tag{39}$$

so the risk-neutral, expected growth rate of A_t is $\hat{\mu}^A - \eta\rho\left(N_t\right)\sigma^A$, which now declines in N_t.

Therefore, as A_t grows, there are two opposing forces that drive P_t. On the one hand, the mechanisms that increase P_t are still there: when A_t directly increases the flow utility of token, or indirectly through N_t, token price increases. On the other hand, through the increase of N_t, the expected growth of A_t under the *risk-neutral* measure declines, which pushes P_t down. If our previous mechanisms work in the early stage of adoption while the channel of N_t-dependent token beta dominates in the later stage of adoption, what we shall see in the equilibrium will be a bubble-like behavior – P_t rises initially, and later as N_t rises, P_t declines because the risk-neutral expectation of A_t growth declines.

We complement Pástor and Veronesi (2009) in that our model allows the increase in the correlation between the SDF and A_t to be purely driven by user base without relying on learning. Admittedly, this depends on the specific functional form of $\rho\left(N_t\right)$. The learning mechanism in Pástor and Veronesi (2009) may serve as a micro-foundation, and there could be other channels that the user base affects the correlation between blockchain technological shock and SDF shock.[16]

4.3   Alternative Tokens and Reflecting Boundary

Many blockchain platforms accommodate not only their native tokens but also other cryptocurrencies. For example, any ERC-20 compatible cryptocurrencies are accepted on Ethereum.[17] To address this issue, we may consider an alternative upper boundary of A_t. Define ψ as the cost of creating a new cryptocurrency that is perfect substitute with the token we study because it functions on the same blockchain and therefore faces the same common blockchain productivity and agent-specific trade needs. This creates a reflecting boundary

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[16]Pástor and Veronesi (2009) discusses network effect in Appendix B, but does not model agent heterogeneity, and thus does not produce endogenous gradual adoption of the new technology (the adoption there is either now or all at a later time).

[17]ERC-20 defines a common list of rules for all tokens or cryptocurrencies should follow on the Ethereum blockchain.

at $\overline{A}$ characterized by a value-matching condition and a smooth-pasting condition:

$$P\left(\overline{A}\right) = \psi \text{ and } P'\left(\overline{A}\right) = 0. \tag{40}$$

When token price increases to ψ, entrepreneurs outside of the model will develop a new cryptocurrency that is compatible with the rules of our blockchain system. So, the price level never increases beyond this value. Because it is a reflecting boundary, we need to rule out jumps of token prices, so the first derivative of $P\left(A_t\right)$ must be zero. Again we have exactly three boundary conditions for a second-order ODE and an endogenous upper boundary that uniquely pins down the solution.

Similarly, we may consider potential competing blockchain systems, and interpret ψ as the cost of creating a new blockchain system and its native token, which together constitute a perfect substitute for our current system. This creates the same reflecting boundary for token price. When token price increases to ψ, entrepreneurs outside of the model will build a new system.

Proposition 6 (**Alternative Boundary**). *The upper boundary condition is given by Equation (40) in the two following cases: (1) the blockchain system accepts alternative tokens or cryptocurrencies that can be developed at a unit cost of ψ; (2) an alternative blockchain system that is a perfect substitute of the current system can be developed at a cost of ψ per unit of its native tokens.*

While our framework accommodates the effect of competition, a careful analysis of crypto industrial organization certainly requires more ingredients, especially those that can distinguish between the entry of multiple cryptocurrencies into one blockchain system and competing blockchain systems. Related is the impact of one platform using another platform's native tokens. We explore these issues in on-going research.

4.4 Token Supply Schedule

In practice, many cryptocurrencies and tokens feature an increasing supply over time (for example, Bitcoin) or state-contingent supply in order to stabilize token price (for example, Basecoin). Our framework can be modified to accommodate this feature, and thus, serve as a platform for experimenting the impact of token supply on user base growth and token

price stability. For example, we may consider the law of motion of token supply M given by an exogenous stochastic process, for example, as follows

$$dM_t = \mu^M (M_t, N_t)\, dt + \sigma^M (M_t, N_t)\, dZ_t^A. \tag{41}$$

A new Markov equilibrium features two aggregate state variable, A_t and M_t.

An alternative formulation is to consider Poisson-arriving incremental of M, which closely resemble the Bitcoin supply schedule. This formulation has the analytical advantage that equilibria between two Poisson arrivals still have only one state variable A_t. We can solve the model in a backward induction fashion, starting from the asymptotic future where token supply has plateaued and moving back sequentially in the Poisson time given the value function from the previous step.

As in many macroeconomic models, our framework features monetary neutrality: doubling the token supply *from now on* simply reduces token price by half and does not impact any real variables. However, neutrality is only achieved if the change of token supply is implemented uniformly and proportionally for any time going forward. If token supply is adjusted on a contingent basis, agents' expectation of token price appreciation will be affected, through which supply adjustments influence user base, token demand, and the total trade surplus realized on the platform.

Finally, we emphasize that to achieve the desirable effects of a token supply schedule, the schedule must be implemented automatically without centralized third-party interventions, so that dispersed agents take the supply process as given when making decisions. Such commitment to rules and protocols highlights a key difference between cryptocurrency supply and money supply by governments – through the discipline of decentralized consensus, blockchain developers can commit to a token supply schedule. A clearly defined mandate for central bankers (e.g., inflation and employment targeting) reflects the push for commitment to a state-contingent monetary policy, but in reality, discretions abound, especially in the face of unforeseeable events and political regime transitions.

5 Institutional Background

In this section, we describe the development of the blockchain technology, and then clarify various concepts associated with cryptocurrency, which are not mutually exclusive and are starting to be used interchangeably. Importantly, we highlight two salient features shared among the majority of cryptocurrencies, crypto-tokens, and platform currencies: first, they are used as media of exchange by design ("token embedding"); second, their typical application scenarios exhibit some forms of network effect ("user-base externality").

Blockchain, Cryptocurrency, and Token. The advances in FinTech and sharing economy is largely driven by the increasing preference for forming peer-to-peer connections that are instantaneous and open, which is transforming how people interact, work together, consume and produce. Blockchain-based applications are part of an attempt to create a financial architecture to reorganize the society into a set of networks of human interactions, allowing peers unknown to and distant from one another to interact, transact, and contract without relying a centralized trusted third party. The technology is believed to potentially avoid single point of failure and even reduce concentration of market power, but still face many challenging issues.[18]

Even though not always necessarily required, a majority of blockchain applications entail the use of cryptocurrencies and tokens. Cryptocurrencies are cryptography-secured digital or virtual currencies. Bitcoin represents the first widely-adopted decentralized cryptocurrency, and popularized the concept.[19] Besides Bitcoin, over 1000 different "altcoins" (stand for for alternative cryptocurrency coins, alternative to Bitcoin) have been introduced over the past few years and many central banks are actively exploring the area for retail and payment systems.[20] Many altcoins such as Litecoin and Dogecoin are variants (forks) from Bitcoin,

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[18]Although Bank of England governor Mark Carney dismissed Bitcoin as an alternative currency, he recognized that the blockchain technology benefits data management by improving resilience by "eliminating central points of failure" and enhancing transparency and auditability while expanding what he called the use of "straight-through processes" including with smart contracts. In particular, "Crypto-assets help point the way to the future of money". See, e.g., beat.10ztalk.com. For various applications of the technology, we refer the readers to Harvey (2016) and Yermack (2017), and for smart contracting, Cong and He (2018).

[19]Many retailers in Japan already accept Bitcoins (e.g., Holden and Subrahmanyam (2017)), not to mention that many ICOs are paid using Bitcoins.

[20]For example, People's Bank of China aims to develop a digital currency system; Bank of Canada and Monetary Authority of Singapore use blockchain for interbank payment systems; Deutsche Bundesbank works on prototype of blockchain-based settlement systems for financial assets; in a controversial move, the

with mofidications to the original open-sourced protocol to enable new features. Others such as Ethereum and Ripple created their own Blockchain and protocol to support the native currency. Cryptocurrencies are typically regarded as payment-focused and primarily associated with their own independent blockchain. In these payment and settlement applications as exemplified by Bitcoin and Ripple, cryptocurrencies obviously act as media of exchange on their respective blockchain platforms.

Meanwhile, Blockchain-based crypto-tokens have also gained popularity. In what is known as Initial Coin Offerings (ICO), entrepreneurs sell "tokens" or "AppCoins" to dispersed investors around the globe.[21] Tokens are representations of claims on issuers' cashflow, rights to redeem issuers' products and services, or media of exchange among blockchain users. They usually operate on top of an existing blockchain infrastructure to facilitates the creation of decentralized applications.[22]

However, it is far from clear how cryptocurrencies and tokens derive values and how they should be adopted given their large volatilities. Consequently, there lacks no critics of the development of cryptocurrency, at least Bitcoin, in both the industry and academia.[23] ICOs are also facing quagmires regarding its legitimacy and distinction from security issuance.[24] In the recent hearing on Capital Markets, Securities, and Investment Wednesday, March 14, 2018, the regulators also appear rather divided on the future of cryptocurrencies, digital currencies, ICOs, and Blockchain development.

Obviously, some tokens derive their value from the company's future cashflow, and thus, serve a function similar to securities (thus termed "security tokens"). The vast majority of ICOs that launched in 2016 and 2017 were "utility tokens", which include many of the highest-profile projects: Filecoin, Golem, 0x, Civic, Raiden, Basic Attention Token (BAT),

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government of Venezuela became the first federal government to issue digital currency and announced on Feb 20, 2018 the presale of its "petro" cryptocurrency — an oil-backed token as a form of legal tender that can be used to pay taxes, fees and other public needs.

[21]While the first ICO in 2013 raised a meager $ 500k and sporadic activities over the next two years. 2016 saw 46 ICOs raising about $ 100m and according to CoinSchedule, in 2017 there were 235 Initial Coin Offerings. The year-end totals came in over $3 billion raised in ICO. In August, 2017, OmiseGO (OMG) and Qtum passed a US$1 billion market cap today, according to coinmarketcap.com, to become the first ERC20 tokens built on the Ethereum network and sold via an ICO to reach the unicorn status.

[22]By "on top of" a blockchain, we mean that one can use smart contract templates, for example on the Ethereum or Waves platform, to create tokens for particular applications, without having to create or modify codes from other blockchain protocols.

[23]Rogoff (2017) and Shiller (2017) are notable representatives, although Cochrane argues otherwise in *The Grumpy Economist*, Nov 20, 2017.

[24]See, for example, "Token Resistance," The Economist, November 11th, 2017.

and more. As we illustrate using some of the aforementioned tokens shortly, utility tokens are often the required media of exchange among users for certain products or services, or represent certain opportunities to provide blockchain services for profit as in the case of "stake tokens". Precisely here lies the key innovation of the blockchain technology: allowing peer-to-peer interactions in decentralized networks, as opposed to designing and auctioning coupons issued by centralized product/service providers – an old phenomenon economists understand relatively well (e.g., pay-in-kind crowdfunding).[25]

In this paper, we focus on the common features shared by cryptocurrencies and utility tokens that serve essentially the role of media of exchange among blockchain users. We thus use "tokens", "cryptocurrencies", etc., interchangeably and often collectively refer to them as "tokens". Next, we highlight these unique features of the blockchain technology that distinguishes the economics of introducing and valuing tokens from what we already know in the literature of monetary economics and asset pricing.

Token Embedding. Many blockchain-based decentralized networks introduce native currencies – a phenomenon we call "token embedding". In the following, we elaborate on the rationales behind such phenomenon and relate them to the issue of money velocity, motivating our formal analysis in Section 2.

First, in the virtual economy, potential users are likely from around the globe, using fiat money issued by and subject to specific countries' legal and economic influences. Transacting in a uniform currency is simply more convenient, free from the transaction costs of currency exchange. For example, it is cheaper to make international payments and settlements using Ripples (XRP) on the Ripple network. Even though Ethereum platform allows other App-Coins and cryptocurrencies (provided that they are ERC-20 compatible), many transactions and fundraising activities are still carried out using Ethers (ETH) because of its convenience and popularity (i.e., widely accepted by Ethereum users).

Second, from a theoretical perspective, it is advantageous to adopt a standard unit of account in the ecosystem because it mitigates the risks of asset-liability mismatches when they are denominated in different units of account (Doepke and Schneider (2017)). This is

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[25]Media often analogizes utility tokens to "corporate coupons", which allow consumers to redeem products or services from the service provider. Although some tokens are indeed corporate coupons, it is thus far neglected that the majority of them are not. Not only the valuation framework differs, but the legal and regulatory implications differ as well.

particularly relevant on a blockchain platform designed for smart contracting. Some argue that the lack of trust in an online space, very much due to the anonymity of participants, implies that trade has to be quid pro quo, so a means of payment is required (Kiyotaki and Wright (1989)).

Then why not just use US dollars or other existing currencies as settlement media? This leads to the third rationale: native currency helps to incentivize miners, validators, and users to contribute to the stability, functionality (provision of decentralized consensus), and prosperity of the ecosystem (Nakamoto (2008)). For example, for blockchain applications where decentralized consensus is achieved through the mechanism of "proof-of-stake", the ownership of native currency entitles platform users to be the consensus generator/recorder; for blockchains relying on "proof-of-work" such as Bitcoins and Filecoins, native tokens are used to reward miners for block creations in the consensus processes; moreover, to profit from providing validation services, OmiseGo tokens (OMG) are required as proof of stakes on the OmiseGo blockchain. If a blockchain application is developed without a native currency, then the incentive of users is no longer directly linked to the platform in question. Practitioners are very well aware of this issue, as Strategic Coin explains in its BAT token launch research report.[26]

Fourth, introducing native currency allows the issuer to collect seigniorage, especially through ICOs (e.g., Canidio (2018)). In contrast to sovereigns who cannot easily commit to a money supply rule, blockchain developers can commit to an algorithmic rule of token supply to generate scarcity. Provided that users need to hold tokens to transact on the platform, a positive token price can arise in an equilibrium, and such value is collected by the developers at ICO, reflecting a form of monopoly rent – the fact that users can only conduct some activities on a particular blockchain platform translates into a high price of its native currency, and more ICO revenues to the developer.

These rationales motivate us to focus on platforms with native tokens. But we still need to ask why cryptocurrency may have a determinant value in the first place. In principle, if one wants to transact on a blockchain platform, one can exchange dollars for its native currency, and make a transfer on the blockchain, and then immediately, the payee may exchange the

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[26]BAT serves as a medium exchange between users, advertisers, and publishers who participate in the Brave browser ecosystem. Advertisers purchase ads using BAT tokens, which are then distributed among both publishers and browser users as compensation for hosting the ads and viewing them, respectively.

native currency back into dollars. If the whole process happens instantaneously, i.e., the velocity of native currency is infinite, then there does not exist a net demand for native currency, so there exists an equilibrium of zero dollar price and equilibria with any positive level of price of native currency. Therefore, we need to pin down a positive demand, so that with a algorithmically controlled supply, token price can be determined. This brings us to the second aspect of token embedding: agents actually need to hold the medium of exchange to profit from on-chain activities. This is indeed the case in practice for at least three reasons, in addition to the obvious practical concern that converting between the native token and other fiat money have high fixed costs—a concept similar to convenience yield of currencies and commodities.

First, a demand may arise because decentralized miners or service providers ("keepers") may have to hold the native currencies to earn the right to serve the system. Proof-of-Stake protocols typically fall in this category. These tokens are sometimes referred to as work tokens or staking tokens, and notable implementations include Keep (off-chain private computation), Filecoin (distributed file storage), Truebit (off-chain computation), Livepeer (distributed video encoding), and Gems (decentralized mechanical Turk). To enforce some sort of mechanism to penalize workers who fail to perform their job to some pre-specified standard, work tokens have to be held. For example, in Filecoin, service providers contractually commit to storing some data with 24/7 access and some minimum bandwidth guarantee for a specified period of time. During the contract term, service providers must "escrow" some number of Filecoin, which can be automatically slashed (taken away) should they fail to perform the service. Staking tokens in OmiseGo are also emerging, where locking up tokens in a smart contract allows a user to access the market place.

Second, blockchains enable the use of smart contracts (Cong and He (2018)). Though not yet widely implemented, smart contracts may involve automated transfers for contingencies specified over an extended period of time, effectively requiring escrowing the tokens.[27] In other words, agents hold cryptocurrency as collateral. While this is similar to the traditional third-party escrow accounts, what it implies is that the tokens are locked up with at least one contracting party.

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[27]Balvers and McDonald (2017) also argues that automated collateral in terms of tokens can help stabilize the purchasing power of cryptocurrency, a point very related to our emphasize on a positive crytocurrency demand.

Third, there are technical and legal limits on how quickly transactions can be validated and accepted. While many protocols such as the Lightening Network and Ethereum process transactions significantly faster than Bitcoin (seconds versus 10-11 minutes), the decentralized nature of the validation means it always takes some amount of time to ensure robustness and synchronization of the consensus. During such confirmation time, cryptocurrency cannot be liquidated by either party of the transfer. For example, a coinbase account requires at least three confirmations—blocks added subsequently—before a transaction shows up and become spendable. Related is the traditional anti-money-laundering practice where one party of the transaction needs to show the funding source and prove it has been in the account for a certain period of time, as seen for example in the KYC (know-your-customer) process for investing in the pre-sale of Dfinity tokens.

Next, we discuss user-base externality, another key feature of decentralized network that our model captures.

User-Base Externality. User-base externality has been well recognized as one of the defining features of P2P platforms, sharing economy, and various decentralized systems. When more people join the platform, individuals enjoy more surplus through interacting with other users because it is easier to find a trade partner. "Trade" here can be very general, encompassing selling products and services and signing a long-term financing contracts. The utility of using cryptocurrencies and crypto-tokens obviously goes up when more people use the blockchain platform.[28] Moreover, UnikoinGold on Unikrn (decentralized token for betting on e-sports and gambling) and Augur (decentralized prediction market) are examples showing that achieving a critical mass is crucial in platform business (e.g., Evans and Schmalensee (2010)).

That said, existing discussions on user-base externality are often static, leaving out intertemporal effects that can be even more important. The fact that a larger user base today helps improve the technology tomorrow, and a larger anticipated user base tomorrow encourages greater investments today are examples of how user-base externality can play an

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[28]According to CoinSchedule, 34.5% of the ICO-financed projects over the past two years focus on infrastructure. While the other top categories included trading and investing at 13.7%, finance at 10.2%, payments at 7.8%, data storage also at 7.8%, and drugs and healthcare at 5.5%, amongst dozens of other industry categories. Regardless the ICO category, these projects share user-base externality as a common attribute, and in terms of user adoption, exhibit a S-shaped development curve – the growth of user base feeds on itself.

inter-temporal role. Filecoin the data storage network, Dfinity the decentralized computation infrastructure, marketplace such as overstock (and its ICO), and infrastructure projects such as Ethereum and LITEX all exhibit user-base externality in both contemporaneous and inter-temporal fashions, as our model highlight.

Commons and Assets with Network Effect. Given token embedding and user-base externality, tokens essentially constitute an asset that delivers owners "dividends" (in terms of user surplus) which increase in the scale of the platform. This type of assets are not restricted to tokens or blockchains. Network effect or user-base externality is prevalent and particularly important in the early stage of adoption for social networks and payment networks such as Facebook, Twitter, YouTube, WeChat, and PayPal. Other examples of such assets include membership of clubs with benefits growing in the network size, and collectibles of limited edition for sports teams.

As such, the insights gleaned from our model may apply to platform currencies in general, such as those used in interactive online games (e.g., World of Warcraft), virtual worlds (e.g., Second Life), and social networks (e.g., Facebook). Our findings also help inform what is likely to happen for other sharing economy applications such as UBER and AirBnB to introduce native tokens. In fact, consistent with our model, when Tencent QQ introduced Q-coin, a case to which our model is applicable, many users and merchants quickly started accepting them even outside the QQ platform (mapped to increase in system trade surplus in our model), tremendously accelerating adoption and token price appreciation.[29]

At a holistic level, this type of asset is important in the creation of commons that underly modern life. TCP/IP, HTTP, GPS, and the English language are some early examples.[30] Instead of having government or private firms facilitating the building of commons, blockchains offer new possibilities and as we show earlier, token embedding plays important roles in growing commons.

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[29]Annual trading volume reached billions of RMB in late 2000s and the government has to intervene. See articles *China bars use of virtual money for trading in real goods* and *QQ: China's New Coin of the Realm?* (WSJ). Halaburda and Sarvary (2016) provide comprehensive discussions on various platform currencies.

[30]Alex Tabarrok gives a nice treatise on the concept in *Economics, Web/Tech* June 4, 2018, "Blockchains and the Opportunity of the Commons.

6   Conclusion

This paper provides the first dynamic pricing model of cryptocurrencies and tokens, taking into consideration the user-base externality and endogenous adoption. Our model highlights a key benefit of introducing cryptocurrencies and crypto-tokens on blockchain platforms: when agents expect future technology or productivity progress, token price appreciation induces more agents to join the platform by serving as an attractive investment. In other words, token capitalizes future growth and speeds up user adoption, which is welfare-enhancing. Tokens can also reduce user base volatility. We characterize the inter-temporal feedback mechanism and show it leads to an S-shaped adoption curve in equilibrium, and token price dynamics crucially depend on the platform productivity, endogenous user adoption, and user heterogeneity. More generally, our framework can be applied to dynamic pricing of assets associated with a platform or system with network externality.

Appendix - Proofs

A1. Proof of Proposition 1

Figure 1 illustrates the equilibrium determination given A_t and μ_t^P, which we take as a snapshot of the dynamic equilibrium with time-varying productivity and expectation of price change. Obviously the response function is

$$R\left(n; A_t, \mu_t^P\right) = 1 - G\left(\underline{u}\left(n; A_t, \mu_t^P\right)\right). \tag{42}$$

The equilibrium N_t is the intersection of the 45° line and the response curve. Define $A(n)$ to be the unique solution to

$$1 - G\left(-\ln(n) + \ln\left(\frac{\phi}{A_t\alpha}\right) - \left(\frac{1-\alpha}{\alpha}\right)\ln\left(\frac{1-\alpha}{r-\mu_t^P}\right)\right) = n, \quad \forall n \in (\epsilon, N_t). \tag{43}$$

for some $N \in (0,1)$. Then by the Least-Upper-Bound-Property of real numbers, $\{A(n), n \in (0,1)\}$ has an infimum, which we denote by $\underline{A}$. When $A_t > \underline{A}$, there is an $n \in (0,1)$ such at the LHS of (49) is greater than the RHS, i.e., the response curve is above the 45° line. Combined with the fact that $R\left(n; A_t, \mu_t^P\right)$ is continuous and $R\left(1; A_t, \mu_t^P\right) \leq 1$, and that he response curve is continuous in $[0,1]$, we conclude that the response curve crosses the 45° line at least once by the Intermediate Value Theorem.

Next, given $\frac{g(u)}{1-G(u)}$ is increasing, we show that the response curve crosses the 45° line exactly once. First note that $R\left(n; A_t, \mu_t^P\right) - n$ either has positive derivative or negative derivative at $n = 0$. If it has positive derivative, then suppose as n increases, the first time the response curve crosses the 45° at n'. Then the derivative of $R\left(n; A_t, \mu_t^P\right) - n$ must be weakly negative at n', i.e.,

$$g\left(\underline{u}\left(n'; A_t, \mu_t^P\right)\right) \leq n' = R\left(n'; A_t, \mu_t^P\right) = 1 - G\left(\underline{u}\left(n'; A_t, \mu_t^P\right)\right) \tag{44}$$

now suppose the response curve next crosses the 45° line from below at $n'' > n'$. Then the

derivative of $R\left(n; A_t, \mu_t^P\right) - n$ at n'' is

$$\frac{g\left(\underline{u}\left(n''; A_t, \mu_t^P\right)\right)}{n''} - 1 \;=\; \frac{g\left(\underline{u}\left(n''; A_t, \mu_t^P\right)\right)}{1 - G\left(\underline{u}\left(n''; A_t, \mu_t^P\right)\right)} - 1 \tag{45}$$

$$<\; \frac{g\left(\underline{u}\left(n'; A_t, \mu_t^P\right)\right)}{1 - G\left(\underline{u}\left(n'; A_t, \mu_t^P\right)\right)} - 1 \tag{46}$$

$$=\; \frac{g\left(\underline{u}\left(n'; A_t, \mu_t^P\right)\right)}{n'} - 1 \tag{47}$$

$$<\; 0 \tag{48}$$

where the first inequality comes from the monotone hazard rate and the fact that $u\left(n; A_t, \mu_t^P\right)$ is decreasing in n for $n \in [\epsilon, 1 + \epsilon]$, and the second inequality follows from (44). This contradicts the presumption that the response curve reaches the 45° line from below. Therefore, we conclude there is a unique equilibrium adoption level n.

Now if $R\left(n; A_t, \mu_t^P\right) - n$ has negative derivative at $n = 0$, then simply taking $n' = 0$ in the above argument, and again we arrive at the conclusion that there cannot be another intersection at $n'' > n'$. In sum, either we only have a degenerate $N_t = 0$ in equilibrium, or we have exactly one $N_t > 0$ in equilibrium.

Finally, to show that a non-degenerate equilibrium N_t is increasing in μ_t^P, consider $\tilde{\mu}_t^P > \mu_t^P$. Suppose the contrary that in equilibrium $\tilde{N}_t \leq N_t$.

Given an equilibrium N_t, the response function satisfies that

$$1 - G\left(-\ln(n) + \ln\left(\frac{\phi}{A_t \alpha}\right) - \left(\frac{1 - \alpha}{\alpha}\right) \ln\left(\frac{1 - \alpha}{r - \mu_t^P}\right)\right) > n, \quad \forall n \in (0, N_t). \tag{49}$$

We know that

$$\tilde{N}_t \;=\; 1 - G\left(\underline{u}\left(\tilde{N}_t; A_t, \tilde{\mu}_t^P\right)\right)$$

$$=\; 1 - G\left(-\ln\left(\tilde{N}_t\right) + \ln\left(\frac{\phi}{A_t \alpha}\right) - \left(\frac{1 - \alpha}{\alpha}\right) \ln\left(\frac{1 - \alpha}{r - \tilde{\mu}_t^P}\right)\right)$$

$$>\; 1 - G\left(-\ln\left(\tilde{N}_t\right) + \ln\left(\frac{\phi}{A_t \alpha}\right) - \left(\frac{1 - \alpha}{\alpha}\right) \ln\left(\frac{1 - \alpha}{r - \mu_t^P}\right)\right)$$

$$>\; \tilde{N}_t, \tag{50}$$

where the first inequality uses $\tilde{\mu}_t^P > \mu_t^P$ and the second inequality uses the fact that $\tilde{N}_t \leq N_t$

and (49). This contradiction implies that the equilibrium N_t has to be increasing in μ_t^P.

A2. Proof of Proposition 3

From Equation (15), $\frac{d^2 P_t}{dA_t^2}$ is Lipschitz continuous in $\frac{dP_t}{dA_t}$, and we can verify that for finite $\underline{U}$, the problem of solving the ODE (15) under the boundary conditions (17) and the value matching part of (19) at A^* satisfies all the regularity conditions in Theorems 4.17 and 4.18 in Jackson (1968). Therefore according to the theorems, there is a unique solution, which in turn implies that our boundary-value problem has a unique solution. As a remark, when $\underline{U} = -\infty$, full adoption only happens when $A_t \to \infty$ in (20). We conjecture the solution is still unique, but the proof is beyond this paper.

A3. Proof of Proposition 5

First, we consider the case without token. Using Itô's lemma, we can differentiate Equation (26), and then, by matching coefficients with Equation (27), we can derive the expressions for μ_t^N and σ_t^N:

$$dN_t = -g\left(\underline{u}_t^{NT}\right) d\underline{u}_t^{NT} - \frac{1}{2} g'\left(\underline{u}_t^{NT}\right) \left\langle d\underline{u}_t^{NT}, d\underline{u}_t^{NT}\right\rangle, \tag{51}$$

where $\left\langle d\underline{u}_t^{NT}, d\underline{u}_t^{NT}\right\rangle$ is the quadratic variation of $d\underline{u}_t^{NT}$. Using Itô's lemma, we differentiate Equation (25)

$$
\begin{aligned}
d\underline{u}_t^{NT} &= -\frac{1}{N_t} dN_t + \frac{1}{2N_t^2} \left\langle dN_t, dN_t\right\rangle - \frac{1}{A_t} dA_t + \frac{1}{2A_t^2} \left\langle dA_t, dA_t\right\rangle \\
&= -\left(\frac{\mu_t^N}{N_t} - \frac{\left(\sigma_t^N\right)^2}{2N_t^2} + \mu^A - \frac{\left(\sigma^A\right)^2}{2}\right) dt - \left(\frac{\sigma_t^N}{N_t} + \sigma^A\right) dZ_t^A
\end{aligned} \tag{52}
$$

Substituting this dynamics into Equation (51), we have

$$
\begin{aligned}
dN_t &= \left[g\left(\underline{u}_t^{NT}\right) \left(\frac{\mu_t^N}{N_t} - \frac{\left(\sigma_t^N\right)^2}{2N_t^2} + \mu^A - \frac{\left(\sigma^A\right)^2}{2}\right) - \frac{1}{2} g'\left(\underline{u}_t^{NT}\right) \left(\frac{\sigma_t^N}{N_t} + \sigma^A\right)^2 \right] dt \\
&\quad + g\left(\underline{u}_t^{NT}\right) \left(\frac{\sigma_t^N}{N_t} + \sigma^A\right) dZ_t^A,
\end{aligned} \tag{53}
$$

By matching coefficients on dZ_t^A with Equation (27), we can solve σ_t^N.

Next, we consider the case with token. Once token is introduced, N_t depends on the expected token price appreciation μ_t^P. which is also a univariate function of state variable A_t because by Itô's lemma, μ_t^P is equal to $\left(\frac{dP_t/P_t}{dA_t/A_t}\right)\mu^A + \frac{1}{2}\frac{d^2P_t/P_t}{dA_t^2/A_t^2}\left(\sigma^A\right)^2$. In equilibrium, its law of motion is given by a diffusion process

$$d\mu_t^P = \mu_t^{\mu^P}dt + \sigma_t^{\mu^P}dZ_t^A. \tag{54}$$

Now, the dynamics of $\underline{u}_t$ becomes

$$
\begin{aligned}
d\underline{u}_t = {}& -\frac{1}{N_t}dN_t + \frac{1}{2N_t^2}\langle dN_t, dN_t\rangle - \frac{1}{A_t}dA_t + \frac{1}{2A_t^2}\langle dA_t, dA_t\rangle \\
& -\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1}{r-\mu_t^P}\right)d\mu_t^P - \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1}{2\left(r-\mu_t^P\right)^2}\right)\langle d\mu_t^P, d\mu_t^P\rangle
\end{aligned}
\tag{55}
$$

Let σ_t^u denote the diffusion of $\underline{u}_t$. By collecting the coefficients on dZ_t^A in Equation (55), we have

$$\sigma_t^u = -\frac{\sigma_t^N}{N_t} - \sigma^A - \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\sigma_t^{\mu^P}}{r-\mu_t^P}\right), \tag{56}$$

which, in comparison with Equation (52), contains an extra term that reflects the volatility of expected token price change. Note that, similar to Equation (51), we have

$$dN_t = -g\left(\underline{u}_t\right)d\underline{u}_t - \frac{1}{2}g'\left(\underline{u}_t\right)\langle d\underline{u}_t, d\underline{u}_t\rangle, \tag{57}$$

so the diffusion of N_t is $-g\left(\underline{u}_t\right)\sigma_t^u$. Matching it with the conjectured diffusion coefficient σ_t^N gives σ_t^N.

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