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Quantifying "Quantitative Tightening" (QT): How Many Rate Hikes Is QT Equivalent To?

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Abstract: How many interest rate hikes is quantitative tightening (QT) equivalent to? In this paper, I examine this question based on the preferred-habitat model in Vayanos and Vila (2021). I define the equivalence between rate hikes and QT such that they both have the same impact on the 10-year yield. Based on the model calibrated to fit the nominal Treasury data between 1999 and 2022, I show that a passive roll-off of $2.2 trillion over three years is equivalent to an increase of 29 basis points in the current federal funds rate at normal times. However, during a crisis period with risk aversion being doubled, it is equivalent to a 74 basis point increase. I also quantify the effect of QT implemented by active sales. Lastly, based on the model-based estimates, I show that if the Treasury were to issue bills to offset maturing securities, the resulting equivalent rate hikes in the current federal funds rate would decrease dramatically to 7.4 (12.6) basis points during normal (crisis) periods.

JEL classification: E43, E44, E52, E58, G12

Key words: monetary policy, quantitative tightening, QT, quantitative easing, QE, rate hikes, preferred-habitat, reserves, reverse repo

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1 Introduction

When the COVID-19 pandemic hit, the Federal Reserve (Fed) lowered interest rates to near zero and launched a suite of emergency lending programs, including an open-ended quantitative easing (QE) program, to support the economy and stabilize financial markets. As a result of this program, the Fed's holdings of Treasury securities and mortgage-backed securities increased from \$4.4 trillion in March 2020 to \$8.5 trillion in March 2022. Since then, the economy has recovered from the severe two-month recession in 2020, but inflation has surged to a 40-year record-high level. To tame elevated inflation, at its March, May, and June 2022 Federal Open Market Committee (FOMC) meetings, the Fed raised the benchmark federal funds rate by 25, 50, and 75 basis points, respectively, to a range between 1.50% and 1.75%. Additionally, at its May FOMC meeting, it was announced that the Fed will start the process of shrinking its \$9 trillion balance sheet on June 1, 2022, a process also known as "quantitative tightening"(QT).[1]

To assess the overall stance of monetary policy, we need to take into consideration the effects of both the interest rate policy and the balance sheet policy. A natural question then arises: what is the equivalence between interest rate hikes and QT? Answering this question is challenging for several reasons. First, the interest rate policy and the balance sheet policy act upon securities of different maturity, with the former influencing primarily short-term interest rates, whereas the latter works to influence primarily longer-term rates. This means a specific definition of "equivalence" between these policy tools is required. Given the specifics of QT (e.g., size and pace), We define a certain sized rate hike as equivalent to QT if both the rate hike and QT have the *same* expected effect on the 10-year yield. Second, our experience with QT is limited: historically, there was only one round of QT,

[1]According to a press release issued in conjunction with the May FOMC statement, the monthly cap for Treasury securities (agency debt and agency mortgage-backed securities) will initially be set at \$30 (\$17.5) billion per month and after three months will increase to \$60 (\$35) billion per month. For details, see Board of Governors of the Federal Reserve System, "Plans for Reducing the Size of the Federal Reserve's Balance Sheet," press release, May 4, 2022, https://www.federalreserve.gov/newsevents/pressreleases/monetary20220504b.htm.

which was initiated in October 2017 and ended abruptly in September 2019 following a liquidity crunch in the repo market. Third, the estimates based on QE—a reversal of QT—are helpful but cannot be used directly without modification because of asymmetries between QT and QE (see our discussion below). Lastly, identifying the effects of QT on interest rates, as distinguished from those of other policy actions (e.g., forward guidance), is empirically challenging. In addition, even if we are able to obtain precise empirical estimates of the effects of QT on interest rates, they cannot be easily adapted for different market conditions in order to conduct an analysis of counterfactuals.

In this paper, we address these challenges by examining the question of the equivalence between rate hikes and QT based on the preferred-habitat model in the seminal work by Vayanos and Vila (2021) (hereafter, V&V). The two channels in the model—a "duration" channel and a "local-supply" channel—through which supply/demand changes could affect bond yields make it especially suitable for examining the effects of QT or QE. Consider QT as an example. Under the duration channel, suppose arbitrageurs need to hold more debt or longer-term debt as a result of QT. The duration of their portfolios would then increase, and they would be more exposed to interest rate risks. As a result, arbitrageurs would require a larger risk premium in exchange for bearing the extra risk, thus driving up long-term yields. Furthermore, such effects could be greatly magnified under the local-supply channel: at heightened risk aversion, arbitrageurs refrain from engaging in active transactions, and the market becomes segmented to the extent that changes in Treasury net supply arising from QT would have the largest effects on the yields of Treasury securities in the maturity sector in which bonds are being rolled off or actively sold as a result of QT.

To infer QT-equivalent rate hikes, we first define what "equivalence" means in our context. Given the specifics of QT (e.g., size and pace), a certain number of rate hikes are defined as equivalent to QT if both rate hikes and QT achieve the *same* effect on the 10-year yield. In this paper, we focus on only nominal Treasury securities, which account for more than 60% of securities held by the Fed. The methodology in this paper can be applied to real Treasury securities or mortgage-backed securities as well. For simplicity, we drop the monthly cap in estimation which binds only occasionally.

We consider the following baseline scenario for estimating QT-equivalent rate hikes in which we assume that the Fed's balance sheet rolls off passively for 3 years, meaning no reinvestment for maturing bonds. Based on data on the Federal Reserve System Open Market Account (SOMA) portfolio,[2] the roll-off over 3 years amounts to a $2.2 trillion reduction in the Fed's balance sheet. We assume normal market conditions between January 1999 and March 2022, which is the sample period used to calibrate the model.

We find that in the baseline scenario under normal market conditions, a $2.2 trillion passive roll-off over 3 years is equivalent to an immediate increase of 29 basis points in the current federal funds rate. In fact, both the roll-off and the 29-basis-point rate hikes have the same impact—an increase of 6 basis points—on the 10-year yield.

We also consider other scenarios with QT of different sizes or duration. For example, if passive roll-off were to last for 7 years, the Fed's balance sheet would be reduced by $3.3 trillion. In this alternative scenario, we show that under normal market conditions, the equivalent increase in the current federal funds rate is 35 basis points.

Besides passive roll-off, we also consider active sales, an alternative way to implement QT, meaning that bonds held in the SOMA portfolio are sold before maturity. Passive roll-off and active sales have two major differences. First, they have different effects on the duration risk borne by the private sector. Under active sales, it is the *remaining* maturity of bonds bought by the private sector that adds to investors' duration risk, whereas under passive roll-off, it is the *initial* maturity at issuance that matters, assuming the Treasury issues new bonds with the same initial maturity to offset maturing bonds. Second, the timing of impact is also different: the impact of active sales is front-loaded, whereas the impact of passive roll-off is back-loaded and takes place when bonds mature.

[2] The data on SOMA holdings can be found at Federal Reserve Bank of New York, "System Open Market Account Holdings of Domestic Securities," April 28, 2022, https://www.newyorkfed.org/markets/soma-holdings.

These two differences pose an interesting trade-off between passive roll-off and active sales. Based on the composition of the SOMA portfolio, we show that in the short run (e.g., 3 years or less), the effects from the duration difference are dominant, and thus passive roll-off has a stronger impact. The impact is greater is because bonds in the SOMA portfolio maturing in the near future are primarily Treasury notes with a longer initial maturity on average. On the other hand, the effects from the timing difference start to dominate in the medium and long run so that QT implemented through active sales has a stronger impact. Here, the impact is greater because earlier sales of long-term bonds have a front-loaded impact, as opposed to the back-loaded impact under passive roll-off.

Our quantitative estimation results suggest that under normal market conditions, QT with active sales is equivalent to an increase of 22 basis points in the current federal funds rate in the baseline scenario in which QT lasts for 3 years, but 47 basis points in the alternative scenario in which ($3.3 trillion) QT lasts for 7 years.

As a counterfactual, we recast the baseline scenario in a crisis period. We model a crisis period as one in which risk aversion of arbitrageurs increases sharply, implying a reduced risk-bearing capacity. Specifically, we assume that risk aversion increases twofold. In this case, the market is more volatile: the 10-year yield volatility increases from 1.3% to 1.7%—an almost 40% increase. At the same time, we find that a passive roll-off of $2.2 trillion QT over three years during a crisis period leads to a much larger effect (arising from the local-supply channel) compared to that under normal market conditions: it is equivalent to an immediate increase of 74 basis points in the current federal funds rate, as opposed to an increase of 29 basis points. One possible trigger for a crisis period with heightened risk aversion could be the shortage of reserves (e.g., the spike in repo rates in September 2019). When things continue to worsen (i.e., risk aversion becomes even greater), both market volatility and the impact of QT may increase dramatically to the extent that the Fed may need to slow down QT, or stop it completely (or even replace it with QE).

Lastly, the estimates of QT-equivalent rate hikes depend on how the Treasury finance the Fed's balance sheet rundown. In the baseline scenario, we assume that the Treasury will issue new securities with the same initial maturity as the maturing bonds. In another counterfactual analysis, we re-estimate QT-equivalent rate hikes when the Treasury issues different types of securities. We show that if the Treasury were to issue bills to offset maturing securities, the resulting equivalent rate hikes in the current federal funds rate would decrease dramatically to 7.4 (12.6) basis points under normal (crisis) market conditions.

This paper is closely related to Crawley et al. (2022) as both papers aim to quantify the equivalence between interest rate hikes and QT. The latter paper finds that reducing the size of the balance sheet by about \$2.5 trillion over the next few years would be roughly equivalent to raising the policy rate a little more than 50 basis points, whereas in this paper a 29-basis-point rate hike is found to be equivalent to a passive roll-off of \$2.2 trillion QT over three years.The similar estimates are obtained using different methodologies. Crawley et al. (2022) use the so-called FRB/US model and define the equivalence between interest rate hikes and QT in terms of their effects on longer-term interest rates, employment, and inflation. By contrast, we use the preferred-habitat model in Vayanos and Vila (2021) and define the equivalence in terms of the effects of both interest rate and balance sheet policies on the 10-year yield.

This paper belongs to the broader literature on the effects of unconventional monetary policy (e.g., QE or QT). Krishnamurthy and Vissing-Jorgensen (2011) decompose the effects of QE into various channels, namely, those of signaling, duration risk, liquidity, safety, etc. Using either event studies around the Fed's QE announcements or regression-based analyses, Gagnon et al. (2011), Krishnamurthy and Vissing-Jorgensen (2011), Hancock and Passmore (2011), Hamilton and Wu (2012), D'Amico and King (2013) find empirical evidence for significant effects of QE on a variety of asset prices.[3] The estimated effects of \$600 billion QE in these studies are in the range of 15 to 20 basis points (William, 2011). By contrast,

[3]See Swanson (2011) for the effects of "Operation Twists", Neely (2010) for the effects of QE on foreign bond yields, Gilchrist and Zakrajšek (2013) on corporate bond spreads, and D'Amico and Seida (2020) for the evolution of the effects of QE or QT over time and across market conditions.

our estimated effects of QT on the 10-year bond yield, renormalized to the size of \$600 billion, is much smaller—about 2.7 basis points.[4] The differences between our estimates for QT and those for QE in the literature may well result from the asymmetries between these two different types of operations. For example, QE typically involves active purchases and is implemented in a crisis period (e.g., the 2008-09 financial crisis or the pandemic), while QT is generally implemented through passive roll-off and in calm periods.

In the rest of the paper, we describe our methodology in details in Section 2. In Section 3, we present our main estimation results. We provide concluding remarks in Section 4.

2 Methodology

In this section, we describe in detail our methodology of estimating QT-equivalent rate hikes based on the V&V model. We first provide a brief sketch of the model and then characterize the impact of rate hikes and QT on bond yields. Lastly, we formally define the equivalence between QT and rate hikes. In the next section, we report our estimation results on QT-equivalent rate hikes based on the calibrated model.

2.1 The Vayanos-Vila Model

Suppose there is a continuum of bonds in zero supply with maturities between 0 and T in the Treasury bond market. Let $P_{t,\tau}$ and $y_{t,\tau}$ denote the time-t price and yield of the Treasury bond with maturity τ. The Vayanos-Vila model has two types of agents: arbitrageurs and preferred-habitat investors. Arbitrageurs maximize a mean-variance objective

[4] Our estimate of 2.7 basis points is obtained as follows $(0.6/15)/(2.2/24.4) \times 6$. In our calculation, we take into account changes in GDP; that is, a \$600 billion QE operation back then was 4% of \$15-trillion GDP, whereas a \$2.2 trillion QT operation now is 9% of \$24.4-trillion GDP.

over instantaneous changes in wealth:

$$\max_{\{X_{t,\tau}\}_{\tau=0}^{T}} E_t\left[dW_t\right] - \frac{a}{2} Var_t\left[dW_t\right],$$
$$s.t., dW_t = \left(W_t - \int_0^T X_{t,\tau} d\tau\right) r_t dt + \int_0^T X_{t,\tau} \frac{dP_{t,\tau}}{P_{t,\tau}} d\tau,$$

where W_t and $X_{t,\tau}$ denote, respectively, their wealth and position in the maturity-τ Treasury bond, and a represents arbitrageurs' risk aversion coefficient, a proxy for their risk-bearing capacity. The short rate r_t follows an exogenous Ornstein–Uhlenbeck process,

$$dr_t = \kappa_r\left(\overline{r} - r_t\right) dt + \sigma_r dB_{r,t},$$

where $\overline{r}$ denotes its long-run average, and κ_r and σ_r are, respectively the mean-reversion and diffusion parameters.

Preferred-habitat investors constitute maturity clienteles: the clientele for maturity τ demand only the bond with maturity τ with a demand below

$$Z_{t,\tau} = \alpha\left(\tau\right) \tau y_{t,\tau} - \beta_{t,\tau},$$

where $\alpha\left(\tau\right)$ and $\beta_{t,\tau}$ represent the demand slope and demand intercept, respectively. The demand intercept $\beta_{t,\tau}$ takes the following form:

$$\beta_{t,\tau} = \theta_0\left(\tau\right) + \theta\left(\tau\right) \beta_t,$$

and is driven by a demand risk factor β_t that follows an Ornstein–Uhlenbeck process:

$$d\beta_t = -\kappa_\beta \beta_t dt + \sigma_\beta dB_{\beta,t}.$$

We use the same specification for $\alpha\left(\tau\right)$, $\theta_0\left(\tau\right)$, and $\theta\left(\tau\right)$ as in Vayanos and Vila (2021).

That is, we set $\alpha(\tau) = \alpha e^{-\delta_\alpha \tau}$, $\theta_0(\tau) = \theta_0\left(e^{-\delta_\alpha \tau} - e^{-\delta_\theta \tau}\right)$, and $\theta(\tau) = \theta\left(e^{-\delta_\alpha \tau} - e^{-\delta_\theta \tau}\right)$ for $\tau \le T$.

By the same argument as in Vayanos and Vila (2021), we can show that bond prices are an exponential-linear function of the short rate r_t and the demand risk factor β_t. In particular, the price of the maturity-τ bond is given by $P_{t,\tau} = \exp\left(-\left[A_r(\tau) r_t + A_\beta(\tau)\beta_t + C(\tau)\right]\right)$, where the coefficients $A_r(\tau)$, $A_\beta(\tau)$, and $C(\tau)$ are endogenously determined (see Appendix A). As a result, its yield $y_{t,\tau}$ is affine in the state variables:

$$y_{t,\tau} = \frac{1}{\tau}\left[A_r(\tau) r_t + A_\beta(\tau)\beta_t + C(\tau)\right].$$

2.2 Rate Hikes

We consider two types of rate hikes in our analysis. First, we consider an immediate increase of Δr in the current federal funds rate. The resulting impact on the maturity-τ yield is given by

$$\Delta y_{t,\tau}^{(I)} = \frac{A_r(\tau)}{\tau} \cdot \Delta r. \tag{1}$$

The second type of rate hikes is through forward guidance about the future path of the short rate. Following Vayanos and Vila (2021), we assume that a forward guidance announcement causes an unanticipated change $\Delta\bar{r}$ in the long-run mean $\bar{r}$, which takes place at time zero and decays deterministically to zero at rate $\kappa_{\bar{r}}$. The impact of the announcement of a rate hike causes the following change in the maturity-τ yield:

$$\Delta y_{t,\tau}^{(II)} = \frac{A_{\bar{r}}(\tau)}{\tau} \cdot \Delta\bar{r} e^{-\kappa_{\bar{r}} t}, \tag{2}$$

where $A_{\bar{r}}(\tau)$ is endogenously determined (see Appendix A for its expression).

2.3 Quantitative Tightening (QT)

We now turn to QT and examine its impact on bond yields. Note that QT can be implemented through either active sales or passive roll-off.

Consider active sales first. Following Vayanos and Vila (2021), we model QT as an unanticipated increase $\Delta\theta_0(\tau)$ in the intercept of preferred-habitat demand, which takes place at time zero and decays deterministically to zero at rate κ_θ. As a starting point, we assume that the SOMA portfolio consists of bonds with maturity τ_{QT} only. This assumption will be relaxed shortly and replaced by the actual maturity distribution of the SOMA portfolio. That is, the unanticipated increase $\Delta\theta_0(\tau)$ is a Dirac function:

$$\Delta\theta_0(\tau) = \Delta_\theta \cdot 1_{\{\tau=\tau_{QT}\}}, \tag{3}$$

where Δ_θ represents the size of QT relative to GDP. The resulting impact on the maturity-τ yield is given by

$$\begin{aligned} \Delta y_{t,\tau}^{QT,active}(\tau_{QT}) &= \frac{A_\theta(\tau)}{\tau} \cdot \Delta_\theta e^{-\kappa_\theta t} \qquad (4) \\ = \quad \Delta_\theta \cdot & \left(\chi_r^{QT}(\tau_{QT}) \frac{\int_0^\tau e^{-\kappa_\theta(\tau-u)} A_r(u)\, du}{\tau} + \chi_\beta^{QT}(\tau_{QT}) \frac{\int_0^\tau e^{-\kappa_\theta(\tau-u)} A_\beta(u)\, du}{\tau} \right) \cdot e^{-\kappa_\theta t}, \end{aligned}$$

where the expressions for $A_\theta(\tau)$, $\chi_r^{QT}(\cdot)$, and $\chi_\beta^{QT}(\cdot)$ are given in Appendix A. We can see that through active sales, QT has a front-loaded impact on bond yields, which is largest upon impact at time 0 and then decays afterward.

In the general case in which the SOMA portfolio has a maturity distribution $\omega(\tau_{QT})$ with a total amount Δ_θ (expressed as a ratio relative to GDP), then the overall impact of QT with active sales is given by

$$\Delta y_{t,\tau}^{QT,active} = \int_0^T \Delta y_{t,\tau}^{QT,active}(\tau_{QT})\, \omega(\tau_{QT})\, d\tau_{QT}. \tag{5}$$

Next, we turn to the alternative implementation of QT: passive roll-off. Under a passive roll-off, the Fed will simply let the maturity-τ_{QT} bonds in the SOMA portfolio expire at date τ_{QT} without reinvestment. Let τ_{QT}^* denote the original maturity of the bonds at issuance. Suppose that the Treasury issues new bonds with the same maturity τ_{QT}^* to offset the maturing issues. Because the Fed does not reinvest, the new bonds with maturity τ_{QT}^* will be held by the private sector. We thus model the impact of a passive roll-off as an unanticipated increase $\Delta\theta_0(\tau)$ that concentrates on maturity τ_{QT}^*:

$$\Delta\theta_0(\tau) = \Delta_\theta \cdot 1_{\{\tau=\tau_{QT}^*\}}. \tag{6}$$

Note that the above shock $\Delta\theta_0(\tau)$ in (6) differs from the previous shock in (3) along two dimensions: (1) The impact of the shock under passive roll-off concentrates on the original maturity at issuance τ_{QT}^*, whereas the shock under active sales concentrates on the remaining maturity τ_{QT}. (2) The timing of the shocks is also different: the former shock under passive roll-off takes place at time τ_{QT} when the bonds mature, whereas the latter shock under active sales occurs at 0, the time of sales.

These two differences pose an interesting trade-off between passive roll-off and active sales. On the one hand, the first difference implies a stronger impact of passive roll-off because original maturity at issuance is always greater than the remaining time to maturity (i.e., $\tau_{QT}^* > \tau_{QT}$), and the resulting supply shock at the long end of the maturity spectrum has a larger impact on the 10-year bond yield. On the other hand, the second difference implies a stronger impact of active sales because the resulting impact is front-loaded without delay. As we will show shortly, if QT has a relatively short duration (e.g., 3 years or less), the first effect dominates, meaning that passive roll-off has a larger impact than active sales. But the reverse is true for QT with a medium to long duration: active sales has a stronger impact because the second effect dominates in the medium to long run.

We assume that the shock decays deterministically to zero at rate κ_θ^*. Because the passive roll-off starts at τ_{QT}, its impact on the maturity-τ yield at time $t \geq \tau_{QT}$ is given by

$$\begin{aligned} & \Delta y_{t,\tau}^{QT,passive}\left(\tau_{QT}, \tau_{QT}^{*}\right) \\ = \; & \Delta_{\theta} \cdot \left(\chi_{r}^{QT}\left(\tau_{QT}^{*}\right) \frac{\int_{0}^{\tau} e^{-\kappa_{\theta}(\tau-u)} A_{r}(u)\, du}{\tau} + \chi_{\beta}^{QT}\left(\tau_{QT}^{*}\right) \frac{\int_{0}^{\tau} e^{-\kappa_{\theta}(\tau-u)} A_{\beta}(u)\, du}{\tau} \right) \cdot e^{-\kappa_{\theta}^{*}\left(t-\tau_{QT}\right)}. \end{aligned}$$

In the general case in which the SOMA portfolio has a maturity distribution $\omega\left(\tau_{QT}, \tau_{QT}^{*}\right)$ with a total amount Δ_{θ} (expressed as a ratio relative to GDP), then the overall impact of QT with active sales is given by

$$\Delta y_{t,\tau}^{QT,passive} = \int_{0}^{T} \int_{0}^{T} \Delta y_{t,\tau}^{QT,passive}\left(\tau_{QT}, \tau_{QT}^{*}\right) \omega\left(\tau_{QT}, \tau_{QT}^{*}\right) d\tau_{QT} d\tau_{QT}^{*}. \tag{7}$$

2.4 Definition of QT-equivalent Rate Hikes

We define the equivalence between QT and rate hikes as follows. For QT and a certain number of rate hikes to be equivalent, they must have the same impact on bond yields of a particular maturity τ^{*}. In our estimation, we choose $\tau^{*} = 10$.[5]

Consider QT with active sales as an example. Suppose QT starts at time 0 and ends at time T_{QT}. Its impact on the maturity-τ^{*} yield at time 0 is given by $\Delta y_{0,\tau^{*}}^{QT,active}$. To find the number of immediate rate hikes in the current federal funds rate that is equivalent to QT, we can solve for Δr such that

$$\Delta y_{0,\tau^{*}}^{(I)} = \frac{A_{r}\left(\tau^{*}\right)}{\tau^{*}} \Delta r = \Delta y_{0,\tau^{*}}^{QT,active}. \tag{8}$$

Similarly, for the second type of rate hikes (i.e., forward guidance), the QT-equivalent rate

[5] We also consider an alternative definition under which the equivalence between QT and rate hikes requires them to have the same impact on the *average* yield within a maturity interval (e.g., between 5 and 15 years). The estimation results are quantitatively similar.

hikes are given by $\Delta\bar{r}$ such that

$$\Delta y_{0,\tau^*}^{(II)} = \frac{A_{\bar{r}}(\tau^*)}{\tau^*}\Delta\bar{r} = \Delta y_{0,\tau^*}^{QT,active}. \tag{9}$$

To find the number of rate hikes that is equivalent to a passive roll-off, we just need to replace $\Delta y_{0,\tau^*}^{QT,active}$ by $\Delta y_{T,\tau^*}^{QT,passive}$. That is,

$$\begin{aligned} \Delta y_{0,\tau^*}^{(I)} &= \frac{A_r(\tau^*)}{\tau^*}\Delta r = \Delta y_{T_{QT},\tau^*}^{QT,passive}, & (10) \\ \Delta y_{0,\tau^*}^{(II)} &= \frac{A_{\bar{r}}(\tau^*)}{\tau^*}\Delta\bar{r} = \Delta y_{T_{QT},\tau^*}^{QT,passive}. & (11) \end{aligned}$$

Note that we evaluate the impact of a passive roll-off on the maturity-τ^* yield at the end of the QT period, T_{QT}, because its impact is back-loaded.

3 Estimation Results

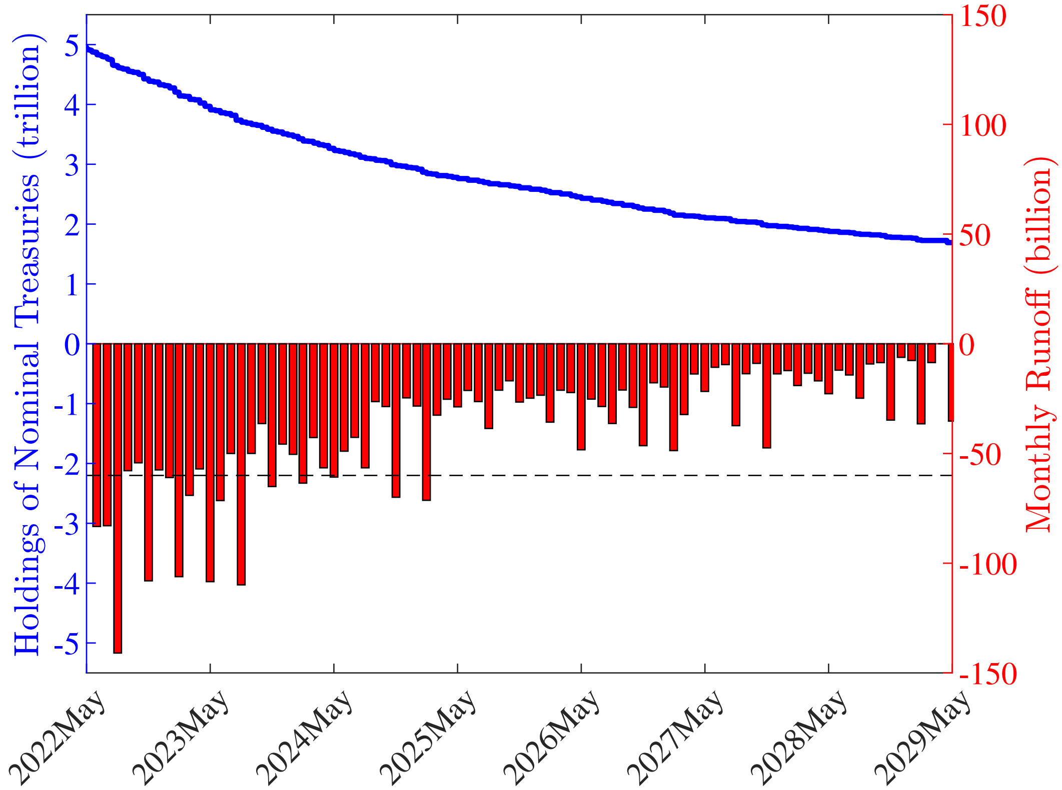
In this section, we first calibrate the model and then use the calibrated model to estimate the QT-equivalent rate hike based on the methodology introduced in the previous section.

3.1 Calibration

We first calibrate the model to fit nominal Treasury data between January 1999 and March 2022. The calibrated model closely matches target moments on yields and trading volume, such as the volatilities of the 1-year yield level and its annual changes, average volatilities across maturities of bond yields and their annual changes, as well as the correlation between annual changes to the 1-year yield and other yields. See Appendix B for details about the calibration and calibrated parameter values. The calibrated model enables us to estimate QT-equivalent rate hikes under normal market conditions during the last two decades.

Next, we consider three scenarios of passive roll-off. In Figure 1, we plot the total amount of nominal Treasury securities held in the SOMA portfolio (solid blue line) under passive roll-off, as well as the monthly amount of securities rolled off (red bars). The dashed line represents the $60 billion cap. As shown in the figure, under passive roll-off, starting at $5 trillion at the end of May 2022, the size of the SOMA portfolio would decrease to $2.8 trillion in 3 years, $2.1 trillion in 5 years, and $1.7 trillion in 7 years. The resulting three scenarios are as follows: the SOMA portfolio runs down by $2.2 trillion within 3 years (Scenario 1), by $2.8 trillion within 5 years (Scenario 2), by $3.3 trillion within 7 years (Scenario 3).

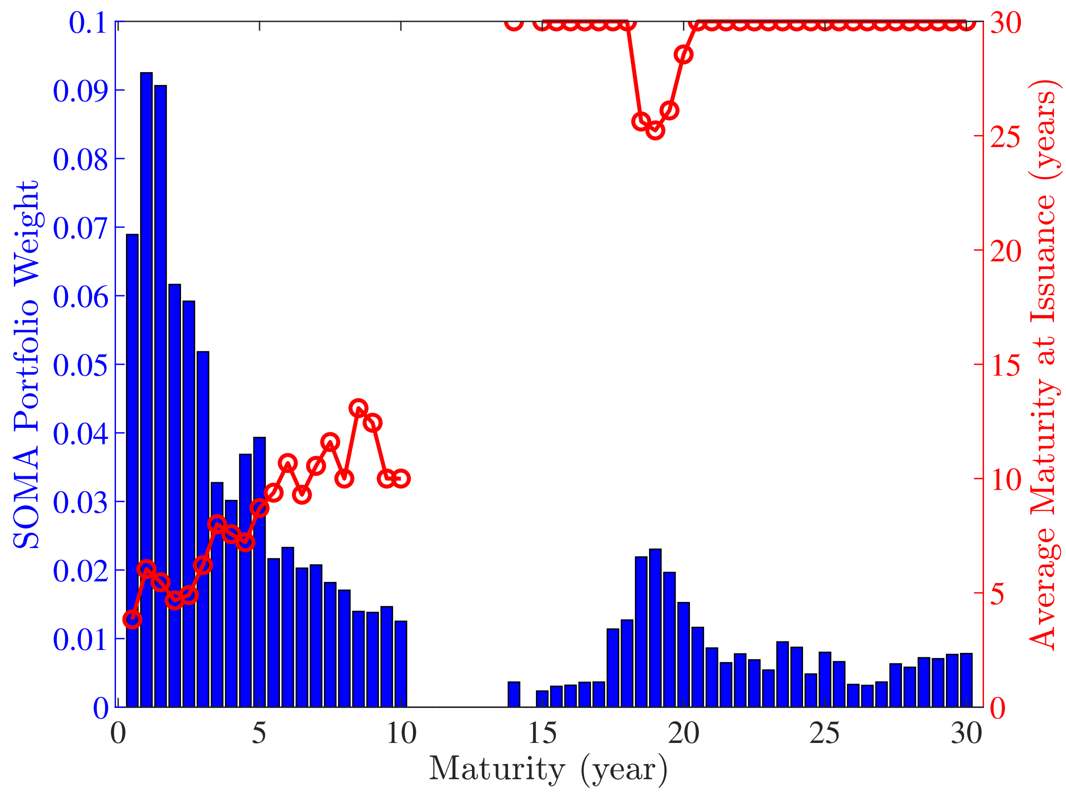
FIGURE 1: Passive Roll-off of SOMA Portfolio



NOTE: This figure plots the total amount of nominal Treasury securities held in the SOMA portfolio (solid blue line) under passive roll-off, as well as the monthly amount of securities rolled off (red bars). The dashed line represents the $60 billion cap.

Besides the primitive model parameters, we also need to calibrate the parameters associated with rate hikes and QT (e.g., decay factors). Consider Scenario 1 as an example in which the Fed's balance sheet is reduced by $2.2 trillion over 3 years. U.S. GDP is $24.4 trillion as of March 2022. Therefore, the size of QT relative to GDP implies that $\Delta_\theta = 2.2/24.4 = 0.09$. To calibrate the decay factor κ_θ^*, we use the maturity distribution of the SOMA portfolio as of the end of March 2022, which is plotted by the blue bars in Figure 2. As shown in the figure, the value-weighted average of maturity is 8 years. In addition, the size of the portfolio under a passive roll-off would decrease from $5 trillion at the end of May 2022 to $2.5 trillion at the end of April 2026. Therefore, we set the decay factor $\kappa_\theta^* = \log 2/4 = 0.17$ so that its half-life is 4 years.

FIGURE 2: Maturity Distribution of SOMA Portfolio (3/30/2022)



NOTE: This figure plots the maturity distribution of the SOMA portfolio as of the end of March 2022 (blue bars) and the value-weighted average maturity at issuance (red circles).

For QT with active sales, we set its decay factor κ_θ to match the half-life of the actual life span of QT. In the baseline scenario, QT spans 3 years, so we set $\kappa_\theta = \log 2/1.5 = 0.46$. To put rate hikes and QT on the same footing, we assume that the policy shocks decay at the same rate; that is, $\kappa_{\overline{r}} = \kappa_{\theta}$. These parameters can be calibrated similarly to match the half-life of QT in other scenarios.

3.2 Estimation Results under Normal Market Conditions

We first estimate QT-equivalent rate hikes under normal market conditions based on the calibrated model. Table 1 reports estimation results for both passive roll-off and active sales in all three scenarios.

Consider passive roll-off in Scenario 1 first. It shows that a passive roll-off of $2.2 trillion within 3 years increases the 10-year yield by 6 basis points. On the other hand, to generate the same impact on the 10-year yield, the current federal funds rate must be increased by 29 basis points, or the future rate path must be increased by 75 basis points within the same period.

The impact is even greater when larger QT is implemented over a longer period. For example, a passive roll-off of $2.8 trillion with 5 years (Scenario 2) or $3.3 trillion within 7 years (Scenario 3) would increase the 10-year yield by about 7 basis points, which is equivalent to an immediate 35-basis-point increase in the current federal funds rate.

We also estimate QT-equivalent rate hikes in terms of their effects on the future path of the federal funds rate. For example, as shown in Table 1, a passive roll-off of $2.2 trillion within 3 years in Scenario 1 is equivalent to a 75-basis-point increase in the long-run mean of the short rate. When QT lasts longer in Scenarios 2 and 3, the effect of rate hikes in the long-run short rate decays more slowly, implying slightly smaller QT-equivalent rate hikes of 63 and 54 basis points, respectively.

Next, we turn to the results with respect to active sales. Interestingly, the impact of active sales in Scenario 1 is actually slightly smaller than that of passive roll-off. For example, QT with active sales is equivalent to an immediate increase of 22 basis points in the current federal funds rate, which is smaller than the 29-basis-point increase under passive roll-off.

Table 1: QT-equivalent Rate Hikes under Normal Market Conditions

Equiv. Impact	Scenario 1 passive	Scenario 1 active	Scenario 2 passive	Scenario 2 active	Scenario 3 passive	Scenario 3 active
10-year yield (basis points)	6.0	4.6	7.2	7.6	7.3	9.8
Current FF rate (basis points)	29.2	22.2	34.7	36.7	35.2	47.2
FF rate path (basis points)	75.0	57.0	63.2	66.9	53.5	71.9
QT size (trillion)	2.2		2.8		3.3	
QT duration (years)	3		5		7	

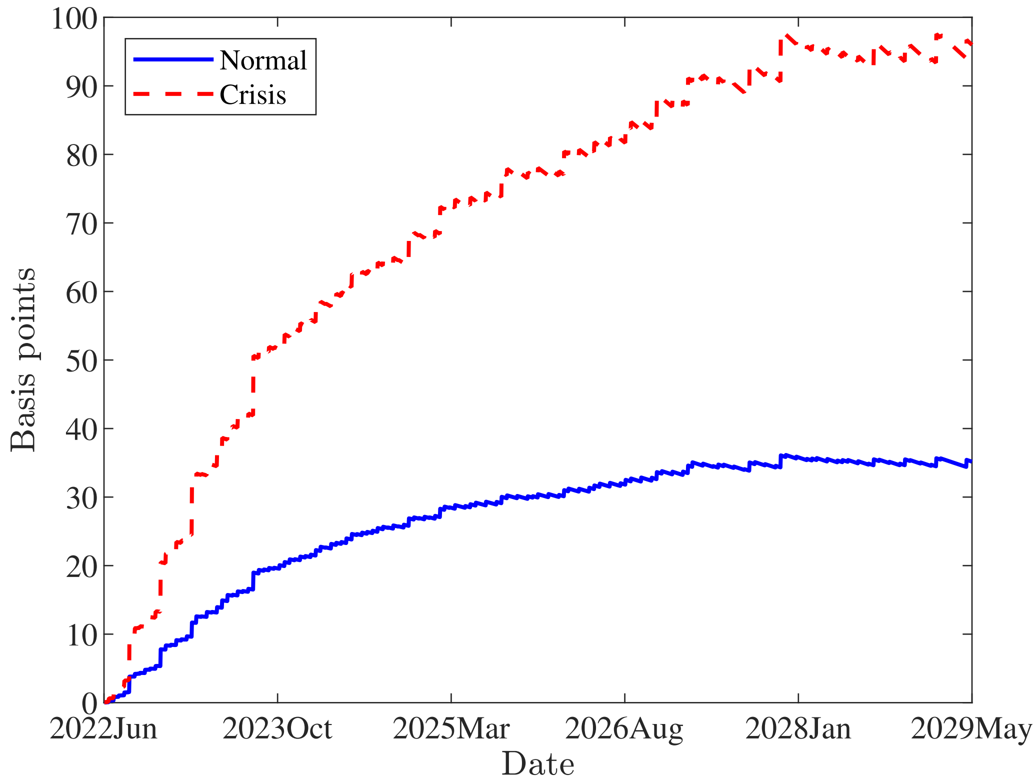
Note: This table reports our estimates of QT-equivalent rate hikes for both passive roll-off (Column "passive") and active sales (Column "active") under normal market conditions. We consider three scenarios: QT—beginning on June 1, 2022—lasts for 3 years in Scenario 1, 5 years in Scenario 2, and 7 years in Scenario 3. We report the equivalent effects of QT in basis points on the 10-year yield (Row "10-year yield"), the current federal funds rate (Row "Current FF rate"), and the path of future federal funds rate (Row "FF rate path").

The above finding of the smaller impact of active sales is intuitive. This is because bonds maturing within 3 years in the SOMA portfolio are primarily Treasury notes with longer maturities at issuance, and the value-weighted average maturity at issuance for those bonds ranges between 4 and 6 years, as shown by the red circles in Figure 2. So under passive roll-off, those bonds will be replaced by new issues with the same and longer initial maturity, which have a greater impact.

By contrast, the impact of active sales starts to exceed that of passive roll-off in Scenarios 2 and 3 as the duration of QT increases. For example, in Scenario 3 when QT lasts for 7 years, QT with active sales is equivalent to an immediate increase of 47 basis points in the current federal funds rate, an impact that is about 30% larger than that of passive roll-off (i.e., 47 basis points versus 36).

As mentioned earlier in the previous section, the larger impact of active sales in the medium to long run is a reflection of the timing difference starting to dominate: the earlier sales of long-dated bonds under active sales have a front-loaded impact, as opposed to the

Figure 3: Federal Funds Rate Hikes Equivalent to Passive Roll-off



Note: This figure plots the estimates of QT-equivalent rate hikes in the current federal funds rate under normal market conditions (blue solid line) or during crisis periods (red dashed line) for passive roll-off. The start date of QT is fixed as June 1, 2022 and the end date varies from June 2, 2022 through June 1, 2029.

back-loaded impact under passive roll-off.

We summarize the results in Table 1. As shown in the table, passive roll-off in Scenarios 1 through 3 is equivalent to a 29.2-, 34.7-, and 35.2-basis-point increase in the current federal funds rate, respectively. To trace the impact of a passive roll-off in real time, we repeat the analysis for QT of an arbitrary duration from 0 to 7 years. That is, for each day between June 1, 2022, and May 31, 2029, suppose. The estimates of the QT-equivalent increases in the current federal funds rate are plotted in Figure 3 (the solid blue line indicates the estimates under normal market conditions).

Table 2: QT-equivalent Rate Hikes under Passive Roll-off
(Normal versus Crisis Market Conditions)

	Scenario 1		Scenario 2		Scenario 3	
Equiv. Impact	normal	crisis	normal	crisis	normal	crisis
10-year yield (basis points)	6.0	9.1	7.2	11.3	7.3	12.0
Current FF rate (basis points)	29.2	74.2	34.7	92.2	35.2	97.6
FF rate path (basis points)	75.0	211.6	63.2	178.0	53.5	153.2
QT size (trillion)	2.2		2.8		3.3	
QT duration (years)	3		5		7	

Note: This table reports our estimates of QT-equivalent rate hikes for both passive roll-off (Column "passive") and active sales (Column "active") during crisis periods. We consider three scenarios: QT—beginning on June 1, 2022—lasts for 3 years in Scenario 1, 5 years in Scenario 2, and 7 years in Scenario 3. We report the equivalent effects of QT in basis points on the 10-year yield (Row "10-year yield"), the current federal funds rate (Row "Current FF rate"), and the path of future federal funds rate (Row "FF rate path").

3.3 Estimation Results during Crisis Periods

It is important to point out that overall market conditions are an important factor in estimating QT-equivalent rate hikes. The estimates in Table 1 are relevant ones under normal market conditions (i.e., the average market condition between January 1999 and March 2022). It is possible that market conditions may worsen substantially as QT progresses. For example, the economy may enter a recession, or the implementation of QT itself may have destabilizing effects (e.g., the repo crisis in September 2019). It would thus be interesting to see how the estimation results change during crisis periods.

To model a crisis period, we increase arbitrageurs' risk aversion. Risk aversion is typically countercyclical and becomes elevated during recessions. Also, risk aversion jumps higher during turbulent market conditions as a result of investors' risk-off sentiment or the reduced risk-bearing capacity of financial intermediaries (see, e.g., Gilchrist et al. (2020) for their analysis of the role of risk aversion around Secondary Market Corporate Credit Facility (SMCCF) announcements).

Specifically, we double arbitrageurs' risk aversion from 11 to 22. With heightened risk aversion, the market becomes more volatile: the 10-year yield volatility increases by almost 40% from 1.3% to 1.7%. At the same time, QT has an outsized effect, as shown in in Table 2. The amount of rate hikes that is equivalent to a passive roll-off in Scenario 1 is estimated to equal an increase of 74 basis points in the current federal funds rate, as opposed to 29 basis points under normal market conditions as shown in Table 1. Similarly, the equivalent change in the rate path (forward guidance) is an increase of 211 basis points, which is almost tripled compared to the counterpart under normal market conditions. The equivalent impact on the 10-year yield increases from 6 basis points to 9. The real-time estimates of the QT-equivalent increases in the current federal funds rate during crisis periods are plotted by the red dashed line in Figure 3.

3.4 Discussion

How long should QT last? Put differently, how much would the Fed allow to roll off from its balance sheet? In the previous 2017-2019 QT cycle, the Fed shrank its balance sheet by \$650 billion to a bit over \$3.8 trillion before it stopped QT following a liquidity crunch in the repo market. Compared with the previous cycle, QT in this cycle has larger caps (\$95 billion a month as opposed to \$50) and will reach its maximum pace sooner (in three months as opposed to 12 months). Inevitably, reserves will likely be drained from the banking system. It remains as a highly debated question that how much can be allowed to roll off before reserves are no longer considered ample. The answer to this question is closely related to the impact of QT on the liability side of the Fed's balance sheet as well as the responses from market players including the U.S. Treasury department. In this subsection, we discuss these implications.

3.4.1 Reserves versus Reverse Repo (RRP)

The liability side of the Fed's balance sheet consists of three major components: (1) reserves held by banks, (2) the Treasury's "checking account" at the Fed (known as the "Treasury General Account", or TGA), and (3) over-night reverse repo (ON RRP) balances held by money market funds (MMFs).

If the Treasury does not issue new securities to offset maturing bonds being rolled off, neither reserves nor ON RRP balances will not change: the Fed's asset holdings should shrink by the same amount as the TGA. On the other hand, if the Treasury does issue new securities, then whether those newly issued securities are purchased by banks or MMFs would have an important bearing on both reserves and ON RRP balances. For example, if it is banks that purchase the newly issued securities, the composition of assets on banks' balance sheet will change: their holdings of Treasury securities increase, but reserves decrease by the same amount. On the other hand, if it is MMFs that purchase the newly issued securities by reducing their investments in the ON RRP facility, the composition of assets on MMFs' balance sheet will change: their holdings of Treasury securities increase, but ON RRP balances decrease by the same amount.

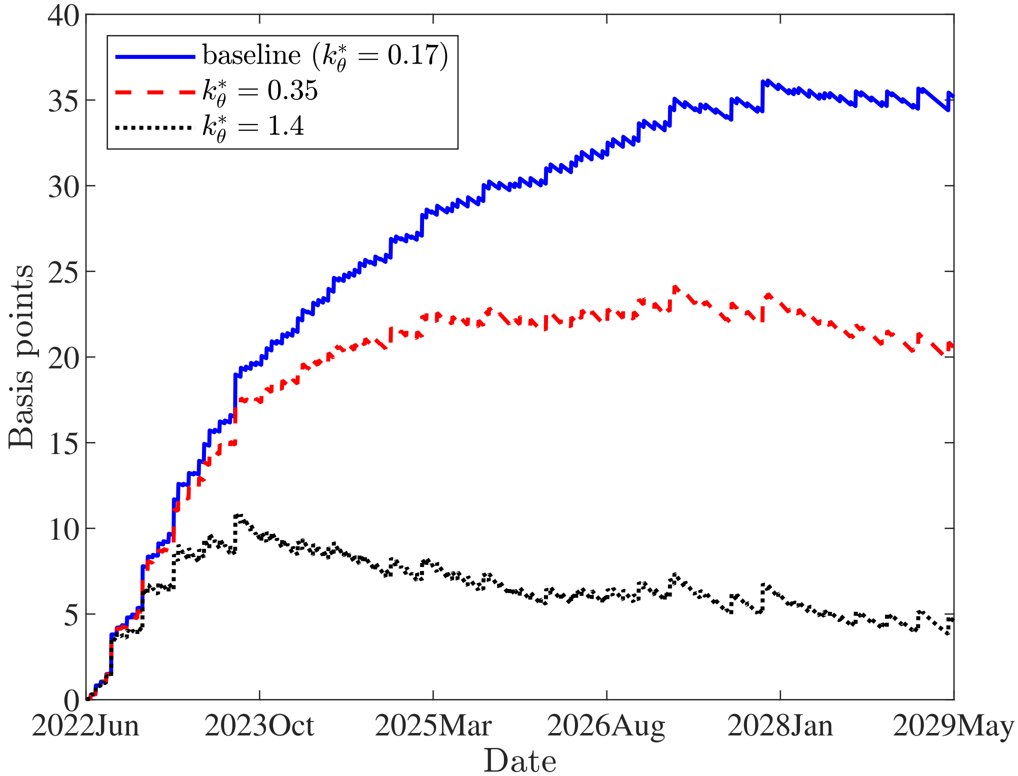
Because MMFs are mandated to invest in short-term money market securities such as Treasury bills or Treasury-backed repos, it is likely that QT may drain reserves faster than ON RRP balances. If reserves fall below a comfortable level for the banking system, the shortage of reserves may cause disruptions in short-term funding market (e.g., the spike in repo rates in September 2019). Such disruptive effects can be captured by heightened risk aversion in the model as discussed in Section 3.3. One takeaway from the findings in Section 3.3 is that the Fed may need to closely monitor overall financial conditions, for example, the volatility of Treasury bond yields, and can slow down or stop QT in response to strains in the financial markets.

3.4.2 How will the Treasury finance the Fed's balance sheet rundown?

As discussed above, how the Treasury will finance the Fed's balance sheet rundown—specifically, whether it will issue new securities and what types of new securities it issues—matters a lot in terms of the impact of QT on the composition of liabilities on the Fed's balance sheet. In our analysis so far, we assume that the Treasury will issue new securities with the same initial maturity as the maturing bonds. The decay factor κ_θ^* is thus calibrated to be 0.17 with a half-life of 4 years. By varying this parameter, we can capture the possibilities of the Treasury issuing different types of securities. For example, if the Treasury were to issue bills to offset all maturing securities, then we would expect more transitory effects owing to changes in the amount of debt held by the private sector. In this case, the decay factor κ_θ^* is calibrated to 1.4 such that its half-life is six months. We also consider an intermediate case with $\kappa_\theta^* = 0.35$ (half-life of 2 years).

Figure 4 plots QT-equivalent rate hikes in the current federal funds rate when the Treasury issues shorter-term securities to offset maturing securities. The blue line represents the baseline scenario where the newly issued securities have the same initial maturity as the maturing ones. The dashed red line and the dotted black line represent the alternative scenarios in which the Treasury issues shorter-term notes and bills, respectively. As expected, QT is shown in the figure to have a much smaller impact when the Treasury issues shorter-term securities because the resulting supply effects are more short-lived. Quantitatively, in the baseline scenario (Scenario 1), the equivalent rate hikes in the current federal funds rate decrease from 29 basis points to 22.5 if the Treasury issues shorter-term notes (red dashed line), and only 7.4 basis points if the Treasury issues bills only (black dotted line). We also find that the equivalent rate hikes during a crisis period in the baseline scenario are 12.6 basis points if the Treasury issues bills only.

Figure 4: Federal Funds Rate Hikes Equivalent to Passive Roll-off (Counterfactual Analysis regarding the Treasury's Response)



Note: This figure plots the estimates of QT-equivalent rate hikes in the current federal funds rate in the three scenarios depending on how the Treasury finances the Fed's balance sheet roll-off, measured by the decay factor κ_θ^*. We consider passive-rolloffs in the following three scenarios: the baseline scenario with $\kappa_\theta^* = 0.17$ (blue solid line), alternative scenarios with $\kappa_\theta^* = 0.35$ (red dashed line) and $\kappa_\theta^* = 1.4$ (black dotted line). As in Figure 3, we fix the start date of QT as June 1, 2022, and vary the end date from June 2, 2022 through June 1, 2029.

4 Conclusion

In this paper, we examine the question of quantifying how many interest rate hikes "quantitative tightening" (QT) is equivalent to. Our model-based estimates are built on the preferred-habitat model in Vayanos and Vila (2021). We define the equivalence between rate hikes and QT such that they both have the same impact on the 10-year yield. In the baseline scenario of a \$2.2 trillion passive roll-off over 3 years, we show that it is equivalent to an increase of 29 basis points in the current federal funds rate at normal times. The amount of equivalent rate hikes increases to 74 basis points in a more volatile market during a crisis period. We also quantify the effect of QT implemented by active sales and estimate QT-equivalent rate hikes in terms of their effects on the future path of the federal funds rate. Lastly, we discuss the impact of QT on the composition of liabilities on the Fed's balance sheet. Based on our model-based estimates, we show that if the Treasury were to issue bills to offset maturing securities, the resulting equivalent rate hikes in the current federal funds rate would decrease dramatically to 7.4 (12.6) basis points under normal (crisis) market conditions.

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Appendices

A Model Solutions

The coefficients $A_r(\tau)$ and $A_\beta(\tau)$ are solutions to the following system of ordinary differential equations

$$\begin{aligned}
& A_r'(\tau) + A_r(\tau)\kappa_r \\
= \; & 1 - a\sigma_r^2 A_r(\tau)\int_0^T \alpha(\tau) A_r(\tau)^2 d\tau - a\sigma_\beta^2 A_\beta(\tau)\int_0^T \alpha(\tau) A_r(\tau) A_\beta(\tau) d\tau,
\end{aligned}$$

and

$$\begin{aligned}
& A_\beta'(\tau) + A_\beta(\tau)\kappa_\beta \\
= \; & a\sigma_r^2 A_r(\tau)\int_0^T \alpha(\tau)(\tau\theta(\tau) - A_\beta(\tau)) A_r(\tau) d\tau \\
& + a\sigma_\beta^2 A_\beta(\tau)\int_0^T \alpha(\tau)(\tau\theta(\tau) - A_\beta(\tau)) A_\beta(\tau) d\tau,
\end{aligned}$$

and

$$\begin{aligned}
& C'(\tau) - A_r(\tau)\kappa_r\overline{r} + \frac{1}{2}A_r(\tau)^2\sigma_r^2 + \frac{1}{2}A_\beta(\tau)^2\sigma_\beta^2 \\
= \; & -a\sigma_r^2 A_r(\tau)\int_0^T (\alpha(\tau)C(\tau) - \theta_0(\tau)) A_r(\tau) d\tau \\
& -a\sigma_\beta^2 A_\beta(\tau)\int_0^T (\alpha(\tau)C(\tau) - \theta_0(\tau)) A_\beta(\tau) d\tau.
\end{aligned}$$

We provide the expressions for the impact of forward guidance on bond yields below. Following the unanticipated change $\Delta\overline{r}$ in the long-run mean $\overline{r}$ at time zero, the bond price is given by

$$P_t^{(\tau)} = \exp\left(-\left[A_r(\tau) r_t + A_\beta(\tau)\beta_t + A_{\overline{r}}(\tau)\Delta\overline{r}e^{-\kappa_{\overline{r}}t} + C(\tau)\right]\right),$$

where $A_{\overline{r}}(\tau)$ is endogenously determined as follows

$$A_{\overline{r}}(\tau) = \chi_r \int_0^\tau e^{-\kappa_{\overline{r}}(\tau-u)} A_r(u)\, du + \chi_\beta \int_0^\tau e^{-\kappa_{\overline{r}}(\tau-u)} A_\beta(u)\, du,$$

and

$$\begin{aligned}
\chi_r &= \frac{\kappa_r}{D}\left[1 + a\sigma_\beta^2 \int_0^T \alpha(\tau)\left(\int_0^\tau e^{-\kappa_{\overline{r}}(\tau-u)} A_\beta(u)\, du\right) A_\beta(\tau)\, d\tau\right], \\
\chi_\beta &= -\frac{\kappa_r}{D}\left[a\sigma_\beta^2 \int_0^T \alpha(\tau)\left(\int_0^\tau e^{-\kappa_{\overline{r}}(\tau-u)} A_r(u)\, du\right) A_\beta(\tau)\, d\tau\right],
\end{aligned}$$

and

$$\begin{aligned}
D = &\left[1 + a\sigma_r^2 \int_0^T \alpha(\tau) A_r(\tau) \int_0^\tau e^{-\kappa_{\overline{r}}(\tau-u)} A_r(u)\, dud\tau\right] \\
&\times \left[1 + a\sigma_\beta^2 \int_0^T \alpha(\tau) A_\beta(\tau) \int_0^\tau e^{-\kappa_{\overline{r}}(\tau-u)} A_\beta(u)\, dud\tau\right] \\
&- \left[a\sigma_r^2 \int_0^T \alpha(\tau) A_r(\tau) \int_0^\tau e^{-\kappa_{\overline{r}}(\tau-u)} A_\beta(u)\, dud\tau\right] \\
&\times \left[a\sigma_\beta^2 \int_0^T \alpha(\tau) A_\beta(\tau) \int_0^\tau e^{-\kappa_{\overline{r}}(\tau-u)} A_r(u)\, dud\tau\right].
\end{aligned}$$

As a result, the impact of forward guidance on the maturity-τ yield at time zero is given by (2); that is,

$$\begin{aligned}
\Delta y_{t,\tau}^{(II)} &= \Delta\overline{r} \cdot \frac{A_{\overline{r}}(\tau)}{\tau} e^{-\kappa_{\overline{r}} t} \\
&= \Delta\overline{r} \cdot \left(\chi_r \frac{\int_0^\tau e^{-\kappa_{\overline{r}}(\tau-u)} A_r(u)\, du}{\tau} + \chi_\beta \frac{\int_0^\tau e^{-\kappa_{\overline{r}}(\tau-u)} A_\beta(u)\, du}{\tau}\right) \cdot e^{-\kappa_{\overline{r}} t}.
\end{aligned}$$

Lastly, we provide the expressions for the impact of QT on bond yields below. Suppose $\Delta\theta_0(\tau) = \Delta_\theta \cdot 1_{\{\tau=\tau_{QT}\}}$ is a Dirac function. Following the unanticipated decrease $\Delta\theta_0(\tau)$ in the intercept of preferred-habitat demand, the bond price is given by

$$P_t^{(\tau)} = \exp\left(-\left[A_r(\tau) r_t + A_\beta(\tau)\beta_t + A_\theta(\tau)\Delta_\theta e^{-\kappa_\theta t} + C(\tau)\right]\right),$$

where $A_\theta(\tau)$ is endogenously determined as follows

$$A_\theta(\tau) = \chi_r^{QT}(\tau_{QT}) \int_0^\tau e^{-\kappa_\theta(\tau-u)} A_r(u)\, du + \chi_\beta^{QT}(\tau_{QT}) \int_0^\tau e^{-\kappa_\theta(\tau-u)} A_\beta(u)\, du,$$

and

$$\begin{aligned}
\chi_r^{QT}(\tau_{QT}) &= \frac{1}{D^{QT}} \left\{ \begin{array}{c} a\sigma_r^2 A_r(\tau_{QT}) \left[1 + a\sigma_\beta^2 \int_0^T \alpha(\tau) \int_0^\tau e^{-\kappa_\theta(\tau-u)} A_r(u)\, du A_\beta(\tau)\, d\tau\right] \\ -a\sigma_\beta^2 A_\beta(\tau_{QT}) \left[a\sigma_r^2 \int_0^T \alpha(\tau) \int_0^\tau e^{-\kappa_\theta(\tau-u)} A_\beta(u)\, du A_r(\tau)\, d\tau\right] \end{array} \right\}, \\
\chi_\beta^{QT}(\tau_{QT}) &= \frac{1}{D^{QT}} \left\{ \begin{array}{c} a\sigma_\beta^2 A_\beta(\tau_{QT}) \left[1 + a\sigma_r^2 \int_0^T \alpha(\tau) \int_0^\tau e^{-\kappa_\theta(\tau-u)} A_r(u)\, du A_r(\tau)\, d\tau\right] \\ -a\sigma_r^2 A_r(\tau_{QT}) \left[a\sigma_\beta^2 \int_0^T \alpha(\tau) \int_0^\tau e^{-\kappa_\theta(\tau-u)} A_r(u)\, du A_\beta(\tau)\, d\tau\right] \end{array} \right\},
\end{aligned}$$

and

$$\begin{aligned}
D^{QT} &= \left[1 + a\sigma_\beta^2 \int_0^T \alpha(\tau) \int_0^\tau e^{-\kappa_\theta(\tau-u)} A_r(u)\, du A_\beta(\tau)\, d\tau\right] \\
&\quad \times \left[1 + a\sigma_r^2 \int_0^T \alpha(\tau) \int_0^\tau e^{-\kappa_\theta(\tau-u)} A_r(u)\, du A_r(\tau)\, d\tau\right] \\
&\quad - \left[a\sigma_r^2 \int_0^T \alpha(\tau) \int_0^\tau e^{-\kappa_\theta(\tau-u)} A_\beta(u)\, du A_r(\tau)\, d\tau\right] \\
&\quad \times \left[a\sigma_\beta^2 \int_0^T \alpha(\tau) \int_0^\tau e^{-\kappa_\theta(\tau-u)} A_r(u)\, du A_\beta(\tau)\, d\tau\right].
\end{aligned}$$

Therefore, the impact of QT on the maturity-τ yield at time zero is given by (4).

B Calibration

In this appendix, we provide more details about model calibration. Following V&V, we calibrate eight parameters $(\kappa_r, \sigma_r, \kappa_\beta, a, \alpha, \theta, \delta_\alpha, \delta_\theta)$ to match target moments on yields and trading volume as well as estimates of demand elasticity from the literature. Specifically, the parameters (κ_r, σ_r) are calibrated to match volatilities of 1-year yield $y_{t,1}$ and of its annual changes $y_{t+1,1} - y_{t,1}$; the parameters $(\kappa_\beta, a\alpha, a\theta)$ to match average volatilities across maturities of bond yields, annual changes, as well as the correlation between annual changes

TABLE B-1: Calibration Results

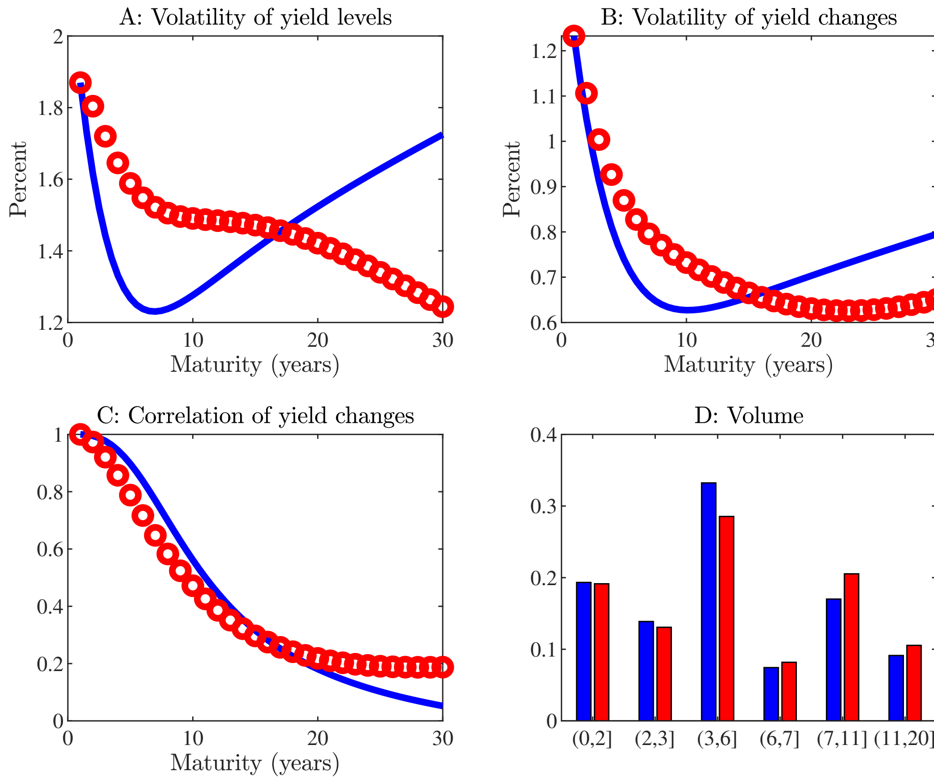
Parameter	Value	Empirical Moments	Value
Mean reversion of r_t		Volatility of 1-year yield levels	
κ_r	0.247	$\sqrt{\mathbb{V}ar\left(y_t^{(1)}\right)}$	1.870
Diffusion of r_t		Volatility of 1-year yield changes	
σ_r	0.016	$\sqrt{\mathbb{V}ar\left(y_{t+1}^{(1)} - y_t^{(1)}\right)}$	1.233
Mean reversion of demand factors		Average volatility of yield levels	
κ_β	0.112	$\frac{1}{30}\sum_{\tau=1}^{30}\sqrt{\mathbb{V}ar\left(y_t^{(\tau)}\right)}$	1.470
Risk aversion $\times$ demand intercept		Average volatility of yield changes	
$a\theta$	4796.2	$\frac{1}{30}\sum_{\tau=1}^{30}\sqrt{\mathbb{V}ar\left(y_{t+1}^{(\tau)} - y_t^{(\tau)}\right)}$	0.734
Risk aversion $\times$ demand slope		Average correlation of yield changes	
$a\alpha$	59.4	$\frac{1}{30}\sum_{\tau=1}^{30}\mathbb{C}orr\left(y_{t+1}^{(1)} - y_t^{(1)}, y_{t+1}^{(\tau)} - y_t^{(\tau)}\right)$	0.414
Demand shock - short maturities		Relative volume - short maturities	
δ_α	0.289	$\frac{\sum_{0<\tau\leq 2} Volume(\tau)}{\sum_{0<\tau\leq 20} Volume(\tau)}$	0.191
Demand shock - long maturities		Relative volume - long maturities	
δ_θ	0.299	$\frac{\sum_{11<\tau\leq 30} Volume(\tau)}{\sum_{0<\tau\leq 30} Volume(\tau)}$	0.105
Demand slope		Demand elasticity	
α	5.21	Estimate in KVJ (2012)	-0.746
Unconditional average of r_t		Average 1-year yield level	
$\overline{r}$	0.016	$\mathbb{A}verage\left(y_t^{(1)}\right)$	1.882
Risk aversion $\times$ demand intercept		Average 7-year yield level	
$a\theta_0$	309.5	$\mathbb{A}verage\left(y_t^{(7)}\right)$	3.064

NOTE: This table reports the calibrated values of model parameters as well as target moments.

to the 1-year yield and to other yields; the parmeters $(\delta_\alpha, \delta_\theta)$ to match the relative volume for maturities of 2 years and below as well as the relative volume for maturities of 11 years and above;[6] and the parameter α is calibrated to be 5.21 using estimates of demand elasticity from Krishnamurthy and Vissing-Jorgensen (2012). Lastly, we calibrate $(\overline{r}, \theta_0)$ to match the empirical average of the 1-year and 7-year Treasury bond yields. As normalization, we set σ_β to σ_r.

[6]For trading volume of Treasury bonds, we use the Federal Reserve 2004 data set, available from the website of the Federal Reserve Bank of New York. We use the data set on nominal Treasury bond volume from April 2013 to March 2022.

Figure B-1: Model-generated and Empirical Moments



Note: This figure plots both model-implied moments (see blue lines or blue bars) as well as those in the data (see red circles or red bars).

Table B-1 reports the calibrated parameters and the empirical moments used to determine them. Based on the results in Table B-1, the risk aversion coefficient of Treasury bond arbitrageurs is calibrated to be 11.4 (i.e., 59.4/5.21).

Figure B-1 plots both model-implied moments (see blue lines or blue bars) as well as those in the data (see red circles or red bars). The model-implied moments fits closely the counterparts in the data.