Collateral Runs

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[1] The views of this presentation are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.

Motivation

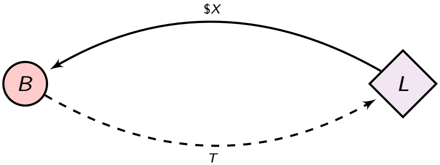
...Thursday had brought an onslaught of new client demands for their cash back, as well as demand for collateral from numerous funding counter parties, leaving the firms with only about $5 billion....

"Okay, where are we in terms of cash we can raise? What collateral can we pledge?"

— Kelly (2009), quoting Sam Molinaro, CFO of Bear Sterns, evening of March 13th 2008.

Motivation – Classic *'Repo runs'*

- Repo contract: Borrower borrows $\$X$ from Lender(s) after pledging collateral T



- Collateral is risky and may take values between $\left(T^D, T^U\right)$
- If low values are likely, lenders may get "spooked" and run to withdraw their cash
- Classic view of repo runs: *Run from liabilities' side*
- **Solution**: Over-collateralization/Zero VaR contracts whereby $X \leq T^D$

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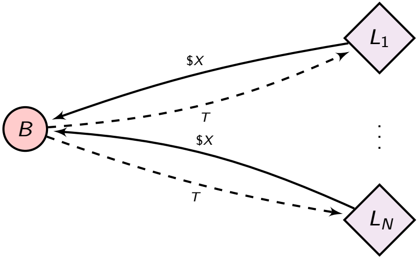

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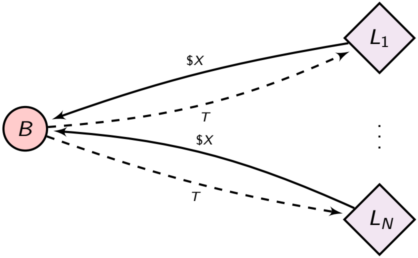

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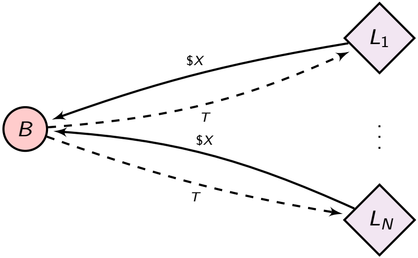

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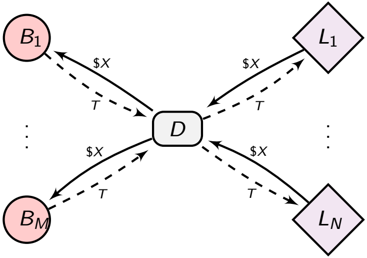
Motivation – Broker dealer and Rehypothecation

- A broker dealer engages in a repo with a lender and a reverse repo with a borrower, and *rehypothecates* the collateral
  - Contrary to other forms of collateralized lending: no limits on repo rehypothecation



Dealer's Balance Sheet

Assets	Liabilities
Reverse repos $\sum_M X$	Repos $\sum_N X$

- Here dealer is matching dollar value of assets (reverse repos) and liabilities (repo)
- Dealer is a veil & faces no "repo" run risk for zero VaR contracts
  - Repo exempt from automatic stay: lender has immediate access to collateral

Inst. Details

Motivation – What is a *'Collateral run'*?

- Dealer may extend fewer funds to B than the ones received from $L \rightarrow X' < X$
- This allows them to extract liquidity potentially used for proprietary trading

- If $\sum_N x - \sum_M x'$ is invested in sufficiently risky assets, borrowers may get "spooked" and **run to withdraw their collateral**
- Note that under a zero VaR contract, $X \leq T^D$, both L and D are fully protected
  $\rightarrow$ Collateral runs can be independent of repo runs

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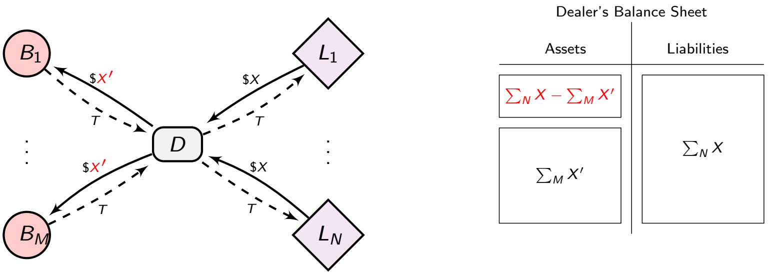

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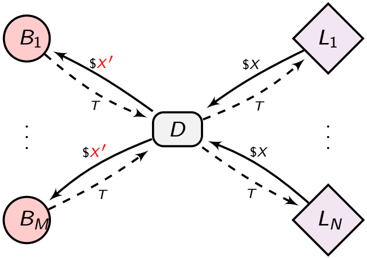

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Literature Review

- Collateralized lending
  - Geanakoplos (2003), Fostel & Geanakoplos (2008, 2015)
- Repo & Rehypothecation
  - Gorton & Metrick (2012), Duffie (2013), Martin et al. (2014), Copeland, Martin & Walker (2015), Krishnamurthy, Nagel & Orlov (2015), Baklanova et al. (2017), Gottardi et al. (2017), Infante (2018)
- Bank Runs & Global Games
  - Diamong & Dybvig (1983), Rochet & Vives (2004), Ahnert et al. (2018), Goldstein & Pauzner (2005), Kashyap, Tsomocos & Vardoulakis (2017)

Model Setup

- 3 periods: $t \in \{0, 1, 2\}$
- 2 assets in perfectly elastic supply:
  - 1 safe asset — valued at 1 in all periods, serves as collateral: T
  - 1 risky asset — payoff in $t = 2$ depends on state $\theta \sim U(0, 1)$ which is realized in $t = 1$

$$\tilde{R} = \begin{cases} R^U > 1 & \text{with prob } \theta \\ R^D \in [0, 1) & \text{with prob } 1 - \theta \end{cases}$$

    valued at 1 in $t = 0$ and in $t = 1$ has liquidation value $\lambda \cdot E(\tilde{R}|\theta)$ with $\lambda < 1$

- 3 types of agents that consume in $t = 2$:
  - Money funds (M): Continuum of "very" risk averse, invest in repo backed by T
  - Hedge funds (H): Continuum of risk neutral, additional preference for T:

$$\eta T > T \rightarrow \text{Incentives to take leverage}$$

  - Dealer (D): Single intermediary with no initial wealth, who intermediates funds and collateral between H and M

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Timeline Overview

- $t = 0$: Initial financing period
  - D sets one period repo contracting terms for both counterparties & invests in risky asset
- $t = 1$: Intermediate period, θ is realized
  - H receives individual noisy signal of state θ & decides whether to roll over its repo or withdraw its collateral
  - Give the fraction of withdrawers, D pays M to recover collateral of withdrawing H & liquidating a fraction of its risky asset at a cost
  - If withdrawals are severe, entire risky asset position is liquidated $\longrightarrow$ collateral run!
- $t = 2$: Final consumption period, if collateral run does not occur
  - Asset outcome realized & contracts settled
  - D protected by limited liability $\longrightarrow$ possible default

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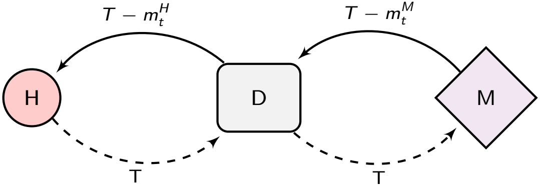
Contracting Terms & Rehypothecation

- D offers take-it-or-leave-it repo contracting terms
- Repo contract at time $t \in \{0, 1\}$ for agent $i \in \{H, M\}$:
  - Margin: degree of overcollateralization $m_t^i \longrightarrow$ initial loan amount $T - m_t^i$
  - Repurchase price: final repayment to repo F_t^i
- Notation: $\Delta m_t := m_t^H - m_t^M$ and $\Delta F_t := F_t^H - F_t^M$
- Initial leg of rehypothecation for single repo/reverse repo pair:



D's balance sheet at $t = 0$

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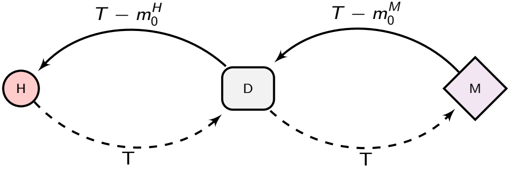
  - D buys risky asset with liquidity windfall Δm_0



- Ingredients to our story:
  1. Rehypothecation of collateral
  2. Liquidity windfall from different contract terms
  3. Proprietary trading in risky-illiquid investments

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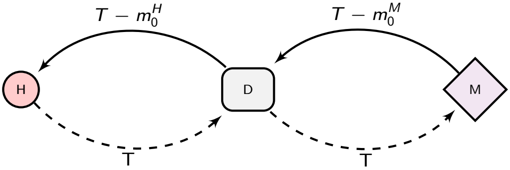

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- H receives noisy signal about θ & decides whether to roll over its repo
  - If H withdraws, D receives ΔF_0
  - If H rolls over, D receives $\Delta F_0 + \Delta m_1$ (case ≥ 0)
- Depending on the portion of H withdrawing, $\mu \in [0, 1]$, one these can happen:
  (a) μ "small": D does not need to sell: $\mu\Delta F_0 + (1-\mu)(\Delta F_0 + \Delta m_1) \geq 0$

  (b) μ "intermediate": D must sell ξ of risky asset position:
  $\mu\Delta F_0 + (1-\mu)(\Delta F_0 + \Delta m_1) + \xi\lambda\mathbb{E}(\tilde{R}|\theta) = 0$

  (c) μ "severe": Full liquidation, M seize collateral & H (sequentially) served with probability f: $f\mu\Delta F_0 + (1-\mu)(\Delta F_0 + \Delta m_1) + \lambda\mathbb{E}(\tilde{R}|\theta) = 0$

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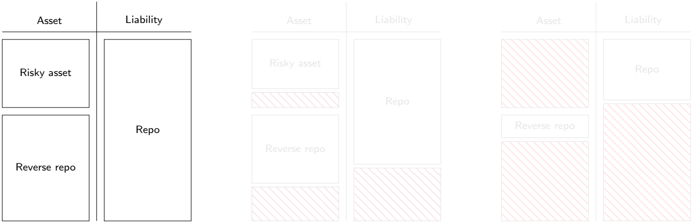

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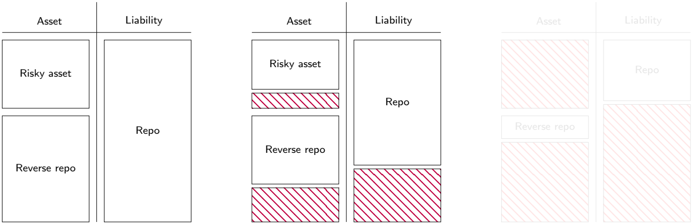

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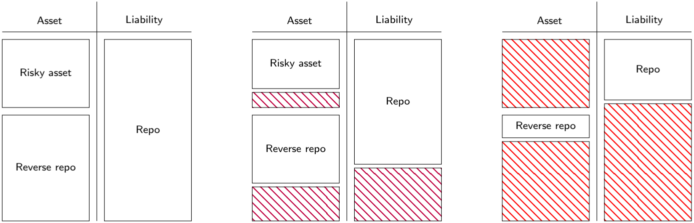

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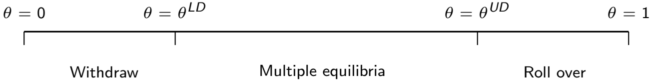
Hedge Fund Incentives & Coordination Failure

- Rolling over:
  - Upside proportional to *levered position* $\eta T - F$
  - But exposure to run risk
- Early withdrawal:
  - Reduces exposure to run risk
  - But upside proportional to *unlevered position* $\eta(T - F)$
- H's decision to withdraw depends on signal over θ **and** beliefs over number of other hedge funds withdrawing$\rightarrow$ Coordination problem!

Global Game

- At beginning of $t = 1$ H receive a noisy signal $x_i = \theta + \epsilon_i$, $\epsilon_i \sim U(-\epsilon, \epsilon)$, i.i.d.
- With complete information about state ($\epsilon_i = 0$)→ multiple equilibria



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  - If signal in-between, coordination failure→ D solvent, but illiquid

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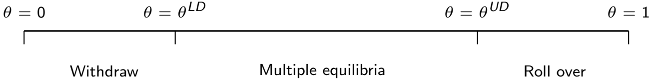

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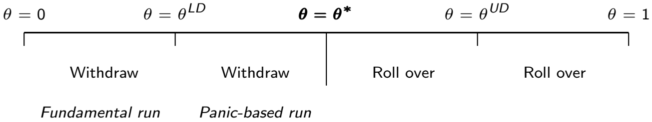
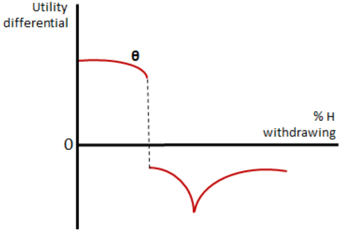


Existence and Uniqueness of Collateral Run

Proposition

Given $0 < \theta^{LD} < \theta^{UD} < 1$ and small noise, there exists a unique θ^* such that a run occurs only for $\theta < \theta^*$

- Define $\nu(\mu, \theta)$ the utility differential between rolling over and withdrawing when μ H withdraw and fundamentals are θ



- θ^* is the level of fundamentals that makes H indifferent

$$V(\theta^*, \Delta m_t, \Delta F_t) = \int_0^1 \nu(\mu, \theta^*) d\mu = 0$$

▸ Details

Threshold Equilibria

- In $t = 0$, D sets contracting terms $(\Delta m_t, \Delta F_t)$ to maximize

$$\int_{\theta^*}^{1} \theta \cdot u(R^U \Delta m_0 + \Delta F_0 + \Delta m_1 + \Delta F_1) d\theta$$

  taking into consideration how the contracting terms affect the probability that a run occurs, θ^*, determined implicitly by

$$V(\theta^*, \Delta m_t, \Delta F_t) = 0$$

- Some intuition behind optimal contracting terms
  - D would like to increase the liquidity windfall, Δm_0, but understand that by doing so she might increase the probability that a run occurs
  - D would like to increase liquidity in the interim period ,Δm_1, but understands that by doing so she only affects those H that want to roll over
- There exist equilibrium contracting terms resulting in Collateral Runs

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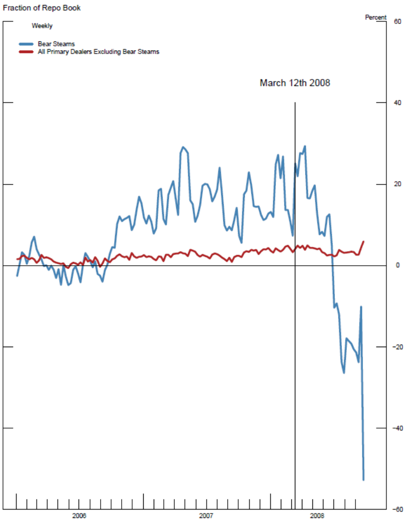
Case Study: Bear Stearns Windfall as a Fraction of Repo Book



Note: Fraction calculated by (Securities Out − (Securities In + Net Position))/ Securities Out
Source: FR 2004C and FR2004A

Data details

Concluding Remarks

- Paper highlights symmetry in dealer balance sheet:
  - Loss of securities introduces runs risk from the asset side
- Use of global games to refine equilibria to eliminate multiplicity
  - Existence of panic-based collateral runs
- Policy prescription:
  - Regulators should consider stability threats from collateral runs
  - Minimum haircut regulation does not fully address stability concerns in repo markets

Back-up slides

Institutional Details

- Repurchase Agreement (repo): secured loans backed by financial assets
- Two main segments in the U.S.:
  - Tri-party repo market ($\approx$ \$1800 Billion):
    Cash investors (i.e., money funds, corporate cash managers) provide secured funding to securities dealers
  - Bilateral repo market :
    Securities dealers provide secured funding to investors and source specific collateral
- Dealers borrow funds to finance inventory and conduct "matched book"
- Repo exempt from automatic stay: cash lenders has immediate access to collateral
- No limits to repo rehypothecation: SEC Rule 15c3-3 does not apply

Back

ν Function

$$\nu(\mu,\theta) = \begin{cases} \theta\left[(\eta-1)T - \Delta F_1\right] + (1-\theta)G_S^D(\mu,\theta) - \eta\Delta m_1 & \mu \in [0,\mu_S) \\ \theta\left[(\eta-1)T - \Delta F_1\right] + (1-\theta)G_I^D(\mu,\theta) - \eta\Delta m_1 & \mu \in [\mu_S,\mu_I) \\ \theta G_I^U(\mu,\theta) + (1-\theta)G_I^D(\mu,\theta) - \eta\Delta m_1 & \mu \in [\mu_I,\mu_R) \\ -\eta\frac{\lambda\overline{R}_\theta\Delta m_0 + \Delta F_0 + \Delta m_1}{\mu} & \mu \in [\mu_R,1] \end{cases}$$

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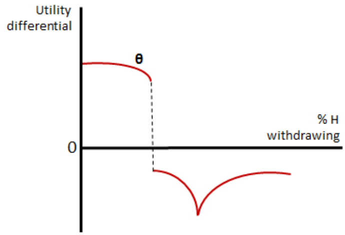
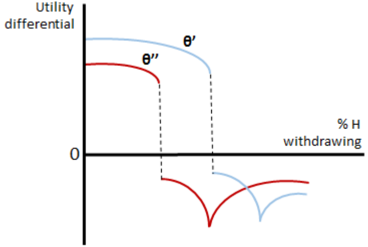
Aside: Technical point

Usual proof requires global strategic complementarities and state monotonicity



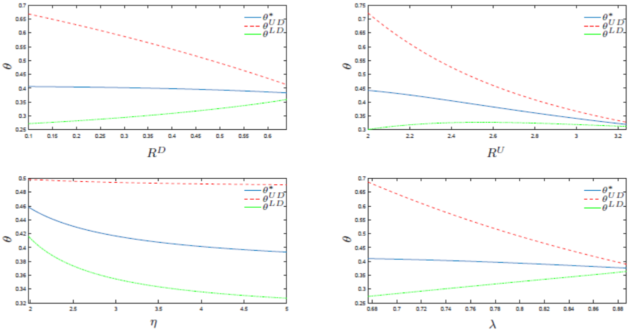
- Bank-run models exhibit one-sided strategic complementarities
- Given that a run occurs, the incentives to withdraw are lower as μ increases
- Goldstein-Pauzner (2005) provide a proof, but fix liquidation value



- We endogenize the liquidation value at the cost of losing global state monotonicity
- Given that a run occurs, the incentives to withdraw are higher as θ increases

Equilibrium Comparative Statics

- We prove in the paper that the dealer will choose under certain condition contracting terms giving rise to *panic-based* collateral runs
- Some comparative statics:



- Increase in asset liquidation value decreases uncertainty and probability of a run
- Increase in η increases uncertainty but decreases the probability of a run.

Estimate of Liquidity Windfall: FR 2004

- FR 2004 reports funds paid or received in short-term financing transactions
- Estimate of cash windfall:

$$CW = (R - m_R) - (RR - m_{RR}) + (L - S) = -\text{"}BoxConstraint\text{"} + \Delta m_0 \leq \Delta m_0$$

- Box Constraint: budget constraint for securities that must be positive (Huh & Infante, 2018)

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