Real Rigidities, Firm Dynamics, and Monetary Nonneutrality: The Role of Demand Shocks

S. Borağan Aruoba, Eugene Oue, Felipe Saffie, and Jonathan L. Willis

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Abstract: We propose a parsimonious framework for real rigidities, in the form of strategic complementarities, that can generate real and nominal dynamics and match key features of the data across several literatures. Existing menu-cost models featuring strategic complementarities require unrealistically volatile shocks to idiosyncratic productivity to be consistent with pricing moments. We develop a simple menu-cost model with strategic complementarities along with idiosyncratic productivity and demand shocks that are disciplined by the data. This approach allows us to overcome previous criticism from analysis of models that employ only an idiosyncratic productivity shock and calibrate solely using data from the price-adjustment literature. Despite its simplicity, the model can generate sizable monetary nonneutrality along with the magnitude of cost pass-through documented in previous studies, while also remaining consistent with micro pricing and markup evidence.

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1 Introduction

Modeling the response of output and prices to monetary policy shocks has been a long-term quest in the macroeconomics literature. Quantitative estimates consistently indicate that monetary policy shocks have persistent impacts on real output (Christiano et al., 1999; Ramey, 2016). When it comes to modeling, early studies (Mankiw, 1985; Akerlof and Yellen, 1985) propose that nominal rigidities, like a menu cost, can lead to real fluctuations. However, subsequent studies such as Golosov and Lucas (2007) show that a menu cost alone is insufficient to generate the observed non-neutrality from quantitative estimates. To this end, Ball and Romer (1990) demonstrate that real rigidities in combination with nominal rigidities can lead to sizable real effects of monetary policy.

Strategic complementarities in pricing represent one promising form of real rigidity used in the money non-neutrality literature. Kimball (1995) introduces a demand system where price elasticity of demand varies with relative price and quantity. Eichenbaum and Fisher (2007) and Smets and Wouters (2007) add strategic complementarities in the form of a Kimball demand system into dynamic stochastic general equilibrium models in order to match the estimated responses of real output to a nominal shock, while also being consistent with micro evidence on the frequency of price adjustment. However, Klenow and Willis (2016) find that a menu-cost model with a Kimball demand system that matches firm-level pricing moments requires idiosyncratic productivity shocks that are much larger than observed in the data.

Elsewhere in the economics literature, the use of strategic complementarities in models has been shown to be important for generating firm-level and aggregate dynamics needed to match the key features of the data. It is used extensively to study exchange-rate pass-through in international macroeconomics (Gopinath and Itskhoki, 2010; Amiti et al., 2019; Berger and Vavra, 2019), variable markups in firm dynamics and international trade (Edmond et al., 2018; Arkolakis et al., 2019), and inflationary dynamics and optimal monetary policy (Harding et al., 2022a,b; Coibion et al., 2012; Blanco, 2021).

In this paper, we propose a simple model featuring strategic complementarities that can generate monetary non-neutrality, while also remaining consistent with the micro and macro evidence. The baseline specification is a menu-cost model with a real rigidity in the form of a Kimball demand system, in which the elasticity of substitution between a given variety and other varieties is decreasing in the relative quantity consumed. The final important element of this model is the specification of two forms of idiosyncratic shocks: productivity and demand. Under a Kimball demand system, idiosyncratic demand influences the desired
markups of firms through its effect on the demand elasticity. Previous studies, including Klenow and Willis (2016), exclusively focus on the role of idiosyncratic productivity shocks.\footnote{Burstein and Hellwig (2007) explore supply and demand shocks under a CES demand system with decreasing returns to scale technology.} Despite its parsimony, the model generates sizable non-neutrality of monetary policy with realistic shocks, while remaining consistent with micro pricing facts. Thus our model opens the door to consistently modeling real (investment, labor, and sales), nominal (pricing and non-neutrality), and competition (markup) dynamics in a parsimonious framework amenable to policy analysis.

The model is calibrated to match the frequency of price adjustment and the serial correlation and standard deviation of idiosyncratic productivity and demand shocks. For the latter productivity and demand moments, we use results from Foster et al. (2008) who point to important, and separate, roles for these shocks using data from Census of Manufactures.\footnote{Recent papers that jointly study and model both productivity and supply shocks are Aruoba et al. (2022) and Carlsson et al. (2022)} We investigate the role of real rigidities by calibrating the model to two additional moments from Foster et al. (2008), that help disentangle demand from supply and discipline the degree of strategic complementarities. The first moment is the correlation between revenue-based total factor productivity (TFPR) and total factor productivity (TFP), or as it is sometimes refer to as, quantity-based TFP (TFPQ). The second moment is the correlation between firm price and TFPQ.

To illustrate the key ingredients of our baseline model relative to a model with CES preferences and TFP shocks, we calibrate four different specifications. The first version is a standard CES model without idiosyncratic demand shocks that is calibrated to match three pricing moments: the frequency of price changes, the fraction of positive changes, and the average size of price changes. The second version is the same CES model, but calibrated to match the autocorrelation and variance of idiosyncratic TFP from Foster et al. (2008) along with the frequency of price changes. The third version goes a step further by adding and calibrating idiosyncratic demand shocks to match the autocorrelation and variance of demand from Foster et al. (2008). The first CES model is consistent with the standard procedure in the pricing literature, i.e., target pricing moments using exogenous shock processes. This first model matches pricing dynamics by construction, but it is inconsistent with firm-level shocks (Klenow and Willis, 2016). The two other CES models are by construction consistent with the idiosyncratic shock processes documented by Foster et al. (2008), but fail at replicating non-targeted pricing moments. The fourth model is our baseline specifi-
cation featuring a Kimball demand system in combination with idiosyncratic demand and productivity shocks with the addition of Corr(TFPQ,TFPR) and Corr(TFPQ,P) as targeted moments. In addition to matching a broader set of targeted moments, our baseline model effortlessly matches three non-targeted pricing moments: the average size of a price change conditional on a change, the fraction of adjustments that are positive, and the dispersion of non-zero price changes. Furthermore, the model delivers a downward-sloping pricing hazard consistent with the data.

The incorporation of a Kimball demand system also allows our model to generate an untargeted cross-sectional markup distribution that closely resembles its empirical counterpart, which is impossible to achieve with CES demand. This is because the desired markup under CES is constant. Although nominal pricing rigidities can generate a non-degenerate distribution around the desired markup, such a model cannot replicate the large cross-sectional variance in the empirical markup distribution without unreasonably large nominal rigidities. In contrast, a Kimball demand system induces more dispersion in firm markups due to variable elasticities as well as the incorporation of demand shocks which affect the desired markup. Additionally, by the virtue of Kimball demand system, the calibrated model produces incomplete cost pass-through (pass-through of 38 percent), in line with the empirical literature.

Our framework overcomes the challenge of using a Kimball demand system in pricing models arising from inconsistency with micro evidence on price setting behavior by firms. The inclusion of both supply and demand shocks is instrumental to this success, through the expanded set of shocks that triggers firms to adjust prices and the magnitude of pass-through from underlying shocks into prices. In fact, a Kimball demand system generates a trade-off between the strength of strategic complementary and the pass-through of productivity shocks. Previous studies using a Kimball demand system included only idiosyncratic productivity shocks. In a typical calibration of the degree of strategic complementary, firms would only pass through 20 to 40 percent of productivity shocks into prices. Therefore, in order to generate price changes consistent with the micro evidence, extremely volatile productive shocks are needed to produce price changes that are aligned with the empirical literature. Our calibrated model implies that firms pass through approximately 62 percent of idiosyncratic demand shocks and 38 percent of productivity shocks into prices. With demand shocks also playing a role in the pricing decisions of firms, our model can generate a distribution of price changes that is similar to the data remaining consistent with micro evidence on the volatility of idiosyncratic shocks.
To study the degree of monetary non-neutrality generated by the model, we expand the framework to incorporate nominal expenditure shocks. On impact, approximately 82 percent of an increase in nominal expenditure is reflected in an increase in real output. The real output response decays with a half-life of 5 months, leaving a cumulative response of 0.40 before eventually fully dissipating after 20 months. The cumulative response of real output is 4 times as large as a simple CES menu cost model without real rigidities calibrated to pricing moments. It is in the upper range of richer models that explore alternative sources of real rigidities without using micro-estimates for their shock processes. The two key features generating this non-neutrality are the Kimball demand system and the presence of both idiosyncratic productivity and demand shocks. The former generates a strategic pricing complementarity under which the firm will temper its price adjustment in response to a shock because of an endogenous change in its desired markup, whereas the latter dampens the selection effect in price adjustments to an aggregate shock and results in larger real output responses.

Overall, our model, calibrated to micro evidence from both the firm-dynamics and the price-adjustment literatures, is able to produce results consistent with the evidence on monetary non-neutrality, cost pass-through, and markups. The combination of menu costs, a Kimball demand system, and idiosyncratic shocks for demand and productivity in the model produces cost pass-through in line with previous studies along with a distribution of markups that is similar to its non-targeted empirical counterpart. Regarding monetary non-neutrality, the findings demonstrate that a model featuring strategic complementarities can produce persistent real output responses to nominal expenditure shocks while also remaining consistent with the micro evidence. Thus, our parsimonious framework proposes a promising path to jointly model pricing and firm level dynamics.

The remainder of the paper is structured as follows. Section 2 introduces a quantitative menu-cost model with idiosyncratic productivity and demand augmented with a Kimball demand system and explores its theoretical properties. Section 3 presents the calibration of the model. Section 4 discusses the model’s implications for non-targeted pricing moments, markup distribution, and non-neutrality of monetary shocks. Finally, Section 5 concludes.

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2 Menu Cost Model

We build a quantitative menu-cost model following Golosov and Lucas (2007). The model features a representative household, a representative final-good producer, and a continuum of monopolistically competitive intermediate-variety producers who face nominal pricing frictions.

2.1 Households

A representative household supplies labor to firms in exchange for wage payments, purchases a complete set of Arrow-Debreu securities, \( B_{t+1} \), and consumes a final good, \( C_t \). It also owns all firms in the economy and receives all accrued profits. The representative household solves the following problem

\[
\max_{C_t, h_t, B_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [\log (C_t) - \chi h_t] \tag{1}
\]

subject to the budget constraint

\[
P_t C_t + Q_t \cdot B_{t+1} \leq B_t + W_t h_t + \Pi_t, \tag{2}
\]

where \( Q_t \) is a vector that contains the prices of the state-contingent securities, \( B_{t+1} \). \( B_t \) represents the payoff of the state-contingent security purchased in period \( t-1 \) that had a non-zero payoff in period \( t \). \( P_t \) and \( W_t \) are the price of the final good and nominal wage, respectively, both of which are taken as given by the households. \( \Pi_t \) denotes the net dividends the household receives from the producers.

Household optimality requires

\[
\frac{W_t}{P_t} = \chi C_t, \tag{3}
\]

and we can also define the household’s stochastic discount factor as

\[
\Xi_{t,t+1} \equiv \beta \frac{C_t}{C_{t+1}}. \tag{4}
\]

2.2 Producers

Production is carried out by a continuum of perfectly-competitive final-good producers, who purchase varieties of intermediate goods and sell a combined final good to the households. The intermediate-good producers are monopolistically competitive as the varieties they pro-
duce are not perfect substitutes.

### 2.2.1 Final Good Producers

A representative final-good firm combines intermediate varieties, $y^i_t$, to produce the final good, $Y_t$, using the Kimball (1995) aggregator. This aggregator is defined implicitly as

$$
\int_0^1 G \left( \frac{n^i_t y^i_t}{Y_t} \right) \, di = 1
$$

where $n^i_t$ represents an idiosyncratic variety-specific demand shifter. Following Dotsey and King (2005), we use the following specification for $G(.)$

$$
G \left( \frac{n^i_t y^i_t}{Y_t} \right) = \frac{\omega}{1 + \omega \psi} \left[ (1 + \psi) \frac{n^i_t y^i_t}{Y_t} - \psi \right]^{\frac{1 + \omega \psi}{\omega (1 + \psi)}} + 1 - \frac{\omega}{1 + \omega \psi}
$$

where (5) and (6) show the only two deviations from a textbook menu cost model: the introduction of the variety-specific demand shifters and the Kimball aggregator. This specification nests the familiar constant elasticity of substitution (CES) Dixit-Stiglitz aggregator when $\psi = 0$. When this is the case, the final good $Y_t$ can be expressed explicitly as

$$
Y_t = \left[ \int_0^1 \left( \frac{1}{n^i_t y^i_t} \right)^{\frac{1}{\omega}} \right]^{\omega},
$$

where the price elasticity of demand is given by $\frac{\omega}{1 - \omega}$, elasticity of substitution is given by $\frac{\omega}{\omega - 1}$ and the gross markup is given by $\omega$. When $\psi \neq 0$, price elasticity of demand, elasticity of substitution and desired markup are no longer constant. We discuss how Kimball aggregation affects firms’ pricing decisions in Section 2.3.

Taking as given variety prices, $p^i_t$, as well as $P_t$, $n^i_t$ and aggregate demand, $Y_t$, the representative final-good producer chooses $y^i_t$ to maximize profits

$$
\max_{y^i_t} 1 - \int_0^1 \frac{p^i_t y^i_t}{P_t Y_t} \, di \quad \text{subject to} \quad \int_0^1 G \left( \frac{n^i_t y^i_t}{Y_t} \right) \, di = 1.
$$

The optimality condition of the final-good producer’s maximization problem implicitly de-
fines the demand function for each variety \( i \)

\[
\frac{n_i^t y_i^t}{Y_t} = \frac{1}{1 + \psi} \left[ \left( \frac{p_i^t}{\lambda_t n_i^t P_t} \right)^{\frac{\omega(1+\psi)}{1-\omega}} + \psi \right],
\]  

(9)

where \( \lambda_t \) is the Lagrangian multiplier on the constraint in the optimization problem, which can be obtained by substituting (9) into (5) as

\[
\lambda_t = \left[ \int_0^1 \left( \frac{p_i^t}{n_i^t P_t} \right)^{\frac{1+\omega \psi}{1-\omega}} d\psi \right]^{\frac{1-\omega}{1+\omega \psi}}.
\]  

(10)

The aggregate price index is derived from the zero-profit condition for the final-good producer as

\[
P_t = \frac{1}{1 + \psi} \left[ \int_0^1 \left( \frac{p_i^t}{n_i^t P_t} \right)^{\frac{1+\omega \psi}{1-\omega}} d\psi \right]^{\frac{1-\omega}{1+\omega \psi}} + \frac{\psi}{1 + \psi} \int_0^1 \frac{p_i^t}{n_i^t} d\psi
\]  

(11)

2.2.2 Intermediate Variety Producers

There is a continuum of intermediate-good producers indexed by \( i \), each producing a differentiated variety \( y_i^t \). Intermediate producers are heterogeneous in their physical productivity, \( z_i^t \), and face demand shocks for their variety, \( n_i^t \). Production technology is linear with labor as the only input

\[
y_i^t = z_i^t l_i^t
\]  

(12)

Idiosyncratic productivity, \( z_i^t \), and idiosyncratic demand, \( n_i^t \), evolve according to a VAR(1) process

\[
\begin{pmatrix}
\log(z_i^t) \\
\log(n_i^t)
\end{pmatrix} =
\begin{bmatrix}
\rho_z & 0 \\
0 & \rho_n
\end{bmatrix}
\begin{pmatrix}
\log(z_{i-1}^t) \\
\log(n_{i-1}^t)
\end{pmatrix} + u_i^t 
\text{where } u_i^t \sim N\left(0, \begin{bmatrix}
\sigma_z^2 & \sigma_{zn} \\
\sigma_{zn} & \sigma_n^2
\end{bmatrix}\right)
\]  

(13)

At the beginning of each period, intermediate-good producers decide whether or not to adjust their nominal prices and if so, by how much. Nominal price adjustments are subject to a fixed cost \( f \) in terms of labor. Given the demand schedule for individual varieties, the
intermediate producers’ gross profit when adjusting their price $p$ is

$$
\pi(p^i_t, z^i_t, n^i_t, S_t) = \left( \frac{p^i_t}{P^i_t} - \frac{W^i_t}{z^i_t P^i_t} \right) \frac{Y^i_t}{n^i_t} \frac{1}{1 + \psi} \left[ \left( \frac{p^i_t}{\lambda^i_t n^i_t P^i_t} \right)^{\frac{\omega(1 + \psi)}{1 - \omega}} + \psi \right], \tag{14}
$$

where $S_t \equiv (P^i_t, W^i_t, Y^i_t, \lambda^i_t)$ collects all aggregate objects the firms need to know, and we assume that the firms know the law of motion for $S_t$.

At the beginning of the period, each intermediate-good producer inherits their price from the previous period $p^i_{t-1}$. At that point, they choose whether or not to change their prices by solving the problem

$$
V(p^i_{t-1}, z^i_t, n^i_t, S_t) = \max \left[ V_N(p^i_{t-1}, z^i_t, n^i_t, S_t), V_A(z^i_t, n^i_t, S_t) \right], \tag{15}
$$

where $V_N(.)$ and $V_A(.)$ are the values for the firm if they do not change and change their prices, respectively.

The value of not adjusting is

$$
V_N(p^i_{t-1}, z^i_t, n^i_t, S_t) = \pi(p^i_{t-1}, z^i_t, n^i_t, S_t) + \mathbb{E}_t \left[ \Xi_{t,t+1} V(p^i_{t-1}, n^i_{t+1}, z^i_{t+1}, S_{t+1}) \right], \tag{16}
$$

which is equal to the flow profit evaluated at last period’s price plus a continuation value. If the firm chooses to adjust its price, it pays the fixed price adjustment cost and chooses $p^i_t$ to maximize the sum of current flow profit and the present discounted value of future profit

$$
V_A(z^i_t, n^i_t, S_t) = -f \frac{W^i_t}{P^i_t} + \max_{p^i_t} \left\{ \pi(p^i_t, z^i_t, n^i_t, S_t) + \mathbb{E}_t \left[ \Xi_{t,t+1} V(p^i_t, n^i_{t+1}, z^i_{t+1}, S_{t+1}) \right] \right\}. \tag{17}
$$

The intermediate-good producers solve this problem taking as given the laws of motion for the idiosyncratic state variables as in (13) and those for the aggregate variables in $S_t$.

### 2.3 Kimball Aggregation and Pricing Decisions

The defining feature of the Kimball demand system is variable price elasticity. In particular, when $\psi < 0$, the price elasticity of demand becomes an increasing function of the relative price of the variety $p/P$, and a decreasing function of idiosyncratic demand, $n^i_t$, and the
effective market share of the variety, \( \frac{ny}{Y} \). The non-constant price elasticity implied by the Kimball demand system has two important consequences. First of all, unlike the CES case where desired markups are constant and independent of idiosyncratic demand, variable price elasticity leads to variable desired markups. Moreover, the desired markup depends on both the firm’s idiosyncratic productivity as well as demand. For example, a firm with lower costs or higher demand would choose to have a higher markup relative to an average firm. Second, the Kimball demand system creates strategic complementarities in pricing among firms, because a deviation from the aggregate price index is costly.

To demonstrate how productivity and demand shocks affect pricing decisions of firms under a Kimball demand system, it is instructive to set \( f = 0 \) and focus on the problem of an individual intermediate-good producer. The first-order condition to the static profit-maximization problem of an intermediate-good producer is

\[
\left( \frac{p^*_i}{\lambda n_i P} \right)^{\omega(1+\psi)} \left[ 1 - \frac{W}{z_i} - p^*_i \right] \left( \frac{(1+\psi)}{1-\omega} \frac{1}{p^*_i} \right) = -\psi \tag{18}
\]

where \( p^*_i \) denotes the optimal price the firm chooses.

Log-linearizing (18) around a symmetric steady state and letting hatted variables denote log-deviations from the steady state yield the following expression for the optimal price

\[
\hat{p}^*_i = \frac{\omega \psi}{\omega \psi - 1} \left( \hat{\lambda} + \hat{P} + \hat{n}_i \right) + \frac{1}{\omega \psi - 1} \hat{z}_i \tag{19}
\]

Letting \( \hat{mc} \equiv 1/\hat{z} \) denote log-deviation in marginal cost, which is inversely proportional to productivity, the price elasticities with respect to cost and demand shocks, respectively, are

\[
\left( \frac{\partial \epsilon}{\partial \left( \frac{ny}{Y} \right) \omega \psi (\omega-1)} \right) = -\frac{\psi \omega}{\left( \frac{ny}{Y} \right) (\omega-1)} \quad \text{and} \quad \left( \frac{\partial \epsilon}{\partial n} \right) = -\frac{\psi \omega Y}{ny (\omega-1)} \cdot
\]

We can also compute the super-elasticity, defined as the elasticity of the demand elasticity with respect to price as

\[
\gamma \equiv \frac{\partial \epsilon}{\partial \epsilon} \frac{p}{\epsilon} = \frac{\omega}{1-\omega} \cdot \frac{\psi (1+\psi)}{\left( \frac{p}{\lambda \pi P} \right) \frac{\psi}{\lambda \pi P} + \psi} = \frac{\omega}{1-\omega} \cdot \frac{\psi}{\lambda \pi P}.
\]

\[4\]The price elasticity of demand is

\[
\epsilon \equiv \frac{dy}{dp} \frac{p}{y} = \frac{\omega (1+\psi) \frac{ny}{Y} - \psi}{1-\omega - \frac{ny}{Y}} = \frac{\omega (1+\psi)}{1-\omega} \left( \frac{p}{\lambda \pi P} \right)^{\frac{\omega(1+\psi)}{1-\omega}}.
\]
given by

\[
\frac{\partial \hat{p}^*_i}{\partial \hat{m}c} = -\frac{1}{\omega \psi - 1} \tag{20}
\]

\[
\frac{\partial \hat{p}^*_i}{\partial \hat{n}_i} = \frac{\omega \psi}{\omega \psi - 1} \tag{21}
\]

In what follows we refer to these as cost and demand pass-through.\(^5\)

When \(\psi = 0\), cost pass-through is complete: price falls one to one with a positive \(z\) shock (negative \(\hat{m}c\) shock). However, when \(\psi < 0\), the pass-through of a cost shock is incomplete. Variable demand elasticity is key to understanding this incomplete pass-through of cost. Consider a firm experiencing a positive productivity shock. As its marginal cost decreases, the firm finds it optimal to reduce its price, which yields greater sales. However, as it moves along the demand curve, with a larger effective market share, its price elasticity decreases, which dampens the increase in revenue from cutting prices. As such, the optimal price cut is smaller than in the CES case. Furthermore, this line of reasoning implies that in the nonlinear solution, the size of the cost pass-through is smaller (larger) for a larger reduction (increase) in cost. Because variable price elasticities attenuate gains from price deviations, the Kimball demand system (with \(\psi < 0\)) generates strategic complementarities in that a firm wants to avoid moving its price too far away from its competitors. This is in line with the early literature such as Ball and Romer (1990) and Caplin and Leahy (1997), which emphasize the importance of strategic complementarities as a real rigidity.

Similarly, and obviously, when \(\psi = 0\), demand pass-through is zero, which is the standard result under CES. When \(\psi < 0\), demand pass-through becomes positive: a firm receiving a demand shock chooses to increase its price or, equivalently, chooses a higher markup over its marginal cost. Under a Kimball demand system, the elasticity of demand decreases in the effective market share. Firms with stronger demand for their product can raise prices without losing as much sales, resulting in higher markups.

The relative importance of demand and cost shocks hinges on the degree of strategic complementarity. Figure 1 plots the pass-through of cost and demand to the optimal frictionless price as a function of \(\psi\), holding \(\omega\) constant at our calibrated value. As \(\psi\) decreases, the pass-through of cost (productivity) shocks decreases and the pass-through of demand shocks increases. This figure makes it clear that the value of \(\psi\) will be critical for determining the pass-through of idiosyncratic shocks. There is overwhelming evidence in the international-trade, international-finance and firm-dynamics literatures that the pass-through from cost

\(^5\)See appendix A for detailed derivations.
Figure 1: Pass-through of Demand and Cost Shocks to Price

Note: This plots pass-through of a small (1%) change in demand and productivity to the optimal frictionless price around a symmetric equilibrium with $\omega = 1.29$.

shocks to prices is less than complete, some of which we turn to in Section 3.1.3. This suggests that a constant elasticity of substitution specification ($\psi = 0$) cannot produce a cost pass-through that matches empirical estimates and strongly points toward a role for strategic complementarities ($\psi < 0$).

2.4 Equilibrium

Money supply, $S_t$, which must be equal to nominal aggregate expenditures, $P_tC_t$, in equilibrium, follows the stochastic process

$$\log (S_t) = \mu + \log (S_{t-1}) + \sigma_S \epsilon_t$$

where $\epsilon_t \sim N(0,1)$, (22)

where money supply grows at rate of $\mu$ every period with stationary fluctuations around it given by $\epsilon_t$. As standard in the literature, because a one-time change in $\epsilon_t$ creates a
permanent change in money balances, we interpret it as a monetary policy shock. This shock is the only source of aggregate uncertainty in our model. In calibrating our model, we set $\sigma_S = 0$ as it has minimal influence on the model-implied moments used for calibration. We explain our computational strategy in more detail in Appendix B.

3 Calibration

Most quantitative papers in the literature that uses menu cost models use a single idiosyncratic shock in their setup. Sometimes this shock directly moves the desired price around (e.g. Caplin and Spulber (1987)), where the authors are agnostic about the fundamental source of this shock. In other instances it is a productivity shock (e.g. Vavra (2014)), but the authors do not use firm-level evidence on productivity to calibrate it. In both cases, the process that drives either the desired price or productivity is typically calibrated to match various moments related to the distribution of firm-level price changes.

In this study, we aim to have a model that respects a broader set of micro-level evidence while also delivering significant monetary non-neutralities. To do so, we introduce two firm-level shocks in our model – productivity and demand. Furthermore, we calibrate the processes for these shocks to be consistent with direct firm-level evidence. This is a significant deviation from the common practice explained above. In order to demonstrate how different pieces of the calibration works and to emphasize that including demand shocks and deviating from CES are key to the success of the calibration, we consider three calibrated models with CES demand in addition to our baseline model with a Kimball demand system. Before turning to the details of these four models, we first review the moments we use from the data, either as calibration moments or as untargeted moments.

3.1 Calibration Targets and Non-Targeted Moments

In this section we report all of the data moments used throughout the paper. It is important to note that a given moment may be a calibration target for some of the models described above while being an untargeted moment in other model specifications.
3.1.1 Firm-Level Productivity and Demand Processes

Using the Census of Manufactures, the seminal work of Foster et al. (2008) estimate firm-level productivity and demand for eleven product markets with minimal vertical differentiation. Using data on sales, quantity sold, and input usage, they estimate the production function of firms assuming Cobb-Douglas technology and recover firm-level physical TFP (TFPQ) as the residual of the following estimation:

$$TFPQ_{it} = \ln q_{it} - \alpha_l \ln l_{it} - \alpha_k \ln k_{it} - \alpha_m \ln m_{it} - \alpha_e \ln e_{it},$$

(23)

where $TFPQ_{it}$ is the firm-level physical TFP of firm $i$ at time $t$, $q_{it}$ is the actual quantity produced by the firm, $l_{it}$ is the labor input, $k_{it}$ is the capital input, $m_{it}$ are the intermediate inputs used in production, and $e_{it}$ is the energy used by the firm. Foster et al. (2008) also estimate revenue-based TFP, which can be obtained using the same method but replacing the dependent variable with the revenue of the firm,

$$TFPR_{it} = \ln p_{it} q_{it} - \alpha_l \ln l_{it} - \alpha_k \ln k_{it} - \alpha_m \ln m_{it} - \alpha_e \ln e_{it}.$$  

(24)

To obtain firm-level estimates of idiosyncratic demand, Foster et al. (2008) estimate the demand function

$$\ln q_{it} = \alpha_0 + \alpha_1 \ln p_{it} + \sum_t \alpha_t \text{YEAR}_t + \alpha_2 \ln(\text{INCOME})_{mt} + n_t,$$

(25)

using an instrumental variable regression, where the log-price, $\ln p_{it}$, is instrumented by the estimate of TFPQ from (23), which serves as a demand shifter. The regression includes time fixed effects and the average income in a plant’s local market, $m$, is defined using the Bureau of Economic Analysis’ Economic Areas. The residual of this equation is interpreted as a pure demand shifter for that firm.

For the eleven products in the analysis, Foster et al. (2008) report average five-yearly autocorrelations of 0.32 and 0.62 for idiosyncratic TFPQ and demand, respectively. The cross-sectional dispersion of TFPQ and demand are 0.26 and 1.16 respectively. This means that demand shocks are more persistent and more dispersed across firms. Furthermore, they report a correlation of −0.54 between firm-level prices and TFPQ and a correlation of 0.75 between firm-level TFPQ and TFPR.

---

6Examples include bread, block ice, and ready-mix concrete.
3.1.2 Pricing Moments

For moments related to micro-level pricing behavior, we reference Vavra (2014) who reports pricing moments using CPI micro-data from the Bureau of Labor Statistics spanning the period from 1988 through 2012.\footnote{The same dataset is widely used in the literature, see Bils and Klenow (2004); Nakamura and Steinsson (2008).} Price data are at the product-outlet level and temporary sales are discarded from the analysis. In his sample, Vavra (2014) reports a monthly frequency of a regular price change to be 11%, of which 65% are upward adjustments. The average size of a price change excluding non-adjustments is 7.7%, and the standard deviation of price changes is 0.075.

3.1.3 Markup and Pass-Through of Cost Shocks to Prices

Following the methods of De Loecker et al. (2020), we estimate the markup distribution of U.S. public firms using Standard and Poor’s Compustat data. To be in line with the sample in Foster et al. (2008), we restrict the analysis to data between 1980 and 2000. In particular, we follow the production approach and compute firm-level markup as the ratio of sales to cost of goods sold, multiplied by the output elasticity of variable inputs estimated at the two-digit NAICS level.\footnote{Following the literature, we exclude the following two-digit industries: utilities, finance and insurance, real estate and rental and leasing, as well as public administration.} In our sample, the average markup is 56% and the median markup is 33%.

A major theoretical implication of a Kimball demand system is the incompleteness of cost pass-through to prices. One of the ways of capturing empirically the magnitude of cost pass-through can be found in the international finance literature. This literature looks at the pass-through of exchange rate shocks to importer prices, with the understanding that the exchange rate movements are exogenous from the viewpoint of importers. The empirical evidence is overwhelmingly in support of an incomplete pass-through of costs even in the medium and long-run: Campa and Goldberg (2005) estimate the long-run pass-through in the US to be 42%, Gopinath and Itskhoki (2010) find it to be between 20% to 40%, and Gopinath et al. (2010) find an aggregate pass-through of 30%. On the other hand, estimation of cost pass-through is more challenging in a purely domestic setting, due to the scarcity of appropriate data and well-identified shocks. To this end, recent studies using merged data on both costs and prices recover cost pass-through estimates that are similar to the international macro evidence. Using Chilean supermarket-supplier merged data, Aruoba et
Table 1: Externally Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount Factor</td>
<td>0.9966</td>
<td>Annual discount rate of 4%</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Labor disutility</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Growth rate of $S$</td>
<td>0.002</td>
<td>Annual inflation rate of 2.4%</td>
</tr>
<tr>
<td>$\sigma_S$</td>
<td>SD of shocks to nom. expenditure</td>
<td>0.0037</td>
<td>Vavra (2014)</td>
</tr>
<tr>
<td>$\sigma_{zn}$</td>
<td>Corr. b/w productivity and demand innov.</td>
<td>0</td>
<td>Foster et al. (2008)</td>
</tr>
</tbody>
</table>

Note: This table displays the externally calibrated parameters in the model.

al. (2022) find that 29% of a supplier price change is passed onto the retail price conditional on a price change at the supermarket level. Carlsson et al. (2022) estimate that between 21% to 33% of innovations to firm productivity are passed through to prices using data on Swedish manufacturing firms. Overall, the evidence from both the open- and closed-economy literature points to incomplete cost pass-through to prices in the range of 20% to 40%.

3.2 Externally Calibrated Parameters

In this section we explain how we fix a subset of parameters. These parameters are kept the same across the four different versions of the model we consider. A period is a month, and we set the monthly discount rate $\beta$ to 0.9966 such that the annual discount rate is 4%. Consistent with the usual choice in the literature (Golosov and Lucas, 2007), the disutility of labor $\chi$ is normalized to 1, so that the nominal wage, $W_t$, is equal to the money supply, $S_t$. The monthly growth rate of the money supply, $\mu$, is 0.2%, which implies an annual inflation rate of approximately 2.4%. Following Foster et al. (2008), we assume that idiosyncratic demand and productivity innovations are uncorrelated. Finally, when we run monetary policy experiments, we set $\sigma_S = 0.0037$ following Vavra (2014) to match the volatility of nominal output growth in the U.S. Table 1 summarizes the five externally calibrated parameters.

9The estimation strategy of Foster et al. (2008) requires demand and productivity to be orthogonal. Using Colombian data and an alternative strategy that relaxes the assumptions about the covariance between demand and supply shocks, Eslava et al. (2022) report a minimal correlation between demand and productivity.
3.3 Internally Calibrated Parameters

The former analysis leaves 7 remaining parameters to be internally calibrated. Four of these parameters govern the AR(1) processes for idiosyncratic productivity \((\rho_z, \sigma_z)\) and idiosyncratic demand \((\rho_n, \sigma_n)\), two additional parameters govern the Kimball demand system \((\omega, \psi)\), and the final parameter is the fixed menu cost \((f)\). In our baseline model, we jointly calibrate these parameters to match a set of empirical moments. Here are more details of how the calibration works for each of the four versions we consider:

- **CES I**: Here we use the standard, or agnostic, approach in the literature and calibrate \((f, \rho_z, \sigma_z)\) to match three pricing moments: frequency of price changes, fraction of positive changes and the average size of price changes. In this version there are no demand shocks. We set \(\omega = 1.33\) to obtain a desired markup of 33%, which is the median markup in our data, and, naturally \(\psi = 0\) so that CES aggregation is obtained.

- **CES II**: In this version we take the first step in trying to be consistent with the firm-dynamics facts from Foster et al. (2008) and calibrate \((\rho_z, \sigma_z)\) to match two moments: the five-yearly autocorrelation and the variance of TFP. We still calibrate \(f\) to match the frequency of price changes, turn off demand, and set \(\psi = 0\) and \(\omega = 1.33\).

- **CES III**: This version adds the firm-level demand shocks to the model and \((\rho_n, \sigma_n)\) to match two moments: the five-yearly autocorrelation and the variance of demand from Foster et al. (2008). The rest of the strategy is identical to CES II.

- **Baseline**: The baseline model jointly calibrates all seven parameters to match seven moments: the four firm dynamics moments CES III uses, two additional moments from Foster et al. (2008): \(\text{Corr}(\text{TFPQ}, P)\) and \(\text{Corr}(\text{TFPQ}, \text{TFPR})\), as well as the frequency of price changes as the only pricing moment.

The model-based moments we need for calibration can only be computed via simulation. To that end, we simulate 20,000 firms for 700 periods and drop the first 100 periods before computing any statistics. Computing moments that are monthly is straightforward. In order to compute moments that have their data counterpart in Foster et al. (2008), we aggregate the simulated data to the corresponding frequency and replicate their methodology. In particular, we aggregate the simulated monthly data into annual frequency by taking simple sums of revenue, sales, and employment. We then construct a panel dataset with the same time structure as Foster et al. (2008), namely five waves of annual observations that are five
years apart. Because labor is the only input in the model, we recover firm-level TFPQ and TFPR as,

\[ TFPQ_{it} = \ln q_{it} - \ln l_{it}, \quad (26) \]
\[ TFPR_{it} = \ln (p_{it}q_{it}) - \ln l_{it}. \quad (27) \]

This is equivalent to mapping our unique inputs to their basket of inputs. We estimate the demand function using the same IV specification as Foster et al. (2008),

\[ \ln(q_{it}) = \beta \ln(p_{it}) + \text{Time FE} + \eta_{it}, \quad (28) \]

where \( \ln(p_{it}) \) is instrumented by \( TFPQ_{it} \) and recover firm-level demand shifters as the residuals, \( \eta_{it} \). At the end of this process, we obtain five-yearly measures that are direct counterparts of those computed by Foster et al. (2008).

Before turning to the results, a discussion on the identification of parameters is in order. While all parameters influence the model’s ability to match all calibration targets, some parameters are more responsible for matching specific target moments. Some of these are quite intuitive. The fixed cost, \( f \), has a significant role in the model-implied frequency of price changes, and the shock-process parameters \( (\rho_z, \sigma_z, \rho_n, \sigma_n) \) are mostly related to the corresponding five-yearly moments from Foster et al. (2008).

What may be less obvious is how \( \psi \) and \( \omega \) are linked to the correlations of TFPQ with prices and TFPQ with TFPR. To understand this, it is instructive to start from a CES demand system (with \( \psi = 0 \)). The profit-maximizing rule in that framework delivers a pricing strategy that sets price as a constant markup over marginal cost. As a result, a firm’s optimal price is inversely proportional to its productivity, that is \( \text{Corr}(P, TFPQ) = -1 \). Moreover in a CES demand system TFPR is equalized across firms and as such \( \text{Corr}(TFPR, TFPQ) = 0 \).

This is because optimizing firms will operate at the point where the marginal product of labor \( (p_{i,t}z_{i,t}) \) is equal to the nominal market wage. Under a Kimball demand system, both productivity and demand factors affect the optimal price of a firm. In particular, deviations from CES, controlled by the parameter \( \psi \), diminish the role of productivity in pricing relatively to demand. This, in turn, would reduce the perfect negative correlation between price and TFPQ since some price changes will be due to demand shocks. The parameter \( \omega \), on the other hand, governs the elasticity of substitution between varieties, with a higher value of \( \omega \) indicating less substitutability across varieties. In a Kimball demand system, more productive firms can charge a higher price if the elasticity of substitution is lower (\( \omega \).
is higher) leading to a higher correlation coefficient between TFPQ and TFPR.

The results of the internal calibration of the four versions of the model (three CES versions and the baseline Kimball version) are reported in Table 2. The top two panels show the moments considered, where targeted moments for each version is shown in bold. The first panel shows pricing moments described in Section 3.1.2 while the second panel shows the firm-dynamics moments from Foster et al. (2008), described in Section 3.1.1. The third panel shows the seven parameters where boldface indicate that the parameters calibrated jointly, while others are fixed as explained earlier.

Starting with CES I, which uses \((\rho_z, \sigma_z, f)\) to match the first three pricing moments in the first panel, we see that it matches those moments very well. However, it completely misses the firm-dynamics moments. The size and persistence of the \(z\) process, which is used to match pricing moments, generates very small and nearly transitory movements in idiosyncratic productivity, as can be seen in the first two rows of the firm-dynamics moments. This version is, by definition, unable to say anything about the remaining firm-dynamics moments due to the absence of demand shocks.

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### Table 2: Internal Calibration

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>CES I</th>
<th>CES II</th>
<th>CES III</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency of price changes</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>0.12</td>
</tr>
<tr>
<td>Fraction of price increases</td>
<td>0.65</td>
<td>0.64</td>
<td>0.61</td>
<td>0.58</td>
<td>0.58</td>
</tr>
<tr>
<td>Size of price changes</td>
<td>0.08</td>
<td>0.08</td>
<td>0.15</td>
<td>0.14</td>
<td>0.07</td>
</tr>
<tr>
<td>5-yearly autocorr of (z^i)</td>
<td>0.32</td>
<td>0.00</td>
<td>0.32</td>
<td>0.32</td>
<td>0.32</td>
</tr>
<tr>
<td>Cross-sectional SD of (z^i)</td>
<td>0.26</td>
<td>0.03</td>
<td>0.26</td>
<td>0.26</td>
<td>0.25</td>
</tr>
<tr>
<td>5-yearly autocorr of (n^i)</td>
<td>0.62</td>
<td>0.01</td>
<td>0.00</td>
<td>0.62</td>
<td>0.62</td>
</tr>
<tr>
<td>Cross-sectional SD of (n^i)</td>
<td>1.16</td>
<td>0.01</td>
<td>0.04</td>
<td>1.18</td>
<td>1.05</td>
</tr>
<tr>
<td>Corr b/w TFPR and TFPQ</td>
<td>0.75</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.74</td>
</tr>
<tr>
<td>Corr b/w price and TFPQ</td>
<td>–0.54</td>
<td>–1.00</td>
<td>–1.00</td>
<td>–1.00</td>
<td>–0.57</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Data</th>
<th>CES I</th>
<th>CES II</th>
<th>CES III</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\psi)</td>
<td>Super-elasticity</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>–1.27</td>
<td></td>
</tr>
<tr>
<td>(\omega)</td>
<td>Elasticity of Substitution</td>
<td>1.33</td>
<td>1.33</td>
<td>1.33</td>
<td>1.29</td>
<td></td>
</tr>
<tr>
<td>(\rho_z)</td>
<td>Persistence of (z^i)</td>
<td>0.66</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
<td></td>
</tr>
<tr>
<td>(\sigma_z)</td>
<td>Standard deviation of (z^i)</td>
<td>0.04</td>
<td>0.05</td>
<td>0.05</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>(\rho_n)</td>
<td>Persistence of (n^i)</td>
<td>–</td>
<td>–</td>
<td>0.992</td>
<td>0.997</td>
<td></td>
</tr>
<tr>
<td>(\sigma_n)</td>
<td>Standard deviation of (n^i)</td>
<td>–</td>
<td>–</td>
<td>0.05</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>(f)</td>
<td>Menu cost</td>
<td>0.01</td>
<td>0.06</td>
<td>0.03</td>
<td>0.03</td>
<td></td>
</tr>
</tbody>
</table>

Note: The top panel of this table compares the targeted moments and model-implied moments for the four model specifications, where the bolded numbers highlight moments that are targeted in the calibration. The bottom panel shows the parameter values for each calibration.
Turning to CES II, where we now give up on matching pricing moments, except for the frequency of price changes, and use \((\rho_z, \sigma_z)\) to match the first two firm-dynamics moments, the calibration is successful in the sense that the targeted moments are matched, including the dynamics of idiosyncratic productivity. However, this version misses the two untargeted pricing moments, especially the last one completely. In an effort to match the more volatile and persistent idiosyncratic productivity process, the model generates price changes that are about twice as large on average than the data. Moreover, due to the absence of demand shocks, the four remaining firm-dynamics moments are also not matched.

CES III attempts to match additional firm dynamics moments by including a idiosyncratic demand shock in addition to a idiosyncratic productivity shock, once again leaving all pricing moments except for the frequency of price changes as untargeted. This version is able to match the four firm-dynamics moments by picking appropriate parameters for the aforementioned shocks. Due to the CES structure, as explained above, the last two moments are still elusive for this version. And just like CES II, it fails to deliver on the untargeted pricing moments.

Finally turning to the baseline model featuring a Kimball demand system, we use the only pricing moment common to all models (frequency of price changes) in addition to all of the six firm-dynamics moments as targets and calibrate all seven parameters. The second panel shows that the model matches all of the firm-dynamics moments very closely. The calibrated process for idiosyncratic demand is highly persistent with a monthly autocorrelation of 0.997, whereas the idiosyncratic productivity process exhibits less persistence with a monthly autocorrelation of 0.98. The standard deviation of the innovation to idiosyncratic productivity (0.06) is higher than that of idiosyncratic demand (0.02). Preferences appear to be very persistent and subject to relatively small shocks when compared to technology. However, the stationary distribution of idiosyncratic demand is highly dispersed due to the persistent nature of the process. The calibrated values of \(\omega\) and \(\psi\) are 1.29 and –1.27, respectively. These parameters are mostly informed by the last two firm dynamics moments. The value of \(\omega\) is not too far from the value we fix in the CES versions based on a 33% markup, though with a Kimball demand system it is no longer interpreted as the desired markup.

To sum up, the baseline model is successfully calibrated in that all targeted moments are matched reasonably well. We next turn to the discussion of how the baseline model performs in matching untargeted moments, including the pricing moments shown in Table 2 in Section 4.1.
3.4 Identification of Model Parameters

We alluded to certain calibration targets as being the main source of information in the calibration of certain parameters. In this section, we make this point explicit. In doing so we also demonstrate that the calibration targets are indeed very informative for the respective parameters by borrowing an exercise from Daruich (2022).

The main idea is to generate variation in the parameter space and investigate how the implied calibration targets are impacted – essentially taking a partial derivative. To do so, we first draw 500 parameter vectors from uniform Sobol points given a hypercube of the parameter space, which generates a quasi-random set of candidate parameter vectors.10 Then, for each parameter vector, we solve and simulate the model to compute the relevant model-implied moments. This allows us to see how each of the seven parameters influences each of the seven calibration targets.

Figure 2 plots the values of each model-implied target moment against the values of the parameter it is assigned to. In particular, we group the values of each parameter in deciles, which we plot on the horizontal axis. Then, for each decile, we show the median value of the associated moment in red circled dots and the 25\textsuperscript{th} and 75\textsuperscript{th} percentiles in blue down-pointing triangles and green up-pointing triangles, respectively. The slope of the scatter plot is informative about the importance of that parameter, whereas the vertical dispersion reveals the influences of all other parameters on a particular moment.

The frequency of price adjustment exhibits a strong negative correlation with the menu cost \( f \). Meanwhile, other parameters also play a role as is evident in the vertical dispersion. For example, for a fixed value of \( f \), larger idiosyncratic shocks generate more frequent price changes. The parameters \((\rho_z, \sigma_z)\) are strongly correlated with the five-yearly autocorrelation and cross-sectional distribution of firm productivity, whereas other parameters play a minimal role as can be seen in the tight vertical variation in the scatter plots. For \((\rho_n, \sigma_n)\), we observe a similar relationship, but there is noticeably more noise in the cross-sectional standard deviation of demand. This is mainly because at a given decile of \( \sigma_n \), the remaining parameters, including \( \rho_n \) are randomly drawn. Since the value of \( \rho_n \) is generally very close to one, the resulting cross-sectional dispersion of demand is very sensitive to the value of \( \rho_n \) in addition to \( \sigma_n \). Lastly, consistent with our reasoning, we recover a strong negative relationship between \text{Corr}(\text{TFPQ},P)\) and \( \psi \). We also observe a weak but visibly positive relationship

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\(10\) A uniform Sobol sequence is a sequence of points that spans the \( n \)-dimensional hypercube in an even and quasi-random manner. For the purpose of the exercise, using quasi-random Sobol numbers are more efficient than drawing random numbers because Sobol numbers are designed to sample the space of possibilities evenly given the total number of draws, whereas a truly random sample is subject to sampling noise.
Figure 2: Identification of Internally-Calibrated Parameters

(a) Price Adjustment Frequency vs. $f$

(b) Corr($TFPQ, P$) vs. $\psi$

(c) Corr($TFPQ, TFP$) vs. $\omega$

(d) Five-yearly AR of $TFPQ$ vs. $\rho_z$

(e) Cross-sectional SD of $TFPQ$ vs. $\sigma_z$

(f) Five-yearly AR of demand vs. $\rho_n$

(g) Cross-sectional SD of demand vs. $\sigma_n$

Note: For each decile of a given parameter plotted on the horizontal axis, the red dot shows the median of the moment that is assigned to the parameter. The blue down-pointing triangles and green up-pointing triangles show the 25th and 75th percentiles respectively.
between \text{Corr}(\text{TFPQ},\text{TFPR}) \text{ and } \omega. \text{ The large variation in this correlation given a value of } \omega \text{ reveals that it is sensitive to the values of other parameters in addition to } \omega. \text{ In particular, we find that } \sigma_n \text{ and } \sigma_z, \text{ which determine the stationary distribution of idiosyncratic productivity and demand, have sizable effects on the level of this correlation. Given that all the parameters except for } \omega \text{ exhibit tight links with their associated targets, we argue that } \omega \text{ can be credibly identified by Corr(TFPQ,TFPR) when all other parameters are fixed and matched to their respective targets.}

\text{They key takeaway from this exercise is that for the most part, the links between the parameters and moments are quite tight. While it is too computationally intensive, if one were to consider a formal generalized method of moments approach to estimating the parameters of interest, this figure seems to suggest that one would obtain fairly tight standard errors for the estimates.}

3.5 External Validity

The work of Foster et al. (2008) is the only study for the U.S. with a systematic estimation of productivity and demand shocks at the firm level for different industries. This estimation requires price and quantity data at the product level along with other information such as inputs. By using a carefully selected set of firms in the Census of Manufactures that produce uniform products, they are able to separately estimate shock processes for productivity and demand. However, the external validity of the estimation of Foster et al. (2008) to the broader economy may be a concern.

\text{In this regard, we first note that we calibrate the strength of strategic complementarity in the baseline model using the correlations between TFPQ, prices, and TFPR from Foster et al. (2008). The calibration yields a cost pass-through of 38\%, which is in line with evidence on the exchange-rate pass-through from the international finance literature as well as recent estimates based on domestic cost-price linked data reviewed in Section (3.1.3), both of which report estimates in the range of 20 to 40 percent.}

\text{Furthermore, several other countries have similar data for a wider set of firms, and researchers have estimated some of the moments that we use for our identification strategy. For instance, Eslava et al. (2013) use Colombian firm-level data covering the entire manufacturing industry to separately identify productivity and demand processes at the firm level. They report similar values for Corr(TFPQ,TFPR) and Corr(TFPQ,P), which are crucial for pinning down the Kimball demand system parameters. Specifically, they report Corr(TFPQ,TFPR) = 0.69 and Corr(TFPQ,P) = −0.65 versus the Foster et al. (2008)
Table 3: Untargeted Pricing Moments

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>CES II</th>
<th>CES III</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Size</td>
<td>0.08</td>
<td>0.15</td>
<td>0.14</td>
<td>0.07</td>
</tr>
<tr>
<td>Fraction Up</td>
<td>0.65</td>
<td>0.61</td>
<td>0.58</td>
<td>0.58</td>
</tr>
<tr>
<td>SD($\Delta p$)</td>
<td>0.08</td>
<td>0.16</td>
<td>0.17</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Note: This table shows the three untargeted moments: average size of adjustment conditional on a price change, the fraction of adjustments that are positive, and the standard deviation of price changes excluding zeros from the empirical data and from model simulated data.

Values of 0.75 and −0.54, respectively.\(^\text{11}\)

Given that idiosyncratic demand and productivity processes determine the ergodic properties of firm growth, we can also compare the cross-sectional dispersion of output growth rates computed from model-simulated data with external evidence to gauge if the estimates from Foster et al. (2008) can be generalized beyond the eleven industries. We benchmark our estimates to Davis et al. (2006), a study using the Longitudinal Business Database and thus a good measure of the U.S. business dynamics. They estimate the cross-sectional standard deviation of firm revenue growth rate to be 0.39 over the period 1982-1997, while in our simulated data this untargeted moment is 0.41. Therefore, we conclude that the estimation of Foster et al. (2008) can be used to shed some light about the general behavior firms.

4 Results

We calibrated our model using firm-dynamics moments from Foster et al. (2008) and one pricing moment, the average frequency of price adjustments. In this section, we turn to investigating the implications of the model in three dimensions: other pricing moments, markup distribution, and monetary non-neutrality, none of which was targeted in our calibration.

4.1 Untargeted Pricing Moments

Table 3 shows three important pricing moments we do not target in the calibration of the baseline model, along with their data counterparts and the results from two CES versions where these pricing moments are not targeted. These are the average size of a price change conditional on a change, the fraction of adjustments that are positive, and the dispersion of

\(^{11}\)Moreover, they estimate a correlation between productivity and demand innovations in the neighborhood of zero, consistent with Foster et al. (2008) and our assumption.
non-zero price changes. In CES II and CES III we calibrate the properties of the stochastic processes for demand and productivity to match the firm dynamics moments. The productivity process, in turn, ends up producing price changes that are about twice as large as what is in the data. The table shows that, unlike the CES versions, the baseline version is actually able to match the three untargeted pricing moments quite well – most importantly the size of price changes is tempered relative to the CES versions. The reason for this is, unlike CES where the pass-through of cost shocks are 100%, with a Kimball demand system the pass-through is much less and therefore for the same level of productivity shocks, price changes are smaller. A back-of-the-envelope calculation using the 38% cost pass-through reported above would suggest that for the same size productivity shocks, the size of price changes will be about 0.053. Because firms also change prices in response to demand fluctuations, the introduction of demand shocks increases this number to around 0.07, which is very close to the moment in the data at 0.08.

In comparison, Klenow and Willis (2016) use a model with a Kimball demand system, but they do not include an idiosyncratic demand shock. They follow the approach in CES I and calibrate their model to match pricing moments. Their conclusion, similar to our findings for CES I, is that the properties of the firm-level productivity process needed for matching the pricing moments is inconsistent with those found in the firm-dynamics literature. The baseline calibration in Klenow and Willis (2016) \( (\theta = 5, \epsilon = 10 \text{ using their specification}) \) translates roughly to parameter values \( \omega = 1.25 \) and \( \psi = -2 \) under our specification of the Kimball aggregator and a cost pass-through of 28%. This is not too far from the pass-through that we obtain in our calibration. Since the cost pass-through is a third of CES, in order to match the same pricing moments, one would need even larger shocks than what we obtained in CES I. This is why Klenow and Willis (2016) conclude that their calibrated model was not consistent with the firm dynamics literature. Our model shows that the inclusion of idiosyncratic demand shocks compensates for the weaker role of productivity shocks. It turns out that idiosyncratic demand and productivity processes that are consistent with firm dynamics estimates can generate realistic price-adjustment facts at a degree of strategic complementarity that is also consistent with firm-level evidence.

One may wonder if the success of the model has much to do with the precise calibration of \( \psi \) or whether any deviation from CES by reducing \( \psi \) below zero would have done the trick. In Figure 3 we plot the average size of non-zero price changes versus different values of \( \psi \), similar to Figure 2, where the vertical variation for a given level of \( \psi \) is due to the differences in other parameters. This figure shows that there is a tight relationship between
the average size of adjustments and \( \psi \). As \( \psi \) falls, the increasing real rigidities (strategic complementarities) make the firms less and less willing to deviate from their competitors, avoiding large price changes. This makes the average size of price changes to fall. Thus, it is even more remarkable that our model matches these moments as good as it does, given that a different value of \( \psi \) from the one calibrated to firm-dynamics moments may have led to a worse fit.

We also examine the hazard function of price change from the model. The hazard of a price change is the probability that a price will change \( t \) periods after the last adjustment, conditional on the price spell lasting \( t \) periods. Empirically, the hazard function is found to be either downward-sloping (Nakamura and Steinsson (2008)) or flat (Klenow and Kryvtsoy (2008)). As pointed out by Nakamura and Steinsson (2008), plain vanilla menu cost models are typically not able to generate hazard functions that are consistent with the empirical evidence. This largely hinges on the calibration of the idiosyncratic processes. In a model with trend inflation, the hazard function is upward-sloping when idiosyncratic shocks are small. Larger idiosyncratic shocks and more persistent idiosyncratic processes flatten the hazard function as they lead to temporary price changes that are often reversed quickly.
When idiosyncratic shocks are sufficiently large, a plain vanilla menu cost is able to generate a downward-sloping hazard. Nakamura and Steinsson (2008) argue that such calibrations are unrealistic, due to the fact that they are inconsistent with micro pricing facts. This conclusion has spurred alternative pricing models that seek to rationalize a downward-sloping hazard, such as Baley and Blanco (2019) who introduce firm-level uncertainty and learning to generate frequent price changes shortly following an adjustment.

Figure 4 plots the hazard function generated by our model across the four specifications. In CES I, which is the approach commonly taken by the literature, the hazard function is increasing over the first few months and flattens out afterwards. This is consistent with the baseline calibration of Nakamura and Steinsson (2008). The other three model specifications, where idiosyncratic processes are calibrated to firm dynamics evidence, instead exhibit a downward-sloping hazard. This shows that calibrations that imply downward-sloping hazards are not necessarily unrealistic. In fact, a simple menu cost model with Kimball aggregation, calibrated to firm dynamics estimates generates a price change hazard that is much more consistent with the empirical counterpart, while remaining consistent with micro pricing facts.
These figures plot the kernel density of the empirical markup distribution from publicly traded firms in the U.S. as well as the kernel density of the markup distribution in the ergodic distribution of the four versions of the model. Both kernel densities are computed using the optimal bandwidth for normal densities.

4.2 Cost Pass-through and Markups

In a flexible-price model with CES and symmetric firms, all prices are set to a fixed markup above marginal cost and thus all firms have the same markup, leading to a degenerate markup distribution. With pricing rigidities, firms that cannot change their price in a period could potentially deviate from the desired markup. As a firm’s marginal cost changes via changes in productivity, so would its price, reflecting the cost change one to one. Since demand shocks do not change prices, they have no impact on markups. Panel (a) of Figure 5 plots the kernel density of markup distribution for our CES versions along with its data counterpart. For all CES versions the markup varies very little and does so symmetrically around \( \omega = 1.33 \), or a net markup of 33%. This is completely at odds with the distribution we obtain from the data, which has a mode just above 0% with a very wide right tail reaching a level of 200%, though markups of as low as −50% are also observed.

The desired markup of a firm facing a Kimball demand system depends on both its idiosyncratic productivity and demand. Because the pass-through of marginal cost to price is incomplete, more productive firms do not pass on their cost advantage to price one-for-one, resulting in higher markups. Also, firms with larger idiosyncratic demand optimally choose higher prices and hence higher markups. The cross-sectional distribution of productivity and
demand, alongside pricing frictions, result in a non-degenerate markup distribution in the model. Panel (b) of Figure (5) plots the kernel density of the cross-sectional distribution of gross markup both from the data and the model. The markup distribution from the model mimics the non-targeted empirical distribution reasonably well. The median gross markup in the model is 1.35, compared to 1.33 in the data. The model-implied markup distribution, however, exhibits lower variance relative to the empirical distribution, much of which stems from the tails. Our model does not generate as many firms that have markups larger than 200%, nor does it deliver markups deep into the negative territory. Matching these extreme tails of the distribution would require additional features such as non-Gaussian shocks to demand or monopolies (large positive markups) and customer capital (negative markups).

The success of our model in replicating the empirical markup distribution is perhaps not very surprising given the work of Arkolakis et al. (2019) and Edmond et al. (2018), among others, who use Kimball demand systems for modeling firm markups. What is remarkable is that our model, calibrated to results from Foster et al. (2008) that relies on selected manufacturing industries, provide a good fit to the distribution of markups estimated for a broader set of industries.

4.3 Monetary Policy and Non-Neutrality

Nominal rigidities alone are typically insufficient at generating sizable real effects to nominal shocks (Caplin and Spulber, 1987; Golosov and Lucas, 2007). A solution put forth by Ball and Romer (1990) is to introduce real rigidities in conjunction with nominal pricing frictions. Since then, the macroeconomics literature has explored the plausibility of various potential sources of real rigidities. Strategic complementarities in pricing induced by a Kimball demand system is one such mechanism. When a monetary policy shock hits the economy, some firms choose not to respond due to the presence of nominal rigidities, which makes monetary policy effective in stimulating real activity, or non-neutral. With strategic complementarities, the firms adjusting their prices choose to adjust less than under CES demand in order to remain closer to their competitors who choose not to adjust their prices, and this real rigidity adds to the degree of non-neutrality. This intuition is formalized in Alvarez et al. (2022), who derive analytic results in a menu cost model featuring strategic complementarity casted as a Mean Field Game and show that complementarity makes the impulse response of output to a nominal shock larger at each horizon.

In spite of its theoretical soundness, the literature has largely dismissed strategic pricing complementarity as an empirically relevant source of real rigidity due to the findings of
Klenow and Willis (2016), who conclude that a menu-cost model featuring a Kimball demand system that is consistent with micro pricing facts and can generate sufficient monetary non-neutrality was inconsistent with the evidence from the firm dynamics literature. In the previous section, we showed that such a model can in fact be simultaneously consistent with both the micro pricing facts and the firm-dynamics literature. With this in mind, we turn to assessing whether our model is able to generate sizable real responses to nominal shocks.

We consider four measures of non-neutrality. The first measure is the unconditional standard deviation of consumption in a long simulation. Our model features no aggregate shocks other than the nominal expenditure shock $\epsilon_t$ in (22), which can also be interpreted as a monetary policy shock. As such, if monetary policy was perfectly neutral, then aggregate consumption would be constant. Thus, the standard deviation of aggregate consumption serves as a measure of deviation from neutrality. The other three measures are obtained from a response of the economy to a one-time change in the nominal expenditure shock, $\epsilon_t$.\(^{12}\)

In particular, we compute the response of real output to a positive nominal expenditure shock of size 0.2%, which doubles the monthly growth rate of aggregate expenditure. The first object we compute from this response is the peak response of output as a fraction of the size of the shock, which in a model like ours without any internal propagation, will happen on impact. The second object we can compute from this response is the half-life of the shock, which tells us how persistent the effect of the shock is. Finally, these two measures can be summarized by the cumulative impulse response (CIR) which adds up the response of output as a fraction of the shock over the period it is non-zero and divides by the number of periods in a year – 12 in our case.

Before we turn to the results, it is worth mentioning that a CES model with Calvo pricing, where firms receive an i.i.d. shock that determines when they can adjust their prices, would yield an output response as a fraction of the shock that is exactly $(1 - \alpha)^h_h$ where $\alpha$ is the probability that firms can change their price and $h$ is the horizon where $h = 1$ is the period of the shock.\(^{13}\) Based on the evidence we presented in Section 3.1.2, $\alpha$ would be set to 0.11 in such a model, indicating that the initial (and peak) response of output would be 89% of the shock. The cumulative response according to the Calvo model would be 0.76.\(^{14}\) This coincides with the result of Alvarez et al. (2016) who show that the CIR can be expressed in

\(^{12}\)If the aggregate price does not respond at all, then the entirety of the increase in nominal expenditure is reflected in an increase in real output, leading to complete transmission to real output. On the other extreme, if the aggregate price is perfectly flexible, the real effect of the nominal shock would be zero.

\(^{13}\)See Appendix A.2 for the derivations.

\(^{14}\)The CIR in the Calvo model is given by $\sum_{t=0}^{\infty} \frac{0.89^t}{12} = 0.76$.
Figure 6: Impulse Response of Real Output to a Nominal Expenditure Shock

Note: This figure plots the impulse response of real output expressed as a fraction of the nominal expenditure shock on the vertical axis and periods elapsed since the shock on the horizontal axis.

terms of the kurtosis and frequency of price changes. According to their formula, the Calvo model has a CIR of 0.76 whereas a menu cost model à la Golosov and Lucas (2007) has CIR of 0.13.

Figure 6 plots the impulse response of real output expressed as a fraction of the size of the shock for the baseline model as well as the three CES versions we introduced earlier. Table 4 contains the four statistics that summarize the degree of monetary non-neutrality we discussed above. CES I (blue dashed line), which was calibrated to pricing moments, deliver a peak response of 0.56, which is substantially lower than the Calvo response. The response is also fairly short-lived with a half-life of 1.25 months. The resulting cumulative impulse response is 0.11, which is close to the number reported in Alvarez et al. (2016) for a Golosov-Lucas type menu cost model. CES II (orange dashed dotted line) which calibrates the model to the productivity process of Foster et al. (2008) produces more non-neutrality with a peak response of 0.69. It is also somewhat more long-lived with a half-life of 2.9 months and has a larger CIR of 0.28 compared to CES I. Neither of these versions delivers
Table 4: Measures of Monetary Non-neutrality

<table>
<thead>
<tr>
<th>Moment</th>
<th>CES I</th>
<th>CES II</th>
<th>CES III</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>SD(C)</td>
<td>0.22%</td>
<td>0.39%</td>
<td>0.44%</td>
<td>0.52%</td>
</tr>
<tr>
<td>Impact</td>
<td>0.56</td>
<td>0.69</td>
<td>0.69</td>
<td>0.82</td>
</tr>
<tr>
<td>Half-life</td>
<td>1.25</td>
<td>2.90</td>
<td>3.46</td>
<td>4.50</td>
</tr>
<tr>
<td>CIR</td>
<td>0.11</td>
<td>0.28</td>
<td>0.30</td>
<td>0.42</td>
</tr>
</tbody>
</table>

Note: This table displays four measures of monetary non-neutrality for the four model calibrations.

Random menu costs were first introduced by Dotsey et al. (1999). Recent works by Nakamura and Steinsson (2010), Midrigan (2011), and Alvarez et al. (2016) explicitly explore the implications of random menu costs on monetary non-neutrality.
Lastly, it is fair to say that a response of this magnitude is considered appropriately large in the literature as summarized in Mongey (2021).

The model exhibits substantial monetary non-neutrality due to two key features. First, as discussed, a Kimball demand system introduces strategic complementarity in pricing among firms as deviation from the prices of competitors are costly. Therefore, firms are hesitant to respond aggressively, if at all, to the nominal shock if a sizeable share of other firms are not responding. Here, we show that a degree of strategic complementarity, as controlled by $\psi$ and summarized by the cost pass-through to prices, that is consistent with empirical evidence contributes to significant non-neutrality. Second, the presence of both productivity and demand shocks plays a role in weakening the selection effect in price adjustments to an aggregate shock which further contributes to a larger output response. As shown in Golosov and Lucas (2007), the real response to nominal shocks hinges not on how many prices adjust but which prices adjust. In the absence of idiosyncratic shocks, prices respond only to aggregate shocks and the only prices that adjust are those that are most out of line with the aggregate shock. Adding idiosyncratic shocks weakens the selection effect as firms respond not only to aggregate shocks but also to disturbances to their idiosyncratic states. Having two idiosyncratic shocks further weakens this self-selection as firms in our model response to two orthogonal shocks in productivity and demand on top of shocks to aggregate nominal expenditure.

5 Conclusion

The advent of rich micro datasets has spurred the advancement of models of firm dynamics over the past twenty years to study a range of macroeconomic and international topics, including monetary non-neutrality, exchange-rate pass-through, and firm markups. From each of these literatures, key modeling ingredients have emerged as important for explaining aggregate and firm-level dynamics. From the exchange-rate pass-through and markup-dynamics

\[\text{(Alvarez et al., 2016).} \]

\[\text{Lastly, it is fair to say that a response of this magnitude is considered appropriately large in the literature as summarized in Mongey (2021).}^{16}\]

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\[\text{not responding. Here, we show that a degree of strategic complementarity, as controlled} \]

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\[\text{and demand shocks plays a role in weakening the selection effect in price adjustments to} \]

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\[\text{Golosov and Lucas (2007), the real response to nominal shocks hinges not on how many} \]

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\[\text{with the aggregate shock. Adding idiosyncratic shocks weakens the selection effect as firms} \]

\[\text{respond not only to aggregate shocks but also to disturbances to their idiosyncratic states.} \]

\[\text{Having two idiosyncratic shocks further weakens this self-selection as firms in our model} \]

\[\text{response to two orthogonal shocks in productivity and demand on top of shocks to aggregate} \]

\[\text{nominal expenditure.} \]

\[\text{\textbf{5 Conclusion}} \]

\[\text{The advent of rich micro datasets has spurred the advancement of models of firm dynamics} \]

\[\text{over the past twenty years to study a range of macroeconomic and international topics,} \]

\[\text{including monetary non-neutrality, exchange-rate pass-through, and firm markups. From each} \]

\[\text{of these literatures, key modeling ingredients have emerged as important for explaining aggregate} \]

\[\text{and firm-level dynamics. From the exchange-rate pass-through and markup-dynamics} \]

\[\text{\textsuperscript{16}As Table A.1 of Mongey (2021) summarizes, menu-cost models without real rigidities typically generate} \]

\[\text{a peak output response in the range of 0.35 to 0.50 (Golosov and Lucas, 2007). Some of the other studies} \]

\[\text{exploring alternative sources of real rigidities find higher numbers. Nakamura and Steinsson (2010) consider} \]

\[\text{a multi-sector model with a round-about production structure and obtain a peak output response of 0.80.} \]

\[\text{Gertler and Leahy (2008) model segmented labor markets and report a real response of 0.75. Burstein and} \]

\[\text{Hellwig (2007) incorporate decreasing returns to scale and wage rigidity and obtain 0.70, whereas Blanco et} \]

\[\text{al. (2022) builds a similar single-product model with DRS and find a peak output response of 0.80. Lastly,} \]

\[\text{Mongey (2021) studies a pricing model with duopoly competition and finds a peak response of 0.74. Relative} \]

\[\text{to the standard setup with monopolistic competition, the duopoly model generates a CIR that is 2.3 times} \]

\[\text{as large.} \]
literatures, models of strategic complementarity have been shown to play an important role. From the price-setting literature, nominal rigidities play a central role, but additional features in the form of real rigidities have also been necessary to approximate the degree of monetary non-neutrality from quantitative estimates.

However, each of these literatures has faced limitations that have prevented the emergence of a single modeling framework able to deliver the main results across all of these areas of study. From the price-setting literature, Klenow and Willis (2016) show that in order to generate realistic price-adjustment moments, a model with strategic complementarities requires a large magnitude of idiosyncratic productivity shocks that is inconsistent with micro evidence. A related limitation across these literatures has been the absence of micro data sets containing both prices and quantities at the firm level. Thus, most of these studies estimate or calibrate the parameters of idiosyncratic productivity processes to match pricing moments, because these pricing datasets lack data on quantities.

We propose a parsimonious framework that brings together the modeling elements from these literatures and also resolves prior limitations on the selection of modeling ingredients. First, we calibrate separate idiosyncratic shock processes for demand and productivity using empirical estimates from the firm-dynamics literature that employ both prices and quantities, along with revenue and inputs. This eliminates the need to calibrate these shock process parameters to produce observed pricing moments without any connection to quantities. Second, this approach allows us to re-investigate the role for strategic complementarities in a richer structure than was used in prior studies.

Our calibrated model is able to generate real and nominal dynamics that match key features of interest across these literatures. The combination of menu costs, a Kimball demand system, and idiosyncratic shocks for demand and productivity in the model produces cost pass-through in line with previous studies along with a distribution of markups that is similar to that estimated from the data. The model also produces simulated moments consistent with non-targeted moments on the size, direction, and dispersion of price changes. And when the model is extended to feature nominal expenditure shocks, it produces real effects from a nominal shock that are within the range of results from other studies of monetary non-neutrality. The two features generating this non-neutrality are the Kimball demand system and the presence of both idiosyncratic productivity and demand shocks, the latter of which dampens the selection effect in price adjustments to an aggregate shock and results in a larger real output response. By calibrating the model to directly match firm-level empirical estimates of the underlying shock processes for productivity and demand, we are
able to avoid the critique of Klenow and Willis (2016), while also highlighting the importance of the joint inclusion of idiosyncratic productivity and demand shock processes in the study of models of strategic complementarities. Thus, we view our work as opening the door to future research that jointly models real (e.g. investment, employment, entry / exit), nominal (price setting) and other (e.g. markup, pass-through) decisions of firms using one unified framework.
References


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A Derivations

A.1 Pass-through

Consider the static optimization problem of an intermediate firm without any pricing frictions. The nominal profit of an intermediate firm is

\[ \pi_i = \left( \frac{p_i}{P} - \frac{W}{Pz_i} \right) Y \frac{1}{n_i} \frac{1}{1 + \psi} \left( \frac{p_i}{\lambda n_i P} \right)^{\frac{\omega(1 + \psi)}{1 - \omega}} + \psi \]  

(29)

The first-order condition with respect to \( p_i \) is given by

\[ \frac{1}{p_i - \frac{W}{z_i}} + \frac{\omega(1 + \psi)}{1 - \omega} \left( \frac{p_i}{\lambda n_i P} \right) ^{\frac{\omega(1 + \psi)}{1 - \omega}} - 1 \frac{1}{\lambda n_i P} = 0 \]  

(30)

\[ \left( \frac{p_i}{\lambda n_i P} \right) ^{\frac{\omega(1 + \psi)}{1 - \omega}} \left[ 1 - \left( \frac{W}{z_i} - p_i \right) \left( \frac{\omega (1 + \psi)}{1 - \omega} \frac{1}{p_i} \right) \right] = -\psi \]  

(31)

Log-linearizing the first-order condition around a symmetric steady state yield

\[ \left( \hat{p}_i - \hat{\lambda} - \hat{P} - \hat{n}_i \right) + \frac{1 + \left( \frac{W}{z_i} - \bar{p}_i \right) \left( \frac{1}{\bar{p}_i} \right)}{1 - \left( \frac{W}{z_i} - p_i \right) \left( \frac{\omega (1 + \psi)}{1 - \omega} \frac{1}{\bar{p}_i} \right)} \hat{p}_i + \frac{W}{z_i} \left( \frac{1}{\bar{p}_i} \right) = 0 \]  

(32)

where hatted variables denote log-deviations from the steady state.

Note that in a symmetric steady state, all firms are identical, have the same market share, and set the same price. Specifically, the optimal price is a fixed markup over cost \( \bar{p}_i = \omega \frac{W}{z_i} \). Substituting this into the log-linearized first-order condition gives

\[ \hat{p}_i = \frac{\omega}{\omega - 1} \left( \hat{\lambda} + \hat{P} + \hat{n}_i \right) + \frac{1}{\omega - 1} \hat{z}_i \]  

(33)

A.2 Transmission of Nominal Shock under Calvo Pricing and CES

Consider a simple model where CES demand and Calvo pricing. Each period, a random fraction \( \alpha \) of firms can adjust their prices freely while the other fraction \( 1 - \alpha \) cannot change their prices. For simplicity, assume that there are no aggregate risks and the economy is initially in a symmetric equilibrium where all firms set nominal prices to \( \bar{p} \) and nominal
expenditure $S = PC$ is equal to $\bar{S}$.

Under the Calvo setup, the aggregate price index $P_t$ can be written as a combination of the optimal reset price $X_t$, which is the price chosen by firms that are adjusting, and the lagged price index, which summarizes the prices of non-adjusters, as follows

$$P_t = \left[ \alpha X_t^{1-\theta} + (1 - \alpha) P_{t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (34)$$

Log-linearizing around the initial steady state where $P_t = X_t = \bar{p}$ and using hatted variables to denote log-deviations from the steady state yields

$$\hat{P}_t^{1-\theta} = \alpha \hat{X}_t^{1-\theta} + (1 - \alpha) \hat{P}_{t-1}^{1-\theta} \quad (35)$$

Suppose that in period $t = 1$, there is an unanticipated permanent shock $\mu > 0$ to nominal expenditure shock $S$, such that $\hat{S}_1 = \mu$. Because the nominal wage is proportional to nominal expenditure, firms with the opportunity to adjust will respond to the shock by increasing their prices by $\mu$ as the optimal markup is a constant $\frac{\theta}{\theta - 1}$ over the marginal cost. In other words, the optimal reset price is $\hat{X} = \mu$.

As such, the aggregate price in period one is given by

$$\hat{P}_1 = \alpha \mu \quad (36)$$

Iterating forward, the aggregate price in period $h$ can be written as

$$\hat{P}_h = \left( \sum_{i=1}^{h} (1 - \alpha)^{i-1} \right) \alpha \mu \quad (37)$$

The response of real output is therefore given by

$$\hat{C}_h = \hat{S} - \hat{P}_h \quad (38)$$

$$= \mu - \left( \sum_{i=1}^{h} (1 - \alpha)^{i-1} \right) \alpha \mu \quad (39)$$

$$= (1 - \alpha)^h \mu \quad (40)$$

which implies that the output response as a fraction of the shock is $(1 - \alpha)^h$ at period $h$. 
B Model Solution

B.1 Rewriting the Problem

Note that firms need to observe $S$ and know its law of motion in order to solve their problem. These relevant aggregate variables in $S$ can be summarized by a single aggregate state variable $P_{t-1}/S_t$.

Because money supply $S_t = P_tC_t$ exhibits positive growth on average, nominal prices are also ever-increasing. We normalize all nominal variables by $S_t$ to ensure that the state variables are stationary. As such, we can rewrite the firm’s profit function

$$
\pi \left( \frac{p^i_t}{S_t}; n^i_t, z^i_t, \frac{P_t}{S_t}, \lambda_t \right) = \left( \frac{p^i_t}{P_t/S_t} - \frac{1}{z^i_t P_t/S_t} \right) \left( \frac{P_t/S_t}{\lambda_t n^i_t (P_t/S_t)} \right)^{\omega (1+\psi)} + \psi
$$

where we also use $Y_t = C_t = P_t/S_t$ from goods market clearing and $W_t = S_t$ from the household’s intratemporal optimality condition.

$P_t/S_t$ and $\lambda_t$ are the collective results of the pricing decisions of all firms. To know these, firms must know the entire firm distribution over the idiosyncratic states which is an infinite-dimensional object. Following the application of the Krusell and Smith (1998) algorithm in menu-cost models (Nakamura and Steinsson, 2010; Midrigan, 2011; Vavra, 2014), we conjecture the following forecasting rules for $P_t/S_t$ and $\lambda_t$

$$
\log \left( \frac{P_t}{S_t} \right) = F \left( \frac{P_{t-1}}{S_t} \right) = \alpha_0 + \alpha_1 \log \left( \frac{P_{t-1}}{S_t} \right) \quad (42)
$$

$$
\log (\lambda_t) = G \left( \frac{P_{t-1}}{S_t} \right) = \beta_0 + \beta_1 \log \left( \frac{P_{t-1}}{S_t} \right) \quad (43)
$$

Using these, the law of motion of the aggregate variable $P_{t-1}/S_t$ is also given by

$$
\log \left( \frac{P_t}{S_{t+1}} \right) = \log \left( \frac{P_t}{S_t} \right) + \log \left( \frac{S_t}{S_{t+1}} \right) = \alpha_0 + \alpha_1 \log \left( \frac{P_{t-1}}{S_t} \right) - (\mu + \sigma_S \epsilon_{t+1}) \quad (44)
$$

Now, we rewrite the intermediate producers’ problem using these state variables. At the beginning of a period, each intermediate producer starts off with a price $p^i_{t-1}/S_t$, idiosyncratic demand $n^i_t$, and idiosyncratic productivity $z^i_t$. They also observe $P_{t-1}/S_t$ and forecast $P_t/S_t$.
and λ\_t using the aforementioned laws of motion. The value of not adjusting is

\[
V_N \left( \frac{p^{i}_{t-1}}{S_t}; n^i_t, z^i_t, \frac{P_{t-1}}{S_t} \right) = \pi \left( \frac{p^{i}_{t-1}}{S_t}; n^i_t, z^i_t, \frac{P_{t-1}}{S_t} \right) + \mathbb{E}_t \left[ \Xi_{t,t+1} \cdot V \left( \frac{p^i_t}{S_{t+1}}, n^{i}_{t+1}, z^{i}_{t+1}, \frac{P_t}{S_{t+1}} \right) \right]
\]

which is equal to the flow profit evaluated at last period’s price adjusted for inflation plus a continuation value.

If the firm chooses to adjust its price, it pays the fixed price adjustment cost and chooses \( p^i_t \) to maximize the sum of the current flow profit and the present discounted value of future profit

\[
V_A \left( n^i_t, z^i_t, \frac{P_{t-1}}{S_t} \right) = -f^i_t \frac{P_t}{S_t} + \max_{p^i_t} \left\{ \pi \left( p^i_t; n^i_t, z^i_t, \frac{P_{t-1}}{S_t} \right) + \mathbb{E}_t \left[ \Xi_{t,t+1} \cdot V \left( \frac{p^i_t}{S_{t+1}}, n^{i}_{t+1}, z^{i}_{t+1}, \frac{P_t}{S_{t+1}} \right) \right] \right\}
\]

A firm chooses to adjust its price if and only if the value of doing so exceeds the value of inaction. Therefore, the value function of the firm is

\[
V \left( \frac{p^{i}_{t-1}}{S_t}; n^i_t, z^i_t, \frac{P_{t-1}}{S_t} \right) = \max \left[ V_N \left( \frac{p^{i}_{t-1}}{S_t}; n^i_t, z^i_t, \frac{P_{t-1}}{S_t} \right), V_A \left( n^i_t, z^i_t, \frac{P_{t-1}}{S_t} \right) \right]
\]

### B.2 Computational Strategy

A sketch of the computation algorithm is as follows. We first make guesses of the coefficients \((\alpha^0_0, \alpha^0_1, \beta^0_0, \beta^0_1)\) in the forecasting equations \( F \) and \( G \). Given the guesses, use value function iteration to solve for the intermediate-good producers’ value functions as well as the optimal pricing rules. Using pricing rules, simulate the model for a large number of periods and obtain simulated sequences of \( \frac{P_t}{S_t} \), \( \lambda_t \), and \( \frac{P_{t-1}}{S_t} \). Estimate the regressions \( F \) and \( G \) with model simulated data and obtain estimated coefficients \((\alpha^1_0, \alpha^1_1, \beta^1_0, \beta^1_1)\) which is then used to update the initial guesses. Repeat this process until the coefficient guesses and sufficiently close to the estimated coefficients from the linear regressions. In doing so, we find that the conjectured law of motion approximates the true law of motion from the model simulation well, as the regression yields an \( R^2 \) larger than 0.99.