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The Transmission of Financial Shocks and Leverage of Financial Institutions: An Endogenous Regime-Switching Framework

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Abstract: We conduct a novel empirical analysis of the role of leverage of financial institutions for the transmission of financial shocks to the macroeconomy. For that purpose, we develop an endogenous regime-switching structural vector autoregressive model with time-varying transition probabilities that depend on the state of the economy. We propose new identification techniques for regime switching models.

Recently developed theoretical models emphasize the role of bank balance sheets for the buildup of financial instabilities and the amplification of financial shocks. We build a market-based measure of leverage of financial institutions employing institution-level data and find empirical evidence that real effects of financial shocks are amplified by the leverage of financial institutions in the financial constraint regime. We also find evidence for a role of heterogeneity of financial institutions including depository financial institutions, globally systemically important banks, and selected nonbank financial institutions. Our results suggest that the leverage ratio is a useful indicator from a policy perspective.

JEL classification: C11, C32, C53, C55, E44, G21

Key words: regime switching, time-varying transition probabilities, financial shocks, leverage, bank and nonbank financial institutions, heterogeneity

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1 Introduction

Since the Global Financial Crisis (GFC) progress has been made in understanding the interactions of financial constraints, financial market instabilities and the macroeconomy and incorporating those in standard macreconomic models, but further work is needed.

There is a growing literature incorporating more elaborate representations of financial markets into structural macroeconomic models, in particular bank balance sheets and leverage of financial institutions. The number of contributions incorporating nonlinearities in that context is, however, still limited.

Our contributions in this paper are to propose an endogenous switching model framework and new identification techniques as well as providing empirical evidence of the relevance of (market-based) leverage of financial institutions for the transmission of shocks to the macroeconomy and the role of heterogeneity of financial institutions including banks and nonbanks in this context.

The GFC illustrated how a financial crisis might arise via the supply side of financial markets. In other situations, imbalances in the financial sector have amplified macroeconomic fluctuations and downturns, for instance during the dot com bubble. While the origin of the current crisis lies in a pandemic - different from the GFC -, financial factors have played an important role at the onset of the current crisis. Financial institutions entered the pandemic crisis better capitalized than the GFC and financial markets calmed due to an early and powerful central bank response. The increasing role of non-banks suggests a particular focus on the heterogeneity of different financial institutions.[1]

With this paper we contribute to the literature in a number of ways with methodological developments as well as the empirical analysis. First, we provide methodological contributions. We propose a regime switching vector autoregressive (RS-VAR) model with time-varying transition probabilities. We show how the Markov-switching structural vector autoregression model framework, proposed in Sims and Zha (2006) and Sims et al. (2008), and employed in Hubrich and Tetlow (2015) can be extended in several dimensions. More precisely, we

[1] Financial institutions entered the crisis better capitalized due to improved regulations following the GFC, in contrast to non-financial firms entered the current crisis with a high level of indebtedness. However, solvency problems of nonfinancial firms will have consequences both for the financial system as well as for the macroeconomy. Therefore, it is important also in the context of the current crisis to understand better the role of banks for the transmission of shocks.

extend previous literature on Markov-Switching models in two important dimensions: 1. We allow for time-varying probabilities in RS-VAR models. Thereby we go beyond the single-equation models suggested in the literature. In regime switching models with time-varying probabilities - sometimes referred to as "endogenous switching" models - the transition probability of being in one regime in the next period, given that we are in a particular regime in this period, can vary over time. Therefore, the transition matrix can depend on the state of the economy. 2. We propose new identification techniques for RS-VAR models, allowing a range of general, non-recursive (over-)identification schemes of the structural shocks. Besides non-recursive zero restrictions, these also include sign restrictions and narrative sign restrictions, thereby bringing the approaches suggested in Antolin-Diaz and Rubio-Ramirez (2018) and Arias, Rubio-Ramirez and Waggoner (2018) to the class of regime switching models. We also allow for different identification schemes in different regimes. One additional contribution is our systematic discussion of identification issues in regime switching models. We employ the recently developed Dynamic Striated Metropolis-Hastings sampler for high-dimensional models to estimate the posterior distribution of the model. This new framework allows us to address the economic questions raised above.

Second, employing this new modelling framework we provide a novel empirical analysis of the role of leverage of financial institutions for the transmission of financial shocks to the macroeconomy. The motivation for our focus on leverage is threefold: First, recent literature on structural macroeconomic models emphasizes the role of bank balance sheets for the build-up of financial instabilities and the amplification of economic downturns. Second, leverage encompasses the entire balance sheet of the financial institution and therefore is a broad indicator for signaling financial vulnerabilities. Third, the leverage ratio is a regulatory tool complementary to the (risk-weighted) capital ratio.

We build a market-based measure of leverage of financial institutions, building on Adrian and Brunnermeier (2016)[2]. We employ financial institution level data to construct a monthly measure of leverage as book assets over market equity. Two arguments suggest a focus on market leverage: First, Market leverage developments can signal a situation where financial institutions need to deleverage quickly, for instance if debt is used to finance asset growth as

[2]See also Paul (2020)

for broker-dealers (see Adrian and Shin (2014)) and if financial institutions rely primarily on short-term funding (see e.g. Adrian et al. (2011) and also the related literature on maturity transformation). More generally, we argue that monitoring market leverage is relevant for a range of reasons including the effect on financial institutions for coping with liquidity shocks and on funding costs (there is evidence that the cost advantage is larger for banks that are thinly capitalized) relevant from a financial stability perspective related to the liquidity regulations introduced in Basel III. Second, market values of equity are more informative about financial institutions' losses compared to book values. Book equity values might not be a timely predictor of bank health. Since book values incorporate information on losses with a delay, financial institutions have time to adjust their book leverage in order to avoid hitting the regulatory limit.[3] Financial institutions (banks and non-bank financial institutions) might be more fragile than their book leverage levels make them appear. Furthermore, market capitalization of a bank is a reflection of the market value of the equity holders' stake, and hence an assessment by market participants of the creditworthiness of the bank as a borrower. Low market-to-book ratios suggest that the assessment of market participants is that banks are more leveraged than their books suggest (see also Adrian et al. (2018)). We highlight the role of the financial fragility implied by market leverage for the transmission of financial shocks in our empirical model, that has been pointed to in a model estimated to match four facts about banks' leverage dynamics (see Begenau et al. (2021)).

We contribute to the empirical literature by, first, presenting empirical evidence for the role of (market) leverage for the amplification of the transmission of financial shocks and the implications for the real economy; second, providing empirical evidence for a different transmission of financial shocks in different regimes; and third, providing evidence of a role of the heterogeneity of financial institutions' leverage for the detrimental real effects of deleveraging of financial institutions and the implications for the probability of persistence of the financial constraint regime. In particular, we consider Depository Financial Institutions, Globally Systemically Important Banks (GSIBs) as well as selected Non-Bank Financial Institutions.

The paper is structured as follows. Section 2 discusses the related theoretical and em-

[3] So far the information content of market equity about book losses has been mostly highlighted in the accounting literature, indicating that banks have flexibility in accounting for losses, consistent with evidence in Blattner et al. (2022). This flexibility in accounting for losses can be even more prominent for non-bank financial institutions that are part of our analysis.

pirical literature, thereby motivating the models estimated in this paper. Section 3 presents our new methodological proposal, outlines the estimation and evaluation of the model and discusses the identification issues that arise for RS-VAR models. Section 4 contains the economic motivation, the data and model specification and the empirical results. Section 5 concludes.

2 Related economic literature and contribution of this paper

We complement previous empirical studies on financial constraints and economic dynamics by providing empirical evidence on the role of leverage of financial institutions for the transmission of financial shocks to the macroeconomy, and in particular market-based leverage. We develop a new regime switching SVAR model framework for this analysis that is motivated by recent theoretical model developments.

Endogenous financial instabilities: Theoretical Models

The theoretical literature has made progress recently in incorporating financial instability and associated nonlinearities into macroeconomic models. Structural models such as Kiyotaki and Moore (1997) as well as Brunnermeier and Sannikov (2014) illustrate how systemic risk might arise endogenously, determined by the choices of the model′s decision makers. In Kiyotaki and Moore (1997) collateral constraints play a key role for the propagation and amplification of shocks while in Brunnermeier and Sannikov (2014) the reduction in the volatility of output and asset prices leads to increased leverage of financial institutions.[4]

Bank balance sheets and leverage of financial institutions

More recently, a number of authors introduced a more sophisticated financial sector into an otherwise standard macroeconomic model. Financial intermediaries' balance sheets have implications for the institutions' access to funds and liquidity that affect their lending activities and thereby economic activity. A fall in the value of a bank's tradable assets and a decline in loan quality can adversely affect the bank′s capital. The fall in asset prices will affect lending activity via the collateral channel. Banks have been found to limit their deposit taking

[4]Other contributions that incorporate financial instabilities and nonlinearities include Mendoza (2010) and He and Krishnamurthy (2019) and Boissay et al. (2016).

in response to a decline in net worth Gertler and Kiyotaki (2010, 2015); Gertler and Karadi (2013).

The importance of the bank capital channel will also depend on the extent to which non-bank financial institutions can substitute lending and liquidity provisions by banks (see e.g. Durdu and Zhong (2019)).

Leverage of financial institutions is an important characteristic in the presence of large and abrupt asset price movements (see e.g. Gertler and Gilchrist (2018) on the role of leverage, and Adrian and Brunnermeier (2016) and Paul (2020) for more details on the mechanisms). The leverage ratio of a bank is the ratio of total assets to shareholder equity. Bank leverage is an indicator for external financing opportunities by banks and for risk-taking by banks. It is a cyclical indicator and can amplify the transmission of shocks.

Asset prices affect the balance sheet and thereby affect leverage both in an accounting sense as well as via resulting changes in the agents' behavior (see Paul (2020)). Financial vulnerabilities build up in boom times, when banks enlarge their balance sheets and increase their leverage, relying more on debt as opposed to equity. In the lead-up to the GFC and Great Recession there was a pronounced rise in leverage in the banking sector. During the GFC stock prices fell dramatically, increasing the already high leverage of banks even further. At the same time, banks' market value of assets in terms of share holdings was also shrinking, reducing the access of banks to external finance (see e.g. Ferrante (2019)).[5] At some point, banks had to sharply reduce the provision of loans to households and firms to obtain liquidity to avoid a bank run and insolvency (see e.g. Gertler et al. (2016) andGertler et al. (2020)). Consequently, banks were more constrained in raising funds and therefore were providing fewer/lower volume loans.[6] Additionally, when banks realized that this shock was not temporary, they deleveraged given a reduced risk tolerance, adding further constraints. [7] The discussion around Basel III and its implementation has aimed to address capital and leverage

[5]Ferrante (2019), building on the framework of Gertler and Karadi (2011), extends a standard New Keynesian model to include a rich financial system in which financially constrained banks lend to firms and homeowners via defaultable long-term loans. In this model financial shocks affecting lending spreads can bring about a widespread recession that has at its core a deterioration in the equity of financial intermediaries and in their leverage capacity.

[6]For a recent paper suggesting a endogenous regime switching DSGE model to analyse financial crises in Mexico, see Benigno et al. (2020).

[7]Note that asset price driven cycles are more likely in market-based banking systems (IMF, 2009). The build-up of leverage is more likely in market-based systems due to the effective use of collateralization and sophisticated risk management and information-sharing strategies.

requirements as lessons from the GFC.

Financial constraints and economic dynamics: empirical evidence

Only a limited number of empirical contributions allow for nonlinearities in the relation between financial constraints and the macroeconomy, while a growing empirical literature has documented stylized facts on the role of financial factors in business cycles and for the development of a financial crisis. [8] Hubrich and Tetlow (2015) investigate whether a financial crisis is just a manifestation of amplified shocks or whether the transmission of shocks does actually change. They find empirical support for the hypothesis of a change in the transmission of financial shocks to the US macroeconomy in episodes of high stress. Hubrich et al. (2013) analyse the effects of financial shocks on the macroeconomy for EU and OECD countries and find evidence for nonlinearities and heterogeneity across countries in the transmission of financial shocks to the macroeconomy. Other studies also highlight empirical nonlinearities arguing that transmission channels may operate differently depending on underlying conditions, e.g. on the credit-to-GDP gap, for instance Aikman et al. (2020) or find different effects depending on the nature of the financial shocks, i.e. whether shocks represent easing or adverse financial conditions (see Barnichon et al. (2019)).

Brunnermeier et al. (2019) employing a structural VAR identified with heteroscedasticity, also find significant output effect of financial stress shocks, measured as spread shocks, including corporate bond (GZ) spread shock that is and an interbank lending spread shock as proxied by the 3-month Eurodollar rate over the 3-month Treasuries. A recent strand of literature studies the distribution of future real GDP growth as a function of current financial and economic conditions or bank capital using quantile regression. They find that the estimated lower quantiles of the distribution of future GDP growth exhibit strong variation as a function of current financial conditions (see for instance Adrian et al. (2019) and Boyarchenko et al. (2020)).[9] We complement that literature by taking a parametric approach.

[8] Credit rises in the run-up to financial crises (Schularick and Taylor, 2012) and recessions associated with financial crises are usually deeper than normal recessions, especially if they are preceded with a build-up of credit (Jordà et al., 2013).

[9] See also Chavleishvili et al. (2021) for how to address macroprudential policy issues with a related approach.

3 The methodology

Most of the methodological literature focuses on models with constant probabilities of Markov switching. Following the seminal paper by Hamilton (1989), a number of contributions extended the basic Markov switching model and its estimation procedure suggested in that paper in different dimensions, see for instance Chauvet (1998), Kim and Nelson (1999), Frühwirth-Schnatter (2006), Sims and Zha (2006) and Sims et al. (2008).

Some papers propose a class of time-varying probability Markov switching regression models, including Filardo (1994), Diebold et al. (1994), Kim (2004), Kim et al. (2008) as well as Bazzi et al. (2017) and Chang et al. (2017). In these papers the probability of regime switching depends on certain variables of interest. They assume a functional form for the dependence of the probability on the state of the economy. Most papers employ a logistic function, sometimes a probit function is used (e.g. Kim et al. (2008)). Only a few recent papers allow for occasionally binding constraints in a VAR context that implies some endogeneity of regime switching, and those include Mavroeidis (2021), Aruoba et al. (2021) and Hayashi and Koeda (2019).[10]

In this paper, we propose a Regime-Switching Vectorautoregressive (RS-VAR) model with time-varying transition probabilities, building on and extending the framework presented in Sims et al. (2008).

We extend previous literature in several dimensions: 1. We allow for a time-varying transition matrix in a regime-switching structural VAR model; 2. we allow for a range of general, non-recursive identification schemes, including sign restrictions and narrative sign restrictions that might be different in different regimes; 3. We highlight and discuss identification issues in Regime-switching Structural VAR models.

3.1 The Regime-Switching Model with time-varying transition matrix

For $1 \leq t \leq T$, let y_t be an n-dimensional vector of endogenous variables, let z_t be a k-dimensional vector of exogenous variables, and let s_t^c and s_t^v be a discrete latent variables with $s_t^c \in \{1, \cdots, h_c\}$ and $s_t^v \in \{1, \cdots, h_v\}$. We propose a structural vector autoregression with

[10]Two recent DSGE model proposals with endogenous switching can be found in Chang et al. (2018), and Benigno et al. (2020).

time-varying transition matrix (RS-SVAR)

$$A_0(s_t^c)y_t = A_+(s_t^c)x_t + \Xi^{-1}(s_t^v)\varepsilon_t, \tag{1}$$

where the predetermined vector x_t is $[y'_{t-1}, \cdots, y'_{t-p}, z'_t]'$ and is of dimension $m = np + k$.[11] The exogenous structural shocks ε_t are n-dimensional and assumed to be standard normal and independent of the regime process s_t^c and s_t^v. The coefficient matrix $A_0(s_t^c)$ is $n \times n$ and invertible, $A_+(s_t^c)$ is $n \times m$, and $\Xi(s_t^v)$ is $n \times n$ and diagonal, with positive diagonal elements. We call s_t^c the coefficient regime and s_t^v the variance regime. We define the overall regime process to be $s_t = h_c(s_t^c - 1) + s_t^v$, which can take on $h = h_c h_v$ distinct values in $\{1, \cdots, h\}$. The coefficient and variance regime processes are assumed to be independent, though this condition could be relaxed.[12]

We will denote the matrix of probabilities governing the transition of the processes s_t^c and s_t^v from time t to time $t+1$ by $P_{t+1|t}^c$ and $P_{t+1|t}^v$, respectively. The matrix $P_{t+1|t}^c$ is $h_c \times h_c$ and the matrix $P_{t+1|t}^v$ is $h_v \times h_v$. The element in row i and column j of these matrices is the probability of transiting from regime j at time t to regime i at time $t+1$. The elements of these matrices are all non-negative and the *columns* of each of these matrices must sum to one. In general, $P_{t+1|t}^c$ and $P_{t+1|t}^v$ can depend on the endogenous variables $y_1, \cdots, y_t$, the exogenous variables $z_1, \cdots, z_t$, and the matrices $A_0(\cdot)$, $A_+(\cdot)$, and $\Xi(\cdot)$. This implies that $P_{t+1|t}^c$ and $P_{t+1|t}^v$ could also depend on the exogenous shocks $\varepsilon_1, \cdots, \varepsilon_t$. In our empirical examples, the transition matrices will depend only on $y_{t-\ell}, \cdots, y_t$, for some fixed non-negative value of ℓ.

In addition to the time-varying transition matrices, to fully specify the regime processes the initial probabilities must be specified. We denote these by p_0^c, which is an h_c-vector with non-negative elements that sum to one, and p_0^v, which is an h_v-vector with non-negative elements that sum to one.

The SVAR parameters of the model will be $A_0(\cdot)$, $A_+(\cdot)$, and $\Xi(\cdot)$. Both the transition matrices and the initial conditions can depend on the SVAR parameters and perhaps some additional vector of parameters that we will denote by q. The transition matrices, of course,

[11] In our empirical application the only exogenous variable is a constant, $z_t = 1$.

[12] The assumption that s_t^c and s_t^v are independent is equivalent to the overall transition matrix from time t to time $t+1$ being $P_{t+1|t} = P_{t+1|t}^c \otimes P_{t+1|t}^v$.

could also depend on the endogenous and exogenous data as described above. We will compactly represent all the parameters by $\theta = (A_0(\cdot), A_+(\cdot), \Xi(\cdot), q)$. For now, the only restrictions on the SVAR parameters are that the $A_0(\cdot)$ are invertible and the $\Xi(\cdot)$ are diagonal matrices with positive diagonal.

3.2 Identification in RS-SVAR models

One of the contributions of this paper is to highlight identification issues and to propose new identification schemes for RS-SVARs. Constant parameter SVAR models with homoskedastic Gaussian shocks are not identified, but constant parameter models with heteroskedastic shocks are identified, at least up to the ordering of the equations and sign of each equation, see Rigobon (2003). A similar result will hold for the RS-SVAR models we consider. For constant parameters structural VAR models with heteroskedastic shocks, the only identification issues are determining the ordering of the equations and the sign of each equation. For RS-SVAR models these identification issues are present, but there also two additional identification issues.

3.2.1 Identification through heteroskedasticity

Before stating our identification result, we need a restriction to pin down the relationship between $A_0(\cdot)$, $A_+(\cdot)$, and $\Xi(\cdot)$. If D is any diagonal matrix with positive diagonal, then the system given by Equation (1) and the system $DA_0(s_t^c)y_t = DA_+(s_t^c)x_t + D\Xi^{-1}(s_t^v)\varepsilon_t$ are observationally equivalent.[13] Thus a restriction is needed to force D to be the identity. In the literature, some authors have chosen the restriction $\Xi(1) = I_n$. While this certainly solves the identification issue, it makes the first variance regime special. We will use the restriction $\sum_{k=1}^{h_v} \Xi^2(k) = I_n$, or $\sum_{k=1}^{h_v} \Xi^{-2}(k) = I_n$. This restriction treats all the variance regimes symmetrically and works better with the usual priors imposed on the $\Xi(\cdot)$. With this restriction, we have the following result.

Proposition 1 *Suppose the span of the predetermined data is all of R^m and the unconditional probability of being each overall regime is not zero for every t. If $h_v > 1$, then, for almost all*

[13]It also must be the case that the new parameters, $DA_0(\cdot)$, $DA_+(\cdot)$, and $\Xi(\cdot)D^{-1}$, must satisfy the restrictions that $DA_0(\cdot)$ be invertible and $\Xi(\cdot)D^{-1}$ be a diagonal matrix with positive diagonal. Since D is diagonal with positive diagonal, both are satisfied.

parameters values, the RS-SVAR model given by Equation (1) *is identified up to the ordering and sign of the equations and the ordering of the regimes.*

Proof. See Appendix A. ■

The hypotheses of Proposition 1 are relatively mild. The first hypothesis is equivalent to the exogenous variables not being collinear and there being at least m observations The second hypothesis says that all the regimes are accessible. If all the initial probabilities are positive or if all the elements of the transition matrices are non-zero, then this hypothesis will be satisfied. Even if neither of these is true, as long at the positions of the zeros in the initial probabilities and the non-zero elements of the transition matrix do not exactly match up, then the hypothesis will be satisfied. In the next section, we discuss the precise meaning of the statement that the model is "identified up to the ordering and sign of the equations and the ordering of the regimes."

3.2.2 Other Identification Issues in RS-VAR models

As we saw in the previous section, multiplication of the system given by Equation (1) by an invertible matrix can result in an observationally equivalent system. In this section, we discuss the need for three more restrictions, all arising from multiplication of the system given by Equation (1) by an invertible matrix. This will make precise what we mean by "identified up to the ordering and sign of the equations and the ordering of the regimes." (see proposition 1).

If one were to permute the rows in Equation (1), then one would obtain an observationally equivalent system. More formally, if Q is a permutation matrix, then the system given by Equation (1) and the system $Q'A_0(s_t^c)y_t = Q'A_+(s_t^c)x_t + Q'\Xi^{-1}(s_t^v)QQ'\varepsilon_t$ are observationally equivalent.[14] So, one must have a restriction that picks a unique ordering of the rows out of the $n!$ possible ordering. Since the rows in Equation (1) correspond to equations and each equation contains a single shock, ordering the rows is referred to as ordering the equations or ordering the shocks. This is also referred to as identifying, or naming, the equations or shocks. For instance, we could order the equations so that the financial shock always appears in the first

[14] If $\sigma(\cdot)$ is a permutation of $(1,\cdots,n)$, then $Q = [e_{\sigma(1)},\cdots,e_{\sigma(n)}]$, where e_j is the j^{th} column of the $n \times n$ identity matrix, is the *column* permutation matrix associated with $\sigma(\cdot)$. Permutation matrices are orthogonal and if A is any $n \times n$ matrix, then AQ permutes the columns A by $\sigma(\cdot)$ and $Q'A$ permutes the rows of A by $\sigma(\cdot)$. So, if Q is a permutation matrix and D is a diagonal matrix, then $Q'DQ$ permutes the diagonal elements of D. Thus, $Q'A_0(\cdot)$ is invertiable, $Q'\Xi^{-1}(\cdot)Q$ is a diagonal matrix with positive diagonal, and $Q'\varepsilon_t$ is standard normal.

equation. In this case we have identified or named the financial shock. If no such restriction is imposed, then we say the system given by Equation (1) is identified up to an ordering of the equations. The restrictions we will employ to order the equations will be discussed in Section 3.3.

If one were to multiply any equation in any coefficient regime in the system given by Equation (1) by minus one, then one would obtain an observationally equivalent system. More formally, if $D(k_c)$, for $1 \leq k_c \leq h_c$, is a diagonal matrix with plus or minus ones along the diagonal, then the system given by Equation (1) and the system given by $D(s_t^c)A_0(s_t^c)y_t = D(s_t^c)A_+(s_t^c)x_t + \Xi^{-1}(s_t^v)\varepsilon_t$ are observationally equivalent. So, one must have a restriction to determine the sign of each equation in each coefficient regime. Such a restriction can be thought of as giving an interpretation to a positive shock. In the constant parameter case, Waggoner and Zha (2003) suggest a class of restrictions for determining the sign of each equation. For each coefficient regime, we will use a restriction from this class of restrictions. In particular, for each coefficient regime and equation, we will restrict the sign of that equation so that the impulse response of some variable at some horizon to a positive shock in that equation has a particular sign. In the above example, where the financial shock is ordered first, one could require that the contemporaneous response of output growth to a positive financial shock to be negative. In addition, one could require that the contemporaneous response of the financial conditions index to a positive financial shock to be positive. These would be consistent with a positive financial shock being detrimental to the outlook in growth. If no such conditions were imposed, then we say the system given by Equation (1) is identified up to sign.

In addition to identifying (or ordering) the equations, one must also identify (or order) both the coefficient and variance regimes. If $\sigma_c(\cdot)$ is a permutation of $(1,\cdots,h_c)$ and $\sigma_v(\cdot)$ is a permutation of $(1,\cdots,h_v)$, then we can define new discrete latent variables by $\tilde{s}_t^c = \sigma_c^{-1}(s_t^c)$ and $\tilde{s}_t^v = \sigma_v^{-1}(s_t^v)$. If Q_c is the $h_c \times h_c$ column permutation matrix associated with $\sigma_c(\cdot)$ and Q_v is the $h_v \times h_v$ column permutation matrix associated with $\sigma_v(\cdot)$, then the transition matrix and initial probabilities for $\tilde{s}_t^c$ are $Q_c' P_{t+1|t}^c Q_c$ and $Q_c' p_0^c$ and the transition matrix and initial probabilities for $\tilde{s}_t^v$ are $Q_v' P_{t+1|t}^v Q_v$ and $Q_v' p_o^v$. Furthermore, the system given by Equation (1) is observationally equivalent to the system given by $\tilde{A}_0(\tilde{s}_t^c)y_t = \tilde{A}_+(\tilde{s}_t^c)x_t + \tilde{\Xi}^{-1}(\tilde{s}_t^v)\varepsilon_t$, where

$\tilde{A}_0(k_c) = A_0(\sigma_c(k_c))$, $\tilde{A}_+(k_c) = A_+(\sigma_c(k_c))$, and $\tilde{\Xi}(k_v) = \Xi(\sigma_v(k_v))$, for $1 \leq k_c \leq h_c$ and $1 \leq k_v \leq h_v$. Thus we must have a restriction for selecting a unique ordering among the $h_c!$ possible orderings of the coefficient regimes and a restriction for selecting a unique ordering among the $h_v!$ possible orderings of the variance regimes. This can be thought of as identifying, or naming, each coefficient regime and each variance regime. If no such restrictions are imposed, then we say the system given by Equation (1) is identified up to an ordering of the regimes. The restrictions that we will employ for the ordering of the regimes will be discussed in Section 3.3.

3.3 Identifying Shocks and Regimes

Even in the constant parameter case with heteroskedastic shocks, coming up with plausible restrictions to identify the equations is difficult. This is one of the disadvantages of identification via heteroskedasticity. In the regime switching case, this difficulty is compounded because the coefficient and variance regimes must also be identified. In this section, we will discuss three different techniques for achieving both of these goals, sign restrictions, zero restrictions, and narrative restrictions. Combinations of these techniques can also be used. Last, we will discuss techniques for identifying the variance regimes.

3.3.1 Sign Restrictions

Sign restrictions on the impulse responses have long been used to identify the shocks in the case of constant parameters with homoskedastic shocks, though this identification is only set identification. In the case of regime switching parameters with heteroskedastic shocks, sign restrictions on the impulse responses can be used to identify both the equations and regimes. Even better, because there are only finitely many ways to identify the equations or regimes, in some cases sign restrictions can uniquely identify the equations or regimes, not just set identify. These ideas can best be explained by an example. For instance, suppose that one of the coefficient regimes is called the financial constraint regime and will be ordered first and that one of the shocks is the financial shock and will be ordered first. Note that the ordering of the equations must be the same across all coefficient regimes, so the financial shock would have to be ordered first in all the coefficient regimes. One could impose the restriction that the contemporaneous impulse response, conditional on being in the financial constraint regime, to a positive financial shock is positive for both financial conditions index and leverage and negative for output growth and interest rates. For some parameter values, in no regime is there a shock whose impulse responses satisfy this pattern, so that parameter would be rejected. In this sense, multiple sign restrictions on the impulse responses to a given shock in a given regime imply that the model is overidentified.[15] For other parameter values, there could be a unique regime and shock whose impulse response satisfied this pattern. In this case the sign restrictions uniquely determine the financial constraint regime and the financial shock and this regime and equation could be ordered first if that were not already the case. Finally, it could be the case that there are multiple regimes or shocks whose impulse response satisfied this pattern. In this case either the financial constraint regime or the financial shock, or both, would not be uniquely determined. As the number of sign restrictions increase, one would expect that the number of parameters that were rejected to increase and the number of parameters that did not uniquely determine both the regime and equation to decrease. If different sign restrictions on the impulse responses to the same shock across all the different regimes are imposed, then either the parameter will be rejected or the regimes will be uniquely determined. Similarly, if different sign restrictions on the impulse responses to all the different shocks, in any regime, are imposed, then either the parameter will be rejected or the equations will be uniquely determined. In this example, only one regime and one equation were being determined, though this idea could easily be extended to determining multiple regimes, i.e. partial identification, and equations or all of the regimes and equations

3.3.2 Zero Restrictions

Zero restrictions on the contemporaneous and predetermined parameters have also long been used to identify the shocks in the case of constant parameters with homoskedastic shocks. These ideas can also be used to identify the regimes and equations. Given a coefficient regime, if the pattern of zero restrictions on the contemporaneous and predetermined parameters in that regime is different from the pattern in all other coefficient regimes, then the given coefficient regime is uniquely determined by the zero restrictions, for almost all parameter values. Given

[15] A single sign restriction on the impulse responses to a given shock in a given coefficient regime does not impose overidentifying restrictions because the sign of any equation in any coefficient regime could be changed. Alternatively, as we saw in Section 3.2.2, a single sign restriction on the impulse response to a given shock in a given regime could be thought of as a restriction determining the sign of the given equation in the given regime.

an equation, if for all other equation there exists a coefficient regime such that the patterns of zero restrictions in those two equations differ in that regime, then the given equation is uniquely determined by the zero restrictions, for almost all parameter values. In the case of heteroskedastic shocks, a single zero restriction overidentifies the model. Using different patterns of zero restrictions across different equations and coefficient regimes is a powerful way of identifying the equations and regimes. However, such restrictions can severely overidentify the model resulting in a poor fit to the data. They should be used judiciously and in combination with sign and narrative restrictions.

3.3.3 Narrative Restrictions

Narrative restrictions were used by Antolín-Díaz and Rubio-Ramírez (2018) to set identify shocks in the case of constant parameters with homoskedastic shocks. Even earlier, Sims and Zha (2006) used similar ideas to identify the regimes in a Markov switching monetary policy model. We describe this procedure in the context of our model. In the middle of September in 2008, Lehman Brothers failed. Our contention is that by October of 2008, there was a high probability that the economy was in what we will call the financial constraint regime, which we will order first. We can use this to uniquely determine the financial constraint regime. In our case, since there are only two coefficient regimes, if we can determine the financial constraint regime, then the other regime, which we call the normal regime, will also be uniquely determined. The first step is to define what we mean when we say that there is high probability that we are in coefficient regime k at time t. To do this one must choose a cutoff probability, and if the smoothed probability that we are in coefficient regime k at time t is greater than the cutoff probability, then we say we there is a high probability that we are in coefficient regime k at time t. If the cutoff probability is greater than or equal to 0.5, then there is either a unique coefficient regime that is of high probability at time t or no coefficient regime that is of high probability at time t. Among the parameter values for which there is a unique coefficient regime that is of high probability in October of 2008, we will order the coefficient regimes so that this regime is first and call the first regime the financial constraint regime. If one was unsure of the exact period that economy was in the financial constraint regime, then one could choose a window about October of 2008, and then say the financial constraint regime was the regime that was in high probability over most of this window. For our models, we found that neither the choice of window about October of 2008 nor the cutoff probability, within reason, affected the determination of the financial constraint regime.

Similar ideas could be used to identify the equations. In our model, we wish to uniquely determine what we will call the financial shock. Again, in October of 2008, there was a massive decline in industrial production and it is our contention that the financial shock caused most of this decline. We first must define what we mean by when we say that "shock k caused most of the decline in industrial production at time t". For each overall regime, one can compute the expected value of each time t shock, conditional on the overall regime at time t. One could then compute the expected contemporaneous impulse response of industrial production to each time t shock, conditional on the overall regime at time t. Finally, using the smoothed probabilities, one could then compute the expected contemporaneous impulse response of industrial production to each time t shock. The time t shock with largest negative expected contemporaneous impulse response of industrial production caused most of the decline in industrial production at time t. Except in knife edge cases, there is a unique shock that caused most of the decline in industrial production in October of 2008 and we will order the equations so that shock is first and call the first shock the financial shock. As with ordering the regimes, if one was unsure of the exact period that the financial shock was dominant, then one could choose a window about October of 2008, and then say the financial shock was the shock that caused the most cumulative decline in industrial production over this window.

3.3.4 Identifying the Variance Regimes

In some ways, identifying the variance regimes is more straightforward. The inverse of the diagonal elements of $\Xi(\cdot)$ directly scale the structural shocks, and thus have an economic interpretation. For instance, in our models we are interested in the financial shock, which we order first. So, we order the variance regimes, after we have ordered the coefficient regimes, so that the first diagonal element of the $\Xi(\cdot)$ are in increasing order. This would imply that the first variance regime would have the most variability, at least in terms of the effect of the financial shock. We should point out that under this ordering, in the first variance regime the impulse response to a financial shock will have the largest response for all variables at all horizons in all coefficient regimes.

3.4 The Transition Matrices and Initial Probabilities

For the variance regime we will use a constant transition matrix and for the coefficient regime we will have time-varying transition matrices of a particular functional form. Our methodology will certainly allow for both the coefficient and variance regimes to have time-varying transition matrices of completely general functional forms, but in the interest of parsimony, we restrict the variance regime to have a constant transition matrix and the diagonal elements of the coefficient regime transition matrix to be a logistic transformation of a linear function of the endogenous variables. The off-diagonal elements of the coefficient regime transition matrix will be a constant times one minus the diagonal element from the same column. For $1 \leq i, j \leq h_v$, we will denote the constant elements in the variance regime transition matrix by $q^v_{i,j}$ and note that $\sum_{i=1}^{h_v} q^v_{i,j} = 1$. Let q_v denote the vector containing all the $q^v_{i,j}$.

Let $p_{t+1|t}(i, j)$ denote the time-varying probability of switching from regime j at time t to regime i at time $t+1$. We assume that the time-varying probability of staying in the j^{th} regime at time $t+1$, given that we are in the j^{th} regime at time t, is of the form

$$p_{t+1|t}(j, j) = \frac{1}{1+\exp(\bar{\gamma}_j - \sum_{k=1}^{\ell} \gamma_{j,k} y_{t-k+1})}. \tag{2}$$

The scalars $\bar{\gamma}_j$ are the location parameters and the n-vectors $\gamma_{j,k}$ are the slope parameters. In keeping with our desire to be parsimonious, in most of our examples, $\ell = 1$ and only a few of the elements of $\gamma_{j,k}$ will be allowed to be non-zero. We will gather all of these parameters that are not restricted to zero into a vector that we will denote by γ.

In the case of only two coefficient regimes, the diagonal elements completely determine the transition matrix. If there are more than two coefficient regimes, then for $i \neq j$,

$$p_{t+1|t}(i, j) = q^c_{i,j}(1 - p_{t+1,t}(j, j)), \tag{3}$$

where the $q^c_{i,j}$ are non-negative constants such that $\sum_{i=1}^{h_c} q^c_{i,j} = 1$, under the convention that $q^c_{j,j} = 0$. Let q_c denote the vector containing all the $q^c_{i,j}$ that are not restricted to be zero.

We will choose the initial probabilities in both the coefficient and variance regime processes so that all the regimes have equal probability. This choice is mandated by the fact that we want the initial probabilities to be invariant to permutations of either of the coefficient or variance regime. If this was not the case, then the initial probabilities would determine, at least partially, the ordering of the coefficient and variance regimes. Unless one was very sure about the initial regime, this would not likely result in a satisfactory condition for ordering the coefficient and variance regimes.

In the case of the variance regime, which has constant transition matrix, choosing the initial variance probabilities to be the ergodic probabilities would also be invariant to permutations of the variance regime. This would be a permissible choice and not increase the number of parameters. However, in most cases this would not deliver substantially different results.

There are no parameters controlling the initial probabilities and the parameters controlling the transition matrices are (q_v, q_c, γ). As stated before, we gather all of these parameters into a vector that we will denote by q.

3.5 The Priors

In this section we describe the priors that we will employ.

For each $1 \leq k_c \leq h_c$, we will use the same Sims-Zha prior on each $(A_0(k_c), A_+(k_c))$. For the hyperparameters in this priors, we will follow the recommendations in Sims and Zha (1998) for monthly data.

For each $1 \leq k_v \leq h_v$, denote the diagonal elements of $\Xi(k_v)$ by $\xi_{k_v,j}$, for $1 \leq j \leq n$. Recall that we use the restriction that either $\sum_{k_v=1}^{h_v} \xi_{k_v,j}^2 = 1$ or $\sum_{k_v=1}^{h_v} \xi_{k_v,j}^{-2} = 1$, for each $1 \leq j \leq n$. We will use the uniform prior across the $\xi_{k_v,j}$. This can be easily implemented as a Dirichlet distribution over $(\xi_{1,j}, \cdots, \xi_{h_v,j})$, for each $1 \leq j \leq n$, with all the Dirichlet hyperparameters equal to one.

For the variance regime transition matrices, the parameters are $q_{i,j}$, for $1 \leq i, j \leq h_v$. We will use a Dirichlet prior on $(q_{1,j}^v, \cdots, q_{h_v,j}^v)$, for each $1 \leq j \leq h_v$. For the off-diagonal elements, we will choose the Dirichlet hyperparameters to all be equal to one. For the diagonal elements, we will choose the hyperparameter to match the desired duration of each variance regime, though we will assume that the duration is the same across all variance regimes.

For the coefficient regime transition matrices, the parameters are (q_c, γ). We will use a

Dirichlet prior on $(q^c_{1,j},\cdots,q^c_{j-1,j},q^c_{j+1,j},\cdots,q^c_{h_v,j})$, for each $1 \le j \le h_v$. We will choose the Dirichlet hyperparameters to all be equal to one, so the distribution will be uniform. For the γ, we will use independent normal distributions. We recommend standardizing all the variables controlling the diagonal elements of the coefficient regime transition matrices. In this case we find that using independent normal distributions works well. Alternatively, if one had prior opinions about the means and variances of the variables controlling the diagonal elements of the coefficient regime transition matrices, then these could be used to set the means and variances of the elements of γ.

Because we assume that the initial probabilities are all equal, there are no parameters associated with the initial parameters.

3.6 The Posterior, Filtered and Smoothed Probabilities

In this section, we give formulas for the posterior, filtered and smoothed probabilities. For completeness, these formulas will be explicitly derived in Appendix B. To derive expressions for the posterior and filtered probabilities, we need the following assumption about the exogenous variables.

Assumption 1 (Exogeniety) *$p(z_{t+1}|S_t,Y_t,Z_t,\theta) = p(z_{t+1}|Y_t,Z_t)$, where $S_t = (s_1,\cdots,s_t)$, $Y_t = (y_1,\cdots,y_t)$, and $Z_t = (z_1,\cdots,z_t)$.*

The idea is that $p(z_{t+1}|Y_t,Z_t)$ is the true, but unknown, distribution of z_{t+1}, conditional on Y_t and Z_t, and knowing the path of regimes or the model parameters provides no additional information. Note that this assumption implies that the conditional distribution of z_t does not depend on the regimes. Under this assumption, if $p(\theta)$ is the prior, then the posterior is proportional to

$$P(\theta|Y_T,Z_T) \propto p(\theta)\prod_{t=1}^{T}\sum_{s_t=1}^{h} p(y_t|s_t,Y_{t-1},Z_t,\theta)p(s_t|Y_{t-1},Z_{t-1},\theta). \tag{4}$$

In order to compute the posterior, we must be able to explicitly compute the conditional likelihood, $p(y_t|s_t,Y_{t-1},Z_t,\theta)$, and the filtered probabilities, $p(s_t|Y_{t-1},Z_{t-1},\theta)$. For RS-SVAR models, the conditional likelihood is normal and easy to compute. Given the initial probabilities, the filtered probabilities can be recursively computed via the Hamilton filter. The recursive formulas are

$$p(s_{t+1}|Y_t, Z_t, \theta) = \sum_{s_t=1}^{h} P_{t+1|t}(s_{t+1}, s_t)p(s_t|Y_t, Z_t, \theta) \tag{5}$$

$$p(s_t|Y_t, Z_t, \theta) = \frac{p(y_t|s_t, Y_{t-1}, Z_t, \theta)p(s_t|Y_{t-1}, Z_{t-1}, \theta)}{\sum_{s_t=1}^{h} p(y_t|s_t, Y_{t-1}, Z_t, \theta)p(s_t|Y_{t-1}, Z_{t-1}, \theta)} \tag{6}$$

Often, one is more interested in the smoothed probabilities, $p(s_t|Y_T, Z_T, \theta)$. This can be done using backward recursion in the Hamilton smoother. The formula for the backward recursion is

$$p(s_t|Y_T, Z_T, \theta) = p(s_t|Y_t, Z_t, \theta) \sum_{s_{t+1}=1}^{h} \frac{p(s_{t+1}|s_t, Y_t, Z_t, \theta)p(s_{t+1}|Y_T, Z_T, \theta)}{p(s_{t+1}|Y_t, Z_t, \theta)}. \tag{7}$$

Since $p(s_T|Y_T, Z_T, \theta)$ can be obtained from the last step of the Hamilton filter, we can start the backward recursion at s_T and then recursively compute the smoothed probabilities.

3.7 Estimation

Having efficient and accurate samplers for simulating the posterior distribution is crucial for Bayesian analysis. There are a number of recent papers proposing new methods to compute posterior distributions, e.g. Durham et al. (2014), Herbst and Schorfheide (2014), Bognanni and Herbst (2018), and Waggoner et al. (2016).

We employ the dynamic striated Metropolis-Hastings (DSMH) sampler (see Waggoner et al. (2016)) [henceforth WWZ (2016)] to our new model class to derive the whole posterior distribution. The DSMH sampler is grounded in the Metropolis-Hastings algorithm, it pools the strengths from the equi-energy and sequential Monte Carlo samplers while avoiding the weaknesses of the standard Metropolis-Hastings algorithm and those of importance sampling. The basic idea of the Dynamic Striated Metropolis-Hastings sampler is to move from a tractable initial distribution one can sample from and transform the initial distribution gradually to the desired posterior distribution through a sequence of stages. Dynamic adjustment is used for sampling form the target distribution at each stage when progressing to the next stage. The sampler utilizes importance weights only for initial draw at each stage to avoid the degeneracy problem of other Sequential Monte Carlo samplers.[16]

4 Empirical Analysis

4.1 Motivation

We complement previous empirical studies on financial constraints and economic dynamics by providing empirical evidence on the role of leverage of financial institutions for the transmission of financial shocks to the macroeconomy using our proposed regimes switching model with new identification techniques. Our motivation to focus on leverage is threefold: First, the recent theoretical literature emphasizes the role of bank balance sheets for the build-up of financial instabilities and the amplification of economic downturns. Second, leverage encompasses the entire balance sheet of the financial institution and therefore is a broad indicator for signaling financial vulnerabilities. Third, we aim to contribute to the discussion regarding the usefulness of the leverage ratio from a financial stability policy perspective.[17]

We empirically investigate the role of balance sheets of financial institutions for the amplification of financial shocks, differences in the transmission of financial shocks in different regimes, and the heterogeneity of financial institutions and implications for the persistence of financial constraint regimes. We build on Adrian and Brunnermeier (2016) and Paul (2019) and construct a novel market measure of leverage of financial institutions. We employ our proposed regime switching structural vector autoregressive (SVAR) model for this investigation.

4.2 Data, model specification and identification

4.2.1 Data

Leverage of financial institutions is an important regulatory indicator of financial sector vulnerabilities, since highly levered financial institutions are less able to absorb losses. Recent literature on structural macroeconomic models emphasizes the role of bank balance sheets for

[16] We use the following settings: 50 stages, 50 striations, 100 Groups, 2000 draws within each group; overall, we have therefore 200000 draws.

[17] Note that the supplementary leverage ratio has been introduced in the US in 2014. The aim is to counterbalance the build-up of systemic risk by limiting the risk weights compression (downweighting of seemingly low risk investments) during booms and therefore add a more countercyclical measure than a risk weighted capital ratio. For a discussion, see e.g. Gambacorta and Karmakar (2018).

the build-up of financial instabilities and the amplification of economic downturns. Furthermore, leverage encompasses the entire balance sheet of the financial institution and therefore is a broad indicator for signaling financial vulnerabilities.[18]

In our empirical analysis we use a market value, micro-data based measure of leverage of financial institutions, building on Adrian and Brunnermeier (2016). We employ the CRSP/Compustat merged database that covers a broad range of publicly listed depository and nondepository institutions, bank holding companies and non-banks.

Market leverage is constructed using market value equity, not the book value, i.e. based on the expected present discounted value of future cash flows of a financial institution, its creditors and its shareholders; in contrast, book values depend on specific accounting rules. Therefore, the leverage measure with market equity that we are using here takes into account that before the global financial crisis the leverage of financial institutions only rose mildly, since as debt went up also the market value of asset prices increased. During the crisis asset prices collapsed, while financial institutions were not able to reduce their accumulated debt burden as quickly leading to a sharp increase in leverage using the market value of equity. Note that our database includes all listed financial institutions, including a broad range of depository and non-depository credit institutions and a range of non-bank institutions, including security brokers and dealers. We use data from the *Fundamentals Quarterly* and *Security Monthly* of the CRSP/Compustat Merged database. We compute a novel monthly market leverage measure based on monthly market equity and quarterly interpolated series for book assets, or - alternatively - liabilities. Our measure builds on Adrian and Brunnermeier (2016), but goes beyond previous literature that uses linear interpolation to convert quarterly book values. We employ monthly call reports data for interpolation of the quarterly book assets and book liabilities. The source of the data used for monthly interpolation of the quarterly book assets and liabilities is a monthly survey of a sample of the commercial banks in the call reports.[19] We compute leverage as book assets over market equity as well as market equity plus book

[18]Under Basel III, the supplementary leverage ratio is introduced as a measure that treats all exposures equally, independent of any risk assessment. The non-risk weighted leverage ratio is intended to avoid that banks lever up their balance sheet by investing in assets that appear in low-risk categories.

[19]The H.8 data from the Federal Reserve presents an estimate of weekly aggregate balance sheets (assets and liabilities) of commercial banks in the United States. The data are based on weekly reports from 875 commercial banks.

liabilities over market equity. [20]

Recent literature has highlighted that book and market leverage diverge substantially during crises (see also Begenau et al. (2021). We provide evidence that market leverage is a useful indicator for monitoring financial institutions since it reflects market developments in a timely way. The advantage of using financial institutions level data also is that they allow us to analyze the economic implications of heterogeneity across financial institutions.[21] In particular, we focus on comparing model specifications with leverage of depository institutions, with leverage of Globally Systemically Important Banks (GSIBs) and with leverage of a particular group of non-bank financial institutions, namely securities brokers and dealers.

In addition to this novel monthly measure of market based leverage, we use US monthly data, seasonally adjusted, for a sample from 1988(12) to 2019(12). We include the following variables: Output growth is measured in terms of growth in industrial production, we include core CPI inflation and the 2-year Treasury rate; market leverage of financial institutions or leverage of particular financial institutions such as Globally Systemically Important Banks (GSIBs) and security brokers and dealers. Market leverage is proxied by book assets over market equity, as described above, and we include a broad financial conditions index including spreads, financial market volatility measures and other financial conditions indicators spanning a broad range of financial markets and financial intermediaries, since we are interested in investigating heterogeneity. It is published by the Federal Reserve Bank of Chicago. We chose this broad measure of financial conditions since we are interested in investigating and comparing the role of heterogeneity of leverage of financial institutions for economic outcomes.

4.2.2 Model specification

We employ our proposed regime switching vector autoregressive (RS-VAR) model with time-varying transition probabilities. We allow for two regimes in the VAR coefficients, which

[20]The latter is often referred to as being a more reliable measure of market leverage since book liabilities are a better proxi for market liabilities than book assets for market assets, and that is what we use for our main empirical analyses.

[21]Note that the treatment of mergers and acquisitions in Compustat is as follows: When firms merge, the financial balance sheet items of the target firm gets absorbed into the balance sheet items of the acquirer. Therefore, when the target firm's data series ends, the acquirer's data series reflects the target financial balance sheet items. This provides the background for how structural changes in the course of the GFC will be reflected in this data set.

we label 'financial constraint' regime and 'normal' times. We make the regime probability dependent on the financial variables in the model since we are primarily interested in the transmission of financial shocks. We also allow for two regimes for the variances that follow a Markov process as a way to model heteroskedasticity.

In our regime switching models with time-varying probabilities - that can also be referred to as "endogenous switching" models - the transition probability of being in one regime in the next period, given that we are in a particular regime in this period, can vary over time. We model the transition matrix to depend on the state of the economy, namely we make it dependent on the financial variables in our system. To identify the regimes and structural shocks, we employ sign restrictions and, alternatively, narrative sign restrictions, thereby extending the approaches suggested in Antolín-Díaz and Rubio-Ramírez (2018) and Arias et al. (2018) to regime switching models. We also allow for different identification schemes in different regimes. We explain the details in the next section.

4.2.3 Identifying Regimes and Shocks

Regime identification In Section 3.3, we discussed how to identify the regimes using narrative restrictions. In this section we give the specific details of how we assigned the regimes to either the financial constraint regime or the normal regime. For each regime, we counted the number of months between 2008(9) and 2009(8), inclusive, that the probability of being in that regime was greater than 0.70. Whichever regime had the larger count was labeled the 'financial constraint' regime and ordered first. The other regime was labeled the 'normal' regime and ordered second. The assignment of the financial constraint regime was robust to varying the cutoff probability from 0.50 through 0.95 and choosing a shorter period around the Global financial crisis, or even only using 2008(10). Put in another way, the data was very informative and clear as to which regime the economy was in during the Global financial crisis.

Shock identification In addition to identifying the regimes, in our RS-VAR model we also need to identify the shocks. In particular, we want to identify the financial shock, which we will order first. We used sign restrictions on the impulse responses to achieve this. The contemporaneous response to a positive financial shock in the financial constraint regime was restricted to be negative for output, inflation and short-term interest rate, but positive for the financial conditions index and leverage. The contemporaneous response to a positive financial shock in the normal regime was restricted to be positive for the financial conditions index only. Overall, in about 20 percent of the draws these sign restrictions uniquely identified the financial shock.

Alternative shock identification We have carried out further empirical analyses using narrative restrictions as an alternative shock identification approach. This is a new class of narrative restrictions developed for constant parameter structural VAR models (see Antonlin-Diaz and Rubio-Ramirez, 2018) that we extend to the regime switching structural VAR model set-up. The structural parameters are constrained in such a way around key historical events that structural shocks and historical decomposition agree with the narrative. Those narrative sign restrictions combine the appeal of narratives with the advantages of sign restrictions. A small number of key historical events (not whole time series) are used for identification, thereby avoiding measurement error in narrative time series that have been presented in earlier literature.

In our implementation we identify the financial shocks as the one that explains most of the variation in output growth during the Global financial crisis. In addition, a considerably smaller set of sign restrictions are imposed than were when only sign restrictions were used. We only impose the positive response of the financial conditions contemporaneously, as well as the negative initial output growth response and the initial positive response of leverage. We present the empirical results using standard sign restrictions as our baseline results in the next few sections, and show some key results for narrative sign restrictions in Section 4.8 and Appendix C.

4.3 Regime probabilities

In the following we use smoothed probabilities as well as the time-varying probabilities and their association to historical events to interpret the different regimes. We use the specification with market leverage of GSIBs as our baseline specification and discuss those results first. We choose a model specification with endogenous regime switching driven by the three financial variables in our model, namely the 2-year Treasury rate, leverage and financial conditions.[22] This choice is motivated by our interest in the financial shock transmission.

Figure 1 presents the smoothed probability of the first coefficient regime allowing for both variance regimes based on our endogenous regime switching specification. We present the estimate of the median of the posterior distribution.[23] We interpret this regime as a "financial constraint" regime; it covers the end of the Savings & Loan crisis, the 1990/91 recession, the Russian debt default, the GFC and the related recession. Note that the filtered probabilities - that are particularly useful for determining the financial constraint regime in real-time - are presented in Figure 2 and are very similar to the smoothed probabilities. We will use the impulse responses to a financial shock presented in the next section to shed light on the economic dynamics in this regime.

Figure 1: Smoothed probabilities of coefficient regime

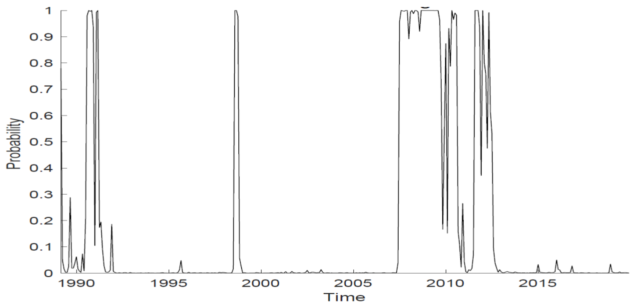
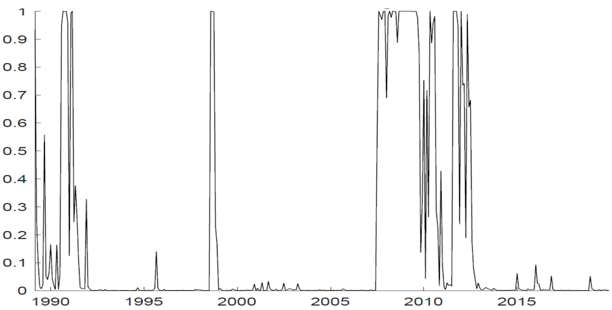


Figure 2: Filtered probabilities of coefficient regime



[22]This model allows for four regimes given the different combinations of variance and coefficient regimes. We focus on one coefficient regime allowing for both variance regimes to occur during this regime in what is presented in the charts.

[23]Note that the error bands for the smoothed probabilities are rather tight around the median.

Figure 3: Time-varying probability of staying in low interest rate regime, conditional of being in that regime

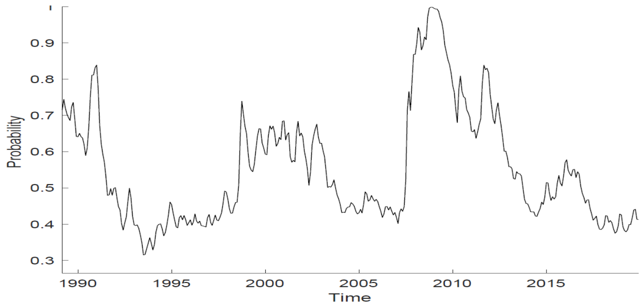


Figure 3 displays the time-varying probability of staying in the financial constraint regime, conditional on being in that regime (the latter probability as indicated in Figure 1). The figure illustrates that the probability of staying in the financial constraint regime is one at the end of 2008 and at the beginning of 2009, but declines sharply in 2009/2010 down to nearly 0.6 percent, increasing again during the European sovereign debt crisis before declining again further.[24] This figure illustrates the value of a model with endogenous regime switching for economic interpretation. Time-varying probabilities with error bands for the specification with GSIBs' leverage are presented in Appendix D and illustrate that uncertainty around this probability of staying in the financial constraint regime is small during the GFC and even thought it increases when the probability is declining the decline, subsequent rise and further decline are significant.

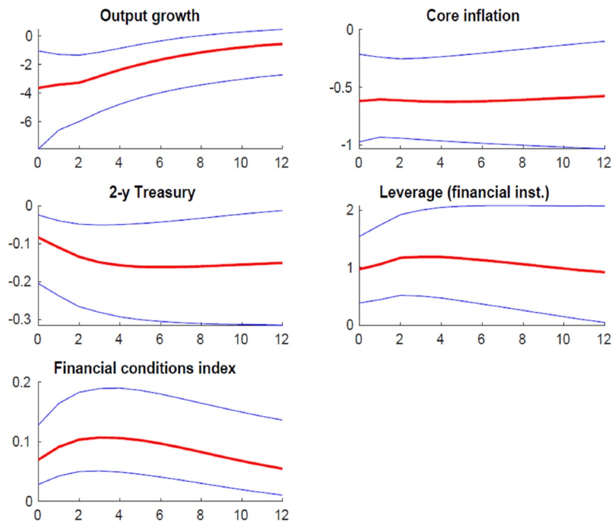
4.4 The transmission of financial shocks

An important contribution of our paper is to shed light on the role of leverage of financial institutions for the transmission of financial shocks to the real economy. Our model specification allows for two coefficient regimes that depend endogenously on the financial variables in the system and for two variance regimes. We now discuss the transmission of a financial shock starting with the financial constraint regime and then compare the results to normal times shock responses.

[24]Note that the time-varying probability of staying in a regime can only be interpreted during the time when there is a high probability of being in that regime corresponding to the smoothed probability depicted in the previous Figure.

First, we investigate the impulse responses to a financial shock in our model when including leverage of the GSIB institutions under supervision of the Federal Reserve Board.[25] Figure 4 presents the impulse responses to a one standard deviation financial shock in the financial constraint regime (for one particular variance regime) conditional on staying in the financial constraint regime. In other words, we assume that the economy stays in the financial constraint regime for 12 months after the shock has hit. The median response of the whole posterior distribution is displayed with 68% error bands.

Figure 4: Impulse Responses to a financial shock in the financial constraint regime, GSIBs



Note: *IRFs conditional on regime; red line: median response, blue lines: lower and upper bound of the 68 percent error bounds; financial shock: 1 std shock to financial conditions index; identification with contemporaneous sign restrictions, median and 68% error bands, output growth (IP), core inflation (CPI), 2-y Treasury yield, market leverage, Financial conditions index (Chicago Fed)*

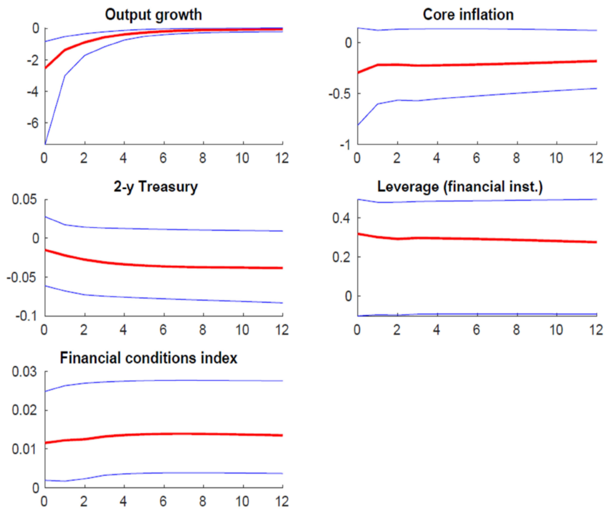
The impulse responses show an output response that turns out to be significantly negative, large and protracted, as would have been expected in the financial constraint regime. We find that leverage of GSIBs initially increases due to a sharp decline in asset prices, and then starts declining since the GSIBs deleverage in response to a financial shock in financial constraints episodes. We interpret that as evidence that deleveraging can lead to amplification effects with adverse implications for the real economy. GSIBs deleverage by liquidating assets, for

[25]Capitalization of GSIBs are regularly monitored in the Financial Stability Report published by the Federal Reserve. They include Bank of America Corporation, the Bank of New York Mellon Corporation, Citigroup Inc., The Goldman Sachs Group, Inc., JP Morgan Chase Co., Morgan Stanley, State Street Corporation, Wells Fargo Company.

instance by carrying out (fire) sales of securities and/or extending fewer loans while existing loans mature. This implies a reduction in overall credit supply. At the same time, it will be more demanding to get external financing due to a decline in collateral value.

Next, we compare the responses of the economy in the financial constraint regime and in normal times. Figure 5 shows the responses to a financial shock in normal times. We find that output growth shows a large negative response in normal times, but that response is non-persistent in contrast to the financial constraint regime. Also, in contrast to the financial constraint regime market leverage remains insignificant over the entire horizon.

Figure 5: Impulse Responses to a financial shock in normal regime. GSIBs



Note: *IRFs conditional on regime; red line: median response, blue lines: lower and upper bound of the 68 percent error bounds; financial shock: 1 std shock to financial conditions index; identification with contemporaneous sign restrictions, median and 68% error bands, output growth (IP), core inflation (CPI), 2-y Treasury yield, market leverage, Financial conditions index (Chicago Fed)*

Since these impulse responses provide an average response for the respective regime and are conditional on the regime, we shed more light on the role of leverage during the GFC using a counterfactual analysis in Section 4.6. Before that we turn to investigating the role of the heterogeneity of financial institutions in terms of leverage for the transmission of financial shocks.

Heterogeneity of financial institutions: Leverage of Depository Institutions

Our second set of results is for depository financial institutions (including commercial banks, savings and loans, and credit unions)[26] from our CRSP/COMPUSTAT database of listed institutions. Again, sign restrictions are only imposed contemporaneously on all endogenous variables in response to a positive financial shock in one of the regimes, and only on one shock in the other regime.

In comparison to the results for the GSIBs our findings for Depository Institutions display a similar median output growth response for a given financial conditions tightening (Figure 6). However, the results do not display asymmetric responses in the tails of the posterior distribution as for the GSIBs, and hence - in contrast to the model with GSIBs leverage - negative growth outcomes appear not to be more likely than positive outcomes. We also find that, as for GSIBs (see Figure 4), market leverage initially increases significantly due to the sharp decline in asset prices and then starts declining gradually since financial institutions deleverage.

The responses to a financial shock in normal times (Figure 7) show large and non-persistent output growth effects and no significant leverage response, similar to the responses we saw for GSIBs (Figure 5).

4.5 Heterogeneity: Securities brokers and dealers

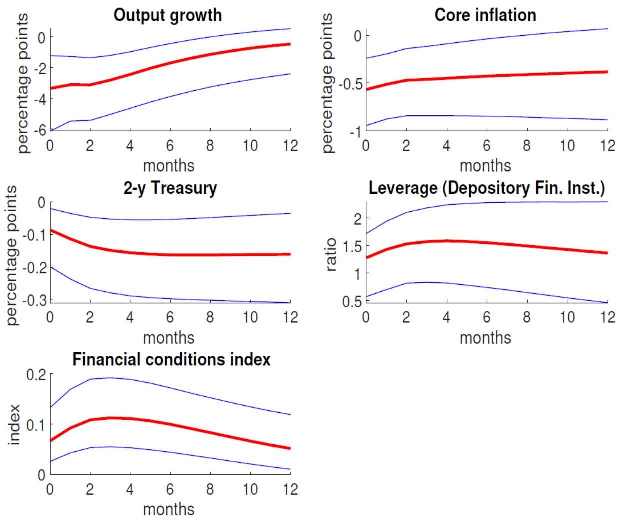
Next we examine the role of heterogeneity by including leverage of security brokers and dealers in the model.

The financial constraint regime is again identified by sign restrictions only contemporaneously on all endogenous variables in one of the regimes, as for previous specifications. As pointed out in Aramonte et al. (2021) higher leverage does not need to correspond to larger balance sheets, but broker dealers clearly used debt to finance asset growth (see Adrian and Shin (2014)).

We find similar results for the real economy implications as before, with a protracted negative output growth in response to a financial shock, with asymmetric responses in the tails

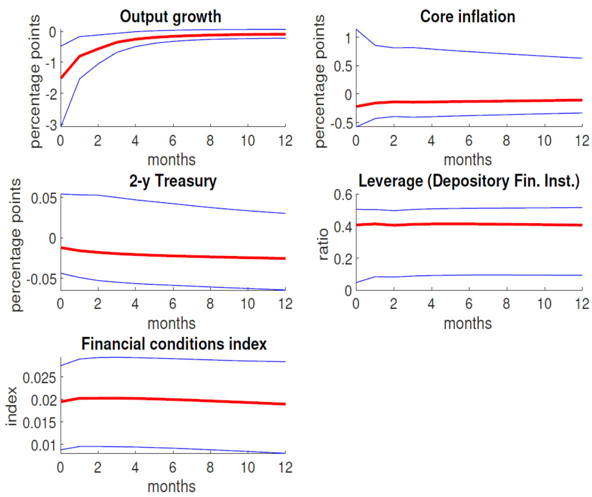
[26]Depository institutions are financial institutions that receive money from depositors to lend out to borrowers such as commercial banks and the other institutions listed here.

Figure 6: Impulse Responses to a financial shock in the financial constraint regime, depository institutions



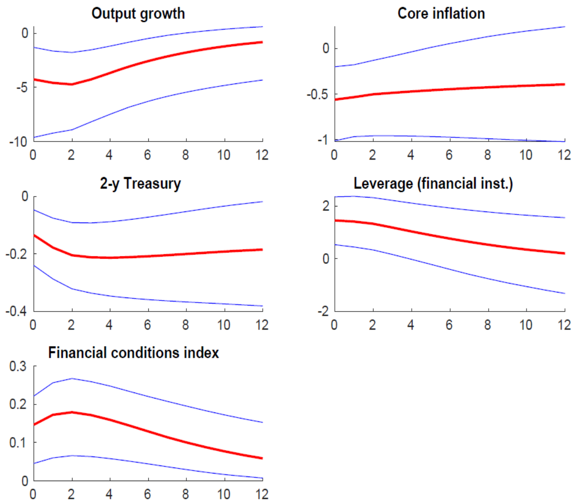
Note: *IRFs conditional on regime; red line: median response, blue lines: lower and upper bound of the 68 percent error bounds; financial shock: 1 std shock to financial conditions index; identification with contemporaneous sign restrictions, median and 68% error bands, output growth (IP), core inflation (CPI), 2-y Treasury yield, market leverage, Financial conditions index (Chicago Fed)*

Figure 7: Impulse Responses to a financial shock in normal regime, depository financial institutions



Note: *IRFs conditional on regime; red line: median response, blue lines: lower and upper bound of the 68 percent error bounds; financial shock: 1 std shock to financial conditions index; identification with contemporaneous sign restrictions, median and 68% error bands, output growth (IP), core inflation (CPI), 2-y Treasury yield, market leverage, Financial conditions index (Chicago Fed)*

Figure 8: Impulse Responses to a financial shock in the financial constraint regime, securities brokers and dealers



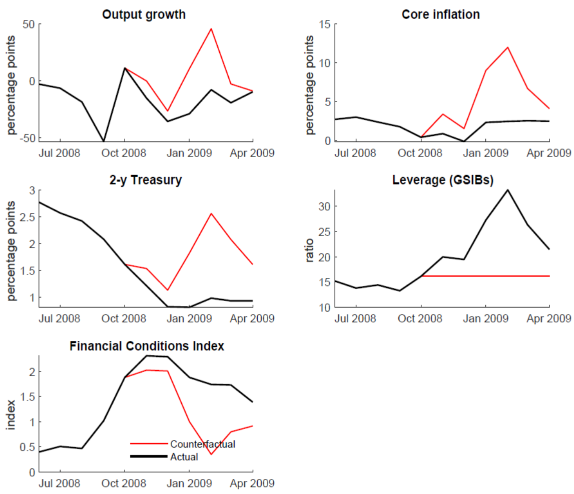
Note: *IRFs conditional on regime; red line: median response, blue lines: lower and upper bound of the 68 percent error bounds; financial shock: 1 std shock to financial conditions index; identification with contemporaneous sign restrictions, median and 68% error bands, output growth (IP), core inflation (CPI), market leverage = assets/market equity, Financial conditions index (Chicago Fed)*

of the posterior distribution as for the GSIBs leverage specification. However, market leverage increases more on impact than for other financial institutions and then immediately declines due to deleveraging. We interpret that as reflecting that The dealers' willingness to take risk amplified the growth of the dealer balance sheets going into the crisis, causing crisis losses and a subsequent sharp contraction of balance sheets post-crisis.

4.6 Counterfactual: Leverage of GISBs

To shed further light on the role of GSIBs leverage in the financial constraint regime, we carry out a counterfactual zooming into the GFC. We simulate the data based on what would have happened if (market) leverage of GSIBs would have remained constant as of October 2008. This experiment is designed to focus on the deleveraging process and its potential adverse effects on the real economy, by avoiding the final sharp increase of market leverage at the onset of the GFC and subsequent fall. As illustrated in Figure 9 this would have implied a less pronounced tightening of the financial conditions in the Fall of 2008 and subsequent quicker recovery. Also, this counterfactual would have implied a dramatically smaller decline

Figure 9: Counterfactual: Leverage constant as of October 2008



Note: Probability of staying in financial constraint regime, red line: counterfactual data with constant GSIBs leverage, black line: actual data; Output growth (IP), core inflation (core CPI), 2-y Treasury yield, leverage (market-based, GSIBs), financial conditions index (Chicago Fed)

in output growth (about 20 percentage points in terms of growth in industrial production) and a subsequent quicker recovery in output growth and related inflation increases despite higher short-term interest rates.

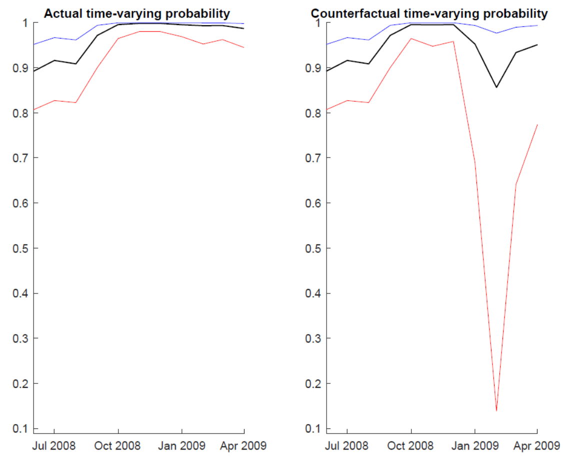
Counterfactual probability of staying in the financial constraint regime: Leverage of GISBs versus Depository Institutions

We also compute the counterfactual probability of staying in the financial constraint regime for the case of keeping leverage of GSIBs constant.

Figure 10 shows that the counterfactual probability of staying in the financial constraint regime (right panel) declines more than the actual probability (left panel). Some of the outcomes indicate a much lower probability of staying in the financial constraint regime under the counterfactual scenario of constant leverage, namely as low as 0.1. This is in line with what might be expected since the median output response might be similar than for other financial institutions, but outcomes might be more detrimental in response to financial shocks for GSIBs that are particularly highly leveraged.

Indeed, this is what is illustrated in Figure 11 for a system including depository financial

Figure 10: Counterfactual probability of staying in the financial constraint regime, GSIBs



Note: *Black line: median time-varying probability of staying in financial constraint regime, black line: upper error bound, red line: lower bound; left panel: actual, right pane: counterfactual*

institutions' leverage where the lower bound probability of staying in the financial constraint regime does not go down below 0.90.

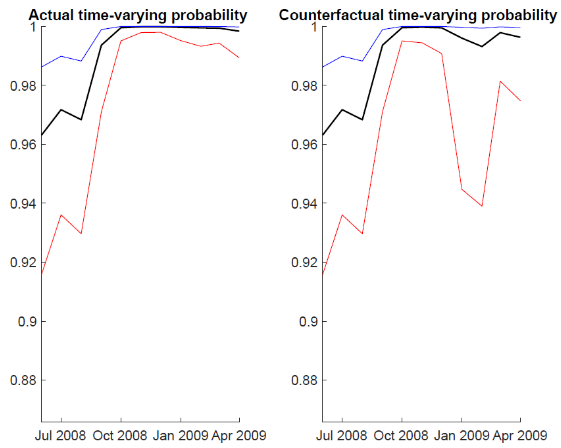
4.7 Market leverage and financial conditions

To illustrate that market leverage and financial conditions take distinct roles for the transmission of financial shocks, we have carried out a number of counterfactuals. We hold the financial conditions index constant as of October 2008 (to make it similar in terms of timing to our other experiment that focuses on the deleveraging process) and compute the counterfactual probability of staying in the financial constraint regime in the model with leverage of GSIBs.

The counterfactual probability of staying in the constraint regime is not going down as in the case of holding leverage constant. We interpret this as evidence that NFCI and leverage provide different characterizations of the financial conditions of the economy and have different implications for the propagation of shocks and the persistence of the constraint regime. It also illustrates that it is not the market price that is the sole driver for our results, since that would be behind both the financial conditions index and leverage.

We have also estimated our model that includes leverage of GSIBs with the GZ spread instead of the the broad financial conditions index to illustrate that it is not the leverage measures

Figure 11: Counterfactual probability of staying in the financial constraint regime, depository financial institutions



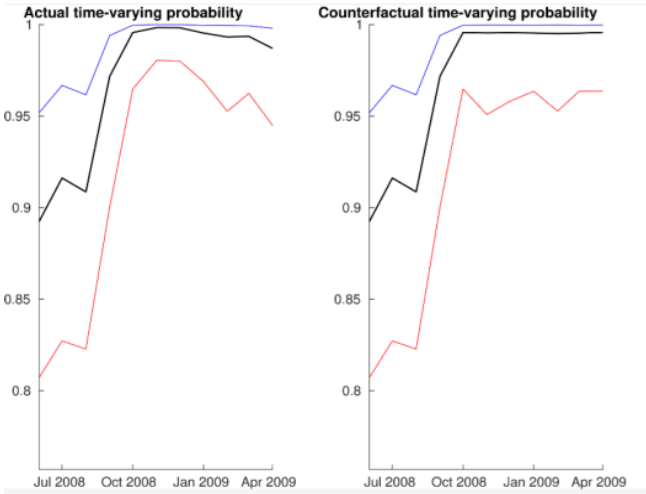
Note: *Black line: median time-varying probability of staying in financial constraint regime, black line: upper error bound, red line: lower bound; left panel: actual, right pane: counterfactual*

or stock market variables related to financial institutions that are behind our results. We get very similar results in that the probability of staying in the constraint regime is declining much more than the actual probability, confirming our result that it helps to prevent the deleveraging process that has adverse implications for the real economy.

4.8 Sensitivity: Narrative restrictions for shock identification

We have investigated the sensitivity of our results to imposing narrative restrictions for shock identification, extending the proposed identification approach by Rubio-Ramirez et al (2018) to our regime switching model. We combine the narrative restrictions with sign restrictions and compare the results with our findings when using standard sign restrictions presented above. We generally find that our results are rather robust when using narrative restrictions. It is noteworthy, that when using narrative restrictions we need fewer sign restrictions than with the standard sign restriction approach. The results for the RS-VAR including leverage of GSIBs are presented in Appendix C.

Figure 12: Counterfactual probability of staying in the financial constraint regime, GSIBs



Note: *Black line: median time-varying probability of staying in financial constraint regime, black line: upper error bound, red line: lower bound; left panel: actual, right pane: counterfactual*

5 Conclusions

We conduct a novel empirical analysis on the role of leverage of financial institutions for the transmission of financial shocks to the macroeconomy. To that end we develop an endogenous regime-switching structural vector autoregressive model with time-varying transition probabilities. First, we allow for the transition probabilities to be dependent on the state of the economy, and thereby to be time-varying. Second, we propose new identification schemes for RS-VAR models, extending sign and narrative restrictions to the regime switching model class. To facilitate economic interpretation, we allow the identification restrictions to differ across regimes. One of our contributions is also to highlight a range of identification issues in the context of regime switching models.

Employing this new modelling framework we provide a novel empirical analysis of the role of leverage of financial institutions for the transmission of financial shocks to the macroeconomy. We construct a new monthly market-based measure of leverage of financial institutions as book assets over market equity, building on Adrian and Brunnermeier (2016) by employing financial institution level data. We motivate our focus on leverage with the recent literature on structural macroeconomic models that emphasizes the role of bank balance sheets for the build-up of financial instabilities and the amplification of economic downturns. Furthermore, market leverage encompasses the entire balance sheet of the financial institution and therefore is a broad indicator for signaling financial vulnerability. Finally, we contribute to the empirical literature by, first, presenting empirical evidence that deleveraging can lead to procyclical financial amplification effects with adverse implications for the real economy in the financial constraint regime. Market leverage provides timely information for monitoring the role of financial institutions in that context. Second, we provide evidence for a role of the heterogeneity of financial institutions' leverage for the detrimental real effects of deleveraging of financial institutions, with implications for the probability of persistence of the financial constraint regime. In particular, we analyse the implications of heterogeneity of depository institutions, GSIBs and non-bank financial institutions.

Our empirical evidence indicates the importance of monitoring market-based leverage of financial institutions for financial stability. Two arguments suggest a focus on market leverage: First, Market leverage developments can signal a situation where financial institutions need to deleverage quickly, for instance if debt is used to finance asset growth as for broker-dealers (see Adrian and Shin (2014)) and if financial institutions rely importantly on short-term funding. More generally, we argue that it is important to monitor market leverage for a range of reasons including the effect on financial institutions for coping with liquidity shocks and on funding costs (there is evidence that the cost advantage is larger for banks that are thinly capitalized) relevant from a financial stability perspective and related to the liquidity regulations introduced in Basel III. Second, market values of equity are more informative about financial institutions' losses compared to book values. Book equity values might not be a timely predictor of bank health, since book values incorporate information on losses with a delay, financial institutions have time to adjust their book leverage in order to avoid hitting the regulatory limit. Financial institutions might be more fragile than their book leverage levels make them appear. Furthermore, market capitalization of a bank is a reflection of the market value of the equity holders' stake, and hence an assessment by market participants of the creditworthiness of the bank as a borrower and the present value of the stream of cash flows that derive from the bank's business activities. Low market-to-book ratios suggest that the assessment of market participants is that banks are more leveraged than their books suggest (see also Adrian et al. (2018)). We highlight the role of the financial fragility implied by market leverage for the transmission of financial shocks in our empirical model, that has been pointed to in an estimated model framework (see Begenau et al. (2021)).

It appears that so far the information content of market equity about book losses has been mostly highlighted in the accounting literature, indicating that banks have flexibility in accounting for losses. One might argue that this flexibility in accounting for losses is even more prominent for non-bank financial institutions, highlighting the relevance of our analysis of heterogeneity of financial institutions for regulatory considerations.

Our results on the role of market-based leverage raise the question how to lower the related financial vulnerability. Begenau et al. (2021), for instance, argue that stricter accounting rules (i.e. faster loan loss recognition) could achieve lower financial fragility and thus mitigate the impact of shocks. Aramonte et al. (2021) develop a conceptual framework that emphasises the central role of leverage fluctuations for the propagation of systemic risk due to non-bank financial institutions. We plan to explore the role of non-bank financial institutions further in future research given the heterogeneity of non-bank financial institutions and in light of the current discussion about the appropriate design of non-bank financial institution regulations.

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A Proof of Proposition 1

The following lemma is key to proving Proposition 1. In both the statement of the lemma and its proof, it will always be assumed that $1 \le k \le r$ and $1 \le m \le s$.

Lemma 1 *Let $A_1, \cdots, A_r$ be invertible $n \times n$ matrices and let $D_1, \cdots, D_s$ be distinct $n \times n$ diagonal matrices such that the diagonal elements of each D_m are distinct and $\sum_{m=1}^{s} D_m = I_n$. If $\tilde{A}_1, \cdots, \tilde{A}_r$ are $n \times n$ matrices, $\tilde{D}_1, \cdots, \tilde{D}_s$ are $n \times n$ diagonal matrices such that $\sum_{m=1}^{s} \tilde{D}_m = I_n$, $\pi(k,m)$ is a function that is a permutation of $\{1, \cdots, s\}$ for each k, and $A_k' D_m A_k = \tilde{A}_k' \tilde{D}_{\pi(k,m)} \tilde{A}_k$, then $\pi(k,m)$ is independent of k, $\tilde{A}_k = E_k Q' A_k$ and $\tilde{D}_{\pi(k,m)} = Q' D_m Q$, where Q is a permutation matrix and E_k is a diagonal matrix with plus or minus ones along the diagonal.*

Proof. Since $\pi(k,m)$ is a permutation of $\{1, \cdots, s\}$ for each k,

$$I_n = (A_k')^{-1} \left(\sum_{m=1}^{s} A_k' D_m A_k \right) A_k^{-1} = (A_k')^{-1} \left(\sum_{m=1}^{s} \tilde{A}_k' \tilde{D}_{\pi(k,m)} \tilde{A}_k \right) A_k^{-1} = (\tilde{A}_k A_k^{-1})' (\tilde{A}_k A_k^{-1}),$$

it follows that $(\tilde{A}_k A_k^{-1})' \equiv \hat{Q}_k$ is an orthogonal matrix and $\tilde{A}_k = \hat{Q}_k' A_k$. Thus,

$$A_k' D_m A_k = \tilde{A}_k' \tilde{D}_{\pi(k,m)} \tilde{A}_k = A_k' \hat{Q}_k \tilde{D}_{\pi(k,m)} \hat{Q}_k' A_k,$$

which implies that $D_m = \hat{Q}_k \tilde{D}_{\pi(k,m)} \hat{Q}_k'$. Because the eigenvalues of a symmetric matrix are unique up to an ordering, the D_m are distinct, and the diagonal elements of each D_m are distinct, the permutation $\pi(k,m)$ does not depend on k. If Q is the column permutation matrix associated with π, then for each k there exists a diagonal matrix E_k, with plus or minus ones along the diagonal, such that $\hat{Q} = Q E_k$. ■

Proof of Proposition 1. Throughout this proof, it will be assumed that $1 \le k \le h_c$, $1 \le m \le h_v$, and $1 \le t \le T$. Let $M(k) = A_0^{-1}(k) A_+(k)$ and $V(k,m) = A_0^{-1}(k) \Xi^{-2}(m) A_0^{-1}(k)'$. For almost all $A_0(k)$, $A_+(k)$ and $\Xi(m)$, the $M(k)$ are distinct, the $V(k,m)$ are distinct, the $\Xi(m)$ are distinct, and the diagonal elements of each $\Xi(m)$ are distinct. So, we can assume that the $M(k)$ are distinct, the $V(k,m)$ are distinct, the $\Xi(m)$ are distinct, and the diagonal elements of each $\Xi(m)$ are distinct.

The distribution of y_t, conditional on x_t, is a mixture of $h = h_c h_v$ normal distributions with means $M(k) x_t$, variances $V(k,m)$, and weights $p((s_t^c, s_t^v) = (k,m) | x_t)$. For a mixture of *distinct* normal distributions with positive weights, the means and variances of the normal distributions in the mixture and their weights are uniquely determined by the distribution of the mixture. If some of the weights were zero, then the normal distributions corresponding to those zero weights would not be determined, though the weights themselves would be. Because the $V(k,m)$ are distinct, the conditional distribution of y_t is a mixture of h distinct normal normal distributions.

If the conditional probabilities, $p((s_t^c, s_t^v) = (k,m)|x_t)$, were zero for all x_t, then the unconditional probabilities, $p((s_t^c, s_t^v) = (k,m))$, would also be zero. So by the hypotheses of Proposition 1, $p((s_t^c, s_t^v) = (k,m)|x_t)$ cannot be identically zero for all t. Because the weight associated with each (k,m) is non-zero for some t, this implies that the normal distributions and their weights are uniquely determined by the conditional distributions of the y_t.

If the system $\tilde{A}_0(\tilde{s}_t^c)y_t = \tilde{A}_+(\tilde{s}_t^c)x_t + \tilde{\Xi}^{-1}(\tilde{s}_t^v)\varepsilon_t$ generates observationally equivalent data as the system given by Equation (1), then there must exist functions $\pi_c(k,m,t) \in \{1,\cdots,h_c)$ and $\pi_v(k,m,t) \in \{1,\cdots,h_v)$ such that

$$M(k)x_t = \tilde{A}_0^{-1}(\pi_c(k,m,t))\tilde{A}_+(\pi_c(k,m,t))x_t \tag{8}$$

$$V(k,m) = \left(\tilde{A}_0^{-1}(\pi_c(k,m,t))\tilde{\Xi}^{-2}(\pi_v(k,m,t))\tilde{A}_0^{-1}(\pi_c(k,m,t))'\right)^{-1} \tag{9}$$

By the hypotheses of Proposition 1, the predetermined data span all of $\mathbb{R}^m$, so Equation (8) implies that

$$M(k) = \tilde{A}_0^{-1}(\pi_c(k,m,t))\tilde{A}_+(\pi_c(k,m,t)) \tag{10}$$

Since the left hand sides of Equations (9) and (10) do not depend on t, neither can the right hand side. Since the both the $M(k)$ and the $V(k,m)$ are distinct, this implies that neither π_c nor π_v depend on t. Since the left hand side of Equation (10) does not depend on m and the $M(k)$ are distinct, π_c does not depend on m. So we can write $\pi_c(k,m,t) = \pi_c(k)$, $\pi_v(k,m,t) = \pi(k,m)$ and Equations (9) becomes

$$V(k,m) = \left(\tilde{A}_0^{-1}(\pi_c(k))\tilde{\Xi}^{-2}(\pi_v(k,m))\tilde{A}_0^{-1}(\pi_c(k))'\right)^{-1} \tag{11}$$

Because the $V(k,m)$ are distinct, it must be the case that $\pi_v(k,m)$ is a permutation of $\{1,\cdots,h_v\}$, for each k. We can now apply Lemma 1 with $r = h_c$, $s = h_v$, and $\pi(k,m) = \pi_v(k,m)$. When the system is normalized so that $\sum_{m=1}^{h_v} \Xi^2(m) = I_n$, we take $A_k = A_0(k)$, $\tilde{A}_k = \tilde{A}_0(\pi_c(k))$, $D_m = \Xi^2(m)$, and $\tilde{D}_m = \tilde{\Xi}^2(m)$. When the system is normalized by $\sum_{m=1}^{h_v} \Xi^{-2}(m) = I_n$, we take $A_k = A_0^{-1}(k)'$, $\tilde{A}_k = \tilde{A}_0^{-1}(\pi_c(k))'$, $D_m = \Xi^{-2}(m)$, and $\tilde{D}_m = \tilde{\Xi}^{-2}(m)$. In either case, we obtain that $\pi_v(k,m)$ does not depend on k and can be written as $\pi_v(k,m) = \pi_v(m)$ and there exists a permutation matrix Q and diagonal matrices E_k, with plus or minus ones along the diagonal, such that $\tilde{A}_0(\pi_c(k)) = E_k Q' A_0(k)$ and $\Xi(\pi_v(m)) = Q'\Xi(m)Q$. Thus, Equation (10) implies that $\tilde{A}_+(\pi_c(k)) = E_k Q' A_+(k)$. So, under the hypotheses of Proposition 1, for almost all $A_0(\cdot)$, $A_+(\cdot)$, and $\Xi(\cdot)$, the system given by Equation (1) is identified up to the sign and ordering of the equations and the ordering of the regimes. ■

B The Likelihood, Posterior, Filtered and Smoothed Probabilities

In this appendix we derive the formulas for computing the the likelihood, the posterior, and filtered and smoothed probabilites. The likelihood is

$$\begin{aligned}
p(Y_T|Z_T,\theta) &= \frac{p(Y_T,Z_T|\theta)}{p(Z_T|\theta)} && (12)\\
&= \frac{\prod_{t=1}^{T} p(y_t,z_t|Y_{t-1},Z_{t-1},\theta)}{p(Z_T|\theta)} && (13)\\
&= \frac{\prod_{t=1}^{T}\sum_{s_t=1}^{h} p(s_t,y_t,z_t|Y_{t-1},Z_{t-1},\theta)}{p(Z_T|\theta)} && (14)\\
&= \frac{\prod_{t=1}^{T}\sum_{s_t=1}^{h} p(y_t|s_t,Y_{t-1},Z_t,\theta)p(z_t|s_t,Y_{t-1},Z_{t-1},\theta)p(s_t|Y_{t-1},Z_{t-1},\theta)}{p(Z_T|\theta)} && (15)\\
&= \frac{\prod_{t=1}^{T} p(z_t|Y_{t-1},Z_{t-1})}{p(Z_T|\theta)} \prod_{t=1}^{T}\sum_{s_t=1}^{h} p(y_t|s_t,Y_{t-1},Z_t,\theta)p(s_t|Y_{t-1},Z_{t-1},\theta), && (16)
\end{aligned}$$

where Equations (12), (13), and (15) follow from Bayes' rule, Equation (14) is obtained by integrating out s_t, and Equation (16) follows from Assumption 1 and a rearrangement of terms. Note that Assumption 1 does not, in general, imply that $p(Z_t|\theta) = p(Z_t)$. If $p(\theta)$ is the prior, then the posterior is

$$\begin{aligned}
P(\theta|Y_T,Z_T) &= \frac{p(Y_T,Z_T,\theta)}{p(Y_T,Z_T)} && (17)\\
&= \frac{p(Y_T|Z_T,\theta)p(Z_T|\theta)p(\theta)}{p(Y_T,Z_T)} && (18)\\
&= \frac{\prod_{t=1}^{T} p(z_t|Y_{t-1},Z_{t-1})}{p(Y_T,Z_T)} p(\theta)\prod_{t=1}^{T}\sum_{s_t=1}^{h} p(y_t|s_t,Y_{t-1},Z_t,\theta)p(s_t|Y_{t-1},Z_{t-1},\theta), && (19)
\end{aligned}$$

where Equations (17) and (18) follow from Bayes rule and Equation (19) follows by substituting the expression for the likelihood and canceling and rearranging terms. So, the posterior is proportional to

$$p(\theta)\prod_{t=1}^{T}\sum_{s_t=1}^{h} p(y_t|s_t,Y_{t-1},Z_t,\theta)p(s_t|Y_{t-1},Z_{t-1},\theta).$$

The recursive formulas for the the Hamilton filter are derived next.

$$\begin{aligned}
p(s_{t+1}|Y_t,Z_t,\theta) &= \sum_{s_t=1}^{h} p(s_{t+1},s_t|Y_t,Z_t,\theta) && (20)\\
&= \sum_{s_t=1}^{h} p(s_{t+1}|s_t,Y_t,Z_t,\theta)p(s_t|Y_t,Z_t,\theta), && (21)
\end{aligned}$$

where Equation (20) is obtained by integrating out s_t and Equation (21) follows from Bayes' rule.

$$\begin{aligned}
p(s_t|Y_1^t,Z_1^t,\theta) &= \frac{p(s_t,y_t,z_t|Y_1^{t-1},Z_1^{t-1},\theta)}{p(y_t,z_t|Y_1^{t-1},Z_1^{t-1},\theta)} && (22)\\
&= \frac{p(s_t,y_t,z_t|Y_1^{t-1},Z_1^{t-1},\theta)}{\sum_{s_t=1}^{h} p(s_t,y_t,z_t|Y_1^{t-1},Z_1^{t-1},\theta)} && (23)\\
&= \frac{p(y_t|s_t,Y_1^{t-1},Z_1^{t},\theta)p(z_t|s_t,Y_1^{t-1},Z_1^{t-1},\theta)p(s_t|Y_1^{t-1},Z_1^{t-1},\theta)}{\sum_{s_t=1}^{h} p(y_t|s_t,Y_1^{t-1},Z_1^{t},\theta)p(z_t|s_t,Y_1^{t-1},Z_1^{t-1},\theta)p(s_t|Y_1^{t-1},Z_1^{t-1},\theta)} && (24)\\
&= \frac{p(z_t|Y_1^{t-1},Z_1^{t-1})p(y_t|s_t,Y_1^{t-1},Z_1^{t},\theta)p(s_t|Y_1^{t-1},Z_1^{t-1},\theta)}{p(z_t|Y_1^{t-1},Z_1^{t-1})\sum_{s_t=1}^{h} p(y_t|s_t,Y_1^{t-1},Z_1^{t},\theta)p(s_t|Y_1^{t-1},Z_1^{t-1},\theta)} && (25)\\
&= \frac{p(y_t|s_t,Y_1^{t-1},Z_1^{t},\theta)p(s_t|Y_1^{t-1},Z_1^{t-1},\theta)}{\sum_{s_t=1}^{h} p(y_t|s_t,Y_1^{t-1},Z_1^{t},\theta)p(s_t|Y_1^{t-1},Z_1^{t-1},\theta)}, && (26)
\end{aligned}$$

where Equations (22) and (24) follow from Bayes' rule, Equation (23) is obtained by integrating out s_t in the denominator, Equation (25) follows from Assumption 1 and rearranging terms, and Equation (26) follows by canceling terms.

Before deriving the formulas for the smoothed probabilities, we must first develop more notation. For $1 \leq \tau \leq t \leq T$, let S_τ^t denote $(s_\tau, s_{\tau+1}, \cdots, s_t)$. We also need the following assumption:

Assumption 2

$$p(s_t|Y_{t-1}, Z_{t-1}, S_{t-1}, \theta) = p(s_t|Y_{t-1}, Z_{t-1}, s_{t-1}, \theta) \tag{27}$$

$$p(y_t|Y_{t-1}, Z_{t-1}, S_{t-1}, \theta) = p(y_t|Y_{t-1}, Z_{t-1}, s_{t-1}, \theta) \tag{28}$$

For the models discussed in this paper, Assumption 2 holds. It is possible to weaken Assumption 2 by replacing s_{t-1} on the right hand sides of Equations (27) and (28) with S_{t-k}^{t-1}, for some fixed value of $k \geq 1$.

It follows from Bayes' rule and Assumptions 1 and 2 Equation (27) that for $1 \leq \tau \leq t \leq T$

$$p(y_t, z_t|S_\tau^t, Y_{t-1}, Z_{t-1}, \theta) = p(y_t|s_t, Y_{t-1}, Z_t, \theta) p(z_t|Y_{t-1}, Z_{t-1}). \tag{29}$$

So, $1 \leq \tau_0 \leq \tau_1 \leq t \leq T$

$$p(y_t, z_t|S_{\tau_0}^t, Y_{t-1}, Z_{t-1}, \theta) = p(y_t, z_t|S_{\tau_1}^t, Y_{t-1}, Z_{t-1}, \theta). \tag{30}$$

For $1 \leq t+1 < \tau \leq T$,

$$p(s_t|S_{t+1}^\tau, Y_\tau, Z_\tau, \theta) = \frac{p(s_t, s_\tau, y_\tau, z_\tau|S_{t+1}^{\tau-1}, Y_{\tau-1}, Z_{\tau-1}, \theta)}{p(s_\tau, y_\tau, z_\tau|S_{t+1}^{\tau-1}, Y_{\tau-1}, Z_{\tau-1}, \theta)} \tag{31}$$

$$= \frac{p(y_\tau, z_\tau|S_t^\tau, Y_{\tau-1}, Z_{\tau-1}, \theta) p(s_\tau|S_t^{\tau-1}, Y_{\tau-1}, Z_{\tau-1}, \theta) p(s_t|S_{t+1}^{\tau-1}, Y_{\tau-1}, Z_{\tau-1}, \theta)}{p(y_\tau, z_\tau|S_{t+1}^\tau, Y_{\tau-1}, Z_{\tau-1}, \theta) p(s_\tau|S_{t+1}^{\tau-1}, Y_{\tau-1}, Z_{\tau-1}, \theta)} \tag{32}$$

$$= \frac{p(s_\tau|s_{\tau-1}, Y_{\tau-1}, Z_{\tau-1}, \theta) p(s_t|S_{t+1}^{\tau-1}, Y_{\tau-1}, Z_{\tau-1}, \theta)}{p(s_\tau|s_{\tau-1}, Y_{\tau-1}, Z_{\tau-1}, \theta)} \tag{33}$$

$$= p(s_t|S_{t+1}^{\tau-1}, Y_{\tau-1}, Z_{\tau-1}, \theta), \tag{34}$$

where Equations (31) and (32) follow from Bayes' rule, Equation (33) follows from Equation (30), cancellation, and Assumption 2 Equation (28), and Equation (34) follows from cancellation. Thus, by a recursive argument,

$$p(s_t|S_{t+1}^T, Y_T, Z_T, \theta) = p(s_t|s_{t+1}, Y_{t+1}, Z_{t+1}, \theta). \tag{35}$$

But

$$p(s_t|s_{t+1}, Y_{t+1}, Z_{t+1}, \theta) = \frac{p(s_t, s_{t+1}, y_{t+1}, z_{t+1}|Y_t, Z_t, \theta)}{p(s_{t+1}, y_{t+1}, z_{t+1}|Y_t, Z_t, \theta)} \tag{36}$$

$$= \frac{p(y_{t+1}, z_{t+1}|s_{t+1}, s_t, Y_t, Z_t, \theta)p(s_{t+1}|s_t, Y_t, Z_t, \theta)p(s_t|Y_t, Z_t, \theta)}{p(y_{t+1}, z_{t+1}|s_{t+1}, Y_t, Z_t, \theta)p(s_{t+1}|Y_t, Z_t, \theta)} \tag{37}$$

$$= \frac{p(s_{t+1}|s_t, Y_t, Z_t, \theta)p(s_t|Y_t, Z_t, \theta)}{p(s_{t+1}|Y_t, Z_t, \theta)}, \tag{38}$$

where Equations (36) and (37) follow from Bayes' rule, and Equation (38) follows from Equation (30) and cancellation. Thus

$$p(s_t|S_{t+1}^T, Y_T, Z_T, \theta) = \frac{p(s_{t+1}|s_t, Y_t, Z_t, \theta)p(s_t|Y_t, Z_t, \theta)}{p(s_{t+1}|Y_t, Z_t, \theta)}. \tag{39}$$

Since

$$p(s_t|Y_T, Z_T, \theta) = \sum_{S_{t+1}^T} p(s_t, S_{t+1}^T|Y_T, Z_T, \theta) \tag{40}$$

$$= \sum_{S_{t+1}^T} p(s_t|S_{t+1}^T, Y_T, Z_T, \theta)p(S_{t+1}^T|Y_T, Z_T, \theta) \tag{41}$$

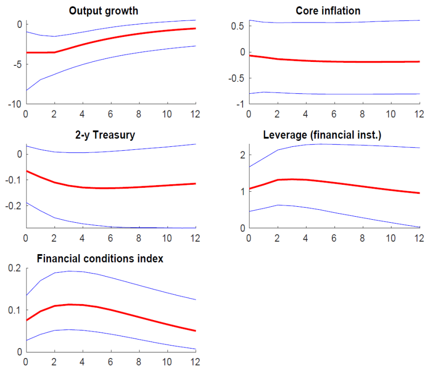
$$= p(s_t|Y_t, Z_t, \theta) \sum_{s_{t+1}=1}^{h} \left(\frac{p(s_{t+1}|s_t, Y_t, Z_t, \theta)p(s_{t+1}|Y_T, Z_T, \theta)}{p(s_{t+1}|Y_t, Z_t, \theta)} \times \sum_{S_{t+2}^T} p(S_{t+2}^T|s_{t+1}, Y_T, Z_T, \theta) \right) \tag{42}$$

$$= p(s_t|Y_t, Z_t, \theta) \sum_{s_{t+1}=1}^{h} \frac{p(s_{t+1}|s_t, Y_t, Z_t, \theta)p(s_{t+1}|Y_T, Z_T, \theta)}{p(s_{t+1}|Y_t, Z_t, \theta)}, \tag{43}$$

where Equation (40) is obtained by integrating out S_{t+1}^T, Equation (41) follows from Bayes' rule, Equation (42) follows from Equation (39) and rearranging terms, and Equation (43) follows because $p(S_{t+2}^T|s_{t+1}, Y_T, Z_T, \theta)$ is a density and integrates (sums) to one.

C Impulse responses with narrative restrictions

Figure A.1: Impulse Responses to a financial shock in the financial constraint regime, narrative sign restrictions, GSIBs

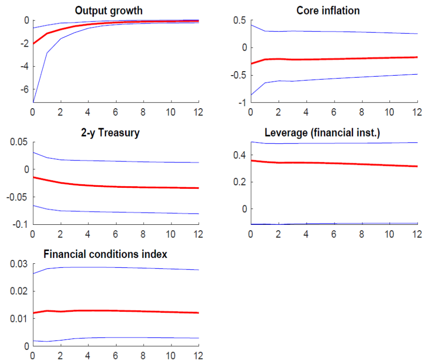


Note: *Impulse Responses to financial shock, narrative sign restrictions, (conditional on staying in) the financial constraint regime; red line: median response, blue lines: lower and upper bound of the 68 percent error bounds; financial shock: 1 std shock to financial conditions index; identification with contemporaneous sign restrictions, median and 68% error bands, output growth (IP), core inflation (CPI), 2-y Treasury yield, market leverage, Financial conditions index (Chicago Fed)*

Figure A.1 presents the results for the financial constraint regime for a model with leverage of GSIBs and identification with narrative sign restrictions, where the shock is identified that explains most of the variation in output growth during GFC.

Responses to a financial shock (median) show a protracted negative output response. Market leverage initially increases, then declines due to deleveraging. We arrive at the same conclusion as in our baseline specification for the GSIBs: Deleveraging can lead to amplification effects with adverse implications for the real economy. In normal times, the responses to a financial shock (median) indicate small, nonpersistent negative output response and insignificant market leverage (Figure A.2) as with the standard sign restriction approach.

Figure A.2: Impulse Responses to a financial shock in normal regime, narrative sign restrictions, GSIBs



Note: *Impulse Responses to financial shock, narrative sign restrictions, (conditional on staying in) the financial constraint regime; red line: median response, blue lines: lower and upper bound of the 68 percent error bounds; financial shock: 1 std shock to financial conditions index; identification with contemporaneous sign restrictions, median and 68% error bands, output growth (IP), core inflation (CPI), 2-y Treasury yield, market leverage, Financial conditions index (Chicago Fed)*

D Time-varying Probabilities with error bands

Figure A.3: Time-varying probability of staying in low interest rate regime with 68 % error bands, conditional of being in that regime, GSIBs