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Economic Diversity and the Resilience of Cities

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Abstract: We show how local worker flow adjustment margins yield a theory-consistent sufficient statistic approximating the welfare effects of local shocks. Furthermore, we isolate a city's insurance value as this approximation's second-order term. Leveraging rich labor flows data across occupations, industries, and cities in France, we estimate spatial and nonspatial flows responses to local labor demand shocks. Less economically diverse French cities experience deeper contractions in gross outflows following negative shocks. In contrast, more economic concentration begets a modestly larger increase in gross worker flows following positive shocks. Altogether, we uncover sizable welfare insurance gains from local economic diversity.

JEL classification: J61, J62, J21

Key words: sufficient statistic, labor flows, concentration, economic diversity, welfare

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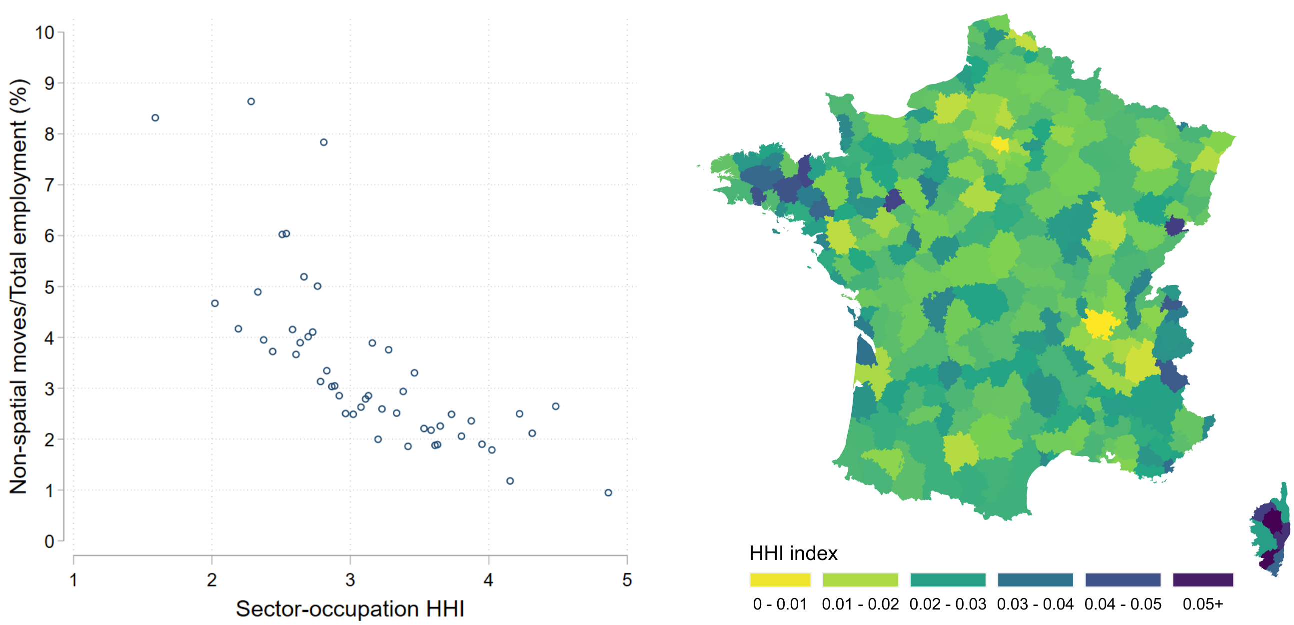
1 Introduction

Are workers living in more economically diverse cities better off or worse off, in the face of labor demand shocks to their industry or occupation? To accurately answer this question, we argue that one needs to take into account the full structure of changes in welfare across all the workers' options, such as staying in the same job, switching locally to another occupation-industry pair or non-employment, or changing city entirely. In this paper, we propose such an approach by deriving sufficient statistics for the second-order approximation of local welfare changes in a standard dynamic discrete choice (DDC) setting. We illustrate our method by using rich worker flows data in France to document sizable welfare insurance gains from economic diversity across cities.

Figure 1. Local Economic Diversity and Nonspatial Labor Reallocation in France

(a) Nonspatial Mobility and Sector-Occupation HHI **(b)** Labor Market Concentration (HHI) Map



***Notes*:** The left panel, Figure 1a, shows a scatter plot of the sector-occupation Herfindahl-Hirschman Index (HHI) index of a commuting zone in France and a measure of nonspatial labor reallocation: the share of local employment that switches to another occupation, sector or both without moving to a new location. The left panel, Figure 1b, shows a map of French commuting zones and their sector-occupation HHI index.

Figure 1 motivates our perspective linking worker flows and local economic diversity to welfare: Figure 1a shows that French cities with more economic diversity as measured by the sector-occupation Herfindahl-Hirschman Index (HHI) index have more local ("nonspatial") worker job flows compared with less economically diverse cities. Figure 1b maps these substantial differences in local industry-occupation diversity across cities in France. In this paper, we measure how spatial and nonspatial worker flow margins respond to labor demand shocks, and how local economic diversity shapes their evolution.

We then combine these estimates as theory-consistent sufficient statistics to measure the welfare value of local economic diversity. We find that more economically diverse French cities are more resilient in response to negative local labor demand shocks, and thereby enjoy a significant welfare insurance advantage.

To infer welfare, we first construct a revealed preference argument using properties of a standard DDC model: we can approximate welfare changes using observed baseline choice probabilities along with the dynamic response of these choice probabilities to a shock. We do so by building on recent advances in the convex analytic properties of DDC models in Chiong, Galichon and Shum (2016). We then present our main theoretical insight using second order approximation of the social surplus in both primal and dual space. To a first order, welfare changes can be approximated as a weighted sum of changes in choice probabilities—the dot product of two data moments: choice probabilities across options and changes in these choice probabilities. To a second order, welfare changes can be approximated by constructing the observed covariance of changes in choices as well as the cross-elasticity between choice alternatives.

In a second step, we implement these insights using data on French workers' employment histories to construct worker flows across location-industry-occupation cells. We examine how flows along different adjustment margins respond to labor demand shocks, and we ask how a city's sectoral and occupational diversity shapes responses to shocks. Specifically, we use a Bartik-style shift-share approach to generate plausibly exogenous labor demand shocks at the city level and "local projections-IV" (LPIV) à la Jordà (2005) to estimate the city-level response to these shocks, tracing both gross and net flows of workers, and separating between spatial and non-spatial adjustment margins.

Our empirical results highlight the significant role of local economic diversity for labor flow responses to positive vs. negative employment shocks. First, following negative employment shocks, we find that more diversified cities experience a significantly smaller fall in within-city churn relative to less diversified cities, which suggests a sort of *hedging* for local workers when local labor market conditions are dire in diversified cities. Moreover, while net worker inflows fall substantially following negative shocks, this inflow contraction is more pronounced for less diversified cities. Second, compared with less concentrated cities, we find that cities with lower sectoral and occupational diversity differ less strikingly in their response to positive employment shocks: Within-city churning flows seem to be similarly impacted by a positive shock for both more diversified and less diversified cities, while net worker inflows are slightly larger in more diversified cities relative to more concentrated cities. In other words, our analysis suggests that more concentrated cities may suffer a more significant penalty on the downside, and a slightly larger boon on the upside.

In a final step, we use our theoretical framework to formalize the link between our empirical estimates and welfare statements. We use the estimated changes in choice probabilities to compute first- and second-order approximations of welfare changes and isolate a sizable insurance value–the second-order approximation term–of local economic diversification in France.

This paper contributes to several research strands: First, our paper contributes methodologically to the literature evaluating the welfare consequences of shocks using approximations and sufficient statistics.[1] Closest to our paper, Kim and Vogel (2020) develop a methodology to approximate first-order welfare differences in the context of a static model using a sufficient statistic approach applied to U.S. commuting zones. Our paper builds on advances in the DDC literature from Chiong, Galichon and Shum (2016) to derive higher order welfare change approximations that leverage the dynamics and composition of worker choice probabilities across more adjustment margins. We illustrate our approach by combining LPIV estimates from rich worker flows data in France to construct theory-consistent welfare measures and to derive the insurance value of local economic diversity.

Second, the paper contributes to an extensive literature on labor flows across cities and sectors in response to employment shocks following the canonical study by Blanchard and Katz (1992).[2] Studies such as Monte, Redding and Rossi-Hansberg (2018) and Marinescu and Rathelot (2018) highlight the heterogeneity in local employment elasticity and worker mobility preferences. The literature on adjustments to trade-induced shocks also highlights the role of both sectoral composition and occupational adjustments. See, for example, Autor et al. (2014), Ebenstein et al. (2014) and Traiberman (2019), among others. Disciplined by the DDC theory, we use rich industry-occupation-location worker flow data in France to document sizable asymmetries in the dynamic adjustment patterns of workers across their spatial and non-spatial options and estimate significant heterogeneity across labor markets with different degrees of economic diversity. We combine these estimates through the lens of our theory to suggest a significant insurance component in the welfare value of economically diverse labor markets.

The remainder of this paper is structured as follows: Section 2 introduces our approach to measure welfare implications of shocks using sufficient statistics, Section 3 introduces the data, provides summary statistics on worker flows in France, and covers our analysis of local projections on worker adjustments. 4 introduces our welfare results, and Section 5 concludes.

[1] See, for example, Deaton (1989), Kim and Vogel (2020), Atkin, Faber and Gonzalez-Navarro (2018), Baqaee and Burstein (2023), Allen et al. (2022), Baqaee and Farhi (2020), Kleinman, Liu and Redding (2021), Porto (2006), Wolf and McKay (2022), and Beraja (2023).

[2] See Dao, Furceri and Loungani (2017) for a recent update of Blanchard and Katz (1992) for the U.S.

2 Theory

We follow the recent literature on the identification of DDC's and exploit the convex analytic structure of the problem (Fosgerau et al., 2021; Chiong, Galichon and Shum, 2016) to approximate the social surplus function, a natural measure of welfare in DDC problems. We first describe the general setup following Chiong, Galichon and Shum (2016) and then turn towards our key approximation result.

2.1 Welfare in Dynamic Discrete Choice Models: An Approximation

Setup. Time is discrete and there are infinite periods. The state variable is $x \in \mathscr{X}$ and agents choose actions between a finite number of actions $y \in \mathscr{Y}$. The single period utility flow of choosing y is given by $\bar{u}_y(x) + \varepsilon_y$ where ε_y denotes an i.i.d additive utility shock. The set of utility shocks is assumed to follow a joint distribution function $Q(\cdot; x)$. We assume conditional independence as in Rust (1987), and we assume that agents are dynamic optimizers that face the following problem:

$$y \in \arg\max_{\tilde{y} \in \mathscr{Y}} \left\{ \bar{u}_{\tilde{y}}(x) + \varepsilon_{\tilde{y}} + \beta \mathbb{E}_{x', \varepsilon'} \left[\bar{V}\left(x', \varepsilon'\right) \mid x, \tilde{y} \right] \right\},$$

where β is the discount factor and primes denote next period values. We define choice-specific continuation values as

$$w_y(x) \equiv \bar{u}_y(x) + \beta \mathbb{E}_{x'} \left[V\left(x'\right) \mid x, y \right]$$

which consist of the per-period utility associated with a given choice y and the continuation value.

Approximating Welfare. In order to derive our approximation result, we first introduce the expected utility of a decision maker, which is common called the social surplus function, i.e.

$$\mathscr{G}(w; x) = \mathbb{E}\left[\max_{y \in \mathscr{Y}} \left(w_y(x) + \varepsilon_y \right) \mid x \right], \tag{1}$$

where the expectation is taken over the distribution of utility shocks. The proposition that follows summarizes the first part of our main result by taking a (second-order) Taylor approximation of the function $\mathscr{G}$. Detailed derivations are in Appendix D.1.

Proposition 1 (Approximate Social Surplus)**.** *We have,*

$$\mathrm{d} \ln \mathscr{G}(w; x) = \sum_y \omega_y(x) \, \mathrm{d} \ln w_y + \frac{1}{2} \sum_y \sum_{y'} \varepsilon^{p,w}_{y,y'} \omega_y(x) \, \mathrm{d} \ln w_y \, \mathrm{d} \ln w_{y'} + o(\cdot)$$

where $\omega_y(x) = \frac{p_y(x) w_y(x)}{\mathscr{G}(w;x)}$ *is a weight that measures the relative contribution of choice* y *to the expected* *utility function $\mathscr{G}$ and where $\varepsilon^{p,w}_{y,y'} \equiv \frac{\partial \ln p_y}{\partial \ln w_{y'}} = \frac{\partial p_y}{\partial w_{y'}} \frac{w_{y'}}{p_y}$ is the elasticity of the choice probability p_y with regard to changes in the choice-specific continuation value w_y.*

Insights and Limitations. Proposition 1 is constructive, in the sense that it details the (empirical) moments necessary to approximate welfare (up to a second-order). Specifically, three inputs are needed: The changes in the continuation values ($\mathrm{d}\ln w_y$), an estimate of the choice cross-elasticity ($\varepsilon^{p,w}_{y,y'}$), as well as baseline weights (ω_y). The second-order terms also point to the role of local economic diversity through the correlation of changes in continuation values ($\mathrm{d}\ln w_y \mathrm{d}\ln w_{y'}$) and the cross-elasticities ($\varepsilon^{p,w}_{y,y'}$). However, there are two obvious challenges to implementing this approximation empirically. First, while choices are observable, continuation values in this dynamic setting are typically not and need to be estimated. Second, the choice cross-elasticity is a high-dimensional object, and while in principle estimation techniques can be developed to estimate it, the dimensionality might make it infeasible to estimate it without further restrictions.

To circumvent the challenge of unobserved continuation values, we turn towards the Legendre-Fenchel conjugate function of $\mathscr{G}$, which is defined as follows,

$$\mathscr{G}^*(p;x) = \sup_{w\in\mathbb{R}^{\mathscr{Y}}} \left\{ \sum_{y\in\mathscr{Y}} p_y(x) w_y(x) - \mathscr{G}(w;x) \right\} \tag{2}$$

where the vector p denotes the choice variable of the convex conjugate function $\mathscr{G}^*$. Therefore, w and p are related in terms of convex duality. Hence, inversion results can be derived to relate the subgradient of the social surplus function and its conjugate to choice probabilities and continuation values, respectively. Moreover, since duality holds we can approximate the convex conjugate for an equivalent welfare approximation, which we summarize in the following proposition:

Proposition 2 (Approximate Conjugate Social Surplus)**.** *We have,*

$$\mathrm{d}\ln \mathscr{G}^*(p;x) = \sum_y \omega^*_y(x)\,\mathrm{d}\ln p_y + \frac{1}{2}\sum_y\sum_{y'} \varepsilon^{w,p}_{y,y'}\omega^*_y(x)\,\mathrm{d}\ln p_y \mathrm{d}\ln p_{y'} + o(\cdot)$$

where $\omega^*_y(x) = \frac{w_y(x)p_y(x)}{\mathscr{G}^*(p;x)}$ is a weight that measures the relative contribution of choice y to the conjugate social surplus function $\mathscr{G}^*$ and where $\varepsilon^{w,p}_{y,y'} \equiv \frac{\partial \ln w_{y'}}{\partial \ln p_y} = \frac{\partial w_{y'}}{\partial p_y}\frac{p_y}{w_{y'}}$ is the inverse elasticity of the choice probability p_y with regard to changes in the choice-specific continuation value w_y.

Proposition 2 is an equivalent, but empirically more convenient approximation of the social surplus. The key difference is that the approximation is in terms of observed choice probabilities ($d\ln p_y$) rather than unobserved continuation values. Finally, the framework can speak to the relationship between insurance and diversification. As we show in more detail in Appendix D.3, the following can be derived:

Corollary 1 (Value of Insurance and Diversification). *Consider a direct shock to choice alternative y ($d\ln w_y \neq 0$), and no confounding shock elsewhere ($d\ln w_{y'} = 0 \quad \forall y' \neq y$), but correlated i.i.d. effects elsewhere such that $\mathbb{E}(d\ln w_{m'} \mid d\ln w_m) = \nu \neq 0$. Assuming a log-linear welfare aggregator across choices, we can (empirically) approximate how much insurance is provided locally as,*

$$Insurance_i = \frac{\nu\theta}{2\mathcal{G}} \sum_{y\in\mathcal{Y}} \pi_y \sum_{y'\neq y\in\mathcal{Y}} \sigma_{yy'}\pi_{y'}.$$

where we have normalized the size of the shock $d\ln w_y = 1$ and assumed θ is a common elasticity of substitution of worker choices across options. Furthermore, this insurance index is proportional to the HHI index when $\pi_y \approx \sum_{y'\neq y\in\mathcal{Y}} \sigma_{yy'}\pi_{y'}$.

We also show in Appendix D.3 that the insurance value of diversification collapses to the HHI index in the case of closed local labor markets. Therefore, we use the HHI statistic as a proxy for economic concentration in the empirical section.

Taking Stock. Proposition 2 offers a empirically convenient representation of the worker's conjugate social surplus. To approximate the first order term, no structural parameters are needed, except to construct appropriate (approximate) welfare weights (ω_y^*) and to identify impulse responses in the choice probabilities ($d\ln p_y$). The second order term poses a bit more of a challenge: it requires estimates of the own and cross-price elasticities ($\varepsilon_{y,y'}^{w,p}$). While in principle, this is a high dimensional object that would be difficult to estimate, we will show that it can be easily calibrated once an iso-elastic labor supply system is assumed. So far, we have discussed the benefits of this approach, but the methodology also features two substantial drawbacks. First, our welfare measure only traces out relative welfare changes across locations and cannot determine the level of welfare. Second, while we characterize the worker's adjustment choices against the backdrop of rich description of a segmented local labor market, we do not explicitly model other dynamic choices, such as consumption-saving choices. In what follows, we apply our theory-derived approach to the study of the resilience of local labor markets.

3 Adjustment to Labor Demand Shocks

Our approximation result provides an empirical strategy to quantify the welfare effects of labor demand shocks on workers. Guided by Proposition 2, we estimate worker flow responses to local labor demand shocks and the role of local sectoral and occupational diversity of labor markets in shaping the responses. To do so, we leverage a large representative sample of employment spells of French workers to construct worker flows across local labor markets segmented by location, industry, and occupation. We also construct a standard labor demand shock, and estimate the impulse response of different worker adjustment margins to these shocks using local projection IVs (LPIV).

In Subsection 3.1, we discuss the construction of the city-industry-occupation worker flows using job spells data and document the salience of both spatial and nonspatial reallocation flows in France. Next, in Subsection 3.2, we discuss our LPIV estimation approach and the labor demand shocks' construction. We then confirm, in Subsection 3.3, that spatial and nonspatial worker adjustment margins respond differently to local shocks. Finally, in Subsection 3.4, we document significant asymmetries and heterogeneity in worker flow responses through the role of city-level economic diversity, further highlighting a link between welfare and economic diversity.

3.1 Data: Worker Flows in France

Data. Our French workers' employment histories data comes from the DADS (Déclarations Annuelles de Données Sociales) administrative panel dataset. The *DADS Fichier Postes*, maintained by the French National Statistical Institute (INSEE), contains Social Security records for all salaried employees in any private and semi-public firm in France.[3] The *DADS Panel* tracks histories for workers born in October in even years, and covers around 4% of private sector workers. The data therefore allows us to track an individual worker across all their non-public sector employment spells from 2005 to 2019.[4] Information on employer establishments includes their 4-digit industry, location at the municipality level, and unique identifier. Thus, we observe workers' transitions across sectors, occupations, and geographical locations–a crucial input for our study.

For each employment spell, the sector is defined as the establishment's 2-digit industry code and the occupation is constructed as the job's 2-digit INSEE occupation code. We use as location the "*Zone d'Emploi*", which is akin to a commuting zone, associated with the establishment zip code. We have a total of 30 occupations, 90 sectors, and 300 geographic locations–which we call "city" for simplicity.

Constructing Worker Flows. To construct the stock of workers in city-sector-occupation (i, s, o) in any quarter t, we start by assigning every active worker to a unique labor market defined for each quarter,

[3] DADS records are compiled from mandatory Social Security employer filings. The DADS excludes self-employed individuals, central government entities ("Fonction Publique d'État"), and domestic services.

[4] We follow Traiberman (2019) and restrict the analysis to individuals ages 23 to 64 years old.

keeping the job with the highest duration.[5] Quarters with no recorded activity are considered as "non-employment" spells. Here, non-employment in a city is a wide category that includes not only local workers that are either looking for a job–the standard unemployment margin in the existing literature, but also workers who exited the private sector labor force. We record as "job-to-job" transitions between quarter t and quarter $t+1$ all situations where a worker's assigned labor market at $t+1$ is different from his time-t labor market. Importantly, and in line with our DDC theoretical framework, we do not limit our investigations to net flows but conduct our analysis using *directed* gross flows for any labor market pair (including non-employment): We construct the stock of employed worker in any labor market, as well as the gross flows of workers between t and $t+1$ for any labor market pair.[6]

Table 1. Quarterly worker reallocation across markets

		Median	SD	p5	p25	p75	p95
1.	$\frac{\text{Flows}_{i,s,o}}{\text{Emp}_i}$	4.54 %	0.98	3.21 %	3.90 %	5.35 %	6.10 %
2.	$\frac{\text{Flows}_{i\to i'}}{\text{Flows}_{i,s,o}}$	48.95 %	9.79	34.32 %	41.48 %	56.14 %	65.65 %
3.	$\frac{\text{Flows}_{s\to s'}}{\text{Flows}_{i,s,o}}$	61.29 %	4.34	53.58 %	58.92 %	63.82 %	66.64 %
4.	$\frac{\text{Flows}_{o\to o'}}{\text{Flows}_{i,s,o}}$	64.91 %	4.74	57.36 %	62.19 %	67.10 %	70.29 %
5.	$\frac{\text{Flows}_{ne\to e}}{\text{Emp}_i}$	3.22 %	1.70	2.10 %	2.70 %	4.21 %	7.62 %

***Notes*:** The displayed values are expressed for the cross-section of locations or cities. Flows are computed at the quarterly frequency. Sub-indices are defined as follows: i denotes cities, s sectors and o occupations. Row 1 shows the share of workers initially in a local labor market switching jobs across any dimension. Rows 2 to 4 display the share of switchers that change at least in the sub-indiced dimension. They do not sum to 100 as they allow for the two other dimensions to change as well, thereby causing overlap. Row 5 shows the share of workers leaving non-employment. It encompasses both new job-seekers and those exiting the labor force.

Anatomy of Worker Reallocation in France. Table 1 presents selected moments of the distribution of city-level flows. The first row computes the share of workers experiencing any type of job-to-job transition as a share of the local employed population. In the median French city, each quarter, 4.5% of workers change labor market affiliation in at least one of the city (i), sector (s) or occupation (o) dimensions. This corresponds approximately to 18.2% of switchers each year.[7] We disaggregate these worker flows into the share of switchers changing at least city i (row 2), sector s (row 3), or occupation o (row 4). We find sizable flows along all dimensions: In the median city, 65 percent of moves involve changing occupation (with or without also changing the other dimensions), while 60 percent involve changing sector, and nearly half involve changing city. We further decompose these flows in Figure A.1,

[5] If a worker has two jobs with the exact same number of days worked, we select the one with the highest total salary. If salaries are also equal, we select the job with the highest number of hours worked.

[6] Because of the private sector scope of the DADS data, transitions into or out of the "non-employment" island in each city include both outright non-employment, but also switches to/from public sector employment or self-employment.

[7] We find a somewhat larger fraction of switchers (18.2%) than Traiberman (2019) reports for Denmark (13.2%). Notably, Traiberman (2019) tracks occupation and sector switches, but not geographic changes.

across all combinations of city, sector, or occupation switches. Moves that are not spatial (Figure A.1a) are more frequent than those that are (Figure A.1b). Occupation switches are by far the most important types of flows. Nonspatial reallocation flows, especially those involving only occupation switches, vary much more across cities than spatial reallocation flows.

3.2 Estimation Methodology

LPIV. We use the local projection method in Jordà (2005) estimate the *average* dynamic response responses to labor demand shocks at the city level for $h = 0, \ldots, 20$ quarters as follows:

$$\Delta y_{i,t+h} = \alpha^h + \gamma_t + \gamma_i + \beta_h \,\text{Shock}_{i,t} + \sum_{m=1}^{8} \gamma_m^h y_{i,t-m} + \sum_{m=1}^{8} \omega_m^h \,\text{Shock}_{i,t-m} + \sum_{m=1}^{8} \delta_m^h \,\text{Z}_{i,t-m} \tag{3}$$

where $\Delta y_{i,t+h} = \log\left(y_{i,t+h}\right) - \log\left(y_{i,t-1}\right)$ and $y_{i,t+h}$ is the cumulative value at time $t+h$ since $t-1$.[8] $\text{Shock}_{i,t}$ is an exogenous local shock and $Z_{i,t}$ denotes controls for various local labor market variables: the share of total flows to local employment, the local labor force, and ratios of city-level local churn, inflows, and outflows relative to all flows. γ_i and γ_t are city- and time-fixed effects accounting for cross-sectional heterogeneity and common macroeconomic shocks. Standard errors are Driscoll-Kraay, meaning that they are robust to forms of spatial and temporal dependence. We allow for $m = 1, \ldots, 8$ quarters of auto-correlation–a standard temporal correlation window of two years.

Constructing Labor Demand Shocks. We use a shift-share approach (Bartik) to generate plausibly exogenous labor demand shocks at the city level. We compute national employment growths for each sector-occupation pair (s, o) and construct the labor demand shock in city i as:

$$\text{Bartik}_{i,t} = \sum_{s,o} \text{share}_{i,2004}^{s,o} \cdot g_{-i,s,o,t}^{\text{national}} \tag{4}$$

where $\text{share}_{i,2004}^{s,o} = E_{i,s,o,t=2004} / \sum_{s',o'} E_{i,s',o',t=2004}$ are within-location (i) shares of sector-occupation (s, o) cells and $g_{-i,s,o,t}$ are leave-one-out sector-occupation employment growths in 2004.[9] Figure A.2 shows the distribution of sector-occupation shocks. Cities are on average exposed to more positive than negative shocks, and some of them experience large changes of up to -5 or +10 %.

[8] When the y outcome concept is a stock (e.g. employment levels) rather than a flow (e.g. location-changing workers), we simply take the log differences in the levels from $t-1$ to t. When the outcome concept is a flow, y_{t+h} is constructed as the cumulative sum of the period-specific flows.

[9] We also conducted the analysis using occupation-sector exposure to rising imports from China following Basco et al. (2024). Our findings are broadly similar and available upon request.

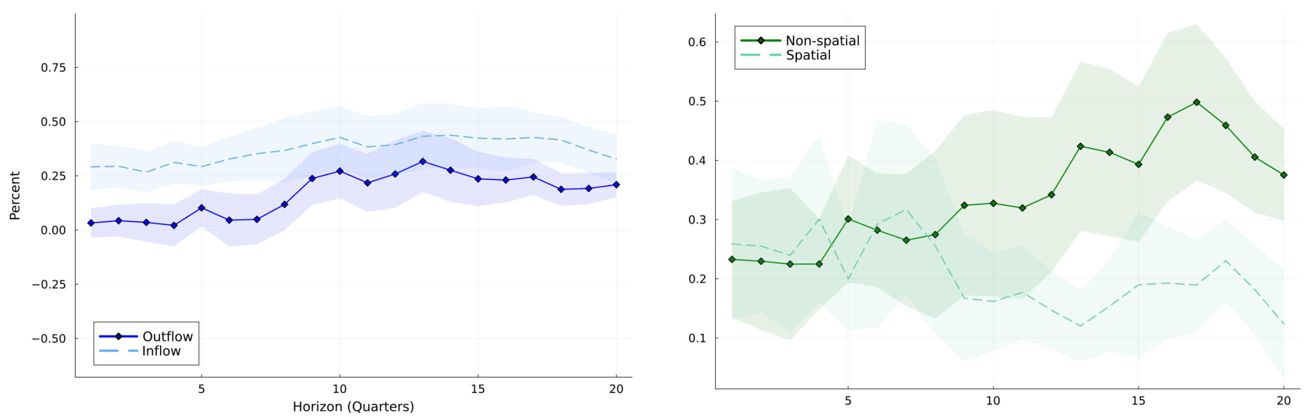
3.3 Beyond Net Employment: The Dynamics of Gross Flows

Our unique worker flows data allows us to further unwrap the black box of net employment responses to a labor demand shock. In particular, our data allows us to display the changes in gross flows that shape net employment responses. City-level adjustments in response to labor demand shocks are presented in Figure 2.[10] The left chart (Figure 2a) shows the average response of gross spatial flows (inflows to and outflow from the city) and the right chart (Figure 2b) shows the average response of spatial and nonspatial flows. Nonspatial flows are defined as the sum of all worker transitions not involving a change of city. Spatial (net) flows are defined as inflows minus outflows, reflecting the employment stock response before including local unemployment changes or private sector employment exits.

Following a labor demand shock, Figure 2a shows that inflows to the city rise significantly on impact, steadily rise, and stay persistently elevated throughout the estimation horizon. In contrast, it takes around two years for outflows from the city to significantly start reacting to the shock before rising rapidly and staying elevated.[11] Figure 2b shows that nonspatial flows are initially as large as spatial flows, but they grow larger than spatial flows after a few quarters.

Figure 2. Average Response to a Bartik shock: Spatial and Nonspatial Flows

(a) Gross Spatial Flows **(b)** Spatial and Nonspatial Flows



***Notes*:** Figure 2a shows the gross flow of workers that move out of a city (solid line) and the gross flow of workers that move in a city (dashed line). Figure 2b shows the net flow of workers that move in a city minus the ones that leave (solid line) and the average flow of workers that reallocate within a city (dashed line).

[10] We present, in Table A.2, results for the additional margins listed in Table 1.

[11] The positive relationship between the labor demand shock and outflows might seem surprising. This might be due to several reasons. Since the outflows respond with a lag of two years, incomers could be replacing some of the current workers. Also, we are not controlling for shocks from neighboring cities.

3.4 The Asymmetric Role of City-Level Economic Diversity

Motivated by the insights from the second-order approximation in Section 2, we posit that city-level economic diversity plays in important role in shaping how worker flows respond to labor demand shocks.[12] We augment the baseline dynamic LPIV with asymmetric and heterogeneous effects. We measure city-level economic concentration (the opposite of diversity) using the standard Herfindahl-Hirschman Index (HHI) defined over sectors and occupations as

$$\mathrm{HHI}_i = \sum_{s,o} \left(\mathrm{share}^{so}_{i,t=2004}\right)^2, \tag{5}$$

where $\mathrm{share}^{s,o}_{i,t=2004}$ is the share of locality i's workers employed in a sector-occupation pair (s,o) in 2004. A lower HHI indicates a more diverse economy with a broader distribution of employment across multiple sectors and occupations, while a higher HHI suggests a more concentrated economy.

Positive shocks ($\mathrm{B}^{+}_{i,t}$) and negative shocks ($\mathrm{B}^{-}_{i,t}$) now enter separately, as workers may respond differently to positive and negative shocks.[13] Both variables only take non-negative values and are interacted with local HHI_i measures:

$$\begin{aligned}\Delta y_{i,t+h} = \alpha^h + \gamma_t + \gamma_i + \beta^-_h \times \mathrm{B}^-_{i,t} + \psi^-_h \left(\mathrm{B}^-_{i,t} \times \mathrm{HHI}_i\right) + \beta^+_h \times \mathrm{B}^+_{i,t} + \psi^+_h \left(\mathrm{B}^+_{i,t} \times \mathrm{HHI}_i\right) \\ + \sum_{m=1}^{8} \gamma^h_m y_{i,t-m} + \sum_{m=1}^{8} \omega^h_m \, \mathrm{B}_{\,i,t-m} + \sum_{m=1}^{8} \delta^h_m \, \mathrm{Z}_{i,t-m} \,.\end{aligned} \tag{6}$$

This specification traces out, for positive shocks and negative shocks separately, how worker flows respond in high vs. low economic diversity cities.[14] One concern could be that the shift-share instrument and the HHI index are correlated, as they both use sectoral and occupational shares in their construction. We show in Figure A.4 that the distribution of Bartik shocks for highly concentrated (high HHI) and highly diversified (low HHI) cities is very similar.

Figure 3 shows our asymmetric estimates for spatial and nonspatial flows, evaluated at the 90th percentile (high HHI) and the 10th percentile (low HHI) of the city-level HHI distribution. The result is striking and confirms, as suggested by our theory, that local economic diversity shapes the transmission of local shocks. We find that local economic diversity (lower HHI) significantly dampens the downside

[12]In Section D.3 in the appendix, we show that, under certain orthogonality conditions in our theoretical framework, the HHI mediates the insurance value of city-level diversity.

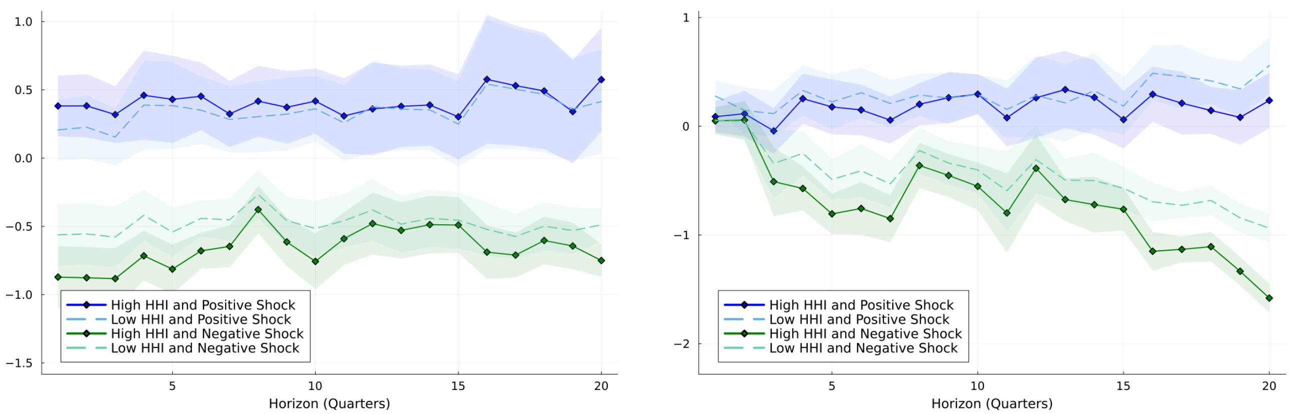
[13]We separate the baseline Bartik shocks into two new sets of variables, $\mathrm{B}^{+}_{i,t}$ and $\mathrm{B}^{-}_{i,t}$, for positive and negative Bartik shocks, respectively. We trim the top and the bottom 0.5% of the HHI for our regressions. Trimming beyond the top and bottom 0.5% does not affect our results. Note that this trimming does not lead us to drop Paris' commuting zone.

[14]Moretti and Yi (2024) highlight the interaction of education and labor market size. We control for local labor force size and focus on illustrating our high-order welfare approximation using city-level diversity. We leave a richer set of interactions and heterogeneous responses for future applications.

of negative labor demand shocks. In response to negative shocks, both local nonspatial churn and net spatial inflows fall less in more diversified (lower HHI) labor markets. However, more concentrated cities do not appear to exhibit an equally outsized response to positive local labor demand shocks, compared with more diversified cities.

Figure 3. Asymmetric and Heterogeneous Response: Positive/Negative Shocks × High/Low HHI

(a) Nonspatial **(b)** Spatial



Notes: Figure 3a shows the differential response of nonspatial flows to a negative/positive labor demand shock for low-HHI (dashed blue/green lines) and high-HHI cities (solid blue/green lines). Figure 3b shows the differential response of spatial flows to a negative/positive labor demand shock for low-HHI (dashed blue/green lines) and high-HHI cities (solid blue/green lines).

4 From Estimation to Welfare

We now leverage the approximation results in Section 2 and the estimated responses using our granular flows data to compute the welfare implications of local economic diversity in response to labor demand shocks.

4.1 Methodology

Setup. Consider a dynamic discrete choice setting as in Section 2. We simply adapt the setting to characterize labor flows between segmented 'local' labor markets. We define the choice set $\mathscr{Y}$ to be sector-occupation-location labor markets: $\mathscr{Y} \equiv \mathscr{S} \times \mathscr{O} \times \mathscr{L}$, where $\mathscr{S}$ refers to the set of sectors in the economy, $\mathscr{O}$, refers to the set of occupations in the economy, and $\mathscr{L}$ refers to the set of locations in the economy. The social surplus is still given by Equation (1) and its convex conjugate is given by (2). However, in order to construct our welfare measures where the change in choice probabilities ($d \ln p_y$)

is being determined by our estimated LPIVs from Section 3, we need to further collapse the worker's adjustment margins: We partition the full set $\mathscr{Y} \times \mathscr{Y}$ of sector-occupation-location transitions into coarser spatial allocation margins: a "stay" margin for workers not changing their sector-occupation-location island, a "local" margin for sector-occupation changes in the same city, and a "spatial" margin for workers who change city.[15]

Approximating Welfare Changes. As discussed in Section 2, to implement Proposition 2, three ingredients are needed: First, a change in conditional choice probabilities ($\mathrm{d}\ln p_y$) along all adjustment margins is needed. For this, we use our LPIV estimates. Second, we need to construct the theory-consistent welfare weights, $\omega^*_y(x) = \frac{w_y(x)p_y(x)}{\mathscr{G}^*(p;x)}$. To do so, we implement the Mass Transport Approach (MTA) first developed in Chiong, Galichon and Shum (2016) that leverages a linear programming formulation to estimate the continuation values as well as the value of the $\mathscr{G}^*(p;x)$ function given a vector for the conditional choice probabilities and discretized approximation of the underlying preference shock. This estimation procedure allows us to directly construct the welfare weights for the baseline period (2006). Finally, to circumvent the challenges associated with the estimation of the cross-elasticity terms, we assume a common labor supply elasticity γ. We obtain:

$$\begin{aligned}\frac{\mathrm{d}\ln \mathscr{G}^*(p;x)}{\mathrm{d}\ln \bar{z}} \approx & \underbrace{\sum_{\substack{\text{y=stay (x),}\\ \text{local (x), spatial (x)}}} \left[\omega^*_y(x)\frac{\mathrm{d}\ln p_y(x)}{\mathrm{d}\ln \bar{z}}\right]}_{\text{first order approximation term}} \\ & \underbrace{+\frac{1}{\gamma}\sum_{\substack{\text{y=stay (x),}\\ \text{local (x), spatial (x)}}} \left[\frac{\omega^*_y(x)}{1-p_y(x)}\left(\frac{\mathrm{d}\ln p_y(x)}{\mathrm{d}\ln \bar{z}}\right)^2\right] - \frac{1}{2\gamma}\sum_{\substack{\text{y,y'=stay (x),}\\ \text{local (x), spatial (x)}}}\sum_{y'\neq y}\left[\frac{\omega^*_y(x)}{p_{y'}(x)}\frac{\mathrm{d}\ln p_y(x)}{\mathrm{d}\ln \bar{z}}\frac{\mathrm{d}\ln p_{y'}(x)}{\mathrm{d}\ln \bar{z}}\right]}_{\text{second order approximation term}}\end{aligned} \tag{7}$$

as derived in Appendix D.2. This approximation states that to evaluate the welfare consequences of a shock, only three ingredients are necessary. First, one needs an estimate of labor supply elasticity γ. Second, baseline choice probabilities, $\{p_{\text{stay}}(x), p_{\text{spatial}}(x), p_{\text{local}}(x)\}$, reflect the relative importance of the different margins in the agent's choice set. Finally, one needs to obtain reliable and causally identified estimates of the impulse response function of these choice probabilities along each margin, i.e. $\left\{\frac{\mathrm{d}\ln p_{\text{stay}|x}(x)}{\mathrm{d}\ln z_\ell}, \frac{\mathrm{d}\ln p_{\text{spatial}}(x)}{\mathrm{d}\ln z_\ell}, \frac{\mathrm{d}\ln p_{\text{local}}(x)}{\mathrm{d}\ln z_\ell}\right\}$.

From Local Projections to Welfare. Following the literature, we assume $\gamma = 2$ and we evaluate the welfare effects of the local labor demand shocks from our estimation stage. Specifically, for a given city

[15] This coarser partitioning of the full set $\mathscr{Y} \times \mathscr{Y}$ of options is akin to assuming a homogeneous response of flows within the same subset of "stay", "local", or "spatial" options.

Table 2. Welfare Effects and Decomposition by Economy Diversification

HHI	Positive Shocks			Negative Shocks		
Deciles	*First order*	*Second order*	*Total*	*First order*	*Second order*	*Total*
1st	3.70	-1.94	1.76	-0.61	-0.03	-0.64
2nd	3.79	-1.50	2.29	-0.62	-0.04	-0.66
3rd	3.87	-1.55	2.32	-0.63	-0.07	-0.69
4th	3.95	-1.69	2.26	-0.64	-0.10	-0.74
5th	4.06	-1.05	3.00	-0.65	-0.13	-0.77
6th	4.13	-0.47	3.66	-0.65	-0.14	-0.79
7th	4.21	-2.37	1.83	-0.66	-0.23	-0.89
8th	4.33	-3.41	0.92	-0.67	-0.31	-0.98
9th	4.51	-3.06	1.46	-0.69	-0.39	-1.08
10th	5.46	-3.97	1.49	-0.78	-0.99	-1.77

Notes: We set $\gamma = 2$ and discount future continuation values relative to the base period using $\beta = 0.9$:

$$\Delta \ln \mathscr{G}^*_\ell \equiv \sum_{t=2006Q1}^{2019Q4} \sum_{h=0}^{20} \beta^{t+h-2004} \left\{ \sum_{\substack{y=\text{ stay,}\\ \text{local, spatial}}} \left[\omega^*_{y|\ell} \Delta \ln p^{y,t+h}_{\ell,t} \right] + \frac{1}{\gamma} \sum_{\substack{y=\text{ stay,}\\ \text{local, spatial}}} \left[\frac{\omega^*_{y|\ell}}{1-p_{y|\ell}} \left(\Delta \ln p^{y,t+h}_{\ell,t} \right)^2 \right] - \frac{1}{2\gamma} \sum_{\substack{y,y'=\text{ stay,}\\ \text{local, spatial}}} \sum_{y' \neq y} \left[\frac{\omega^*_{y|\ell}}{p_{n|\ell}} \Delta \ln p^{n,t+h}_{\ell,t} \Delta \ln p^{y,t+h}_{\ell,t} \right] \right\}.$$

The first column denotes HHI deciles. The welfare changes are in percentage changes relative to the baseline period for the average shock in each HHI decile bin, separately reporting the effect of positive and negative shocks. The next three columns present, for positive shocks: the FOA term and the SOA term of the welfare formula along with their sum. The last three columns present similar terms for negative shocks.

ℓ, we know its sequence of annual local labor demand shock realizations $\left\{ \mathrm{B}^+_{\ell,t}, \mathrm{B}^-_{\ell,t} \right\}_t$ and its baseline sector-occupation economic diversification index HHI_ℓ. We then use the LPIV estimates to predict, for the realized average shocks $\left\{ \tilde{\mathrm{B}}^+_t, \tilde{\mathrm{B}}^-_t \right\}$, the implied path of changes in choice probabilities for each margin over the periods h in estimation horizon: $\left\{ \Delta \ln p^{\text{stay},t+h}_{\ell,t}, \Delta \ln p^{\text{spatial},t+h}_{\ell,t}, \Delta \ln p^{\text{local},t+h}_{\ell,t} \right\}_h$.[16] Finally, we use a discount factor $\beta = .9$ to cumulate the welfare effects over time across the history of shocks.

4.2 Results

Our welfare results are summarized in Table 2. We separately report the effect of positive and negative shocks for all HHI deciles.[17] The first-order approximation term (FOA) captures the direct welfare impact of shocks, while the second-order approximation term (SOA) reflects the insurance value that

[16] $\tilde{\mathrm{B}}^+_t$ and $\tilde{\mathrm{B}}^-_t$ are simple unweighted averages of $\left\{ \mathrm{B}^+_{\ell,t} \right\}_\ell$ and $\left\{ \mathrm{B}^-_{\ell,t} \right\}_\ell$. In order to limit the impact of HHI outliers on our post-estimation inference, we use a city's HHI decile average HHI instead of its own HHI. Due to pending disclosure limitations, the current results employ an auxiliary regression based on the response of the stock of employment at the local labor markets to approximate $\Delta \ln p^{\text{stay},t+h}_{\ell,t}$.

[17] We plot, in Figure A.6 in the appendix, the city-level values that are averaged decile-by-decile to create the decile-level values reported in Table 2.

comes from a city's economic diversity. Overall, our findings highlight that economic diversity is a key determinant of the heterogeneity in welfare outcomes observed across cities.

Focusing first on positive shocks, our sufficient-statistic framework reveals that more concentrated cities (with largest HHI values, or in higher deciles) exhibit larger first-order gains than more diversified cities, hinting at possible gains from specialization. That said, more negative values for SOA terms significantly alter these gains. The SOA terms are in fact large enough to revert the rising FOA gains with HHI in the upper half of HHI deciles. These findings suggest that lack of economic diversification also comes with negative insurance value, especially for the most specialized cities.

The effects of negative shocks strikingly confirm that economic diversity plays a substantial role in providing insurance against adverse economic shocks. Indeed, while the FOA loss from a negative shock does not vary substantially along the HHI distribution, we observe large negative second-order welfare effects (compared to the magnitude of the FOA terms), which are strongly increasing in magnitude with HHI. In other words, more diversified cities are associated with significantly smaller welfare losses from negative shocks, due to smaller change from the SOA term measuring "insurance value". This result echoes our empirical findings in section 3.4 where we showed that, in response to negative shocks, local non-spatial churn and net spatial inflows fall less in more diversified labor markets.

Our results underscore the importance of economic diversification as a strategy for enhancing economic resilience: A more diversified economy may facilitate labor reallocation, reduce costly spatial mobility, and allow for more stable employment outcomes in the face of sector- or occupation-specific downturns.[18] Perhaps more importantly, our results illustrate the applicability and usefulness of the higher-order sufficient-statistic approach using granular worker flows that we put forward in this paper.

5 Conclusion

This paper introduces a second-order sufficient-statistics framework to analyze the role of city-level economic diversity on workers' welfare, and applies it using rich worker flows data in France to uncover substantial welfare insurance gains from economic diversity in the face of local labor demand shocks.

To a first-order approximation, workers located in more specialized cities experience larger welfare gains in response to a positive labor demand shock, suggesting possible gains from specialization. However, a second-order term capturing the insurance value of economic diversity reveals that high economic concentration comes with a larger penalty in response to negative shocks. As such, our results

[18] The spatial variation in these welfare effects is depicted in Figures A.7 and A.8 in the appendix. These figures show the distribution of welfare outcomes under both the "first-order only" approach and the welfare outcomes when accounting for the insurance value captured by the SOA term.

underscore that economic diversity plays a crucial role in buffering against adverse economic shocks.

We hope that our approach and results will be useful to guide future empirical and quantitative investigations of local labor markets responses to economic shocks, and contribute to a better understanding of the role of city-level characteristics in shaping the local response to broader national economic fluctuations.

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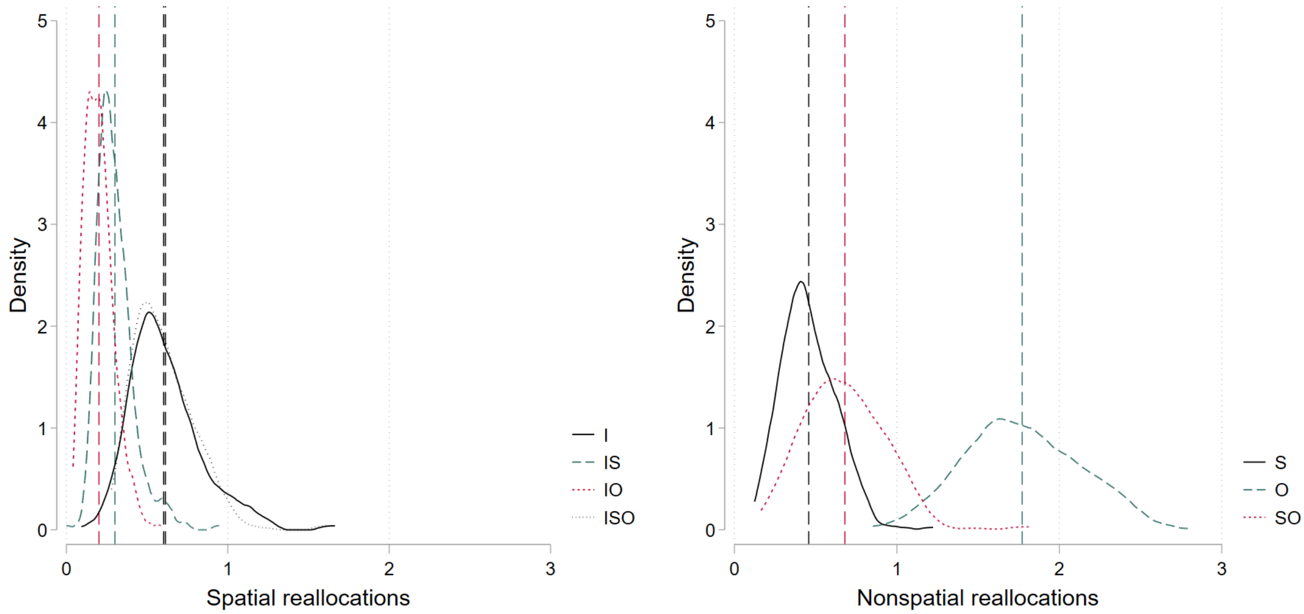
A Additional Figures



(a) Spatial mobility

(b) Nonspatial mobility

Figure A.1. Mobility of workers across cities, sectors and occupations

***Notes*:** Figure (a) illustrates the distribution (density) of workers who change cities (spatial mobility). The lines decompose this mobility into different combinations of transitions across sectors and occupations. Specifically, i represents moves that are purely geographic, *is* includes both geographic and sectoral changes, io represents geographic and occupational changes, and iso captures changes across all three dimensions: geographic, sectoral, and occupational. Figure (b) displays the density of workers who remain in the same city (nonspatial mobility). The lines break down non-geographic transitions, with s representing sector-only changes, o representing occupation-only changes, and so for workers who switch both sectors and occupations without changing location.

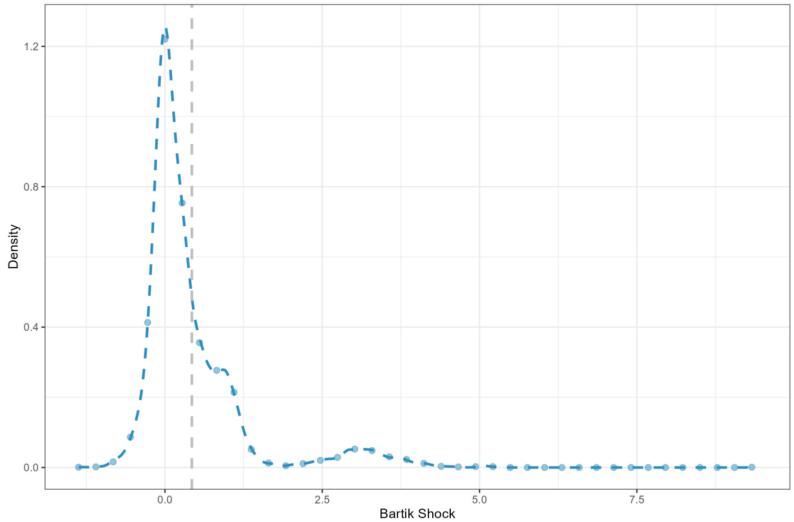


Figure A.2. Distribution of Bartik shocks

***Notes*:** This figure shows the distribution of labor demand shocks as constructed in Equation (4). The distribution captures the range of positive and negative shocks affecting cities, highlighting the heterogeneity in local labor demand changes across different regions.

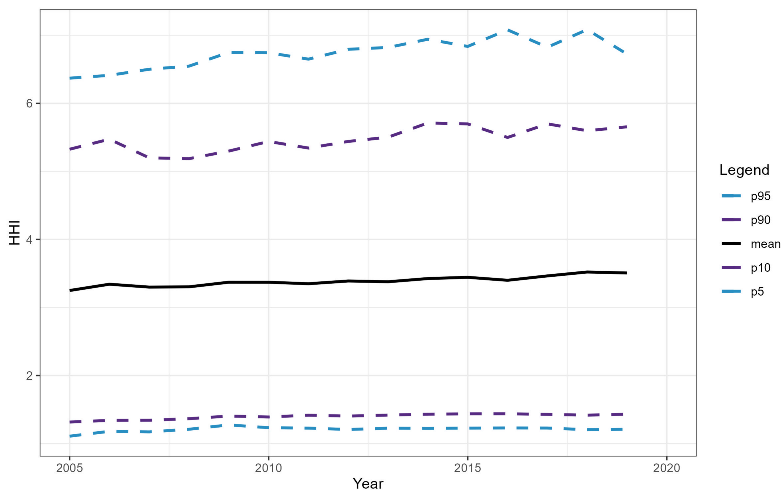


Figure A.3. Distribution of HHI over the time series

***Notes*:** This figure presents the distribution of the Herfindahl-Hirschman Index (HHI) for local labor markets over time. It highlights key percentiles, including the 95th (p95), 90th (p90), mean, 10th (p10), and 5th (p5) values, offering insight into the variation in economic concentration across cities throughout the panel period.

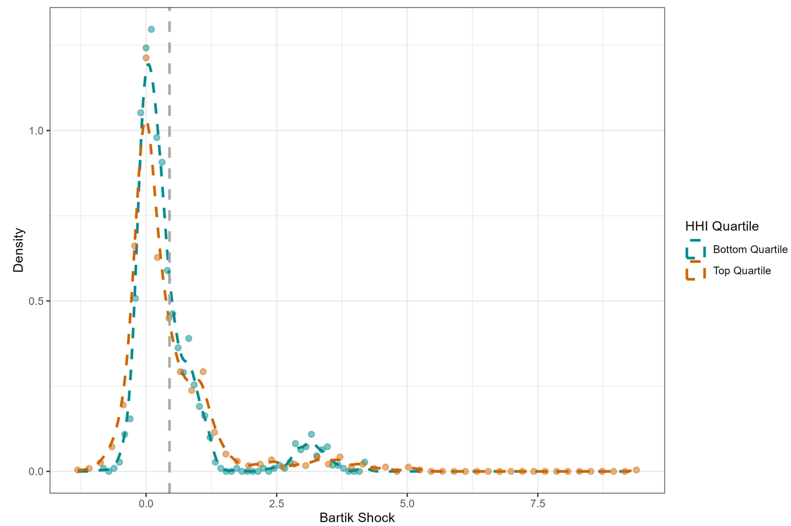
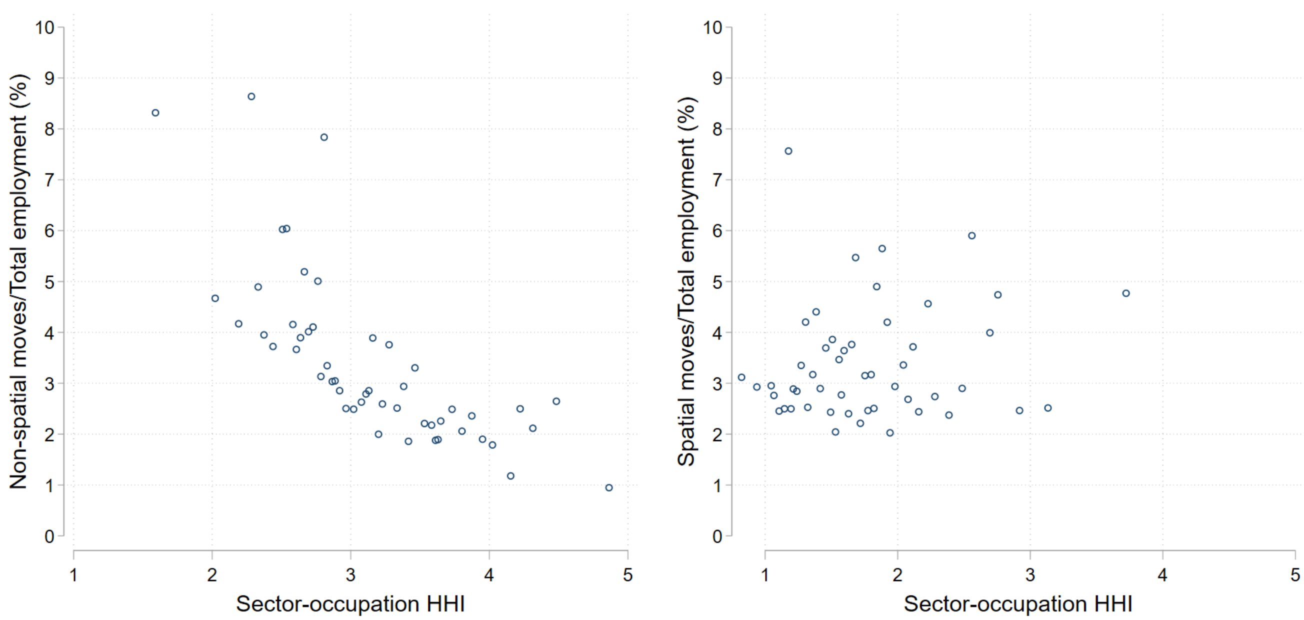


Figure A.4. Density of Bartik shock by HHI Top and Bottom Quartile

***Notes*:** This figure shows the distribution of Bartik shocks, as constructed in Equation (4), categorized by the bottom and top quartiles of the Herfindahl-Hirschman Index (HHI) distribution. It compares how labor demand shocks are distributed across cities with low and high economic concentration.



(a) HHI and nonspatial flows

(b) HHI and spatial flows

Figure A.5. HHI and flows

***Notes*:** Figure (a) displays the correlation between the Herfindahl-Hirschman Index (HHI) and the share of local employment that transitions to another occupation, sector, or both, without leaving the locality (nonspatial flows). Figure (b) shows the correlation between the HHI and the share of local employment that involves geographic mobility (spatial flows), reflecting how economic concentration influences worker relocation across cities.

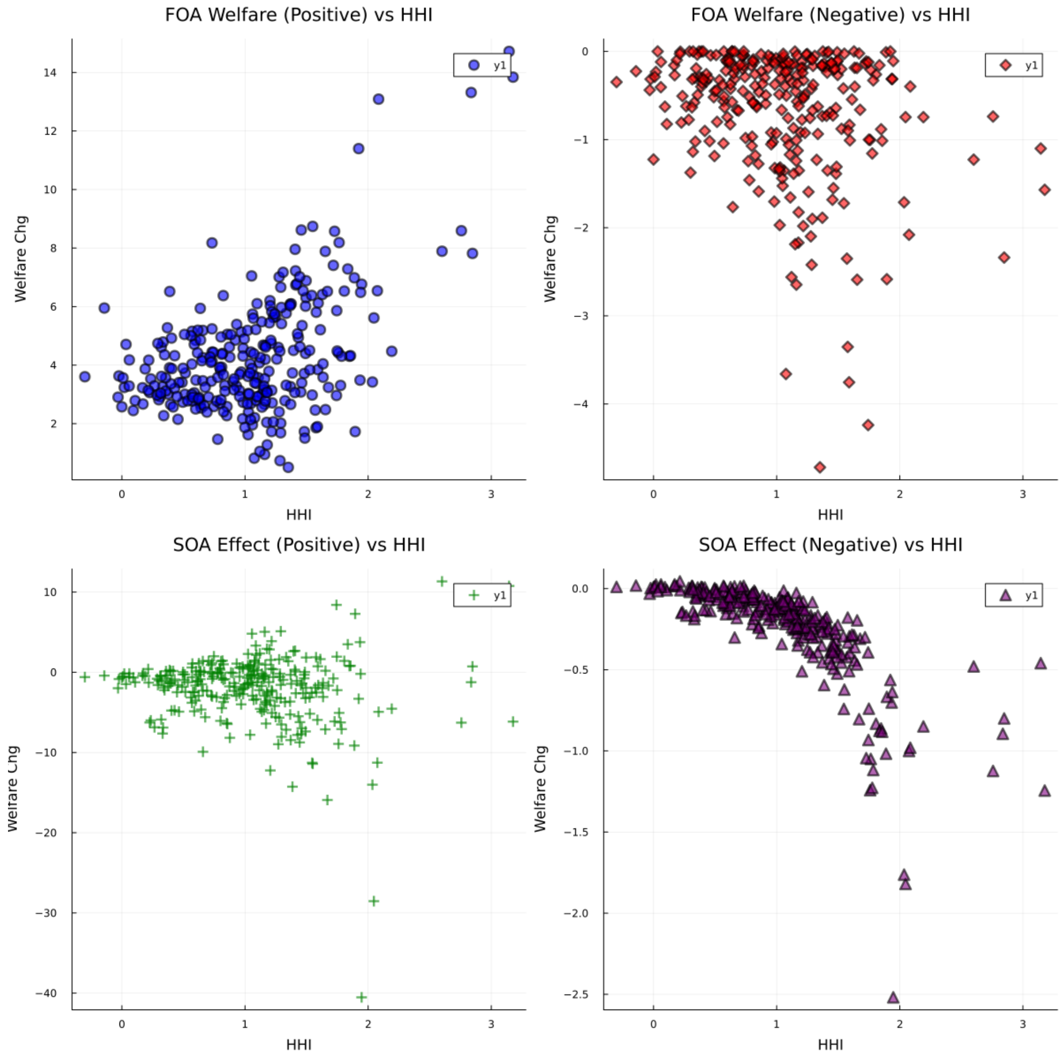


Figure A.6. Welfare components: FOA, SOA, HHI

***Notes*:** This figure presents scatterplots illustrating the relationship between the first-order approximation (FOA) and second-order approximation (SOA) terms, as constructed in Section 4. The FOA terms represent the direct welfare impact of labor demand shocks, capturing the immediate effects on local labor markets. The SOA terms measure the "insurance value" derived from economic diversification, reflecting how cities with different levels of sectoral and occupational diversity buffer the effects of these shocks.

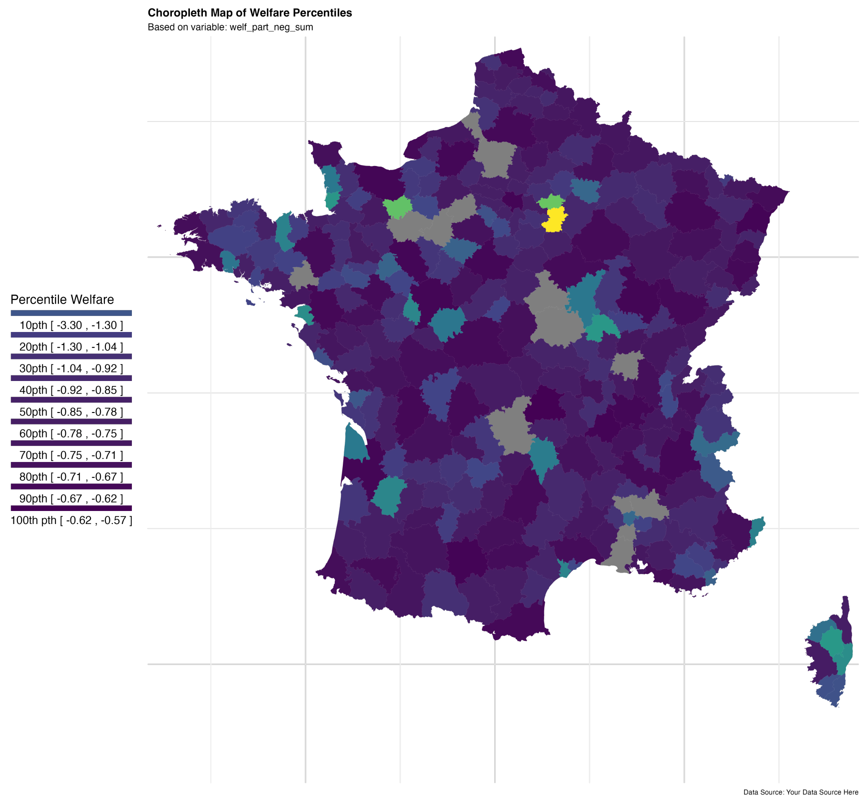


Figure A.7. Heterogeneous Welfare Effects (Negative)

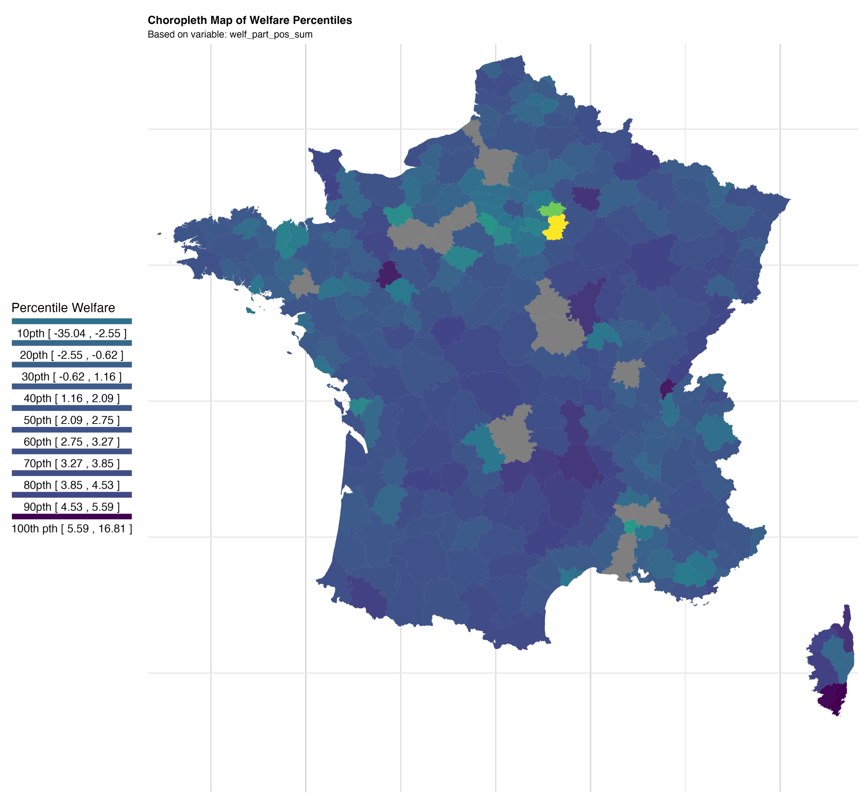


Figure A.8. Heterogeneous Insurance (Positive)

B Additional Tables

Table A.1. QUARTERLY WORKER REALLOCATION ACROSS MARKETS

	Median	SD	p5	p25	p75	p95
Emp_i	693	6,396	170	380	1,633	5,252
ΔEmp_i	0.32 %	0.26	-0.11%	0.16 %	0.47 %	0.75 %
$\frac{\text{Flow}_{i'\to i}}{\text{Emp}_i}$	1.67 %	0.59	1.04 %	1.32 %	2.08 %	2.89 %
$\frac{\text{Flow}_{i\to i'}}{\text{Emp}_i}$	1.63 %	0.58	1.01 %	1.32 %	2.00 %	2.91 %
$\frac{\text{Flow}_{s,o}}{\text{Emp}_i}$	2.96 %	0.66	2.11 %	2.57 %	3.48 %	4.17 %
$\frac{\text{Flow}_{i'\to i}-\text{Flow}_{i\to i'}}{\text{Emp}_i}$	0.03 %	0.13	-0.14 %	-0.05 %	0.12 %	0.26 %
$\frac{\text{Flow}_{ne\to e}-\text{Flow}_{e\to ne}}{\text{Emp}_i}$	0.32 %	0.32	-0.11 %	0.16 %	0.50 %	0.86 %

***Notes*:** The displayed statistics are computed for the cross-section of French cities. The first row displays the average distribution of employment size across local labor markets. The second row displays the average quarterly growth rate of employment. The following three rows show the share of spatial inflows, spatial outflows and non-spatial flows. The last term does not display any flow direction since it does not involve any spatial component. Hence, non-spatial outflows are equivalent to non-spatial inflows. Finally, the last two rows display the spatial reallocation, i.e. spatial inflow minus spatial outflow, and the participation reallocation, i.e. employment inflow minus non-employment inflow.

Table A.2. RESPONSE TO A LABOR DEMAND SHOCK

Emp	$\sum \text{Flow}_{i'\to i}$	$\sum \text{Flow}_{i\to i'}$	$\sum \text{Flow}_{i'\to i} - \sum \text{Flow}_{i\to i'}$	$\sum \text{Flow}_{s,o}$	$\sum \text{Flow}_{s,o} - \left[\sum \text{Flow}_{i'\to i} - \sum \text{Flow}_{i\to i'}\right]$
0.40***	0.37***	0.19**	0.18***	0.40***	0.23**
(0.10)	(0.11)	(0.09)	(0.06)	(0.09)	(0.09)

***Notes*:** Notes: LPIV regressions of different outcome variables on shift-share instrument at the commuting zone level (Equation (3)). All columns indicate the cumulative responses at year 5 or quarter 20. The first column provides the response of the employment stock. The second column provides the spatial inflow. The third colmn provides the impact on the spatial outflow. The fourth column provides the impact on the net flow. The fifth column provides the impact on all non-spatial adjustments (reallocation across sectors or occupations locally). The last column provides the difference between non-spatial adjustments and netflows. Robust standard errors in parentheses.

Table A.3. Effect of Bartik on worker reallocation: asymmetry and HHI

	$Q1$	$Q2$	$Q3$	$Year1$	$Year2$	$Year3$	$Year4$	$Year5$
$Emp_i -$ **Flow**$_{i' \to i} -$ **Flow**$_{i \to i'}$								
Bartik$^+$	0.71∗∗∗	0.9∗∗∗	0.6∗∗∗	0.46∗∗	0.69∗∗∗	0.66∗∗∗	0.76∗∗∗	0.82∗∗∗
	0.12	0.11	0.2	0.2	0.13	0.13	0.14	0.17
Bartik$^-$	−0.36∗∗∗	−0.72∗∗∗	−0.75∗∗∗	−0.94∗∗∗	−0.67∗∗∗	−1.22∗∗∗	−1.08∗∗∗	−0.92∗∗∗
	0.1	0.15	0.14	0.22	0.19	0.14	0.14	0.17
Bartik$^+$ × HHI$_{so}$	−0.02	−0.09∗∗∗	−0.04	−0.04	0.03	0.01	−0.04	−0.08∗∗
	0.04	0.03	0.04	0.04	0.04	0.04	0.03	0.04
Bartik$^-$ × HHI$_{so}$	−0.05	−0.08∗∗	−0.09∗∗∗	−0.11∗∗∗	−0.09	−0.15∗∗∗	−0.17∗∗∗	−0.1∗∗
	0.03	0.04	0.02	0.03	0.07	0.03	0.05	0.04
Flow$_{i \to i'}$								
Bartik$^+$	0.3∗∗∗	0.34∗∗∗	0.16	0.17	0.35∗∗∗	0.24∗	0.36∗∗∗	0.26∗∗
	0.09	0.07	0.12	0.11	0.12	0.13	0.12	0.11
Bartik$^-$	0.19∗∗∗	0.06	0.04	−0.29∗	−0.14	−0.38∗∗∗	−0.29∗∗∗	−0.42∗∗∗
	0.07	0.06	0.09	0.17	0.09	0.09	0.11	0.1
Bartik$^+$ × HHI$_{so}$	−0.01	−0.05∗∗∗	−0.01	−0.04∗∗	−0.02	−0.01	−0.04∗∗	−0.04
	0.02	0.02	0.01	0.02	0.02	0.02	0.02	0.02
Bartik$^-$ × HHI$_{so}$	−0.0	−0.0	0.0	−0.04	−0.08∗∗	−0.08∗∗∗	−0.08∗∗	−0.08∗∗∗
	0.01	0.02	0.02	0.03	0.03	0.02	0.04	0.03
Flow$_{s,o}$								
Bartik$^+$	0.19∗∗	0.15	0.17	0.1	0.36∗∗	0.37∗∗	0.32∗∗	0.27∗∗
	0.08	0.1	0.11	0.11	0.18	0.17	0.13	0.13
Bartik$^-$	−0.19∗∗∗	−0.46∗∗∗	−0.45∗∗∗	−0.48∗∗∗	−0.32∗∗∗	−0.45∗∗∗	−0.36∗∗∗	−0.39∗∗∗
	0.05	0.11	0.11	0.11	0.08	0.09	0.06	0.08
Bartik$^+$ × HHI$_{so}$	0.03	0.04	0.04	0.04∗	0.02	0.01	0.02	0.01
	0.03	0.03	0.03	0.02	0.02	0.02	0.02	0.02
Bartik$^-$ × HHI$_{so}$	−0.06∗∗	−0.07∗∗∗	−0.08∗∗∗	−0.07∗∗∗	−0.07∗∗∗	−0.06∗∗∗	−0.06∗∗∗	−0.05∗∗∗
	0.03	0.03	0.03	0.03	0.03	0.02	0.02	0.01
Emp_i								
Bartik$^+$	0.43∗∗∗	0.42∗∗∗	0.56∗∗∗	0.59∗∗∗	0.55∗∗∗	0.46∗∗∗	0.5∗∗∗	0.53∗∗∗
	0.11	0.1	0.11	0.16	0.21	0.13	0.11	0.14
Bartik$^-$	−0.2∗∗	−0.32∗∗∗	−0.62∗∗∗	−0.6∗∗∗	−0.48∗∗	−0.63∗∗∗	−0.75∗∗∗	−0.53∗∗∗
	0.09	0.06	0.08	0.12	0.19	0.08	0.09	0.11
Bartik$^+$ × HHI$_{so}$	0.01	0.02	−0.02	−0.05	−0.0	0.0	−0.02	−0.03
	0.02	0.02	0.02	0.03	0.06	0.03	0.03	0.02
Bartik$^-$ × HHI$_{so}$	0.04	0.01	−0.07∗∗∗	−0.07∗∗∗	0.02	−0.04∗∗	−0.08∗∗∗	−0.03∗∗
	0.03	0.02	0.01	0.02	0.05	0.02	0.01	0.01
Observations	14535	14250	13965	13680	12540	11400	10260	9120

***Notes*:** Notes: LPIV regressions of different outcome variables on shift-share instrument at the commuting zone level (Equation (6)). The first section provides the response of non-spatial adjustments. The next section provides the response of spatial netflows. The last section provides the difference between non-spatial and spatial adjustments. Robust standard errors in parentheses.

C Additional Results

To complement the analysis in the main text, we also introduce an LPIV that identifies the heterogeneous response, but without focusing on the asymmetric response. The estimating equation is specified as follows:

$$\Delta y_{i,t+h} = \alpha^h + \gamma_t + \gamma_i + \beta_h \,\text{Shock}_{i,t} + \psi_h \left(\text{Shock}_{i,t} \times \text{HHI}_i\right) + \sum_{m=1}^{8} \gamma_m^h y_{i,t-m} + \sum_{m=1}^{8} \omega_m^h \,\text{Shock}_{i,t-m} + \sum_{m=1}^{8} \delta_m^h Z_{i,t-m}$$

In this equation, $\Delta y_{i,t+h}$ represents the change in the outcome variable of interest for location i at horizon h, allowing us to model the response over a specified time horizon. The intercept α^h is a time-specific constant that adjusts for general trends in the data, while the fixed effects γ_t and γ_i control for temporal and spatial heterogeneity. These fixed effects are essential to ensure that the estimated response to the shock is not biased by time-invariant location-specific factors or broader macroeconomic trends. The key term, $\beta_h \,\text{Shock}_{i,t}$, captures the direct effect of the shock at time t on the outcome at time $t+h$. The shock $\text{Shock}_{i,t}$ is an exogenous disturbance affecting location i and is assumed to vary across locations and time. The term is furthermore interacted with the local HHI_i as introduced in the main text.

To account for persistence in the outcome and the potential for shocks to have lasting effects, the model includes lags of both the outcome variable and the shock itself. The terms $\sum_{m=1}^{8} \gamma_m^h y_{i,t-m}$ represent the influence of past values of the outcome on current changes, while the terms $\sum_{m=1}^{8} \omega_m^h \,\text{Shock}_{i,t-m}$ capture the dynamic effects of past shocks. These lagged terms allow us to model the full dynamic response of the outcome to shocks, recognizing that the effect of a shock may persist over multiple periods.

Table A.4. Effect of Bartik on worker reallocation

	Q1	Q2	Q3	Year 1	Year 2	Year 3	Year 4	Year 5
Emp$_i$								
Bartik	0.30***	0.37***	0.61***	0.61***	0.53***	0.43**	0.60**	0.65***
	(0.09)	(0.07)	(0.09)	(0.14)	(0.13)	(0.16)	(0.24)	(0.17)
Bartik × HHI$_{so}$	0.02	0.01	-0.05***	-0.06**	-0.03*	-0.00	-0.05	-0.05*
	(0.02)	(0.01)	(0.02)	(0.02)	(0.02)	(0.03)	(0.04)	(0.03)
Flow$_{s,o}$								
Bartik	0.22***	0.35***	0.36***	0.34***	0.35***	0.34**	0.35**	0.43***
	(0.06)	(0.10)	(0.10)	(0.10)	(0.11)	(0.13)	(0.15)	(0.16)
Bartik × HHI$_{so}$	-0.02	-0.03	-0.03*	-0.03***	-0.02*	-0.01	-0.00	-0.02
	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)
Flow$_{i' \to i}$								
Bartik	0.07	0.12**	0.35***	0.43***	0.37***	0.26*	0.41***	0.55***
	(0.05)	(0.06)	(0.12)	(0.15)	(0.12)	(0.14)	(0.14)	(0.18)
Bartik × HHI$_{so}$	0.02***	0.04***	-0.02	-0.05**	-0.02	0.00	-0.02	-0.06**
	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)
Flow$_{i \to i'}$								
Bartik	0.03	0.12**	0.05	0.23**	0.36***	0.37**	0.38***	0.56***
	(0.04)	(0.05)	(0.07)	(0.10)	(0.10)	(0.18)	(0.14)	(0.12)
Bartik × HHI$_{so}$	-0.00	-0.02**	0.00	-0.04*	-0.06***	-0.04	-0.04*	-0.09***
	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.03)	(0.02)	(0.02)
Observations	14,535	14,250	13,965	13,680	12,540	11,400	10,260	9,120

Notes: Notes: LPIV regressions of different outcome variables on shift-share instrument at the commuting zone level (Equation (C)) interacted with the HHI. The first section provides the response of the employment stock. The second section provides the response of non-spatial adjustments. The next section provides the response for outflows. The last section provides the response of the inflow margin. Robust standard errors in parentheses.

Table A.5. Effect of Bartik on worker reallocation

	Q1	Q2	Q3	Year 1	Year 2	Year 3	Year 4	Year 5
$\text{Flow}_{i'\to i} - \text{Flow}_{i\to i'}$								
Bartik	0.03	0.02	0.32**	0.22**	-0.01	-0.13*	0.01	-0.04
	(0.05)	(0.06)	(0.14)	(0.09)	(0.09)	(0.07)	(0.08)	(0.06)
Bartik × HHI_{so}	0.03**	0.05***	-0.02	-0.01	0.05***	0.05***	0.02	0.04**
	(0.09)	(0.01)	(0.02)	(0.01)	(0.02)	(0.01)	(0.02)	(0.01)
$\text{Flow}_{s,o} - [\text{Flow}_{i'\to i} - \text{Flow}_{i\to i'}]$								
Bartik	0.20***	0.37***	0.06	0.14	0.36***	0.48***	0.35**	0.47***
	(0.07)	(0.09)	(0.10)	(0.08)	(0.10)	(0.17)	(0.17)	(0.11)
Bartik × HHI_{so}	-0.05**	-0.08***	-0.01	-0.02	-0.07***	-0.06**	-0.02	-0.05**
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.04)	(0.02)
$\text{Flow}_{i,s,o}$								
Bartik	0.33***	0.58***	0.76***	1.00***	1.07***	0.96**	1.13***	1.51***
	(0.10)	(0.14)	(0.19)	(0.31)	(0.26)	(0.42)	(0.39)	(0.44)
Bartik × HHI_{so}	0.00	-0.00	-0.05*	-0.11**	-0.10***	-0.04	-0.06	-0.16**
	(0.03)	(0.03)	(0.03)	(0.05)	(0.04)	(0.06)	(0.05)	(0.06)
$\text{Flow}_{ne\to e} - \text{Flow}_{e\to ne}$								
Bartik	0.30***	0.43***	0.31**	0.37***	0.50***	0.52***	0.54**	0.68***
	(0.08)	(0.07)	(0.14)	(0.13)	(0.09)	(0.17)	(0.12)	(0.09)
Bartik × HHI_{so}	-0.01	-0.05***	-0.04	-0.05**	-0.07***	-0.04	-0.07*	-0.09***
	(0.01)	(0.02)	(0.03)	(0.02)	(0.02)	(0.03)	(0.03)	(0.02)
$Emp_i - \text{Flow}_{i'\to i} - \text{Flow}_{i\to i'}$								
Bartik	0.53***	0.80***	0.70***	0.74***	0.88***	0.89***	0.93**	1.12***
	(0.12)	(0.13)	(0.15)	(0.19)	(0.18)	(0.27)	(0.35)	(0.26)
Bartik × HHI_{so}	-0.03	-0.08***	-0.07**	-0.08***	-0.10***	-0.06**	-0.07	-0.11***
	(0.03)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.05)	(0.03)
Observations	14,535	14,250	13,965	13,680	12,540	11,400	10,260	9,120

D Additional Derivations

This appendix section provides additional derivations for the results in Section 2 and 4 as well as a motivation for the utilization of HHI indices.

D.1 Derivations for Section 2

In this subsection, we provide additional derivations for the results in Section 2. Specifically, we present the derivations for the approximation results. To begin, it will be helpful to reiterate key results from Chiong, Galichon and Shum (2016), specifically, their Theorem 1, which relates conditional choice probabilities and continuation values to the subdifferential of the $\mathscr{G}$ function and the convex conjugate of the $\mathscr{G}$ function, i.e.

$$p \in \partial \mathscr{G}(w)$$

and,

$$w \in \partial \mathscr{G}^*(p)$$

where the former equation is a generalization of the Williams-Daly-Zachery (WDZ) theorem as discussed in Chiong, Galichon and Shum (2016) and the latter generalizes results in Hotz and Miller (1993). We then start by approximating the $\mathscr{G}^*$ function around w_0,

$$\mathscr{G}(w) = \mathscr{G}(w^0) + \sum_y \frac{\partial \mathscr{G}(w)}{\partial w_y}\left(w_y - w_y^0\right) + \frac{1}{2}\sum_y\sum_{y'} \frac{\partial^2 \mathscr{G}(w)}{\partial w_y \partial w_{y'}}\left(w_y - w_y^0\right)\left(w_{y'} - w_{y'}^0\right) + o\left(\cdot\right)$$

$$d\mathscr{G}(w) = \sum_y \frac{\partial \mathscr{G}(w)}{\partial w_y} dw_y + \frac{1}{2}\sum_y\sum_{y'} \frac{\partial^2 \mathscr{G}(w)}{\partial w_y \partial w_{y'}} dw_y dw_{y'} + o\left(\cdot\right)$$

and similarly, for the convex conjugate function,

$$\mathscr{G}^*(p) = \mathscr{G}^*(p^0) + \sum_y \frac{\partial \mathscr{G}^*(p)}{\partial p_y}\left(p_y - p_y^0\right) + \frac{1}{2}\sum_y\sum_{y'} \frac{\partial^2 \mathscr{G}^*(p)}{\partial p_y \partial p_{y'}}\left(p_y - p_y^0\right)\left(p_{y'} - p_{y'}^0\right) + o\left(\cdot\right)$$

$$d\mathscr{G}^*(p) = \sum_y \frac{\partial \mathscr{G}^*(p)}{\partial p_y} dp_y + \frac{1}{2}\sum_y\sum_{y'} \frac{\partial^2 \mathscr{G}^*(p)}{\partial p_y \partial p_{y'}} dp_y dp_{y'} + o\left(\cdot\right)$$

In terms of log changes we have,

$$d\ln \mathscr{G}(w) = \sum_y \frac{\frac{\partial \mathscr{G}(w)}{\partial w_y} w_y}{\mathscr{G}(w)} d\ln w_y + \frac{1}{2}\sum_y\sum_{y'} \frac{\frac{\partial^2 \mathscr{G}(w)}{\partial w_y \partial w_{y'}} w_y w_{y'}}{\mathscr{G}(w)} d\ln w_y d\ln w_{y'} + o\left(\cdot\right)$$

$$d\ln \mathscr{G}^*(p) = \sum_y \frac{\frac{\partial \mathscr{G}^*(p)}{\partial p_y} p_y}{\mathscr{G}^*(p)} d\ln p_y + \frac{1}{2}\sum_y\sum_{y'} \frac{\frac{\partial^2 \mathscr{G}^*(p)}{\partial p_y \partial p_{y'}} p_y p_{y'}}{\mathscr{G}^*(p)} d\ln p_y d\ln p_{y'} + o\left(\cdot\right)$$

Applying Theorem 1 from Chiong, Galichon and Shum (2016), we obtain,

$$d \ln \mathscr{G}(w) = \sum_y \frac{p_y w_y}{\mathscr{G}(w)} d \ln w_y + \frac{1}{2} \sum_y \sum_{y'} \frac{\frac{\partial p_y}{\partial w_{y'}} \frac{w_{y'}}{p_y} p_y w_y}{\mathscr{G}(w)} d \ln w_y d \ln w_{y'} + o(\cdot)$$

$$d \ln \mathscr{G}^*(p) = \sum_y \frac{w_y p_y}{\mathscr{G}^*(p)} d \ln p_y + \frac{1}{2} \sum_y \sum_{y'} \frac{\frac{\partial w_y}{\partial p_{y'}} \frac{p_{y'}}{w_y} w_y p_y}{\mathscr{G}^*(p)} d \ln p_y d \ln p_{y'} + o(\cdot)$$

Defining the cross-elasticity, $\varepsilon^{p,w}_{y,y'} \equiv \frac{\partial \ln p_y}{\partial \ln w_{y'}} = \frac{\partial p_y}{\partial w_{y'}} \frac{w_{y'}}{p_y}$ and $\varepsilon^{w,p}_{y,y'} \equiv \frac{\partial \ln w_y}{\partial \ln p_{y'}} = \frac{\partial w_y}{\partial p_{y'}} \frac{p_{y'}}{w_y}$,

$$d \ln \mathscr{G}(w) = \sum_y \frac{p_y w_y}{\mathscr{G}(w)} d \ln w_y + \frac{1}{2} \sum_y \sum_{y'} \varepsilon^{p,w}_{y,y'} \frac{p_y w_y}{\mathscr{G}(w)} d \ln w_y d \ln w_{y'} + o(\cdot)$$

$$d \ln \mathscr{G}^*(p) = \sum_y \frac{w_y p_y}{\mathscr{G}^*(p)} d \ln p_y + \frac{1}{2} \sum_y \sum_{y'} \varepsilon^{w,p}_{y,y'} \frac{w_y p_y}{\mathscr{G}^*(p)} d \ln p_y d \ln p_{y'} + o(\cdot)$$

Rewrite in terms of generic weights,$\omega_y \equiv \frac{w_y p_y}{\mathscr{G}(w)}$ and $\omega^*_y \equiv \frac{w_y p_y}{\mathscr{G}^*(p)}$,

$$d \ln \mathscr{G}(w) = \sum_y \omega_y d \ln w_y + \frac{1}{2} \sum_y \sum_{y'} \varepsilon^{p,w}_{y,y'} \omega_y d \ln w_y d \ln w_{y'} + o(\cdot)$$

$$d \ln \mathscr{G}^*(p) = \sum_y \omega^*_y d \ln p_y + \frac{1}{2} \sum_y \sum_{y'} \varepsilon^{w,p}_{y,y'} \omega^*_y d \ln p_y d \ln p_{y'} + o(\cdot)$$

D.2 Derivations for Section 4

Turning towards an isoelastic labor supply system, i.e.

$$p_y = \frac{w_y^\gamma}{\sum_y w_y^\gamma}$$

We can totally differentiate and obtain,

$$\varepsilon^{w,p}_{y,y'} \equiv \frac{\partial \ln w_y}{\partial \ln p_{y'}} = \left(\frac{\partial p_y}{\partial w_{y'}} \frac{w_{y'}}{p_y} \right)^{-1} = \left(-p_{y'} \gamma \right)^{-1} \qquad \text{if} \qquad y = y'$$

$$\varepsilon^{w,p}_{y,y'} \equiv \frac{\partial \ln w_y}{\partial \ln p_{y'}} = \left(\frac{\partial p_y}{\partial w_{y'}} \frac{w_{y'}}{p_y} \right)^{-1} = \left(\gamma - p_y \gamma \right)^{-1} \qquad \text{if} \qquad y \neq y'$$

Substituting, we obtain,

$$d \ln \mathscr{G}^*(p) = \sum_y \omega^*_y d \ln p_y + \frac{1}{\gamma} \sum_y \frac{\omega^*_y}{1 - p_y} \left(d \ln p_y \right)^2 - \frac{1}{2\gamma} \sum_y \sum_{y' \neq y} \frac{\omega^*_y}{p_{y'}} d \ln p_y d \ln p_{y'} + o(\cdot)$$

Assuming, $y \in \mathscr{Y} = [\text{Stay}, \text{Local}, \text{Spatial}]$, we obtain,

$$\frac{\mathrm{d}\ln\mathscr{G}^{*}(p;x)}{\mathrm{d}\ln\bar{x}} \approx \sum_{\substack{\text{y=stay,}\\ \text{local, spatial}}} \left[\omega^{*}_{Y}(x)\frac{\mathrm{d}\ln p_{y}(x)}{\mathrm{d}\ln\bar{x}}\right] + \frac{1}{\gamma}\sum_{\substack{\text{y=stay,}\\ \text{local, spatial}}} \left[\frac{\omega^{*}_{Y}(x)}{1-p_{y}(x)}\left(\frac{\mathrm{d}\ln p_{y}(x)}{\mathrm{d}\ln\bar{x}}\right)^{2}\right]$$
$$-\frac{1}{2\gamma}\sum_{\substack{\text{y=stay,}\\ \text{local, spatial}}}\sum_{y'\neq y}\left[\frac{\omega^{*}_{y}(x)}{p_{y'}(x)}\frac{\mathrm{d}\ln p_{y}(x)}{\mathrm{d}\ln\bar{x}}\frac{\mathrm{d}\ln p_{n}(x)}{\mathrm{d}\ln\bar{x}}\right]$$

as reported in Section 4.

D.3 An index to measure insurance through diversity

We simplify the baseline model in the theory section in order to connect diversity indices and the insurance value of the local choice set. Consider a direct shock to choice alternative y ($d\ln w_y \neq 0$), no direct shock elsewhere ($d\ln w_{y'} = 0$), but correlated effects elsewhere ($E\left(d\ln w_{y'}|d\ln w_y\right) \neq 0$), the formula then becomes,

$$\frac{d\ln\mathscr{G}(w;x)}{d\ln w_y} = \frac{\sigma_y w_y}{\mathscr{G}} + \frac{1}{2}\sum_{y'\neq y}\frac{\sigma_{y'}w_{y'}}{\mathscr{G}}\theta_{y'y}E\left(d\ln w_{y'}|d\ln w_y\right) + O(\cdot)$$

Furthermore, let the correlated effects be iid distributed, such that, $E\left(d\ln w_{y'}|d\ln w_y\right) = \nu$ and assume furthermore, that there exists a constant elasticity of substitution between labor markets, θ. Additionally, normalize the size of the shock $d\ln w_y = 1$ and assume that in normal times all workers would have remained attached to their current labor market, i.e. $\frac{\sigma_y w_y}{\mathscr{G}} = 1$. Using a log-linear welfare aggregator, [1]

$$\frac{d\mathscr{W}'_i}{\mathscr{W}'_i} = \sum_{y\in\mathscr{Y}}\frac{d\mathscr{G}(w;x)'}{\mathscr{G}(w;x)'} \times \pi_y$$

Combining, the effect of a negative shock depends on the relative population-weighted insurance given from local and non-local labor markets, i.e.

$$\frac{d\mathscr{W}'_i}{\mathscr{W}'_i} = \sum_{y\in\mathscr{Y}}\pi_y \times \left(\frac{\nu\theta}{2}\sum_{y'\neq y\in\mathscr{Y}}\frac{\sigma_{y'}w_{y'}}{\mathscr{G}} + \frac{\nu\theta}{2}\sum_{y'\notin\mathscr{Y}}\frac{\sigma_{y'}w_{y'}}{\mathscr{G}}\right)$$

Since ω_y is just a measure of the overall attractiveness of a local labor market, in steady state it should correspond to the local share of workers working in that sector (i.e. π_y). We can empirically approximate how much insurance is provided locally by measuring,

$$\text{Ind}_i = \sum_{y\in\mathscr{Y}}\pi_y \times \left(\frac{\nu\theta}{2}\frac{\sum_{y'\neq y\in\mathscr{Y}}\sigma_{yy'}\pi_{y'}}{\mathscr{G}}\right).$$

[1] This can be derived from a first-order approximation initial period spatial allocation problem,

Relationship to HHI This index takes into account labor mobility, in particular to what extent workers across different local labor markets can reach large sectors of the local economy given their usual mobility pattern. However, it is also intimately related to the HHI given by

$$\mathrm{HHI}_i = \sum_{y' \in \mathscr{Y}} \left(\pi_{y'}\right)^2.$$

Re-arranging the formula above and assuming symmetric option values ($\mathscr{G}_y = \mathscr{G} \quad \forall y$) yields,

$$\mathrm{Ind}_i = \frac{\nu\theta}{2\mathscr{G}} \sum_{y \in \mathscr{Y}} \pi_y \sum_{y' \neq y \in \mathscr{Y}} \sigma_{yy'} \pi_{y'}$$

which in turn simplifies directly to the HHI in a closed-labor market where the $\sum_{y' \neq y \in \mathscr{Y}} \sigma_{yy'} \pi_{y'}$ simplifies to π_y. This is to say that HHI is an appropriate index to measure labor market diversity through the lens of our model. In what follows, we will utilize HHI as a proxy for local labor market diversity.