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Monetary Policy over the Lifecycle

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Abstract: A tighter monetary policy is associated with higher nominal and real interest rates on deposits and loans, weaker performance of equities, lower aggregate investment and consumption, and slower growth in employment and wages. We propose a quantitative lifecycle model that reproduces these responses and find that an asset substitution, or Tobin effect, plays a central role in understanding the responses of stock prices and aggregate investment. We use the model to analyze how a household's exposure to monetary policy varies with its age and find that monetary policy has important distributional effects. The sign, magnitude, and persistence of household consumption responses depend on age, and the pattern of these responses implies that a tighter monetary policy increases inequality in net worth and consumption.

JEL classification: E52, E62, G51, D15

Key words: monetary policy, lifecycle, portfolio choice, nominal government debt, Tobin effect

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1 Introduction

Monetary policy is generally thought to operate through financial markets. A higher policy interest rate increases nominal and real interest rates on liquid assets like deposits and government debt and increases borrowing costs. These changes in turn have a negative effect on the performance of the stock and real estate markets and depress aggregate consumption, investment, output and employment. Formal empirical evidence supporting this perspective is provided by the large literature on monetary policy shocks (see Christiano et al., 2005).

Given that the size and composition of household disposable income and wealth differs across households, it is likely that they have heterogeneous exposures to monetary policy shocks. Auclert (2019) raises this possibility and decomposes individual exposures to monetary policy into four components: income, unexpected inflation, unhedged real interest rate, and intertemporal substitution.

One reason household exposures to monetary policy differ is uninsured earnings risk. Households face the risk of a long sequence of low labor income realizations, save to self-insure against this risk and this action influences their exposure to monetary policy. Considerable progress has been made in developing quantitative structural monetary models of uninsured earnings risk. Some recent examples of the heterogeneous agent New Keynesian (HANK) research initiative includes Kaplan et al. (2018), Bayer et al. (2019), Gornemann et al. (2021) and Luetticke (2021). A merit of these models is that they reproduce key cross-sectional observations about earnings and wealth inequality.

Uninsured earnings risk is one of the most important sources of income and wealth inequality. Still, other dimensions of household heterogeneity, such as age, gender, family structure, race and education also affect a household's exposure to monetary policy shocks. Indeed, empirical evidence from local projections presented here suggests that monetary policy induces redistribution across age groups.

The main objective of our paper is to show that modeling routine savings and asset portfolio decisions over the lifecycle produces new insights into the transmission channels and distributional effects of monetary policy. A monetary policy tightening in our model crowds in nominally denominated government debt, but crowds out investment and depresses share prices due to an asset substitution effect analyzed in Tobin (1969) and more recently in Hu et al. (2021). Household income and asset portfolios vary by age and it follows that monetary policy also produces redistribution across age groups. According to our results a surprise tightening in monetary policy reduces the consumption of a 31-year old working-age household but increases the consumption of a 71-year old retired household. Since the latter household has higher net worth, wealth and consumption inequality both rise. Moreover, because monetary policy affects the investment opportunities of these two households in different ways, the increase in consumption inequality is persistent.

To make these points we show that the size and composition of household income and wealth exhibit large variation over the lifecycle in Japanese data. Then we document how household exposures to monetary policy vary over the lifecycle by estimating impulse responses to a tightening in monetary policy by age group. Both the signs and magnitudes of the responses of disposable income and consumption vary with age of the household head. Moreover, the age profile of these responses is broadly consistent with what one would expect from the age profiles of asset holdings and income.

Other empirical results presented here indicate that monetary policy also has important effects on aggregate economic activity. We are interested in understanding the economic mechanisms that induce a household of a particular age to change its consumption and asset allocations plans in a particular way and we are also interested in understanding how their collective decisions get reflected in aggregate economic activity. We propose a model to investigate these questions.

Our model has the following structure. Households can borrow or lend liquid and illiquid assets but illiquid assets are costly to adjust. Young working-age households have low initial resources and face a hump-shaped labor efficiency profile and mandatory retirement at the age of 68. Retirees receive public pensions. Individuals face idiosyncratic mortality risk that is increasing in age. Government debt is a nominal liquid liability that is held constant in all periods and the monetary authority sets the nominal interest rate on government debt and other liquid securities. Finally, the model includes nominal price rigidities as in Rotemberg (1996).

We find that the model's macro responses are consistent with the empirical responses in Japanese data. A tightening in monetary policy produces declines in aggregate consumption, investment and stock prices in both our model and the data and the magnitudes of the responses are about the same.

We also analyze how and why monetary policy shocks induce redistribution across age groups. A monetary policy tightening has a negative impact on the consumption of retirees aged 85+. Households in this age group experience declines in most or even all four of the exposures described by Auclert (2019). They have short planning horizons so their consumption response is large. Consumption increases, in contrast, for households aged between 57 and 81. Their response is driven primarily by increases in the two-asset cash-flow components. A decrease in inflation increases the value of their large holdings of nominally denominated liquid assets and the prospect of higher future real interests rates also benefits them because they are savers. Consumption of younger working age households falls. The intertemporal substitution component is important for this group.

This pattern of exposures to monetary policy implies that a monetary policy tightening increases both wealth and consumption inequality. High net worth households close to retirement see their net worth increase. Young working-age households and older retirees are less affluent and their net worth falls. The increase in consumption inequality is persistent because monetary policy has heterogeneous effects on household investment opportunities and this influences how households choose to adjust consumption over the remainder of their life. For example, a household of age 71 increases the share of liquid assets in its portfolio and this allows it to enjoy more consumption at older ages. A household that experiences the same shock at age 31, in contrast, faces a lower return on its preferred investment strategy which involves borrowing liquid securities to purchase illiquid securities. It copes with the shock by adjusting its portfolio but still experiences persistently low consumption. The persistent increases in wealth and consumption inequality produced by our model are consistent with empirical findings in Coibion et al. (2017) for the U.S.

Finally, we link the household level responses back to the aggregate responses. Asset substitution plays a central role in our model. All households find it optimal to reduce their positions in illiquid assets following a tightening in monetary policy and it follows that aggregate investment falls. Our model has nominal price rigidities and imperfect competition and a tightening in monetary policy increases profits. However, this dividend effect is outweighed by changes in discounting and stock prices fall persistently. Thus, our model produces a conditional version of the Campbell and Shiller (1988) observation.

Related literature Our paper has ties to the large but primarily theoretical literature on monetary policy in overlapping generations (OLG) models as in Wallace (1980). Several recent papers, including Hu et al. (2021) and Sterk and Tenreyro (2018), have analyzed monetary policy in tractable OLG models. We propose a quantitative OLG model that allows us to analyze the quantitative significance of asset substitution and redistribution channels of monetary policy at each stage of the lifecycle.

Previous work has focused on particular subsets of the four household exposures described by Auclert (2019). Kaplan et al. (2018) analyze substitution and household balance sheet exposures but abstract from unexpected inflation exposures. Hagedorn (2021) emphasizes that monetary and fiscal policy jointly determine the price level in incomplete markets models with nominal government debt and Doepke and Schneider (2006) analyze unexpected inflation exposures by age of the household. Other research has analyzed how changes in real returns influence household portfolio choices of liquid and illiquid assets. Berger et al. (2017) analyze how households consumption decisions respond to changes in the real return on illiquid (housing) assets in a lifecycle framework. Garriga and Hedlund (2020) analyze the joint exposure of individual real asset cash flows and labor income cash flows in an infinite horizon real economy with illiquid assets and leverage. Finally, Wong (2019) and Garriga et al. (2021) analyze nominal and real-asset cash-flow exposures to monetary policy. A merit of our framework is that all four exposures are active. This allows us to analyze their individual and joint contributions to consumption at each point of the lifecycle.

Our paper is also related to recent work by Bielecki et al. (2022) who analyze monetary policy in a NK OLG framework with housing. Households decide how much to save but do not make portfolio choices. In our model, the portfolio choice decision is endogenous and we find that modeling this decision matters not only for the response of consumption and inequality to monetary policy shocks but also for the sign of the response of aggregate investment. In our model, when the nominal interest rate increases, consumption responses differ by age, but all households choose to invest less in illiquid assets and aggregate investment falls.

Both the empirical and model results presented here suggest that the highest net worth households increase their consumption when the nominal interest rate increases. Households with low net worth have high levels of debt and tend to reduce their consumption. These results are consistent with empirical evidence in Cloyne et al. (2020) for the U.S. and the U.K. and Holm et al. (2020) for Norway. The peak response of nondurable consumption expenditures of outright homeowners is smaller and has a different sign as compared to homeowners with mortgages in Cloyne et al. (2020). Holm et al. (2020) find that households who are net borrowers reduce their consumption in response to a monetary policy tightening but that households who are net creditors increase their consumption.

Our results also suggest that asset substitution may be an alternative mechanism for resolving the puzzling responses of investment and stock prices to monetary policy in HANK frameworks. Broer et al. (2020) and Kaplan et al. (2018) find that countercyclical profits induce countercyclical responses in investment and hours to monetary policy shocks in their

HANK models and propose particular remedies. In our model wages are flexible, profits increase when monetary policy is tightened, and there are no restrictions on how households allocate dividends from profits between liquid and illiquid assets. Still, investment, hours and stock prices all fall when monetary policy is tightened in our model and asset substitution effects play a central role in this result.

The remainder of the paper is organized as follows. Section 2 motivates our analysis of monetary policy over the lifecycle and provides some intuition for the asset substitution channel of monetary policy. Section 3 describes the model and Section 4 explains how we parameterize the model. Section 5 reports the aggregate responses to a monetary policy shock in our model and in Japanese data. In Section 6 we switch gears and analyze how and why monetary policy induces redistribution across age groups. This section also links the micro responses to the aggregate responses and documents the individual roles of nominal price rigidities and the asset substitution channel in our results. Section 7 conducts a robustness analysis and Section 8 contains our concluding remarks.

2 Motivation

2.1 Net worth, liquid and illiquid assets by age

One aim of our paper is to analyze how and why a household's exposure to monetary policy varies with its age. To motivate our analysis we start by showing that there is large age-based variation in the size and composition of household net worth. We focus on Japan because nationally representative survey data is available on household portfolios of physical and financial assets by age group. The Japanese National Survey of Family Income and Expenditure (NSFIE), which is conducted every five years, provides detailed information on household holdings of physical assets including real estate and durable goods as well as a range of financial asset categories by 10-year age group.

Table 1 reports our imputations of household net worth and holdings of liquid and illiquid assets by age group using data from the 2014 NSFIE. Age refers to the age of the household head and asset holdings are relative to peak pretax income over the lifecycle, which occurs in the group aged 50–59. Liquid assets include liquid securities like deposits, and illiquid assets consist of less liquid real and financial assets like residential real estate and equity. Our strategy for classifying assets into these two categories follows Kaplan et al. (2018) with one important exception. We assign all household borrowing to the liquid asset

Table 1: Household net worth, liquid and illiquid asset holdings by age in Japan

Age	Net worth	Liquid assets	Illiquid assets
Under 30	0.65	-0.08	0.73
30–39	1.60	-0.58	2.18
40–49	2.58	-0.31	2.90
50–59	4.52	0.76	3.76
60–69	6.29	1.70	4.60
70+	6.01	1.77	4.25

Note: Liquid assets are net of total borrowing and illiquid assets are gross. Net worth and assets are divided by peak income of the 50–59 year old age group. The main data source is the 2014 NSFIE. More details on the construction of the data can be found in Appendix C.

category.[1] Thus, liquid asset holdings are net of all household borrowing and illiquid asset holdings are gross. This way of organizing the data is consistent with the structure of our model.

Table 1 has four main properties. First, net worth is hump-shaped. It increases steadily during working ages, peaking at over 6 times peak income. Full public pensions become available at age 65 and the 60–69 age group has the highest net worth. Net worth then falls during retirement as older households draw down their savings. Second, households aged 49 or below have negative net holdings of liquid assets but positive holdings of illiquid assets. In other words, they are taking leveraged long positions in illiquid assets. Third, older working-age individuals and retirees have positive holdings of both liquid and illiquid assets. Fourth, net worth falls during retirement but individuals maintain their asset holdings until late in life.[2]

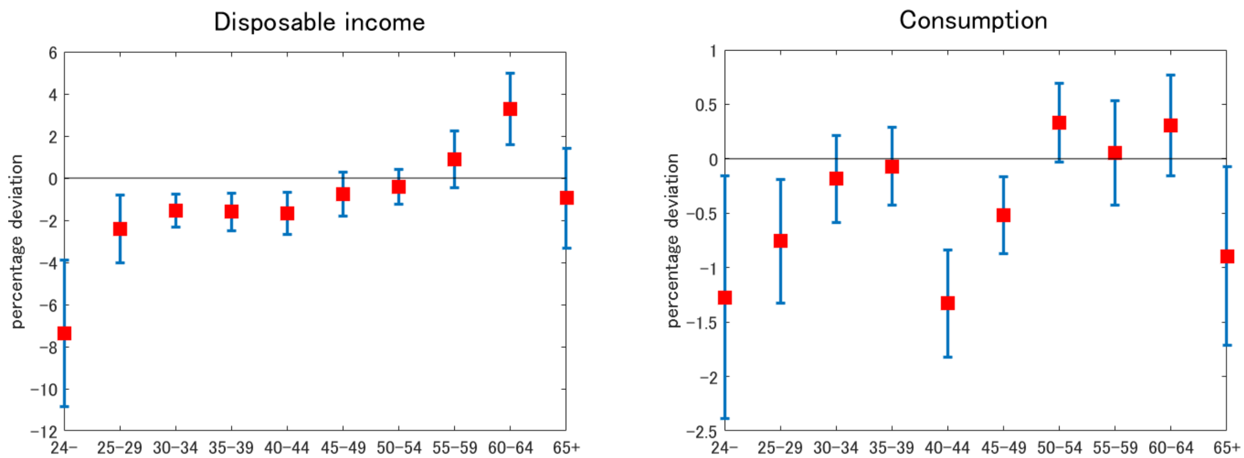
Households' other sources of income also vary by age. Wages initially increase with age, peak at around age 50, and then decline rapidly from age 55 on (see Braun and Joines 2015).[3] After retirement public pensions are an important source of household income.

[1] In Japan most mortgages have adjustable interest rates and the costs of prepaying a mortgage in Japan are small. So it is not unreasonable to treat a mortgage as a liquid liability. Appendix C provides more details on how we decompose total assets into liquid and illiquid categories.

[2] Murata (2019) reports results from the Japanese Study of Aging and Retirement (JSTAR) panel data that suggest that the decline in net worth for older couples is larger than what is reported here. The NSFIE data reported here includes both couples and also multi-generational households.

[3] Many Japanese companies impose mandatory retirement at either 55 or 60 years of age on regular employees. Companies are required to offer a fixed-term contracts after mandatory retirement until the employee qualifies for public pension benefits at the age of 65. However, salary and fringe benefits of re-employment contracts are often less attractive. A recent supreme court ruling in 2019 largely reaffirmed this practice (see Puckett 2019).

Figure 1: Responses of household disposable income and consumption by age group to a tightening in monetary policy.



Notes: Cumulative responses in the impact year, monthly Japanese data (FIES), the monetary policy shocks estimated by Kubota and Shintani (2021) who use high frequency identification. Vertical lines are 90% confidence intervals.

2.2 Household responses to monetary policy shocks by age

Japan also collects monthly data on consumption and disposable income by age group that can be used to analyze how different age groups respond to monetary policy shocks. Figure 1 reports the cumulative responses in the impact year of household disposable income and consumption to a tightening in monetary policy.[4] The sample period is 1992–2019, consumption consists of non-durables plus services net of social expenditures, and the monetary policy shocks are from Kubota and Shintani (2021). They use a high frequency identification (HFI) scheme similar to Gürkaynak et al. (2005) and estimate monetary policy shocks by measuring the response of high frequency interest rate futures data to announced changes in monetary policy. The data source for our household level data is working households from the Family Income and Expenditure Survey (FIES).[5] More details on the construction of the data and empirical methodology can be found in Appendix F.

The results reported in Figure 1 are consistent with our conjecture that household exposures to monetary policy vary by age. Income falls sharply when monetary policy is tightened for households under age 25. The magnitude of the decline in disposable income is smaller for older age groups and is about zero for households in the age 50–54 age group.

[4]The responses of macroeconomic variables to this shock are reported in Section 5.

[5]The FIES is a nationally representative rotating panel that follows sampled households for 6 months. We use working households because the FIES data only reports monthly disposable income for working households.

Disposable income increases for households between 55 and 64 years of age, then turns slightly negative for the 65+ age group. This final result should be interpreted with some caution, because our sample consists of working households and many households in the 65+ age group are omitted from our sample because they are retired.

The overall pattern of consumption responses by age group is similar to disposable income, but the consumption responses are noisier and the confidence intervals in many cases do not rule out the possibility of no consumption response. Consumption falls sharply in the youngest age group. This is not surprising given that this age-group has the lowest net worth as shown in Table 1. Observe, next, that the size of the consumption decline is considerably smaller for households between the ages of 25 and 39 but that there is a dip in the consumption response of the 40–44 age group. The strong decline in consumption of this age group may reflect the fact that borrowing of liquid assets is U-shaped in Table 1. Finally, observe that three out of four of the 50+ age groups exhibit positive consumption responses. These age groups have positive net positions in liquid securities and have the highest net worth in Table 1.

2.3 Asset-substitution channel of monetary policy

Another objective of our paper is to analyze the quantitative significance of the asset-substitution transmission channel of monetary policy. This transmission channel is either absent or weak in most other quantitative monetary models, so we illustrate how it works with a simple example. In a two-period flexible price OLG model with nominal government debt and capital accumulation along the lines of Hu et al. (2021) or Braun and Ikeda (2022), the equilibrium price level and capital stock are determined by the following two equations. The first equation is the asset market clearing condition

$$\frac{d_t^n}{P_t} + k_{t+1} = w_t, \tag{1}$$

where d_t^n is the (exogenous) per capita stock of nominal government debt, P_t is the price level, k_{t+1} is per capita capital, $w_t = f(k_t) - k_t f'(k_t)$ is per capita income of the young, and $f(\cdot)$ is a standard production function with $f' > 0$ and $f'' < 0$. Per capita income in this model is also aggregate asset demand, because households only consume in the second period of their life. The second equation is the Fisher equation

$$f'(k_{t+1}) = R_t \frac{P_t}{P_{t+1}}, \tag{2}$$

where R_t is the nominal interest rate on government debt set by the central bank, which we assume is also exogenous for the purposes of this discussion. Under the assumption that the stock of nominal government debt is fixed, Hu et al. (2021) provide a set of regularity conditions for existence and uniqueness of an equilibrium with the following properties. A temporary increase in R_t decreases k_{t+1}, which can be seen from equation (2) for given P_t/P_{t+1}. This in turn decreases P_t from equation (1) since w_t is pre-determined and d_t^n is fixed. The decrease in P_t mitigates the increase in the real rate, R_tP_t/P_{t+1}, caused by the monetary tightening, but does not fully offset it. Thus, a higher nominal interest rate pushes the price level down and decreases private capital formation. The strength and duration of this asset-substitution effect depends on the specific details of the model and we now propose a lifecycle model where this channel enhances the empirical performance of the model.[6]

3 The Model

The model we propose strikes a compromise in terms of its microeconomic detail. On the one hand, it captures the main sources of income and savings over the lifecycle. On the other hand, it abstracts from idiosyncratic risk factors that produce cross-sectional heterogeneity within an age group such as uninsured earnings and medical expenditure risk. These simplifications allow us to readily discern how monetary policy affects households at different stages of the lifecycle and significantly reduce the computational burden.[7]

We consider an OLG economy with representative cohorts along the lines of Braun et al. (2009) and Braun and Joines (2015) and extend these papers in two ways. First, households can save and/or borrow two assets that differ in terms of their liquidity services. Illiquid assets such as homes and equity offer a high return but are costly to acquire and sell. Liquid assets offer a lower return but are costless to adjust. Depending on where households are

[6] Hagedorn (2021) refers to this model of price level determination as the Demand Theory of the Price Level (DTPL). In Braun and Ikeda (2022) we compare and contrast the DTPL with the Fiscal Theory of the Price Level.

[7] Solving our model once takes about four hours using the Matlab parallel computing toolbox and a 16 core Intel CPU.

in their lifecycle, they choose to borrow liquid assets to purchase illiquid assets or hold positive amounts of both assets. Second, the model includes nominal government debt, nominal price rigidities and a central bank that pursues a nominal interest rate rule.

3.1 Demographic structure

The economy has an OLG structure that evolves in discrete time with a period length of one year. Let j denote the age of the individual as $j = 1, ..., J$. We start keeping track of individuals at age 21 and individuals survive until at most age 120. Thus, the model age of $j = 1$ corresponds to age 21, that of $J = 100$ corresponds to age 120 and $J = 100$ cohorts are active in a given year. We consider an economy with a stationary population distribution and no population growth. Let $N_{j,t}$ be the number of individuals of age j in period t. The population by age is constant, $N_{j,t} = N_j$, for all t, and the aggregate population is $N = \sum_{j=1}^{J} N_j$. The population of each age is defined recursively as $N_{j+1} = \psi_j N_j$, where ψ_j is the conditional probability that an individual of age j survives to the next year.[8]

3.2 Households

Individuals are organized into households. Each household consists of one individual (adult) and children. The number of children varies with the age of the adult and the age of the household is indexed by the age of the adult. Adults face mortality risk and have no bequest motives.[9] At the beginning of each period the adult learns whether she will die at the end of the current period as in Braun et al. (2016). Let $z^i_{j,t} \in \{0, 1\}$ index the survival state for the i^{th} household where a value of zero denotes the death state.[10] Under our assumption, households consume all of their resources in their final year of life and there are no accidental bequests. This, in turn, reduces the costs of death and helps the model to reproduce the empirical observation that households retain wealth until late in life. Death is the only source of idiosyncratic risk faced by households and there are only two types of households in any age cohort: surviving households ($z^i_{j,t} = 1$) and non-surviving households ($z^i_{j,t} = 0$).

[8] Note also that the unconditional probability of surviving from birth to age $j = 2, ..., J$ is given by $\Psi_j = \psi_{j-1}\Psi_{j-1}$ where $\Psi_1 = 1$ and that the population share of each cohort is given by $\mu_j = \frac{\Psi_j}{\sum_{j=1}^{J} \Psi_j}$.

[9] Horioka (2021) in a survey article concludes that the assumptions of the selfish lifecycle model work particularly well in Japan but less well in other countries such as the U.S. One reason for this is bequest taxes. Bequest taxes are higher and exemptions are much lower in Japan than in the U.S.

[10] Children only affect consumption demand as we explain in more detail below. Thus, we do not track their membership or reassign them if the household is dissolved due to a death event.

Households work, consume, and save for retirement. A household of age j in period t earns an after-tax wage rate of $(1-\tau^w)w_t\epsilon_j$, where τ^w denotes a labor-income tax rate and ϵ_j is the efficiency of labor of an age-j household.[11] All cohorts face the same age-efficiency profile and the efficiency index ϵ_j is assumed to drop to zero for all $j \geq J_r$, where J_r is the mandatory retirement age.

Households provision for retirement by acquiring liquid and illiquid assets. They may save or borrow using either assets. Liquid assets earn the nominal interest rate R_{t-1} between period $t-1$ to t and its after-tax real return is given by $\tilde{R}_{t-1}/\pi_t$, where $\tilde{R}_{t-1} \equiv 1+(1-\tau^a)(R_{t-1}-1)$, $\pi_t = P_t/P_{t-1}$ and τ^a is the tax rate on asset income. The real return on illiquid assets in period t is R^a_t and its after-tax return is $\tilde{R}^a_t \equiv 1+(1-\tau^a)(R^a_t-1)$.

From the perspective of the household the only distinction between liquid and illiquid assets is that households face costs of adjusting their holdings of illiquid assets as in Aiyagari and Gertler (1991) and Kaplan and Violante (2014). When we parameterize our model, we follow Kaplan et al. (2018) and include physical assets such as homes and durable goods and illiquid financial assets such as equities in our measure of illiquid assets. So the adjustment costs can be interpreted as representing service flows to the financial service sector when, for instance, a household purchases or sells a home. Following Kaplan et al. (2018), we also abstract from the service flow of utility services provided by physical assets. Thus, the benefit from holding illiquid assets is entirely pecuniary in our model. Adjustment costs on holdings illiquid assets are given by

$$\chi(a_{j,t}, a_{j-1,t-1}, z) = \begin{cases} \frac{\gamma_a(z)}{2}(a_{j,t}-a_{j-1,t-1})^2, & a_{j-1,t-1} > 0 \\ \frac{\gamma_a(z)}{2}a^2_{j,t}, & a_{j-1,t-1} = 0, \end{cases} \tag{3}$$

where $a_{j,t}$ denotes the holdings of illiquid assets in the end of period t and $\gamma_a(z) \geq 0$ is a parameter that governs the size of the adjustment costs for $z = z^i_{j,t} \in \{0,1\}$. These costs have two main features. First, they vary with the level of the change in assets. Second, they depend on whether the household experiences the death event in the current period. Our specification of adjustment costs has several attractive features. It creates a wedge between the after-tax return on liquid and illiquid assets even though there is no aggregate uncertainty in the model. In addition, the incidence of the adjustment costs varies systematically with age in a way that is consistent with how one might to expect

[11] Given that there is only one type of heterogeneity in a cohort, to conserve on notation we do not explicitly index the identity of each household of age j in period t in the ensuing discussion unless it is required to avoid confusion.

them to vary over the lifecycle. For instance, newly formed households face relatively high costs of accumulating illiquid assets and older working-age households who experience the death event face a relatively high cost of liquidating their relatively large holdings of illiquid assets. Finally, the costs are convex, so they don't create kinks in the household's budget set. Essentially, these costs can be interpreted as those pertaining to a representative individual of a given age and we parameterize these costs to fit the age profile of average asset holdings reported in Table 1.

Given these definitions, the decisions of a surviving household of age-j in period t (i.e., a household with $z^i_{j,t} = 1$) are constrained by

$$\begin{aligned}(1+\tau^c)c_{j,t} + a_{j,t} + \chi(a_{j,t}, a_{j-1,t-1}, 1) + d_{j,t} \\ \leq \tilde{R}^a_t a_{j-1,t-1} + \frac{\tilde{R}_{t-1}}{\pi_t} d_{j-1,t-1} + (1-\tau^w) w_t \epsilon_j h_{j,t} + b_{j,t} + \xi_t,\end{aligned} \tag{4}$$

where $c_{j,t}$ is total household consumption for a household of age j in period t, τ^c is a consumption tax rate, $d_{j,t}$ denotes holdings of the liquid assets, expressed in terms of the final good, at the end of period t, $h_{j,t}$ denotes hours worked, $b_{j,t}$ denotes public pension (social security) benefits, ξ_t is a lump-sum government transfer, and $\chi(\cdot)$ is the transaction cost of adjusting individual holdings of the illiquid assets.[12] We wish to emphasize that there are no ad hoc restrictions on borrowing of surviving households. They are free to borrow against their future earnings and they are also free to take leveraged long positions on illiquid assets, which have a higher return in equilibrium. The only constraint on borrowing of surviving households is the natural borrowing constraint.

If instead the household is in its final period of life ($z^i_{j,t} = 0$), the event is publicly observed by lenders and borrowing is not possible. Thus, the optimal strategy for the household is to consume all of its income and wealth during the current period

$$(1+\tau^c)c_{j,t} = \tilde{R}^a_t a_{j-1,t-1} + \frac{\tilde{R}_{t-1}}{\pi_t} d_{j-1,t-1} + (1-\tau^w) w_t \epsilon_j h_{j,t} + b_{j,t} + \xi_t - \chi(0, a_{j-1,t-1}, 0). \tag{5}$$

The period utility function for a household of age j in period t is given by

$$u(c_{j,t}, h_{j,t}; \eta_j) = \frac{\eta_j \left(c_{j,t}/\eta_j\right)^{1-\sigma}}{1-\sigma} - \frac{\upsilon}{1+1/\nu} h_{j,t}^{1+1/\nu}, \tag{6}$$

[12] We are omitting here the dependence of individual choices on the survival event to save on notation. Formally, we have for $z^i_{j,t} \in \{0,1\}$: $c_{j,t}(z^i_{j,t})$, $a_{j,t}(z^i_{j,t})$, $d_{j,t}(z^i_{j,t})$, and $h_{j,t}(z^i_{j,t})$. In what follows this dependence is only made explicit when required.

where $\sigma > 0$ is the inverse of the elasticity of intertemporal substitution, $\nu > 0$ governs the Frisch elasticity of labor supply, $\upsilon > 0$ is a labor disutility parameter, and η_j is a family scale, which we assume is time-invariant. In the model, children are essentially age-specific deterministic demand shocks to household consumption.

We assume that working-age households belong to a labor union. The union respects their marginal utilities and wages are flexible. We analyze the symmetric equilibrium. Thus, hours worked are identical for all workers in period t, $h_{j,t} = \bar{h}_t$ for all $j < J_r$ with $\bar{h}_t$ given by

$$(1 - \tau^w)\bar{\epsilon}_t w_t = \upsilon \bar{\lambda}_t^{-1} \bar{h}_t^{\frac{1}{\nu}}, \tag{7}$$

where $\bar{\lambda}_t$ is the weighted average of the marginal utilities of working households and $\bar{\epsilon}$ is the weighted average of the efficiency of labor. More details on the labor supply decision and the derivation of equation (7) can be found in Appendix A.3. This specification implies that workers who experience a shock are unable to self-insure by adjusting their hours worked differently from the average worker. It is worth pointing out that earnings vary by age because the efficiency of a worker's labor depends on the worker's age.

Under these assumptions the household's optimal choices are given by the solution to

$$\begin{aligned} U_j(a_{j-1,t-1}, d_{j-1,t-1}, z_{j,t}) = \max_{\{c_{j,t}, a_{j,t}, d_{j,t}\}} \Big\{ & u\left(c_{j,t}, \bar{h}_t; \eta_j\right) \\ & + \beta z_{j,t} \left[(1 - \psi_{j+1}) U_{j+1}(a_{j,t}, d_{j,t}, 0) + \psi_{j+1} U_{j+1}(a_{j,t}, d_{j,t}, 1)\right] \Big\}, \end{aligned} \tag{8}$$

subject to equations (4) and (5) for $z_{j,t} \in \{0, 1\}$ and for $j = 1, ..J - 1$, and $z_{J,t} = 0$, where $\beta > 0$ is the preference discount factor and ψ_{j+1} is the conditional probability that a household of age $j + 1$ survives to the next period.[13] Note that we have imposed no restrictions on the sign or magnitude of asset holdings beyond the natural borrowing constraint. It is thus conceivable, for instance, that households might want to borrow both types of assets. However, in the equilibria analyzed here, the return on illiquid assets exceeds the return on liquid assets and all private borrowing is in the form of liquid assets. Appendix A.1 derives the optimality conditions for the household problem. In addition, Appendix A.2 provides an analytical characterization of the liquidity premium and optimal portfolios using a simpler version of the household problem.

[13] There is a theoretical possibility that adjustment costs on illiquid assets could exceed the size of beginning of period illiquid assets. Our strategy for parameterizing the adjustment costs on illiquid assets rules this possibility out.

3.3 Production of goods and services

The production of goods and services is organized into four sectors.

Final good sector. Firms in this sector are perfectly competitive and combine a continuum of intermediate goods, $\{Y_t(i)\}_{i\in(0,1)}$, to produce a homogeneous final good Y_t, using the production technology, $Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{\theta}} di\right]^{\theta}$ with $\theta > 1$. Let $P_t(i)$ denote the price of intermediate good i, and P_t denote the price of the final good. Final good firms are price takers in input markets and it follows that demand for intermediate good i is

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\theta}{\theta-1}} Y_t. \tag{9}$$

The final good is either consumed by households or used as an input in the capital good sector.

Intermediate goods sector. Firms in this sector are monopolistically competitive and each firm produces a unique good indexed by $i \in (0, 1)$. Intermediate goods firm i produces $Y_t(i)$ by combining capital $K_t(i)$ and effective labor $H_t(i)$ with a Cobb-Douglas production function

$$Y_t(i) = K_t(i)^{\alpha} H_t(i)^{1-\alpha}, \quad 0 < \alpha < 1. \tag{10}$$

Intermediate goods firm i faces demand curve (9), and sets its price $P_t(i)$ to maximize profits subject to a quadratic price adjustment cost function. The optimality condition of this problem is derived in Appendix A.4. In a symmetric equilibrium, the condition can be expressed as

$$(\pi_t - 1)\pi_t = \frac{1}{\gamma}\frac{\theta}{\theta - 1}(mc_t - 1) + \Lambda_{t,t+1}\frac{Y_{t+1}}{Y_t}(\pi_{t+1} - 1)\pi_{t+1}, \tag{11}$$

where $\pi_t = P_t/P_{t-1}$ is the gross inflation rate. Equation (11) is the New Keynesian Phillips curve that relates the current inflation rate π_t to the real marginal cost mc_t and the future inflation rate π_{t+1}. The firms are subsidized so that there is no steady-state markup distortion, i.e. the steady-state marginal cost becomes unity, as implied by equation (11).

In a symmetric equilibrium the aggregate output is

$$Y_t = K_t^{\alpha} H_t^{1-\alpha}, \tag{12}$$

where K_t denotes the aggregate capital and H_t denotes the aggregate effective labor. The aggregate total profits of intermediate goods firms in period t, $\Omega_t \equiv \int_{i\in(0,1)} \Omega_t(i)di$, are given by

$$\Omega_t = \left[\theta - mc_t - \frac{\gamma}{2}\left(\pi_t - 1\right)^2\right] Y_t. \tag{13}$$

Capital good sector. Capital good firms are perfectly competitive and use a linearly homogeneous production technology to produce capital. The representative firm purchases $(1-\delta)K_t$ units of old (depreciated) capital from the mutual fund and I_t units of the final good from the final good firms, and uses the two inputs to produce K_{t+1} units of new capital that is sold back to the mutual fund. Then, the conventional investment identity obtains

$$K_{t+1} = (1-\delta)K_t + I_t. \tag{14}$$

Mutual fund sector. Our economy has two types of illiquid assets – capital and shares in intermediate goods firms – and there is no aggregate uncertainty in the model after time-zero. Thus, a no arbitrage argument implies that the return on the two illiquid assets is the same in all periods except possibly time-zero when their returns will differ if an aggregate time-zero shock occurs. We allocate ownership and the potential time-zero capital gains and losses among households by assuming that households invest in a mutual fund produced by perfectly competitive financial service firms. Each firm holds the market portfolio of the two illiquid assets and pays households the market return on illiquid assets.

To derive the market return on illiquid assets note that the return on capital in period t is given by

$$R_t^k = r_t^k + 1 - \delta. \tag{15}$$

The one period return from investing one unit of the period $t-1$ final good into shares is

$$R_t^v = \frac{\Omega_t + V_t}{V_{t-1}}, \tag{16}$$

where V_t is the share price. We assume that the return on capital and equity is subject to a corporate tax as well as an asset-income tax paid by households. Liquid assets, in contrast, will consist primarily of government debt in equilibrium and are taxed once at the household level. To reduce the notational burden, we assume that corporate taxes are paid by the mutual fund. Let τ^k denote the corporate tax rate. Then, perfect competition leads to the no-arbitrage conditions

$$R_t^a - 1 = (1-\tau^k)(R_t^k - 1) = (1-\tau^k)(R_t^v - 1), \tag{17}$$

for all $t > 0$. From the second equality in equation (17) the share price is given by

$$V_t = \sum_{i=1}^{\infty} \left(\prod_{j=1}^{i} \frac{1}{R_{t+j}^k} \right) \Omega_{t+i}. \tag{18}$$

Hence, the discount factor $\Lambda_{t,t+1}$ in equation (11) is given by $\Lambda_{t,t+1} = 1/R_{t+1}^k$.

We will analyze the dynamic responses to shocks to monetary policy by assuming that the economy is in a steady state in all periods prior to $t = 0$ and that an unexpected shock hits the economy in period $t = 0$. Equation (17) does not obtain in period $t = 0$. A time-zero shock creates a wedge between the ex ante and ex post return of each illiquid asset and thereby produces an unexpected capital gain or loss. Both the sign and size of these time-zero revaluation effects will generally be different for the two illiquid assets.

3.4 Government

The government consists of a central bank and a fiscal authority.

Central bank. The central bank sets the nominal interest rate R_t following a simple rule that depends on the current inflation rate and the past nominal interest rate

$$\log\left(\frac{R_t}{R}\right) = \rho_r \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_r)\phi_\pi \log(\pi_t) + \epsilon_t, \tag{19}$$

where R is a constant and ϵ_t is an i.i.d. monetary policy shock. The parameter ρ_r governs the inertia of the nominal interest rate, and the parameter $\phi_\pi > 1$ captures the central bank's stance on inflation. A high ϕ_π implies a strong anti-inflation stance and vice versa.

Fiscal authority. The fiscal authority raises revenue by taxing consumption, labor income, capital income, and mutual funds. Total tax revenue is

$$T_t = \sum_{j=1}^{J} \left[\tau^c \bar{c}_{j,t} + \tau^{ka}(R_t^k - 1)a_{j-1,t-1} + \tau^a \frac{(R_{t-1}-1)}{\pi_t} d_{j-1,t-1} + \tau^w w_t \epsilon_j \bar{h}_t \right] N_{j,t}, \tag{20}$$

where $\bar{c}_{j,t} = \psi_{j,t} c_{j,t}(1) + (1-\psi_{j,t}) c_{j,t}(0)$ is the average consumption by surviving and non-surviving households and $\tau^{ka} = \tau^a + \tau^k - \tau^a \tau^k$ is the total tax rate on illiquid assets.

Let D_t^n denote the face value of nominal government debt issued in period t. Then aggregate government expenditures consist of government purchases G_t, nominal interest payments on its debt, net of new issuance, $(R_{t-1} D_{t-1}^n - D_t^n)/P_t$, subsidies to intermediate goods firms, $\tau^f Y_t = (\theta - 1) Y_t$, public pension benefits, $B_t \equiv \sum_{j=J_r}^{J} b_{j,t} N_{j,t}$, and lump-sum transfers to households, $\Xi_t \equiv \sum_{j=1}^{J} \xi_t N_{j,t}$. It follows that the government flow budget constraint is given by

$$G_t + \frac{R_{t-1} D_{t-1}^n - D_t^n}{P_t} + \tau^f Y_t + B_t + \Xi_t = T_t, \tag{21}$$

and the government bond market clearing condition is given by

$$\frac{D_t^n}{P_t} = D_t \equiv \sum_{j=1}^{J} \bar{d}_{j,t} N_{j,t}, \tag{22}$$

where $\bar{d}_{j,t} = \psi_j d_{j,t}(1) + (1 - \psi_j) d_{j,t}(0)$ is the average government bond holdings by surviving and non-surviving households.[14]

We assume that the fiscal authority is passive in that it does not adjust the size of its nominal debt when monetary policy is changed. In other words, nominal government debt is constant: $D_{t-1}^n = D_t^n$, for $t = 0, 1, ...$ Consequently, a change in monetary policy affects both tax revenues and the real value of government debt. The size of the lump-sum transfer, ξ_t, is adjusted in each period to close the government budget constraint, equation (21). This assumption matters because in our economy changing the size of the lump-sum transfer induces redistribution. This same assumption is maintained in other models where monetary policy has distributional effects such as Sterk and Tenreyro (2018), Hagedorn et al. (2019) and Hu et al. (2021).

Benefits from the pubic pension program are modeled in the same way as Braun et al. (2009). A household starts receiving a public pension benefit at the mandatory retirement age of J_r. The real size of the benefit during the household's retirement is constant at a

[14]Because $d_{j,t}(0) = 0$, the aggregate bond can be arranged as

$$D_t \equiv \sum_{j=1}^{J} \left[\psi_j d_{j,t}(1) + (1 - \psi_j) d_{j,t}(0)\right] N_{j,t} = \sum_{j=1}^{J} \psi_j d_{j,t}(1) N_{j,t} = \sum_{j=1}^{J} d_{j,t}(1) N_{j+1,t+1}.$$

level that is proportional to its average real wage income before retirement

$$b_{j,s+j-1} = \begin{cases} 0 & \text{for } j = 1, ..., J_r - 1 \\ \lambda \left(\frac{1}{J_r - 1} \sum_{j=1}^{J_r-1} w_{s+j-1} \epsilon_j \bar{h}_{s+t-j} \right) & \text{for } j = J_r, ..., J, \end{cases} \tag{23}$$

where λ is the replacement ratio of the pension benefit and s is the household's birth year. Thus, the public pension system implicitly assumes perfect inflation indexation of pension benefits.

3.5 Competitive equilibrium

In the impulse response analysis that follows, we will assume that the shock arrives at the beginning of time zero and that households have perfect foresight about the subsequent evolution of prices and government policy.[15] Consequently, perfect foresight is assumed in the following definition of a competitive equilibrium.

Given prices, all firms maximize their profits, all households maximize their utility, and all markets clear. Appendix B provides specific details on the definition and algorithms used to compute a steady state and dynamic equilibria for this economy. Here we simply state the two market clearing conditions that have not yet been reported.

First, the aggregate household illiquid assets, denoted by $A_t \equiv \sum_{j=1}^{J} \bar{a}_{j,t} N_{j,t}$ with $\bar{a}_{j,t} = \psi_j a_{j,t}(1) + (1 - \psi_j) a_{j,t}(0)$, are equal to the sum of capital and the value of all ownership shares of intermediate goods firms

$$A_t = K_{t+1} + V_t. \tag{24}$$

Second, as shown in Appendix B.1, Walras' Law implies the market clearing condition for the final good

$$C_t + I_t + G_t = Y_t - \frac{\gamma}{2} (\pi_t - 1)^2 Y_t - X_t, \tag{25}$$

where $X_t = \sum_{j=1}^{J} \bar{\chi}_{j,t} N_{j,t}$, with $\bar{\chi}_{j,t} = \psi_j \chi_{j,t}(1) + (1 - \psi_j) \chi_{j,t}(0)$, is the aggregate cost of adjusting illiquid assets. Observe that the aggregate costs of price and illiquid asset adjustments are modeled as explicit resource costs and consequently subtracted from aggregate output.

[15] Boppart et al. (2017) provide a justification for using this approach in heterogeneous agent economies.

Table 2: Aggregate stocks of liquid and illiquid assets relative to GDP in Japan

Liquid assets and liabilities	
Currency and domestic deposits	1.74
Bonds (total public and private)	0.052
Consumer credit	-0.069
Total net liquid assets as defined in Kaplan et al. (2018)	1.73
Total net liquid assets in our model	1.23
Illiquid assets and liabilities	
Household physical assets	2.02
Equity and options	0.49
Insurance and private pensions	0.99
Mortgages and installment credit	-0.37
Other non-housing loans	-0.12
Total net illiquid assets as defined in Kaplan et al. (2018)	3.01
Total net illiquid assets in our model	3.50
Net worth	4.73

Note: The financial data are taken from the Flow of Funds Accounts (FFA) for the fiscal year 2014. The stock of household physical assets is the 2014 end of calendar year value from the Japanese National Income and Product Accounts (NIPA). All variables are expressed as a multiple of GDP for the fiscal year 2014.

4 Model parameterization and assessment

4.1 Capital, saving and debt

Our general equilibrium model has implications for household-level holdings of liquid and illiquid assets by age and also the aggregate net stocks of liquid and illiquid assets. We parameterize the model to reproduce the aggregate magnitudes of net liquid and illiquid assets in Japanese macro data. We then assess the model by comparing its implications for household-level holdings of liquid and illiquid assets by age with Japanese micro survey data.

Before we calibrate the model, we first need to classify household assets and liabilities as either liquid or illiquid. Our classification scheme is similar in spirit to that used by Kaplan et al. (2018). Table 2 provides an overview of the main components of the two categories.[16]

A comparison of the results in Table 2 with the similar results for U.S. data for the year

[16]See Appendix C for complete details.

2004 reported in Kaplan et al. (2018) (See Table 2 of their paper) reveals some important distinctions between the U.S. and Japan. The biggest difference between the two countries is that Japanese households hold a lot more liquid assets compared to Americans. The net stock of liquid assets in Japan is 1.73 but only 0.26 in the U.S., where all variables are expressed as a multiple of GDP. Japanese hold more deposits and currency than Americans. Deposits (plus currency) are 1.86 in Japan but only 0.23 in the U.S. In addition, over 90% of Japanese government debt is held domestically. So this difference between the two countries reflects the fact that Japanese households are indirectly holding a large amount of government debt.

The net stock of illiquid assets is about the same in Japan and the U.S. Illiquid assets are 3.0 in Japan and 2.9 in the U.S. However, Japanese households have smaller direct holdings of equity than Americans (0.49 in Japan versus 1.61 in the U.S.) and Japanese hold more physical assets. Physical assets are 1.53 in Japan versus 1.32 in the U.S.[17]

Given that Japanese households have much higher levels of liquid wealth than Americans while illiquid wealth is about the same in the two countries, it follows then that Japanese households have higher net worth (relative to GDP).[18] Aggregate household net worth in Japan is 4.73 times GDP and only 3.18 times GDP in the U.S.

An important distinction between our model and Kaplan et al. (2018) is that individuals save and borrow for different reasons. In our model, individuals save to smooth consumption over the lifecycle and only face life-expectancy risk. Individuals in our model borrow for two reasons. First, young individuals borrow against their future higher income. Second, some individuals in our model borrow liquid assets and use them to purchase illiquid assets because this strategy enhances the overall return on their portfolio of liquid and illiquid assets. In Kaplan et al. (2018), in contrast, individuals borrow if they experience negative earnings shocks. This difference in the savings motive affects how we organize the data. In our model, all borrowing by individuals is made in the form of liquid assets in equilibrium and we rearrange the data to reflect this property. When calibrating the model's net aggregate stock of liquid and illiquid assets, we assign all household borrowing to the liquid asset category.[19] This adjustment results in a lower total stock of aggregate household

[17]The remaining difference in holdings of illiquid assets is insurance assets which are not reported for the U.S. in Kaplan et al. (2018).

[18]We define net worth to be the sum of illiquid and liquid assets and abstract from human wealth throughout the paper.

[19]This property of the model captures the main features of Japanese data. The biggest component of illiquid liabilities using the Kaplan et al. (2018) scheme is mortgage debt. As discussed above most Japanese mortgages have adjustable interest rates and the costs of prepaying a mortgage in Japan are small.

(net) liquid assets of 1.23 and higher aggregate stock of (gross) illiquid assets of 3.5. We reproduce these two targets in the model by varying the stock of government debt and the preference discount factor. The resulting net debt-GDP ratio in the model is of 1.23 and the resulting value of β is 0.996 as reported in Table 3.

4.2 Model parameters

Table 3 reports the entire parameterization of the model. The remainder of our calibration strategy follows Braun and Joines (2015). In particular, we adjust Japanese National Income and Product Accounts (NIPA) data to recognize some differences between our model and the data. For instance, our model has no external sector and no government investment. The specific adjustments follow the strategy of Hayashi and Prescott (2002).

Starting with demographics, we assume that new households are formed at age 21 and the size of the household is parameterized in the same way as Braun et al. (2009). In the model, individuals face mandatory retirement at age 68 ($J_r = 48$). This is two years older than the age where one can qualify for full public pension benefits in Japan and is chosen to be consistent with the effective labor-market exit age in 2014 for Japan estimated by the OECD.[20] Finally, the maximum lifespan is set to 120 years ($J = 100$).

We use the same depreciation rate as Braun and Joines (2015). But we use a smaller value of the capital share parameter $\alpha = 0.3$. This value in conjunction with the rest of our parameterization results in an after-tax return on illiquid assets of 1.80% per annum. The parameter θ governs the elasticity of substitution of intermediate goods. We set this parameter to produce a gross markup of 1.05 in a steady state if there were no government subsidies.

Many real business cycle and NK models assume that preferences are additively separable in consumption and leisure and posit log-preferences over consumption. Lifecycle analyses though often set the relative-risk aversion coefficient on consumption at a higher level of about 3 (see Brown and Finkelstein, 2008). We set σ to this value. Hours worked in our model are determined in a way that is close to the representative agent framework and there is no distinction between extensive and intensive labor supply decisions. It follows that the Frisch labor supply elasticity in our model, given by ν, reflects the combined effects of adjustments along both margins. We set this parameter to 2.[21]

[20] See Pensions at a Glance OECD, 2015.

[21] We assign zero weight to individuals in their death year when computing $\bar{\lambda}$ in equation (7). If we include them and set $\nu = 1$, the model results are essentially indistinguishable from what is reported here.

Table 3: Parameterization of the model

Parameter	Description	Value
Demographics		
J_r	Retirement age	48 (Age 68)
J	Maximum lifespan	100 (Age 120)
$\{\psi_j\}_{j=1}^{J}$	Survival probabilities	Braun and Joines (2015)
Technology		
θ	Gross markup	1.05
γ	Price adjustment cost	41.2
α	Capital share parameter	0.30
δ	Depreciation rate	0.102
$\gamma_a(0)$	Cost of adjusting illiquid assets in death year	0.0723
$\gamma_a(1)$	Cost of adjusting illiquid assets in non-death year	0.203
Preferences		
σ	Inverse elasticity of intertemporal substitution	3
ν	Frisch labor supply elasticity	2
υ	Preference weight on leisure	6.9
β	Preference discount factor	0.996
Monetary Policy		
ρ_r	Interest rule persistence	0.35
ϕ_π	Interest rule inflation elasticity	2
Fiscal Policy		
τ^c	Consumption tax rate	0.05
τ^k	Corporate tax rate	0.35
τ^a	Tax rate on asset income	0.2
τ^w	Tax rate on labor income	0.232
τ^f	Subsidy to intermediate goods firms	$\theta - 1$
G/Y	Government share of output	0.16
λ	Public pension replacement ratio	0.094
D/Y	Net government debt output ratio	1.23

Household-level adjustment costs on illiquid assets produce a small liquidity premium of 0.13 percent and help the model to reproduce the main features of the empirical age profiles of net worth and its components reported in Table 1. The model's implications for net worth and its components are discussed in Section 4.3.2. The specific values of the two parameters that govern the age profile of adjustment costs on illiquid assets were selected after experimenting with a range of values for each parameter. We varied $\gamma_a(1)$ so that the initial cost of acquiring illiquid assets for 21 year old households ranged from 2% to 5% of their end of period assets. The parameter $\gamma_a(0)$ was chosen by varying the maximum costs for a household in their death year of liquidating all illiquid assets from 10% to 25% of total assets. The baseline targets of 2 percent of assets for survivors and 20 percent of assets for non-survivors help the model to account for the long number of years that working-age households borrow and induce retirees to hold positive amounts of liquid assets until relatively late in life. Appendix D reports the age profiles of adjustment costs on illiquid assets and discussion about our strategy for parameterizing the two parameters that govern their size.

The extent of nominal price rigidities in the model is determined by γ, the cost of price adjustment for intermediate goods firms. We set $\gamma = 41.2$, which implies that intermediate goods prices adjust on average every 2 years.[22]

The monetary and fiscal policy parameters are set in the following way. It is common to allow for persistence in the central bank's interest rate rule. We set the serial correlation to $\rho_r = 0.35 = 0.77^4$. The inflation elasticity is set to $\phi_\pi = 2$. Both of these choices are common choices used elsewhere in the literature.

In terms of fiscal policy variables we set the net Japanese government debt ratio to 1.23, which is the size of net liquid assets held by households in the 2014 Flow of Funds Accounts (FFA) data discussed above. Intermediate goods firms receive a subsidy that is chosen so that the steady-state markup is zero. Capital income is taxed twice in Japan. The overall tax rate on capital income faced by households in the model is 48%, which is a combination of a corporate profits tax rate (τ^k) of 35% and a 20% personal tax rate on asset income (τ^a). The consumption tax rate is set to 5% and the labor income tax rate is set to 23.2%. The personal tax rate on labor income, government purchases relative to output and the replacement rate of the public pension are calibrated using the same targets as Braun and Joines (2015).

4.3 Steady-state equilibrium

When analyzing the dynamic properties of our model we will often consider deviations from the steady-state equilibrium. Here we report the main properties of the steady-state equilibrium. Since many of the model moments we document here are not explicit calibration targets, comparing these moments with the data provides a way to assess the model.

[22] Using a log-linearized version of the model we can map back and forth between γ and the corresponding Calvo parameter and derive the average duration of price changes.

4.3.1 Aggregate moments

The steady state of our model reproduces some of the main features of the Japanese economy including the composition of aggregate output, the size of aggregate net worth and the main revenue sources and expenditures of the public sector. Lump-sum transfers are used to close the government budget constraint and amount to only -0.2 percent of output. More information is provided in Table 9 of Appendix I. Here we discuss the main gaps between the model and Japanese data.

The pre-tax premium on illiquid assets is 1.47 percent in the model. It consists of a liquidity premium of 0.13 percent and a tax treatment premium of 1.34 that arises because illiquid assets are subject to both the corporate profits tax and the household tax on asset income. For purposes of comparison, Damodaran (2020) estimates that the overall equity premium in Japan is 5.4 percent. Given that there is no aggregate uncertainty in the model and no individual-specific earnings risk, or limited participation effects, it is to be expected that the excess return on illiquid assets in the model is smaller than the excess return on equity in Japanese data.

The model's steady state also understates the aggregate amount of borrowing in Japanese FFA data. In the model, household borrowing is 35 percent of GDP whereas in our FFA data it is 56 percent. This result is striking because the model imposes a weak natural borrowing constraint on individual borrowing. Still, the model abstracts from individual earnings and medical expense risk and introducing other risk factors could produce higher aggregate borrowing in the model.

Finally, the steady-state level of the real interest rate on government debt may be too high in the model. The nominal yield on 10-year Japanese government bonds has been close to zero and Japan's inflation rate has been also close to zero (as of writing in 2021). The model, in contrast, produces a pretax real return on government debt of 2.25 percent. We are not too concerned about this gap between the model and Japanese data because this interest rate is also the interest rate on private loans in the model.

4.3.2 Net worth and asset portfolios by age

Table 4 compares the model's steady-state age profiles of net worth, liquid assets and illiquid assets with the data we previously discussed in Table 1. The results are expressed as a fraction of peak (pre-tax) income of the 50–59 year old age group.[23]

[23] The lifetime peak in income occurs in the 50–59 year old age group in both the data and our model.

Table 4: Household net worth, liquid and illiquid asset holdings by age: model and data

	Net worth		Liquid assets		Illiquid assets	
Age	Model	Data	Model	Data	Model	Data
Under 30	0.06	0.65	-0.59	-0.08	0.65	0.73
30–39	0.97	1.60	-0.78	-0.58	1.76	2.18
40–49	2.87	2.58	0.18	-0.31	2.69	2.90
50–59	5.39	4.52	2.05	0.76	3.34	3.76
60–69	7.13	6.29	3.47	1.70	3.66	4.60
70+	4.28	6.01	1.11	1.77	3.28	4.25

Note: Liquid assets are net of total borrowing and illiquid assets are gross. Net worth and assets are relative to pretax income of the 50–59 year old age group. The main source for the data is the 2014 NSFIE. More details on the construction of the data can be found in Appendix C.

As we explained in Section 2, our data has four main properties. First, the age profile of net worth is hump-shaped. Second, households under age 50 hold leveraged long positions in illiquid assets. Third, older working-age households and retirees have positive holdings of both liquid and illiquid assets. Fourth, households maintain their asset holdings until late in life.

Our model reproduces each of these properties of our data. The hump-shaped pattern of saving over the lifecycle produced by the model reflects primarily the hump-shaped age-earning profile and the fact that Japan's public pension insurance program provides incomplete coverage. The reason younger households are taking leveraged long positions in illiquid assets in the model is because their mortality risk is relatively low and thus investing in illiquid assets is a good deal for them. On balance, the benefits of a higher expected return on illiquid assets exceed the cost of having to suddenly liquidate their holdings if they experience a death event. Older working-age households and retirees choose to hold a diversified portfolio of liquid and illiquid assets. Working-age individuals experience a sudden income loss at retirement and they plan for retirement by acquiring liquid assets. Once they retire, they immediately start to draw down their holdings of liquid and illiquid assets. Finally, net worth declines monotonically during retirement in the model. The pace of decline is gradual and net worth remains high until late in life. In our NSFIE sample the average age of the 70+ age group is 76. Households of this age have net worth that is more than five times their peak earnings in the model.

We are using a parsimonious, convex two parameter model of adjustment costs to reproduce the age profiles of net worth and holdings of liquid and illiquid assets and some gaps between the model and data are bound to arise. The most notable gap is that the decline in net worth between the two oldest age groups is smaller in the data as compared to the model. We believe that this gap, at least, partly reflects measurement issues in the NSFIE. Our NSFIE sample consists of all households including multi-generational households, which are still relatively common in Japan and have very different age profiles compared to older couples.[24] For instance, results reported in Murata (2019) using the first wave of the JSTAR panel data set indicate that net worth of households with 2 or more members continues to increase as the household head moves into retirement, but that older couples draw down their net worth as they age. Net worth of retired older couples falls by over 30% between its peak at age 64 and age 76 and the annual rate of decline is about 2.7% per year.[25] For purposes of comparison, net worth in our model falls by 3.3 percent per year during the first twelve years of retirement.

Our model has a simple mechanism that induces households to retain assets until late in life. Households know their death year has arrived at the start of their final year of life and have the opportunity to consume their wealth before they pass away and the marginal cost of selling illiquid assets is lower in the death state. The literature has offered several specific reasons for why households retain wealth until late in life. Kopecky and Koreshkova (2014) highlight the role of nursing home risk and Lockwood (2018) finds evidence that bequests are a luxury good. We have omitted these motives from the model for reasons of tractability. We compute dynamic responses in Section 6 and this adds substantial computational burden compared to these previous papers that conduct respectively steady state and partial equilibrium analyses. Bequest motives and nursing home risk are less important in Japan than the U.S. Japan has a universal public long-term care insurance program and imposes large taxes on bequests. However, modeling either mechanism would further dampen the speed that households draw down their net worth during retirement in the model.

A second gap between the model and the data is that households between the ages of 40–69 have higher holdings of liquid assets and higher net worth in the model as compared to the NSFIE survey. These gaps may reflect under-reporting biases in the NSFIE as we explain in Appendix C. We have calibrated the model to reproduce aggregate net worth in

[24]We thank Fumio Hayashi for making this point.

[25]See Appendix Figure 3 of Murata (2019). The annual rate of decline is imputed by us using data reported in the figure.

the FFA. So the model results in Table 4 can be interpreted as an estimate of the size of missing assets in the NSFIE survey.

4.3.3 Marginal propensities to consume

The average marginal propensity to comsume (MPC) in our model is 0.05 for a one year 50,000 yen increase in the lump-sum transfer and rises to 0.09 if the transfer is increased by this amount for two years.[26] Average MPCs of this magnitude are somewhat smaller than recent estimates using Japanese data. Koga and Matsumura (2020) estimate that the average MPC out of transitory income shocks is 0.15 using data from the Japanese Household Panel Survey. MPCs increase monotonically with age in the model and age profiles of MPCs can be found in Appendix E.

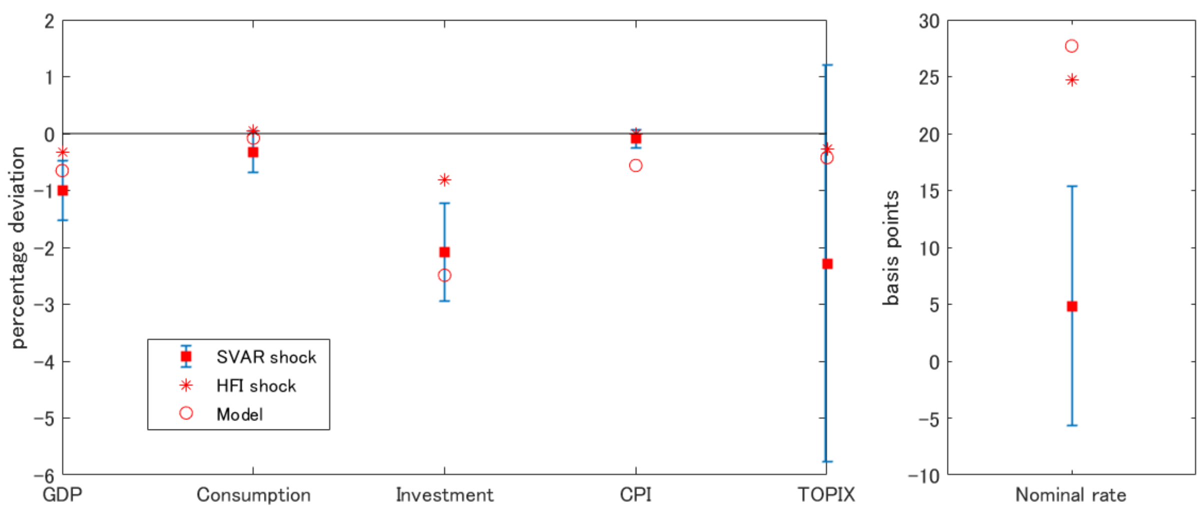
5 The aggregate effects of monetary policy

This section compares the response of aggregate variables in our model to a tightening in monetary policy with the responses in Japanese data using two identification schemes. We need to dispense with a conceptual issue before we conduct the comparison. The empirical impulse responses are estimated using the 1992–2019 sample period and the Japanese policy interest rate has been close to zero during this period. How has monetary policy affected economic activity? Our maintained hypothesis in the model is that the effective lower bound (ELB) of nominal interest rates has not constrained the ability of the central bank to achieve its policy objectives because the unconventional monetary policy tools at its disposal have allowed it to implement a negative (shadow) policy rate. Our maintained hypothesis is supported by research by Ikeda et al. (2020) who estimate the shadow policy rate at the ELB in Japan and find that unconventional monetary policy has been, if anything, a more effective stabilization tool than conventional monetary policy at horizons of one year and beyond. We discuss the robustness of our conclusions to this maintained assumption in Section 7.

Figure 2 reports data responses of GDP, consumption, investment, CPI, and the TOPIX stock price index to a tightening in monetary policy in the impact year. We use local projections with quarterly data from 1992–2019 and report cumulative empirical responses to monetary policy shocks for two different identification schemes. The "SVAR" results are

[26] The size of the MPCs is virtually identical if the size of the increase in transfers is 5,000 yen instead.

Figure 2: Impulse responses by age to a 1% tightening in monetary policy: data and model.



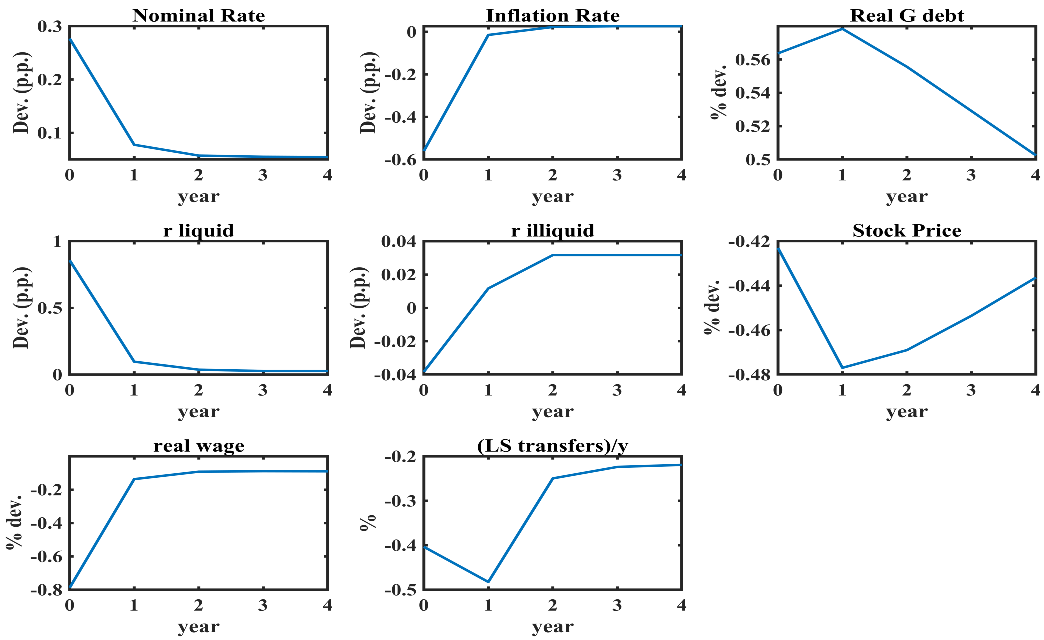
Notes: The two data responses are cumulative effects over four quarters with the exception of TOPIX, which is the response of the stock price index in the impact quarter. The SVAR shock identification is from Ikeda et al. (2020) and the HFI shock identification is from Kubota and Shintani (2021). The sample period is 1992-2019 and the vertical lines are 90% confidence intervals for local projections using the SVAR identification scheme. "Model" refers to our baseline model.

from Ikeda et al. (2020) who estimate an endogenous regime-switching VAR and use sign restrictions to identify shocks to monetary policy. In contrast, the "HFI" results use the high frequency proxy variable identification scheme we discussed in Section 2. The vertical lines in the figure are 90 percent confidence intervals for the SVAR scheme. We do not report confidence intervals for the HFI scheme because they are considerably larger and uninformative. Finally, the results labeled "Model" refer to our baseline model.

The model and SVAR shock size is $\epsilon_0 = 0.01$. However, the HFI scheme is conceptually different and we adjust the size of the HFI shock so that it produces the same sized response in the 10-year bond yield as the SVAR scheme. See Appendix F for more details.

A comparison of the impact responses in the model and the data indicate that the model is in good accord with these two empirical schemes for identifying monetary policy shocks. The signs and magnitudes of the responses of GDP, consumption and investment in the model are all close to the empirical responses. The model predicts a somewhat larger decline in the price level compared to the two empirical schemes, but it reproduces the decline in the TOPIX Japanese stock price index. We discuss this point in Section 6.5, but it is worth noting here that our lifecycle model produces declines in aggregate investment and stock prices using a very simple macro-economic structure. We abstract from aggregate adjustment costs on investment, nominal wages are flexible and all profits are paid out as

Figure 3: Impulse responses by age to a 1% tightening in monetary policy in the model.

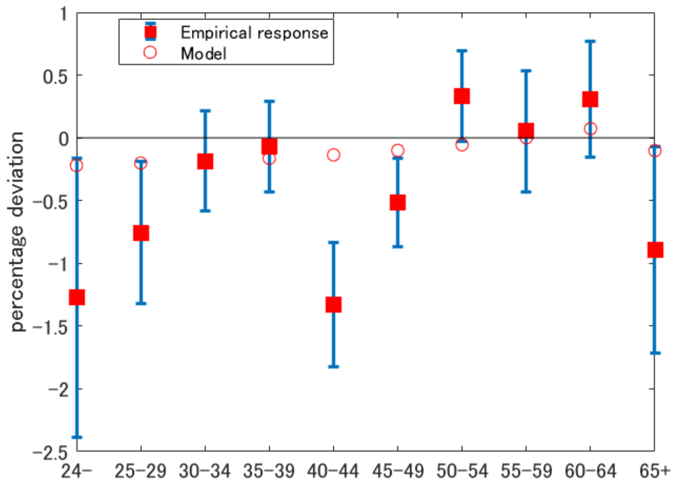


dividends to shareholders. Moreover, from Table 4 we know that all age groups are holding positive amounts of illiquid assets and as we describe below all age groups choose to reduce their holdings of illiquid assets when monetary policy is tightened.

Finally, observe that the size of the 1-year nominal government bond yield (nominal interest rate) response is about the same for the HFI scheme (25 bp) and the model (28 bp), but is much smaller using the SVAR identification scheme (5 bp). This difference likely reflects the fact that the SVAR scheme imposes the ELB. The HFI scheme identifies monetary policy shocks directly off the response of interest rate futures in a narrow window around an announced change in monetary policy, but doesn't impose the ELB. The SVAR scheme, in contrast, imposes the ELB and measures the impact of unconventional monetary policy (UMP) on economic activity by inferring the value of the shadow policy rate from multiple macroeconomic indicators. Under this identification scheme the response of the 1-year rate to a surprise easing in MP is constrained by the ELB in most of the sample period and it is consequently not surprising that the average (local perturbation) response of the 1-year rate is smaller than the HFI scheme or our model.

Notes: "r liquid" and "r illiquid" are pre-tax real interest rates and G debt is real government debt. It will be helpful in our ensuing discussion of micro responses to know the complete range of aggregate disturbances experienced by households and Figure 3 reports

Figure 4: Responses of consumption by 5-year age group to a tightening in monetary policy model and data.



Notes: Cumulative consumption responses in the impact year, monthly Japanese data (FIES), high frequency identification based on Kubota and Shintani (2021). Vertical lines are 90% confidence intervals.

this information. The figure has two notable properties. First, the asset substitution channel of monetary policy is readily discernible in the impact period. A monetary policy tightening increases the real value of government debt and increases the real return on liquid assets, but reduces the real return on illiquid assets. Second, the real wage rate and lump-sum transfers decline persistently.

6 Monetary policy over the lifecycle

We now analyze how a tightening in monetary policy affects the situation of households at different stages of the lifecycle.

6.1 Responses of consumption by age

We start by comparing the empirical age profile of consumption responses previously reported in Figure 1 with consumption responses in our model. Figure 4 conducts this comparison. The empirical responses for consumption are noisy, but the overall age profile of consumption responses is consistent with our model. The youngest households experience particularly large declines in consumption and the size of the consumption responses decline with age and turn positive as the household head approaches retirement.

Our finding that the consumption response of younger working-age households is particularly large is consistent other results in the literature. Wong (2019) finds that consumption of younger working-age households is particularly sensitive to changes in monetary policy because interest rates have a big impact on their decision to purchase their first home and their monthly expenses if they already own a home and Bachmann et al. (2021) find that consumption of younger working-age households is particularly sensitive to temporary changes in German value added taxes.

One difference between our results and other results in the literature is that we find that the *sign* of the consumption response differs across age groups both in our model and in Japanese data. Many HANK models, in contrast, have the property that all households reduce their consumption when monetary policy is tightened (see e.g. Luetticke, 2021).[27]

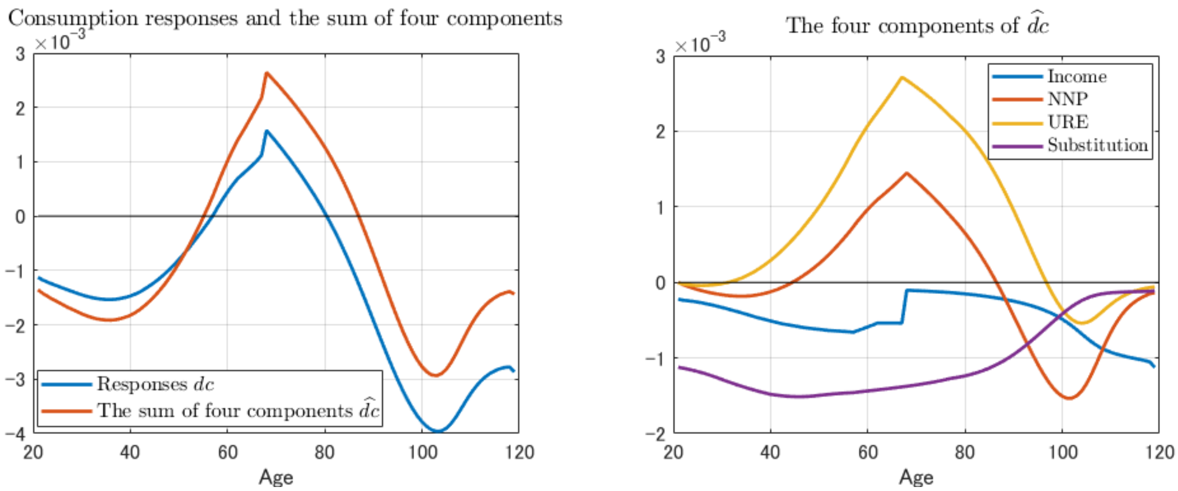
Monetary policy risk exposures by age. We next decompose the age profile of consumption responses in the impact period into income, wealth and intertemporal substitution effects. Household income exposures vary by age because earnings and government income depend on a household's age. Wealth exposures also vary by age since the sign of the consumption response depends on age. Wealth effects can be further decomposed into capital gains/losses due to unexpected inflation emphasized in Doepke and Schneider (2006) and gains/losses due to unexpected changes in real interest rates which play a central role in the work of Guerrieri et al. (2020) and Garriga and Hedlund (2020). Finally, we are interested in the role of intertemporal substitution effects which are important in representative agent New Keynesian (RANK) models.

A formula proposed by Auclert (2019) provides a convenient way to decompose the model response of consumption by age into four risk exposures. The income component captures the impact of changes in labor and net government income. The net nominal position (NNP) component measures the effect of a surprise change in the inflation rate on the real value of nominal assets and liabilities, and the unhedged interest rate exposure (URE) component reflects the effects of a change in the real interest rate on net asset cash flows that are maturing.[28] The URE component can be important, for instance, for retired households who are drawing down their holdings of assets to consume. Finally, the intertemporal substitution component captures the tradeoff between consumption and

[27] We do not compare model and FIES disposable income responses because the model concept of disposable assets and income is different from FIES data. In spite of these differences the age profiles of disposable income responses in the model and data are very similar.

[28] In our model all financial and physical assets are one-period assets. However, the URE component also includes human capital in assets and future consumption plans in liabilities.

Figure 5: Decomposition of consumption responses by-age to a tightening in monetary policy in year 0.



Note: dc denotes the level of the consumption deviation from steady state by age and $\widehat{dc}$ is the sum of the four components: income, NNP, URE, and intertemporal substitution.

saving arising from a change in the real interest rate.

Let dc denote the difference in consumption from a steady state and define $\widehat{dc}$ to be the sum of the four components. Under the specific assumptions maintained by Auclert (2019), the response of dc coincides with $\widehat{dc}$. The left panel of Figure 5 reports the impact responses of dc and $\widehat{dc}$ by age in our model. Inspection of this panel indicates that the Auclert decomposition is not exact in our model.[29] Still, we believe that it is a useful device for measuring the relative contributions of these four factors to the age profile of consumption responses.

The right panel of Figure 5 plots the four components of $\widehat{dc}$. This decomposition suggests that the URE component is more important than the NNP (surprise inflation) component analyzed by Doepke and Schneider (2006) for most age groups. The gap between the two components is particularly pronounced among households who are close to retirement. These households are benefiting from lower unexpected inflation because they have high holdings of liquid securities. In addition, they are savers and benefit from higher interest rates in future periods. Our results indicate that this second benefit is particularly large. Notice, next, that the sign of these two components reverses for long-lived retirees. For this age group, the capital losses due to lower inflation are particularly important because they are borrowing liquid assets to finance their holdings of illiquid assets and a tighter

[29] Appendix G reports the formula for this decomposition and explains why this decomposition is not exact for our model.

monetary policy increases the real value of their loans.

Kaplan et al. (2018) perform a similar decomposition and find that the overall contribution of the intertemporal substitution component is small. In their HANK model a large mass of workers have positive but low holdings of liquid assets and income and asset cash flow exposures are more important for them than intertermporal substitution in this region of the state space. In our model the contribution of these channels depends on the age of the household. Intertemporal substitution is the most important component for working-age households under 55 years of age and it is also influential at other stages of the lifecycle. For instance, the intertermporal substitution component is about as large in absolute magnitude as the NNP component at age 95.

Finally, observe that all four components are negative for individuals older than 96. Beyond this age the roles of substitution, and the two asset components become less important because household planning horizons are short and asset holdings are small. Net government transfers which have fallen in response to the shock are particularly significant for individuals in this age group.

6.2 Responses of net worth and portfolios by age

Luetticke (2021) finds that a household's net worth has a big influence on how it adjusts its asset portfolio to a change in monetary policy in a HANK model where households use liquid and illiquid assets to self-insure against earnings risk. Luck matters here too, but in a different way. In our model a household's age when the monetary policy shock arrives affects its investment opportunities. For some age groups the shock changes future asset returns in a way that dovetails perfectly with the household's desired asset allocation plan and its investment opportunities improve moving forward. In other age groups the household's future investment opportunities deteriorate and adjustments are aimed instead at mitigating their loss.

In the discussion that follows, it will be helpful to recall from our discussion in Section 4.3.2 that the household-level steady-state asset portfolios have three properties. First, surviving households of all ages hold positive levels of illiquid assets. Second, younger working-age households and older retirees borrow liquid assets. That is, they hold leveraged long positions in illiquid assets. Third, households with ages 21–29 have low net worth.[30]

Table 5 shows how households adjust their portfolios in the impact period in response

[30] In fact, households with ages 21–24 have negative steady-state net worth.

Table 5: Portfolio adjustments by age to a tighter monetary policy

Age group	Liquid assets	Illiquid assets	Leverage	Net worth
21–25	0.39	-3.04	2.75	-35.09
26–30	-0.47	-0.88	1.37	-8.43
31–35	-0.70	-0.55	1.26	-2.15
36–40	-1.08	-0.41	1.27	-2.25
41–45	-3.63	-0.33	2.51	-0.53
46–50	-0.21	-0.29	0.00	-0.26
51–55	0.24	-0.26	0.00	-0.09
56–60	0.39	-0.24	0.00	0.04
61–65	0.50	-0.22	0.00	0.13
66–70	0.66	-0.22	0.00	0.21
71–75	0.83	-0.22	0.00	0.20
76–80	1.06	-0.23	0.00	0.13
81–85	3.74	-0.25	0.00	0.04
86–90	2.59	-0.28	-2.31	-0.12
91–95	0.34	-0.36	0.02	-0.40
96+	5.02	-5.06	0.01	0.74

Note: These are asset holdings at the end of the impact year. Liquid, illiquid assets and net worth are expressed as percentage deviations from the steady state for the given age group. The sign of the liquid asset response is adjusted so that a positive number implies that the household has increased holdings (or reduced borrowing for borrowers). Leverage is expressed as a percentage change from the steady-state leverage ratio.

to a tightening in monetary policy. The responses for liquid and illiquid assets and net worth reported in this table are percentage deviations from the steady state at the *end* of the impact year for the given age group. The sign of the responses of liquid assets and net worth are adjusted so that a positive number corresponds to higher holdings for savers and less borrowing for borrowers. Table 5 also reports the leverage ratio which is defined as the percentage change in the ratio of liquid to illiquid assets for households who are borrowers.

Perhaps the most noteworthy feature of Table 5 is that households of all ages choose to reduce their holdings of illiquid assets. This response reflects the asset substitution effect. A monetary policy tightening has a persistent and distinct impact on the real returns on the two assets. Inspection of the responses of these returns in Figure 3 reveals that both returns are above their steady-state levels in period 1, but the deviation of the real return of liquid assets is larger and all households respond by investing less in the illiquid assets.

Household adjustments to liquid assets exhibit considerable heterogeneity. Households in the 21–25 age group reduce their borrowing. Yet, the leverage ratio on their portfolio increases as they also reduce the holdings of illiquid assets. Households in the age groups between 26 and 39, however, increase their borrowing. These households suffer from labor income declines and capital losses on their leveraged long positions on illiquid assets, leaving the impact year with lower net wealth.

Households between the ages 56 and 85 have positive holdings of both liquid and illiquid assets when the monetary policy shock arrives and exit the impact period with higher net worth. They benefit from a capital gain on impact because the higher real return on their holdings of liquid assets exceeds the decline in the return on their holdings of illiquid assets. From their perspective, liquid assets are a more attractive investment. Thus, they reduce their allocation to illiquid assets and increase their allocation to liquid assets.

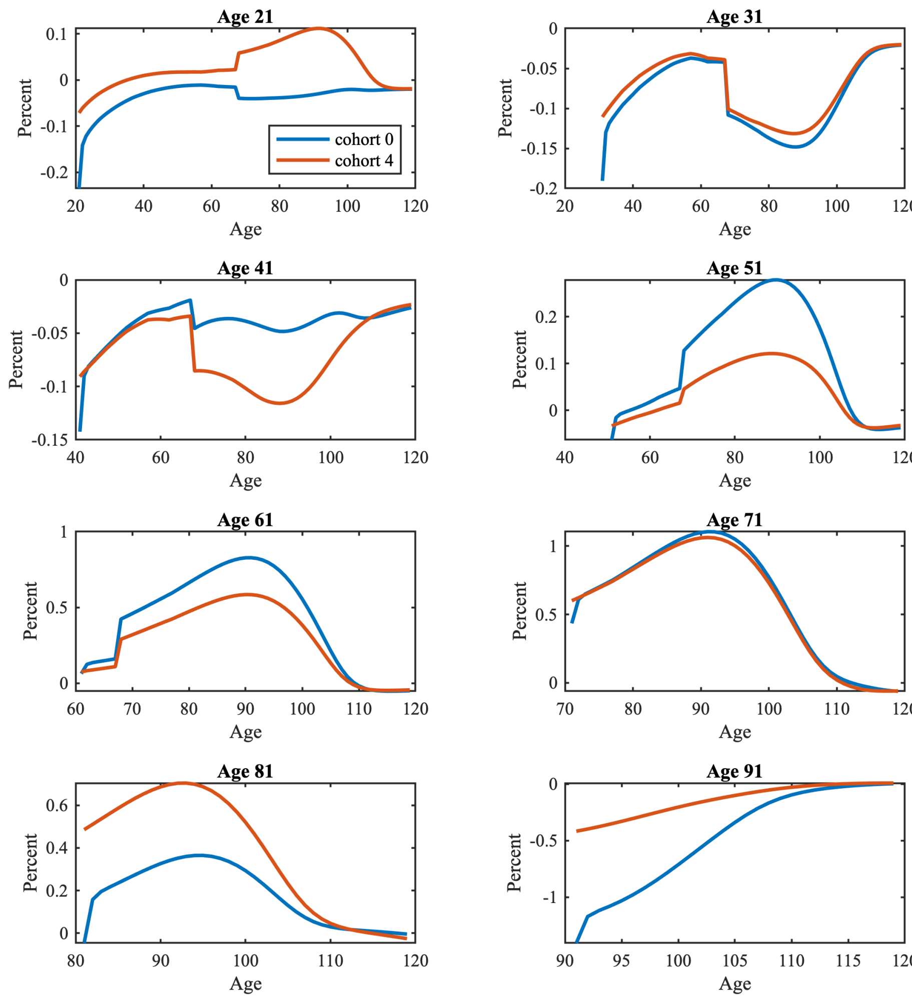
Households over age 85 are also borrowers. Even in this age group one can discern heterogeneous responses to a monetary policy tightening. All households in the 86+ age group reduce their borrowing. Leverage ratios fall for households with ages 86–90. However, 91+ year old households increase their leverage ratios. For this age group the expected benefit of a higher return on their leveraged portfolio exceeds the cost of experiencing a death event and having to liquidate their position in illiquid assets.

6.3 Cohort effects

We have seen that monetary policy shocks have heterogeneous effects on household consumption and portfolio choices. Up to this point though we have focused primarily on responses in the impact year. Guerrieri et al. (2020) document that changes in the price of illiquid (housing) assets induce persistent impacts on consumption. Our model has this same property. However, as we now proceed to explain, the sign and shape of the age profile of consumption responses depends on the age of the household when the shock arrives. To illustrate this point, consider first members of the cohort who are 71 years old when the monetary policy shock arrives. We choose this cohort because the decomposition of consumption we performed above suggests that wealth effects are particularly pronounced for households around this age. These households have just retired and have high and positive holdings of both liquid and illiquid assets.

Figure 6 displays age-consumption profiles of households in a cohort starting from the age specified in the title of the panel. The line labeled cohort 0 shows the age-consumption profile for the cohort with that age (e.g. 21 years old in the upper left panel) in the

Figure 6: Consumption-age profiles of selected cohorts.



Note: The figure reports consumption deviations from steady state as a percentage of steady-state disposable income for an household of the age listed on the horizontal axis. Cohort 0 has the age listed in the title of the panel when the shock arrives in time zero. Cohort 4 reaches the age specified in the title of the panel five years later.

year that the shock arrives. The consumption-age profiles are reported as the deviation of consumption at the age listed on the horizontal axis from its steady-state level relative to steady-state disposable income at the same age. The panel titled "Age 71" of Figure 6 shows that the monetary policy tightening has a positive and hump-shaped effect on the consumption of households who are 71 years old in the impact period. Surprisingly, the peak deviation of their consumption occurs at age 90 or nearly 20 years after the shock arrives. To see why their consumption responds in this way it is helpful to decompose the response of the age profile of consumption of a 71 year-old household into three components using the household's flow budget constraint: changes in cash flows from illiquid assets, changes in cash flow from liquid assets and changes in net cash flows from the government. Appendix H performs this decomposition and shows that cash flows from liquid assets exhibit particularly large, positive and persistent deviations from the steady-state profile. Returns on liquid and illiquid assets are slightly above their steady-state levels for many periods but the deviation of liquid asset holdings is larger (see Figure 14 in Appendix H). Households in this cohort respond by persistently tilting their portfolio towards liquid assets.

Why does the peak consumption deviation occur at age 90? This is because the households' preferred asset allocation strategy changes if they survive to age 91. Up until this point they preferred to hold positive amounts of both assets. But, after age 90 they prefer to leverage up on illiquid assets and this investment strategy is not as lucrative because borrowing costs are now higher. Still, the households have accumulated more wealth at this juncture of the lifecycle and they are able to enjoy higher consumption in each remaining period of their life, if they pass away prior to age 108.[31]

Next we turn to describe how consumption-age profiles respond to the monetary policy tightening for households in other age groups. Notice that the age 51 and 61 cohort 0 households also experience large and hump-shaped increases in their consumption-age profile deviations. However, the cohort that is of age 91 at the time that monetary policy is tightened experiences a large loss in wealth and it responds by shifting its entire age-consumption profile down. Consumption declines by over 2 percent on impact and is down by more than 0.5 percent in all subsequent years of life (conditional on surviving). The shape of the consumption deviations is monotonic in this cohort. This reflects the fact that mortality risk is high and increasing rapidly with age.

The cohort whose age is 21 when the nominal interest rate rises also experiences a relatively large and persistent decline in its subsequent age-consumption profile. Households are born with zero wealth so the MPC of this cohort is large in the impact period. However, the monetary policy shock is affecting not only this cohort's current income but also the return on its preferred investment strategy.

An alternative way to measure the size and persistence of cohort effects is to compare two cohorts who attain the same age in different years. The second line in each panel, labeled cohort 4, shows the consumption-age profile of the cohort that reaches the age specified in the title of the panel five years later. In most cases members of cohort 4 have smaller consumption-age profile deviations (in absolute value) from steady state. This is

[31] See Appendix H for more details.

not true in all instances. For example, at age 41, the deviation in the consumption-age profile of cohort 4 is larger than the deviation of cohort 0. The reason why this occurs is because cohort 4 households experienced the shock at age 36. At that stage of their lifecycle their preferred investment strategy was to borrow liquid assets and invest them in illiquid assets. The monetary policy tightening lowered the return from that investment strategy and they have arrived at age 41 with lower wealth compared to cohort 0. The cohort 4 consumption-age profile deviation is also larger at age 81. At this stage of the lifecycle cohort 0 is worse off than cohort 4. Members of cohort 0 were too old to enjoy the big positive effects that a tighter monetary had on the asset portfolio of the younger cohort.

6.4 Inequality

The heterogeneous asset allocation and consumption responses that we have documented above suggest that monetary policy influences wealth and consumption inequality. Table 6 reports the size and persistence of these effects using Gini coefficients expressed as percentage point differences from steady state in each year. Inequality in net worth increases by 0.2 percentage points in the impact year and then gradually declines in subsequent years. Wealth inequality increases because the highest wealth households, who are close to age 65, have positive returns on their portfolios while younger and older households have negative returns. Consumption inequality increases by 0.082 percentage points on impact and continues to increase for the ensuing four years. Consumption inequality increases because some age groups are reducing their consumption while other age groups are increasing their consumption. The reason consumption inequality increases by less than wealth inequality is because the shock is transient and households smooth consumption. Consumption inequality is also more persistent than wealth inequality. A tighter monetary policy enhances the investment opportunities of the most affluence cohorts and they prefer to invest more now in order to consume more later in life.

6.5 The asset substitution channel of monetary policy

We complete our discussion of monetary policy over the lifecycle by analyzing the quantitative significance of the asset substitution channel of monetary policy.

We start by discussing how the household level savings and asset allocation responses get reflected in the aggregate market clearing conditions for liquid and illiquid assets and

Table 6: Wealth and consumption inequality (Gini coefficients)

Year	0	1	2	3	4
Wealth Gini	0.20	0.20	0.19	0.19	0.17
Consumption Gini	0.082	0.084	0.085	0.086	0.086

Note: Figures in the table show a percentage point change in the Gini coefficient from its steady state, associated with a shock of size 0.01 to monetary policy. For instance a value of 0.20 implies an increase in the Gini coefficient from 0.409 to 0.411.

then relate these adjustments to the response of aggregate investment and share prices. We have already seen one aspect of the asset substitution effect. Table 5 indicates that all households invest less in illiquid assets in period 0 and it follows immediately that aggregate investment and the capital fall in response to the monetary policy tightening. This response is due to the asset substitution effect. Both the real supply of and the real return on liquid assets increase following the monetary policy tightening and this crowds out demand for illiquid assets. One cannot directly ascertain how aggregate holdings of liquid assets respond in Table 5 because not all households respond in the same way, but it is easy to infer the response of aggregate real holdings of liquid assets because nominal government debt is constant and the price level falls. This is why the real stock of government debt increases in Figure 3. The decline in the price level is persistent and it follows that the real return on liquid securities must increase persistently to induce households to hold more of the liquid assets in future periods as well.

The asset substitution effect also has a strong influence on the price of shares in intermediate goods producers. A higher real supply of liquid assets and a higher future real return on them means that share prices fall not just on impact but also in a persistent fashion. The response of share prices is unambiguously negative if prices are flexible, because the markup is constant under that assumption.

What is novel about our specification is that the price of shares is falling persistently in a model where intermediate goods producers face convex costs of adjusting their prices. A standard property of NK models is that a monetary policy tightening pushes profits up and higher profits put upward pressure on stock prices. This dividend effect is dominant in infinite horizon RANK and HANK models such as those analyzed by Broer et al. (2020) and Kaplan et al. (2018) and the price of shares increases when monetary policy is tightened. Put differently, a novel feature of our model is that it is consistent with the Campbell and Shiller (1988) observation that discounting is more important for stock price movements compared to movements in dividends. Dividends (profits) go up in our model, yet the price of shares in intermediate goods firms falls persistently due to changes in discounting induced by asset substitution.

Our model features both nominal rigidities and competing assets with different liquidity properties. How important are these two channels of monetary policy? Figure 7 reports aggregate responses to a monetary policy tightening for the baseline and flexible price specifications of the model. The distinction between the two specifications is particularly pronounced in the first period. In most cases, the signs of the responses are identical, but the magnitudes are very different. Modeling nominal price rigidities reduces the response of the nominal price level and amplifies the response of most real variables.[32] Investment, stock prices, wages, hours and output all experience larger declines in the baseline specification compared to the flexible price specification. Real government debt, in contrast, increases by more under flexible prices.

Figure 7 also shows that asset substitution is inducing persistence in the responses of real government debt, investment, stock prices and ultimately consumption. Real government debt is persistently high in both specifications, and in our model higher real debt crowds out household demand for competing assets. Aggregate investment exhibits a small but persistent decline that is of about the same magnitude in both specifications in years 1–4 and stock prices are also persistently depressed. These persistent responses of different asset classes induce persistence in aggregate consumption for the reasons that we described in Section 6.3. The persistent cohort effects that we discussed above are having small but persistent effects on aggregate economic activity in both specifications.

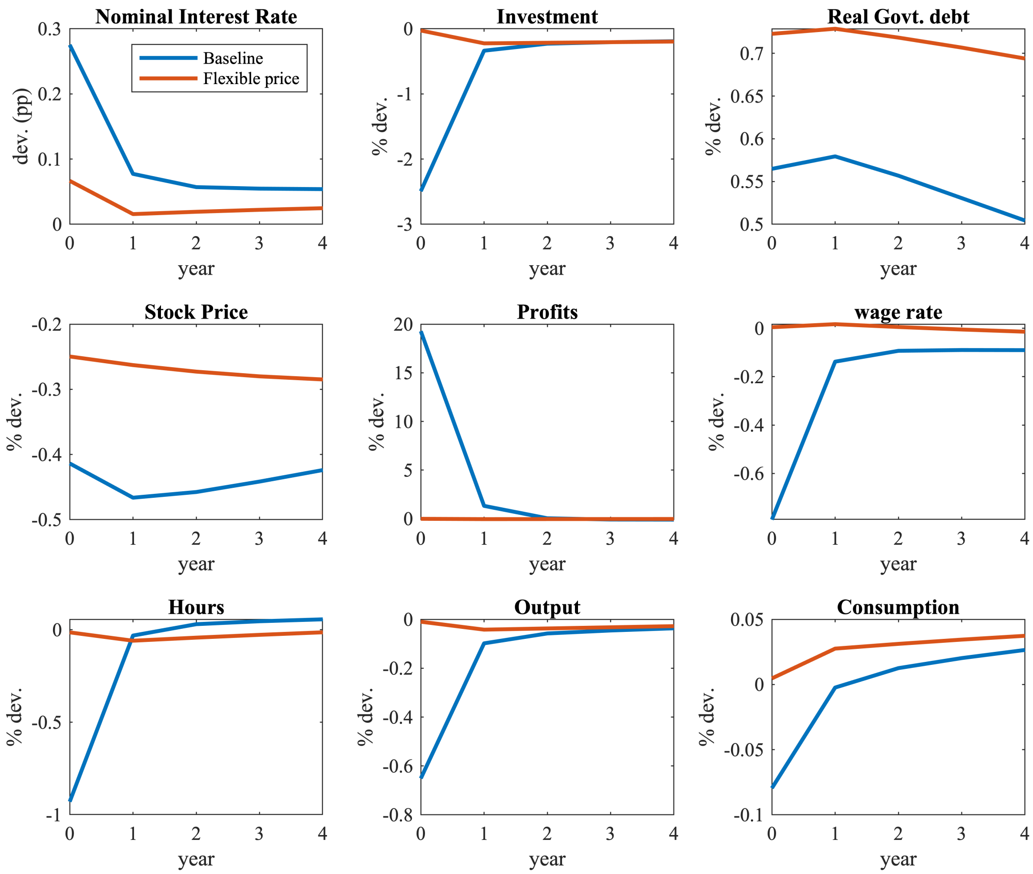
The most important difference between the two specifications is the response of consumption. Aggregate consumption falls in the baseline specification but increases under flexible prices. Table 7 provides more information on how household adjustments by age group determine the aggregate consumption responses in the impact period. Specifically, it reports the contribution of household level responses of net worth, illiquid asset holdings and consumption for three age groups.[33] The specific age ranges are chosen so that the sign of the consumption response is the same for all households in a given age group in the baseline specification.

Inspection of Table 7 reveals that consumption and net worth increases in the 57–80

[32] The (inverse) price responses can be inferred from the government debt responses in Figure 7 because the stock of nominal government debt is held fixed.

[33] The responses reported here are population weighted averages of individual responses.

Figure 7: Responses of aggregate variables to a tightening in monetary policy: baseline and flexible price specifications.



Notes: Responses to an innovation of size 0.01 in the interest rate targeting rule.

age group and the increases are particularly pronounced in the flexible price specification. This is the age group with the largest holdings of liquid assets and as we documented above they experience large capital gains on their positions when monetary policy is tightened. Their capital gains are larger when prices are flexible, because the price level falls more and stock prices fall less in the impact period. The other two age groups, experience declines in net worth and respond by reducing their consumption in both specifications. However, the declines are a bit smaller in the flexible price specification. Finally, households in all three age groups prefer to reduce their investments in illiquid assets in both the baseline and flexible price specifications.

To conclude this section, we wish to point out that the strength of the asset substitution channel depends on how the government budget constraint is balanced. Our baseline simulation assumes that lump–sum transfers adjust in each period. If we assume instead that

Table 7: Population shares and impact responses of average consumption and net worth responses to a tighter monetary policy by age group

Age	Population Share	Consumption	Net worth	Illiquid assets
Baseline				
21–56	0.563	-0.194	-0.436	-0.812
57–80	0.328	0.130	0.147	-0.224
81–120	0.108	-0.283	-0.162	-0.303
21–120	1.000	-0.098	-0.215	-0.564
Flexible Price				
21–56	0.563	-0.065	-0.596	-0.257
57–80	0.328	0.176	0.260	-0.066
81–120	0.108	-0.279	-0.230	-0.091
21–120	1.000	-0.009	-0.275	-0.176

Note: Population shares are percentages of the total population. Consumption and net worth are averages of percentage deviations from the aggregate steady-state value by age in the impact year.

the government accumulates new debt during the first 100 years of the transition and then balances its budget with higher lump-sum taxes from year 101 on, all households alive at the time of the shock escape higher taxes. The resulting positive wealth effect attenuates the declines in investment, hours, output and consumption in the baseline specification and flips the sign of the investment, hours and output responses under flexible prices.

The message of this example is general. A tightening in monetary policy also induces the asset substitution effect if consumption and/or wage taxes are increased instead and the strength of the asset substitution effect depends on when taxes are increased. A merit of analyzing the asset substitution channel in a quantitative overlapping generations framework is that one be explicit about the range and scale of government tax and transfer policies. This in turn increases our confidence that the size of the fiscal adjustments induced by a change in monetary policy described here are empirically relevant. The only caveat is government purchases. Government purchases have no productivity enhancing function in our model and if a tightening in monetary policy is accompanied by reductions in government purchases, more resources are available for the private sector and the resulting wealth effects will attenuate the size of the asset substitution channel of monetary policy.

7 Robustness

Taxing the old. In our baseline analysis, the government budget constraint is closed by adjusting the lump-sum transfer in each period. Changes in the lump-sum transfer are important for the consumption response of the oldest households in the model. They have short planning horizons and a lower lump-sum transfer associated with tighter monetary policy has a large impact on their disposable income (Figure 5). This raises the question of how robust our conclusions are to our strategy for balancing the government budget constraint. We investigate this question by performing a counterfactual where the lump-sum transfer to all households over age 75 is held fixed at its steady-state value. Changing the government financing scheme in this manner does not alter our main findings. Consumption of the oldest retirees still declines when monetary policy is tightened and consumption of households close to retirement continues to increase. The responses of the macro aggregates are also close to the baseline responses (Table 11 in Appendix I). The main difference is that consumption inequality increases by more (Table 11). Consumption of the oldest households increases, but young households face higher lump-sum taxes since the tax base is narrower and their consumption declines by more. The second effect is more pronounced and the consumption Gini increases by more than in the baseline scenario.

Effective lower bound on the nominal interest rate. When comparing impulse response functions in our model with Japanese data, we have maintained the assumption that unconventional policies are equally effective in stabilizing the economy at one-year horizon or longer when the nominal interest rate is at its effective lower bound. One way to investigate the robustness of our conclusions to this assumption is to consider a monetary policy shock in the model under the assumption of a nominal interest rate peg by setting $\phi_\pi = 0$. Using the information in Table 10 in Appendix I and Figure 4, we can see that the fit of the model improves using the SVAR empirical shock responses as a metric. In the model aggregate consumption declines by twice as much on impact, the price of equity declines by more and the inflation rate falls by less than the baseline. All of these differences improve the model's empirical fit.

Easier monetary policy. Given the large variation of households' exposures to a monetary policy shock, it is interesting to ascertain whether there are important asymmetries in the impacts of tighter as compared to a looser monetary policy shock. An easier monetary policy shock of the same magnitude produces small but meaningful differences in the absolute magnitudes of the aggregate responses (Table 10 in Appendix I). The absolute magnitudes of the responses of output, hours, and the inflation rate are larger, but the responses of consumption and investment are smaller compared to the baseline.

Higher government debt. Given the important role that asset income plays in determining a household's overall exposure to monetary policy, it is interesting to understand how the results change when we posit a higher steady-state debt-output level in the model. Increasing the debt-output level from its baseline level of 1.23 to 1.5 has a large impact on the model's steady-state. The ratio of private illiquid assets to output falls from 3.5 to 3.3 and private borrowing is also crowded out by higher government borrowing. These declines are associated with higher returns on both liquid and illiquid assets.

Households close to the age of 68 have a higher share of liquid assets in their portfolio and are consequently better hedged against a tightening in monetary policy. However, younger workers and older retirees are more exposed to shock. Net worth and consumption of the first group increases by more and net worth and consumption of the other two groups fall by more in the impact period relative to the baseline (Table 10 in Appendix I) and a tightening in monetary policy increases the consumption Gini by more (Table 11 in comparison to the baseline scenario.

8 Conclusion

Our aim here has been to propose a quantitative model of money with a detailed lifecycle micro-structure. We have seen that modeling lifecycle consumption and asset allocation decisions provides new insights into the transmission channels of monetary policy and that monetary policy induces persistent redistribution across the age distribution. In our current research, we are analyzing how the feedback rules of the central bank and fiscal authority influence aggregate economic activity during a demographic transition to an older age distribution. Demographic change is a gradual, but persistent impulse and our preliminary results suggest that the reaction of government policy has large and persistent effects on macroeconomic activity during the demographic transition.

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Appendixes (For online publication)

A Model

A.1 Household problem

Consider the household problem (8). Let $\lambda_{j,t}(z)$ denote a Lagrange multiplier on the budget constraint (4) for $z = 1$ or (5) for $z = 0$. The first-order conditions with respect to $c_{j,t}$, $a_{j,t}$, and $d_{j,t}$ are given respectively by

$$\lambda_{j,t}(z) = \frac{1}{1+\tau_t^c}\left(\frac{c_{j,t}(z)}{\eta_j}\right)^{-\sigma}, \tag{26}$$

$$\begin{aligned}\lambda_{j,t}(1)\left(1+\gamma_a(1)\Delta a_{j,t}\right) \\ = \beta\Bigg[(1-\psi_{j+1,t+1})\frac{\partial U_{j+1}(a_{j,t},d_{j,t},0)}{\partial a_{j,t}} + \psi_{j+1,t+1}\frac{\partial U_{j+1}(a_{j,t},d_{j,t},1)}{\partial a_{j,t}}\Bigg],\end{aligned} \tag{27}$$

$$\lambda_{j,t}(1) = \beta\Bigg[(1-\psi_{j+1,t+1})\frac{\partial U_{j+1}(a_{j,t},d_{j,t},0)}{\partial d_{j,t}} + \psi_{j+1,t+1}\frac{\partial U_{j+1}(a_{j,t},d_{j,t},1)}{\partial d_{j,t}}\Bigg], \tag{28}$$

where $\Delta a_{j,t} = a_{j,t}(1) - a_{j-1,t-1}$. For $z = 0$, conditions (27) and (28) are replaced by $a_{j,t} = 0$ and $d_{j,t} = 0$. The envelope conditions imply

$$\frac{\partial U_{j+1}(a_{j,t},d_{j,t},1)}{\partial a_{j,t}} = \lambda_{j+1,t+1}(1)\left(\tilde{R}^a_{t+1} + \gamma_a(1)\Delta a_{j+1,t+1}\right), \tag{29}$$

$$\frac{\partial U_{j+1}(a_{j,t},d_{j,t},0)}{\partial a_{j,t}} = \lambda_{j+1,t+1}(0)\left(\tilde{R}^a_{t+1} - \gamma_a(0)a_{j,t}\right), \tag{30}$$

$$\frac{\partial U_{j+1}(a_{j,t},d_{j,t},z_{j+1})}{\partial d_{j,t}} = \lambda_{j+1,t+1}(z_{j+1})\frac{\tilde{R}_t}{\pi_{t+1}}, \quad z_{j+1}\in\{0,1\} \tag{31}$$

Arranging conditions (26)-(31) yields

$$\begin{aligned}\left(1+\gamma_a(1)\Delta a_{j,t}\right)\left(\frac{c_{j,t}(1)}{\eta_{j,t}}\right)^{-\sigma} = \beta\Bigg[(1-\psi_{j+1,t+1})\left(\frac{c_{j+1,t+1}(0)}{\eta_{j+1}}\right)^{-\sigma}\left(\tilde{R}^a_{t+1} - \gamma_a(0)a_{j,t}\right) \\ + \psi_{j+1,t+1}\left(\frac{c_{j+1,t+1}(1)}{\eta_{j+1}}\right)^{-\sigma}\left(\tilde{R}^a_{t+1} + \gamma_a(1)\Delta a_{j+1,t+1}\right)\Bigg],\end{aligned} \tag{32}$$

$$\left(\frac{c_{j,t}(1)}{\eta_{j,t}}\right)^{-\sigma} = \beta\Bigg[(1-\psi_{j+1,t+1})\left(\frac{c_{j+1,t+1}(0)}{\eta_{j+1}}\right)^{-\sigma} + \psi_{j+1,t+1}\left(\frac{c_{j+1,t+1}(1)}{\eta_{j+1}}\right)^{-\sigma}\Bigg]\frac{\tilde{R}_t}{\pi_{t+1}}. \tag{33}$$

In the state $z = 0$, the household is in the final period of life and consumes all of its wealth

$$c_{j,t}(0) = \frac{\tilde{R}^a_t a_{j-1,t-1} - \chi(0,a_{j-1,t-1},0) + \frac{\tilde{R}_{t-1}}{\pi_t}d_{j-1,t-1} + (1-\tau^w)w_t\epsilon_j h_{j,t} + b_{j,t} + \xi_t}{1+\tau_t^c}. \tag{34}$$

A.2 Liquidity premium and asset portfolios

Suppose that adjustment costs on illiquid assets are the same for surviving and non-surviving households: $\gamma_a(1) = \gamma_a(0) \equiv \gamma_a$. In addition, consider a stationary equilibrium so that time subscripts are dropped. Then, combining equations (32) and (33) yields

$$\Delta a_{j+1} = \frac{\tilde{R}}{\pi \kappa_{j+1}} \Delta a_j - \frac{1}{\gamma_a \kappa_{j+1}} \left(\tilde{R}^a - \frac{\tilde{R}}{\pi} \right) + \frac{1 - \kappa_{j+1}}{\kappa_{j+1}} a_j, \tag{35}$$

where $\Delta a_j \equiv a_j(1) - a_{j-1}$ and $0 < \kappa_{j+1} < 1$, given by

$$\kappa_{j+1} = \frac{\psi_{j+1} \left(c_{j+1}(1)/\eta_{j+1} \right)^{-\sigma}}{\left(1 - \psi_{j+1}\right) \left(c_{j+1}(0)/\eta_{j+1} \right)^{-\sigma} + \psi_{j+1} \left(c_{j+1}(1)/\eta_{j+1} \right)^{-\sigma}}.$$

From equation (35) we can establish two results. In doing so, we guess and verify that the initial real asset holding is positive $a_1 > 0$. Since $a_0 = 0$, an increase in the real asset in the initial age is also positive: $\Delta a_1 > 0$. The first result is that the interest rate spread between illiquid assets and liquid assets has to be positive

$$\tilde{R}^a - \frac{\tilde{R}}{\pi} > 0.$$

To show this, suppose contrarily that the interest rate spread is non-positive: $\tilde{R}^a - \tilde{R}/\pi \leq 0$. Then, because $\Delta a_1 > 0$, equation (35) implies $\Delta a_j > 0$ for all $j = 2, ..., J$, so that $a_J > 0$, violating the terminal condition. Hence, the interest rate spread has to be positive in equilibrium. Second, because of the positive interest rate spread, individuals will prefer to borrow at the same interest rate as the liquid assets. The second result implies $a_j \geq 0$ for all $j = 1, ..., J$. Hence, the household may leverage illiquid assets by borrowing liquid assets: $a_j > 0$ and $d_j < 0$. Since the problem of the illiquid asset choice is smooth in the initial age, illiquid asset holdings are positive in the initial age $a_1 > 0$, verifying the guess assumed in this discussion.

A.3 Labor supply decision

Working households belong to a labor union and work for an identical amount of hours $h_{j,t} = \bar{h}_t$ for all $j = 1, ..., J_r - 1$. The labor union consists of a continuum of union groups $l \in (0, 1)$ and distributes total hours worked $\sum_{j=1}^{J_r-1} \bar{h}_t N_{j,t}$ among union groups. Each union group l has a one-to-one linear technology that transforms hours per worker into specific labor supply per worker $\tilde{h}_t(l)$. An employment agency combines a continuum of specific labor and produces homogeneous labor $\tilde{H}_t = \tilde{h}_t \sum_{j=1}^{J_r-1} N_{j,t}$ following the technology: $\tilde{h}_t =$

$\left[\int_0^1 \tilde{h}_t(l)^{\frac{1}{\theta_w}} dl\right]^{\theta_w}$, where $\theta_w > 1$ is a wage markup. The employment agency is competitive and it follows that demand for specific labor l is given by

$$\tilde{h}_t(l) = \left(\frac{W_t(l)}{W_t}\right)^{-\frac{\theta_w}{\theta_w - 1}} \tilde{h}_t, \tag{36}$$

where $W_t(l)$ is nominal wage for specific labor supply $\tilde{h}_t(l)$.

Each union group chooses $W_t(l)$ to maximize the benefits of the members of the labor union, i.e. working households. Then, the problem of the union group is given by

$$\max_{\{W_t(l)\}} \; (1 - \tau^w) \sum_{j=1}^{J_r - 1} \frac{W_t(l)}{P_t} \epsilon_j \tilde{h}_t(l) \mu_{j,t}^w - \sum_{j=1}^{J_r - 1} \left(\frac{\psi_{j,t}}{\lambda_{j,t}(1)} + \frac{1 - \psi_{j,t}}{\lambda_{j,t}(0)}\right) \frac{\upsilon}{1 + \frac{1}{\nu}} \tilde{h}_t(l)^{1 + \frac{1}{\nu}} \mu_{j,t}^w,$$

subject to the labor demand curve (36), where $\lambda_{j,t}(z)$ with $z \in \{0, 1\}$ is a Lagrange multiplier, given by equation (26), and $\mu_{j,t}^w = N_{j,t} / \sum_{j=1}^{J_r - 1} N_{j,t}$ is the ratio of population with age j to the working population. The second term of the problem is the weighted average of disutility of supplying labor, which is transformed into the units of the final good, over working households. This statement of the problem weights the disutilities of both surviving households and also households who experience a death shock. Our baseline specification, however, assumes that only surviving households receive weight and $\psi_{j,t}$ is set to 1 for all j in the problem. The first-order condition of this problem is equation (7) with $\bar{\lambda}_t$ and $\bar{\epsilon}_t$ are given, respectively, by[34]

$$\bar{\lambda}_t = \left[\sum_{j=1}^{J_r - 1} \left(\frac{\psi_{j,t}}{\lambda_{j,t}(1)} + \frac{1 - \psi_{j,t}}{\lambda_{j,t}(0)}\right) \mu_{j,t}^w\right]^{-1}$$

$$\bar{\epsilon}_t = \sum_{j=1}^{J_r - 1} \epsilon_j \mu_{j,t}^w.$$

A.4 Intermediate goods firms

The i-th intermediate goods firm's problem can be solved in two steps. First, it minimizes the real costs of production, $w_t H_t(i) + r_t^k K_t(i)$, subject to equation (10), where w_t is the real wage per effective unit of labor and r_t^k is the rental rate of capital. Let $K_t \equiv \int_{i \in (0,1)} K_t(i) di$ and $H_t = \int_{i \in (0,1)} H_t(i) di$ denote aggregate capital and aggregate labor in effective units,

[34] Equation (7) abstracts from a markup because it is not identified under our additive preference structure.

respectively. Then the cost minimizing input demands can be expressed as

$$w_t = mc_t(1-\alpha)\left(\frac{K_t(i)}{H_t(i)}\right)^{\alpha} = mc_t(1-\alpha)\left(\frac{K_t}{H_t}\right)^{\alpha}, \tag{37}$$

$$r_t^k = mc_t\alpha\left(\frac{K_t(i)}{H_t(i)}\right)^{\alpha-1} = mc_t\alpha\left(\frac{K_t}{H_t}\right)^{\alpha-1}, \tag{38}$$

where mc_t is real marginal cost and the second equality follows from the linear-homogeneity of the production function.

Second, the firm chooses $P_t(i)$ to maximize the present value of profits, $\Pi_t(i)+\Lambda_{t,t+1}\Pi_{t+1}(i)+$ ..., subject to equation (9), where the discount factor, $\Lambda_{t,t+1}$, is derived from preferences in the next subsection, and where the period-t profits, $\Omega_t(i)$, are given by

$$\Omega_t(i) = (1+\tau^f)\frac{P_t(i)}{P_t}Y_t(i) - mc_tY_t(i) - \frac{\gamma}{2}\left(\frac{P_t(i)}{P_{t-1}(i)}-1\right)^2 Y_t, \quad \gamma > 0. \tag{39}$$

In equation (39) τ^f is a subsidy and the quadratic term is the price adjustment cost. We assume that the subsidy is set at $\tau^f = \theta - 1$, so that marginal cost is one in the steady state. The optimality condition for the firm's price setting problem is:

$$\gamma\left(\frac{P_t(i)}{P_{t-1}(i)}-1\right)\frac{Y_t}{P_{t-1}(i)} = \frac{\theta}{\theta-1}\frac{mc_tY_t}{P_t} - \frac{1+\tau^f}{\theta-1}\frac{Y_t}{P_t} + \Lambda_{t,t+1}\gamma\left(\frac{P_{t+1}(i)}{P_t(i)}-1\right)\frac{P_{t+1}(i)}{P_t(i)^2}Y_{t+1}.$$

In a symmetric equilibrium the previous equation simplifies to equation (11), which is a nonlinear version of the NK Phillips curve.

B Competitive equilibrium

B.1 Derivation of the final goods market equilibrium condition

To derive the final goods market clearing condition, observe that the household budget constraints given by equations (4)–(5) hold with equality in equilibrium and can be summed over j, to obtain

$$\begin{aligned}
&(1+\tau_t^c)C_t + A_t + X_t + D_t \\
=&\tilde{R}_t^a\sum_{j=1}^{J} a_{j-1,t-1}(1)N_{j,t} + \frac{\tilde{R}_{t-1}}{\pi_t}\sum_{j=1}^{J} d_{j-1,t-1}(1)N_{j,t} + (1-\tau^w)w_tH_t + B_t + \Xi_t, \\
=&\tilde{R}_t^a A_{t-1} + \frac{\tilde{R}_{t-1}}{\pi_t}D_{t-1} + (1-\tau^w)w_tH_t + B_t + \Xi_t,
\end{aligned}$$

where $H_t = \sum_{j=1}^{J} \epsilon_j \bar{h}_t N_{j,t}$ and

$$X_t = \sum_{j=1}^{J} \left[\psi_{j,t}\chi\left(a_{j,t}(1), a_{j-1,t-1}(1), 1\right) + (1-\psi_{j,t})\chi\left(0, a_{j-1,t-1}(1), 0\right)\right] N_{j,t}$$

with $a_{0,t-1} = 0$. Substituting the government budget constraint, equation (21), into the previous equation and using the formulas for the after-tax interest rates we obtain

$$\begin{aligned}
&(1+\tau_t^c)C_t + A_t + X_t \\
=&[1+(1-\tau^{ka})(R_t^k - 1)]A_{t-1} - \tau^a \frac{R_t}{\pi_t} D_{t-1} + (1-\tau^w) w_t H_t + T_t - G_t - \tau^f Y_t.
\end{aligned}$$

Further substituting the tax equation (20) for T_t into this condition yields

$$C_t + A_t + X_t = R_t^k A_{t-1} + w_t H_t - G_t - \tau^f Y_t.$$

Recall that equation (17) implies $R_t^k = (\Omega_t + V_t)/V_{t-1}$. Thus, income from illiquid assets can be expressed as

$$\begin{aligned}
R_t^k A_{t-1} =& R_t^k (K_t + V_{t-1}), \\
=& [r_t^k + 1 - \delta] K_t + \Omega_t + V_t \\
=& r_t^k K_t + K_{t+1} - I_t + \Omega_t + V_t.
\end{aligned}$$

Substituting (37), (38), (13), and (24) for w_t, r_t^k, Ω_t, and A_t, respectively, into the budget constraint yields[35]

$$C_t + I_t + G_t = Y_t - \frac{\gamma}{2}\left(\pi_t - 1\right)^2 Y_t - X_t, \tag{40}$$

B.2 Stationary equilibrium

In the economy with non-zero population growth, the aggregate variables such as Y_t grow at the rate of the population growth of $n_t = N_t/N_{t-1}$ in a stationary equilibrium. We scale aggregate variables in capital letters by using population and denote corresponding

[35] Substituting equation (24) into the aggregate household budget constraint yields

$$C_t + I_t + G_t = r_t^k K_t + w_t H_t + \Omega_t - \tau^f Y_t - X_t.$$

Substituting (13) into this equation

$$C_t + I_t + G_t = r_t^k K_t + w_t H_t + \left[\theta - mc_t - \frac{\gamma}{2}\left(\pi_t - 1\right)^2\right] Y_t - \tau^f Y_t - X_t.$$

Note that equations (38) and (37) imply $r_t^k K_t + w_t H_t = mc_t Y_t$. And note that $\tau^f = \theta - 1$. Substituting these into the above equation yields equation (40).

per-capita variables as their small letters, e.g., $y_t = Y_t/N_t$.

In a stationary equilibrium the inflation rate is at its target value of $\pi = 1$. This implies that the marginal cost is unity, $mc = 1$, from equation (11), and the price adjustment cost is zero. Instead of pinning down per-capita hours worked $h = H/N$, we normalize it to be unity in a stationary equilibrium and set the coefficient of disutility of labor, υ, to satisfy $h = 1$. Note that hours in efficiency units per total population and hours per working population are related as follows: $h = \sum_{j=1}^{J_r-1} \epsilon_j \bar{h} \mu_j$. Under the normalization of $h = 1$, hours per working population is given by

$$\bar{h} = \frac{1}{\sum_{j=1}^{J_r-1} \epsilon_j \mu_j}.$$

Only when $\sum_{j=1}^{J_r-1} \epsilon_j \mu_j = 1$ e.g. for normalization, the two variables coincide: $h = \bar{h} = 1$.

Fix r^k, $\tilde{R}$, and ξ.[36] The return on capital is $R^k = r^k + 1 - \delta$. After the deduction of the capital income tax, the return on illiquid assets is given by $R^a = 1 + (1 - \tau^k)(R^k - 1)$. The interest rate income tax τ^a is also imposed. So, the after-tax return is $\tilde{R}^a = 1 + (1 - \tau^a)(R^a - 1)$. The capital-labor ratio is given by $K/H = (r^k/\alpha)^{-1/(1-\alpha)}$ from equation (38). The output is given by $y = (K/H)^\alpha h$ with $h = 1$ by normalization. The real wage is given by $w = (1 - \alpha)(K/H)^\alpha$ from equation (37). The pension benefit b is given by

$$b = \lambda \frac{1}{j_r - 1} \sum_{j=1}^{j_r-1} w \epsilon_j h_j,$$

where $h_j = \bar{h}$. The liquid asset holding is $d = (D/Y)y$, where D/Y is a targeted net government debt output ratio. The government spending is $g = (G/Y)y$, where G/Y is a targeted government expenditure output ratio.

Consider a backward shooting algorithm for $c_J(0)$ and $a_{J-1}(1)$. Fix $c_J(0) > 0$ and $a_{J-1}(1) \geq 0$. Consumption $c_{J-1}(1)$ is given by equation (33) with $\psi_J = 0$ as

$$c_{J-1}(1) = \left(\frac{\beta \tilde{R}}{\pi} \right)^{-\frac{1}{\sigma}} \frac{\eta_{J-1}}{\eta_J} c_J(0).$$

From equation (32) with $\psi_J = 0$, we obtain $a_{J-2}(1)$ as

$$a_{J-2}(1) = a_{J-1}(1) - \frac{1}{\gamma_a(1)} \left[\beta \left(\frac{c_J(0)}{c_{J-1}(1)} \frac{\eta_{J-1}}{\eta_J} \right)^{-\sigma} \left(\tilde{R}^a - \gamma_a(0) a_{J-1}(1) \right) - 1 \right].$$

[36] Practically the initial values for r^k and ξ are set at the corresponding steady-state values with no cost of adjusting illiquid assets.

The liquid asset holding $d_{J-1}(1)$ is given by equation (34) as

$$d_{j-1}(1) = \frac{1}{\tilde{R}} \left\{ c_j(0) + \chi(0, a_{j-1}(1), 0) - \tilde{R}^a a_{j-1}(1) - \left[(1-\tau^w) w \epsilon_j \bar{h} + b + \xi\right] \right\},$$

for $j = J$. From the budget constraint (4), the liquid asset holding $d_{J-2}(1)$ is given as

$$d_{j-1}(1) = \frac{1}{\tilde{R}} \Big\{ c_j(1) + a_j(1) + \chi(0, a_j(1), a_{j-1}(1)) + d_j(1) \\ - \tilde{R}^a a_{j-1}(1) - \left[(1-\tau^w) w \epsilon_j \bar{h} + b + \xi\right] \Big\}, \quad (41)$$

for $j = J-1$. Then, $c_{J-1}(0)$ is given by equation (34). With $c_{J-1}(0)$ and $c_{J-1}(1)$ on hand, $c_{J-2}(1)$ is given by equation (33) as

$$c_j(1) = \left(\frac{1}{\beta \tilde{R}}\right)^{\frac{1}{\sigma}} \left(\frac{\eta_j}{\eta_{j+1}}\right) \left[(1-\psi_{j+1}) c_{j+1}(0)^{-\sigma} + \psi_{j+1} c_{j+1}(1)^{-\sigma}\right]^{-\frac{1}{\sigma}}, \quad (42)$$

for $j = J-2$. Then, Δa_j is given by equation (32) and the real asset holding in the previous period is given by $a_{j-1}(1) = a_j(1) - \Delta a_j$ for $j = J-2$. Then, the liquid asset holding in the previous period is given by (41), the consumption $c_{j-1}(0)$ is given by equation (34), and the consumption $c_{j-1}(1)$ is given by equation (42). Continuing this process for $j = J-3, ..., 1$ yields the initial asset holdings of $a_{-1}(1)$ and $d_{-1}(1)$. Consumption $c_J(0)$ and the real asset holding $a_{J-1}(1)$ are adjusted so as to satisfy $d_{-1}(1) = 0$ and $a_{-1}(1) = 0$.

The loop for r^k, $\tilde{R}$, and ξ is closed as follows. Let τ_t denote per capita tax revenue: $\tau_t = T_t/N_t$. From equation (21), the updated value of the lump-sum transfer ξ' is given by

$$\xi' = \tau - g - \left(\frac{R}{\pi n} - 1\right) d - \tau^f y - b \sum_{j=j_r}^{J} \mu_j.$$

where $R = 1 + (\tilde{R} - 1)/(1 - \tau^a)$, n is the gross growth rate of population, and τ is given by equation (20) as

$$\tau = \tau^c \sum_{j=1}^{J} \bar{c}_j \mu_j + \tau^{ka}(r^k - \delta) \sum_{j=1}^{J} a_{j-1}(1) \mu_j + \tau^a \frac{R-1}{\pi} \sum_{j=1}^{J} d_{j-1}(1) \mu_j + \tau^w w \sum_{j=1}^{J} \epsilon_j \bar{h} \mu_j, \quad (43)$$

where $\bar{c}_j = \psi_j c_j(1) + (1 - \psi_j) c_j(0)$. From equation (22),

$$d_t = \sum_{j=1}^{J} d_{j,t}(1) \mu_{j+1,t+1} n_{t+1},$$

with $d_{J,t}(1) = 0$, so that in evaluating τ, $\sum_{j=1}^{J} d_{j-1}(1)\mu_j$ in equation (43) should be set at

$$\sum_{j=1}^{J} d_{j-1}(1)\mu_j = d/n.$$

The capital stock is given by $k = (a - v)/n$ from equation (24), where the illiquid asset holding is given by $a \equiv \sum_{j=1}^{J} a_{j-1}(1)\mu_j/n$ and the value of firms v is given by combining equation (18) and (13) as

$$v = \frac{(\theta - 1)y}{R^k/n - 1}.$$

The updated value of r^k is given by

$$r^{k\prime} = \alpha k^{\alpha-1} h^{1-\alpha},$$

with $h = 1$. The value of the liquid asset holding, implied from the household optimization problem is $d' = \sum_{j=1}^{J} d_{j-1}(1)\mu_j/n$. The corresponding value that is consistent with the target value of debt output ratio is d. Since households are willing to hold liquid assets more as the interest rate increases, adjust the updated value of $\tilde{R}'$ as follows: increase (decrease) $\tilde{R}'$ if $d' < d$ ($d' > d$). In doing so, make sure that there is a positive spread between $\tilde{R}^a$ and $\tilde{R}/\pi$: $\tilde{R}^a - \tilde{R}/\pi > 0$.

Guessed values for r^k, $\tilde{R}$, and ξ are adjusted until $|r^k - r^{k\prime}| + |d - d'| + |\xi - \xi'|$ becomes close enough to zero. This completes the description of the computation of the stationary equilibrium.

B.3 Dynamic equilibrium

Consider the computation of a transition path from period t_s to period t_e. Without loss of generality we assume that the economy reaches a stationary equilibrium in period t_e. In what follows we will assume that the initial condition is also a steady state, that is, that the economy is in steady state in period $t_s - 1$. In order to induce a transition we assume that in period t_s an MIT shock arrives. The MIT shock could consist of a perturbation to the central bank's nominal interest rate targeting rule, and/or some element of fiscal policy.

In the main text we have assumed that the population distribution is stationary with zero population growth in the dynamic equilibrium. However, here we relax that assumption and allow for the possibility of time-varying population during the transition. In particular, the size of the population in period t follows $N_t = n_t N_{t-1}$ where n_t is the population growth rate and survival probabilities ψ_t can depend on t during the transition. In what follows our use of the term steady state when referring to the initial or terminal condition consists of a steady-state population distribution with a constant population as well as the steady-state price system and allocations that we defined in the previous section.

The aggregate state variables in period $t = t_s$ consist of the per-capita capital stock k_t, per-capita value of equity in intermediate goods firms v_{t-1}, per capita government debt d_{t-1}, and the nominal interest rate R_{t-1}. The initial conditions for age-specific variables are liquid asset holdings $\{d_{j,t-1}\}_{j=1}^J$ and illiquid asset holdings $\{a_{j,t-1}\}_{j=1}^J$. Recall that illiquid assets consist of the capital stock and equity in intermediate goods firms and that each age-group holds the market portfolio of the two underlying assets. The market portfolio shares of capital stock and equity are k_{t_s}/a_{t_s-1} and v_{t_s-1}/a_{t_s-1}, respectively, where $a_{t_s-1} = k_{t_s} + v_{t_s-1}$.

We solve this two point boundary problem by guessing and verifying a path for $\{r_t^k, \pi_t, \xi_t, \gamma_{y,t+1}, \gamma_{k,t+1}\}_{t=t_s}^{t_e}$ and $R_{t_s}^a$, where $\gamma_{y,t+1} = y_{t+1}/y_t$, $\gamma_{k,t+1} = k_{t+1}/k_t$, and $R_{t_s}^a$ is the ex post return on the market portfolio of illiquid assets in the initial period.

Since the initial condition is a steady state, a sequence of capital is given by $k_{t+1} = \gamma_{k,t+1} k_t$ with $k_{t_s} = k$. From the law of motion for capital (14), the newly produced investment good in per capita is given by

$$i_t = n_{t+1} k_{t+1} - (1-\delta) k_t.$$

where $i_t \equiv I_t/N_t$. The real return on illiquid assets is given by the arbitrage condition (17) as $R_{t+1}^a = 1 + (1-\tau^k)(R_{t+1}^k - 1)$, where $R_{t+1}^k = r_{t+1}^k + 1 - \delta$. Its after-tax return is given as $\tilde{R}_s^a = 1 + (1-\tau^a)(R_s^a - 1)$ for $s = t_s+1, t_s+2, \ldots$. For $s = t_s$, the guessed value of $R_{t_s}^a$ is used to compute its after-tax return. Compute the nominal interest rate R_t using the monetary policy rule (19) for $t = t_s, \ldots, t_e$. Compute the marginal cost mc_t using the Phillips curve (11) as

$$mc_t = 1 + \frac{\gamma(\theta-1)}{\theta}\left[(\pi_t - 1)\pi_t - \frac{\gamma_{Y,t+1} n_{t+1}}{R_{t+1}^k}(\pi_{t+1} - 1)\pi_{t+1}\right].$$

for $t = t_s, \ldots, t_e$. Compute k_t/h_t from equation (38) and compute w_t from equation (37). Since k_t is known, hours per capita in efficiency units h_t can be computed from the ratio k_t/h_t. Hours per working population is given by $\bar{h}_t = h_t / \sum_{j=1}^{J_r-1} \epsilon_j \mu_{j,t}$.

Given these prices and aggregate variables, solve the household problem for those with age $j = 2, \ldots, J$ in period $t = t_s$ and those who are born in period $t = t_s, \ldots, t_e$. The solution yields $\{a_{j,t}(z), d_{j,t}(z), c_{j,t}(z)\}_{j=1}^J$ for $t = t_s, \ldots, t_e$ and for $z \in \{0, 1\}$. From these individual decisions, the illiquid asset holding, the liquid asset holding, and consumption in per capita terms are given, respectively, by $a_t = \sum_{j=1}^{J}[\psi_{j,t}a_{j,t}(1) + (1-\psi_{j,t})a_{j,t}(0)]\mu_{j,t}$, $d_t = \sum_{j=1}^{J}[\psi_{j,t}d_{j,t}(1) + (1-\psi_{j,t})d_{j,t}(0)]\mu_{j,t}$, and $c_t = \sum_{j=1}^{J}[\psi_{j,t}c_{j,t}(1) + (1-\psi_{j,t})c_{j,t}(0)]\mu_{j,t}$, where $\mu_{j,t} = N_{j,t}/N_t$ is the share of population with age j.

Now we are in a position to aggregate the economy and derive conditions to confirm whether the initially guessed values for $R^a_{t_s}$ and $\{r^k_t, \pi_t, \xi_t, \gamma_{y,t+1}, \gamma_{k,t+1}\}_{t=t_s}^{t_e}$ are in an equilibrium. By using the endogenously computed d_t and the exogenously given $d^n_t = D^n_t/N_t$, the price level P_t can be computed from the liquid asset market clearing condition (22) for all t. This yields the updated sequence of the inflation rate, $\pi'_t = P_t/P_{t-1}$ for all t, where P_{t_s-1} is in the initial steady state. From equation (22), using $d_{0,t-1}(1) = d_{J,t-1}(1) = 0$, the liquid asset market clearing condition is written as

$$\frac{d^n_{t-1}}{P_{t-1}} = \sum_{j=1}^{J} d_{j-1,t-1}(1)\mu_{j,t}n_t.$$

Then, the liquid asset term in the tax equation (20) can be written as

$$\begin{aligned}\tau^a \frac{R_{t-1}-1}{\pi_t}\sum_{j=1}^{J} d_{j-1,t-1}(1)N_{j,t} =& \tau^a \frac{R_{t-1}-1}{P_t/P_{t-1}}\sum_{j=1}^{J} d_{j-1,t-1}(1)\mu_{j,t}N_t \\ =& \tau^a \frac{R_{t-1}-1}{P_t}\frac{d^n_{t-1}}{n_t}N_t.\end{aligned}$$

From equations (20) and (21), the updated value of the lump-sum transfer ξ'_t is given by: for $t = t_s+1, ..., t_e$

$$\begin{aligned}\xi'_t =& \tau^c_t \sum_{j=1}^{J} \bar{c}_{j,t}\mu_{j,t} + \tau^{ka}(R^k_t - 1)\sum_{j=1}^{J} a_{j-1,t-1}(1)\mu_{j,t} + \tau^a \frac{(R_{t-1}-1)d^n_{t-1}/n_t}{P_t} \\ &+ \tau^w w_t \sum_{j=1}^{J} \epsilon_j \bar{h}_t \mu_{j,t} - g_t - \frac{R_{t-1}d^n_{t-1}/n_t - d^n_t}{P_t} - \tau^f y_t - \sum_{j=j_r}^{J} b_{j,t}\mu_{j,t}. \end{aligned} \tag{44}$$

where $g_t = G_t/N_t$ and $\tau^{ka} = \tau^k + \tau^a - \tau^k\tau^a$. In period $t = t_s$ when the monetary policy shock hits, the returns earned by investing in capital and ownership shares can be different so that the lump-sum transfer is given by:

$$\xi'_{t_s} = \sum_{j=1}^{J} \left[\tau_t^c \bar{c}_{j,t} + \frac{\tau^{ka}}{1-\tau^k}(R^a_{t_s} - 1)a_{j-1,t_s-1} + \tau^w w_{t_s}\epsilon_j \bar{h}_{t_s} \right] \mu_{j,t_s} + \tau^a \frac{(R_{t_s-1}-1)d^n_{t_s-1}/n_{t_s}}{P_{t_s}}$$
$$- g_{t_s} - \frac{R_{t_s-1}d^n_{t_s-1}/n_{t_s} - d^n_{t_s}}{P_{t_s}} - \tau^f y_{t_s} - \sum_{j=j_r}^{J} b_{j,t_s}\mu_{j,t_s}.$$

From equation (24), the updated value of the capital stock is given by

$$k'_{t+1} = (a_t - v_t)/n_{t+1},$$

where $v_t \equiv V_t/N_t$ is the per-capita value of the sum of the intermediate goods firms and investment good firms. Its aggregate value V_t is given by (18) with Ω_t given by (13). Thus, v_t can be computed forward using

$$v_t = \frac{\left[\theta - mc_{t+1} - \frac{\gamma}{2}(\pi_{t+1}-1)^2\right] y_{t+1}}{R^k_{t+1}/n_{t+1}} + \frac{v_{t+1}}{R^k_{t+1}/n_{t+1}},$$

With a sequence of k'_{t+1} in hand, the updated gross growth rate of capital is simply computed as $\gamma'_{k,t+1} = k'_{t+1}/k'_t$. From the wage Phillips curve (7), the updated value of the hours worked $\bar{h}'_t$ can be computed. The updated value for the net return on capital $r_t^{k\prime}$ is given by equation (38). The updated output is given by

$$y'_t = (k'_t)^\alpha \left(\sum_{j=1}^{J} \epsilon_j \bar{h}'_t \mu_{j,t} \right)^{1-\alpha},$$

for all $t = t_s, ..., t_e$. Then, the updated value of $\gamma_{y,t+1}$ is given by $\gamma'_{y,t+1} = y'_{t+1}/y'_t$. The updated value of $R^a_{t_s}$ is the weighted average of the returns on capital and equity

$$R^{a\prime}_{t_s} = 1 + (1-\tau^k)\Bigg\{ \frac{k_{t_s}n_{t_s}}{k_{t_s}n_{t_s} + v_{t_s-1}}(r^k_{t_s} + 1 - \delta)$$
$$+ \frac{v_{t_s-1}}{k_{t_s}n_{t_s} + v_{t_s-1}} \frac{\left(v_{t_s} + \left[\theta - mc_{t_s} - \frac{\gamma}{2}(\pi_{t_s}-1)^2\right] y_{t_s}\right) n_{t_s}}{v_{t_s-1}} - 1 \Bigg\},$$

where $k_{t_s} = k$ and $v_{t_s-1} = v$. Finally, the updated value of pension benefits is computed as:

$$b_{j,t} = \lambda \left(\frac{1}{j_r - 1} \sum_{i=1}^{j_r-1} w_{t-j+i}\epsilon_i \bar{h}_{t-j+i} \right).$$

for $j = j_r, ..., J$ and $t = t_s, t_s+1, ...$

The values of $\{r^k_t, \pi_t, \xi_t, \gamma_{Y,t+1}\}_{t=t_s}^{t_e}$ and $R^a_{t_s}$ are adjusted until $\max_{t\in\{t_s,...,t_e\}}(|r^k_t - r^{k\prime}_t| +$

$|\pi_t - \pi'_t| + |\xi_t - \xi'_t| + |\gamma_{Y,t+1} - \gamma'_{Y,t+1}| + |\gamma_{k,t+1} - \gamma'_{k,t}|) + |R^a_{t_s} - R^{a\prime}_{t_s}|$ becomes close enough to zero. This completes the computation of the transition.

C Data measures of liquid and illiquid assets

We calibrate two model parameters, β, the preference discount factor and d/y the public debt output ratio, to data on aggregate stocks of (net) liquid and (gross) illiquid assets. Here we provide more detail on how we construct the aggregate stocks of liquid and illiquid assets. We have two alternative data sources. First, we can construct them using aggregate data from the Japanese FFA for financial variables and data from the Japanese NIPA accounts for aggregate holdings of physical assets. Secondly, we derive aggregate stocks from the NSFIE, which is nationally representative and conducted once every five years. We describe each of these strategies in turn. Table 8 reports assets and liabilities as a fraction of GDP. The GDP shares in the column with the heading FFA/NIPA are constructed using 2014 fiscal year FFA data for the household sector from the Bank of Japan and using 2014 NIPA accounts for calendar year GDP and holdings stocks of physical assets held by households. We discuss construction of the data in this column first. In Japanese FFA data the household sector consists of households and private unincorporated enterprises. Aggregate liquid assets consist of household holdings of cash, domestic deposits, and public and private debt securities. This amounts to 1.79 times GDP. Liquid liabilities, which consist of consumer credit, are 0.069 of GDP. Net liquid assets relative to GDP are then 1.73 using the classification scheme of Kaplan et al. (2018). As explained in the main text it is more suitable in our model to treat all household borrowing as liquid borrowing. Under this assumption net liquid assets are 1.23.

Illiquid assets have two components: physical and financial assets. Physical assets are the end-of-calendar-year stock of household (and unincorporated private business) non-financial assets taken from the 2014 NIPA and amount to 2.02 times GDP. Financial assets include household holdings of: non-life insurance reserves, life insurance reserves, annuity entitlements, private pensions (defined benefit and defined contribution), and equity and options from the FFA. The resulting magnitude of illiquid financial assets is 3.50 times GDP. Illiquid liabilities consist of net non-financial sector loans, installment credit and non-financial sector loans, mortgages, and other loans by financial institutions plus non-housing loans by public financial institutions. Total illiquid liabilities constructed in this way amount to 0.49 of GDP. Net illiquid assets are then 3.01 using the classification scheme

Table 8: Liquid and illiquid assets relative to GDP in FFA/NIPA and NSFIE

	FFA/NIPA	NSFIE
A. Liquid assets and liabilities		
Liquid assets		
Currency and domestic deposits	1.74	0.99
Debt securities (total public and private)	0.052	0.047
Total liquid assets	1.79	1.04
Liquid liabilities		
Consumer credit	-0.069	-0.025
Total liquid liabilities	-0.069	-0.025
Net liquid assets: Kaplan et al. (2018)	1.73	1.01
Net liquid assets: Our model	1.23	0.85
B. Illiquid assets and liabilities		
Illiquid assets		
Physical assets	2.02	2.17
Equity and options	0.49	0.19
Insurance	0.99	0.30
Non-life insurance reserves	0.09	NA
Life insurance	0.88	0.30
Life insurance reserves	0.39	NA
Annuity entitlements	0.18	NA
Private pensions	0.31	NA
Total illiquid assets	3.50	2.66
Illiquid liabilities		
Installment credit and net non-financial sector loans	-0.00	-0.01
Mortgages	-0.37	-0.34
Other loans fin. inst. and non-home loans public inst.	-0.11	NA
Total illiquid liabilities	-0.49	-0.35
Net illiquid assets: Kaplan et al. (2018)	3.01	2.31
Net illiquid assets: Our model	3.50	2.66
C. Net worth	4.73	3.32

Note: Data are expressed as a multiple of GDP. Results under the heading FFA/NIPA are based on Flow of Funds and NIPA aggregate data. Data under the heading NSFIE are constructed from the National Family Income and Expenditure Survey and the Family Income and Expenditure Survey.

of Kaplan et al. (2018) and 3.50 using our classification scheme. Summing together net liquid and illiquid assets implies that total household net-worth is 4.73 times GDP.

An alternative way to construct these aggregates is to aggregate up micro survey data. Our primary source for the results reported in the final column of Table 8 is the 2014 NSFIE. An attractive feature of this survey is that it reports family income, financial and physical assets, and liabilities by 10 year age group of the household head. This survey is large and nationally representative but is only conducted at 5 year intervals. The most recent survey for which data is publicly available as of 2020 is from the year 2014. This is why we have chosen to use the year 2014 as our reference point.

A much smaller household survey called the Family Income and Expenditure Survey (FIES) is conducted on a quarterly basis. This survey provides more detail on the composition of financial assets and liabilities than the NSFIE but doesn't report results for holdings of physical assets. We use data from the FIES 2019 end of year survey to impute shares of financial liquid assets and liabilities by 10-year age group when these categories are not available in the NSFIE. Namely, we impute liquid and illiquid shares of financial securities and equity using their shares in the 2019 FIES for each 10 year age group. The 2014 NSFIE reports mortgages but does not provide detail on other liabilities. We thus impute non-mortgage illiquid liabilities and liquid liabilities by 10-year age group using their shares in the 2019 FSIE. A final issue is that holdings of cash are not reported in the NSFIE. We impute cash holdings assuming that they are 9% of deposits, which is the ratio of cash to deposits in the aggregate FFA data.

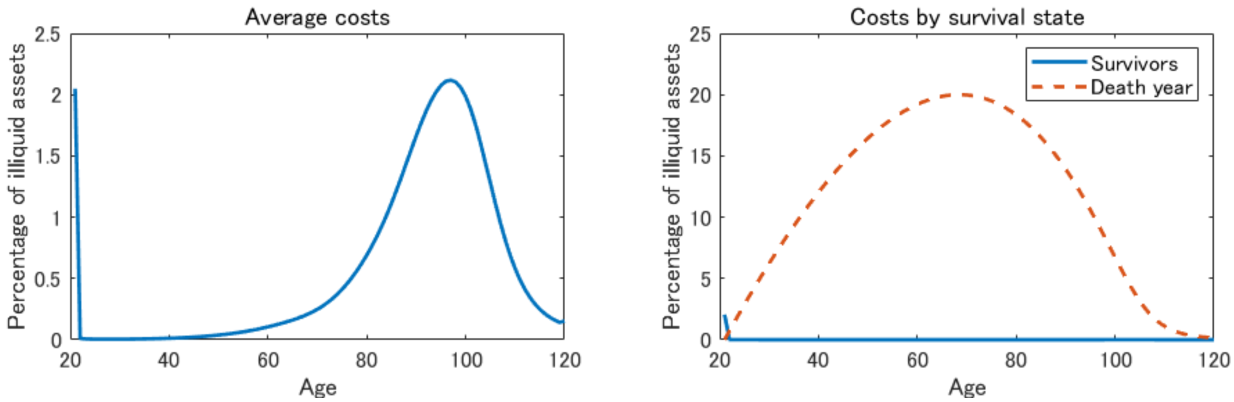
With household level data on liquid and illiquid assets in hand, we then construct aggregate NSFIE assets by multiplying average per-household levels of each variable times the total number of households in Japan in 2014 and then dividing by calendar year nominal GDP for the year 2014. A comparison of the two columns of data in Table 8 reveals that the size of net liquid and illiquid assets is smaller in the NSFIE than in the FFA/NIPA. For instance, liquid assets are 1.79 times GDP in the FFA/NIPA and only 1.04 times GDP in the NFSIE. Deposits in the FFA, in particular, are much higher than in the NSFIE.

Illiquid assets are also smaller in the NSFIE as compared to the FFA. Physical assets are about the same in the two datasets, but, holdings of life insurance and equity are much smaller in the NSFIE. A final and smaller difference is that illiquid liabilities are also smaller in the NSFIE. This second difference appears to be primarily due to non–mortgage lending by financial institutions and non-housing loans by public financial institutions, categories that are not broken out in the NSFIE. These differences translate into smaller average net worth and a larger share of illiquid assets in total (net) assets. Net worth is 3.32 in the NSFIE while it is 4.73 in the FFA data. Using definitions suitable for our model, the share of illiquid assets in total assets is 0.76 for the NSFIE and 0.74 for the FFA.

Takayama and Kitamura (1994) report that the size of average household financial assets is substantially smaller in the NSFIE survey than the FFA data using the NSFIE in earlier years. They suggest three reasons for the gap. First, a disproportionately large number of high wealth households may be refusing to participate in the NSFIE. Second, some self-employed respondents may be confused about what to report and are not reporting assets and liabilities of their businesses. Third, there may be measurement error because the household sector is treated as a residual in the SNA/FFA commodity flow method.

The results in Table 8 are consistent with the claim that the NSFIE may not be ade-

Figure 8: Adjustment costs on illiquid assets relative to total asset holdings by age



quately capturing assets and *liabilities* of proprietorships. Not only are deposits small in the NSFIE, but the NSFIE doesn't appear to be capturing other loans of financial institutions and non-home loans of public institutions. We conjecture that most of these loans are to proprietorships. Finally, our results also suggest that respondents in the NSFIE are also under-stating the value of their private pensions and annuities.

D Costs of adjusting illiquid assets

Here we report the size of the baseline steady-state age profile of adjustment costs and provide some intuition for how they work. Figure 8 reports the age profile of adjustment costs on illiquid assets relative to total assets using the baseline parameterization of the model which assumes that the maximum cost at any age is two percent of assets conditional on surviving and 20 percent of assets conditional on it being the death year. The right panel of the figure reports the adjustment costs by survival state. It has two main properties. Newly formed households of age 21 have the highest adjustment costs conditional on survival. This is because they enter the economy with no assets. Newly retired households (age 68) have the highest adjustment costs if they experience a death event shock at this age because they have the largest holdings of illiquid assets. Put differently, a 68 year old household who discovers that this is the last year of its life and quickly liquidates its large stock of illiquid assets pays a fee of 20 percent of total assets.

The left panel of Figure 8, which shows the age profile of the average costs of adjusting illiquid assets, has two modes. The first mode occurs at age 21 and is a cost of acquiring assets. The adjustment costs then fall sharply and are less than 0.5 percent of total assets until age 76. The second mode in average adjustment costs occurs at age 98 and is 2.3

percent of total assets. Households older than age 76 face an interesting trade off. On the one hand, they are attracted to the higher return offered by illiquid assets. On the other hand, mortality risk is increasing and it is costly for them to have to rapidly liquidate their entire holdings of illiquid assets if they discover that this is their death year.

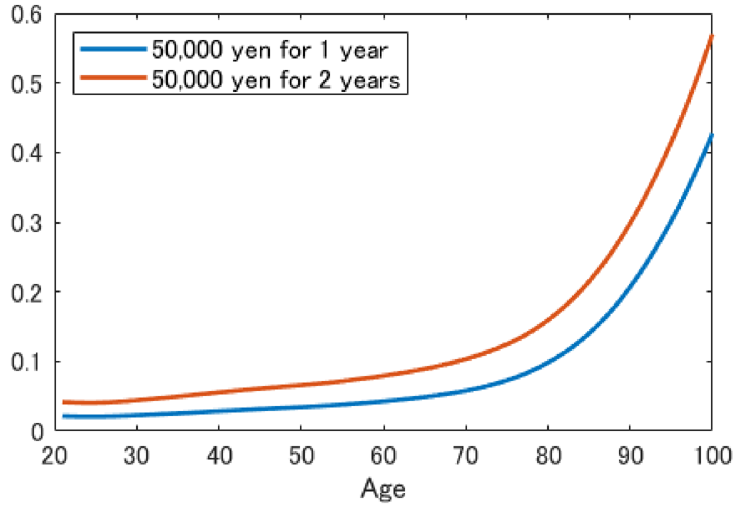
Having described the size of the adjustment costs by age, we are now in a position to discuss what they represent and how we parameterized them. The adjustment cost for survivors captures in a simple way that young households may not be very sophisticated purchasers of a home or car and allocate more resources to acquiring them. The peak cost occurs at age 21 and is two percent of total assets. For purposes of comparison taxes, broker commissions and other fees of purchasing a home in Japan are about 6-7 percent of the purchase price. The second and larger cost is the cost of liquidating illiquid assets in the death year. In the model households of age 68 pay the peak cost of 20 percent of total assets. We believe that this is a simple way to capture the following considerations. Average commissions for mediating residential real estate sales in Japan are 4.1 percent (see Shirakawa and Okoshi (2017)). Construction costs in Japan are commensurate with costs in other advanced economies, but land prices are relatively high. Moreover, residential structure lifespans are as short as 15 years and it is common to demolish the existing structure at the time of sale (see Koo and Sasaki (2008)). These factors are likely to be particularly important for 68 year old households who pass away. On average the age of their home is young, but the resale value of the structure is low or even negative if it has to be demolished. Finally, we don't specifically model bequests but exemptions are low in Japan and bequest taxes are progressive ranging from 10 percent to 55 percent of assets in excess of the exemption.[37] Holdings of real estate and equity are registered and face the full burden of this tax.

E Marginal propensities to consume

Figure 9 reports the marginal propensity to consume for households with ages between 21–100 for two scenarios. The first scenario increases the lump-sum transfer by 50,000 yen for one year and the second scenario increases it by the same amount for two years. These are partial equilibrium marginal propensities to consume that hold prices and government policy variables fixed at their steady-state values. However, the individual responses are dynamic. Households optimally choose how to divide up the bonus to their income among

[37]The formula for exemptions provides a base exemption of \$30,000 plus \$6,000 per legal heir.

Figure 9: Partial equilibrium marginal propensities to consume by age



consumption in all future periods of their life. The marginal propensities to consume are calculated as expected values over the two survival states.

Figure 9 shows that the pattern of the marginal propensity to consume increases monotonically with age. The main reason a household's MPC increases with age is because its planning horizon becomes shorter.[38] They face relatively high mortality risk and if this is their death year, they consume all of their wealth. Average life expectancy in the model is 83 years. So the fraction of households with high MPCs is small.

F Empirical impulse responses

Local projections. Let Y_t denote a variable of our interest in period t. For example, it is the real consumption for a specific age group or the GDP. Since we are interested in the cumulative effects of a monetary policy shock over a year while the data we use are monthly or quarterly, we consider local projections with the following specification:

$$\frac{\sum_{h=0}^{H-1} \log(Y_{t+h})}{H} = \alpha + \beta \epsilon_t + \sum_{k=1}^{K} \gamma_k X_{t-k} + u_t \tag{45}$$

where H is the horizon over which the effect of the monetary policy shock ϵ_t is estimated, including its impact effect in period t, and X_{t-k} is a column vector that consists of lagged

[38] We don't report MPCs for households over age 100 in Figure 9 because their share of the population is very small.

variables and seasonal dummies.[39] Coefficient β captures the effect of the monetary policy shock of $\epsilon_t = 1$ as shown by

$$\beta = \frac{1}{H} \sum_{h=0}^{H-1} \{E_t [\log(Y_{t+h})|\epsilon_t = 1] - E_t [\log(Y_{t+h})|\epsilon_t = 0]\}$$

For monthly data we set $H = 12$ and for quarterly data we set $H = 4$. The lagged variables include $\log(Y_{t-k})$. Following the VAR analyses conducted by Miyao (2002) and Kubota and Shintani (2022) for Japan, we include a stock price index in X_{t-k} as well for the impulse responses of the macro variables shown in Figure 2. We set the number of lags as $K = 12$ for monthly data and $K = 4$ for quarterly data. Since the lagged variables are included in our specification of local projections, the standard error of the estimate of β can be computed by using the usual heteroscedasticity-robust Eicker-Huber-White standard error, as shown by Montiel Olea and Plagborg-Møller (2021).

Monetary policy shocks. We use two measures of monetary policy shocks. The first measure is a surprise to interest rate futures before and after a monetary policy announcement, estimated by Kubota and Shintani (2021) using high-frequency data for Japan. Specifically, we use the target factor proposed by Gürkaynak et al. (2005), which reflects the expectations up to one year ahead, aggregated over monthly frequency and used by Kubota and Shintani (2022). For estimation with quarterly data, we simply sum up the monthly monetary policy shocks over the corresponding quarter.

Our second measure of monetary policy shocks is that estimated by Ikeda et al. (2020). They use a non-linear SVAR and identify monetary policy shocks for Japan by combining an identification method that takes advantage of the presence of the effective lower bound, newly proposed by Mavroeidis (2021), with sign restrictions. Since the monetary policy shocks are not point-identified but identified as a set, we use the middle value of the set as our monetary policy measure.

Since the first measure of monetary policy shocks is available from 1992 to 2019, the sample period of our analyses in empirical responses is set to 1992–2019. This sample stating year happens to coincide with the period of the burst of the so-called asset price "bubble" in Japan and the starting period of the so-called "lost decade."

[39] Even if an independent variable in (45) is given as $\log(\sum_{h=0}^{H-1} Y_{t+h})$, it would not change our result significantly since it is equivalent to using the left-hand-side of (45) up to the first-order approximation and under the assumption of $1/H = Y_{t+j}/\sum_{t=0}^{H-1} Y_{t+h}$ for all $j = 0, .., H-1$.

Data. For the data by age group, used in Figure 1, the data source is the Family Income and Expenditure Survey (FIES). We use the disposable income reported in the FIES, which is available only for the category of working households. The consumption used in Figure 1 is non-durable consumption for working households. Specifically, our measure of non-durable consumption is defined as total consumption net of durable consumption and unknown consumption and gifts or transfers in the category of other consumption. Our measure of durable consumption consists of repairs and maintenance (house, garden and other facilities and their repair), household durable goods, interior furnishings and decorations, bedding, domestic utensils, private transportation (purchases of automobiles and motorcycles, and their maintenance fees and services), text books, and entertainment durables. The coverage of the FIES changed slightly in 2000: before 2000 the FIES excludes fishery and farmer households, but after 2000 it includes those households. We use year-on-year growth rates to connect the series before 2000 with those after 2000. Both consumption and disposable income are deflated by using the CPI index.

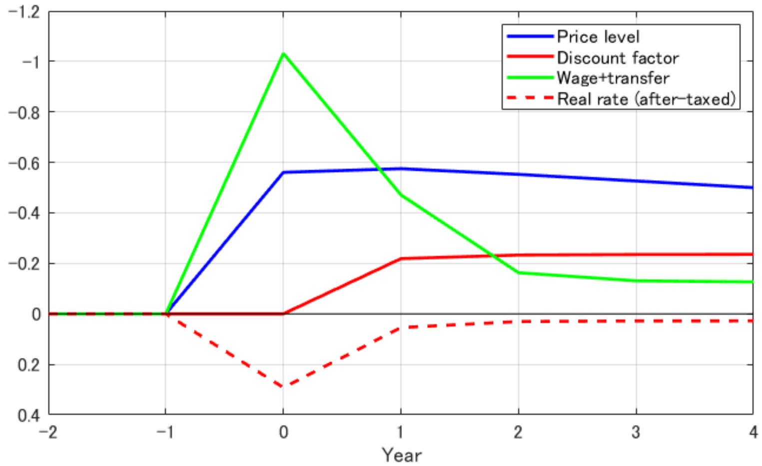
For the aggregate data used in Figure 2, GDP is the real GDP, consumption is the real private consumption, investment is the sum of the real private non-residential investment and the real private residential investment, CPI is the CPI index excluding the effect of changes in consumption tax rates, TOPIX is a stock price index that measures the overall trend in the stock market in Japan, covering an extensive proportion of the Japanese stock market, and the nominal rate is one-year government bond yield.

G The decomposition of consumption responses

Applicability of decomposition. The decomposition formula developed by Auclert (2019) holds up to first-order approximation under specific assumptions about a model and a monetary policy shock. The assumptions hold for a one-time transitory monetary policy shock in a standard RANK model without persistence. But, our model does not satisfy the assumptions strictly as the model has some degree of persistence and has positive nominal government debt, and it is solved nonlinearly. Yet, the model may have features that are not so away from the assumptions.

Figure 10 plots the impulse responses of the price level P_t, the ex-post real interest rate R_t/π_{t+1}, the real discount factor R_0^k/R_t^k, and real wage plus real unearned income $w_t + \xi_t$ to a tightening in monetary policy in $t = 0$ in our model. The vertical axis is flipped for ease of comparison with Figure 1 of Auclert (2019). Although the response of the wage

Figure 10: Impulse responses of aggregate variables



plus transfers is somewhat persistent, the responses of the variables are broadly in line with those assumed in Auclert (2019): one time permanent changes in the price level and the discount factor; one time transitory changes in the real rate and the wage plus transfers.

Decomposition formula. Consider the impact response of consumption by an age-j household to a tightening in monetary policy in $t = 0$. The decomposition formula developed by Auclert (2019) can be written as

$$dc_{j,0} = \underbrace{\widehat{MPC}_j \left(d\mathcal{Y}_{j,0} - NNP_{j,0}\frac{dP_0}{P} + URE_{j,0}\frac{d\tilde{R}^r_1}{\tilde{R}^r} \right) - \frac{c_j}{\sigma}\left(1 - \widehat{MPC}_j\right)\frac{d\tilde{R}^r_1}{\tilde{R}^r}}_{\widehat{dc}_{j,0}} + \text{disc}_j, \quad (46)$$

where $dc_{j,0} \equiv c_{j,0} - c_j$ is the difference in consumption by an age-j household from steady state, dP_0/P is the deviation of the price level from steady state, $d\tilde{R}^r_1/\tilde{R}^r$ is the deviation of the ex-post real interest rate $\tilde{R}^r_1 = \tilde{R}_0/\pi_1$, and disc_j is the discrepancy between $dc_{j,0}$ and $\widehat{dc}_{j,0}$. In formula (46), there are four key statistics: the modified marginal propensity to consume, $\widehat{MPC}_j$, a change in earned and unearned income, $d\mathcal{Y}_{j,0}$, net nominal position,

$NNP_{j,0}$, and unhedged interest rate exposure, $URE_{j,0}$, which are defined, respectively, as

$$\begin{aligned}
\widehat{MPC}_j &\equiv \frac{\partial c_j/\partial \xi}{1+(1-\tau^w)w\epsilon_j(\partial h/\partial \xi)},\\
d\mathcal{Y}_j &\equiv d\xi_0 + db_{j,0} + (1-\tau^w)h\epsilon_j dw_0 + (1-\tau^w)w\epsilon_j dh,\\
NNP_j &\equiv \frac{\tilde{R}_{-1}}{\pi_0}d_{j-1,-1},\\
URE_{j,0} &\equiv (1-\tau^w)w_0\epsilon_j h_0 + b_{j,0} + \xi_0 - (1+\tau^c)c_{j,0} + \frac{\tilde{R}_{-1}}{\pi_0}d_{j-1,-1} + a^m_{j,0},
\end{aligned}$$

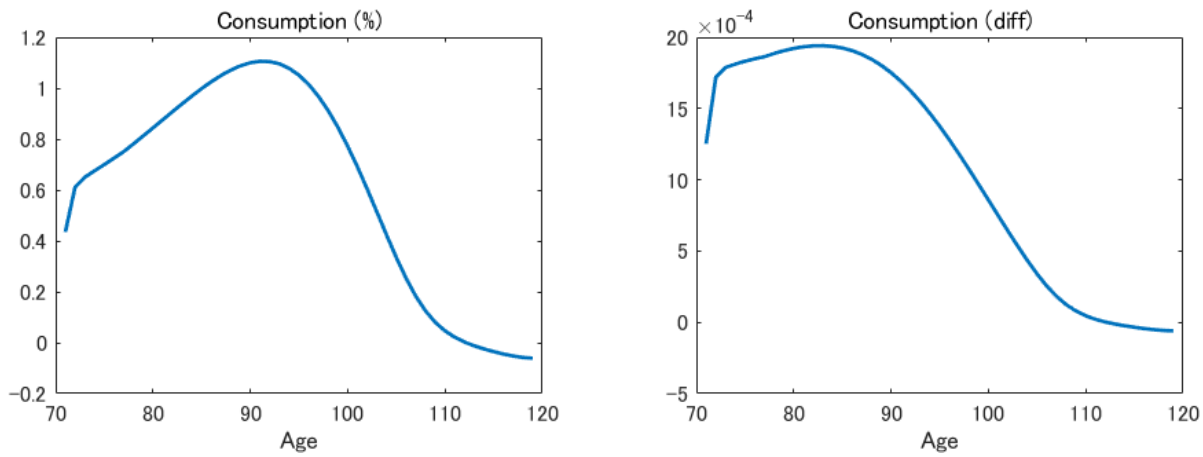
where $\partial c_j/\partial \xi$ denotes the partial equilibrium marginal propensity to consume for an age-j household, $\partial h/\partial \xi$ denotes the partial equilibrium marginal propensity to supply labor, and $a^m_{j,0}$ is the illiquid assets maturing in period 0, held by an age-j household. The partial equilibrium marginal propensity to consume is already calculated in Figure 9 for all ages. Since the hours worked are determined by the labor union to which all working households belong, an increase in ξ for a single household does not affect hours worked, and thus $\partial h/\partial \xi = 0$. The illiquid assets maturing in period 0 is given as $a^m_{j,0} = \mu^a_{j,0}A^m_0$, where $\mu^a_{j,0}$ is a ratio of the illiquid assets held by an age-j household to the total illiquid assets, given by $\mu^a_{j,0} \equiv a_{j-1,-1}/A_{-1}$, and A^m_0 is the aggregate illiquid assets maturing in period 0, given by $A^m_0 \equiv \left[1+(1-\tau^a)(1-\tau^k)(r^k_0-\delta)\right]K_0 + (1-\tau^a)(1-\tau^k)\Omega_0$. The four components shown in Figure 5 are then given by

$$\begin{aligned}
\mathit{Income} &= \widehat{MPC}_j \times d\mathcal{Y}_{j,0},\\
\mathit{NNP} &= -\widehat{MPC}_j \times NNP_{j,0} \times \frac{dP_0}{P},\\
\mathit{URE} &= \widehat{MPC}_j \times URE_{j,0} \times \frac{d\tilde{R}^r_1}{\tilde{R}^r},\\
\mathit{Substitution} &= -\frac{c_j}{\sigma}\left(1-\widehat{MPC}_j\right) \times \frac{d\tilde{R}^r_1}{\tilde{R}^r}.
\end{aligned}$$

H The effects of a tighter monetary policy on age-consumption profile of a 71 year old household.

This appendix provides more details about why a tightening in monetary policy produces hump-shaped consumption a response for households who are close to retirement age by analyzing the response of the cohort that is aged 71 when monetary policy is tightened. The left panel of Figure 11 plots the response of the household's consumption in terms of a difference from steady state for each age consumption, $c_{j,t} - c_j$, divided by the steady-state disposable income in each age, $(\tilde{R}^a - 1)a_{j-1} + (\tilde{R} - 1)d_{j-1} + b_j + \xi$. The response is hump-shaped and persistent with peak arrived more than 20 years later after the shock hits. The response of consumption in terms of a difference from steady state, $c_{j,t} - c_j$, is also hump-shaped as shown in the right panel of Figure 11.

Figure 11: Impulse responses by a household aged 71



To understand why the response of consumption is hump-shaped, let's start considering the steady-state age profiles of consumption $c_{j,t}$, illiquid asset holdings $a_{j,t}$, and liquid asset holdings $d_{j,t}$ (Figure 12). Since the mortality risk is relatively high after age 71, in spite of consumption smoothing motive, the age consumption profile is decreasing in age. The illiquid asset holding is also decreasing in age as the household draws down the illiquid assets for consumption. The household draws down the liquid assets more quickly than the illiquid assets. After age 85 the household borrows to keep the illiquid asset position. It is important to keep in mind that it is relative to this age profile of consumption (the left panel of Figure 12) that the response of consumption is hump-shaped.

Consumption of a household with age $j \geq 71$ in period t is given by

$$c_{j,t} = \underbrace{(\tilde{R}^a_t - 1)a_{j-1,t-1} - \triangle a_{j,t}}_{\text{Illiquid asset}} + \underbrace{(\tilde{R}_t - 1)d_{j-1,t-1} - \triangle d_{j,t}}_{\text{Liquid asset}} + \underbrace{b_{j,t} + \xi_t}_{\text{Government}}, \tag{47}$$

where $\triangle a_{j,t} = a_{j,t} - a_{j-1,t-1}$ and $\triangle d_{j,t} = d_{j,t} - d_{j-1,t-1}$. Figure 13 plots a decomposition of the consumption response into an illiquid asset cash flow component, $(\tilde{R}^a_t - 1)a_{j-1,t-1} - \triangle a_{j,t}$, a liquid asset cashflow component, $(\tilde{R}_t - 1)d_{j-1,t-1} - \triangle d_{j,t}$, and the government transfer component, $b_{j,t} + \xi_t$, in terms of a differences from steady state, from $t = 1$ (age 72) onward, where the impact period of $t = 0$ (age 71) is omitted because the responses in period $t = 0$

Figure 12: Age profiles of consumption and asset holdings

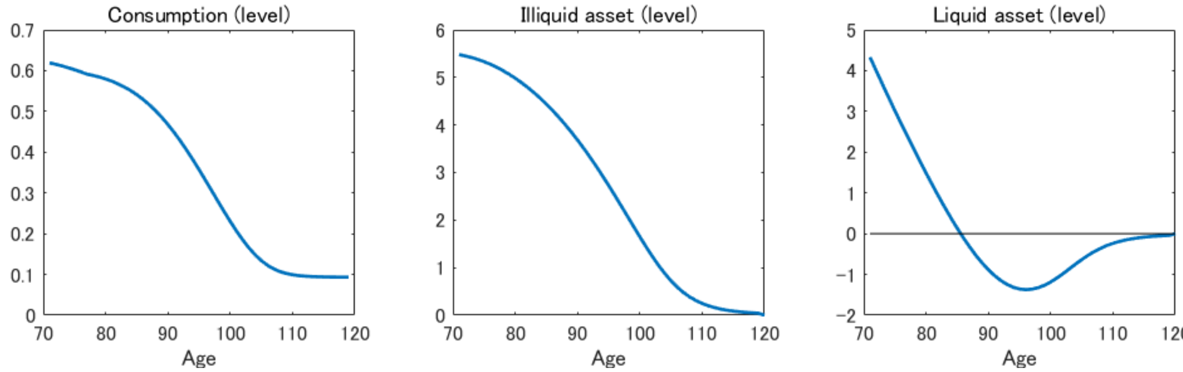
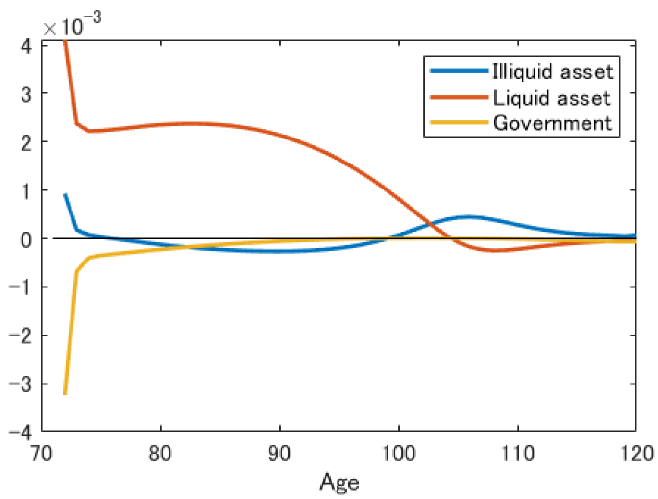


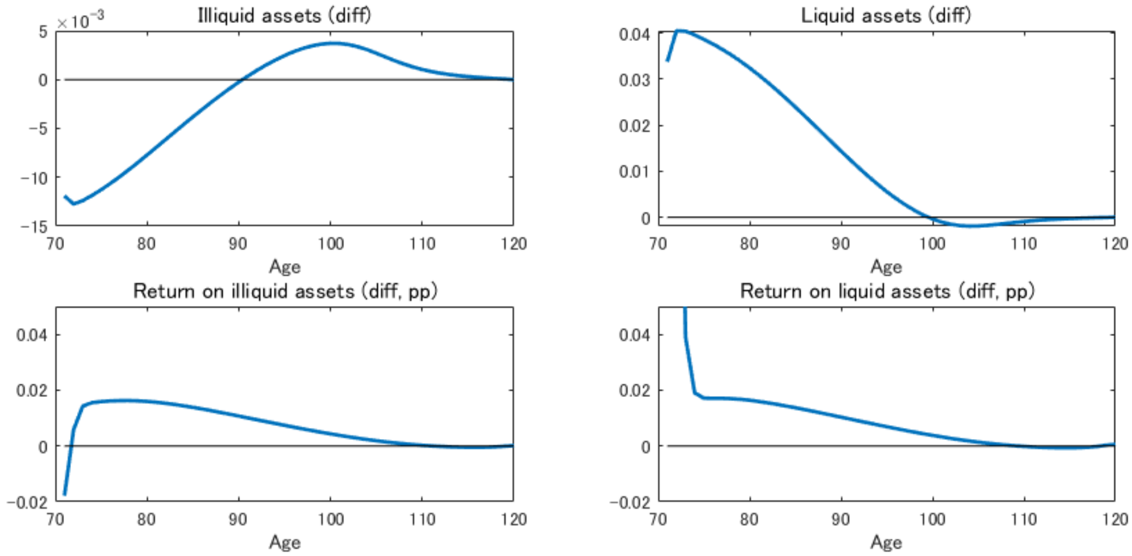
Figure 13: Decomposition of the consumption response (diff)



are much greater than those in the remaining periods. Figure 13 indicates that the liquid asset cash flow component is driving the hump-shaped response of consumption deviations from steady state.

In response to a tightening in monetary policy, the household reduces its holdings of illiquid assets and increases its holdings of liquid assets (Figure 14) in a persistent fashion. Its preferred asset allocation strategy is to hold positive amounts of both assets up until age 85 and tilting its portfolio towards liquid assets is attractive to it given the relatively high deviation of liquid assets from their steady-state level. The consumption difference peaks at age 83 and then begins to fall. This decline reflects the fact that beyond age 85 the household's preferred asset allocation strategy is to take a leveraged long position in illiquid securities and this strategy is not so attractive because it now is facing relatively

Figure 14: Illiquid and liquid asset holdings (diff)



high borrowing costs as shown in the bottom right panel of Figure 14.

Then, why does the real return on liquid assets remain relatively high? Since the nominal debt is fixed and the price level is the same in the initial and terminal steady states, a monetary policy tightening causes deflation initially but it is followed by inflation later. The central bank in the model responds to this small increase in the inflation rate by raising the nominal interest rate.

I Additional results

Table 9: Model aggregate steady-state moments

Variable	description	steady-state value
$R^k - 1$	return on capital	3.72%
$R/\pi - 1$	return on government debt	2.25%
$\tilde{R}^a - 1$	After–tax return on illiquid assets	1.93%
$\tilde{R}/\pi - 1$	After–tax return on liquid assets	1.80%
D^G/Y	Gross liquid assets relative to output	1.58
$(D^G - D)/Y$	Private borrowing relative to output	0.35
$(A + D - D^G)/Y$	Net stock of illiquid assets relative to output	3.15
$(A + D)/Y$	Net worth	4.73
K/Y	Capital-output ratio	2.16
V/Y	Value of shares relative to output	1.34
Ξ/Y	Lump–sum transfers relative to output	-0.002
B/Y	Social Security outlays relative to output	0.091
C/Y	Consumption share of output	0.67
I/Y	Investment share of output	0.24
$\bar{\gamma}_a/y$	Financial services share of output	0.011

Table 10: Impact responses of aggregate variables to monetary policy innovation under alternative scenarios.

A. Response of goods and labor market variables

Scenario	Y	C	I	H	w	r^k	π
Baseline	-0.651	-0.080	-2.49	-0.929	-0.788	-0.238	-0.561
Lump-sum	-0.633	-0.063	-2.46	-0.903	-0.790	-0.235	-0.557
$\phi_\pi = 0$	-0.882	-0.164	-3.04	-1.26	-1.16	-0.334	-0.499
Negative	0.725	0.062	2.21	1.037	0.811	0.258	0.581
d/y=1.5	-0.659	-0.076	-2.60	-0.940	-0.790	-0.246	-0.560

B. Response of financial and fiscal variables

Scenario	R	r^d	r^a	Spread	V	Ω	ξ	d
Baseline	0.278	0.855	-0.034	-0.889	-0.423	19.24	-0.403	0.564
Lump-sum	0.282	0.856	-0.040	-0.897	-0.440	19.15	-0.457	0.560
$\phi_\pi = 0$	1.028	1.55	-0.145	-1.69	-0.902	28.51	-0.658	0.501
Negative	-0.253	-0.842	-0.018	0.825	0.406	-23.31	-0.053	-0.578
d/y=1.5	0.278	0.857	-0.032	-0.888	-0.454	19.33	-0.884	0.563

Note: "Baseline" is the baseline scenario. "Lump-sum" assumes that lump-sum taxes are fixed at their steady-state level for households aged 76+ and $\phi_\pi = 0$ assumes that the coefficient on the inflation rate in the interest rate targeting rule is 0. "Negative" assumes that the monetary policy innovation is -0.01 instead and "d/y=1.5" assumes that the debt output ratio is 1.5 instead of its baseline value of 1.23.

Table 11: Wealth and consumption inequality under alternative scenarios

Wealth Inequality					
Year	0	1	2	3	4
Baseline	0.20	0.20	0.19	0.19	0.17
Lump-sum	0.20	0.20	0.19	0.19	0.17
$\phi_\pi = 0$	0.30	0.30	0.28	0.26	0.24
Negative	-0.20	-0.20	-0.19	-0.18	-0.17
d/y=1.5	0.20	0.20	0.19	0.18	0.17
Consumption Inequality					
Year	0	1	2	3	4
Baseline	0.082	0.084	0.085	0.086	0.086
Lump-sum	0.093	0.094	0.094	0.094	0.093
$\phi_\pi = 0$	0.086	0.086	0.086	0.086	0.086
Negative	-0.084	-0.085	-0.086	-0.087	-0.087
d/y=1.5	0.086	0.089	0.090	0.091	0.092

Note: Figures in the table show a percentage point change in the Gini coefficients from its stedy state, associated with a shock of size 0.01 to monetary policy.