“Sowing the Wind” Monetary Policy *

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Abstract

Seeking to mitigate slumps, leaning monetary policy raises short-term interest rates during booms to rein in financial excesses. We develop a model with speculative booms and demand recessions in which such leaning *exacerbates* slumps (“sows the wind”). During a speculative boom, unexpected rate hikes harm levered investors, leaving the economy more vulnerable to a crash. Systematic rate hikes run the economy cold during booms, deepening demand recessions. To avoid unexpected falls in asset prices, optimal policy involves a steepening of the yield curve, achieving macroprudential benefits by signaling policy rate hikes if speculation worsens.

Keywords: Speculation, leverage, financial stability, monetary policy, leaning against the wind

JEL Codes: E12, E21, E30, G01, G11

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Why then did the Federal Reserve raise interest rates in 1928? The principal reason was the Fed's ongoing concern about speculation on Wall Street.... The market crash of October 1929 showed, if anyone doubted it, that a concerted effort by the Fed can bring down stock prices. But the cost of this 'victory' was very high.'

Bernanke (2004), Money, Gold, and the Great Depression.

1. Introduction

High asset prices and leverage portend costly financial crises (Greenwood et al. 2022). Tighter monetary policy appears to increase risk premia and may reduce risk taking (Gertler and Karadi 2015).[1] These observations point to the potential benefits from central banks raising the policy rate to lean against the wind during speculative booms—a view going back to Kindleberger (1973) and Minsky (1977) and recently emphasized by Kashyap and Stein (2023). However, tightening monetary policy during speculative booms can trigger or exacerbate the very crisis central banks seek to avoid (Schularick et al. 2021; Grimm et al. 2023a; Jimenez et al. 2023), as illustrated by the Great Depression (Bernanke 2004). The banking distress in 2023 following rapid rate hikes were a pointed reminder of the financial stability risks associated with monetary tightening after a period of accommodative policy and falling risk premia (Jiang et al. 2023). We develop a model consistent with such evidence regarding the interaction of monetary policy and financial stability. We study optimal monetary policy balancing Kindleberger-Minsky's perspective with Bernanke's caution about undesirable tightening.

Leaning monetary policy responds to speculative booms by raising interest rates to rein in financial excesses. The goal is to sacrifice a bit of output in good times to bolster financial stability and mitigate the slumps. Could leaning in fact exacerbate the slumps, incurring the cost of lower output during booms while making recessions *worse*? If so, how should optimal monetary policy take financial stability concerns into account? We uncover two mechanisms through which leaning monetary policy can become a "lose-lose" endeavour (worse booms and worse busts). First, announcing a leaning policy once a speculative boom is underway reduces asset prices when investors are highly levered, deepening the very crash that policymakers are seeking to soften. Second, recessions are more severe if investors anticipate that monetary policy will lean against a speculative boom during the recovery. A systematic policy of hiking rates when speculation emerges is a commitment to run the economy "cold" during recoveries—

[1] See also Bernanke and Kuttner (2005) and Jiménez et al. (2014). For reviews of the empirical evidence connecting low policy rates and high risk taking, see Boissay et al. (2023) and Kashyap and Stein (2023).

the opposite of the common prescription to ameliorate recessions by committing to run the economy hot during expansions.

Echoing Bernanke's caution, these announcement and anticipation effects suggest that raising interest rates to reduce financial excesses may in fact be "sowing the wind," reaped in the form of more severe recessions. However, echoing Kindelberger and Minsky, these two intertemporal costs of leaning *do not* imply that a central bank is relegated to ignoring the build-up of financial vulnerabilities and waiting until after a crisis to clean up. Macroprudential (regulatory) tightening during a boom does not sow the wind, if monetary policy is adjusted to offset the decline in asset prices from tighter regulations. Moreover, even when regulatory tightening is infeasible, our optimal policy analysis shows that monetary policy can achieve macroprudential benefits by phasing in tightening over time, with the policy rate rising if the speculation worsens.

These thoughts are formalized by studying a speculative boom followed by a financial crash and recession. The model captures well the key mechanisms cited by leaning proponents (Borio and Lowe 2002). We build on the risk-centric framework of Caballero and Simsek (2020a,b). High-valuation and low-valuation investors (optimists and pessimists) trade with each other in financial markets.[2] A lower optimist wealth share in the recession implies lower asset prices and aggregate demand. Leaning monetary policy during the boom raises interest rates and reduces asset prices, seeking to create space to cut interest rates in the recession. Leaning does in fact dampen the recession *conditional on* two crucial variables: (i) the optimist wealth share just prior to the recession, and (ii) the risky asset price in the recession, as a function of optimists' wealth share in the recession. These two variables can be considered as (i) the balance-sheet strength of high-valuation investors *prior to recession*, and (ii) the demand for risk exposure *during recession* for a given balance-sheet strength of high-valuation investors. These two variables are endogenous and negatively impacted by raising the policy rate. The first variable is harmed by discretionary (unexpected) leaning: Adopting leaning once the boom is underway, when high-valuation investors have a levered position in the risky asset, immediately and persistently reduces optimists' wealth during the boom, exacerbating the subsequent recession (the "announcement effect"). The second variable is harmed by systematic leaning: During recessions, investors anticipate aggregate demand and the risky asset's price will be capped by

[2] The belief disagreements between optimists and pessimists creates connections—key to our main results—between financial-market sentiment and the discretionary and systematic components of monetary policy (Kashyap and Stein 2023). As Caballero and Simsek (2020b) note, the persistence of belief disagreements implies that the optimists can also be considered as intermedaries (as opposed to households) or relatively risk-tolerant investors.

leaning monetary policy in a subsequent boom (the "anticipation effect"), depressing demand for the risky asset during the recession even conditional on optimists' wealth share. Our paper studies the positive and normative consequences of the announcement and anticipation effects and characterizes optimal monetary policy taking these effects into account.

Sowing through announcement. The announcement effect can be illustrated using a few key relations in the model. Let α denote optimists' wealth share. Optimists take a levered trading position in the risk asset, funded by borrowing from the pessimists. As a speculative boom begins $(s = 1)$, monetary policy is accommodative, risk premia are falling, credit-to-GDP is rising, and wealth is accumulating in the hands of the optimists. The central bank is gradually raising the policy rate over time, to offset the higher aggregate demand that would arise from rising asset prices. Correspondingly, the risky asset price is $Q_1(\alpha) = Q^*$, which stabilizes activity with the resource utilization that would obtain absent nominal rigidities.[3]

As the boom continues, the central bank has the opportunity to adopt a leaning monetary policy rule $(s = 2)$. If the central bank opts to maintain its previous accommodative policy, $Q_2(\alpha_2) = Q^*$, as shown by the red dotted horizontal line in the left panel of Figure 1. If the central bank instead adopts leaning, it raises the policy rate to reduce the risky asset price to $Q_2(\alpha) < Q^*$, as shown by the blue solid line.[4] Just prior to the adoption of leaning, optimists' wealth share is $\bar{\boldsymbol{\alpha}}$ and optimists hold a levered position in the risky asset with gross leverage $\omega_1^o > 1$. If the adoption of leaning left unchanged the optimist wealth share, the asset price would fall to $Q_2(\bar{\boldsymbol{\alpha}}) < Q^*$. This, however, cannot be an equilibrium, as the lower asset price induces a decline in optimists' wealth share (as indicated by the horizontal arrows) and thus further reduces the risky asset price. The new, lower equilibrium optimist wealth share and risky asset price are shown by the black square, where the risky asset price induced by monetary policy (in blue) and the optimist wealth share induced by a given risky asset price (purple) intersect. The decline in optimists' wealth share is persistent and exacerbates the subsequent recession, highlighting that the design of monetary policy tightening must take into account how the new strategy's announcement affects levered, high-valuation investors. Even for small amounts of tightening, the announcement effect implies a first-order welfare loss by reducing utilization during the recession, when the economy is already far from efficient utilization.[5]

[3] A monetary policy that achieves an asset price Q^* is still accommodative because it pursues this stabilization of activity without regard to the financial vulnerabilities associated with compressed risk premia and high leverage.

[4] The goal of a leaning monetary policy is to create space to cut the policy rate and dampen the fall in the asset price when the recession $s = 3$ arrives.

[5] The first-order costs of the announcement effect arise (only) from the impact on activity during the recession. Small reductions in utilization during the boom have second-order welfare effects, because

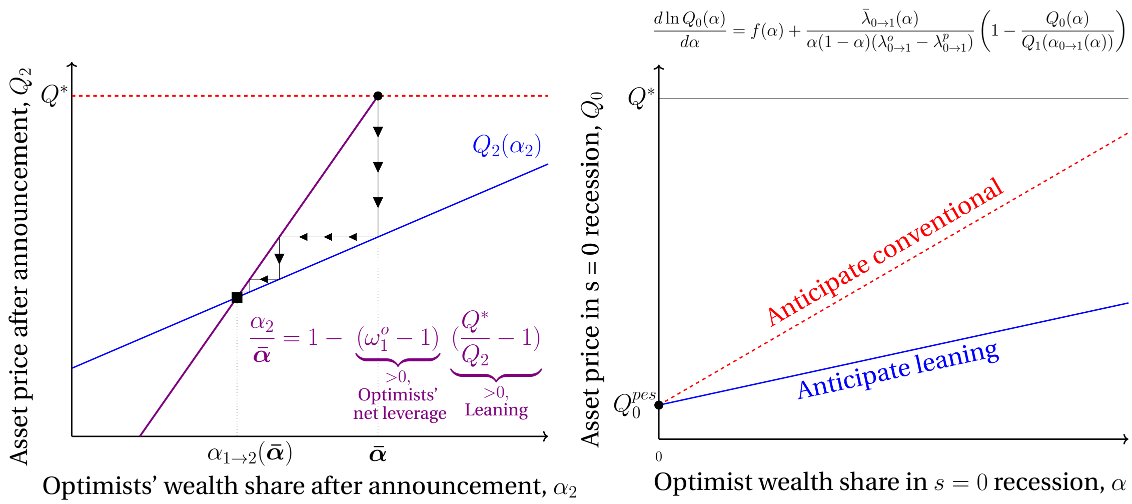

Figure 1: **Announcement and anticipation effects.** The left panel illustrates the decline in optimists' wealth share when a leaning monetary policy is adopted during the boom. The right panel illustrates how the risky asset price—and hence output—is depressed during a recession if investors anticipate leaning during the subsequent boom.

In contrast, directly tightening regulatory policy (lowering the leverage limit) reduces vulnerabilities without harming optimists' wealth share, because monetary policy can be adjusted to offset any decline in the risky asset price while still maintaining the recession-softening benefit of lower leverage. Under leaning monetary policy, utilization and welfare are lower through the boom and recession, relative to when regulatory policy is directly tightened. The announcement effect implies monetary policy and regulation are *not* close substitutes for the achievement of macroprudential goals: Regulatory policy has a clear advantage.

Sowing through anticipation. Systematic leaning also sows the wind, as we show using a simple bust-boom-bust cycle. During the initial recession ($s = 0$), investors anticipate that monetary policy will lean against a subsequent boom ($s = 1$). A systematic policy of leaning during the boom ($Q_1(\alpha) < Q^*$) caps the price appreciation of the risky asset when the boom arises, implying a lower risky asset price ($Q_0(\alpha)$) and hence lower output in the recession. This result follows because the recession-state risky asset price is increasing in the optimist wealth share, with a slope that is increasing in the price appreciation of the risky asset if a boom arrives:

$$\frac{d \ln Q_0(\alpha)}{d\alpha} \approx \underbrace{\frac{\bar{\lambda}_{0\to1}(\alpha)}{\alpha(1-\alpha)(\lambda^o_{0\to1} - \lambda^p_{0\to1})}}_{\text{Risk-adjusted, weighted arrival rate of boom}} \underbrace{\left(1 - \frac{Q_0(\alpha)}{Q_1(\alpha_{0\to1}(\alpha))}\right)}_{\text{Capital gain if boom arrives}} \tag{1}$$

utilization during the boom is at its efficient level under conventional policy.

More aggressive leaning during the boom ($Q_1(.)$ lower) therefore implies a shallower slope of $Q_0(\alpha)$. In addition, the risky asset price Q_0^{pes} in the all-pessimist economy ($\alpha = 0$) during the initial recession is weakly higher when investors anticipate conventional policy during the subsequent boom. As illustrated in the right panel of Figure 1, leaning during the boom implies a lower risky asset price $Q_0(\alpha)$ and hence lower output during the recession.

Optimal policy with "sowing" effects. We characterize optimal policy taking into account these announcement and anticipation effects, when the central bank adopts a new policy strategy during the boom (taking into account the announcement effect) and when designing systematic policy (with the anticipation effect). Optimal policy does use monetary policy for macroprudential objectives, rather than ignoring financial stability concerns and focusing only on stabilizing output during the boom. However, optimal policy is *very different* from the standard "leaning" approach of tightening monetary policy to lower the risky asset price by raising the policy rate and shifting upward the entire term structure of interest rates. Optimal policy—either adopted unexpectedly during a speculative boom or, under systematic policy, when a speculative boom begins—implies initially *no* fall in the risky asset price. That is, initially, the risky asset price is the same as would obtain under an accommodative monetary policy that focuses exclusively on stabilizing output during the boom. However, over time, if the boom continues, the risky asset price falls, as the stance of monetary policy tightens. This approach generates macroprudential benefits—the recession is dampened—but mitigates the announcement and anticipation effects. Achieving this path of the risky asset price entails a "bear steepening," in the following sense: *Forward* expected policy rates—the expected policy rate over a fixed period, beginning at some future date—rise, while the expected policy rate over short horizons falls.

This general insight—avoiding an initial decline in the risky asset price through a bear steepening—is a key aspect of optimal policy with both the announcement and anticipation effects. The similarity of optimal discretionary and systematic policy is remarkable because the planner's problem is very different in these two scenarios. The announcement effect transmits entirely through a persistent decline in optimists' wealth (what we call a "high-valuation balance-sheet channel"), while the anticipation effect arises entirely through the expectation of a cap on the risky asset price without any effect on optimists' wealth during the initial recession (a "cap-upside price channel"). Nonetheless, both the announcement effect and the anticipation effect are ameliorated by reducing initial declines in the risky asset price.

Importantly, our model gives myriad advantages to leaning policies. In the model, a bust always (eventually) follows the speculative boom and is always severe enough to push the economy against the zero lower bound for nominal interest rates. Also, prices are completely fixed, so the central bank can focus completely on intertemporal tradeoffs for stabilizing utilization. In reality, exuberant asset prices sometimes reflect strong fundamentals or revert to normal levels without a crisis. Less severe busts that do not reduce the neutral interest rate below zero could be addressed "ex-post" by cutting interest rates after the bust arrives without leaning during the boom. In addition, leaning can result in prolonged undershooting of the inflation target during the boom, a theoretically and empirically relevant concern (Barlevy 2022).

Our paper bridges two major strands of the literature on leaning. One strand emphasizes the benefits of announcing leaning policies before a speculative boom begins (Borio and Lowe 2002; Caballero and Simsek 2020a; Fontanier 2022; Boissay et al. 2023; Kashyap and Stein 2023). In our setting, the detrimental effects of unexpectedly announcing leaning during the boom are consistent with the view that leaning is more beneficial when adopted before high-valuation investors have levered up. This strand of the literature sees a systematic leaning policy as contributing positively to aggregate demand during financial busts and thereby ameliorating them. Our paper offers a different perspective in this regard. We show how leaning can worsen the severity of recessions, even when implemented systematically or before the boom begins. Hence, our paper complements a second strand of the literature that is skeptical about leaning, viewing such policies as worsening aggregate demand during the bust (Svensson 2017). A contribution to this second strand of the literature is that, in our paper, leaning aggravates recessions through financial markets and speculation.

Our paper formally models the concerns in Bernanke and Gertler (2001) about the unintended consequences of a surprise adoption of leaning once a boom is underway. Allen et al. (2022), in a framework with speculation and risk-shifting, find that leaning is more effective when it disproportionately discourages riskier investments. Our paper is also complementary with Galí (2014), in which leaning can increase the size of an asset price bubble. Our model builds on the framework for speculation in Simsek (2013) and Caballero and Simsek (2020b). Wealth losses for high-valuation investors from unexpected monetary policy tightening is also emphasized by Kekre and Lenel (2022), which studies monetary policy transmission.

The rest of the paper is structured as follows. Section 2 presents the model. Section 3 describes monetary policy. Section 4 analyzes the announcement of leaning during a boom. Section 5 studies optimal policy taking into account the announcement effect. Section 6 studies how the anticipation of leaning during booms affects aggregate demand during recessions. A final section concludes.

2. Model

Time is continuous $t \in [0, \infty)$. There is a single consumption good and a single factor of production, capital. There are four states $s \in \{1, 2, 3, 4\} \equiv S$. As we describe in greater detail below, the growth rate of capital (before depreciation) is high in states 1 and 2, low in state 3, and high again in state 4. Therefore, states 1 and 2 are labeled the boom, state 3 is the recession, and state 4 is referred to as the recovery. In our main analysis, we assume that the policy rule in place in $s = 1$ is the traditional output-stabilization rule in which the central bank sets the interest rate equal to the neutral rate consistent with full resource utilization during the boom. The defining feature of state 2 (distinguishing it from state 1) is that, when the transition to state 2 occurs, the central bank changes its policy rule. In $s = 2$, the central bank can adopt a leaning against the wind policy or, of course, retain the inherited policy rule. The change of policy rule in state 2 allows us to study the unexpected adoption of leaning monetary policy during a boom or "discretionary" leaning. We will later analyze systematic leaning monetary policy—"preannounced" policy set before time 0—including in Section 6.

The economy transitions across states according to a Poisson process. From state 1, the economy can enter into state 2 (change in policy rule while the economy is booming) or directly into state 3 (recession). From state 2, the economy can transition into recession. The recession is followed by recovery (state 4) in which the central bank implements conventional policy. $s = 4$ is an absorbing state (the economy ends).[6] There are three types of agents: the optimists o, the pessimists p, and the planner or central bank (pl). According to agent type $i \in \{o, p, pl\}$, the transition from state s to s' occurs at Poisson arrival rate $\lambda^i_{s \to s'}$.

Persistence of beliefs. We now describe investors' beliefs. During the boom states, optimists perceive a lower Poisson rate for the arrival of the recession, relative to pessimists: $\lambda^o_{s \to 3} < \lambda^p_{s \to 3}$, for $s \in \{1, 2\}$. During the recession ($s = 3$), optimists perceive a higher rate of arrival of the recovery, relative to pessimists: $\lambda^o_{3 \to 4} > \lambda^p_{3 \to 4}$. Optimists are agents that value the risky asset more than pessimists regardless of the state of the economy (during both the boom and recession). Because we are interested first in studying the adoption of leaning during the boom (rather than systematic or preannounced policy), it is important that the transition to state 2 be at least partly unexpected. For simplicity, we assume that the transition to state 2 is *fully unexpected* by all the investors: $\lambda^i_{1 \to 2} = 0$ for $i \in \{o, p\}$.

Capital and depreciation. The growth rate of capital before depreciation is denoted by

[6] In Section 6, we add a recession as a state $s = 0$, so that we can study how a recession is affected by expectations about monetary policy during a subsequent speculative boom.

g_s and varies across the 4 states. The parameters satisfy $g_3 < \min(g_1, g_2, g_4)$. To clarify that state 2 differs from state 1 only with regard to the policy rule, we set $g_1 = g_2$ and $\lambda^i_{1\to3} = \lambda^i_{2\to3}$ for all agent types. Our main points do not depend on this assumption. The capital stock at time t in state $s \in S$ is denoted $k_{t,s}$. Capital utilization $\eta_{t,s}$ is endogenous. Output of the consumption good is $y_{t,s} = Ak_{t,s}\eta_{t,s}$. The depreciation function $\delta(\eta_{t,s})$ is increasing, convex, and differentiable.[7]

Financial assets, leverage, and portfolio choice. There are two financial assets. The risky asset is a claim on all output, with total value $Q_{t,s}k_{t,s}$, where $Q_{t,s}$ is the price per unit of capital. The return on the risky asset absent state transition is

$$r_{t,s} = \underbrace{\frac{y_{t,s}}{Q_{t,s}k_{t,s}}}_{\text{Dividend yield}} + \underbrace{g_s - \delta(\eta_s) + \frac{\dot{Q}_{t,s}}{Q_{t,s}}}_{\text{Expected growth}}. \tag{2}$$

The other asset is a risk-free asset in zero net supply with instantaneous return $r^f_{t,s}$. Investor i has financial wealth $a^i_{t,s}$ and chooses consumption $c^i_{t,s}$ and the share of wealth allocated to the risky asset $\omega^i_{t,s}$ (which also captures the investors' leverage). There is a leverage limit in the boom states, $\omega^i_{t,s} \leq \bar{\omega}$ for $s \in \{1, 2\}$. There is no leverage constraint in other states, which highlights that dynamics are not driven, as in much of literature, by a binding leverage constraint in the bad state.[8] The (boom-state) leverage constraint has an important role in leaning against the wind. Without a leverage constraint in the good state, leaning policies in general are ineffective in dampening the subsequent recession because they cause optimists to take on more leverage. To assure market clearing, $\bar{\omega} \geq 1$. Because $\bar{\omega} = 1$ imposes the absence of leverage-fueled speculation, we further assume $\bar{\omega}$ is strictly above 1. Define investor i's wealth share as

$$\alpha^i_{t,s} = \frac{a^i_{t,s}}{Q_{t,s}k_{t,s}} \text{ for } i \in \{o, p\}. \tag{3}$$

The wealth share of the optimists, $\alpha^o_{t,s} = 1 - \alpha^p_{t,s}$, is the key state variable of the model. Investors maximize the discounted utility of consumption and have log preferences with discount rate ρ. Asset market clearing requires $a^o_{t,s} + a^p_{t,s} = \omega^o_{t,s}a^o_{t,s} + \omega^p_{t,s}a^p_{t,s} = Q_{t,s}k_{t,s}$.

Nominal rigidities and equilibrium in the goods market. Without price rigidites, firms set $\delta'(\eta_{t,s})Q_{t,s} = A$. That is, the marginal cost of depreciation equals the marginal benefit of increasing utilization. Prices are assumed to be completely fixed.[9] With price rigidities, as

[7] In the numerical analysis in Section 4.3, the depreciation function is differentiable almost everywhere and weakly convex.

[8] For normative analyses of models with leverage constraints in bad states, see Lorenzoni (2008), Dávila and Korinek (2017), and Fontanier (2022), among many others.

[9] For analysis of speculative frenzies with partly rigid prices, see Barlevy (2022).

in New Keynesian models, firms meet any level of demand at the fixed price so long as the price exceeds the marginal cost. The consumption good is the numeraire. Thus, firms produce $y_{t,s} = \eta_{t,s} A k_{t,s} = c^o_{t,s} + c^p_{t,s}$ so long as $\delta'(\eta_{t,s}) Q_{t,s} \leq A$.

Equilibrium. The history $\mathcal{H}^t$ of aggregate shocks is the arrival times of any state transitions that occurred between $[0, t]$. The equilibrium is a sequence $\{r^f_{t,s}, \eta_{t,s}, Q_{t,s}, r_{t,s}, \alpha_{t,s}, c^o_{t,s}, c^p_{t,s}\}_{t \in [0,\infty), s \in S}$ for each history $\mathcal{H}^t$ consistent with investors' maximizing their expected utility and market clearing for the consumption good, the risky asset, and the risk-free asset. Monetary policy is a rule for the policy rate $\{r^f_{t,s}(\mathcal{H}^t)\}_{t \in [0,\infty), s \in S}$ chosen subject to the zero lower bound constraint, $r^f_{t,s}(\mathcal{H}^t) \geq 0$. Associated with the monetary policy rule are the utilization rate and asset price, $\{\eta_{t,s}(\mathcal{H}^t), Q_{t,s}(\mathcal{H}^t)\}_{t \in [0,\infty), s \in S}$, determined by the monetary policy rule and equilibrium conditions. Because prices are fixed, the central bank, by setting the nominal interest rate, also sets the real risk-free interest rate. The economy's initial condition is the optimist wealth share at $t = 0$.

Equilibrium in good markets. Output is connected to the asset price according to

$$A \eta_{t,s} k_{t,s} = y_{t,s} = c^o_{t,s} + c^p_{t,s} = \rho(a^o_{t,s} + a^p_{t,s}) = \rho Q_{t,s} k_{t,s}, \tag{4}$$

where the first equality follows from the production function, the second equality from market clearing in the goods market, the third equality from investors' optimal consumption-saving decision under log preferences, and the fourth equality from market clearing in financial markets.

Equilibrium in asset markets. The simple arithmetic of the budget constraint implies that the wealth share of investor i after a transition from state s to s' is

$$\frac{\alpha^i_{t,s'}}{\alpha^i_{t,s}} = 1 + \left(\omega^i_{t,s} - 1\right)\left(1 - \frac{Q_{t,s}}{Q_{t,s'}}\right). \tag{5}$$

A transition that decreases the asset price also reduces an investor's wealth share if the investor has leverage greater than 1. Investors choose leverage such that their wealth share after transition satisfies

$$r_{t,s} + \lambda^i_{s \to s'} \frac{\alpha^i_{t,s}}{\alpha^i_{t,s'}} \frac{Q_{t,s'} - Q_{t,s}}{Q_{t,s'}} \geq r^f_{t,s}, \tag{6}$$

which holds with equality if the leverage constraint is not binding.[10] Absent transition,

$$\frac{\dot{\alpha}^i_{t,s}}{\alpha^i_{t,s}} = \lambda^p_{s \to s'} \frac{\alpha^p_{t,s}}{\alpha^p_{t,s'}} \left(1 - \frac{\alpha^i_{t,s'}}{\alpha^i_{t,s}}\right). \tag{7}$$

[10]Eq. (6) applies for $(s, s') \in \{(1,3), (2,3), (3,4)\}$, consistent with the state transition possibilities and beliefs specified earlier in this section.

The wealth-weighted average belief about the transition rate from s to s' is

$$\bar{\lambda}_{t,s\to s'} = \alpha^o_{t,s}\lambda^o_{s\to s'} + \alpha^p_{t,s}\lambda^p_{s\to s'}. \tag{8}$$

During the boom ($s \in \{1,2\}$), the risk premium $rp^i_{t,s}$, or expected excess return, under the belief of agent type $i \in \{o,p,pl\}$ is

$$rp^i_{t,s} = \underbrace{r_{t,s} - r^f_{t,s}}_{\substack{\text{Excess return absent}\\ \text{state transition}}} + \lambda^i_{s\to 3}\underbrace{\left(\frac{Q_{t,3}}{Q_{t,s}} - 1\right)}_{\substack{\text{Capital loss}\\ \text{upon recession}}} \tag{9}$$

The excess return absent state transition is determined by applying (6) to the pessimists, for whom the leverage constraint is not binding. The perceived risk premium (9) differs across agent types. All agents agree on the excess return absent state transition and the capital loss upon recession, but they disagree on the arrival rate of recession, with pessimists giving more weight than optimists to the capital loss from recession. When we evaluate the risk premium and other belief-dependent variables for the planner, we assume the planner's beliefs are an average of the optimists and pessimists, with $\lambda^{pl}_{s\to s'} = 0.5(\lambda^o_{s\to s'} + \lambda^p_{s\to s'})$.

In the remainder of the paper, we often drop the time and agent-type subscripts and denote optimists' wealth share as α. The remaining variables are described as functions of optimists' wealth share. For example, $Q_s(\alpha)$ denotes the asset price in state s with optimist wealth share α. If optimists' wealth share just prior to a state transition from s to s' is equal to α, we denote their wealth share at the start of s' as $\alpha_{s\to s'}(\alpha)$.

3. Monetary policy

We next introduce leaning monetary policy. During the recession, utilization is below its efficient level and monetary policy is constrained by the zero lower bound. In the initial stage of the boom ($s = 1$), the central bank achieves full resource utilization. As the speculative boom continues ($s = 2$), the central bank can choose to depress the asset price and utilization during the boom (lean against the wind) to attempt to soften the decline in optimists' wealth share when the recession ($s = 3$) arrives.

Absent nominal rigidities, utilization is at the (statically) efficient level η^* satisfying $\delta'(\eta^*)\eta^* = \rho$. The asset price consistent with efficient utilization is $Q^* = A\eta^*/\rho$. Conventional output-stabilization policy sets the policy rate to the neutral rate $r^*_s(\alpha)$ consistent with efficient asset prices: $Q_s(\alpha) = Q^*, r^f_s(\alpha) = r^*_s(\alpha)$ or $Q_s(\alpha) < Q^*, r^f_s(\alpha) = 0$. We characterize the neutral rate below. The central bank follows conventional output-stabilization policy in states $s \in \{1,3,4\}$ and a feasible rule $\{r_2^f(\alpha), Q_2(\alpha), \eta_2(\alpha)\}$ in state 2. Parametric assumptions ensure that the neutral rate is positive during the boom and recovery states, and negative during the recession.[11]

Recession and recovery. During the recovery, the central bank, pursuing conventional policy, sets the risk-free rate equal to the positive, constant neutral rate ($r_4^f(\alpha) = r_4^* > 0, Q_4 = Q^*$). During the recession, the central bank is constrained to set the risk-free rate equal to 0. From Eqs. (6) and (7),

$$r_3(\alpha) + \bar{\lambda}_{3\to 4}(\alpha)\left(1 - \frac{Q_3(\alpha)}{Q^*}\right) = r_3^f(\alpha) \tag{10}$$

To achieve an efficient asset price $Q_3(\alpha) = Q^*$ in the recession, the policymaker would have to set the risk-free rate equal to the constant neutral interest rate $r_3^* = \rho + g_3 - \delta(\eta^*) < 0$, where g_3 is low enough such that this neutral rate is negative. Thus, under conventional policy, the central bank is constrained to set the risk-free rate equal to 0. The asset price $Q_3(\alpha) < Q^*$ is determined according to a differential equation derived from (7) and (10), shown in Internet Appendix A. The asset price is below its efficient level $Q_3(\alpha) < Q^*$ and is increasing in the optimists' wealth share, $\frac{dQ_3(\alpha)}{d\alpha} > 0$ for $\alpha \in (0,1)$. From Eq. (4), utilization is below its efficient level and increasing in the optimist wealth share.

Monetary policy during the boom. In state $s = 1$, the central bank follows a conventional approach and sets $r_1^f(\alpha) = r_1^*(\alpha)$. Such a monetary policy stabilizes activity and the risky asset price at the levels that would obtain absent nominal rigidities. However, this monetary policy can still be described as accommodative, because it pursues the stabilization of activity without regard to the financial vulnerabilities associated with compressed risk premia and high leverage, as described further below.

In $s = 2$, the central bank can choose to lean against the wind, tightening the stance of monetary policy and setting the policy rate to achieve a lower asset price. As shown in Appendix A, conditional on the optimist wealth share just prior the recession, a policy that reduces the asset price during the boom can soften the optimist wealth share decline when a recession arrives. Hence, the central bank can consider a leaning monetary policy that targets the risky asset price $Q_2(\alpha)$ during the boom, with a goal of affecting the severity of the recession by shifting upward the mapping $\alpha_{2\to 3}(\alpha)$. We consider Q-targeting rules satisfying an upper and lower bound. First, $Q_2(\alpha) \leq Q^*$ for $\alpha \in [0,1]$. If this inequality holds strictly for all optimist wealth shares, then the Q-targeting rule coincides with conventional monetary policy. Second, the Q-targeting rule must have the *cure-no-worse-than-disease* property: $Q_2(\alpha) > Q_3(\alpha)$, for

[11] See Internet Appendix A.

$\alpha \in [0,1]$. A targeting rule $Q_2(.)$ has the property of not being worse than the disease if the rule induces asset prices above the asset prices in the recession.

Because an essential aspect of leaning monetary policy is a reduction in the risky asset price during the boom, Q-targeting rules are a natural way to describe leaning policies in the model. In addition, communications and meeting transcripts suggest that central banks set policy rates with the goal of achieving a certain level of accommodation or restrictiveness through the levels of risky asset prices (Cieslak and Vissing-Jorgensen 2020). Making the Q-targeting rule a function of the economy's state variable α is consistent with suggestions by researchers and policymakers that monetary policy rules incorporate credit measures capturing the quantity of debt in the economy (e.g., Gourio et al. 2018). In our setting, when the leverage constraint binds for optimists, optimists' wealth share α is proportional to and perfectly correlated with credit-to-total assets, because credit-to-total-assets is $(\bar{\omega}-1)\alpha$ during the boom state with sufficient leaning.[12]

Implementation. The central bank would implement the Q-targeting rule $Q_2(\alpha)$ by setting the risk-free rate equal to the risk-adjusted return on capital:

$$r_2^f(\alpha) = \underbrace{\rho + g_2 - \delta\left(\frac{Q_2(\alpha)}{Q^*}\eta^*\right) + \frac{d\ln(Q_2(\alpha))}{d\alpha}\dot{\alpha}_2(\alpha)}_{\substack{\text{Return on capital absent recession arrival, } r_2(\alpha) \\ \text{Equation (2)}}} - \underbrace{\frac{\lambda^p_{2\to 3}(1-\alpha)}{1-\alpha_{2\to 3}(\alpha)}\left(\frac{Q_2(\alpha)}{Q_3(\alpha_{2\to 3}(\alpha))}-1\right)}_{\substack{\text{Recession risk term,} \\ \text{Equation (6)}}} \quad (11)$$

where optimists' wealth share if the recession arrives $\alpha_{2\to 3}(\alpha)$ and the change in optimists' wealth share absent transition $\dot{\alpha}_2(\alpha)$ are obtained using (5) and (7). The expression (11) is itself obtained from (6), applied to the pessimists, for whom the leverage constraint never binds.

The return on capital, $r_2(\alpha)$, includes a dividend yield $y_2/(Q_2 k_2)$ equal to the discount rate, consistent with the Eq. (4). Overall, the return on capital has four terms: the discount rate (ρ), the exogenous growth rate of capital g_2, capital depreciation $\delta(\eta_2)$, and price appreciation absent recession. Higher discount rate (i.e., dividend yield) and exogenous growth, lower depreciation, and higher price appreciation increase agents' demand for the risky asset. The return on capital is adjusted for the risk of recession through the recession risk term in Eq. (6). This term captures how pessimists' demand for the risky asset is reduced by the risk of the fall in the asset price (from $Q_2(\alpha)$ to $Q_3(\alpha_{2\to 3}(\alpha))$) in the event of a recession, which pessimists' view as arriving at rate $\lambda^p_{2\to 3}$. The multiplier $(1-\alpha)/(1-\alpha_{2\to 3}(\alpha)) < 1$ reflects risk compensation: Conditional on the arrival of a recession, pessimists give *less* weight to the fall in asset price than optimists do,

[12] There is a threshold $Q_s^{bind}(\alpha)$ such that optimists' leverage constraint binds if $Q_s(\alpha) \leq Q_s^{bind}(\alpha)$, as shown in Internet Appendix A.

because pessimists experience a *relative* wealth gain when the recession arrives.

A benefit of leaning. The macroprudential benefit of leaning monetary policy is that a lower target for $Q_2(\alpha)$ implies higher optimist wealth during the recession, conditional on the recession arriving when optimists' wealth share is α. In Appendix A, we show that lower $Q_2(\alpha)$ shifts upward $\alpha_{2\to 3}(\alpha)$. Thus, in our model, leaning monetary policy is effective in dampening the recession *conditional on the optimist wealth share just prior to the recession.* Of course, the optimist wealth share just prior to the recession is endogenous. Our analysis of sowing-the-wind announcement effects in Section 4 is focused on how the announcement of leaning once the boom is underway causes an immediate and persistent decline in optimists' wealth share.

A cost of leaning. Beyond any such announcement effects, leaning reduces optimists' wealth share at the time the recession arrives by slowing the rate at which optimists' accumulate wealth over time during the boom. From (7),

$$\frac{\dot{\alpha}_2(\alpha)}{\alpha} = \lambda^p_{2\to 3}\frac{1-\alpha}{\alpha}\frac{\alpha - \alpha_{2\to 3}(\alpha)}{1-\alpha_{2\to 3}(\alpha)}. \tag{12}$$

This cost of leaning is the necessary counterpart to the macroprudential benefit of leaning: Choosing a targeting rule with larger macroprudential benefits (higher $\alpha_{2\to 3}(\alpha)$) implies that, as long as the boom continues, the growth rate of optimists' wealth $\dot{\alpha}_2(\alpha)$ will be lower. A lower growth rate of optimist wealth during the boom implies, all else equal, lower optimist wealth in the recession, and more so, the longer the boom lasts. This intertemporal cost of leaning is present whether the adoption of leaning is unexpected or systematic.

Remark. The belief differences between optimists and pessimists and the resulting speculation in financial markets are necessary for there to be any potential welfare benefit from leaning policies. If there were no belief differences during the boom (i.e., $\lambda^o_{s\to 3} = \lambda^p_{s\to 3}$ for $s \in \{1, 2\}$), reducing the risky asset price during the boom would reduce utilization during the boom (from (4), which does not depend on belief differences) but with no benefit to utilization during the recession.

Welfare. To understand the welfare implications of leaning and to characterize optimal policy, we use the gap value function that captures the (expected and properly discounted) losses due to demand-driven deviations from efficient utilization. This welfare function is an exact version of the quadratic loss measure used in the New Keynesian literature. As Caballero and Simsek (2020a) emphasize, this gap variable matches well central banks' mandates and is based on a measure of instantaneous welfare losses from underutilization that all agent types agree on, regardless of their belief differences about state transition rates. In the boom states, with $s \in \{1, 2\}$, the gap value w^{pl}_s under the planner's beliefs satisfies

$$\rho w_s^{pl}(\alpha) = \underbrace{W(Q_s(\alpha))}_{\substack{\text{Welfare flow}\\ \text{from utilization}}} + \lambda_{s\to 3}^{pl} \underbrace{\left(w_3^{pl}(\alpha_{s\to 3}(\alpha)) - w_s^{pl}(\alpha)\right)}_{\substack{\text{Welfare loss}\\ \text{upon recession}}} + \underbrace{\frac{\partial w_s^{pl}(\alpha)}{\partial \alpha}\dot{\alpha}_s(\alpha)}_{\substack{\text{Welfare gain}\\ \text{absent recession}}}, \tag{13}$$

where $W(Q) \equiv \ln \frac{Q}{Q^*} - \frac{1}{\rho}(\delta(\frac{Q}{Q^*}\eta^*) - \delta(\eta^*))$. $W(Q)$ is strictly concave and achieves a maximum value $W(Q^*) = 0$ when $Q = Q^*$. This function captures the instantaneous welfare losses when the asset price and hence utilization deviate from their efficient levels. The gap value thus corresponds to the discounted expected value of welfare losses due to demand recessions. In the differential equation (13) that determines the gap value, monetary policy affects three terms directly: the risky asset price $Q_s(\alpha)$ and hence the instantaneous utilization measure $W(Q_s(\alpha))$, the change in optimists' wealth share upon recession $\alpha_{s\to 3}$, and the growth rate of optimists' wealth share $\dot{\alpha}_s$.

Illustration: A simple Q-targeting rule. We now illustrate leaning monetary policy using a numerical example. The parameter values are described in Appendix B. We focus on a simple, but general, class of Q-targeting rules: a convex combination of the risky asset price under conventional monetary policy and the risky asset price in the recession. The targeting rule is

$$Q_2(\alpha) = \psi Q_3(\alpha) + (1 - \psi)Q^*, \tag{14}$$

for $\psi \in [0, 1)$. Higher values of ψ correspond to more aggressive leaning. This class of rules nests conventional policy ($\psi = 0$) and converges to the *cure-no-worse-than-disease* constraint as ψ approaches 1. Figure 3, top left panel, shows this Q-targeting rule for $\psi = 0.5$ (blue solid line), halfway between the upper and lower bounds on the Q rule. The leverage constraint is binding (a key condition for leaning to be worthwhile) when the price of the risky asset is below $Q_2^{bind}(\alpha)$, shown by the yellow dotted line.[13] The green dashed line shows the cure-no-worse-than-disease constraint.

The top right panel shows the risk-free rate that would implement this Q-targeting rule. The central bank sets a higher-than-neutral risk-free rate to reduce the risky asset price to its desired level. The light dashed-dot lines in the middle panel show the return on capital absent the arrival of a recession, a key component of the risk-free rate (expression (11)). The return on risky capital absent recession is slightly increased if the risky asset price is increasing over time absent recession under the central bank's Q-targeting rule.

The macroprudential benefits of the example Q-targeting rule are shown in the bottom left

[13] For α with $Q_2(\alpha) \geq Q_2^{bind}(\alpha)$, expression (IA.10) shows that the decline in optimists' wealth share if recession arrives is the same as under conventional policy (i.e., the leaning Q-targeting rule has no macroprudential benefit).

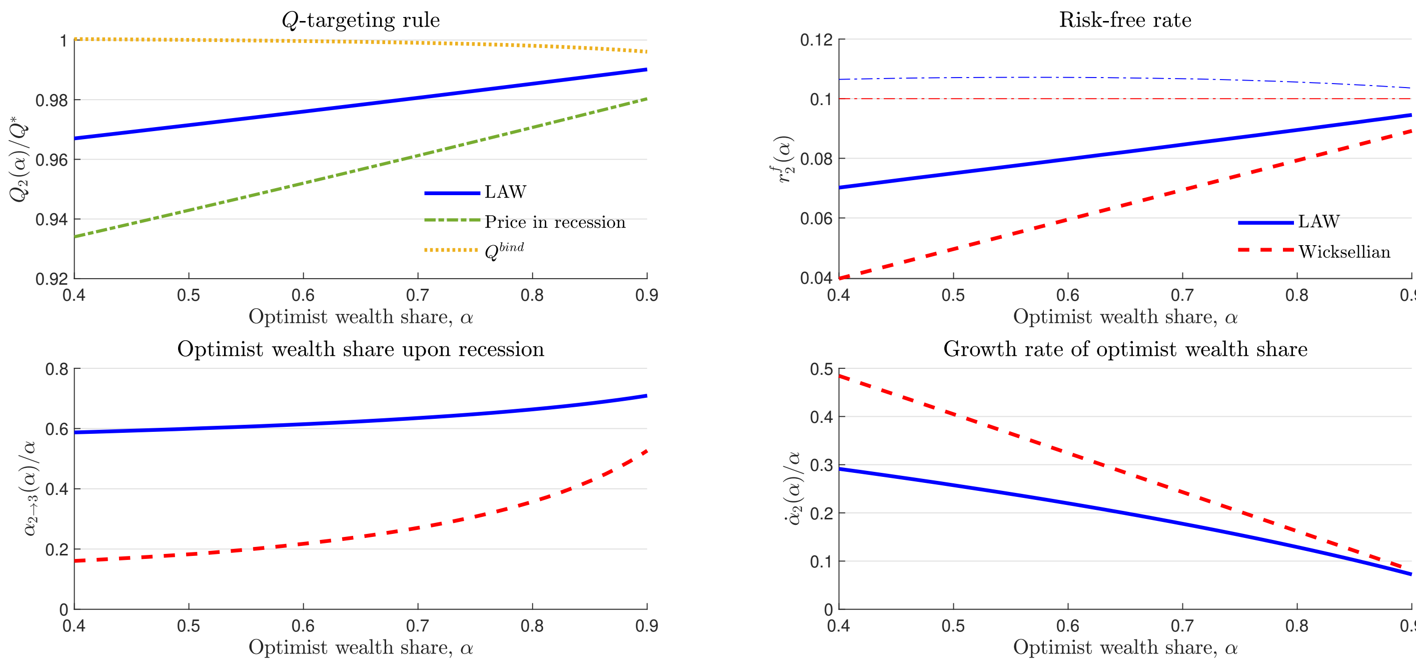

Figure 2: **Leaning monetary policy: An illustration.** *Top row:* The leaning Q-targeting rule (14) and the implementing risk-free rate are shown in the top left and right panels, respectively. The thin dash-dot lines in the top right panel show the return on capital absent transition to recession. *Bottom row:* A macroprudential benefit (bottom left panel) and cost (bottom right) are shown.

panel of Figure 2. Optimists' wealth share upon the arrival of recession falls less, the lower is the asset price under the Q-targeting rule. Overall, by setting a higher interest rate during the boom, the central bank lowers the risky asset price and thus creates room to cut the interest rate when the recession arrives, ameliorating the recession's severity. Costs of leaning monetary policy arise through a low asset price in the boom, which depresses utilization during the boom and also reduces the growth rate of optimists' wealth share (12), as shown in the bottom right panel. In the next section, we will turn to additional costs of leaning monetary policy that arise when the central bank announces such a policy once the boom is underway.

Empirical relevance of announcing leaning during a speculative boom. Our analysis of discretionary leaning monetary policy focuses on the (at least partly) unexpected adoption of leaning once a speculative boom is underway. An unexpected announcement of leaning during the boom is consistent with the historical pattern that central banks have remained inattentive to the buildup of speculative risks when setting their target for short-term rates, taking up the discussion of leaning against the wind only when a large number of indicators are "flashing red" (Kashyap and Stein 2023). Well-known examples include the lead up to the Great Depression and the debate among US policymakers in 2012-2013, which culminated in the "Taper Tantrum." (Moreover, even if policymakers can partly dampen the announcement effect by adopting leaning during a time of relatively low speculation and leverage, the sowing channel associated with systematic leaning policy—which we call the "anticipation effect" and study in Section 6—would be relevant as long as there were an ongoing sequence of boom-bust cycles.)

4. Sowing the wind: Announcement effect

4.1 Macroprudential *monetary policy* "sows the wind"

As described above, leaning monetary policy in our framework does have a channel through which it supports economic stabilization: *Conditional on optimists' wealth share just prior to the recession's arrival*, a leaning policy during the boom implies a higher optimist wealth share following the recession's arrival, thereby ameliorating the recession. In this section, however, we show that the announcement of leaning monetary policy in the midst of the boom "sows the wind" by reducing optimists' wealth share, upon announcement and persistently over time. This announcement effect incurs a first-order welfare loss, even for small doses of leaning monetary policy.

The leaning announcement in $s = 2$ reduces the risky asset's price and thereby leads to a fall in optimists' wealth share, because optimists have a levered position in the risky asset.[14] Let $\bar{\boldsymbol{\alpha}}$ denote the optimists' wealth share just prior to the monetary-policy announcement at the start of $s = 2$. The new optimist wealth share $\alpha_{1\to 2}(\bar{\boldsymbol{\alpha}})$ following the adoption of the Q-targeting rule $Q_2(.)$ solves the fixed point condition:

$$\alpha_{1\to 2}(\bar{\boldsymbol{\alpha}}) = \bar{\boldsymbol{\alpha}}\left[1 - (\omega_1(\bar{\boldsymbol{\alpha}}) - 1)\left(\frac{Q^*}{Q_2(\alpha_{1\to 2}(\bar{\boldsymbol{\alpha}}))} - 1\right)\right] \tag{15}$$

From the perspective of the central bank choosing a monetary policy in $s = 2$, the optimist wealth share and leverage prior to the announcement ($\bar{\boldsymbol{\alpha}}$ and $\omega_1(\bar{\boldsymbol{\alpha}})$ in equation (15)) are predetermined and therefore unaffected by the choice of the Q-targeting rule. The asset price prior to announcement (in the numerator of the right hand side) is Q^*, because we assume the central bank initially has followed a conventional policy in $s = 1$. Importantly, for the determination of $\alpha_{1\to 2}(\bar{\boldsymbol{\alpha}})$, optimists have a levered position in the risk asset, with $\omega_1(\bar{\boldsymbol{\alpha}}) > 1$, so long as the economy features a non-degenerate mix of optimists and pessimists (i.e., $\bar{\boldsymbol{\alpha}} \in (0, 1)$). Under certain parametric assumptions in Appendix A, for any $\bar{\boldsymbol{\alpha}}$, there is a unique $\alpha_{1\to 2}(\bar{\boldsymbol{\alpha}})$ satisfying

[14] The decline in the risky asset's price causes a larger wealth loss for optimists than for pessimists (the pessimists may in fact have a capital gain, if they have a short position in the risky asset, with $\omega_1^p < 0$).

(15). Moreover, the fixed point condition (15) implies the following result.

Proposition 1 **(Announcement effect)**

(i) Announcing a leaning monetary policy with $Q_2(\bar{\boldsymbol{\alpha}}) < Q^*$ *causes an immediate decline in the optimist wealth share,* $\alpha_{1\to 2}(\bar{\boldsymbol{\alpha}}) < \bar{\boldsymbol{\alpha}}$.

(ii) Announcing a leaning monetary policy with $Q_2(\bar{\boldsymbol{\alpha}}) = Q^*$ *causes no decline in optimists' wealth share,* $\alpha_{1\to 2}(\bar{\boldsymbol{\alpha}}) = \bar{\boldsymbol{\alpha}}$.

This result establishes the effect on optimists' wealth share of adopting a leaning monetary policy during the boom. Part (i) points to how leaning monetary policy may "sow the wind." Part (ii) states that a leaning policy without bite at the time of announcement—that is, a leaning policy that has no macroprudential benefit if a recession arrives *immediately*—avoids the decline in optimists' wealth share. Both parts of the proposition are key for the design of optimal policy in the presence of announcement effects (Section 5).[15]

We next turn to dynamic implications for optimists' wealth share and utilization and the normative consequences. To facilitate the analysis, we introduce the possibility of a "speculative lull." If a speculative lull occurs at t, investors delever for an instant, with $\omega_{t,1} = 1$. If the transition from $s = 1$ to $s = 2$ involves a change in policy simultaneously with a speculative lull, then an announcement of leaning monetary policy (with $Q_2(\bar{\alpha}) < Q^*$) does not lead to a fall in optimists' wealth share. In this scenario, the central bank establishes a leaning monetary policy without the "sowing" announcement effect. The speculative lull is economically relevant because the positive and normative implications of leaning policies announced during a speculative lull are the same as if leaning policy were systematic or "preannounced" before the $s = 1$ boom.[16] We will also see below in Section 4.2 that the dynamics induced by announcing leaning monetary policy during a speculative lull have important similarities to the dynamics induced, even without a lull, by directly tightening the leverage limit through regulation.

Without a speculative lull, welfare upon announcing a leaning Q-targeting rule $Q_2(.)$ is $w_2^{pl}(\alpha_{1\to 2}(\bar{\boldsymbol{\alpha}}))$. Welfare with a speculative lull is $w_2^{pl}(\bar{\boldsymbol{\alpha}})$. Denoting the the welfare difference (due

[15]As shown in the proof in Internet Appendix A, the characterization of the decline in optimists' wealth share can be extended: Following the announcement of a leaning policy, the new optimist wealth share is strictly above the level if a recession had arrived instead, with $\alpha_{1\to 2}(\bar{\boldsymbol{\alpha}}) > \alpha_{1\to 3}(\bar{\boldsymbol{\alpha}})$.

[16]The economy's dynamics when leaning is announced during a speculative lull with optimist wealth share $\bar{\boldsymbol{\alpha}}$, are the same as when policy is "preannounced" before time 0 and the exogenous initial condition of the economy (invariant to monetary policy) is time-0 optimist wealth share $\bar{\boldsymbol{\alpha}}$.

to the Announcement Effect) as $\Delta w_2^{pl}(\bar{\boldsymbol{\alpha}}) \equiv w_2^{pl}(\alpha_{1\to 2}(\bar{\boldsymbol{\alpha}})) - w_2^{pl}(\bar{\boldsymbol{\alpha}})$,

$$\Delta w_2^{pl}(\bar{\boldsymbol{\alpha}}) = \int_{\tau_2}^{\infty} e^{-(\rho+\lambda_{2\to 3}^{pl})t} \Bigg[\underbrace{W(Q_2(\alpha_{t,2})) - W(Q_2(\alpha_{t,2}^{lull}))}_{\substack{\text{Welfare flow from different} \\ \text{utilization during boom at time } t}} + \lambda_{2\to 3}^{pl} \underbrace{(w_3^{pl}(\alpha_{2\to 3}(\alpha_{t,2}) - w_3^{pl}(\alpha_{2\to 3}(\alpha_{t,2}^{lull})))}_{\substack{\text{Welfare effect of different} \\ \text{expected utilization during recession}}} \Bigg] dt$$

where $\alpha_{t,2}^{lull}$ is the path for optimists' wealth share with a speculative lull, τ_2 is the time of transition to $s = 2$, and $\alpha_{t,2}$ continues to denote the optimist wealth share without a lull.

Proposition 2 (Announcement effect reduces welfare).

Consider a leaning monetary policy with Q-targeting rule satisfying $Q_2(\bar{\boldsymbol{\alpha}}) < Q^*$ and $Q_2(\alpha)$ weakly increasing in α. The announcement effect reduces welfare:

$$\Delta w_2^{pl}(\bar{\boldsymbol{\alpha}}) < 0. \tag{16}$$

Optimists' wealth share and utilization with the announcement effect is lower in the boom $(s = 2)$ and the recession $(s = 3)$ than with a speculative lull, $\forall\, t$ regardless of when the recession arises:

$$\alpha_{t,2} < \alpha_{t,2}^{lull} \qquad \textit{and} \qquad \alpha_{t,3} < \alpha_{t,3}^{lull}, \tag{17}$$

$$\eta_{t,2} \le \eta_{t,2}^{lull} \qquad \textit{and} \qquad \eta_{t,3} < \eta_{t,3}^{lull}. \tag{18}$$

The announcement effect—the initial fall of optimists' wealth share when leaning monetary policy is adopted without a speculative lull—implies a lower path for the optimist wealth share throughout the boom and recession (expression (17)), for each potential realization of the arrival times of recession and recovery. This result obtains because of three features of the equilibrium: (i) adopting leaning monetary policy without a speculative lull causes a fall, on impact, in optimists' wealth share; (ii) the law of motion for the optimist wealth share ($\dot{\alpha}_s(\alpha)$, $s \in \{2, 3, 4\}$) is identical whether or not a speculative lull occurred at the time of the leaning announcement; and (iii) optimists' wealth share upon recession is increasing in optimists' wealth share just prior to recession ($\alpha_{2\to 3}(\alpha)$ is increasing in α, per Internet Appendix Lemma 2). The lower path for optimists' wealth share during the recession implies that utilization is strictly lower in the recession as a result of the announcement effect (second inequality in (18)).

These results—the lower path for optimists' wealth share during boom and recession and lower utilization during the recession when leaning is introduced without a speculative lull—do not depend on the assumption that the Q-targeting rule is weakly increasing in α. With the natural additional assumption that $Q_2(\alpha)$ is weakly increasing (implying that the risky asset price weakly appreciates over time during the boom), the lower path for optimists' wealth share during the boom also implies weakly lower utilization during the boom. Consequently, with the announcement effect reduces utilization during boom and recession, it follows that the announcement effect reduces welfare.

We next characterize the magnitude of these welfare losses. We focus on the size of the welfare losses with small doses of leaning, defined as follows. Many types of leaning policies can be described by the following family of Q-targeting rules: $Q_2(\alpha; \psi)$, with $Q_2(\alpha; \psi)$ strictly decreasing in ψ for all $\alpha \in (0, 1)$, and with $Q_2(\alpha; \psi)$ differentiable in ψ. That is, higher values of ψ imply more aggressive leaning, with $\psi = 0$ corresponding to conventional policy $(Q_2(\alpha; 0) = Q^*)$. We say that there are small amounts of leaning with $\psi \approx 0$, in the neighborhood of conventional policy.

Proposition 3 **(Sowing effects even for small leaning).**

Even for small amounts of tightening, the announcement effect implies a first-order welfare loss, as formalized by:

$$\frac{d\Delta w_2^{pl}(\bar{\boldsymbol{\alpha}})}{d\psi}|_{\psi=0} = \frac{dw_2^{pl}(\alpha_{1\to 2}(\bar{\boldsymbol{\alpha}}), Q_2(., \psi))}{d\psi}|_{\psi=0} - \frac{dw_2^{pl}(\bar{\boldsymbol{\alpha}}, Q_2(., \psi))}{d\psi}|_{\psi=0} < 0. \quad (19)$$

The first-order welfare loss from the announcement effect arises *only* because of lower recession-state utilization when leaning monetary policy is announced without a speculative lull. Lower utilization during the boom state implies only a second-order welfare loss when small doses of leaning monetary policy are used, as formalized by:

$$\frac{dW(Q_2(\alpha_{t,2}, \psi))}{d\psi}|_{\psi=0} = \frac{dW(Q_2(\alpha_{t,2}^{lull}, \psi))}{d\psi}|_{\psi=0} = 0, \quad (20)$$

which obtains because the instantaneous welfare function $W(Q)$ is maximized at $Q = Q^*$. Even so, the announcement effect implies a first-order loss in gap value, through utilization during the recession:

$$\underbrace{\frac{dw_3^{pl}(\alpha_{2\to 3}(\alpha_{t,2}; Q_2(., \psi)))}{d\psi}|_{\psi=0}}_{\substack{\text{Welfare effect of} \\ \text{leaning without lull}}} < \underbrace{\frac{dw_3^{pl}(\alpha_{2\to 3}(\alpha_{t,2}^{lull}; Q_2(., \psi)))}{d\psi}|_{\psi=0}}_{\substack{\text{Welfare effect of} \\ \text{leaning without lull}}} \quad (21)$$

The announcement effect's welfare cost under small doses of leaning through lower utilization *during the recession* are first order, whereas the cost of lower utilization *during the boom* are second order. Put differently, under conventional policy, utilization is at its efficient level during the boom (implying small welfare losses from small changes in utilization) but utilization is strictly below its efficient level during the recession.

4.1.1 Risk premium and sowing

We next analyze how the announcement of leaning monetary policy affects the risk premium, or, the expected return on the risky asset less the risk-free rate. During the boom states, the risk premium under the beliefs of agent type $i \in \{pl, o, p\}$ is

$$rp_s^i(\alpha) = \underbrace{\rho + g_s - \delta(\frac{Q_s(\alpha)}{Q^*}\eta^*) + \frac{\dot{Q}_s(\alpha)}{Q_s(\alpha)}}_{\text{Return absent recession}} + \lambda_{s\to 3}^i \underbrace{\left(\frac{Q_3(\alpha_{s\to 3}(\alpha))}{Q_s(\alpha)} - 1\right)}_{\text{Return upon recession}} - r_s^f(\alpha). \tag{22}$$

The next result shows that the announcement effect increases risk premia. For tractability, we consider a class of leaning monetary policy targeting rules parametrized by ζ and satisfying

$$\frac{Q_3(\alpha_{2\to 3}(\alpha))}{Q_2(\alpha;\zeta)} = \zeta, \tag{23}$$

with $\zeta < 1$. Eq. (23) has a simple interpretation: ζ is the gross return on the risky asset when a recession arrives. In the Internet Appendix, we construct $Q_2(\alpha;\zeta)$.

Proposition 4 (Risk premium).

Under the Q-targeting rule Eq. (23) in which the gross return on the risky asset upon recession is $\zeta < 1$, the risk premium under belief type $i \in \{o, p, pl\}$ is

$$rp_2^i(\alpha) = \lambda_{2\to 3}^p \frac{1-\alpha}{1-\alpha_{2\to 3}(\alpha)}\left(\zeta^{-1} - 1\right) + \lambda_{2\to 3}^i\left(\zeta - 1\right) \tag{24}$$

which is decreasing in α. Hence, the announcement effect, by inducing a lower path for optimists' wealth share (Proposition 2), induces a higher path for the risk premium (under all agents' beliefs).

The announcement effect raises the risk premium because it shifts wealth from the (levered) optimists to the pessimists, thereby increasing the wealth-weighted-average arrival rate of recession. Although the risk premium perceived by each agent type $i \in \{o, p, pl\}$ differs (expression (9)), all agent types agree about the sign and magnitude of the change in risk premium due to the announcement effect. For each agent type i, the perceived risk premium is decreasing in the arrival rate of recession, $\lambda_{2\to 3}^i$. As the central bank approaches extreme leaning (i.e., as ζ

converges to 1, making the cure as bad as the disease), the risk premium vanishes, because the boom is becoming as bad as the recession.

Proposition 4 sheds light on the debate about whether central banks should tighten monetary policy to increase risk premia when risk premia are low. Some proponents of leaning monetary policy suggest that increasing the risk premium from too-low levels is the mechanism through which leaning supports financial stability and welfare. Our results enrich this perspective, in a way that partly supports this view but also partly challenges it. In our risk-centric framework, the welfare consequences of increasing the risk premium through leaning monetary policy depend on how the risk premium is raised. Any rise in the risk premium due to the announcement effect— through the *balance-sheet channel* associated with capital losses for levered, high-valuation investors—are associated with a reduction in welfare and exacerbate the recession, following Propositions 2 to 4. That said, as discussed in Section 5 later, leaning under optimal policy may raise the risk premium and any such increase—which occurs through a discounting channel related to the level, slope and curvature of the yield curve, rather than by reducing the optimist wealth share though an announcement effect—is welfare-enhancing.

4.2 Macroprudential *regulation* "does not sow the wind"

The analysis so far has focused on a central bank altering its monetary policy rule, with the regulatory policy (the leverage limit $\bar{\omega}$) constant and exogenous. However, central banks also often have some control over regulatory policy, raising questions about the roles of monetary and regulatory policies in responding to the buildup of financial vulnerabilities. We next consider the implications of leaning through macroprudential regulation—that is, reducing the leverage limit—during the $s = 2$ boom. We allow the central bank to set a state-dependent regulatory limit $\tilde{\omega}(\alpha)$ that varies with optimists' wealth share. The central bank announces this rule at the start of $s = 2$ and the rule remains in place for the remainder of the boom.

Regulatory tightening has a macroprudential benefit, by dampening the decline in the optimist wealth share when the recession arrives. That is, $\alpha_{2\to 3}(\alpha)$ is decreasing in the leverage limit $\tilde{\omega}(\alpha)$ (equation (IA.10)). However, unlike a leaning monetary policy, announcing a regulatory tightening during the boom does *not* induce a decline in the risky asset price and optimists' wealth share. Under conventional policy, monetary policy can respond to the surprise regulatory tightening by adjusting the risk-free rate rule to maintain a risky asset price equal to the efficient level. Thus, relative to leaning monetary policy, regulatory tightening avoids two costs. First, a tighter regulation does not reduce utilization during the boom (Caballero and Simsek 2020b). Second, when implemented during the boom, unexpected regulatory tightening does not reduce optimists' wealth share. In sum, regulatory tightening does not sow the wind.

Proposition 5 (Regulatory policy).

A decrease the leverage limit $\tilde{\omega}(\alpha)$ during the boom has a macroprudential benefit (i.e., $\frac{d\alpha_{2\to3}(\alpha)}{d[-\tilde{\omega}(\alpha)]} \geq 0$ and strictly so if the leverage limit is binding) but implies no *change in optimists' wealth share upon announcement (i.e., $\alpha_{1\to2}(\bar{\boldsymbol{\alpha}}) = \bar{\boldsymbol{\alpha}}$).*

Underlying this result is that the central bank can fully stabilize utilization and the price of the risky asset when the economy is away from the zero lower bound.[17] An assumption in Internet Appendix A guarantees that the neutral risk-free rate during the boom is strictly positive and the central bank has room to offset any decline in the neutral rate from the announcement of a tightening of the leverage limit.

We next study the direction and magnitude of the welfare cost of leaning against speculation using monetary policy (relative to leaning by directly tightening regulation). To do so, we consider a type of regulatory tightening that achieves macroprudential benefits equivalent to the leaning monetary policy with Q-targeting rule $Q_2(\alpha)$. That is, the α-dependent leverage limit $\tilde{\omega}(\alpha)$ is such that the decline in optimists' wealth share upon recession is the same as under monetary leaning. The leverage limit $\tilde{\omega}(.)$ is constructed such that $\alpha_{2\to3}(\alpha)$ is the same: (i) when the leaning targeting rule is $Q_2(\alpha)$ (with the leverage limit $\bar{\omega}$ unchanged); and (ii) when the leverage limit is $\tilde{\omega}(\alpha)$ and the central bank pursues conventional monetary policy. Appendix A proves the existence of such a leverage limit $\tilde{\omega}(\alpha)$, which has sensible properties. For example, $\frac{d\tilde{\omega}(\alpha)}{dQ_2(\alpha)} > 0$, or, less aggressive monetary leaning can be mimicked by a looser leverage limit.[18]

By construction, $\alpha_{2\to3}(\alpha)$ is the same under leaning monetary policy and under this state-dependent leverage limit. Hence, the law of motion $\dot{\alpha}_2(\alpha)$ is also the same for these two prudential policies. With these observations, one can obtain the following result.

Corollary to Propositions 2-5.

Under leaning monetary policy: (i) Welfare with targeting rule $Q_2(\alpha)$ is lower than when the leverage limit is directly tightened to $\tilde{\omega}(\alpha)$. (ii) Optimists' wealth share is strictly lower than under regulatory tightening, for all t.

[17] This resonates as a sort of "divine coincidence" property of monetary policy in New Keynesian models, i.e. there is no trade-off between the stabilization of good price inflation (asset prices in this model) and the stabilization of the welfare-relevant output gap (the utilization gap of this model) for central banks.

[18] The monetary-policy-mimicking leverage limit is constructed similarly to the regulatory-policy-mimicking monetary policy in Caballero and Simsek (2020a).

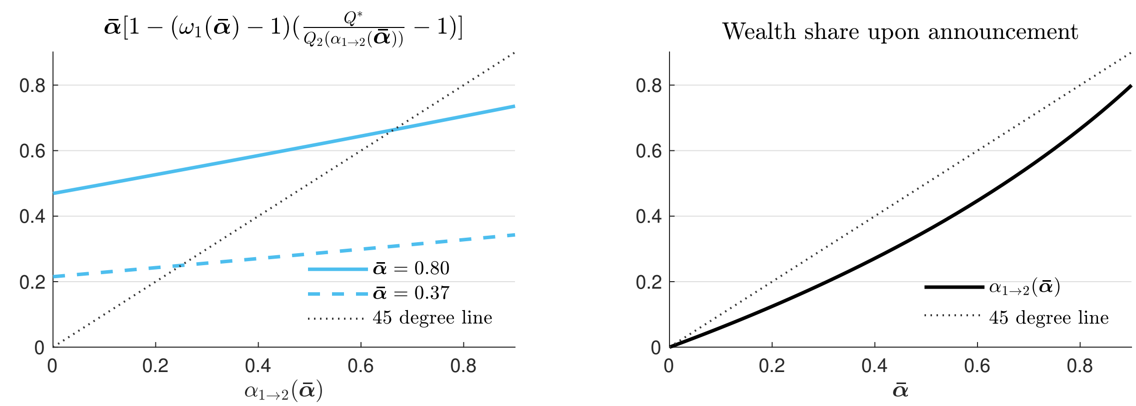

Figure 3: **Equilibrium with leaning-against-the-wind monetary policy**. The left panel illustrates the fixed point condition (15), for two different values of the optimist wealth share $\bar{\alpha}$. The right panel shows the solution to the fixed point condition, as a function of $\bar{\alpha}$.

Even for small amounts of tightening, the difference in welfare under leaning monetary policy relative to regulatory tightening is first-order.

Because leaning monetary policy sows the wind through the announcement effect on optimists' wealth share, but regulatory tightening with the same macroprudential benefits does not, monetary tightening and regulatory tightening are not substitutes for achieving financial stability: regulatory tightening is better than monetary tightening alone for dampening speculation, and the welfare difference is first-order even for small doses of tightening.

4.3 Numerical illustration

Figure 3, left panel, illustrates the decline in optimists' wealth share when the central bank adopts a leaning monetary policy in $s = 2$. The solid blue line shows the right hand side of equation (15), conditional on the optimist wealth share prior to the announcement equal to $\bar{\alpha} = 0.8$. That is, the solid blue line shows the post-announcement optimist wealth share if the new asset price is $Q_2(\alpha_{1\to2}(\bar{\alpha}))$. The intersection of this line with the 45-degree line is the post-announcement equilibrium optimist wealth share. Consistent with Proposition 1, the adoption of leaning monetary policy entails an immediate decline in the optimist wealth share (the blue line intersects the dotted line at a value $\alpha_{1\to2}(\bar{\alpha}) < \alpha$). The blue dashed line illustrates the same fixed point problem, but for a lower optimist wealth share prior to the change in monetary policy.

The right panel of Figure 3 shows the equilibrium optimist wealth share after announcing leaning, as a function of the wealth share just prior to the announcement ($\bar{\alpha}$). Leaning causes the

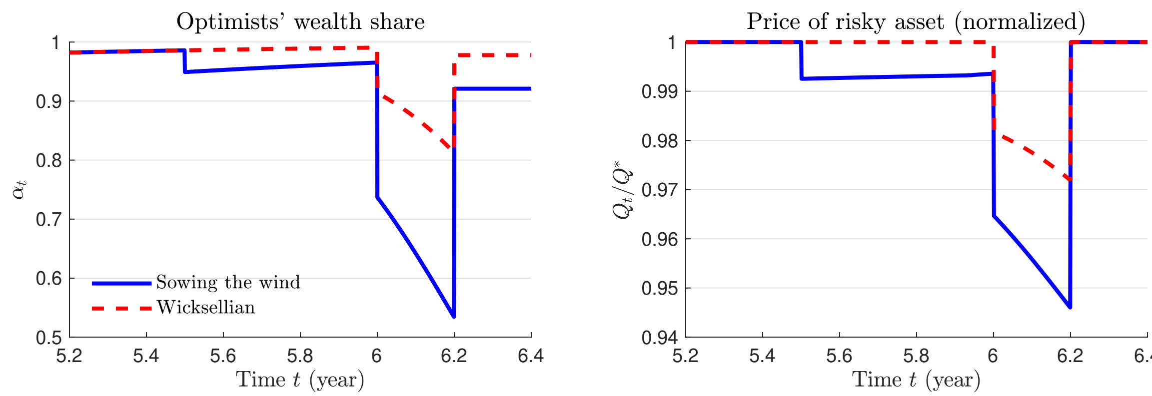

Figure 4: **Simulation of the equilibrium path under alternative monetary policies.** This figure shows the simulated path of the economy when the recession arrives at $t = 6$ years and the recovery arrives at $t = 6.2$ years. The red dashed lines correspond to conventional monetary policy, with the policy rate equal to the neutral rate during the boom. The blue solid lines show paths when the central bank adopts leaning monetary policy at $t = 5.5$ years.

optimist wealth share to fall if there is a non-degenerate mix of optimists and pessimists. If the economy initially consists of only pessimists ($\bar{\alpha} = 0$), then there is no leverage or speculation and a leaning announcement has no effect on the optimist wealth share. Similarly, when optimists are present but scarce (low $\bar{\alpha}$), there is little room for the optimist wealth share to decline. In contrast, when the economy's wealth is more evenly divided between optimists and pessimists, the decline in the optimist wealth share is larger.

Figure 4 shows the dynamic consequences of the decline in optimists' wealth share when leaning is announced. The figure presents a simulation of the economy's evolution under alternative monetary policies. The simulation presumes that the optimist wealth share at $t = 0$ is 0.45 and that the duration of the boom is 6 years, approximately the average of the boom duration expected by optimists and pessimists.

The red dashed lines show outcomes if the central bank follows a conventional monetary policy throughout the boom, with the policy rate equal to the neutral rate. During the boom, optimists' wealth share gradually rises. When the recession arrives (at $t = 6$ years), optimists' wealth share declines and then trends downward throughout the recession until the recovery (at $t = 6.2$). A longer recession would imply a greater decline in optimists' wealth share.

The blue solid lines in Figure 4 show outcomes if leaning is announced at the start of $s = 2$ (at $t = 5.5$). The announcement of the leaning Q-targeting rule (shown in Figure 2, left panel) triggers a decline in the optimist wealth share. During the remainder of the boom, the risky asset price and hence utilization are below their efficient levels for two reasons. First, for any optimist wealth share, the leaning policy requires a risky asset price below Q^*. Second, the announcement reduces the optimist wealth share and the leaning rule requires a lower asset price, the lower is the optimist wealth share. When the recession arrives, the optimist wealth share falls to a much lower level than under conventional policy. Correspondingly, the risky asset price and utilization are lower during the recession under leaning than under conventional policy. This illustrates how leaning monetary policy sows the wind.

After the recession arrives, the lower optimist wealth share under leaning monetary policy relative to under conventional policy reflects three forces, all of which point to worse outcomes with leaning. First, the already mentioned fall in the optimist wealth share upon announcement. Second, the growth rate of optimists' wealth share (conditional on the optimists' wealth share) $\dot{\alpha}_2(\alpha)$ is lower with leaning than under conventional policy (expression (12), shown in the bottom right panel of Figure 2). This second force has negligible impact over small time periods, but it accumulates over time following the announcement as long as the boom continues. Put differently, under leaning monetary policy, optimists wealth share would grow faster during $s = 2$ if it followed the same law of motion as under conventional policy. Third, when the recession arrives, the optimist wealth share falls by a greater magnitude under leaning policy than under conventional policy. This third force reflects that a higher optimist wealth share during the boom is itself a kind of macroprudential insurance: Because the recessionary price $Q_3(\alpha)$ is increasing in the optimist wealth share, a higher optimist wealth during the boom dampens speculation and implies that the asset price has less room to fall.

Figure 5 examines the welfare implications. The left panel shows the gap value (13) under alternative leaning policies, less the gap value under conventional policy. This difference in gap values is shown as a function of the strength of leaning ψ, holding constant the economy's "initial condition" at the start of $s = 2$ (optimists' wealth share $\bar{\alpha} = 0.8$). The black dash-dot line corresponds to regulatory policy. Small doses of regulatory tightening increase welfare and large doses, even more so. With a speculative lull (green dotted line), leaning monetary policy can achieve almost the same welfare gain as regulatory policy. In contrast, with the announcement effect (i.e., without a speculative lull), small doses of LAW *reduce* welfare relative to conventional policy and large doses much more so, as shown by the blue line. The right panel presents a similar analysis, but varying optimists' wealth share $\bar{\alpha}$ and holding constant the strength of leaning ($\psi = 0.5$). With the announcement effect, leaning reduces welfare for all initial optimist wealth shares relative to conventional policy.

Next we study the optimal Q-targeting rule of the form (14). We examine gap values when, conditional on optimists' wealth share just prior to recession $\bar{\alpha}$, the strength of leaning ψ is chosen to maximize the gap value. We define the optimal leaning policy within the class (14)

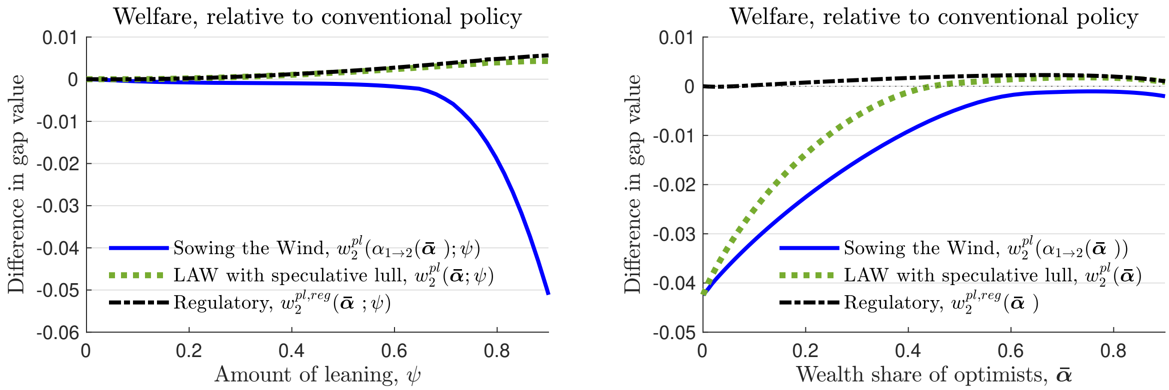

Figure 5: **Gap value under alternative monetary and regulatory policies.** Each panel shows the gap value under alternative policies less the gap value under conventional policy. $w_2^{pl}(\alpha;\psi)$ is the gap value under the leaning monetary policy associated with Q-targeting rule (14), with ψ parametrizing the strength of leaning. $w_2^{pl,reg}(\alpha;\psi)$ is the gap value under a tighter leverage limit $\tilde{\omega}(\alpha)$ that achieves the same macroprudential benefit (i.e., the same $\alpha_{2\to3}(\alpha)$ mapping) as the leaning monetary policy associated with Q-targeting rule (14). In the left panel, $\bar{\alpha} = 0.8$. In the right panel, $\psi = 0.5$.

and taking into account the announcement effect, $\psi^*(\bar{\alpha})$, as

$$\psi^*(\bar{\alpha}) = \arg\max_{\psi} w_2^{pl}(\alpha_{1\to2}(\bar{\alpha};\psi),\psi) \tag{25}$$

where $\alpha_{1\to2}(\bar{\alpha},\psi)$ is the solution to (15) conditional on the Q-targeting rule $Q_2(\alpha;\psi)$. Similarly, with this class (14), the optimal strength of leaning with a speculative lull is

$$\psi^{*,lull}(\bar{\alpha}) = \arg\max_{\psi} w_2^{pl}(\bar{\alpha};\psi). \tag{26}$$

The optimal strength of a direct regulatory tightening is:

$$\psi^{*,reg}(\bar{\alpha}) = \arg\max_{\psi} w_2^{pl,reg}(\bar{\alpha};\psi). \tag{27}$$

As shown in the top left panel of Figure 6, for each $\bar{\alpha}$, the optimal strength of leaning is highest when using direct regulatory tightening, intermediate with a speculative lull, and lowest when using leaning monetary taking into account the announcement effect. For this general class of targeting rules, with the announcement effect, the central bank would not lean (at all). In contrast, with a lull, the optimal strength of leaning through monetary policy is positive. The top right panel of Figure 6 examines the welfare consequences of sowing when the planner sets the optimal strength of leaning, taking into account the announcement effect. That is, the blue line shows the gap value when $\psi(\bar{\alpha}) = \psi^*(\bar{\alpha})$ and the central bank does not benefit from a speculative lull. The dotted green line shows the gap value when $\psi(\bar{\alpha}) = \psi^{*,lull}(\bar{\alpha})$ and the central bank does benefit from a speculative lull. When the strength of leaning is set optimally, the announcement effect implies a welfare loss, which is meaningful for intermediate and high values of the optimist wealth share.

A growing body of empirical evidence suggests that low risk premia, especially in conjuction with high amounts of borrowing, are associated with a substantially higher probability of a subsequent financial crash and economic slump (Greenwood et al. 2022). Consequently, researchers and policymakers have argued that central banks should lean more aggressively when risk premia are low.[19] Our results provide support for this view *if* leaning is through regulatory policy or preannounced, as shown in the bottom panels of Figure 6.

Summary. The results so far underscore the cost of waiting until the speculative boom is underway before adopting a leaning monetary policy that raises interest rates and reduces risky asset prices. Adopting such a leaning monetary policy during the boom weakly reduces the optimist wealth share, implying persistently lower optimist wealth share and utilization. Therefore, there is a first-order welfare cost to adopting leaning monetary policy during the boom (without a speculative lull), relative to preannouncing leaning or announcing during a speculative lull. Moreover, due to the announcement effect, adopting a leaning policy during the boom can lead to a *more* severe recession than maintaining conventional policy, even though leaning reduces the fall in optimists' wealth share when the recession arrives. If the central bank can announce leaning during a lull or use macroprudential regulation to lean, the optimal strength of leaning is greater, the more compressed is the risk premium on the risky asset. However, even when the risk premium is very low, a central bank considering whether to adopt leaning during a boom may optimally choose not to lean once the announcement effect is taken into account. Overall, in seeking to lean against the wind, the central bank may in fact be *sowing* it and later reap the whirlwind when the recession arrives.

5. Optimal policy with announcement effect

We now study fully optimal monetary policy taking into account the announcement effect. (Section 4.3 above studied the optimal strength of leaning confining attention to a given class of targeting rules.) Given an optimist wealth share $\bar{\alpha}$ just prior to the transition to $s = 2$, the planner's problem is:

$$\max_{\{Q_2(\alpha)\}_{\alpha\in[0,1]},\alpha_{1\to 2}} w_2^{pl}(\alpha_{1\to 2}) \tag{28}$$

subject to:

[19] A salient example—which also highlights the empirical relevance of the adoption of leaning once a speculative boom is underway—is the calls to tighten monetary policy to cool down overheated financial markets in 2012-13 (Stein 2013).

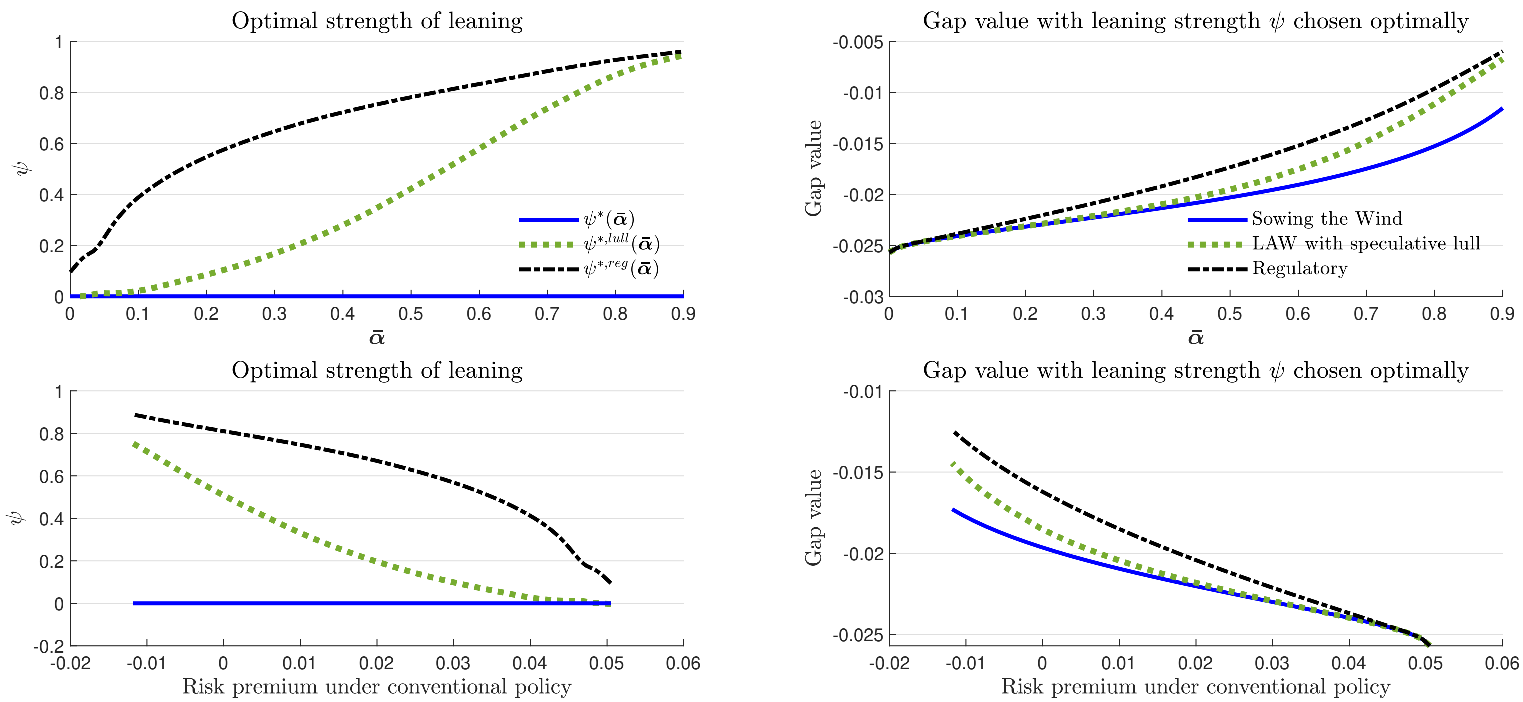

Figure 6: **Optimal leaning, within a class of simple Q targeting rules.**

$$\alpha_{1\to 2} = \bar{\boldsymbol{\alpha}}\left(1-(\omega_1^o(\bar{\boldsymbol{\alpha}})-1)\left(\frac{Q^*}{Q_2(\alpha_{1\to 2})}-1\right)\right) \tag{29}$$

$$\dot{\alpha}_2(\alpha) = \frac{\alpha(1-\alpha)\lambda^p_{2\to 3}}{1-\alpha_{2\to 3}(\alpha)}(1-\frac{\alpha_{2\to 3}(\alpha)}{\alpha}), \forall\alpha\in[0,1] \tag{30}$$

as well as the constraint $Q_2(\alpha) \in (Q_3(\alpha), Q^*], \forall\alpha \in [0,1]$.[20] The planner jointly chooses a monetary policy $Q_2(\alpha)$ and the optimist wealth share $\alpha_{1\to 2}$. The wealth share must be consistent with the sowing-the-wind state transition equation (29). Equation (30) governs the growth rate of optimists wealth after the new policy is in place, which is relevant for the gap value (13).

It is useful to consider first the central bank's problem without the sowing-the-wind constraint (29). This would correspond to the central bank's problem with a speculative lull or as if policy would have been "preannounced before the boom begins," which is the focus of Caballero and Simsek (2020a). Lower $Q_2(\alpha)$ has the macroprudential benefit of weakly increasing optimists' wealth share if a recession arrives ($\alpha_{2\to 3}(\alpha)$, Eq. (IA.10)) at the expense of lower utilization ($W(Q_2(\alpha))$ falls) and slower growth of optimists' wealth share ($\dot{\alpha}_2(\alpha)$ falls). These benefits and costs appear clearly on the right hand side of the gap value equation (13). We denote the optimal Q-targeting rule in the case of a lull—the argument of the maximum in the

[20]The constraint $Q_2(\alpha) > Q_3(\alpha)$ requires that the optimal policy satisfies the cure-no-worse-than-disease principle (Section 3). The maximand in Eq. (28), $w_2^{pl}(\alpha_{1\to 2})$, is the solution to (13) for $s = 2$.

planner's problem ignoring (29)—as $Q_2^{lull}(\alpha)$. The optimal policy rule $Q_2^{lull}(\alpha)$ clearly would not depend on $\bar{\boldsymbol{\alpha}}$.[21]

The problem of optimal policy once the boom is underway, however, is quite different because of the sowing-the-wind constraint. In setting $Q_2(\alpha)$ the central bank has to additionally take into account how this choice might affect the optimist wealth share ($\alpha_{1\to2}(\bar{\boldsymbol{\alpha}})$) when the new monetary policy rule is announced. Moreover, the optimal monetary policy in $s = 2$ depends on the optimist wealth share at the time of transition to $s = 2$. We denote the optimal Q-targeting rule taking the announcement effect into account—the argument of the maximum in the planner's problem including (29)—as $Q_2^{opt}(\alpha)$.

The next result characterizes optimal policy.

Proposition 6 (Optimal policy with announcement effect).

(i) Optimal policy involves no initial decline in the price of the risky asset: $Q_2^{opt}(\bar{\boldsymbol{\alpha}}) = Q^*$. *Therefore it has no macroprudential benefit immediately upon adoption.*

(ii) For an optimist wealth share $\alpha > \bar{\boldsymbol{\alpha}}$*, the optimal policy is the same as when the sowing constraint is not present:* $Q_2^{opt}(\alpha) = Q_2^{lull}(\alpha)$.

On impact, by avoiding a decline in the risky asset price, optimal policy—even though announced unexpectedly—does not reduce optimists' wealth share and hence does not sow the wind. Over time, the asset price falls, generating a macroprudential benefit (i.e., the optimist wealth share upon recession $\alpha_{2\to3}(\alpha)$ is higher than under conventional policy).[22] However, this subsequent fall in the asset price is *expected* and hence does not reduce optimists' wealth share.

Although the announcement of optimal policy is unexpected and involves a change in monetary policy (in the direction of tighter policy) there is no fall in the asset price upon announcement. To achieve this, monetary policy induces a bear steepening of the yield curve. The central bank commits to hike the policy rate over time if speculation continues (i.e., absent a transition to recession); the expectation of these hikes rotates the yield curve upward. In contrast, the expected policy rate over short horizons falls, to avoid an immediate fall in the risky asset price due to expected future hikes. This result is formalized next.

Consider the expected policy rate from the time of the policy change τ_2 over horizon h

[21] The planner's problem ignoring (29) can be written recursively and readily calculated numerically using standard recursive methods.

[22] Under optimal policy, the asset price equals Q^* immediately upon adoption and subsequently falls, if there is an $\alpha > \bar{\boldsymbol{\alpha}}$ such that optimal policy without the sowing constraint involves leaning (i.e., $Q^{lull}(\alpha) < Q^*$).

(taking into account the potential for state transitions):

$$\bar{r}_h^i \equiv \mathbb{E}_{\tau_2}^i \left[\frac{1}{h} \int_{\tau_2}^{\tau_2+h} r_{t,s}^f dt \right], \tag{31}$$

where $\mathbb{E}^i$ is the expectation operator under the beliefs of type $i \in \{o, p, pl\}$.[23] We denote the expected risk-free rate under optimal policy by $\bar{r}_h^{opt,i}$. The expected risk-free rate with a lull, under the optimal policy ignoring the announcement effect, is denoted by $\bar{r}_h^{lull,i}$. The next results capture how Bernanke's caution shows up in optimal monetary policy when the announcement effect is taken into account.

Proposition 7 characterizes the implications for the expected average risk-free rate under the optimal policy that takes into account the announcement effect, relative to optimal policy with a lull.

Proposition 7 **(Optimal policy and the expected path of the policy rate).** *Under optimal policy, at the time of the policy announcement, the expected policy rate curve under any agents' belief $i \in \{o, p, pl\}$ is lower and steeper than under optimal policy without taking into account the sowing constraint:*

$$\bar{r}_h^{opt,i} = \bar{r}_h^{lull,i} - \frac{1}{h} \ln \left(\frac{Q^*}{Q_2^{lull}(\bar{\boldsymbol{\alpha}})} \right). \tag{32}$$

If $Q_2^{lull}(\bar{\boldsymbol{\alpha}}) < Q^*$, then $\bar{r}_h^{opt,i} < \bar{r}_h^{lull,i}$ ("lower") *and $\frac{d\bar{r}_h^{opt,i}}{dh} > \frac{d\bar{r}_h^{lull,i}}{dh}$* ("steeper").

Proposition 7 shows that taking into account the sowing constraint requires a looser stance of monetary policy (with less hiking of the policy rate in the shorter run), relative to optimal preannounced policy (or optimal policy with a lull). If optimal policy with a lull involves reducing the risky asset price (leaning, with $Q_2^{lull}(\bar{\boldsymbol{\alpha}}) < Q^*$) when the policy change is announced, then the expected policy rate curve when the optimal policy $Q_2^{opt}(.)$ is announced is strictly lower and strictly steeper than under optimal policy with a lull. Moreover, from Eq. (32), it is clear that for sufficiently short horizons h, the expected policy rate path is below the neutral rate. The amount of steepening under optimal policy is greater, the lower is the risky asset price (the more aggressive is the leaning) under optimal policy with a lull. The expected policy rate curve with optimal policy when the central bank does not benefit from a lull differs from the expected policy rate curve under optimal policy with a lull, by an amount that does not depend on which agents' beliefs are used to calculate the expected path of policy, because all agent types anticipate a decline in the risky asset price of the same magnitude and same timing.

[23]The expected policy rate curve is the expectations component of the zero-coupon yield curve.

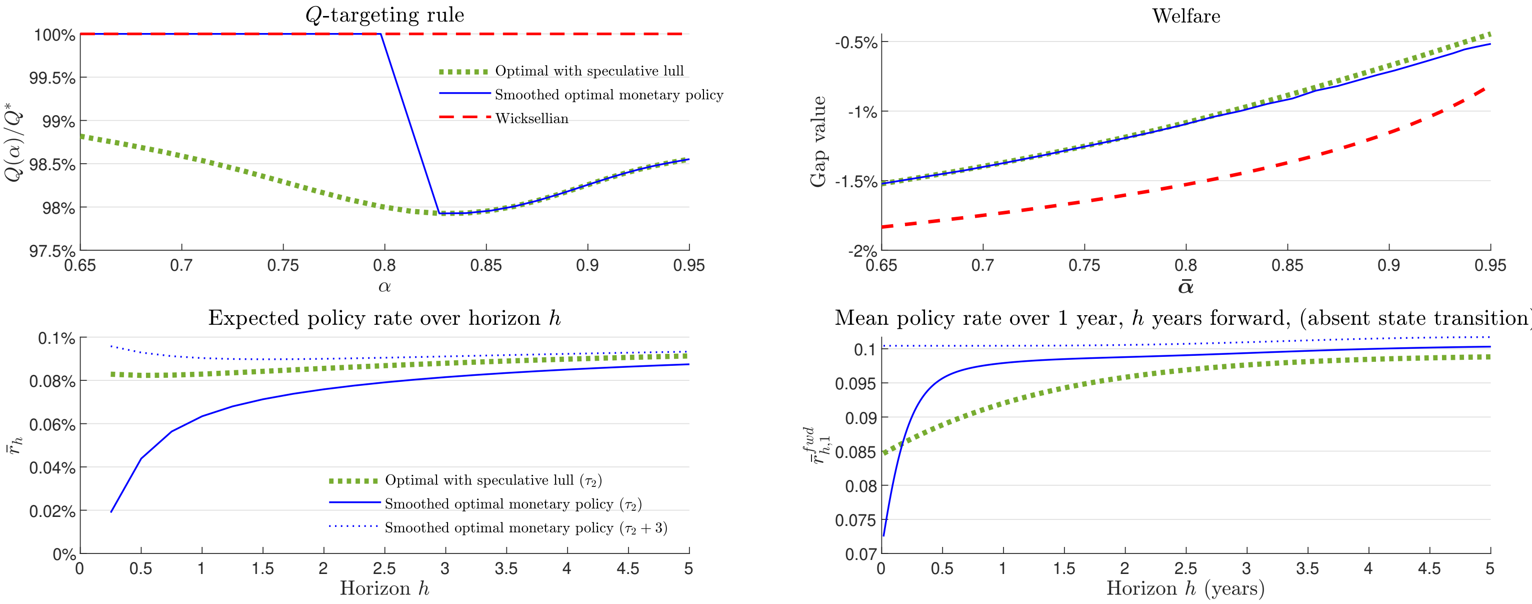

Figure 7: **Optimal monetary policy with announcement effect**

Numerical illustration. Figure 7, top left panel, illustrates how the optimal Q-targeting rule (taking into account the announcement effect) differs significantly from the optimal rule if the central bank can announce during a speculative lull (and thus ignore the announcement effect). The green dotted line shows optimal policy with a lull, $Q_1^{lull}(\alpha)$. The blue solid line shows the optimal policy, $Q_1^{opt}(\alpha)$. Specifically, the blue solid line is a smoothed approximation to the optimal policy in Proposition 6.[24] As shown in the top right panel of Figure 7, the smoothed approximation to optimal policy achieves nearly the same welfare as the optimal policy when the central bank benefits from a lull. The optimal monetary policy also differs significantly from conventional monetary policy, as shown by the dashed red lines in the top panels. Optimal monetary policy involves a generally tighter stance of monetary policy than conventional policy and achieves higher welfare.

The bottom panels of Figure 7 display the implementation of optimal policy and the consequences for the expected path of the policy rate. The bottom left panel shows the expected policy rate over various horizons, $\bar{r}_h$, under alternative policies and at different times. The green dotted line shows the expected policy rate under the optimal policy when the central bank announces during a lull and can ignore the announcement effect. The solid blue line shows the expected policy rate curve under optimal policy, at the time of announcement, when the

[24] In the approximation to optimal policy shown in Figure 7, $Q_2(\bar{\boldsymbol{\alpha}}) = Q^*$, as in Proposition 6. We assume that $Q_2(\alpha) = Q_2^{lull}(\alpha)$ for $\alpha > \bar{\alpha} + \hat{\alpha}$, with $\hat{\alpha}$ small ($\hat{\alpha} = 0.01$). Between $\bar{\alpha}$ and $\bar{\alpha} + \hat{\alpha}$, the adjustment is linear in α.

central bank does not benefit from a lull. Taking into account the announcement effect implies less leaning—with a uniformly lower expected average policy rate at all horizons h—and also a much stepper expected policy rate curve, consistent with Proposition 7. That is, monetary tightening is "backloaded" under optimal policy. The expected policy rate curve under optimal policy, 3 years after announcement, is shown by the blue dotted line in the bottom left panel. If the speculative boom continues, the central bank raises the policy rate and the expected policy rate curve "bear flattens" (i.e., the blue dotted line is uniformly above the blue solid line, with larger increases at shorter horizons).

To further illustrate the implications of optimal policy for the expected path of the policy rate, the bottom right panel studies the *forward* policy rate curve, which, by construction, is a leading indicator of the future stance of policy. The forward policy rate (at τ_2) over m years, beginning h years ahead and conditional on no state transition, is

$$\bar{r}^{fwd}_{h,m} = \int_{\tau_2+h}^{\tau_2+h+m} r^f_{t,2} dt. \tag{33}$$

Note that, for simplicity, Eq. (33) conditions on no state transition, with the economy remaining in the boom state.[25] The red dashed line in the bottom right panel shows the forward policy rate over 1 year, beginning h years ahead, under conventional monetary policy. The forward curve under conventional policy is upward sloping because the neutral rate of interest is increasing in the optimist wealth share, which rises gradually over time during the boom. The blue solid line shows the forward curve under optimal policy. Optimal policy involves a "bear steepening" of the forward rate curve, with forward rates under optimal policy above forward rates under conventional policy, except at very short horizons.

6. Sowing the wind: Anticipation effect

In this section, we study systematic leaning policies that are in place across business cycles. Such systematic policies avoid the announcement effect, because investors anticipate leaning—and hence are not taken by surprise when monetary policy leans during a speculative boom. However, we show that systematic leaning can also sow the wind, through the anticipation effect: When investors expect monetary policy to lean when a speculative boom is incipient, recessions are exacerbated. Put differently, a systematic policy of leaning as a speculative boom begins is equivalent to promising to run the economy "cold" during booms, the opposite of the

[25] A version of the forward rate that takes into account the possibility of state transitions is defined and shown in Internet Appendix B, and the results are qualitatively very similar.

common prescription to ameliorate recessions by promising to run the economy hot during booms. Interestingly, although the anticipation effect and the announcement effect are very different, the optimal systematic leaning policy has important similarities with the optimal leaning policy if announced unexpectedly amid the speculative boom.

6.1 Anticipation effect

The main mechanism of the anticipation effect can be understood by assuming that the economy begins in a recession, rather than a boom. We study how this initial recession is affected by the anticipation of leaning during the subsequent boom. We also study the design of optimal leaning policy taking this anticipation effect into account. We modify the model by assuming that at time $t = 0$, the state is $s = 0$, a recessionary state. Optimists believe the boom state will arrive sooner than pessimists do: $\lambda^o_{0\to1} > \lambda^p_{0\to1}$. The monetary policy that will be in place during the $s = 1$ boom is preannounced prior to $t = 0$, with full commitment and no opportunity for the central bank to change monetary policy. This assumption allows an analysis of the anticipation effect separate from an unanticipated change in monetary policy (announcement effect). The remainder of the model is unchanged. For tractability, we assume that the recession state is always followed by a speculative boom. This assumption facilitates the analysis, but all that is required for the anticipation effect is some possibility that the recession is followed by a speculative boom. Consistent with some possibility of a speculative boom after a lower-for-longer liquidity-trap bust, recent empirical work (Grimm et al. 2023b, among others) highlights that persistently accommodative monetary policy (intended to ameliorate recessions) tends to be associated with the emergence of speculation and financial vulnerabilities.

Let $\underline{\boldsymbol{\alpha}}$ denote optimists' wealth share just prior to the arrival of the $s = 1$ boom. We assume that the central bank can choose a systematic $s = 1$ leaning monetary policy $Q_1(\alpha; \underline{\boldsymbol{\alpha}})$ that is contingent on optimists' wealth share in $s = 0$ just prior to the boom's arrival. When choosing its systematic policy, the central bank does not know what the optimists' wealth share just prior to the boom's arrival ($\underline{\boldsymbol{\alpha}}$) will be, but the central bank can, in principle, condition its boom-period monetary policy on $\underline{\boldsymbol{\alpha}}$. This conditioning will be a key feature of the optimal systematic policy, as discussed below. Of course, the central bank can also choose not to condition on ($\underline{\boldsymbol{\alpha}}$). Once the $s = 1$ boom arrives, the optimists' wealth share is $\alpha_{0\to1}(\underline{\boldsymbol{\alpha}}) = \frac{\lambda^o_{0\to1}}{\lambda^o_{0\to1}(\underline{\boldsymbol{\alpha}})}$ (see Corollary to Lemma 1 in the Internet Appendix).

Using (7) and (8), the risky asset price $Q_0(\alpha)$ in state $s = 0$ satisfies the differential equation

$$\underbrace{\rho + g_0 - \delta(\frac{Q_0(\alpha)}{Q^*}\eta^*) - \frac{Q_0'(\alpha)}{Q_0(\alpha)}\alpha(1-\alpha)(\lambda^o_{0\to1} - \lambda^p_{0\to1})}_{\substack{\text{Instantaneous return on capital} \\ \text{if recession continues}}} + \underbrace{\bar{\lambda}_{0\to1}(\alpha)\left(1 - \frac{Q_0(\alpha)}{Q_1(\alpha_{0\to1}(\alpha);\alpha)}\right)}_{\substack{\text{Risk-adjusted boom-arrival term,} \\ \text{conditional on } Q_1(.)}} = 0, \tag{34}$$

and the boundary condition for $Q_0(0)$, the risky asset price conditional on an all-pessimist economy. This boundary condition is given by (34) for $\alpha = 0$. We compare outcomes during the initial recession but varying the degree of leaning during the boom. We say that a Q-targeting rule $Q_1(\alpha;\underline{\boldsymbol{\alpha}})$ is more aggressive than $\hat{Q}_1(\alpha;\underline{\boldsymbol{\alpha}})$ if $Q_1(\alpha;\underline{\boldsymbol{\alpha}}) < \hat{Q}_1(\alpha;\underline{\boldsymbol{\alpha}})$ for all $(\alpha,\underline{\boldsymbol{\alpha}}) \in (0,1)\times(0,1)$. We then obtain the following result:

Proposition 8 (Anticipation effects).

When investors anticipate more aggressive leaning in monetary policy in the subsequent boom: the risky asset price $Q_0(\alpha)$ *and utilization* $\eta_0(\alpha)$ *are strictly lower, for all* $\alpha \in (0,1)$*; and, during the initial recession, the time-paths for the risky asset price and utilization,* $\{Q_{t,0}, \eta_{t,0}\}_{t>0}$*, are strictly lower.*

We provide a short heuristic proof here, with the full proof in the appendix. The elasticity of the recessionary risky asset price to the optimist wealth share, $d\ln Q_0(\alpha)/d\alpha$, is higher under more aggressive policy, for all α and $Q_0(\alpha)$. Hence, as the $s = 1$ leaning policy becomes more aggressive, the recessionary risky asset price for a given optimist wealth share has to fall, in order to guarantee that it converges to the asset price in the all-pessimist economy (as $\alpha \to 0$).

Figure 8 illustrates the anticipation effect in our numerical model. We consider the class of simple Q-targeting rules of the form (14), $Q_1(\alpha;\underline{\boldsymbol{\alpha}}) = \psi Q_3(\alpha) + (1-\psi)Q^*$, and we set $\psi = 0.5$. (This Q-targeting rule does not vary with optimists' wealth share just prior to the boom.) The anticipation effect implies lower risky asset prices during the $s = 0$ recession, as shown in the left panel. Conditional on optimists' wealth share at the start of the boom and preannounced policy that avoids announcement effects, this leaning monetary policy *increases* the $s = 1$ gap value for intermediate and high values of α (green dotted line in Figure 5, right panel). In this *conditional* sense, leaning monetary policy does achieve a welfare-enhancing macroprudential benefit. However, taking anticipation effects into account, the same leaning policy *reduces* the $s = 0$ gap value (Figure 8, right panel).

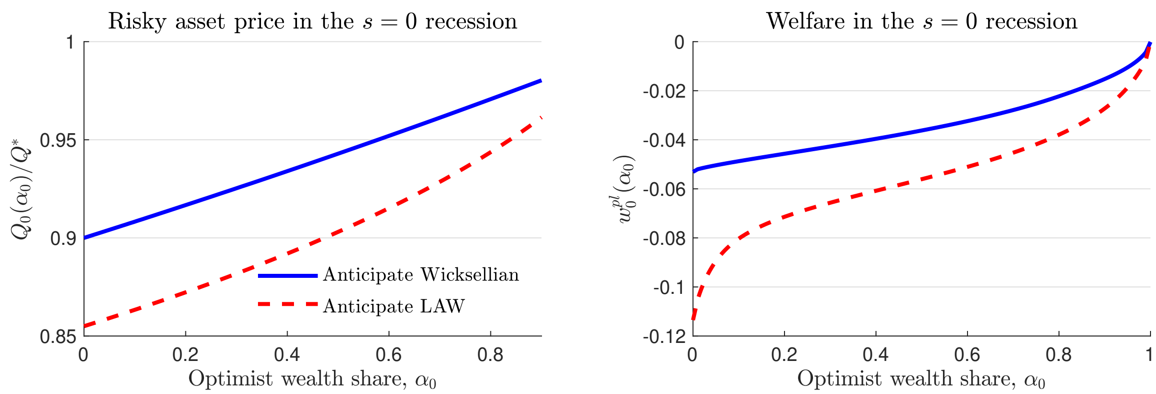

Figure 8: **Sowing through systematic leaning policy: The anticipation effect.**

6.2 Optimal systematic leaning

Next, we study optimal policy with the anticipation effect. The planner's problem is:

$$\max_{\{Q_1(\alpha;\underline{\alpha})\}_{(\alpha,\bar{\alpha})\in[0,1]\times[0,1]}} w_0^{pl}(\alpha) \tag{35}$$

subject to $\dot{\alpha}_0(\alpha) = \alpha(1-\alpha)(\lambda^p_{0\to1} - \lambda^o_{0\to1})$ and (34). The maximand in (35) is the solution to a differential equation of the form (13), but for $s = 0$, as specified in the proof of the next result. In Section 5, we discussed the planner's problem at $s = 1$ taking the wealth share α as given (as with a speculative lull). The optimal policy in that scenario was denoted $Q_1^{lull}(\alpha)$. For given optimist wealth share, lower values of $Q_1(\alpha;\underline{\alpha})$ imply a lower growth rate of optimists' wealth share and lower utilization during the boom but also greater macroprudential benefits if the $s = 3$ recession arrives. The solution to the planner's problem during the boom, assuming a speculative lull, balanced these tradeoffs. With anticipation effects, there are additional costs of leaning. For given α, lowering $Q_1(\alpha;\underline{\alpha})$ (uniformly in a neighborhood of α) reduces utilization during the recession whenever optimists' wealth share in the recession is above $\alpha^{-1}_{0\to1}(\alpha)$.[26] We therefore refer to (34) as a sowing-the-wind constraint, because more aggressive leaning during the boom reduces the risky asset price and utilization during the recession.

The next results characterize optimal policy with anticipation effects.

Proposition 9 (Optimal policy with anticipation effects).

(i) Systematic optimal policy involves no downward pressure on the risky asset price when the

[26] If the optimists wealth share in $s = 0$ is below $\alpha^{-1}_{0\to1}(\alpha) = \frac{\lambda^p_{0\to1}}{\lambda_{0\to1}(\alpha)}\alpha$, then the optimist wealth share in the recession will always be below the level such that if a boom arrives, the new optimist wealth share will be α.

speculative boom initially arrives: $Q_1^{opt}(\alpha;\underline{\boldsymbol{\alpha}}) = Q^*$ *for* $\alpha \leq \alpha_{0\to1}(\underline{\boldsymbol{\alpha}}) = \underline{\boldsymbol{\alpha}}\frac{\lambda^o_{0\to1}}{\lambda_{0\to1}(\underline{\boldsymbol{\alpha}})}$. *Therefore it has no immediate macroprudential benefit when the speculative boom is incipient.*

(ii) For an optimist wealth share $\alpha > \alpha_{0\to1}(\underline{\boldsymbol{\alpha}})$, *the systematic optimal policy is the same as when the sowing constraint is not present:* $Q_1^{opt}(\alpha;\underline{\boldsymbol{\alpha}}) = Q_1^{lull}(\alpha)$.

As a speculative boom begins, the optimal systematic policy allows the risky asset price to rise all the way to the price Q^* consistent with the economy operating at full capacity. Beyond the beginning of the boom, the central bank leans as much as would be optimal if there were no sowing constraint (i.e., the optimal policy if there were a speculative lull right before the boom arrived). The next result describes how to implement this optimal systematic policy.

Corollary to Proposition 9. *(Optimal expected policy path).*
(i) (Steeper early in the boom.) When the speculative boom ($s = 1$) *begins, under optimal policy, the expected policy rate curve under any agents' belief* $i \in \{o, p, pl\}$ *is lower and steeper than under optimal policy without taking into account the anticipation sowing constraint* (34)*:*

$$\bar{r}^{opt,i}_{h,s=1} = \bar{r}^{lull,i}_{h,s=1} - \frac{1}{h}\ln\left(\frac{Q^*}{Q_1^{lull}\left(\frac{\lambda^o_{0\to1}}{\lambda^o_{0\to1}(\underline{\boldsymbol{\alpha}})}\underline{\boldsymbol{\alpha}}\right)}\right). \tag{36}$$

(ii) (Lower during the initial recession.) During the initial recession ($s = 0$)*, under the optimal policy, the expected policy rate curve is lower than under optimal policy without taking into account the anticipation sowing constraint* (34)*. That is, for any* $h > 0$, $\bar{r}^{opt,i}_{h,s=0} < \bar{r}^{lull,i}_{h,s=0}$.

Taking into account the anticipation effect implies a smaller rise in the expected path of policy, relative to the optimal policy ignoring the anticipation effect. Moreover, at the start of the boom, the expected policy path is steeper under optimal systematic policy relative to optimal policy when sowing constraints are ignored. The back-loading or delay in rate hikes during the speculative boom under optimal systematic policy also implies that, during the initial recession, the expected policy rate curve is flatter.

The planner's problem with anticipation effects is very different than the problem with an announcement effect. With anticipation effects, the planner chooses a systematic or preannonced leaning policy for boom states, taking into account how such policy could depress utilization during an initial recession. In contrast, the planner's problem with announcement effects is to choose a discretionary leaning policy for the boom, taking into account how the unexpected announcement generates disproportionate capital losses for high-valuation investors, without taking into account the effects on prior recessions. Nonetheless, the form of optimal policies with announcement effects and with anticipation effects share important similarities. Set the risky asset price equal to Q^* instantaneously (at the start of the boom, in the case of systematic policy, or at the time of announcement, in the case of discretionary policy during the boom) and then subsequently lean against the wind in the way that optimally trades off macroprudential costs and benefits while ignoring announcement and anticipation effects.

7. Conclusion

Using a model with speculative booms and demand recessions, we study two intertemporal costs of leaning monetary policy and characterize optimal policy. Unexpected rate hikes when leaning is adopted during a speculative boom generates losses for levered investors, worsening the crash that leaning is meant to soften. With systematic leaning, recessions are more severe because investors anticipate the central bank will lean (run the economy cold) during a subsequent boom. Thus, leaning against the wind by raising short-term interest rates may in fact be sowing the wind, consistent with long-held concerns about the unintended consequences of leaning monetary policy.

However, the announcement and anticipation effects we uncover do not imply that a central bank is relegated to ignoring the build-up of financial vulnerabilities and waiting until after a crisis to clean up. Regulatory tightening, even if announced unexpectedly during a speculative boom, does not sow the wind if monetary policy is optimally adjusted to offset the decline in asset prices from tighter regulations. Moreover, even when regulatory tightening is infeasible, our optimal policy analysis shows that monetary policy can achieve macroprudential benefits by phasing in tightening over time, with the policy rate rising if the speculation continues. Optimal monetary policy is implemented through a "bear steepening" of the expected path of the policy rate. Forward policy rates—the expected policy rate over a fixed period, beginning at some future date—rise, while the expected policy rate over short horizons falls. This principle applies both when monetary policy is changed unexpectedly during a speculative boom and, under systematic optimal policy, when a speculative boom is incipient.

One direction for further research is to understand how shadow banks affect the transmission of leaning monetary policy and the implications for optimal monetary policy. An often-cited benefit of leaning monetary policy is to "get in all of the cracks" of the financial system, including shadow banks (Stein 2013). The get-in-all-the-cracks property of leaning monetary policy, however, becomes a disadvantage in regard to the announcement effect: Because shadow banks can take risks that the regulated banking sector cannot, they may experience even larger losses if leaning is unexpectedly adopted.

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Internet Appendix for "Sowing the Wind" Monetary Policy

Jonathan Goldberg and David López-Salido

A. Proofs and derivations

Q-targeting rule in $s = 2$. We restrict attention to Q-targeting rules in the $s = 2$ state that satisfy $\frac{Q_2'(\alpha)}{Q_2(\alpha)} < 1$. That is, in $s = 2$, the elasticity of the asset price to the optimist wealth share is not too high. This assumption holds in our numerical analyses and is a sufficient condition (it can be relaxed).

Neutral rates. To guarantee a negative neutral rate during the recession and a positive neutral rate during the recovery, we assume $\delta(0) - (\rho + \lambda^p_{3\to4}) < g_3 < \delta(\eta^*) - \rho < g_4$. To ensure that the neutral rate is strictly positive during the boom states, we assume, for $s \in \{1, 2\}$, $\rho + g_s - \delta(\eta^*) - \lambda^p_{s\to3}(\frac{Q^*}{Q_3(0)} - 1) > 0$.

Risky asset price during the recession. The risky asset price in the recession state $s = 3$ satisfies

$$\frac{d \ln Q_3(\alpha)}{d\alpha} = \frac{1}{\alpha(1-\alpha)(\lambda^o_{3\to4} - \lambda^p_{3\to4})}\left(\rho + g_3 - \delta\left(\frac{Q_3(\alpha)}{Q^*}\eta^*\right) + \bar{\lambda}_{3\to4}(\alpha)\left(1 - \frac{Q_3(\alpha)}{Q^*}\right)\right) \quad \text{(IA.1)}$$

and the boundary conditions

$$\rho + g_3 - \delta\left(\frac{Q_3(0)}{Q^*}\eta^*\right) + \lambda^p_{3\to4}\left(1 - \frac{Q_3(0)}{Q^*}\right) = 0 \quad \text{(IA.2)}$$

and

$$\rho + g_3 - \delta\left(\frac{Q_3(1)}{Q^*}\eta^*\right) + \lambda^o_{3\to4}\left(1 - \frac{Q_3(1)}{Q^*}\right) = 0 \quad \text{(IA.3)}$$

The assumptions regarding g_3 (see "Neutral rates" above) ensure existence and uniqueness of $Q_3(\alpha)$ satisfying (IA.1)-(IA.3) with $Q_3(\alpha) \in (0, Q^*)$ and $Q_3'(\alpha) > 0$ for $\alpha \in [0, 1]$.

Leverage limit. We assume $\bar{\omega} < \hat{\omega}$, where $\hat{\omega} = \frac{1}{1 - \frac{Q_3(0)}{2Q^*}}$. This assumption requires that the leverage limit is below a threshold $\hat{\omega}$, with this threshold strictly greater than 1 (because $Q_3(0) > 0$) and increasing in $Q_3(0)/Q^*$, or, the ratio of the recessionary asset price in the all-pessimist economy to the efficient asset price. This is a sufficient condition and can be relaxed.

Proof of Proposition 1. We first sketch the proof. If $Q_2(\bar{\boldsymbol{\alpha}}) = Q^*$, then $\alpha_{1\to2}(\bar{\boldsymbol{\alpha}}) = \bar{\boldsymbol{\alpha}}$ is a solution to (15). For the case $Q_2(\bar{\boldsymbol{\alpha}}) < Q^*$, a heuristic version of the proof can be seen immediately from the left panel of Figure 1. The optimist wealth share after announcement (conditional on asset price Q_2 and shown in purple) is increasing in Q_2 and includes the point $(\bar{\boldsymbol{\alpha}}, Q^*)$. Hence, a Q-target with $Q_2(\bar{\boldsymbol{\alpha}}) < Q^*$ implies a new optimist wealth share $\alpha_{1\to 2}(\bar{\boldsymbol{\alpha}}) < \bar{\boldsymbol{\alpha}}$.

Next, we proceed to the complete proof of the existence and uniqueness of $\alpha_{1\to 2}(\alpha)$ and the properties claimed in Proposition 1. Optimist leverage in the $s = 1$ boom is $\omega_1^o(\alpha) = \min\{\bar{\omega}, \omega_1^{unc}(\alpha)\} \in (1, \infty)$, where the optimist leverage, if unconstrained in boom state $s \in \{1, 2\}$, is

$$\omega_s^{unc}(\alpha) = 1 + \frac{\frac{\lambda^o_{s\to 3}}{\lambda_{s\to 3}(\alpha)} - 1}{1 - \frac{Q_s(\alpha)}{Q_3\left(\frac{\lambda^o_{s\to 3}}{\lambda_{s\to 3}(\alpha)}\alpha\right)}} > 1 \tag{IA.4}$$

Using this result ($\omega_1^o(\alpha) > 1$) and (15), $Q_2(\bar{\boldsymbol{\alpha}}) = Q^*$ implies $\alpha_{1\to 2}(\bar{\boldsymbol{\alpha}}) = \bar{\boldsymbol{\alpha}}$. The remainder of the proof focuses on the case $Q_2(\bar{\boldsymbol{\alpha}}) < Q^*$. For $(\alpha_2, \alpha_1) \in (0,1) \times (0,1)$, define the function $f(\alpha_2, \alpha_1)$ as

$$f(\alpha_2, \alpha_1) \equiv \alpha_1 \left(1 - (\bar{\omega} - 1)\left(\frac{Q^*}{Q_2(\alpha_2)} - 1\right)\right). \tag{IA.5}$$

The optimist wealth share following adoption, $\alpha_{1\to 2}(\bar{\boldsymbol{\alpha}})$, satisfies the fixed-point condition $f(\alpha_{1\to 2}(\bar{\boldsymbol{\alpha}}); \bar{\boldsymbol{\alpha}}) = \alpha_{1\to 2}(\bar{\boldsymbol{\alpha}})$, as can be seen by comparing (15) and (IA.5). To complete the proof, we will show that (i) $f(\alpha_{1\to 3}(\bar{\boldsymbol{\alpha}}); \bar{\boldsymbol{\alpha}}) > \alpha_{1\to 3}(\alpha)$; (ii) $f(\bar{\boldsymbol{\alpha}}; \bar{\boldsymbol{\alpha}}) < \bar{\boldsymbol{\alpha}}$; and (iii) if $\alpha_2 \in (0,1)$ satisfies $f(\alpha_2; \bar{\boldsymbol{\alpha}}) = \alpha_2$, it holds that $f_{\alpha_2}(\alpha_2, \bar{\boldsymbol{\alpha}}) < 1$.

Starting with claim (i): The optimist wealth share $\alpha_{1\to 3}(\alpha)$ that obtains if the economy transitions from $s = 1$ directly into a recession satisfies the fixed-point condition:

$$\alpha_{1\to 3}(\alpha) = \alpha\left(1 - (\bar{\omega} - 1)\left(\frac{Q^*}{Q_3(\alpha_{1\to 3}(\alpha))} - 1\right)\right), \tag{IA.6}$$

where α is the optimist wealth share just prior to recession. Combining (IA.5), (IA.6), and $Q_2(\alpha) > Q_3(\alpha)$ ("cure-no-worse-than-disease" assumption), one obtains $f(\alpha_{1\to 3}(\alpha), \alpha) > \alpha_{1\to 3}(\alpha)$, for $\alpha \in (0,1)$. Thus, we have condition (i).

Condition (ii) follows immediately from $Q_2(\alpha) < Q^*$ and $\bar{\omega} > 1$.

For condition (iii), differentiate (IA.5) to obtain

$$f_{\alpha_2}(\alpha_2, \alpha_1) = \alpha_1(\bar{\omega} - 1)\frac{Q^*}{Q_2(\alpha_2)^2} Q_2'(\alpha_2). \tag{IA.7}$$

Eq. (IA.7) and $f(\alpha_2, \alpha_1) = \alpha_2$ imply

$$f_{\alpha_2}(\alpha_2, \alpha_1) = (\alpha_1 \bar{\omega} - \alpha_2) \frac{Q_2'(\alpha_2)}{Q_2(\alpha_2)}. \tag{IA.8}$$

Using again $f(\alpha_2, \alpha_1) = \alpha_2$, we have

$$\alpha_1 \bar{\omega} - \alpha_2 = (\alpha_1 - \alpha_2) \left(\frac{Q^*}{Q^* - Q_2(\alpha_2)} \right). \tag{IA.9}$$

Next, note that $\bar{\omega} < \hat{\omega}$ implies $\bar{\omega} < 1 + \frac{Q_3(0)}{Q^*}$, because $Q_3(0) < Q^*$. In addition, $\bar{\omega} < 1 + \frac{Q_3(0)}{Q^*}$ implies $\alpha - \alpha_{1\to 2}(\alpha) < 1 - \frac{Q_2(\alpha_{1\to 2}(\alpha))}{Q^*}$ for any $\alpha \in (0,1)$ for which the leverage constraint is binding (i.e., with $\omega_1^{unc}(\alpha) > \bar{\omega}$). Combining this result with Eqs. (IA.8) and (IA.9), we obtain $f_{\alpha_2}(\alpha_2, \bar{\boldsymbol{\alpha}}) < 1$ if $f(\alpha_2, \bar{\boldsymbol{\alpha}}) = \alpha_2$ (claim (iii)).

The macroprudential benefit of leaning monetary policy. When changing its policy rule during $s = 2$, the central bank takes into account the macroprudential benefits of tighter monetary policy: When the leverage constraint is binding, a lower asset price during the boom reduces the severity of the subsequent recession (Caballero and Simsek 2020a), as explained in the next result, Lemma 1.

Lemma 1. If a recession arrives, optimists' wealth share falls: $\alpha_{s\to 3}(\alpha) < \alpha$, for $s \in \{1, 2\}$ and $\alpha \in (0,1)$. Specifically,

$$\alpha_{s\to 3}(\alpha) = \begin{cases} \alpha \frac{\lambda_{s\to 3}^o}{\lambda_{s\to 3}(\alpha)} & \text{if} \quad \omega_s^o(\alpha) < \bar{\omega} \\ \alpha \left[1 - (\bar{\omega} - 1) \left(\frac{Q_s(\alpha)}{Q_3(\alpha_{s\to 3}(\alpha))} - 1 \right) \right] & \text{if} \quad \omega_s^o(\alpha) = \bar{\omega}, \end{cases} \tag{IA.10}$$

with $\frac{d\alpha_{s\to 3}(\alpha)}{dQ_s(\alpha)} \leq 0$, and strictly so if the leverage limit is binding. There is a threshold $Q_s^{bind}(\alpha)$ such that the leverage constraint binds if and only if $Q_s(\alpha) \leq Q_s^{bind}(\alpha)$.

Discussion of Lemma 1. When the leverage constraint binds during the boom, the optimist wealth share if a recession arrives $(\alpha_{s\to 3}(\alpha))$ is determined according to the bottom expression in (IA.10). A policy that reduces $Q_2(\alpha)$ therefore softens the optimist wealth share decline if a recession arrives, so long as $Q_2(\alpha) < Q_s^{bind}(\alpha)$ (i.e., $Q_2(\alpha)$ is low enough that the leverage constraint is binding). A higher optimist wealth share during the recession is associated with a higher asset price and higher utilization during the recession, when monetary policy is constrained.

Proof of Lemma 1. First consider the case in which optimists' leverage constraint is not binding during the boom state $s \in \{1,2\}$. In this case, (6) holds with equality for all investors (optimists and pessimists). This implies $\lambda^o_{s\to 3}\frac{\alpha}{\alpha_{s\to 3}(\alpha)} = \lambda^p_{s\to 3}\frac{1-\alpha}{1-\alpha_{s\to 3}(\alpha)}$. This result and the definition of $\bar{\lambda}_{s\to 3}$ (Eq. (8)) imply the top line of (IA.10). The bottom line follows from (5). Finally, using (5) and the top line of (IA.10), we obtain that the leverage constraint in $s \in \{1,2\}$ binds for optimists when their wealth share is α if and only if $Q_s(\alpha) \le Q^{bind}_s(\alpha)$, defined as the solution to

$$\frac{\lambda^o_{s\to 3}}{\bar{\lambda}_{s\to 3}(\alpha)} = 1 - (\bar{\omega}-1)\left(\frac{Q^{bind}_s(\alpha)}{Q_3(\frac{\lambda^o_{s\to 3}}{\bar{\lambda}_{s\to 3}(\alpha)}\alpha)} - 1\right). \tag{IA.11}$$

Next, we consider the case of a binding leverage constraint $\omega^o_s(\alpha) = \bar{\omega}$. Define $f^{s\to 3}(\alpha_3, \alpha)$ as follows:

$$f^{s\to 3}(\alpha_3,\alpha) = \alpha\left[1 - (\bar{\omega}-1)\left(\frac{Q_s(\alpha)}{Q_3(\alpha_3)} - 1\right)\right]. \tag{IA.12}$$

For α for which the leverage constraint is binding, $\alpha_{s\to 3}(\alpha) = f^{s\to 3}(\alpha_{s\to 3}(\alpha),\alpha)$. Analogously to Proposition 1, to complete the proof, we will show that (i) $f^{s\to 3}(\alpha\frac{\lambda^o_{s\to 3}}{\bar{\lambda}_{s\to 3}(\alpha)};\alpha) > \alpha\frac{\lambda^o_{s\to 3}}{\bar{\lambda}_{s\to 3}(\alpha)}$; (ii) $f^{s\to 3}(\alpha;\alpha) < \alpha$; and (iii) if $\alpha_3 \in (0,1)$ satisfies $f^{s\to 3}(\alpha_3;\alpha) = \alpha_3$, it holds that $f^{s\to 3}_{\alpha_3}(\alpha_3,\alpha) < 1$.

With a binding leverage constraint,

$$f^{s\to 3}(\alpha\frac{\lambda^o_{s\to 3}}{\bar{\lambda}_{s\to 3}(\alpha)};\alpha) = \alpha\left[1 - (\bar{\omega}-1)\left(\frac{Q_s(\alpha)}{Q_3(\alpha\frac{\lambda^o_{s\to 3}}{\bar{\lambda}_{s\to 3}(\alpha)})} - 1\right)\right]. \tag{IA.13}$$

Combining this expression with $Q_s(\alpha) < Q^{bind}_s(\alpha)$, and using (IA.11), we have claim (i). Next, substituting $\alpha_3 = \alpha$ into (IA.12), observe that

$$f^{s\to 3}(\alpha,\alpha) = \alpha\left[1 - (\bar{\omega}-1)\left(\frac{Q_s(\alpha)}{Q_3(\alpha)} - 1\right)\right]. \tag{IA.14}$$

Claim (ii) immediately follows from (IA.14), $Q_3(\alpha) < \min\{Q_2(\alpha), Q^*\}$ (cure-no-worse-than-disease assumption), and $\bar{\omega} > 1$. For claim (iii), analogously to (IA.7), using (IA.12) we have

$$f^{s\to 3}_{\alpha_3}(\alpha_3,\alpha) = \alpha(\bar{\omega}-1)\frac{Q_s(\alpha)}{Q_3(\alpha_3)^2}Q_3'(\alpha_3). \tag{IA.15}$$

We assume $\frac{Q_3'(\alpha)}{Q_3(\alpha)} < 1$. (Similarly to $\frac{Q_2'(\alpha)}{Q_2(\alpha)} < 1$, this is a sufficient condition.) The leverage-limit assumption $\bar{\omega} < \hat{\omega}$ then implies that expression (IA.15) is strictly less than 1 whenever $f(\alpha_3,\alpha) = \alpha_3$.

From (IA.10), it follows that $\frac{d\alpha_{s\to 3}(\alpha)}{dQ_s(\alpha)} = 0$ if the leverage constraint is not binding. Otherwise

$$\frac{d\alpha_{s\to 3}(\alpha)}{dQ_s(\alpha)} = -\frac{\alpha(\bar{\omega}-1)\frac{1}{Q_3(\alpha_{s\to 3}(\alpha))}}{1-f^{s\to 3}_{\alpha_3}(\alpha_{s\to 3}(\alpha),\alpha)} < 0, \tag{IA.16}$$

where claim (iii) above guarantees $f^{s\to 3}_{\alpha_3}(\alpha_{2\to 3}(\alpha),\alpha) < 1$ and hence implies the inequality in (IA.16). Note that (IA.16) implies that the elasticity of the optimist wealth share upon recession to the asset price during the boom is:

$$\frac{d\alpha_{s\to 3}(\alpha)}{dQ_s(\alpha)}\frac{Q_s(\alpha)}{\alpha_{s\to 3}(\alpha)} = -\frac{\alpha}{\alpha_{s\to 3}(\alpha)}\frac{1}{1-f^{s\to 3}_{\alpha_3}(\alpha_{2\to 3}(\alpha),\alpha)}\frac{(\bar{\omega}-1)Q_s(\alpha)}{Q_3(\alpha_{2\to 3}(\alpha))} < 0, \tag{IA.17}$$

Corollary to Lemma 1. In the $s = 0$ recession in Section 6, the absence of a leverage limit implies, via a very similar argument to the proof of Lemma A1, that $\alpha_{0\to 1}(\alpha) = \frac{\lambda^o_{0\to 1}}{\lambda_{0\to 1}(\alpha)}\alpha$.

Lemma 2. For given optimist wealth share α during the $s = 2$ boom, optimists' wealth share upon recession, $\alpha_{2\to 3}(\alpha)$, is increasing in α.

Proof of Lemma 2. First, suppose that the leverage constraint is not binding: $\omega^{unc}_1(\alpha) < \bar{\omega}$, with $\omega^{unc}_1(\alpha)$ given by (IA.4). Then, from the top line of (IA.10), $\alpha_{2\to 3}(\alpha) = (1 + (\frac{1}{\alpha}-1)\frac{\lambda^p_{2\to 3}}{\lambda^o_{2\to 3}})^{-1}$, which is increasing in α. Second, for α for which the leverage constraint is binding: Taking the derivative of the bottom line of (IA.10) with respect to α, we have

$$\frac{d\alpha_{s\to 3}(\alpha)}{d\alpha} = \frac{1}{1-f^{s\to 3}_{\alpha_3}(\alpha_{s\to 3}(\alpha),\alpha)}\left(1-(\bar{\omega}-1)\left(\frac{Q_s(\alpha)}{Q_3(\alpha_{s\to 3}(\alpha))}-1\right)-\alpha(\bar{\omega}-1)\frac{Q_s'(\alpha)}{Q_3(\alpha_{s\to 3}(\alpha))}\right) \tag{IA.18}$$

Recall from claim (iii) in the Proof of Lemma 1 that $f^{s\to 3}_{\alpha_3}(\alpha_{s\to 3}(\alpha),\alpha) < 1$. Note that if $Q_s'(\alpha) \le 0$, then expression (IA.18) is strictly positive, and the proof of the lemma is complete. We will next address the case of $Q_s'(\alpha) > 0$. Re-arranging (IA.18), the condition $\frac{d\alpha_{s\to 3}(\alpha)}{d\alpha} > 0$ is equivalent to

$$\frac{\bar{\omega}}{\bar{\omega}-1} > \frac{Q_s(\alpha)}{Q_3(\alpha_{1\to 3}(\alpha))}\left(1+\alpha\frac{Q_s'(\alpha)}{Q_s(\alpha)}\right) \tag{IA.19}$$

Then, $Q_2(\alpha) \le Q^*$ and $Q_3(0) \le Q_3(\alpha_{1\to 3}(\alpha))$ and the assumptions $Q_2'(\alpha)/Q_2(\alpha) < 1$ and $\bar{\omega} < \hat{\omega}$ together imply (IA.19), completing the proof of the lemma.

Proof of Proposition 2. From proposition 1, $Q_2(\bar{\boldsymbol{\alpha}}) < Q^*$ implies $\alpha_{1\to 2}(\bar{\boldsymbol{\alpha}}) < \bar{\boldsymbol{\alpha}}$. Thus, if the transition from state $s = 1$ to $s = 2$ occurs at time τ_2, then $\alpha_{\tau_2,2} < \alpha^{lull}_{\tau_2,2}$. From (12) and (IA.10), the law of motion $\dot{\alpha}_2(\alpha)$ is the same with and without a speculative lull, because $Q_2(\alpha)$ and $\alpha_{2\to 3}(\alpha)$

are the same with and without a lull. These results imply $\alpha_{t,2} < \alpha_{t,2}^{lull}$, for $t \geq \tau_2$. $Q_2(\alpha)$ is assumed in the proposition to be weakly increasing in α. Therefore, $\alpha_{t,2} < \alpha_{t,2}^{lull}$ implies $\eta_{t,2} \leq \eta_{t,2}^{lull}$. Next, denote the arrival time of recession by τ_3. By Lemma 2 (i.e., $\alpha_{2\to3}(\alpha)$ increasing in α) and using $\alpha_{t,2} < \alpha_{t,2}^{lull}$, we have that optimists' wealth share at the start of the recession is lower than with a lull $\alpha_{\tau_3,3} < \alpha_{\tau_3,3}^{lull}$. The law of motion $\dot{\alpha}_3(\alpha) = -(\lambda_{3\to4}^{o} - \bar{\lambda}_{3\to4}(\alpha))$ is the same whether or not there was a lull in $s = 2$, thus implying $\alpha_{t,3} < \alpha_{t,3}^{lull}$ for $t > \tau_3$. Because the recessionary asset price $Q_3(\alpha)$ is increasing in α, the result $\alpha_{t,3} < \alpha_{t,3}^{lull}$ immediately implies $\eta_{t,3} < \eta_{t,3}^{lull}$. The welfare cost of the announcement effect ($\Delta w_2^{pl}(\bar{\boldsymbol{\alpha}})$) is the appropriately discounted difference of instantaneous welfare flows with and without a lull at the time when the central bank adopts leaning. Because $W(Q)$ is increasing in Q for $Q < Q^*$, we obtain that, for all $t > \tau_2$ and for every possible history $\mathcal{H}^t$, the instantaneous welfare flow is weakly lower due the announcement effect, and strictly so during the state $s = 3$.

Proof of Proposition 3. Taking the derivative of $\Delta w_2^{pl}(\bar{\boldsymbol{\alpha}})$ with respect to ψ,

$$\frac{dw_2^{pl}(\alpha_{1\to2}(\bar{\boldsymbol{\alpha}}), Q_2(.))}{d\psi} - \frac{dw_2^{pl}(\bar{\boldsymbol{\alpha}}; Q_2(.))}{d\psi} = \int_{\tau_2}^{\infty} e^{-(\rho+\lambda_{2\to3}^{pl})(t-\tau_2)} \Bigg[\frac{dW(Q_2(\alpha_{t,2}))}{d\psi} - \frac{dW(Q_2(\alpha_{t,2}^{lull}))}{d\psi} + \lambda_{2\to3}^{pl}\left(\frac{dw_3^{pl}(\alpha_{2\to3}(\alpha_{t,2}; Q_2(.)))}{d\psi} - \frac{dw_3^{pl}(\alpha_{2\to3}(\alpha_{t,2}^{lull}; Q_2(.)))}{d\psi}\right)\Bigg] dt \tag{IA.20}$$

The welfare function $W(.)$ satisfies $W'(Q^*) = 0$, implying (20). Using the chain rule,

$$\frac{dw_3^{pl}(\alpha_{2\to3}(\alpha_{t,2}; Q_2(.)))}{d\psi}\Big|_{\psi=0} = \underbrace{\frac{dw_3^{pl}(\alpha)}{d\alpha}\Big|_{\alpha=\alpha_{2\to3}(\alpha_{t,2};Q^*)}}_{\chi^1} \underbrace{\frac{d\alpha_{2\to3}(\alpha)}{d\alpha}\Big|_{\alpha=\alpha_{t,2},\psi=0}}_{\chi^2} \underbrace{\frac{d\alpha_{t,2}}{d\psi}\Big|_{\psi=0}}_{\chi^3} + \underbrace{\frac{dw_3^{pl}(\alpha)}{d\alpha}\Big|_{\alpha=\alpha_{2\to3}(\alpha_{t,2};Q^*)}}_{\chi^4} \underbrace{\frac{d\alpha_{2\to3}(\alpha)}{d\psi}\Big|_{\alpha=\alpha_{t,2},\psi=0}}_{\chi^5}. \tag{IA.21}$$

Similarly,

$$\frac{dw_3^{pl}(\alpha_{2\to3}(\alpha_{t,2}^{lull}; Q_2(.)))}{d\psi}\Big|_{\psi=0} = \underbrace{\frac{dw_3^{pl}(\alpha)}{d\alpha}\Big|_{\alpha=\alpha_{2\to3}(\alpha_{t,2}^{lull};Q^*)}}_{\chi^{1,lull}} \underbrace{\frac{d\alpha_{2\to3}(\alpha)}{d\alpha}\Big|_{\alpha=\alpha_{t,2}^{lull},\psi=0}}_{\chi^{2,lull}} \underbrace{\frac{d\alpha_{t,2}^{lull}}{d\psi}\Big|_{\psi=0}}_{\chi^{3,lull}} + \underbrace{\frac{dw_3^{pl}(\alpha)}{d\alpha}\Big|_{\alpha=\alpha_{2\to3}(\alpha_{t,2}^{lull};Q^*)}}_{\chi^{4,lull}} \underbrace{\frac{d\alpha_{2\to3}(\alpha)}{d\psi}\Big|_{\alpha=\alpha_{t,2}^{lull},\psi=0}}_{\chi^{5,lull}}. \tag{IA.22}$$

There are five derivatives in (IA.21) (labeled χ^1, χ^2, ..., χ^5) and five related derivatives in (IA.22) (labeled $\chi^{1,lull}$, $\chi^{2,lull}$, ..., $\chi^{5,lull}$). The proof proceeds by showing the following claims. Claim (i): $\chi^3 < \chi^{3,lull}$ (or, $\frac{d\alpha_{t,2}}{d\psi}|_{\psi=0} < \frac{d\alpha^{lull}_{t,2}}{d\psi}|_{\psi=0}$ for $t \geq \tau_2$). Claim (ii): $\chi_1 > 0$ and $\chi_2 > 0$. Claim (iii): $\chi^j = \chi^{j,lull}$ for $j \in \{1,2,4,5\}$.

The persistent fall in optimists' wealth share due to the announcement effect (Propositions 1 and 2) does not become vanishingly small in the neighborhood of conventional policy, in the sense made precise by claim (i). To verify claim (i), note from (15):

$$\frac{d\alpha_{1\to 2}(\alpha)}{d\psi} = \alpha\left(\omega_1^o(\alpha)-1\right)\frac{Q^*}{Q_2(\alpha_{1\to 2}(\alpha))^2}\frac{dQ_2(\alpha_{1\to 2}(\alpha);\psi)}{d\psi}\frac{1}{1-f_{\alpha_2}(\alpha_{1\to 2}(\alpha),\alpha)}. \tag{IA.23}$$

By assumption (see the explanation of Q-targeting rules indexed by ψ in the main text, just before the Proposition), $\frac{dQ_2(\alpha_2;\psi)}{d\psi} < 0$. By the usual argument, $f_{\alpha_2}(\alpha_{1\to 2}(\alpha),\alpha) < 1$ (see Proposition 1). Hence (IA.23), evaluated at $\psi = 0$, is strictly negative. Thus, $\frac{d\alpha_{\tau_2,2}}{d\psi}|_{\psi=0} < 0$ (that is, a marginal increase in ψ, starting from $\psi = 0$, strictly reduces optimists' wealth share when $s = 2$ arrives). In contrast, with a lull, there is no change in optimists' wealth share when $s = 2$ arrives: $\alpha^{lull}_{1\to 2}(\alpha) = \alpha$, and hence $\frac{d\alpha^{lull}_{\tau_2,2}}{d\psi} = 0$. Next, note that the law of motion $\dot{\alpha}_2(\alpha;Q_2(.))$ is the same whether Q_2 is announced in a lull or not; moreover, $\alpha_{t,2}|_{\psi=0} = \alpha^{lull}_{t,2}|_{\psi=0}$. Hence, $\frac{d^2\alpha_{t,2}}{dtd\psi}|_{\psi=0} = \frac{d^2\alpha^{lull}_{t,2}}{dtd\psi}|_{\psi=0}$. Combining this result with $\frac{d\alpha_{\tau_2,2}}{d\psi}|_{\psi=0} < 0$ and $\frac{d\alpha^{lull}_{\tau_2,2}}{d\psi} = 0$, we have claim (i).

For claim (ii), we will show $\frac{dw_3^{pl}(\alpha)}{d\alpha} > 0$, for $\alpha \in (0,1)$ and also $\frac{d\alpha_{2\to 3}(\alpha)}{d\alpha}|_{\alpha=\alpha_{t,2},\psi=0} > 0$. We can write the gap value in $s = 3$ as

$$w_3^{pl}(\alpha) = \int_0^\infty e^{-(\rho+\lambda^{pl}_{3\to 4})t}\left(W(Q(\alpha_{t,3})) + \lambda^{pl}_{3\to 4}w_4^{pl}(\alpha_{3\to 4}(\alpha_{3,t})\right)dt. \tag{IA.24}$$

where $\alpha_{t,3}$ satisfies $\dot{\alpha}_{t,3} = \alpha_{t,3}(1-\alpha_{t,3})(\lambda^o_{3\to 4} - \lambda^p_{3\to 4})$ and the initial value $\alpha_{t=0,3} = \alpha$. Noting that $w_4^{pl}(\alpha) = 0$ for $\alpha \in [0,1]$, and taking the derivative of (IA.24) with respect to α,

$$\frac{dw_3^{pl}(\alpha)}{d\alpha} = \int_0^\infty e^{-(\rho+\lambda^{pl}_{3\to 4})t}\frac{dW(Q(\alpha_{t,3}))}{d\alpha}dt > 0, \tag{IA.25}$$

where the inequality obtains because $\frac{dW(Q(\alpha_{t,3}))}{d\alpha} > 0$ for all $\alpha_{t,3} \in [0,1]$. The second part of Claim (ii), $\frac{d\alpha_{2\to 3}(\alpha)}{d\alpha}|_{\alpha=\alpha_{t,2},\psi=0} > 0$, follows from Lemma 2 in this Internet Appendix.

For claim (iii), observe that the mappings from optimists' wealth share to the $s = 3$ gap value and the change in optimist wealth share when $s = 3$ arrives ($w_3^{pl}(\alpha)$ and $\alpha_{2\to 3}(\alpha)$) are the same with and without a lull. Hence, evaluated at $\psi = 0$ (or equivalently $Q_2(\alpha) = Q^*$), $\chi^j = \chi^{j,lull}$ for $j \in \{1,2,4,5\}$.

Risk premium. Using a discrete-time approximation with periods of length Δ, the risk premium $rp^i_{t,s}$ during $s \in \{1, 2\}$ under the beliefs of agent type i is

$$rp^i_{t,s}\Delta = \exp(-\lambda^i_{s\to 3}\Delta) r_{t,s}\Delta + \lambda^i_{s\to 3}\Delta \exp(-\lambda^i_{s\to 3}\Delta)\left(\frac{Q_3(\alpha_{s\to 3}(\alpha_{t,s}))}{Q_s(\alpha_{t,s})} - 1\right) - r^f_{t,s}\Delta + \mathcal{O}(\Delta), \tag{IA.26}$$

where $\mathcal{O}(\Delta)$ represents terms that satisfy $\lim_{\Delta\to 0} \mathcal{O}(\Delta)/\Delta = 0$. The terms on the right hand side of (IA.26)—excluding the risk-free rate—sum to the expected return on the risky asset over period Δ. During this period, state transition arrives at Poisson rate $\lambda^i_{s\to 3}$. The probability of no state transition over the period of length Δ is $\exp(-\lambda^i_{s\to 3}\Delta)$. The probability of a (single) state transition is $\lambda^i_{s\to 3}\Delta \exp(-\lambda^i_{s\to 3}\Delta)$; and $\mathcal{O}(\Delta)$ term captures that an event with Poisson arrival rate $\lambda < \infty$ occurs two or more times over a period of length Δ with a probability that vanishes to zero as $\Delta \to 0$. The terms $(rp^i_{t,s}\Delta,\ r_{t,s}\Delta,\ r^f_{t,s}\Delta)$ are the risk premium, return absent state transition, and risk-free rate, respectively. $(\frac{Q_3(\alpha_{s\to 3}(\alpha_{t,s}))}{Q_s(\alpha_{t,s})} - 1)$ is the return upon state transition. Dividing both sides by Δ and taking the limit as $\Delta \to 0$ implies

$$rp^i_{t,s} = r_{t,s} + \lambda^i_{s\to 3}\left(\frac{Q_3(\alpha_{s\to 3}(\alpha_{t,s}))}{Q_s(\alpha_{t,s})} - 1\right) - r^f_{t,s}. \tag{IA.27}$$

Proof of Proposition 4. We construct the $Q_2(\alpha)$ targeting rule consistent with Eq. (23):

$$Q_2(\alpha;\zeta) = \begin{cases} \frac{1}{\zeta} Q_3\left(\alpha \frac{\lambda^o_{2\to 3}}{\lambda_{2\to 3}(\alpha)}\right), & \text{if } Q_3(\alpha\frac{\lambda^o_{2\to 3}}{\lambda_{2\to 3}(\alpha)}) \geq \zeta Q^{bind}_2(\alpha) \\ \frac{1}{\zeta} Q_3\left(\alpha(1 - (\bar{\omega} - 1)(\zeta^{-1} - 1))\right), & \text{otherwise.} \end{cases} \tag{IA.28}$$

To see that this targeting rule implies a gross return of ζ upon recession ($s = 3$), first suppose $Q_3(\alpha\frac{\lambda^o_{2\to 3}}{\lambda_{2\to 3}(\alpha)}) \geq \zeta Q^{bind}_2(\alpha)$ and hence $Q_2(\alpha;\zeta)$ is given by the top line of (IA.28). In this case, the leverage constraint is not binding and hence $\alpha_{2\to 3}(\alpha)$ is given by the top line of Eq. (IA.10), implying $\frac{Q_3(\alpha_{2\to 3}(\alpha))}{Q_2(\alpha)} = \zeta$. Next, suppose that instead $Q_2(\alpha;\zeta)$ is given by the bottom line of (IA.28). Substituting into the bottom line of (IA.10), we obtain that $\alpha_{2\to 3}(\alpha) = \alpha(1 - (\bar{\omega} - 1)(\zeta^{-1} - 1))$ is a fixed point of (IA.10) and hence $\frac{Q_3(\alpha_{2\to 3}(\alpha))}{Q_2(\alpha)} = \zeta$.

Next, substitute (6), for $i = p$, and (23) into (IA.27). Note that (6) holds with equality for pessimists, for whom the leverage constraint never binds. We thereby have:

$$rp^i_2(\alpha) = \lambda^p_{2\to 3}\frac{1-\alpha}{1-\alpha_{2\to 3}(\alpha)}\left(\zeta^{-1} - 1\right) + \lambda^i_{2\to 3}\left(\zeta - 1\right) \tag{IA.29}$$

Because $\zeta < 1$ and $\lambda^p_{2\to 3} > 0$, this expression $rp^i_2(\alpha)$ is decreasing in α if and only if $\frac{1-\alpha}{1-\alpha_{2\to 3}(\alpha)}$ is decreasing in α.

If $Q_3(\alpha \frac{\lambda^o_{s\to 3}}{\lambda_{2\to 3}(\alpha)}) \geq \zeta Q_2^{bind}(\alpha)$, then

$$\frac{1-\alpha}{1-\alpha_{2\to 3}(\alpha)} = \frac{1-\alpha}{1-\frac{\lambda^o_{2\to 3}}{\lambda^o_{2\to 3}+\lambda^p_{2\to 3}(\frac{1}{\alpha}-1)}} \tag{IA.30}$$

Taking the derivative of (IA.30),

$$\frac{d\left[\frac{1-\alpha}{1-\alpha_{2\to 3}(\alpha)}\right]}{d\alpha} = \frac{\lambda^o_{2\to 3}}{\lambda^p_{2\to 3}} - 1 < 0, \tag{IA.31}$$

where the inequality obtains because $\lambda^o_{2\to 3} < \lambda^p_{2\to 3}$. If $Q_3(\alpha \frac{\lambda^o_{2\to 3}}{\lambda_{2\to 3}(\alpha)}) < \zeta Q_2^{bind}(\alpha)$, then

$$\frac{1-\alpha}{1-\alpha_{2\to 3}(\alpha)} = \frac{1-\alpha}{1-\alpha(1-(\bar{\omega}-1)(\zeta^{-1}-1))}. \tag{IA.32}$$

The expression (IA.30) is decreasing in α because $(1-(\bar{\omega}-1)(\zeta^{-1}-1)) < 1$ (i.e., $\zeta < 1$, with the asset price and optimist wealth share falling upon recession).

Proof of Proposition 5 and Corollary to Propositions 2-5. Under conventional monetary policy, the central bank maintains full utilization of resources in the $s = 2$ boom (with $Q_2(\alpha) = Q^*$), regardless of the leverage limit $\tilde{\omega}(\alpha)$, so long as the neutral rate in $s = 2$ remains weakly positive. First, we show that the neutral rate is indeed positive during $s = 2$ even if the central bank tightens regulation (i.e., $\tilde{\omega}(\alpha) \in [1, \bar{\omega})$) and hence the central bank can achieve full utilization by adjusting the risk-free rate. Substituting $Q_2(\alpha) = Q^*$ into (11), the neutral rate in $s = 2$ is:

$$r_2^f(\alpha) = \rho + g_2 - \delta(\eta^*) - \lambda^p_{2\to 3}\frac{(1-\alpha)}{1-\alpha_{2\to 3}(\alpha)}\left(\frac{Q^*}{Q_3(\alpha_{2\to 3}(\alpha))} - 1\right) > 0 \tag{IA.33}$$

The assumption $\rho + g_s - \delta(\eta^*) - \lambda^p_{s\to 3}(\frac{Q^*}{Q_3(0)} - 1) > 0$ (see "Neutral rates" earlier in the Internet Appendix) guarantees the inequality in (IA.33), for any $\alpha_{2\to 3}(\alpha) \in [0, \alpha]$.

Next, consider the following version of (IA.12):

$$\tilde{f}^{s\to 3}(\alpha_3, \alpha) = \alpha\left[1 - (\tilde{\omega}(\alpha) - 1)\left(\frac{Q^*}{Q_3(\alpha_3)} - 1\right)\right]. \tag{IA.34}$$

We then have

$$\frac{d\alpha_{2\to 3}(\alpha)}{d\tilde{\omega}(\alpha)} = -\frac{\alpha\left(\frac{Q^*}{Q_3(\alpha_{2\to 3}(\alpha))} - 1\right)}{1 - \tilde{f}^{s\to 3}_{\alpha_3}(\alpha_{2\to 3}(\alpha), \alpha)} < 0, \tag{IA.35}$$

where the inequality obtains because claim (iii) in the proof of Lemma 1 implies that $1-\tilde{f}_{\alpha_3}^{s\to 3} > 0$ for $\tilde{\omega}(\alpha) \leq \bar{\omega}$. Thus, we have shown that announcing the leverage limit $\tilde{\omega}(\alpha)$ along with a Q-targeting rule $Q_2(\alpha) = Q^*$ is feasible (i.e., the neutral rate, which is the implementing risk-free rate, is positive). Hence, such an announcement implies no decline in the risky asset price and no change in optimists' wealth share.

Finally, we construct the leverage limit $\tilde{\omega}(\alpha)$ that achieves the same macroprudential benefit (same $\alpha_{2\to 3}(\alpha)$ mapping) as the leaning monetary policy with Q-targeting rule $Q_2(\alpha)$ and leverage limit $\bar{\omega}$. (This leverage limit $\tilde{\omega}(\alpha)$ is akin to the converse of "Prudential Monetary Policy" in Caballero and Simsek (2020a): We start with a leaning policy and derive the macroprudentially-equivalent leverage constraint.) Under leaning monetary policy with Q-targeting rule $Q_2(\alpha)$,

$$\alpha_{2\to 3}(\alpha; Q_2(\alpha)) = \alpha\left[1 - (\omega_2^o(\alpha) - 1)\left(\frac{Q_2(\alpha)}{Q_3(\alpha_{2\to 3}(\alpha; Q_2(\alpha)))} - 1\right)\right], \tag{IA.36}$$

where $\omega_2^o(\alpha) = \bar{\omega}$ if $Q_2(\alpha) < Q_2^{bind}(\alpha)$ and otherwise $\omega_2^o(\alpha) = \omega_2^{unc}(\alpha)$. If conventional monetary policy is used (i.e., $Q_2(\alpha) = Q^*$, $r_2^f(\alpha) = r_2^{f,*}$) but the leverage constraint is reduced to $\tilde{\omega}(\alpha)$, then

$$\alpha_{2\to 3}(\alpha; \tilde{\omega}(\alpha)) = \alpha\left[1 - (\tilde{\omega}(\alpha) - 1)\left(\frac{Q^*}{Q_3(\alpha_{2\to 3}(\alpha; \tilde{\omega}(\alpha)))} - 1\right)\right]. \tag{IA.37}$$

Thus, $\alpha_{2\to 3}(\alpha; Q_2(\alpha)) = \alpha_{2\to 3}(\alpha; \tilde{\omega}(\alpha))$ can be achieved if

$$\tilde{\omega}(\alpha) = 1 + (\omega_2^o(\alpha) - 1)\frac{Q_2(\alpha) - Q_3(\alpha_{2\to 3}(\alpha; Q_2(\alpha)))}{Q^* - Q_3(\alpha_{2\to 3}(\alpha; Q_2(\alpha)))}. \tag{IA.38}$$

Proof of Proposition 6. Note that $\bar{\boldsymbol{\alpha}}$ appears in (29), but not in the remainder of the planner's problem, and hence the optimal Q-targeting rule with a lull, $Q_2^{lull}(\alpha)$, does not depend on $\bar{\boldsymbol{\alpha}}$. Consider the Q-targeting rule

$$Q_2^{opt}(\alpha) = \begin{cases} Q^*, & \text{if } \alpha \leq \bar{\boldsymbol{\alpha}} \\ Q_2^{lull}(\alpha), & \text{if } \alpha > \bar{\boldsymbol{\alpha}}. \end{cases} \tag{IA.39}$$

To prove the proposition, we show that, taking the sowing constraint (29) into account, $Q_2^{opt}(\alpha)$ achieves the same gap value as if the central bank announced $Q_2^{lull}(\alpha)$ during a lull. From the top line of (IA.39) and Proposition 1, at the time of the policy announcement, $Q_{\tau_2,2} = Q^*$ and there is no change in the optimist wealth share (i.e., $\alpha_{1\to 2}(\bar{\boldsymbol{\alpha}}) = \bar{\boldsymbol{\alpha}}$). Thus, at τ_2, the optimist wealth share when $Q_2^{opt}(.)$ is announced unexpectedly without a lull is the same as the wealth share when $Q_2^{lull}(.)$ if announced during a lull. Beyond τ_2, the growth rate of the optimist wealth share, absent transition ($\dot{\alpha}_s(\alpha)$), the change in optimist wealth share upon recession ($\alpha_{2\to 3}(\alpha)$), and the welfare flow from utilization ($W(Q_2(\alpha))$) are identical under $Q_2^{opt}(.)$ and $Q_2^{lull}(.)$. Therefore, for a set of histories with measure 1, the time path for the welfare flow from utilization $\{W(Q_t)\}_{t\in[\tau_2,\infty)}$ coincide with the time path for welfare flow under the central bank's problem with the speculative lull. That is, the paths differ only if the recession arrives *exactly* at $t = \tau_2$—an event with measure zero for the continuous-time Poisson process. (The event has measure zero because $\dot{\alpha}_2(\bar{\boldsymbol{\alpha}}) > 0$ if $Q_2(\bar{\boldsymbol{\alpha}}) = Q^*$ as in (IA.39), for $\bar{\boldsymbol{\alpha}} < 1$. That is, there is no steady state with $\alpha_{t,2} = \bar{\boldsymbol{\alpha}}$.) Using a discrete-time approximation to (11) with periods of length Δ, the implementing risk-free return between τ and $\tau + \Delta$ is

$$r_{\tau_2}^{f,\Delta} = (\rho+g_2-\delta(\eta^*))\Delta+\ln(Q_2^{lull}(\bar{\boldsymbol{\alpha}})/Q^*)-\lambda_{2\to 3}^{p}\Delta\exp(-\lambda_{2\to 3}^{p}\Delta)\frac{(1-\bar{\boldsymbol{\alpha}})}{1-\alpha_{2\to 3}(\bar{\boldsymbol{\alpha}})}\left(\frac{Q_2(\bar{\boldsymbol{\alpha}})}{Q_3(\alpha_{2\to 3}(\bar{\boldsymbol{\alpha}}))}-1\right). \tag{IA.40}$$

As $\Delta \to 0$, $r_{\tau_2}^{f,\Delta} = \ln(Q_2^{lull}(\bar{\boldsymbol{\alpha}})/Q^*)$. For $t > \tau_2$, the interest rate is given by (11).

Proof of Proposition 7. Using the discrete-time approximation to Eq. (31),

$$\bar{r}_h^{opt,i} = \frac{1}{h}\mathbb{E}_{\tau_2}^{i}\left(\sum_{j=0}^{\frac{h}{\Delta}-1} r_{\tau_2+j\Delta,s}^{f,\Delta}\right) \tag{IA.41}$$

where $r_{t,2}^{f,\Delta}$ is defined in the proof of Proposition 6. From Eq. (IA.40), note $r_{\tau_2}^{f,\Delta} = \ln(Q_2^{lull}(\bar{\boldsymbol{\alpha}})/Q^*))$. Denote the risk-free return between t and $t+\Delta$ when $Q_2^{lull}(.)$ is announced during a lull by $r_t^{f,lull,\Delta}$. The proof of Proposition 6 shows that, for $t > \tau_2$, under optimal policy announced without a lull and under Q_2^{lull} announced with a lull, the economies coincide over a set of histories with measure 1. Hence, $\mathbb{E}_{\tau_2}^{i}[r_{t,s}^{f,\Delta}] = \mathbb{E}_{\tau_2}^{i}[r_{t,s}^{f,lull,\Delta}]$ for $t > \tau_2$. Substituting into (IA.41), one obtains (32).

Proof of Proposition 8. We assume that if Q-targeting rule $Q_1(\alpha, \bar{\boldsymbol{\alpha}})$ is more aggressive than $\hat{Q}_1(\alpha, \bar{\boldsymbol{\alpha}})$, then $Q_1(0,0) \leq \hat{Q}_1(0,0)$ (that is, the risky asset price in the all-pessimist economy under the more aggressive leaning policy is weakly lower than under the less aggressive policy). This assumption covers the boundary case of an all-pessimist economy, whereas the definition of more-aggressive policy in the main text addressed $\alpha, \boldsymbol{\alpha} \in (0,1)\times(0,1)$. We also assume (consistent with "Q-targeting rule in $s = 2$" at the beginning of the Internet Appendix) that $Q_1(\alpha, \bar{\boldsymbol{\alpha}})$ is differentiable in α and $\bar{\boldsymbol{\alpha}}$, so that $Q_0(\alpha)$ (the solution to (34)) is Lipschitz continuous.

First part: Let $Q_s^{pes} = Q_s(0)$ denote the state-s risky asset price in the all-pessimist economy

$(\alpha = 0)$. Q_0^{pes} is obtained by setting $\alpha = 0$ in (34):

$$\rho + g_0 - \delta\left(\frac{Q_0^{pes}}{Q^*}\eta^*\right) + \lambda_{0\to1}^p\left(1 - \frac{Q_0^{pes}}{Q_1^{pes}}\right) = 0. \tag{IA.42}$$

This expression (IA.42) is decreasing in Q_0^{pes} and increasing in Q_1^{pes}. Therefore, Q_0^{pes} is increasing in Q_1^{pes}. With leaning, $Q_1^{pes} \leq Q^*$ (the boom-period asset price is weakly below its value under Wicksellian policy, because a weakly lower asset price for given optimist wealth share is the essential aspect and part of the definition of leaning monetary policy). Next, re-arranging (34), we have

$$\frac{d\ln Q_0(\alpha)}{d\alpha} = \frac{1}{\alpha(1-\alpha)(\lambda_{0\to1}^o - \lambda_{0\to1}^p)}\left(\rho + g_0 - \delta\left(\frac{Q_0(\alpha)}{Q^*}\eta^*\right) + \bar{\lambda}_{0\to1}(\alpha)\left(1 - \frac{Q_0(\alpha)}{Q_1(\alpha_{0\to1}(\alpha);\alpha)}\right)\right), \tag{IA.43}$$

Note that $\alpha_{0\to1}(.)$ does not depend on the Q-targeting rule $Q_1(.,.)$ because there is no leverage limit in the recession. There exists a unique solution $Q_0(.)$ to (IA.43), subject to the boundary condition for the all-pessimist economy, under both the less aggressive and more aggressive $s = 1$ Q-targeting rules. Moreover, because (IA.43) is decreasing in $Q_0(\alpha)$ and increasing in $Q_1(\alpha_{0\to1}(\alpha, \alpha))$, the $s = 0$ risky asset price under the more aggressive rule must be lower than the risky asset price under the less aggressive rule.

The second part of the proposition obtains because, in the absence of a leverage limit during the initial recession, the path for the optimist wealth share $\alpha_{t,s=0}$ is determined by the initial condition $\alpha_{t=0,s=0}$ and $\dot{\alpha}_0(\alpha) = \alpha(1-\alpha)(\lambda_{0\to1}^p - \lambda_{0\to1}^o) < 0$, which do not depend on the anticipated monetary policy in the subsequent boom.

Proof of Proposition 9. Under the planner's beliefs, the gap value $w_0^{pl}(\alpha)$ satisfies

$$\rho w_0^{pl}(\alpha) = W(Q_0(\alpha)) + \lambda_{0\to1}^{pl}\left(w_1^{pl}(\alpha_{0\to1}(\alpha)) - w_0^{pl}(\alpha)\right) + \frac{\partial w_0^{pl}(\alpha)}{\partial\alpha}\alpha(1-\alpha)(\lambda_{0\to1}^p - \lambda_{0\to1}^o). \tag{IA.44}$$

Let $Q_1^{lull}(\alpha)$ be the argument of the problem (28), when the announcement-effect constraint (29) is not imposed. That is, $Q_1^{lull}(.)$ maximizes the expected future discounted value from $s = 1$ onward under a systematic policy—that is a policy in place prior to $s = 1$ and not unexpectedly announced during the boom. For any $\alpha \in [0, 1]$ and $\underline{\boldsymbol{\alpha}} \neq \alpha$, changing $Q_1(\alpha, \underline{\boldsymbol{\alpha}})$ has no effect on the path of utilization and the welfare flow $W_{t,s=0}$ during the initial recession. Moreover, a policy that would set $Q_1(\underline{\boldsymbol{\alpha}}; \underline{\boldsymbol{\alpha}}) = Q^*$, for all $\underline{\boldsymbol{\alpha}} \in [0, 1]$, would imply a uniformly (weakly) higher $Q_0(\alpha)$ than any other Q-targeting rule (because, from Eq. (34), it follows that the slope of $\ln Q_0(\alpha)$ is increasing in $Q_1(\alpha; \alpha)$). Finally, observe that $w_1^{pl}(\alpha)$ is unaffected by $Q_1(\alpha, \alpha)$ (i.e., changing

$Q_1(\alpha, \alpha)$ changes the evolution of the optimist wealth share over a set of histories of measure 0, by a similar argument as the proof of Proposition 6.)

Proof of Corollary to Proposition 9. The proof of part (i) is analogous to the proof of Proposition 7. For part (ii), define the expected policy rate curve as $\bar{r}^i_{h,s=0} = \frac{1}{h}\mathbb{E}^i_{t=0}\int_0^h [r^f_{t,s}dt]$. Using the discrete-time approximation, we have

$$\bar{r}^{opt,i}_{h,s=0} = \bar{r}^{lull,i}_{h,s=0} + \frac{1}{h}\sum_{j=0}^{\frac{h}{\Delta}-1} \underbrace{\exp(-\lambda^i_{0\to 1} j\Delta)\Delta\lambda^i_{0\to 1}\exp(-\Delta\lambda^i_{0\to 1})}_{\substack{\text{Probability of } s=1 \text{ arriving} \\ \text{between time } j\Delta \text{ and } (j+1)\Delta}}(r^{f,\Delta}_{j\Delta,s=1} - r^{f,lull,\Delta}_{j\Delta,s=1}) \tag{IA.45}$$

where $r^{f,\Delta}_{t,s=1}$ is the risk-free return between t and $t+\Delta$ under the optimal policy (as described in Proposition 9) if the boom arrives at time t. Similarly, $r^{f,lull,\Delta}_{t,s=1}$ is the risk-free return under a policy $Q_1(\alpha, \bar{\boldsymbol{\alpha}}) = Q^{lull}_1(\alpha)$, $\forall \alpha \in [0,1]$, if the boom arrives at time t. Eq. (IA.45) reflects that, for any history, the risk-free rate and evolution of the optimist wealth share are identical except at the time that boom arrives. As $\Delta \to 0$, $r^{f,\Delta}_{j\Delta,s=1} - r^{f,lull,\Delta}_{j\Delta,s=1} = \ln Q^* - \ln(Q^{lull}_1(\frac{\boldsymbol{\lambda}^o_{0\to 1}}{\boldsymbol{\lambda}^o_{0\to 1}(\boldsymbol{\alpha})}\underline{\boldsymbol{\alpha}})) < 0$.

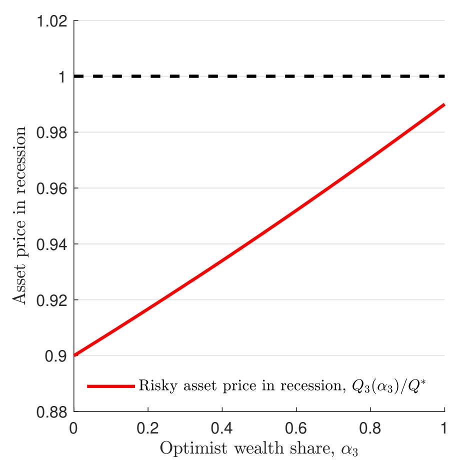
B. Parameter values and additional numerical results

The parameter values for the numerical exercises are shown in Table IA.I. See also Caballero and Simsek (2020a). Figure IA.1 shows the risky asset price in the recession state, $s = 3$. Figure IA.2 shows the forward rate curve in $s = 2$, absent state transition (left panel of Figure IA.2, from Eq. (33), and as also shown in the bottom right panel of Figure 7 in the main text) and taking into account the possibility of state transition. The forward expected policy rate (at time τ) over m years, beginning h years ahead, taking into account the possibility of state transitions, is $\mathbb{E}^i_\tau[\int_{\tau+h}^{\tau+h+m} r^f_{t,s}dt]$. Accounting for state transitions reduces the differences among the forward rate curves, but implies qualitatively similar results as conditioning on the absence of state transition.

Table IA.I: **Parameter values**

Parameter	Description	Value
$\underline{\eta}$	Reducing utilization below $\underline{\eta}$ does not reduce depreciation	0.97
$\underline{\delta}$	Depreciation rate when utililization is low (i.e., $\eta < \underline{\eta}$)	0.04
ϵ	Depreciation elasticity parameter	20
$\bar{\delta}$	Depreciation parameter (chosen to normalize $\eta^* = 1$)	0.087
ρ	Discount rate	0.04
A	Productivity level (normalization)	1
$g_1 = g_2 = g_4$	Growth rate of capital during boom and recovery	0.101
$g_0 = g_2$	Growth rate of capital during recessions	-0.049
$(\lambda^o_{s\to 3},\quad \lambda^p_{s\to 3})$	Poisson transition rate from boom ($s \in \{1,2\}$) to recession	(0.09, 0.9)
$(\lambda^o_{3\to 4},\quad \lambda^p_{3\to 4})$	Poisson transition rate from recession $s = 3$ to recovery $s = 4$	(4.97, 0.49)
$(\lambda^o_{0\to 1},\quad \lambda^p_{0\to 1})$	Transition rate: Initial recession to boom (Section 6 of main text)	(4.97, 0.49)

Figure IA.1: **Risky asset price during recession**



The risky asset price in the recession is below the efficient price Q^* and is increasing in the optimist wealth share.

Figure IA.2: **Forward rate curves, with and without accounting for state transitions**