A General Equilibrium Analysis of Check Float

James McAndrews and William Roberds

Federal Reserve Bank of Atlanta
Working Paper 97-4
October 1997

Abstract: Households and businesses in the United States prefer to use check payment over less costly, electronic means of payment. Earlier studies have focused on check "float," that is, the time lag between receipt and clearing, as a potential explanation for the continued popularity of checks. An underlying assumption of these studies is that check float operates as a pure transfer from payee to payor. We construct a simple general equilibrium model in which payments are made by check. In general equilibrium, check float does not act as a pure transfer. If float can be priced into market transactions, then it has no effect on equilibrium allocations. If float is not priced into market transactions, then it acts as a distorting tax. Our results are consistent with the view that float is a significant factor behind the continued popularity of check payment. Our results are also consistent with recent data that indicate that the average value of float (per check) is small.

JEL classification: E58, G21, G28

Key words: checks, float, payments

The authors thank Zsolt Becsi and Scott Freeman for helpful discussions leading up to this paper. They also thank John Bryant, Jerry Dwyer, Bob Eisenbeis, Marco Espinosa, Steve Russell and Bruce Smith for helpful comments. The views expressed here are those of the authors and not necessarily those of the Federal Reserve Banks of Atlanta or New York or the Federal Reserve System. Any remaining errors are the authors' responsibility.

Please address questions regarding content to James McAndrews, Research Department, Federal Reserve Bank of New York, 33 Liberty Street, New York, New York 10045-0001, 212/720-5000, jamie.mcandrews@ny.frb.org, and William Roberds, Research Department, Federal Reserve Bank of Atlanta, 104 Marietta Street, N.W., Atlanta, Georgia 30303-2713, 404/521-8970, 404/521-8956 (fax), william.roberds@atl.frb.org.

Questions regarding subscriptions to the Federal Reserve Bank of Atlanta working paper series should be addressed to the Public Affairs Department, Federal Reserve Bank of Atlanta, 104 Marietta Street, N.W., Atlanta, Georgia 30303-2713, 404/521-8020. The full text of this paper may be downloaded (in PDF format) from the Atlanta Fed's World-Wide Web site at http://www.frbatlanta.org/publica/work_papers/.

1. INTRODUCTION

Payment by check is by far the most popular form of noncash payment in the U.S. In 1995, 63 billion payments or roughly 80 percent of noncash payments were made by check. The value of all checks written in 1995 was $73.5 trillion, comprising about 87 percent of "small-value" non-cash payments.[1]

While popularity of check payment is beyond dispute, its efficiency is not. Wells (1996, 5) estimates that the average cost of a payment by check averages roughly $1.60 more than the cost of a payment made electronically via the Fed's automated clearinghouse (ACH) system ($2.90 per payment by check vs. $1.30 per payment by ACH in 1993 dollars). Yet despite this cost disparity and despite increased opportunities to use ACH and other electronic forms of payment, checks have remained the predominant form of noncash payment in the U.S. And by some measures, the popularity of check payment has actually increased in recent years.[2]

Does this continued use of an apparently inefficient means of payment constitute a market failure? According to an influential study by Humphrey and Berger (1990), the answer to this question is "yes." Humphrey and Berger identify check *float* as a potential cause of market failure. Float is defined as the time lag between the receipt of a check as payment and its clearing. Until the check clears, the writer of the check has access to the funds and can earn interest on these funds. *Ceteris paribus*, the presence of float leads to a transfer of interest income from the receiver of the check to the check writer. A sufficiently large transfer could lead to a preference for using checks over electronic

[1] Figures are from Bank for International Settlements (1996). "Small-value" payments exclude "large-value" payments made over the Fedwire and CHIPS networks.
[2] See Humphrey, Pulley, and Vesala (1996).

methods of payment which are not subject to float. Using 1987 data, Humphrey and Berger estimate that the average amount of the float transfer more than compensates for the cost advantage of ACH over checks.

Humphrey and Berger's characterization of check float is challenged by Wells (1996). Using 1993 data, Wells estimates that the average value of check float has fallen quite dramatically in recent years, i.e., from $1.04 per check in 1987 to $.09 per check in 1993. The decrease in the average float value has resulted from both reductions in check processing lags and in nominal short-term interest rates. Since the average value of float is small relative to the differential in average cost between payment by check and payment by ACH, Wells argues that check float cannot be a source of "market failure" in the market for payments.

A potential limitation of the analysis in this area has been the partial-equilibrium nature of its theoretical underpinnings. For example, both Humphrey and Berger (1990, 51) and Wells (1996, 4) characterize float as a pure transfer of income from the check receiver to check writer.[3] As noted above, this is true in a partial equilibrium sense. In general equilibrium, however, it stands to reason that beneficiaries of float will use their additional income to income to bid up prices of the goods they wish to purchase, potentially leading to distortions in the absence of complete markets.

Below, we present a model that allows for examination of the costs of float in a general equilibrium environment. The model is an adaptation of Freeman's (1996) model of banknote issue. While highly abstract, the model incorporates some relevant features of the U.S. banking and payments systems. Our findings are as follows.

First, if the presence of float is sufficiently widespread (though not universal) throughout the economy, then presence of float may have no effect on equilibrium allocations. That is, in some cases the income effect of the float transfer may be precisely offset by substitution effects.

Second, we show that the first result can also hold for the case where banking markets are characterized by imperfect competition.

Third, a combination of par check valuation and differential clearing lags can cause float to have allocational consequences. In this case, float inflates the relative prices of goods which are purchased with checks drawn on remote locations. The effect is a distortion of relative prices rather than a transfer.

These results are consistent with Humphrey and Berger's (1990) view that float is a significant factor behind the continued use of checks. However, our results are also consistent with Wells' (1996) findings that the average value of float is small. Our results imply that so long as the *marginal* value of float is high for some check writers, it would be advantageous for many people to use checks, even though the value of float might appear inconsequential when averaged across all checks.

2. THE MODEL

2.1 Institutional Environment

A formal model of check float requires certain key ingredients. First, the model must contain an environment in which economic agents have an incentive to trade, i.e., an "Edgeworth box." Second, there should be a role for "checks," i.e., transfers of inside

---

[3] Humphrey and Berger also note that if additional costs must be incurred in order to generate or reduce float, then float can result in welfare losses. These effects are discussed in Section 5 below.

money or privately issued debt, as a medium of exchange. Third, in order to model agents' incentive to capture the benefits of float, the model should provide a role for both non-interest bearing government debt or outside money, and for interest-bearing assets such as government bonds.

To produce a model with these features we will adapt Freeman's (1996) model, in which payments are made using privately issued debt. There are a number of distinctions between our setup and Freeman's, however. These distinctions result from our placing certain institutional constraints on the model environment. The constraints are intended to mimic some noteworthy features of U.S. banking and payments institutions, particularly as they apply to the use of checks as a payments medium.

First, as in Freeman's model, agents in our model will make payments using inside money. However, in our model, only banks may create inside money, and inside money cannot be issued in circulating (banknote) form. Instead, inside money can only be created as demand deposits at banks. When a demand deposit claim is transferred in order to make a purchase, then the transfer must be cleared through the banking system in order for the purchase to be valid. This restriction mimics historical restrictions on private banknote issue in the U.S.

Second, our model presumes a special role for outside money. That is, outside money will be the only acceptable medium for settlement of interbank transactions, when the transactions do not take place through a private clearing arrangement. While settlement in outside money is not a legal requirement in the U.S., it is common expectation and a de facto requirement that such transactions be settled in outside funds.[4] This requirement will not hold for interbank transactions made through a private clearing arrangement.

Third, we take as a legal requirement that checks must be honored at par value in interbank exchange. This restriction has applied to virtually all interbank exchanges in the U.S. since the 1920s.[5]

Fourth, in keeping with U.S. banking history, we assume that banking markets are "geographically dispersed," and that banks are prohibited from branching into all banking markets. Further, banks cannot form private clearing organizations across all banking markets.

2.2 Preferences, Endowments, and Technology

Following Freeman (1996), the economy takes place on I+1 isolated locations known as "islands," where I is a large, even number. The first I islands are each inhabited by a large number of two-period lived people. On each island, N new people are born in each period $t \geq 1$. In the first period there is a group of N people (the "initial old") who live for only one period. Each person born on one of these islands is endowed when young with y units of a good that is unique to that island. The good is not transportable across islands, and is nonstorable in the sense that it vanishes at the end of the period if not

---

[4] In models where banks can hold risky portfolios, settlement on the books of a central bank (i.e., in outside funds) can be value-enhancing if settlement arrangements allow for some risk-shifting from commercial banks to the central bank. This incentive is modeled by Emmons (1995) and Kahn and Roberts (1996). This incentive cannot be captured in the setup described below, since banks hold riskless portfolios.

[5] Duprey and Nelson (1986) describe the transition from nonpar to par checking.

consumed. Instead, people wishing to make transactions will journey to other islands in order to obtain their desired consumption good.

The first *I* islands are split into "archipelagoes." An archipelago will define a local banking market. Each archipelago contains an equal number of two types of islands, "debtor" and "creditor" islands (see Figure 1).

People born on debtor islands ("debtors") wish to consume some of their own endowment good and also the good of a creditor island when young, and nothing when old. On every debtor island, some debtors may want to consume creditor goods found on creditor islands within their own archipelago, and others may wish to consume creditor goods found on creditor islands of other archipelagoes. The utility of a debtor is given by $v(c_t, d_t)$, where c_t represents the debtor's consumption of the creditor good, and d_t the consumption of their own endowment good.

People born on creditor islands ("creditors") wish to consume some of their own endowment good while young, and also a good found on a particular debtor island while old. In contrast to debtors, creditors never wish to buy goods from outside their own archipelago. Creditors also cannot directly consume debtors' endowment goods. A debtor's endowment must first be transformed by a production process (described below) before it can be consumed by creditors. A creditor's utility is given by $u(C_t, D_{t+1})$, where D_{t+1} represents the creditor's consumption of the debtor good, and C_t represents the consumption of their own endowment good. The utility functions of both the debtors and creditors are sufficiently well-behaved so that interior optima obtain for both types.

The I+1[st] island is known as the "central island." On the central island there are commercial banks ("banks"), a central bank, and a government. Banks are infinitely lived, and their utility is given by $\sum_{t=0}^{\infty}\beta^t e_t$ where e_t represents period t consumption and β is a discount factor. Banks are endowed with technologies that allow them to keep perfect records and "clear" payments across islands and archipelagoes. However, due to branching restrictions, not every bank can communicate with every archipelago. In particular, each bank can communicate with ("has a branch on") only a fraction of the archipelagoes. Bank branches are also endowed with a production technology that can transform debtors' endowment goods into goods that can be consumed by creditors. The production process converts one unit of endowment into one unit of a consumption good.

In addition to record-keeping and production technologies, banks are endowed with an initial stock of one-period-maturity government bonds B_0 and an initial stock of central-bank issued fiat money M_0. Fiat money cannot be counterfeited, is unbacked, and can be costlessly exchanged. Fiat money is "intrinsically useless," but as discussed above it has a special function in settlement of interbank obligations. Government debt has the same properties as fiat money, with two distinctions. First, debt pays a gross nominal interest rate R_t per period. Second, it cannot be used to settle interbank obligations.

The initial old creditors on the creditor islands are endowed with claims that aggregate to $M_0 + B_0$, the sum of the initial assets owned by the banks.

The stock of outstanding government debt is financed via seignorage. In steady state, the central bank purchases sufficient government bonds in each period so that the stock of outside money expands at a constant rate μ per period. Assuming that the government runs a deficit net-of-interest G_t, its budget constraint is given by

$$B_t = R_t B_{t-1} + G_t - (M_t - M_{t-1}) \qquad (1)$$

If G_t is not too large and grows at a rate μ in steady state, (1) can be solved forward in the usual way (see e.g., Sargent (1987)) to yield

$$B_t = \frac{(1+\mu)^t}{r-\mu}\left(\mu M_0 - (1+\mu)G_0\right) \tag{2}$$

where in steady state $R_t = R > 1$, and $r = R-1 > \mu$[6] In steady state, the government budget is balanced in a net-present-value sense, and the ratio of money to bonds is given by

$$\frac{M_t}{B_t} = \frac{(r-\mu)M_0}{\mu M_0 - (1+\mu)G_0} \tag{3}$$

2.3 Trading

As in Freeman (1996), each period will consist of three stages. In the first stage, new debtors and creditors are born, receive their endowment good, and consume as much of their endowment good as they desire. In the second stage, young (generation *t*) debtors and old (generation *t*-1) creditors travel to other islands in order to consume their desired creditor and debtor goods, respectively. We assume that this travel takes place in a uniform and symmetric fashion, so that there are always the same number of people on every island. In the third stage, debtors and creditors consume the goods of the islands they have traded with.

This setup generates a role for money as a medium of exchange since barter is physically impossible. Note that outside money alone is insufficient for exchange since the young debtors must travel to remote islands before they have a chance to exchange their endowment good for money. Exchange is possible, however, if young debtors can issue

[6] In other words, we are analyzing "dynamically efficient" equilibria in which the real interest rate (r-μ) exceeds the growth rate of the economy (zero). For analyses of related models which consider dynamically inefficient equilibria see, e.g., Espinosa and Russell (1996) or Bullard and Russell (1997).

debt in exchange for creditor goods and if there is a technology for monitoring and enforcing ("clearing") all such debt contracts. Banks have access to such technology and this allows exchange to occur.

Our model differs from Freeman's in that debtors are legally precluded from paying for their purchases with their own IOUs. Instead, debtors must draw on funds on deposit with a bank by writing a check. They obtain these funds by depositing a portion of their endowment good at a local bank, which transforms their endowment good into a form that can be consumed by creditors.[7] A check cannot circulate: to be a valid form of payment, a check payment must be cleared and settled through the banking system. Clearing involves presentation of the check to the bank on which it is drawn. We will initially assume that the operational costs of clearing are negligible.

Settlement of a check payment occurs when the paying bank transfers assets of equal value to the presenting bank. Settlement can occur in two ways. First, if the banks involved are members of a clearinghouse, then the banks in this clearinghouse can net out payments against each other. Due to the symmetry of payment patterns in our model, each bank's net position against all other banks in the clearinghouse will be zero.

Restrictions on branching imply that the banks cannot form economy-wide clearinghouses. Instead, a certain fraction w of check payments presented to each bank are from banks outside the clearinghouse. Such payments must be settled by transfer of outside money to the presenting bank.

[7] For purposes of tractability, we assume that bank branches on debtor islands are willing to buy and sell local goods. A better name for banks in this environment might be "zaibatsu." We also assume that depositors do not withdraw their bank funds in cash, although it is possible to modify the model to accommodate both cash and non-cash payments (see Freeman (1996, 113-114)).

Young creditors who receive check payments from debtors deposit these checks at a bank that has a branch on their island. These creditors draw on their bank funds when they purchase debtor goods during the next period. Since creditors never travel outside their own archipelago, we assume that all checks written by creditors are cleared through the “local” clearinghouse.

3. EQUILIBRIUM

In this section we assume that there are a sufficiently many banks’ branches on each island so that banks are “perfectly competitive” in the following sense. Banks compete with each other by offering depositors interest on deposits held more than one period. Since profits are zero in equilibrium, banks end up paying all profits from their bond portfolio to depositors. We also assume that deficits net-of-interest G_t are zero.

3.1 Equilibrium without float

If there is no float then checks written at time *t* must be cleared and during period *t.* Interest on deposits therefore accrues to creditors.

In order to trade, a young debtor must have some funds on deposit at a local bank branch. The bank is willing to take a certain portion of the debtor’s endowment as a deposit. That is if the debtor deposits $y-d_t$ debtor goods with the bank, the debtor’s bank balance h_t will be given by

$$h_t = (y-d_t)p_t \tag{4}$$

where p_t is the money price of a debtor good on a debtor island. The budget constraint faced by debtors will thus be

$$yp_t = d_t p_t + h_t = d_t p_t + c_t p_t^* \tag{5}$$

where p_t^* is the money price of creditor good on a creditor island. The first-order condition for the debtor's utility maximization problem will thus be

$$v_c / v_d = p_t^* / p_t \tag{6}$$

where subscripts indicate partial derivatives.

Creditors sell a certain portion of their endowment good while young, receiving check payments in return. In the next period, they use the deposits plus accrued interest to purchase debtor goods. Hence a creditor's budget constraints are defined by

$$\begin{aligned} y_t p_t^* &= C_t p_t^* + l_t \\ l_t(1+f_t) &= D_{t+1} p_{t+1} \end{aligned} \tag{7}$$

where l_t is the period-*t* bank balance of a young creditor, and f_t is the (contract) interest rate on deposits. The first-order condition for utility maximization by a creditor is

$$u_C / u_D = (p_t^* / p_{t+1})(1+f_t) \tag{8}$$

Before deriving equilibrium conditions, we consider the balance sheet of a typical bank at the various stages within a period. At stage 1 of every period, the bank's balance sheet will look like the following (all entries are stated on a per-capita basis):

Assets	Liabilities + NW
Bonds B_t	$(1+f_{t-1})l_{t-1}$ deposits of old creditors
Outside money M_t	NW (=0 in equilibrium)

At stage 2 of every period, the bank's balance sheet will be:

Assets	Liabilities + NW
Bonds B_t	h_t deposits of young debtors
Outside money M_t	$(1+f_{t-1})l_{t-1}$ deposits of old creditors
Goods of young debtors worth $p_t(y-d_t)$	NW (=0 in equilibrium)

At stage 3, both debtors' and creditors' purchases have been cleared and settled, implying that the bank's balance sheet will be:

Assets	Liabilities + NW
Bonds B_t Outside money M_t	l_t deposits of young creditors NW (=0 in equilibrium)

From the balance sheets above, we can derive several useful relationships. First, from the stage 3 balance sheet it follows that the value of young creditors' deposits equals the value of the bank's outside assets, i.e.,

$$l_t = M_t + B_t \tag{9}$$

Since the value of funds paid by debtors for creditor goods equals the value of funds received by creditors, it follows that in equilibrium

$$h_t = l_t \tag{10}$$

Finally, comparing the bank's stage 3 balance sheet at time *t* to its stage 1 balance sheet at time *t+1*, we obtain

$$(1+f_t) = \frac{M_{t+1} + B_{t+1}}{M_t + B_t} = (1+m) \tag{11}$$

To solve for the prices p_t and p_t^*, we substitute (9), (10), and (11) into budget constraints (5) and (7) to obtain

$$p_t = \frac{M_t + B_t}{y - d_t} \tag{12}$$

$$p_t^* = \frac{M_t + B_t}{c_t} \tag{13}$$

Substituting the market prices for debtor and creditor goods into first-order conditions (6) and (8), we obtain the following conditions

$$\frac{v_c}{v_d} = \frac{p_t^*}{p_t} = \frac{y - d_t}{c_t} \tag{14}$$

$$\frac{u_C}{u_D} = \frac{p_t^*}{p_{t+1}}(1+\phi_t) = \left(\frac{y-d_{t+1}}{c_t}\right)\left(\frac{M_t + B_t}{M_{t+1} + B_{t+1}}\right)(1+\mu) = \left(\frac{y-d_{t+1}}{c_t}\right) \tag{15}$$

Equations (14) and (15), together with the resource constraints

$$d_t + D_t = y = c_t + C_t \tag{16}$$

determine the (stationary) equilibrium values of (c_t, d_t, C_t, D_t). Clearing of the market for outside money requires that the demand for outside money as a settlement medium equal its supply, i.e., that

$$M_t = \omega h_t = \omega(M_t + B_t) \Rightarrow M_t / B_t = \omega / (1+\omega) \tag{17}$$

Consistency of equilibrium thus requires that the money-bond ratio in (17) be identical to that implied by the present-value budget balance condition (3).

3.2 Properties of the no-float equilibrium

Inspection of equilibrium conditions (14), (15), and (16) reveals that the equilibrium values of (c_t, d_t, C_t, D_t) are unaffected by the growth rate of the money stock μ i.e., money is superneutral. The equilibrium allocation in this case is also efficient in the sense that it solves the social planner's problem of maximizing a population-weighted sum of utilities in steady state, subject to resource constraints (16). This problem has first-order condition[8]

$$u_C / u_D = v_c / v_d$$

which is implied by equilibrium conditions (14) and (15).

3.2 Equilibrium with float

[8] See Green (1997) for a derivation and a discussion.

We now consider a version of the model where all young debtors buy goods in “remote” locations, i.e., in archipelagoes other than their own. Checks written at these remote locations take one period to clear, so that debtors can collect interest on their deposits until their checks clear in the next period. We assume that these checks clear in the first stage of the period. Checks written by creditors are cleared through the local clearinghouse and clear within the same period.

In this version of the model, the budget constraints for a representative debtor are

$$\begin{aligned} yp_t &= d_t p_t + h_t \\ h_t(1+\mathrm{f}_t) &= c_t p_t^* \end{aligned} \tag{18}$$

A representative creditor has budget constraint

$$y_t p_t^* = C_t p_t^* + l_t = C_t p_t^* + D_{t+1} p_{t+1} \tag{19}$$

These budget constraints imply the following first-order conditions for the debtor and creditor, respectively:

$$v_c / v_d = (p_t^* / p_t)\big(1/(1+\mathrm{f}_t)\big) \tag{20}$$

$$u_C / u_D = p_t^* / p_{t+1} \tag{21}$$

To solve for p_t^*, we note that the amount of the debtors’ funds available for purchasing creditor goods must equal the value of the purchase, i.e.,

$$h_t(1+\mathrm{f}_t) = M_{t+1} + B_{t+1} = c_t p_t^* \tag{22}$$

which implies

$$p_t^* = \frac{M_{t+1} + B_{t+1}}{c_t} \tag{23}$$

Similarly, the value of the young debtors’ goods purchased by the old creditors must equal the amount of deposits held by the old creditors, implying

$$p_t = \frac{M_t + B_t}{y - d_t} \tag{24}$$

Substituting the expressions for prices into first-order conditions (20) and (21), and using the fact that $\phi_t = \mu$ we obtain

$$\frac{v_c}{v_d} = \frac{p_t^*}{p_t(1+\phi_t)} = \left(\frac{M_{t+1}+B_{t+1}}{(1+\mu)(M_t+B_t)}\right)\left(\frac{y-d_t}{c_t}\right) = \left(\frac{y-d_t}{c_t}\right) \tag{25}$$

$$\frac{u_C}{u_D} = \frac{p_t^*}{p_{t+1}} = \left(\frac{M_{t+1}+B_{t+1}}{c_t}\right)\left(\frac{y-d_{t+1}}{M_{t+1}+B_{t+1}}\right) = \left(\frac{y-d_{t+1}}{c_t}\right) \tag{26}$$

Equations (25) and (26) are identical to equilibrium conditions (14) and (15) from the no-float model; hence they imply the same equilibrium allocation as in the no-float model. As was the case with the no-float model, money is superneutral and the equilibrium allocation is efficient.[9]

The reason that float fails to have any effect on equilibrium allocations is that the income effect resulting from transferring the float to debtors is precisely offset by a substitution effect. The debtors use their float income to bid up the price of creditor goods (see equation (23)), which negates the effect of the float transfer for the debtors and compensates the creditors for the loss of interest income.

4. EXTENSIONS

4.1 Imperfect Competition

So far we have assumed that the market for banking and payment services is perfectly competitive. While this assumption is a reasonable one for most modern-day

---

[9] Freeman (1996, 113) derives a similar result.

urban banking markets, competition has historically been restricted in many banking markets in the U.S., particularly in rural areas.

To formally investigate the effects of imperfect competition, we consider an extreme case of the model where each archipelago has only one bank. The single bank is legally constrained to offer its customers a standard deposit contract (customers have full access to funds on deposit), but the bank retains all earnings on its bond portfolio. For purposes of tractability, we assume that the bank is only interested in consuming creditor goods. The bank obtains these goods by writing checks on itself. We also assume that the bank is sufficiently impatient so that it uses its bond income for immediate consumption.

First consider the case where there is no float so that checks clear in the same period. The amount of income available to the bank is the interest on its previous period's bond portfolio. Hence in steady state its budget constraint is

$$c_t^B p_t^* = rB_{t-1} \tag{27}$$

where c_t^B denotes the banker's consumption of the creditor good. The market-clearing condition requires that the value of funds available to purchase the creditor good equal expenditures on the creditor good, i.e., that

$$p_t^* = \frac{M_t + B_t}{c_t + c_t^B} = \frac{M_t + B_t}{y - C_t} \tag{28}$$

A similar constraint applies to the debtor good. Note however that old creditors at time *t* do not have claim to the bank's entire stock of outside assets $M_t + B_t$, since the bank retains the earnings on the bonds for its own consumption. It follows that

$$p_t(y - d_t) = M_t + B_t - rB_{t-1} \Leftrightarrow y_t - d_t = \frac{M_{t-1} + B_{t-1}}{p_t} \tag{29}$$

Since neither debtors nor creditors receive interest on their deposits, their budget constraints are given by (5) and (19), and their first-order conditions by (6) and (21) respectively. Substituting market-clearing conditions (28) and (29) into the debtor and creditor's first-order conditions, we obtain

$$\frac{v_c}{v_d} = \left(\frac{y - d_t}{y - C_t}\right)(1 + \mu) \tag{30}$$

$$\frac{u_C}{u_D} = \frac{y - d_{t+1}}{y - C_t} \tag{31}$$

Substituting market-clearing conditions (28) into the bank's budget constraint (27) and using the expression for the money-bond ratio (3), we obtain

$$c_t^B = \frac{rB_{t-1}}{p_t^*} = \frac{rB_{t-1}(y - C_t)}{M_t + B_t} = \frac{\mu}{1 + \mu}(y - C_t) \tag{32}$$

Equations (30), (31), and (32), together with resource constraints, determine the equilibrium values of $(c_t, d_t, c_t^B, C_t, D_t)$. This equilibrium is not efficient in the sense discussed above, and money is not superneutral. The bank's appropriation of profits from its bond portfolio has the effect of placing a distortionary tax on purchases of creditor goods, which encourages debtors to substitute consumption away from creditor goods and towards their own endowment good. The bank uses revenues from this "tax" to finance its own purchases so there is no offsetting income effect.

We now consider the case where purchases of creditor goods are subject to float. Formally, we model float as meaning that checks written to purchase a creditor good take an extra period to clear. In this case, the bank pays for its purchases of the creditor good with the interest on the current period's bond portfolio, so its budget constraint becomes

$$c_t^B p_t^* = rB_t \tag{33}$$

and the market-clearing condition for the debtor and creditor good are

$$p_t = \frac{M_t + B_t}{y - d_t} \tag{34}$$

$$p_t^* = \frac{M_{t+1} + B_{t+1}}{c_t} \tag{35}$$

The budget constraints for debtors and creditors and their first-order conditions remain as before. Substituting the market-clearing conditions into the first-order conditions and into the bank's budget constraint (33) we again obtain

$$\frac{v_c}{v_d} = \left(\frac{y - d_t}{y - C_t}\right)(1 + \mu) \tag{36}$$

$$\frac{u_C}{u_D} = \frac{y - d_{t+1}}{y - C_t} \tag{37}$$

$$c_t^B = \frac{\mu}{1 + \mu}(y - C_t) \tag{38}$$

Equilibrium conditions (36), (37), and (38) coincide exactly with equations (30), (31) and (32). That is, the equilibrium allocations with float are identical to those without float. Although the presence of float gives the banker has a higher nominal income in every period, the value of this income is bid away in equilibrium. The resulting equilibrium is inefficient, but no more or less so than the equilibrium without float.

4.2 Distortionary Public Finance

Similar effects on equilibrium allocations can also arise when the government runs positive deficits net-of-interest G_t. Suppose that in each period, a portion of the newly created outside money is deposited in banks and used to buy creditor goods. The creditor goods purchased by the government are not transferred to other people but are used by the government to produce goods such as "national defense."

We first consider the case without float. If the banking industry is competitive, then the stage 1 balance sheet of the representative bank will incorporate both government and private sector deposits:

Assets	Liabilities + NW
Bonds B_t	$(1+\mathrm{f}_{t-1})l_{t-1}$ deposits of old creditors
Outside money M_t	G_t government deposits
	NW (=0 in equilibrium)

Similarly the stage 2 balance sheet will be:

Assets	Liabilities + NW
Bonds B_t	h_t deposits of young debtors
Outside money M_t	$(1+\mathrm{f}_{t-1})l_{t-1}$ deposits of old creditors
Goods of young debtors worth $p_t(y-d_t)$	G_t government deposits
	NW (=0 in equilibrium)

After clearing and settlement, all claims on banks are held by young creditors, so that the bank's stage 3 balance sheet is given by:

Assets	Liabilities + NW
Bonds B_t	l_t deposits of young creditors
Outside money M_t	NW (=0 in equilibrium)

Since both debtors and government are now in the market for creditor goods, equating the value of the funds paid for the creditor goods to the value of the funds received implies

$$h_t = l_t + G_t \tag{39}$$

Comparing the bank's stage 3 balance sheet at time t to its stage 1 balance sheet at time $t+1$, we obtain

$$(1+\phi_t) = \frac{M_{t+1} + B_{t+1} - G_t}{M_t + B_t} \tag{40}$$

We solve for ϕ by substituting the government's budget constraint (2) into (40) which yields

$$(1+\phi_t) = \left(\frac{rM_0 - (1+r)G_0}{rM_0 - (1+\mu)G_0}\right)(1+\mu) \equiv 1+\mu' < 1+\mu \tag{41}$$

To solve for the prices p_t and p_t^*, we substitute (9), (39) and (41) into budget constraints (5) and (7) to obtain

$$p_t = \frac{(M_{t-1} + B_{t-1})(1+\mu')}{y - d_t} \tag{42}$$

$$p_t^* = \frac{M_t + B_t}{y - C_t} \tag{43}$$

Using market-clearing conditions (42) and (43), together with first-order conditions (6) and (8) we obtain the following equilibrium conditions

$$\frac{v_c}{v_d} = \frac{p_t^*}{p_t} = \left(\frac{y - d_t}{c_t}\right)\left(\frac{1+\mu}{1+\mu'}\right) \tag{44}$$

$$\frac{u_C}{u_D} = \frac{p_t^*}{p_{t+1}}(1+\phi_t) = \left(\frac{y - d_{t+1}}{y - C_t}\right) \tag{45}$$

In equilibrium, government consumption c_t^G is given by

$$c_t^G = \frac{G_t}{p_t^*} = \frac{G_t}{M_t + B_t} c_t = \frac{\mu M_0 - (r-\mu)B_0}{(1+\mu)(M_0 + B_0)} c_t \tag{46}$$

As was the case with monopoly banking, this equilibrium is inefficient and money is not superneutral. Once again seignorage acts as a distorting tax on the purchase of creditor goods. The key difference between this model and that of the previous sector is that here income from seignorage is diverted to governmental and not private use. In the limiting case where there is no government debt and where all government deficits are financed directly by money creation, so that

$$G_t = M_t - M_{t-1} \quad (47)$$

then it is easy to show that the equilibrium allocation implied by equations (44), (45), and (46) is identical to the equilibrium allocation of the monopoly model of the previous section.

It is also possible to modify this version of the model to accommodate check float, by allowing a one-period clearing lag for checks written to purchase creditor goods. As with previous versions of the model, introducing float into the model does not change equilibrium allocations. Since the derivation is essentially a repetition of that of the previous section it is omitted.

4.3 Remote Disbursement

The analysis above implies a benign view of float. If delays occur in clearing and settling checks (or any similar form of payment), then this analysis suggests that markets can adjust so that the costs associated with these delays are appropriately priced. Equilibrium allocations are unaffected by float, and are efficient so long as (1) the banking industry is competitive, and (2) deficits are not financed by money creation.

Is this Panglossian view of float applicable to the U.S. payment system? The obvious problem with this view is the additional processing costs associated with use of a form of payment that involves float (checks) over other forms of payment that do not. Even abstracting from cost considerations, we think that the answer to this question may be “no,” due to the effects of certain institutional frictions. A potentially important friction results from the fact that lags in check clearing vary according to the distance between the paying bank and the collecting bank, yet the costs associated with such varying delays are not always priced. Wells (1996) notes that in business-to-business payments, the effects of float are often subject to negotiation and may be internalized via private contracts. Such internalization is rare when payments involve consumers, however.

The competitive/no-deficit version of our model can capture the effect of unpriced differential clearing times if we divide each generation of debtors into two classes, according to the time it takes their checks to clear. The first class, known as "local debtors," always journeys when young to islands in the same archipelago. Hence checks written by these debtors at time t will clear before the end of period t. The second class of debtors, known as "remote debtors," always journeys when young to islands in another archipelago. Checks written by young debtors to creditors on these remote islands always take two days to clear. Since every bank is required to honor checks on all other banks at par, creditors' banks cannot discount the value of checks written by remote debtors.[10]

If the creditor does not price the float, this gives an advantage to remote debtors. The budget set of local debtors is constrained by the no-float equation (9), while that of remote debtors is constrained by the float equations (18). The typical creditor sells to both remote and local debtors. The creditor's budget constraint is given by

$$y p_t^* \mathrm{y}_t = C_t p_t^* \mathrm{y}_t + D_{t+1} p_{t+1} \qquad (48)$$

where y_t represents the average time t+1 value of payments received at time t. The creditor's budget constraint says that the time t+1 value of the creditors' goods sold in period t equals the value of goods purchase in period t. In symmetric equilibrium, the creditor sells λc_t to local debtors and $(1-\lambda)c'_t$ to remote debtors, where λ is the proportion of local debtors and primes denote consumption by remote debtors. Hence we have

$$\psi_t = \frac{\lambda c_t(1+\phi_t)+(1-\lambda)c'_t}{\lambda c_t+(1-\lambda)c'_t} = \frac{\lambda c_t(1+\phi_t)+(1-\lambda)c'_t}{y-C_t} \tag{49}$$

Budget constraints (9), (18), and (48) imply the following respective first-order conditions for local debtors, remote debtors, and creditors.

$$v_c / v_d = p_t^* / p_t \tag{50}$$

$$v'_c / v'_d = (p_t^* / p_t)\big(1/(1+\phi_t)\big) \tag{51}$$

$$u_C / u_D = (p_{t+1}^* / p_t)\psi_t \tag{52}$$

Market-clearing conditions are given by

$$p_t^* = \frac{M_{t+1}+B_{t+1}}{\lambda c_t(1+\phi_t)+(1-\lambda)c'_t} \tag{53}$$

$$p_t = \frac{M_t+B_t}{y-\lambda d_t-(1-\lambda)d'_t} \tag{54}$$

Substituting the market-clearing conditions into the first-order conditions, we obtain

$$\frac{v_c}{{}_d} = \frac{y-\underline{d}_t}{{}_{-t}}\left(\frac{1 \quad \mu}{\psi_t}\right) \tag{55}$$

$$\frac{v'_c}{v'_d} = \left(\frac{y-\underline{d}_t}{\underline{c}_t}\right)\left(\frac{1}{\psi_t}\right)$$

$$\frac{u_C}{u_D} = \left(\frac{y - \underline{d}_{t+1}}{\underline{c}_t} \right) \qquad (57)$$

where $\underline{c}_t$ and $\underline{d}_t$ are weighted averages of debtor consumption of creditor and debtor goods, i.e.,

$$\underline{c}_t \equiv \lambda c_t + (1-\lambda) c_t' \qquad (58)$$

$$\underline{d}_t \equiv \lambda d_t + (1-\lambda) d_t' \qquad (59)$$

Conditions (55)-(57) imply that the "remote disbursement" equilibrium allocation differs from the efficient allocation so long as $\lambda \in (0,1)$. Note that the equilibrium goes to the no-float equilibrium as the measure of local debtors λ goes to one, and to the float equilibrium as λ goes to zero. For $\lambda \in (0,1)$, however, the presence of unpriced float leads to an inefficient equilibrium.

In general, the three equilibrium conditions (55)-(57) plus the two resource constraints are insufficient to determine equilibrium values of $(c_t, d_t, c_t', d_t', C_t, D_t)$. However, the remote disbursement is easily solved for the special case where the utility of both creditors and debtors is time-separable and logarithmic, i.e.,

$$v(c,d) = \log c + \log d \qquad (60)$$

$$u(C,D) = \log c + \log D \qquad (61)$$

For this case, the creditors' and debtors' demand for their respective endowment goods is price inelastic and given by *y*/2. Using this fact, we obtain the following stationary equilibrium allocation:

$$C = D = d = d' = y/2 \qquad (62)$$

$$c = \frac{y}{2\big(\lambda + (1-\lambda)(1+\mu)\big)} < \frac{y}{2} \qquad (63)$$

$$c' = \frac{(1+\mu)y}{2\left(\lambda + (1-\lambda)(1+\mu)\right)} > \frac{y}{2} \qquad (64)$$

For positive rates of inflation μ, it is easy to show that the consumption of remote debtors is increasing in λ. As the proportion of remote debtors grows and λ falls to zero, the advantage of being a remote debtor is increasingly bid away until c' reaches its efficient value *y/2*.

We also note that for this case, the average amount of check float in the economy is given by $(1-\lambda)c'\mu$, which is the measure of remote debtors (=proportion of checks written by remote debtors), times the amount of their checks, times the interest received by the remote debtors while they are waiting for their checks to clear. This figure can be quite small if λ is close to unity, yet the marginal benefit of float to "remote debtors" can be quite large. On the other hand, a small average amount of float implies small price distortions and hence, small losses in aggregate welfare.

5. IMPLICATIONS

The analysis above shows that there may be both less and more to the problem of float than has previously been thought. Sections 3, 4.1, and 4.2 show that in general equilibrium, the market can offer a natural remedy to the float problem through price effects. That is, to the extent that the buyers in a particular market are beneficiaries of float, the market prices will adjust so as to offset the income effect afforded by float. In the case where float beneficiaries and non-beneficiaries are buyers in mutually exclusive markets, then the price effect perfectly offsets the income effect. Thus, in equilibrium float may not represent a transfer of real income. Depositors would give up nothing in terms of their real purchasing power, if they were to agree to switch en masse from a form of payment that allowed for float (such as checks) to a form of payment that did not (such as ACH). Individual depositors, on the other hand, would lack an incentive to unilaterally give up their access to (unpriced) float.

Section 4.3 shows that if there is differential access to float by participants on the same side of a given market, and that if float is unpriced, then float-induced distortions can result despite low average values of float as reported by Wells (1996). The last result suggests that float may remain a strong motive behind the continued use of checks. Specifically, if the marginal benefit of float is high for some group of check-writers, these check-writers have no incentive, either collectively or individually, to switch to a form of payment that denies them this float benefit (again, as long this benefit is not priced). Other check-writers who are not the beneficiaries of float would also not gain by switching payment systems, unless the check-writers who are generating the float were also to change payment systems. This view is more in keeping with the “market-failure” characterization of check float advanced by Humphrey and Berger (1990).

As noted above, a small average float amount implies that the welfare costs associated with float-induced price distortions are also small. However, as Humphrey and Berger (1990,51) point out, another social cost associated with the use of check payment is the resource cost of activities designed to generate or reduce float. Our model abstracts from such costs, but can easily be modified to accommodate their effects. For example, in the log-utility example of “remote disbursement” model of Section 4.3, suppose that $r>0$ is the resource cost of providing remote disbursement services to the remote debtors, and that qr is the fraction of these costs directly borne by remote debtors (for the local debtors this cost is prohibitively high). Since this resource cost of remote disbursement does not depend on the value of checks, the equilibrium conditions would not change as a result of this cost. As long as the marginal value of remote disbursement exceeds its cost to the remote debtor, i.e.,

$$c' - qr > c \tag{65}$$

then it will pay for remote debtors to take advantage of remote disbursement. In this case, the efficiency losses from stemming from these costs would only compound the efficiency losses resulting from the float distortion. A similar argument would apply to the costs of accelerated presentment, in that interpretation of the model.

If the cost of processing ACH payments is significantly lower than the cost of processing checks, then it might appear that banks should be able to entice their customers into using the lower-cost form of payment by charging them lower fees. However, there are two reasons why this might not be the case. First, in cases where the marginal value of float to the customer is high, the cost saving might be insufficient to compensate for the loss of the float benefit. The convention of par check valuation means that the float benefit cannot enter into the bank's pricing formula for clearing checks. If a payee is willing to accept a check drawn on a remote location at par value, the payee's bank cannot discount the check to compensate for the value lost by float. Second, as long as the cost of float is not priced into market transactions, this blunts the incentives of all bank customers to use lower cost means of payment.

This discussion is not meant to imply that the marginal value of float is the only possible reason for the continued use of checks. Other contributing factors that have been suggested include the distribution of check processing costs (under current arrangements the collecting bank pays a significant portion thereof), the convenience associated with check payment, and the high initial costs with associated with moving from paper-based to electronic payments. More detailed empirical analysis is clearly needed to sort out the relative magnitudes of these effects.

References

Bank for International Settlements. *Statistics on Payment Systems in the Group of Ten Countries*. Basle, 1996.

Bullard, James and Steven Russell. "How Costly is Sustained Low Inflation for the U.S. Economy?" Manuscript, Federal Reserve Bank of St. Louis, April 1997.

Duprey, James N. and Clarence W. Nelson. "A Visible Hand: The Fed's Involvement in the Check Payments System." Federal Reserve Bank of Minneapolis *Quarterly Review* 10 (Spring 1986): 18-29.

Emmons, William R. "Interbank Netting Agreements and the Distribution of Bank Default Risk." Federal Reserve Bank of St. Louis Working Paper 95-016A, 1995.

Espinosa, Marco and Steven Russell. "Conventional Monetary-Policy Wisdom in the Diamond Model." Manuscript, December 1996.

Freeman, Scott. "Clearinghouse Banks and Banknote Over-Issue." *Journal of Monetary Economics* 38 (July 1996): 101-15.

Green, Edward J. "Money and Debt in the Structure of Payments." Bank of Japan *Monetary and Economic Studies* 15 (May 1997): 63-87.

Humphrey, David B. and Allen N. Berger. "Market Failure and Resource Use: Economic Incentives to Use Different Payment Instruments." In *The U.S. Payment System: Efficiency, Risk and the Role of the Federal Reserve: Proceedings of a Symposium on the U.S. Payment system Sponsored by the Federal Reserve Bank of Richmond*, ed. David B. Humphrey, pp.45-86. Boston: Kluwer Academic Publishers, 1990.

Humphrey, David B., Lawrence B. Pulley, and Jukka M. Vesala. "Cash, Paper, and Electronic Payments: A Cross-Country Analysis." *Journal of Money, Credit, and Banking,* 28 (November 1996): 914-39.

Kahn, Charles M. and William Roberds. "Payments System Settlement and Bank Incentives." Federal Reserve Bank of Atlanta Working Paper 96-10, 1996.

Lacker, Jeffrey M. "The Check Float Puzzle." Unpublished Manuscript, Federal Reserve Bank of Richmond, September 1997.

Sargent, Thomas J. *Macroeconomic Theory.* 2nd Ed., Orlando: Academic Press, 1987.

Wells, Kirsten E. "Are Checks Overused?" Federal Reserve Bank of Minneapolis *Quarterly Review* 20 (Fall 1996): 2-12.

Figure 1: Movement of Agents in the Model