Financial Fragility with SAM?

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Abstract

Shared Appreciation Mortgages (SAMs) feature mortgage payments that adjust with house prices. Such mortgage contracts can stave off home owner default by providing payment relief in the wake of a large house price shock. SAMs have been hailed as an innovative solution that could prevent the next foreclosure crisis, act as a work-out tool during a crisis, and alleviate fiscal pressure during a downturn. They have inspired Fintech companies to offer home equity contracts. However, the home owner's gains are the mortgage lender's losses. We consider a model with financial intermediaries who channel savings from saver households to borrower households. The financial sector has limited risk bearing capacity. SAMs pass through more aggregate house price risk and lead to financial fragility when the shock happens in periods of low intermediary capital. We compare house prices, mortgage rates, the size of the mortgage sector, default and refinancing rates, as well as borrower and saver consumption between an economy with standard mortgage contracts and an economy with SAMs.

1 Introduction

The $10 trillion market in U.S. mortgage debt is the largest consumer debt market in the world and the second largest fixed income market. Mortgages are not only the largest liability for U.S. households, they are also the largest asset of the U.S. financial sector. Banks and credit unions hold $3 trillion in mortgage loans directly on their balance sheets in the form of whole loans. They hold an additional $2.2 trillion in the form of agency mortgage-backed securities.[1] Given the exposure of the financial sector to mortgages, large house price declines and the default wave that accompanies them can severely hurt the solvency of the U.S. financial system. This became painfully clear during the Great Financial Crisis of 2008-2011. In addition, interest rate increases may lead to large valuation losses on mortgage debt for financial intermediaries and represent an important source of risk going forward.

In this paper we study the allocation of house price and interest rate risk in the mortgage market between mortgage borrowers, financial intermediaries, and savers. The standard 30-year fixed-rate freely prepayable mortgage (FRM) stipulates a particular allocation of house price and interest rate risk between borrowers and lenders. Borrower home equity absorbs the initial house price declines. A large enough price decline pushes the home owner under-water. A sufficiently high loan-to-value ratio, possibly coupled with an income shock, may lead the home owner to default on the mortgage, inflicting losses on the lender. The lender bears the risk of large price declines. During the GFC, U.S. house prices fell 30% nationwide, and by much more in some regions. The financial sector had written out-of-the-money put options on aggregate house prices with more than $5 trillion in face value, and the risk materialized. Seven million home owners were foreclosed upon. About 25% of U.S. home owners were were underwater by 2010; most were unable to refinance their mortgage. Even if they continued to service their mortgage, the inability to refinance hampered home owners' propensity to consume and short-circuited the stimulative consumption response from lower mortgage rates that

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[1]Including insurance companies, money market mutual funds, broker-dealers, and mortgage REITS in the definition of the financial sector adds another $1.5 trillion to the financial sector's agency MBS holdings. Adding the Federal Reserve Bank and the GSE portfolios adds a further $2 trillion and increases the share of the financial sector's holdings of agency MBS to nearly 80%.

policy makers hoped for.

This episode led many to ask whether a fundamentally different mortgage finance system could not lead to a better sharing of risk between borrowers and lenders.[2] Several alternatives to the standard FRM are already widely available. One such contract is the adjustable rate mortgage (ARM). The ARM features nearly perfect pass-through of monetary policy rates but still faces the refinancing problem associated with being underwater. It also remains a standard debt contract. Another now infamous product is the option-ARM, which gives the borrower low and flexible mortgage payments in the initial phase of the mortgage.[3]

Going beyond the existing contract menu, some have called for automatically refinancing mortgages or ratchet mortgages, whose interest rate only adjusts down (Kalotay, 2015).[4] Eberly and Krishnamurthy (2014) propose a mortgage contract that automatically refinances from a FRM into an ARM, even when the loan is underwater. They argue that government programs that subsidize payment reductions to underwater households can be welfare enhancing since they mitigate the costs of foreclosures and support consumption during the crisis.

The most well known proposal is the shared appreciation mortgage (SAM). The SAM indexes the mortgage interest payments to house price changes. In the fully symmetric version, house price decreases lower mortgage payments while house price increases raise them. The mortgage payment indexation to house prices makes the contract more equity-like. The advantage of such a contract is that the mortgage borrower receives payment relief in bad states of the world, which can substantially reduce mortgage default and the associated deadweight losses to society. The main point of this paper is to quantitatively evaluate the benefits of SAMs to home owners and to weigh them against the costs of increased systemic risk they impose on financial intermediaries. Mortgage write downs in bad aggregate states could increase financial fragility at inopportune times. In other words, whether SAMs present a better risk sharing arrangement to the overall economy

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[2]The New York Federal Reserve Bank organized a two-day conference on this topic in May 2015 with participants from academia and policy circles.

[3]Piskorski and Tchistyi (2011) study optimal mortgage contract design in a partial equilibrium model with stochastic house prices and show that option-ARM implements the optimal contract.

[4]The automatically refinancing mortgages address the failure os some households to refinance despite having a strong rate incentives to do so (Campbell, 2006, 2013; Keys, Pope, and Pope, 2017).

than FRMs is an open question. We argue for a need to shift the debate from one on *household risk management* to one on *economy-wide risk management*.

We set up a model with mortgage borrowers and lenders. Initially, the lender is an amalgam of financial intermediaries and saving households. Later, we separate out these two types of agents. Borrowers face income shocks as well as idiosyncratic depreciation shocks. Households optimally default on their mortgages when the value of the house falls below the amount owed on the mortgage. In addition to aggregate income risk, the cross-sectional dispersion of the depreciation shocks represent a second source of aggregate risk. We refer to the dispersion shocks as "uncertainty shocks." House prices and mortgage interest rates are endogenously determined. Borrowers and savers are risk averse, but with a different degree of patience, risk aversion, and willingness to inter-temporally substitute consumption.

We start by solving an economy with FRMs, captured here as perpetuities with geometrically declining payments and a duration calibrated to actual FRM. These mortgages are long-term, defaultable, and prepayable contracts. We separately keep track of the mortgage balance, which allows us to enforce loan-to-value constraints only at mortgage origination but not subsequent to origination. The mortgage balance is the amount the borrower owes when she refinances the mortgage. We introduce a cost of refinancing. We contrast this economy to an economy with SAMs. What distinguishes a SAM from a regular mortgage in our model is that the mortgage payments and/or the mortgage principal are indexed to house prices. We consider several types of house price indexation of mortgage payments: to aggregate, local, or individual-level house price fluctuations.

By tying mortgage payments and principal to individual house valuation shocks, SAMs can eliminate virtually all defaults. Intermediate levels of indexation, which can be thought of as local house price indexation, reduce defaults by less but help to deal with moral hazard issues. Indexation to aggregate house price dynamics offer even less insurance to the individual borrower. By switching off indexation of mortgage payments but not indexation of the mortgage principal, we capture features of a FRM-SAM combination where the SAM entails no periodic payments but entitles the lender to a share of the appreciation of the property upon termination of the mortgage. Initially, we consider a

symmetric SAM contract where payments adjust both up and down with house prices. Later we explore how the equilibrium changes when periodic payments can only go below those associated with a regular mortgage and principal adjustment is only upward, to compensate the lender for the downside payment protection. For all cases, we compare the equilibrium size of the mortgage market, house price level and volatility, mortgage interest rate and volatility, the default rate, the prepayment rate, and the consumption level and volatility of borrowers and lenders.

Our main result is that indexation of mortgage debt to aggregate house prices re-allocates house price risk from borrowers to lenders. In the benchmark model without any indexation, borrower consumption is roughly twice as volatile as saver consumption. House prices strongly move with aggregate income shocks and have a large effect on borrower wealth and consumption. When mortgage debt is marked to changes in house prices, the correlation of borrower wealth with house prices decreases, while the correlation of saver wealth with house prices increases. As a result, the volatility of consumption of borrowers and lenders is roughly equalized, while aggregate consumption volatility remains constant. In addition, default and mortgage interest rates become less volatile, while mortgage debt volatility increases. When we further allow mortgage debt to be indexed to idiosyncratic house price risk, mortgage default becomes unattractive and the default rate drops to zero. Lower equilibrium interest rates that reflect this lower default risk lead to higher house prices and more mortgage debt. Our results demonstrate that the nature of the mortgage contract has large quantitative implications for equilibrium interest rates and house prices, as well as the quantity of mortgage debt. It further has significant effects on the distribution of house price risk between borrowers and lenders.

Related Literature   This paper contributes to the literature that studies innovative mortgage contracts. In early work, Shiller and Weiss (1999) discuss the idea of home equity insurance policies. The idea of SAMs was discussed in a series of papers by Caplin, Chan, Freeman, and Tracy (1997); Caplin, Carr, Pollock, and Tong (2007); Caplin, Cunningham, Engler, and Pollock (2008). They envision a SAM as a second mortgage in addition to a conventional FRM with a smaller principal balance. The SAM has no in-

terest payments and its principal needs to be repaid upon termination (e.g., sale of the house). At that point the borrower shares a fraction of the house value appreciation with the lender, but only if the house has appreciated in value. The result is lower monthly mortgage payments throughout the life of the loan, which enhances affordability, and a better sharing of housing risk. They emphasize that SAMs are not only a valuable work-out tool after a default has taken place, but are also useful to prevent a mortgage crisis in the first place.[5] Mian (2013) and Mian and Sufi (2014) introduce a version of the SAM, which they call the Shared Responsibility Mortgage (SRMs). The SRM replaces a FRM rather than being an additional mortgage. It features mortgage payments that adjust down when the local house price index goes down, and back up when house prices bounce back, but never above the initial FRM payment. To compensate the lender for the lost payments upon house price declines, the lender receives 5% of the home value appreciation. They argue that foreclosure avoidance raises house prices in a SRM world and shares wealth losses more equitably between borrowers and lenders. When borrowers have higher marginal propensities to consume out of wealth than lenders, this more equitable sharing increases aggregate consumption and reduces job losses that would be associated with low aggregate demand. The authors argue that SRMs would reduce the need for counter-cyclical fiscal policy and give lenders an incentive to "lean against the wind" by charging higher mortgage rates when house price appreciation seems excessive.

Shared appreciation mortgages have graduated from the realm of the hypothetical. They have been offered to faculty at Stanford University for leasehold purchases for fifteen years (Landvoigt, Piazzesi, and Schneider, 2014). More recently, several fintech companies such as FirstRex and EquityKey have been offering home equity products where they offer cash today for a share in the future home value appreciation.[6] These

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[5]Among the implementation challenges are (i) the uncertain holding period of SAMs, (ii) returns on investment that decline with the holding period, and (iii) the tax treatment of SAM lenders/investors. The first issue could be solved by a maximum maturity provision of say 15 years. The second issue can be solved by replacing the lender's fixed appreciation share by a shared-equity rate. For example, instead of 40% of the total appreciation, the investor would have a 4% shared-equity rate. If the holding period of the SAM is 10 years and the original SAM principal represented 20% of the home value, the lender is entitled to the maximum of the SAM principal and $20\% \times (1.04)^{10} = 29.6\%$ of the terminal home *value*. This scheme delivers an annual rate of return to the lender that is constant rather than declining in the holding period. The authors refer to this variant as SAMANTHA, a SAM with A New Treatment of Housing Appreciation.

[6]EquityKey started issuing such shared equity contracts in the early 2000s. It was bought by a Belgian retail bank in 2006. the founders bought the business back from the Belgian bank after the housing crisis

products are presented as an alternative to home equity lines of credit, closed-end second mortgages, reverse mortgages for older home owners, or to help finance the borrower's downpayment at the time of home purchase. They allow the home owner to tap into her home equity without taking on a new debt contract. Essentially, the home owner writes a call option on the local house price index (to avoid moral hazard issues) with strike price equal to the current house price value and receives the upfront option premium in exchange. Our work sheds new light on the equilibrium implications of introducing home equity products.

Kung (2015) studies the effect of the disappearance of non-agency mortgages for house prices, mortgage rates and default rates in an industrial organization model of the Los Angeles housing market. He also evaluates the hypothetical introduction of shared appreciation mortgages in the 2003-07 housing boom. He finds that symmetric SAMs would have enjoyed substantial uptake, partially supplanting non-agency loans, and would have further exacerbated the boom. They would not have mitigated the bust. Our model is an equilibrium model of the entire U.S. market with an endogenous risk-free rate rather than of a single city where households face an exogenously specified outside option of moving elsewhere and constant interest rates. Our lenders are not risk neutral, and charge an endogenously determined risk premium on mortgages. When lenders are risk neutral, they are assumed to be better able to bear house price risk than risk averse households. That seems like a fine assumption when all house price risk is idiosyncratic. However, banks may be severely negatively affected by aggregate house price declines and SAMs may exacerbate that financial fragility.

Hull (2015) studies house price-indexed mortgage contracts in a simple incomplete markets equilibrium model. He finds that such mortgages are associated with lower mortgage default rates and higher mortgage interest rates than standard mortgages. Our analysis features aggregate risk, long-term prepayable mortgage debt and an intermediary

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and resumed its activities. In 2016, the company closed its dorrs after the hedge fund that funded the operations lost interest. FirstREX changed its name to Unison Home Ownership Investors in December 2016. It has been making home ownership investments since March 2004. Its main product offers up to half of the downpayment in exchange for a share of the future appreciation. The larger downpayment eliminates the need for mortgage insurance. Its product is used alongside a traditional mortgage, just like the original SAM contract. Unison is active in 13 U.S. states and plans to add 8 more states in 2017. It is funded by 8 lenders.

sector that is risk averse.

Elenev, Landvoigt, and Van Nieuwerburgh (2016b) studies the role the default insurance provided by the government-sponsored enterprizes, Fannie Mae and Freddie Mac. They consider an increase in the price of insurance that restores the absorption of mortgage default risk by the private sector and show it leads to an allocation that is a Pareto improvement. This paper introduces SAMs, REO housing stock dynamics, and long-term mortgages whose rate does not automatically readjusts every period. Greenwald (2016) studies the interaction between the payment-to-income and the loan-to-value constraint in a model of monetary shock transmission through the mortgage market but without default. Corbae and Quintin (2014) investigate the effect of mortgage product innovation relating to relaxed underwriting criteria in a general equilibrium model with default. Guren and McQuade (2016) study the interaction of foreclosures and house prices in a model with search.

Our paper also relates to the literature that studies the amplification of business cycle shocks provided by credit frictions. E.g., Bernanke and Gertler (1989), Bernanke, Gertler, and Gilchrist (1996), Kiyotaki and Moore (1997), and Gertler and Karadi (2011). A second generation of models has added nonlinear dynamics and a richer financial sector. E.g., Brunnermeier and Sannikov (2014), He and Krishnamurthy (2012), He and Krishnamurty (2013), He and Krishnamurthy (2014), Gârleanu and Pedersen (2011), Adrian and Boyarchenko (2012), Maggiori (2013), Moreira and Savov (2016), and Elenev, Landvoigt, and Van Nieuwerburgh (2016a). Recent work has begun to empirically test these intermediary-based asset pricing models (Adrian, Etula, and Muir, 2014; He, Kelly, and Manela, 2017). Our solution uses a state-of-the-art global non-linear solution technique of a problem with occasionally binding constraints.

Finally, we connect to a recent empirical work has found strong consumption responses and lower default rates (Fuster and Willen, 2015) to exogenously lowered mortgage interest rates (Di Maggio et al. 2017) and to higher house prices (Mian and Sufi, 2009; Mian, Rao, and Sufi, 2013).

2 Model

This section presents the theoretical model.

2.1 Endowments

The two consumption goods in the economy — nondurable consumption and housing services — are provided by two Lucas trees. For nondurable consumption, each type j receives a fixed share s_j of the overall endowment Y_t, which cannot be traded. The overall endowment grows at a deterministic rate g, and is subject to temporary but persistent shocks $\tilde{y}_t$:

$$Y_t = Y_{t-1}\exp(g + \tilde{y}_t),$$

where $\mathbb{E}(\exp(\tilde{y}_t)) = 1$ and

$$\tilde{y}_t = (1 - \rho_y)\mu_y + \rho_y\tilde{y}_{t-1} + \sigma_y\varepsilon_{y,t}, \qquad\qquad \varepsilon_{y,t} \sim N(0,1).$$

The $\varepsilon_{y,t}$ can be interpreted as transitory shocks to the level of TFP. Shares of the housing tree are in fixed supply. Shares of the tree produce housing services proportional to the stock, growing at the same rate g as the nondurable endowment. Housing also requires a maintenance cost proportional to its value, where the proportion depends on the holder of the housing asset.

Linear taxes on labor τ are devoted to government spending G_t that provides no utility for the households.

2.2 Demographics and Preferences

The economy is populated by a continuum of agents of three types: borrowers (denoted B), depositors (denoted D), and intermediaries (denoted I). The measure of type j in the population is denoted χ_j, with $\chi_B + \chi_D + \chi_I = 1$. Households are able to trade a complete set of state-dependent securities with households of their own type, providing perfect insurance against idiosyncratic consumption risk, but cannot trade these securities with members of the other types. Housing capital is divided among the three types of

households in constant shares, $\bar{K} = \bar{K}^B + \bar{K}^I + \bar{K}^D$. Households can only trade housing capital with members of their own type.

Each agent of type $j \in \{B, D, I\}$ has preferences following Epstein and Zin (1989), so that lifetime utility is given by

$$U_t^j = \left\{ (1 - \beta_j) \left(u_t^j \right)^{1 - 1/\psi} + \beta_j \left(\mathbb{E}_t \left[\left(U_{t+1}^j \right)^{1 - \sigma_j} \right] \right)^{\frac{1 - 1/\psi}{1 - \sigma_j}} \right\}^{\frac{1}{1 - 1/\psi}} \tag{1}$$

$$u_t^j = \left(C_t^j \right)^{1 - \xi} \left(H_t^j \right)^\xi \tag{2}$$

where C_t^j is nondurable consumption and H_t^j is housing services. Housing capital produces housing services with a linear technology.

2.3 Financial Technology

There are two financial assets in the economy: mortgages that can be traded between the borrower and the intermediary, and deposits that can be traded between the depositor and the intermediary.

2.3.1 Mortgage Technology

Mortgage contracts are modeled as nominal perpetuities with payments that decline geometrically, so that one unit of debt yields the payment stream $1, \delta, \delta^2, \dots$ until prepayment or default. The interest portion of mortgage payments can be deducted from taxes. New mortgages face a loan-to-value constraint (shown below in (6)) that is applied at origination only, so that borrowers to do not have to delever if they violate the constraint later on.

Borrower Reoptimization Non-defaulting borrowers can choose at any time to refinance their housing and debt holdings. If a refinancing borrower previously held a mortgage, she must first prepay the principal balance on the existing loan before taking on a new loan. The transaction cost of obtaining a new loan is proportional to the balance on the new loan M_t^*, given by $\kappa_{i,t} M_t^*$, where $\kappa_{i,t}$ is drawn i.i.d. across borrowers and time from a distribution with c.d.f. Γ_κ. Since these costs likely stand in for non-monetary fric-

tions such as inertia, these costs are rebated to borrowers, and do not impose an aggregate resource cost. We assume that borrowers must commit in advance to a refinancing policy that can depend in an unrestricted way on $\kappa_{i,t}$ and all aggregate variables, but cannot depend on the borrower's individual loan characteristics. This setup keeps the problem tractable by removing the distribution of loans as a state variable while maintaining the realistic feature that a fraction of borrowers choose to refinance in each period and that this fraction responds endogenously to the state of the economy.

We verify that the optimal plan for the borrower is to refinance whenever $\kappa_{i,t} \leq \bar{\kappa}_t$, where $\bar{\kappa}_t$ is a threshold cost that makes the borrower indifferent between refinancing and not refinancing. The fraction of non-defaulting borrowers who choose to refinance is therefore

$$Z_{R,t} = \Gamma_\kappa(\bar{\kappa}_t).$$

Once the threshold cost (equivalently, refinancing rate) is known, the total transaction cost per unit of debt is defined by

$$\Psi_t(Z_{R,t}) = \int^{\bar{\kappa}_t} \kappa \, d\Gamma_\kappa = \int^{\Gamma_\kappa^{-1}(Z_{R,t})} \kappa \, d\Gamma_\kappa.$$

Borrower Default and Mortgage Indexation Before deciding whether or not to refinance a loan, borrowers decide whether to or not to default on the loan, in which case the housing collateral used to back the loan is seized by the intermediary. To allow for an aggregated model in which the default rate responds endogenously to macroeconomic conditions, we introduce shocks $\omega_{i,t}$ to the quality of borrowers' houses, drawn i.i.d. across borrowers and time from a distribution with c.d.f. $\Gamma_{\omega,t}$, with $\mathbb{E}_t(\omega_{i,t}) = 1$ and $\text{Var}_t(\omega_{i,t}) = \sigma_{\omega,t}^2$. The cross-sectional dispersion of the house quality shocks $\sigma_{\omega,t}$ follows a mean-reverting stochastic process

$$\log \sigma_{\omega,t} = (1 - \rho_u) \log \bar{\sigma}_\omega + \rho_u \log \sigma_{\omega,t-1} + \sigma_u \varepsilon_{u,t}, \qquad \varepsilon_{u,t} \sim N(0,1).$$

where the uncertainty shocks $\varepsilon_{u,t}$ represent a second source of aggregate risk in addition to the endowment shocks. Borrowers must commit to a default plan that can depend in

an unrestricted way on $\omega_{i,t}$ and the aggregate states, but not on a borrower's individual loan conditions.

In addition to the standard mortgage contracts defined above, we introduce Shared Appreciation Mortgages whose payments are indexed to house prices. To capture different types of risk sharing, we allow mortgage contracts to potentially insure households in two ways. First, mortgages can be indexed to aggregate house prices. Specifically, each period, the principal balance and promised payment on each existing loan is multiplied by

$$\zeta_{p,t} = \iota_p \left(\frac{p_t}{p_{t-1}} \right) + (1 - \iota_p) \tag{3}$$

where p_t is the aggregate house price. The special cases $\iota_p = 0$ and $\iota_p = 1$ correspond to the cases of no insurance and complete insurance against aggregate house price risk.

Second, mortgage contracts can be indexed to individual movements in house prices $\omega_{i,t}$. Specifically, each period, the principal balance and promised payment on a loan backed by a house that receives shock $\omega_{i,t}$ are multiplied by

$$\zeta_\omega(\omega) = \iota_\omega \omega + (1 - \iota_\omega).$$

We verify that the optimal plan for the borrower is to default whenever $\omega_{i,t} \leq \bar{\omega}_t$, where $\bar{\omega}_t$ is the threshold shock that makes the borrower indifferent between defaulting and not defaulting. Note that the level of the threshold generally depends on the aggregate state and the level of indexation. Given $\bar{\omega}_t$, the fraction of non-defaulting borrowers is

$$Z_{N,t} = 1 - \Gamma_{\omega,t}(\bar{\omega}_t).$$

Since non-defaulting borrowers are those who receive relatively good shocks, the share of borrower housing kept by non-defaulting households is

$$Z_{K,t} = \int_{\bar{\omega}_t} \omega \, d\Gamma_{\omega,t},$$

and the outstanding borrower debt by non-defaulting borrowers is

$$Z_{A,t} = \int_{\bar{\omega}_t} \zeta_\omega(\omega) \, d\Gamma_{\omega,t} = \iota_\omega \int_{\bar{\omega}_t} \omega d\Gamma_{\omega,t} + (1 - \iota_\omega)\Gamma_{\omega,t}(\bar{\omega}_t) = \iota_\omega Z_{K,t} + (1 - \iota_\omega)Z_{N,t}. \quad (4)$$

Since the model does not distinguish between shocks to local house prices and "basis risk" to an individual house, indexation to local house prices would correspond to partial indexation, $0 < \iota_\omega < 1$. Intuitively, with zero indexation to idiosyncratic shocks, defaulting is attractive for borrowers if the value of the houses lost in foreclosure (fraction $1 - Z_{K,t}$) is smaller than the value of debt shed in default (fraction $1 - Z_{A,t}$). Equation (4) shows that increasing indexation shrinks this difference and therefore makes defaulting less attractive for borrowers. It is easy to show that for the case of full indexation to idiosyncratic shocks, $\iota_\omega = 1$, one gets $Z_{N,t} = Z_{A,t} = Z_{K,t} = 1$, i.e. borrowers optimally do not default on any payments in that case.

2.3.2 REO Sector

The housing collateral backing defaulted loans is seized by the intermediary and rented out as REO ("real estate owned") housing to the borrower. Housing in this state incurs a larger maintenance cost designed to capture losses from foreclosure, $\nu^{REO} > \nu^K$. With probability S^{REO} per period, REO housing is sold back to borrowers as owner-occupied housing. The existing stock of REO housing is denoted by K_t^{REO}, and the value of a unit of REO-owned housing is denoted p_t^{REO}.

2.3.3 Deposit Technology

Deposits in the model take the form of risk-free one-period loans issued from the depositor to the intermediary, where the price of these loans is denoted q_t^f, implying the interest rate $1/q_t^f$. Intermediaries must satisfy a leverage constraint (defined below by (15)) stating that their promised deposit repayments must be collateralized by their existing loan portfolio and ownership of REO housing.

2.4 Borrower's Problem

Given this structure, the individual borrower's problem aggregates to that of a single representative borrower. The endogenous state variables are the promised payment A_t^B, the face value of principal M_t^B, and the stock of borrower-owned housing K_t^B. The borrower's control variables are nondurable consumption C_t^B, housing service consumption H_t^B, the amount of housing K_t^* and new loans M_t^* taken on by refinancers, and the fraction of housholds that refinance $Z_{R,t}$ and default $1 - Z_{N,t}$, to maximize (1) subject to the budget constraint

$$
C_t^B = \underbrace{(1-\tau_t)Y_t^B}_{\text{income}} + \underbrace{Z_{R,t}\left(Z_{N,t}M_t^* - \delta Z_{A,t}M_t^B\right)}_{\text{net new borrowing}} - \underbrace{(1-\delta)Z_{A,t}M_t^B}_{\text{principal payment}} - \underbrace{(1-\tau)Z_{A,t}A_t^B}_{\text{interest payment}}
$$
$$
- \underbrace{p_t\left[Z_{R,t}Z_{N,t}K_t^* + \left(\nu^K - Z_{R,t}\right)Z_{K,t}K_t^B\right]}_{\text{owned housing}} - \underbrace{\rho_t\left(H_t^B - K_t^B\right)}_{\text{rental housing}}
$$
$$
- \underbrace{\left(\Psi(Z_{R,t}) - \bar\Psi_t\right)Z_{N,t}M_t^*}_{\text{net transaction costs}} \tag{5}
$$

the loan-to-value constraint

$$
M_t^* \le \phi^K p_t K_t^* \tag{6}
$$

and the laws of motion

$$
M_{t+1}^B = \pi_{t+1}^{-1}\zeta_{p,t+1}\left[Z_{R,t}Z_{N,t}M_t^* + \delta(1-Z_{R,t})Z_{A,t}M_t^B\right] \tag{7}
$$
$$
A_{t+1}^B = \pi_{t+1}^{-1}\zeta_{p,t+1}\left[Z_{R,t}Z_{N,t}r_t^*M_t^* + \delta(1-Z_{R,t})Z_{A,t}A_t^B\right] \tag{8}
$$
$$
K_{t+1}^B = Z_{R,t}Z_{N,t}K_t^* + (1-Z_{R,t})Z_{K,t}K_t^B \tag{9}
$$

where r_t^* is the current interest rate on mortgages, τ is the income tax rate, ν^K is the depreciation rate on owner-occupied housing, ρ_t is the rental rate for housing services, and $\bar\Psi_t$ is a subsidy that rebates transaction costs back to borrowers.

2.5 Intermediary's Problem

Intermediaries in the model lend to borrowers, invest in and rent REO housing, issue deposits, and trade in the secondary market for mortgage debt. The continuum of in-

termediaries can also be aggregated to a representative intermediary. Although each mortgage with a different interest rate has a different secondary market price, we show in the appendix that any portfolio of loans can be replicated using only two instruments: an interest-only (IO) strip, and a principal-only (PO) strip. In equilibrium, beginning-of-holdings of the IO and PO strips will correspond to the total promised interest payments and principal balances that are the state variables of the borrower's problem, and will therefore be denoted A_t^I and M_t^I, respectively. Denote new lending by intermediaries in terms of face value by L_t^*. Intermediaries can immediately sell these new loans to other intermediaries in the secondary market. Then the end-of-period *supply* of PO and IO strips is given by

$$\hat{M}_t^I = L_t^* + \delta(1 - Z_{R,t})Z_{A,t}M_t^I$$

$$\hat{A}_t^I = r_t^* L_t^* + \delta(1 - Z_{R,t})Z_{A,t}A_t^I.$$

These claims trade at market prices q_t^M and q_t^A, respectively. Denote intermediary *demand* for PO and IO strips, and therefore the end-of-period holdings of these claims, by $\tilde{M}_t^I$ and $\tilde{A}_t^I$, respectively. The laws of motion for these variables depend on the level of indexation. Since they are nominal contracts, they also need to be adjusted for inflation:

$$M_{t+1}^I = \pi_{t+1}^{-1}\zeta_{p,t+1}\tilde{M}_t^I \tag{10}$$

$$A_{t+1}^I = \pi_{t+1}^{-1}\zeta_{p,t+1}\tilde{A}_t^I. \tag{11}$$

The endogenous state variables for the intermediary are liquid wealth W_t^I, the beginning-of-period holdings of PO and IO strips, M_t^I and A_t^I, and the stock of REO housing K_t^{REO}. The law of motion of the REO housing stock is

$$K_{t+1}^{REO} = (1 - S^{REO})K_t^{REO} + I_t^{REO},$$

where I_t^{REO} are new REO purchases by REO subsidiaries that are fully owned and operated by intermediaries. Per unit of face value outstanding, the recovery value of housing

from foreclosed borrowers is

$$X_t = \frac{(1 - Z_{K,t})K_t^B(p_t^{REO} - \nu^{REO}p_t)}{M_t^B}. \tag{12}$$

Note that X_t is taken as fixed by each individual intermediary, who does not internalize the effect of their debt issuance on the overall recovery rate. Then beginning-of-period liquid wealth is given by

$$W_t^I = \underbrace{\left[X_t + Z_{A,t}\Big((1-\delta) + \delta Z_{R,t}\Big)\right]M_t^I + Z_{A,t}A_t^I}_{\text{payments on existing debt}}$$

$$+ \underbrace{\delta(1 - Z_{R,t})Z_{A,t}\left(q_t^A A_t^I + q_t^M M_t^I\right)}_{\text{sales of IO and PO strips}} - \underbrace{\pi_t^{-1}B_t^I}_{\text{deposit redemptions}}. \tag{13}$$

The intermediary chooses nondurable consumption C_t^I, new lending L_t^*, end-of-period holdings of PO and IO strips, $\tilde{M}_t^I$ and $\tilde{A}_t^I$, new deposits B_{t+1}^I, and new purchases of REO housing I_t^{REO} to maximize (1) subject to the budget constraint

$$C_t^I = W_t^I + \underbrace{(1 - \tau)Y_t^I}_{\text{income}} + \underbrace{q_t^f B_{t+1}^I}_{\text{new deposits}} - \underbrace{(1 - r_t^* q_t^A - q_t^M)L_t^*}_{\text{net new debt}} - \underbrace{q_t^A \tilde{A}_t^I}_{\text{IO strips}} - \underbrace{q_t^M \tilde{M}_t^I}_{\text{PO strips}}$$

$$+ \underbrace{\left[\rho_t + \left(S^{REO} - \nu^{REO}\right)p_t\right]K_t^{REO}}_{\text{REO income}} - \underbrace{p_t^{REO}I_t^{REO}}_{\text{REO investment}} - \underbrace{\nu^K p_t H_t^I}_{\text{own housing maint.}}, \tag{14}$$

and the leverage constraint

$$B_{t+1}^I \le \phi^I \left(q_t^A \tilde{A}_t^I + q_t^M \tilde{M}_t^I\right) + \phi^{REO} p_t^{REO}\left[(1 - S^{REO})K_t^{REO} + I_t^{REO}\right]. \tag{15}$$

2.6 Depositor's Problem

The depositor's problem can also be aggregated, so that the representative depositor chooses nondurable consumption C_t^D and deposits B_t^D to maximize (1) subject to the budget constraint

$$C_t^D \le \underbrace{(1 - \tau)Y_t^D}_{\text{disp. income}} - \underbrace{\left(q_t^f B_{t+1}^D - \pi_t^{-1}B_t^D\right)}_{\text{net deposit iss.}} - \underbrace{\nu^K p_t H_t^D}_{\text{own housing maint.}}. \tag{16}$$

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and a restriction that deposits must be positive: $B_t^D \geq 0$.

2.7 Equilibrium

Given a sequence of endowment and uncertainty shock realizations $(Y_t, \sigma_{\omega,t})$, a competitive equilibrium in this model is defined as a sequence of borrower allocations $(M_t^B, A_t^B, K_t^B, C_t^B, H_t^B, K_t^*, M_t^*, Z_{R,t}, \bar{\omega}_t)$, depositor allocations (C_t^D, B_t^D), intermediary allocations $(M_t^I, A_t^I, K_t^{REO}, W_t^I, C_t^I, L_t^*, I_t^{REO}, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I)$, and prices $(r_t^*, q_t^A, q_t^M, q_t^f, p_t, p_t^{REO}, \rho_t)$ such that borrowers, intermediaries, and depositors optimize, and all markets clear[7]

$$
\begin{aligned}
\text{New mortgages:} \quad & Z_{R,t} Z_{N,t} M_t^* = L_t^* \\
\text{PO strips:} \quad & \tilde{M}_t^I = \hat{M}_t^I \\
\text{IO strips:} \quad & \tilde{A}_t^I = \hat{A}_t^I \\
\text{Deposits:} \quad & B_{t+1}^I = B_{t+1}^D \\
\text{Housing Purchases:} \quad & Z_{R,t} Z_{N,t} K_t^* = S^{REO} K_t^{REO} + Z_{R,t} Z_{K,t} K_t^B \\
\text{Housing Services:} \quad & H_t^B = K_t^B + K_t^{REO} = \bar{K}^B \\
\text{REO Purchases:} \quad & I_t^{REO} = (1 - Z_{K,t}) K_t^B \\
\text{Resources:} \quad & Y_t = C_t^B + C_t^I + C_t^D + G_t \\
& \quad + \underbrace{\nu^K p_t (Z_{K,t} K_t^B + \bar{K}^I + \bar{K}^D) + \nu^{REO} p_t \left[K_t^{REO} + (1 - Z_{K,t}) K_t^B \right]}_{\text{housing expenditure}}
\end{aligned}
$$

The resource constraint states that the endowment income of the economy, Y_t, is either spent on nondurable consumption, government consumption, or housing consumption. Housing consumption consists of maintenance payments for houses owned by borrowers, $Z_{K,t} K_t^B$, houses owned by REO firms, K_t^{REO}, and houses of foreclosed borrowers that are bought by REO firms $(1 - Z_{K,t}) K_t^B$. Government consumption consists of income taxes net of the mortgage interest deduction

$$
G_t = \tau (Y_t - Z_{A,t} A_t^B).
$$

---

[7]Intermediaries and depositors consume their fixed endowment of housing services each period, $H_t^j = \bar{K}^j$, for $j = I, D$.

3   Model Solution

3.1   Borrower Optimality

The optimality condition for new debt is

$$1 = \Omega_{M,t} + r_t^* \Omega_{A,t} + \lambda_t^{LTV}$$

which relates the benefit of taking on additional debt — \$1 today — against the continuation cost of holding debt in the future, plus the shadow cost of tightening the LTV constraint. The optimality condition for housing services consumption is

$$\rho_t = u_{c,t}^{-1} u_{h,t}$$

which simply sets the rental rate to be the marginal rate of substitution between housing services and nondurables.

The borrower's optimality condition for new housing capital is

$$p_t = \frac{\mathbb{E}_t\left\{ \Lambda_{t+1}^B \left[\rho_{t+1} + Z_{K,t+1} p_{t+1} \left(1 - \nu^K - (1 - Z_{R,t+1}) \lambda_{t+1}^{LTV} \phi^K \right) \right] \right\}}{1 - \lambda_t^{LTV} \phi^K}.$$

The numerator represents the present value of holding an extra unit of housing next period: the rental service flow, plus the continuation value of the housing if the borrower chooses not to default, net of the maintenance cost. The continuation value needs to be adjusted by $(1 - Z_{R,t+1})\lambda_{t+1}^{LTV} \phi^K$ because if the borrower does not choose to refinance, which occurs with probability $Z_{R,t+1}$, then she does not use the unit of housing to collateralize a new loan, and therefore does not receive the collateral benefit.

The optimality condition for refinancing rate is

$$Z_{R,t} = \Gamma \left\{ \underbrace{(1 - \Omega_{M,t} - \bar{r}_t \Omega_{A,t}) \left(1 - \frac{\delta Z_{A,t} M_t}{Z_{N,t} M_t^*} \right)}_{\text{equity extraction incentive}} - \underbrace{\Omega_{A,t} \left(r_t^* - \bar{r}_t \right)}_{\text{interest rate incentive}} \right.$$

$$-\underbrace{p_t \mathcal{C}_t \left(\frac{Z_{N,t} K_t^* - Z_{K,t} K_t^B}{Z_{N,t} M_t^*} \right)}_{\text{collateral expense}} \Bigg\}$$

where $\bar{r}_t = A_t^B / M_t^B$ is the average interest rate on existing debt. The "equity extraction incentive" term represents the net gain from obtaining additional debt at the *existing* interest rate, while "interest rate incentive" term represents the gain from moving from the existing to new interest rate. The final "collateral expense" term occurs because housing trades at a premium relative to the present value of its housing service flows due to its use as collateral, so that refinancing is less desirable when taking on new debt would require paying a high cost for collateral.

The optimality condition for default rate is

$$\zeta_\omega(\bar{\omega}_t) \left[\underbrace{\Big(\delta Z_{R,t} + (1-\delta) \Big) M_t + (1-\tau) A_t}_{\text{current payment}} + \underbrace{\delta(1 - Z_{R,t}) \left(\Omega_{M,t} M_t + \Omega_{A,t} A_t \right)}_{\text{continuation cost of debt}} \right]$$

$$= \underbrace{\left(1 - \nu^K - (1 - Z_{R,t}) \lambda_t^{LTV} \phi^K \right) p_t \bar{\omega}_t K_t^B}_{\text{continuation value of housing}}$$

This expression relates the benefit of defaulting on debt — eliminating both the current payment and continuation cost, after indexation — against the cost of losing a unit of housing a marginal unit of housing (at the threshold idiosyncratic shock level $\bar{\omega}_t$), as well as the cost of not being able to use that lost unit of housing to finance new borrowing under a refinancing.

The marginal continuation costs are defined by the fixed point expressions

$$\Omega_{M,t} = \mathbb{E}_t \left\{ \Lambda_{t+1}^B \pi_{t+1}^{-1} \zeta_{p,t+1} Z_{A,t+1} \Big[(1-\delta) + \delta Z_{R,t+1} + \delta(1 - Z_{R,t+1}) \Omega_{M,t+1} \Big] \right\}$$

$$\Omega_{A,t} = \mathbb{E}_t \left\{ \Lambda_{t+1}^B \pi_{t+1}^{-1} \zeta_{p,t+1} Z_{A,t+1} \Big[(1-\tau) + \delta(1 - Z_{R,t+1}) \Omega_{A,t+1} \Big] \right\}$$

where an extra unit of principal requires a payment of $(1 - \delta)$ in the case of non-default, plus payment of the face value of prepaid debt and the continuation cost of non-prepaid debt, while an extra promised payment requires a tax-deductable payment on non-defaulted debt plus the continuation cost if the debt is not prepaid.

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3.2 Intermediary Optimality

The optimality condition for new debt is

$$1 = q_t^M + r_t^* q_t^A$$

which balances the cost of issuing new debt — $1 today — against the value of the loan obtained — 1 unit of PO strip plus r_t^* units of the IO strip. The optimality condition for deposits is

$$q_t^f = \mathbb{E}_t\left[\Lambda_{t+1}^I \pi_{t+1}^{-1}\right] + \lambda_t^I$$

where λ_t^I is the multiplier on the intermediary's leverage constraint. This is simply the intermediary's nominal Euler equation plus a wedge that is nonzero when the leverage constraint is binding.

The optimality condition for REO housing is

$$p_t^{REO} = \frac{\mathbb{E}_t\left\{\Lambda_{t+1}^I\left[\rho_{t+1} - \nu^{REO}p_{t+1} + S^{REO}p_{t+1} + (1 - S^{REO})p_{t+1}^{REO}\right]\right\}}{1 - \phi^{I,REO}\lambda_t^I}.$$

The numerator represents the present discounted value of holding a unit of REO housing next period. This term is in turn made up of the rent charged to borrowers, the maintenance cost, and the value of the housing next period, both the portion sold back to the borrowers, and the portion kept in the REO state. The denominator represents a collateral premium for REO housing, which can be used to collateralize deposits.

The optimality conditions for IO and PO strip holdings are

$$q_t^A = \frac{\mathbb{E}_t\left\{\Lambda_{t+1}^I \pi_{t+1}^{-1}\zeta_{p,t+1}\left[Z_{A,t+1}\left(1 + \delta(1 - Z_{R,t+1})q_{A,t+1}^A\right)\right]\right\}}{(1 - \phi^A \lambda_t^I)}$$

$$q_t^M = \frac{\mathbb{E}_t\left\{\Lambda_{t+1}^I \pi_{t+1}^{-1}\zeta_{p,t+1}\left[X_{t+1} + Z_{A,t+1}\left((1 - \delta) + \delta Z_{R,t+1} + \delta(1 - Z_{R,t+1})q_{t+1}^M\right)\right]\right\}}{(1 - \phi^M \lambda_t^I)}.$$

Both securities issue cash flows that are nominal (discounted by inflation) and indexed to house prices (discounted by $\zeta_{p,t+1}$). Both securities can also be used to collateralize

deposits, leading to the collateral premia in the denominators. The IO strip's next-period payoff is equal to \$1 for loans that do not default, with a continuation value of q_{t+1}^A for loans that do not prepay or mature. The PO strip's next-period payoff is the recovery value for defaulting debt X_{t+1} plus the payoff from loans that do not default: the principal payment $1 - \delta$, plus the face value of prepaying debt, plus the continuation value q_{t+1}^M for loans that do not mature or prepay.

3.3 Depositor Optimality

The depositor's sole optimality condition for deposits

$$q_t^f = \mathbb{E}_t\left[\Lambda_{t+1}^D \pi_{t+1}^{-1}\right]$$

ensures that the depositor's nominal Euler equation is at an interior solution.

4 Calibration

This section describes the calibration procedure for key variables, and presents the full set of parameter values in Table 1. For the idiosyncratic housing quality shock distribution, we parameterize $\Gamma_{\omega,t}$ as a log-normal distribution, so that

$$Z_{N,t} = \int_{\bar{\omega}}^{\infty} dF(\omega) = 1 - \Phi\left(\frac{\log \bar{\omega}_t + \sigma_{\omega,t}^2/2}{\sigma_{\omega,t}}\right)$$
$$Z_{K,t} = \int_{\bar{\omega}}^{\infty} \omega dF(\omega) = \Phi\left(\frac{\sigma_{\omega,t}^2/2 - \log \bar{\omega}_t}{\sigma_{\omega,t}}\right)$$

where Φ denotes the standard normal distribution function. The average variance $\bar{\sigma}_\omega$ is calibrated to match a 2% average annualized default rate.

For the prepayment cost distribution, we assume a mixture distribution, so that with probability 3/4, the borrower draws an infinite prepayment cost, while with probability 1/4, the borrower draws from a logistic distribution with mean μ_κ and scale s_κ. This parameterization ensures that the approximate *annualized* prepayment rate, $cpr_t = 4Z_{R,t}$

Table 1: Parameter Values: Baseline Calibration (Quarterly)

Parameter	Name	Value	Internal	Target/Source
Demographics and Preferences				
Frac. borrowers	χ_B	0.470	N	
Frac. depositors	χ_D	0.510	N	
Borr. inc. share	s_B	0.380	N	
Saver inc. share	s_I	0.520	N	
Borr. discount factor	β_B	0.969	N	
Saver discount factor	β_I	0.992	N	2% real rate
Saver discount factor	β_I	0.995	N	2% real rate
EIS	ψ	1.000	N	
Housing preference	ξ	0.185	Y	1998 SCF
Housing and Mortgages				
Housing stock	$\log \bar{H}^B$	2.173	Y	$p_{ss} = 1$
Saver demand	$\log \bar{H}^I$	0.000	N	
Mortgage decay	δ	0.992	N	30-year duration
Tax rate	τ	0.198	N	
LTV Limit	ϕ^K	0.800	N	
Issuance cost mean	μ_κ	0.165	Y	$Z_{R,ss} = 4.5\%$
Issuance cost scale	s_κ	0.031	N	
Maint. cost (owner)	ν^K	0.005	N	
Maint. cost (REO)	ν^{REO}	0.033	Y	$p_{ss}^{REO}/p_{ss} = 0.88$
REO sale rate	S^{REO}	0.167	N	6Q to sale
Avg. St. Dev.	$\log \bar{\sigma}_\omega$	0.249	N	
Technology				
Inflation rate	$\bar{\pi}$	1.008	N	3% inflation
TFP persistence	ρ_{TFP}	0.964	N	
TFP st. dev.	σ_{TFP}	0.008	N	
Uncertainty persistence	ρ_u	0.900	N	
Uncertainty st. dev.	σ_u	0.050	N	

has a logistic form

$$cpr_t = \frac{1}{1 + \exp\left(\frac{\bar{\kappa}_t - \mu_\kappa}{s_\kappa}\right)}$$

that has shown to fit well in prepayment regressions. We then calibrate μ_κ so that the steady state prepayment rate is equal to 4.5% (quarterly), consistent with the average prepayment rate on mortgages in Fannie Mae 30-Year Fixed Rate MBS pools (code: FNM30) over the period 1994-2015 (source: eMBS).

We choose the housing preference parameter ξ to match a ratio of housing wealth to income for borrowers of 8.89, consistent with the same ratio for "borrowers" — households with a house and mortgage but less than two months' income in liquid assets — in the 1998 SCF. We then calibrate the borrower housing stock so that the price of housing is equal to unity in the steady state. We calibrate the maintenance cost in the REO state, ν^{REO}, so that the ratio of the value of REO housing to owner-occupied housing is 88%.

Results

Our goal is to understand how indexation of mortgage balance and repayments to aggregate and local house price risk affects equilibrium prices and quantities. To this end, we solve three different versions of the model with different levels of indexation: (i) no indexation corresponding to $\iota_p = \iota_\omega = 0$, which is the benchmark, (ii) only aggregate indexation, such that $\iota_p = 1$ and $\iota_\omega = 0$, and (iii) aggregate and local indexation, which we parameterize as $\iota_p = 1$ and $\iota_\omega = 0.5$. We choose $\iota_\omega < 1$ to capture the idea that the ω-shocks represent both local and truly house-idiosyncratic variation, with the latter not being included in the indexation.

Effects of indexation   To gain a basic understanding of the first-order effects of introducing indexation, we conduct a long simulation for each of the three model economies. Table 2 shows first and second moments of key prices and quantities computed using the simulated time series.

The table shows that aggregate indexation has no effect on the steady state of the model. This is not surprising, since at the steady state the aggregate house price is

Variable	Benchmark		Agg. Indexation		Full Indexation	
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
$\log p_t$	0.000	0.030	0.000	0.030	0.012	0.030
$\log R_{m,t}$ (Ann.)	0.056	0.002	0.056	0.002	0.051	0.000
$\log M_t$	0.622	0.033	0.622	0.035	0.660	0.029
$Z_{N,t}$ (Ann.)	0.020	0.015	0.020	0.015	0.000	0.000
$\log C_t^B$	-1.274	0.039	-1.274	0.031	-1.271	0.031
$\log C_t^I$	-2.580	0.036	-2.580	0.052	-2.582	0.050
$Z_{R,t}$ (Ann.)	0.152	0.008	0.152	0.003	0.144	0.001
$\log C_t$	-0.277	0.031	-0.277	0.031	-0.276	0.031
$\log A_t$	-3.632	0.022	-3.632	0.033	-3.699	0.031
K_t^{REO}/H^B	0.014	0.008	0.014	0.008	0.000	0.000

constant and therefore indexation to changes does not affect variable means in a first-order approximation of the model's dynamics. However, we can see that the aggregate house price becomes more volatile with indexation, while mortgage and default rates become less volatile. Further, as one would expect, aggregate mortgage principal (M_t) and effective mortgage debt owed (A_t) become substantially more volatile as they are marked to house price changes each period.

Furthermore, in the benchmark model borrower consumption volatility is almost twice as large as lender consumption volatility. When mortgage debt is indexed to aggregate house price changes, borrower and lender consumption volatility are almost equalized, while aggregate consumption volatility remains unchanged.

When mortgage debt is also partially indexed to idiosyncratic house prices as in the third panel of table 2, the default rate drops to almost zero. As a result, mortgage rates decline by 2 percentage points and also become less volatile. House prices and total mortgage principal increase, while the present value of future repayments declines. The refinancing rate falls compared to the benchmark model, since borrowers in the benchmark model choose to refinance more often to replace their collateral lost to default.

Dynamic response to TFP and uncertainty shocks Our model features two types of aggregate risk: endowment (TFP) shocks and shocks to the cross-sectional dispersion of house values ($\sigma_{\omega,t}$). In the following, we analyze the model's dynamic responses to

understand the interaction of the different types of shocks with indexation.

Figure 1 compares the response to a TFP shock in the benchmark economy and the one with aggregate indexation. The responses confirm the insights gained from studying the volatilities in table 2. The response of default rates to TFP shocks is greatly muted. Mortgage rates, which reflect default risk, also react less strongly in the indexed economy. We can further clearly see the stronger response of mortgage debt. In the benchmark model, debt only rises slowly in response to a positive shock as borrowers take advantage of lower rates through refis. In the indexation economy, mortgage debt jumps up on impact to reflect the rise in house prices. The bottom two panels clearly show the reallocation of house price risk between borrowers and savers. In the benchmark economy, borrower consumption responds more strongly as borrower wealth loads more directly on aggregate house prices. In the economy with indexation, the gains in wealth and consumption are distributed more equally across borrowers and lenders.

Figure 2 shows the response of the same variables to a rise in σ_ω. The first-order effect is an increase in the default rate. The overall response of both types of economies to the shock is similar. Indexation to aggregate prices does not reduce defaults caused by greater idiosyncratic risk. Even though the response of prices and mortgage rates is similar, the drop in the aggregate house price reduces mortgage debt in the economy with indexation and reallocates house price risk from borrowers to savers.

Figures 3 and 4 compare impulse responses to both types of shocks for the model with aggregate indexation to the model with full indexation. The main take-away from figure 3 is that aggregate indexation already eliminates most of the sensitivity of defaults to TFP shocks. Going to full indexation reduces the response to almost zero. There is no difference in the response of mortgage debt and consumption between the two economies. However, as can be seen in figure 4, adding full indexation has a large effect on the responses to uncertainty shocks. Recall that these shocks do not affect the mean of borrower house values, but only increase their dispersion. Without mortgage debt indexed to these shocks, a rise in the dispersion causes borrowers to optimally default on the mortgages secured by the houses that received the worst shocks. With full indexation, this default option loses its value, the default rate drops to zero, and the shocks to σ_ω
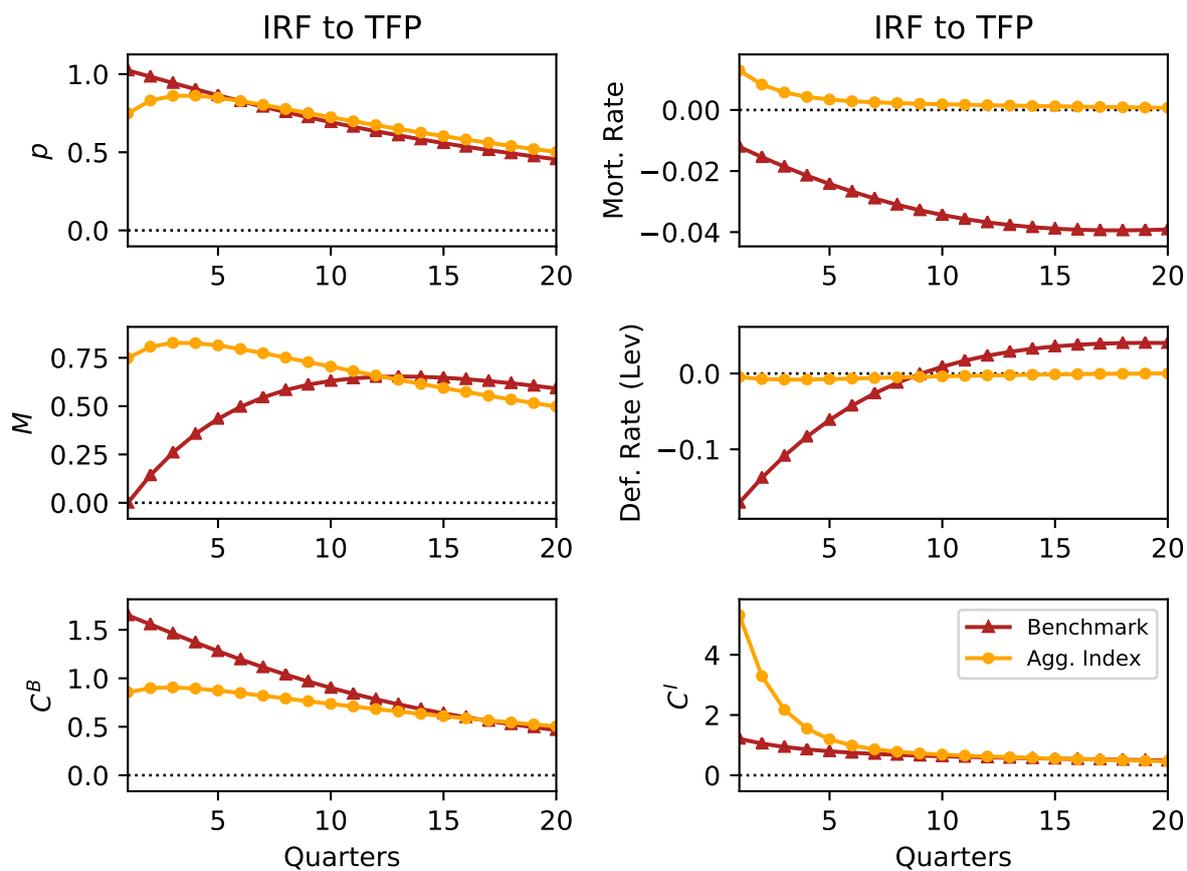
Figure 1: IRF to 1% Shock to TFP: Benchmark vs. Aggregate Indexation Economy

Note: A value of 1 represents a 1% increase relative to steady state, except for "Def. Rate" which is measured in percentage points at an annualized rate.
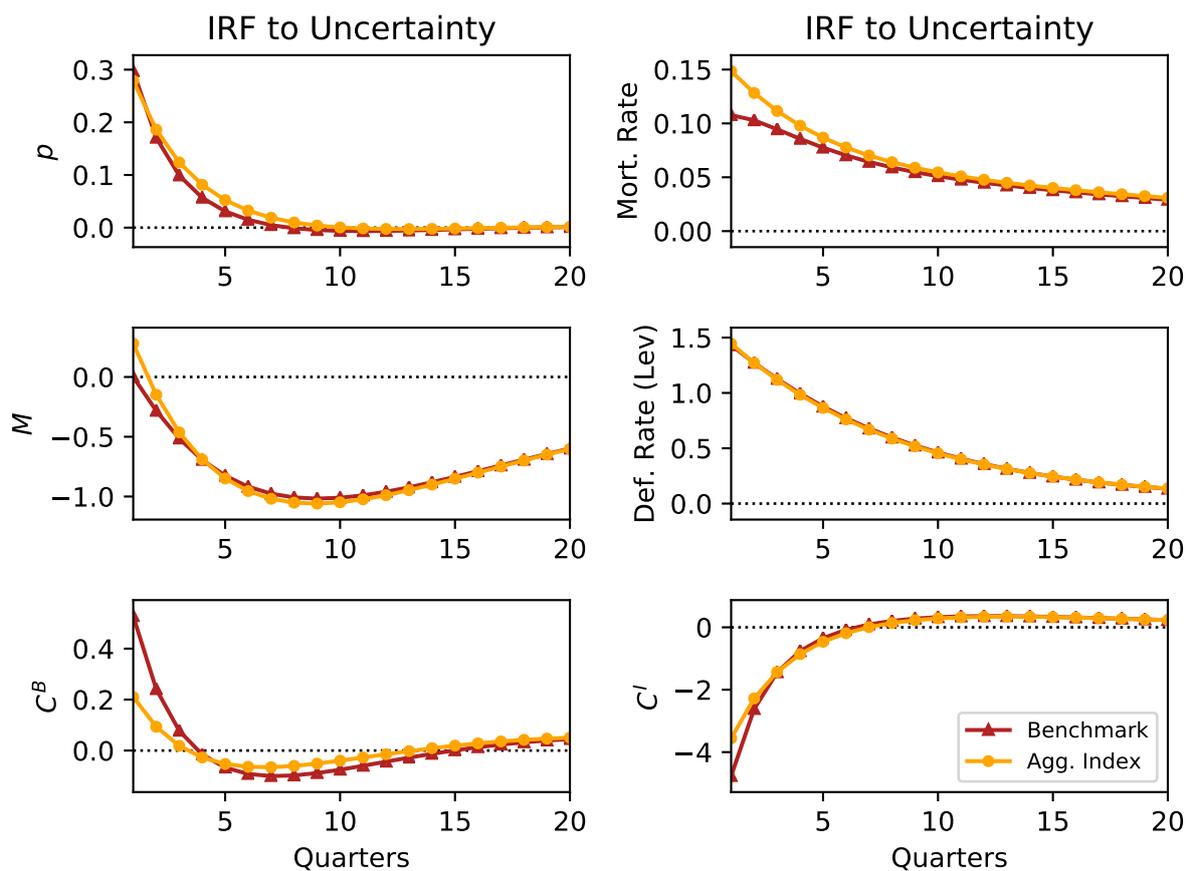
Figure 2: IRF to 10% Shock to Uncertainty: Benchmark vs. Aggregate Indexation Economy

Note: A value of 1 represents a 1% increase relative to steady state, except for "Def. Rate", which is measured in percentage points at an annualized rate.
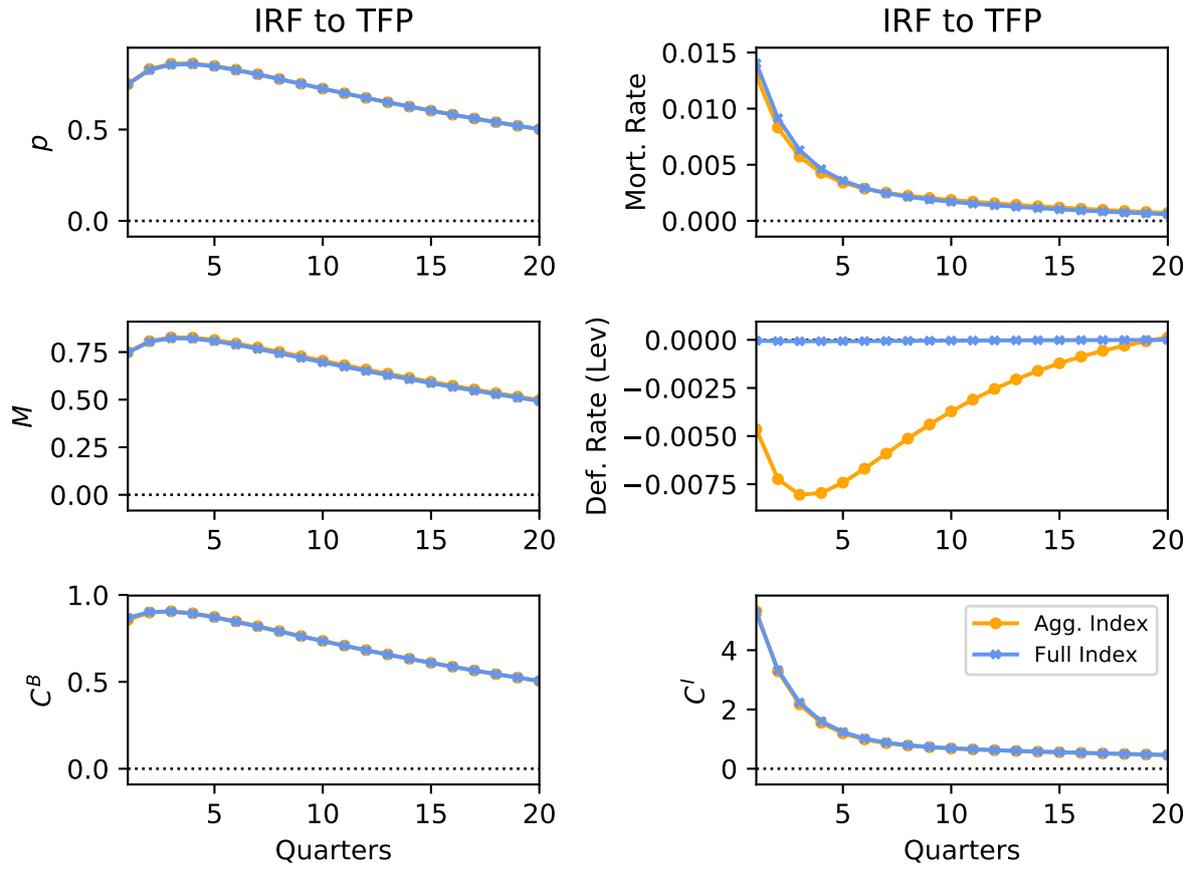
Figure 3: IRF to 1% Shock to TFP: Aggregate Indexation vs. Full Indexation Economy

Note: A value of 1 represents a 1% increase relative to steady state, except for "Def. Rate", which is measured in percentage points at an annualized rate.
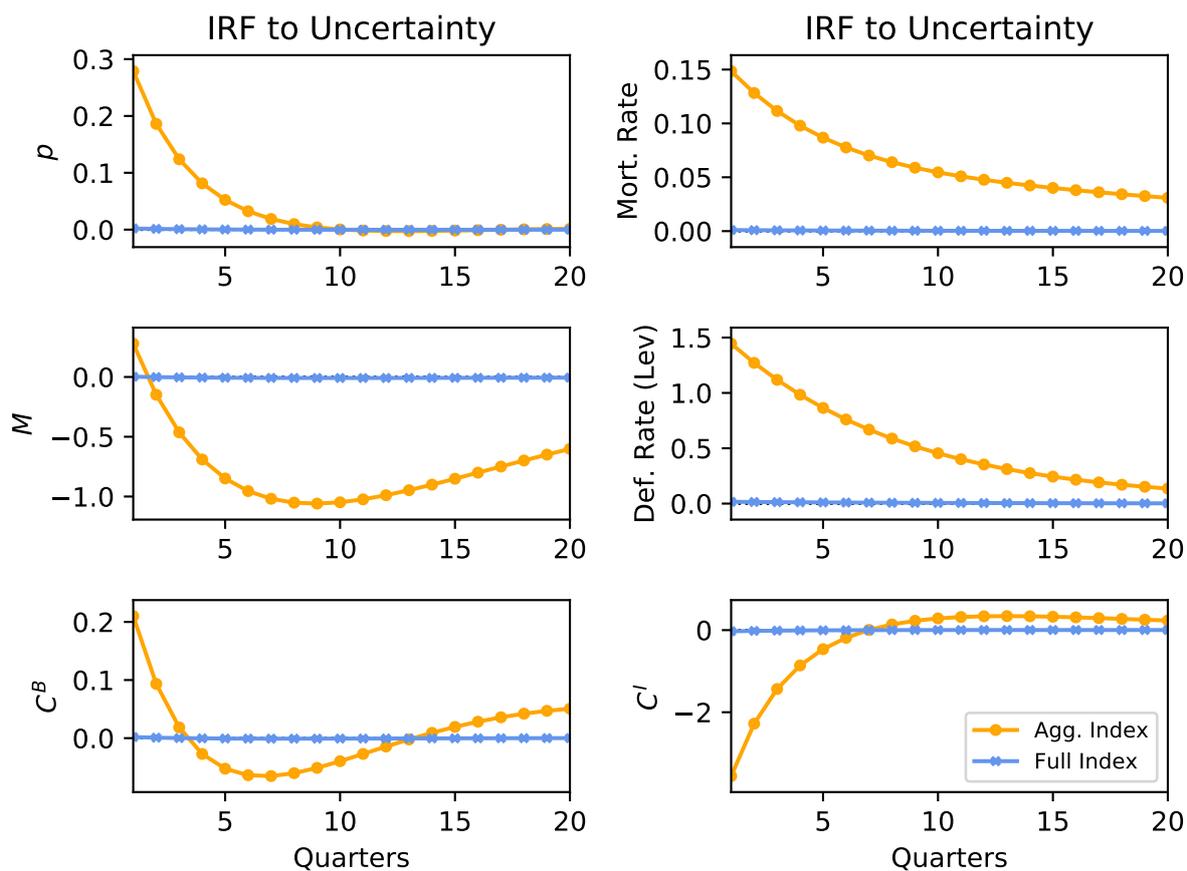
become irrelevant for the model's dynamics.

Figure 4: IRF to 10% Shock to Uncertainty: Aggregate Indexation vs. Full Indexation Economy

Note: A value of 1 represents a 1% increase relative to steady state, except for "Def. Rate", which is measured in percentage points at an annualized rate.

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A Appendix

A.1 Aggregation of Intermediary Problem

Before aggregating across loans, we must treat the distribution over $m_t(r)$, the start-of-period balance of a loan with interest rate r, as a state variable. In addition, the intermediary can freely choose her end-of-period holdings of these loans $\tilde{m}_t(r)$ by trading in the secondary market at price $q^m(r)$. In this case, the intermediary's problem is to choose nondurable consumption C_t^I, new debt issuance L_t^*, new deposits B_{t+1}^I, new REO investment I_t^{REO}, and end-of-period loan holdings $\tilde{m}_t(r)$ to maximize (1) subject to the budget constraint

$$
C_t^I = \underbrace{(1-\tau)Y_t^I}_{\text{income}} + \underbrace{\int \left[X_t + Z_{A,t}\Big(r + (1-\delta) + \delta Z_{R,t}\Big) \right] m_t(r)\, dr}_{\text{payments on existing debt}} - \underbrace{(1 - q_t^m(r_t^*))L_t^*}_{\text{net new debt}}
$$
$$
+ \underbrace{q_t^f B_{t+1}^I - \pi_t^{-1} B_t^I}_{\text{net deposits}} - \underbrace{\int q_t^m(r) \Big[\tilde{m}_t(r) - \delta(1 - Z_{R,t}) Z_{A,t} m_t(r) \Big] dr}_{\text{secondary market trades}} \tag{17}
$$
$$
+ \underbrace{\left[\rho_t + \big(S^{REO} - \nu^{REO} \big) p_t \right] K_t^{REO}}_{\text{REO income}} - \underbrace{p_t^{REO} \Big[I_t^{REO} - X_t A_t^I \Big]}_{\text{REO investment}}
$$

and the leverage constraint

$$
q_t^f B_t^* \leq \phi^M \int q_t^m(r) \tilde{m}_t(r)\, dr + \phi^{REO} p_t^{REO} \tilde{K}_t^{REO}
$$

with the laws of motion

$$
m_{t+1}(r) = \pi_{t+1}^{-1} \zeta_{p,t+1} \tilde{m}_t(r)
$$
$$
K_{t+1}^{REO} = (1 - S^{REO}) K_t^{REO} + (1 - Z_{K,t}) K_t^B
$$

and where the recovery rate X_t is defined as before. From the optimality condition for end-of-period holdings for loans with a given interest rate $\tilde{m}_t(r)$, we obtain

$$
q_t^m(r) = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^I \pi_{t+1}^{-1} \zeta_{p,t+1} \left[X_{t+1} + Z_{A,t+1}\Big(r + (1-\delta) + \delta Z_{R,t+1} + \delta(1 - Z_{R,t+1}) q_{t+1}^m(r) \Big) \right] \right\}}{1 - \lambda_t^I \phi^M}
$$

where λ_t^I is the multiplier on the intermediary's leverage constraint. To obtain aggregation, we can split $q_t(r)$ into an interest-only strip with value q_t^M and a principal-only strip with value q_t^A, so that

$$q_t^m(r) = rq_t^A + q_t^M.$$

Substituting into the equilibrium condition for $q_t^m(r)$ verifies the conjecture and yields

$$q_t^A = \frac{\mathbb{E}_t\left\{\Lambda_{t+1}^I \Upsilon_{t+1}^M Z_{A,t+1}\left[1 + \delta(1 - Z_{R,t+1})q_{t+1}^A\right]\right\}}{1 - \lambda_t^I \phi^M}$$

$$q_t^M = \frac{\mathbb{E}_t\left\{\Lambda_{t+1}^I \Upsilon_{t+1}^M\left[X_{t+1} + Z_{A,t+1}\left((1 - \delta) + \delta Z_{R,t+1} + \delta(1 - Z_{R,t+1})q_{t+1}^M\right)\right]\right\}}{1 - \lambda_t^I \phi^M}.$$

Importantly, due to our assumption on the prepayment behavior of borrowers (ensuring a constant $Z_{R,t}$ across the r distribution), the prices q_t^A and q_t^M are independent of r. Substituting into the budget constraint, and applying the identities

$$M_t^I = \int m_t(r)\, dr$$

$$A_t^I = \int rm_t(r)\, dr$$

now yields the aggregated budget constraint (14) and leverage constraint (15).