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Abstract: Empirical support for the long-run Fisher effect, a hypothesis that a permanent change in inflation leads to an equal change in the nominal interest rate, has been hard to come by. This paper provides a plausible explanation of why past studies have been unable to find support for the long-run Fisher effect. This paper argues that the necessary permanent change to the inflation rate following a monetary shock has not occurred in the industrialized countries of Australia, Austria, Belgium, Canada, Denmark, France, Germany, Greece, Ireland, Italy, Japan, the Netherlands, Norway, Sweden, Switzerland, the United Kingdom, and the United States. Instead, this paper shows that inflation in these countries follows a mean-reverting, fractionally integrated, long-memory process, not the nonstationary inflation process that is integrated of order one or larger found in previous studies of the Fisher effect. Applying a bivariate maximum likelihood estimator to a fractionally integrated model of inflation and the nominal interest rate, the inflation rate in all seventeen countries is found to be a highly persistent, fractionally integrated process with a positive differencing parameter significantly less than one. Hence, in the long run, inflation in these countries will be unaffected by a monetary shock, and a test of the long-run Fisher effect will be invalid and uninformative as to the truthfulness of the long-run Fisher effect hypothesis.

JEL classification: C2; E4

Key words: Fisher effect, fractional integration, long memory

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1 Introduction

The long-run Fisher effect hypothesis is a proposition that states a permanent change in the inflation rate will cause nominal interest rates to move one for one with the change in inflation. Thus, the real interest rate will remain unchanged in response to a monetary shock if the Fisher effect holds (Fisher, 1930). Unfortunately, empirical support for the long-run Fisher effect has been hard to come by (see Weber, 1994; King and Watson, 1997; Koustas and Serletis, 1999; and Rapach, 2003). The objective of this paper is to provide an explanation as to why these previous studies of the long-run Fisher effect have been unable to support this widely held theory.

Past empirical studies of the long-run Fisher effect have employed variations of the Fisher and Seater (1993) bivariate, vector-autoregression test of long-run (super)neutrality. The key to being able to apply this reduced form test is finding inflation to follow a non-stationary process integrated of order one or larger. Most tests of non-stationarity contain in some form or another a test for a unit-root. If inflation contains a unit-root an exogenous monetary shock will permanently change inflation. The long-run response of the nominal interest rate to a permanent change in inflation will then depend on the relative orders of integration in inflation and the nominal interest rate. It is the argument of this paper that the necessary permanent change to inflation has not taken place; i.e., inflation is not integrated of order one or larger. Consequently, a test of the long-run Fisher effect will not be valid and any inference made as to whether the hypothesis holds or not will be unsubstantiated.

There is a class of models where the order of integration is a real number and whose response to an exogenous shock can be very long lived but not permanent. Called fractionally integrated models, they not only nest the unit-root behavior within it, but they also display stationary and non-stationary, mean-reverting dynamics, along with long-memory and anti-persistent dependencies. For example, a fractionally integrated process with an order of integration greater than 1/2 reverts back to its mean following an exogenous shock. However, the rate at which the process reverts to its mean is so slow that its variance explodes. Although nonstationary in the sense that its variance is infinite, this fractionally integrated process is still stationary in the mean-reverting context. If inflation follows such a process it would be difficult to distinguish the long-lasting, but finite, impact of a shock from that of a unit-root process's permanent impact. This strong persistence in a fractionally integrated inflation series could cause one to mistakenly conclude that the affect of the monetary shock on inflation is permanent and incorrectly proceed with a test of the long-run Fisher effect.

In this paper a bivariate fractionally integrated estimator is applied to the inflation and nominal interest rate series of 17 developed countries. In all 17 countries, the post-war inflation series is found to follow a mean-reverting, fractionally integrated, long-memory process. The null hypothesis that the order of integration in inflation equals one is clearly rejected for every country. These findings of a mean-reverting, fractionally integrated, inflation series are robust to monthly and quarterly measures of the consumer price index, and to quarterly inflation series calculated from the gross domestic price deflator. It follows from the fractional integration results for inflation that the long-run Fisher hypothesis cannot be tested in these 17 industrialized countries during the post-war period.

Our findings for US inflation are also robust to regime shifts associated with changes in the Federal Reserve's monetary policy. Neither the Fed's October 1979 decision to move away from interest-rate smoothing, nor its October 1982 decision to weight monetary aggregates less heavily when setting monetary policy, affects the stationary, long-memory behavior of US inflation. Hence, a test of the long-run Fisher effect hypothesis in the US cannot be carried out with the bivariate reduced form approach in either of the monetary regimes.

The remainder of the paper is organized as follows. In Sections 2 we extend Bae et al. (2005) relative order of integration conditions for testing long-run money neutrality to the long-run Fisher effect hypothesis. These conditions are then tested in Section 3 by estimating the seventeen country's inflation and nominal interest rate's fractional order of integration. We conclude in Section 4 by summarizing our empirical findings and commenting on the implications a fractionally integrated inflation series has on monetary policy.

2 Integration Conditions

Bae et al. (2005) extends the relative order of integration restrictions of Fisher and Seater (1993) for testing long-run neutrality to a bivariate fractionally integrated, autoregressive, moving average model of the nominal and real variable. Table 2 lists these fractional orders of integration conditions in terms of the long-run Fisher effect hypothesis, with d_π representing the order of integration of inflation, d_R, the order of integration of nominal interest rates, and, $\gamma_{R\pi}$, the long-run derivative of nominal interest rate to a change in inflation. The L in Table 2 represents the lag operator, $L^j x(t) \equiv x(t-j)$, where, $j = 0, 1, 2, \ldots$ In the fourth column of Table 2 we list the outcomes each case has on $\gamma_{R\pi}$ and the long-run Fisher effect hypothesis. There are three possible outcomes: (1) the long-run

Case	Relative Order of Integration	Economic Meaning	Fisher Effect
(i)	$0 < d_\pi < 1$	Monetary shocks do not permanently change π.	Cannot be tested
(ii)	$0 < d_R < 1 \le d_\pi$	Monetary shocks permanently change π, and do not permanently change R.	Reject
(iii)	$1 \le d_\pi = d_R$	Monetary shocks permanently change π and R.	$\gamma_{R\pi}$
(iv)	$1 \le d_R < d_\pi$	Monetary shocks permanently change $(1-L)^{d_\pi - 1}\pi$, and do not permanently change $(1-L)^{d_\pi - 1}R$.	Reject
(v)	$1 \le d_\pi < d_R$	Monetary shocks do permanently change $(1-L)^{d_R - 1}\pi$.	$\gamma_{R\pi}$

Table 1: The relative fractional orders of integration of inflation and the nominal interest rate when testing the long-run Fisher effect hypothesis.

Fisher effect can be tested by conducting the hypothesis test, $H_0 : \gamma_{R\pi} = 1$, (2) the long-run Fisher effect cannot be tested (neither acceptance nor rejection of the hypothesis is possible), and (3) the long-run Fisher effect is rejected outright.

Our interest in this paper rests with Case (i). Under Case (i), Fisher and Seater's (1993) reduced-form approach to testing the long-run Fisher hypothesis suffers from the Lucas (1972) and Sargent (1971) critique. Lucas and Sargent both point out that a test of long-run neutrality will not be valid if a permanent change to the nominal variable has not taken place. Letting $\epsilon(t)$ represent an exogenous monetary shock, Lucas and Sargent's point can be understood in terms of the long-run derivative:

$$\gamma_{R\pi} \equiv \lim_{k\to\infty} \frac{\partial R(t+k)/\partial\epsilon(t)}{\partial\pi(t+k)/\partial\epsilon(t)},$$

where $R(t)$ is the nominal interest rate at time period t, and $\pi(t)$ is the inflation rate at time t. When a permanent change to inflation does not occur, $\partial\pi(t+k)/\partial\epsilon(t) \to 0$, as $k \to \infty$, causing $\gamma_{R\pi}$ to not exist. Because $\gamma_{R\pi}$ does not exist, it follows that the long-run Fisher effect cannot be tested.

To test if a permanent change has occurred in a fractionally integrated inflation series, we look at the Wold representation of inflation, $(1-L)\pi(t) = A(L)\epsilon(t)$, where $\epsilon(t)$ is white noise,

$$A(L) \quad \sim \quad (1-L)^{1-d_\pi} = F(d_\pi - 1, 1; 1; L), \qquad \text{as } L \to 1,$$

where $F(d_\pi - 1, 1, 1, ; L)$ is the hypergeometric function (Gradshteyn and Ryzhik; 1994, p. 1066), and $\sim$ means the ratio consisting of the left and righthand side values equals one.

Known as the infinite cumulative impulse response function, $A(1)$ measures the long-run impact a unitary, exogenous, monetary shock has on inflation. If inflation is a mean-reverting process, in other words, the long-run impact of an exogenous monetary shock to π is zero, then $A(1)$ will equal zero.

From the properties of $F(d_\pi - 1, 1, 1, ; L)$ found in Gradshteyn and Ryzhik, (1994, p. 1066), $A(1)$ will:

1. Converge to zero when $d_\pi < 1$.
2. Diverge from zero when $d_\pi \geq 1$.

Thus, when $d_\pi \geq 1$, an unexpected monetary shock will permanently change inflation so that the long-run Fisher effect hypothesis can be tested. Whereas, when $d_\pi < 1$, the effect of monetary shock on inflation dissipates over time until it reaches zero.

In contrast to the exponential rate of decay in the autocorrelation function of a short-memory process, a fractionally integrated process's autocorrelation decays at the slower hyperbolic rate, $\tau^{2d_\pi - 1}$, as $\tau \to \infty$. It follows then that there is strong long-range dependence in a fractionally integrated inflation series, causing a monetary shock to affect inflation for a very long time. If fact, the impact is so long lived that when $1/2 < d_\pi < 1$ the autocorrelations are not summable, causing the variance to explode. However, even under the infinite variance case the monetary shock will not permanently affect inflation. These are the circumstances of Case (i), where the long-run Fisher effect cannot be tested.

In Case (ii), because $1 \leq d_\pi$, inflation will be permanently affected by an exogenous monetary shock. However, the effect of the shock on the nominal interest rate will not be permanent. Because $d_R < 1$, the nominal interest rate follows a mean reverting, fractionally integrated process that when perturbed slowly returns to its pre-monetary shock level. Thus, in Case (ii) the long-run Fisher effect hypothesis can be tested but it is rejected outright since the nominal interest rate is only temporarily affected by a permanent change in inflation.

Except for the fractional nature of the orders of integration, Case (iii) is the same necessary condition found in previous tests of the long-run Fisher effect (see King and Watson, 1997; Koustas and Serletis, 1999; and Rapach, 2003). Because in Case (iii), d_π and d_R are both greater than or equal to one, an exogenous monetary shock permanently affects both inflation and the nominal interest rate. Under this scenario, the Fisher hypothesis can only be tested by estimating the long-run derivative between the nominal interest rate and inflation, $\gamma_{R\pi}$, and testing whether it equals one (Fisher effect holds) or not.

Case (iv) can be understood in a similar manner to Case (ii). By differencing both inflation and the nominal interest rate $d_\pi - 1$ times, the differenced inflation series, $\pi^* = (1-L)^{d_\pi - 1}\pi$, will by definition be a unit-root process. However, the differenced nominal interest rate series, $R^* = (1-L)^{d_\pi - 1}R$, order of integration will be less than one since $1 \leq d_R < d_\pi$. The relative relationship between π and R orders of integration carry over to the relative orders of integration between π^* and R^*. How this affects the long-run derivative can be understood by writing the long-run derivative in terms of the fractional differencing operators:[1]

$$\gamma_{R\pi} \propto \left. \frac{(1-L)^{1-d_R}}{(1-L)^{1-d_\pi}} \right|_{L=1} = (1-L)^{d_\pi - d_R}\Big|_{L=1}.$$

Because $\gamma_{R\pi}$ is a ratio of fractional differencing operators, applying the differencing operator $d_\pi - 1$ times to R and π does not change $\gamma_{R\pi}$. We can now write $\gamma_{R\pi}$ in terms of R^* order of integration:

$$\gamma_{R\pi} \propto (1-L)^{d_\pi - (d_\pi - 1) - d_R + (d_\pi - 1)}\Big|_{L=1} = (1-L)^{1-d_{R^*}}\Big|_{L=1}$$

where $d_{R^*} = d_R - d_\pi + 1$. Since, $d_{\pi^*} = 1$, a shock to inflation will permanently affect π^*, and hence, the long-run derivative, $\gamma_{R\pi}$, exists. However, since $d_{R^*} < 1$, the shock will only temporarily affect R^*. It follows then that under Case (iv), $\gamma_{R\pi} = 0$, and the long-run Fisher effect is rejected.

In Case (v), the long-run derivative, $\gamma_{R\pi}$, exists since $d_\pi \geq 1$. Its value will be $\gamma_{R\pi} \propto (1-L)^{d_\pi - d_R}|_{L=1}$. Because the relative orders of integration under Case (v) are $d_\pi < d_R$, it follows from the properties of the hypergeometric function that, $(1-L)^{d_\pi - d_R}$, diverges from zero as $L \rightarrow 1$. Thus, to determine if the long-run Fisher effect holds under Case (v) one must estimate $\gamma_{R\pi}$, as in Case (iii), and test whether its value equals one.

3 Order of fractional integration

We desire to test the long-run Fisher effect hypothesis in 17 developed countries; Australia, Austria, Belgium, Canada, Denmark, France, Germany, Greece, Ireland, Italy, Japan, the Netherlands, Norway, Sweden, Switzerland, the United Kingdom, and the United States. Each country's price and interest rate data is the quarterly series published in the International Financial Series database. The inflation rate equals the log difference in the country's Consumer Price Index. The nominal interest rate, R, for Australia, Belgium, Canada, the UK, and the US, is the annual rate of return of its Treasury Bill; for Austria, Denmark,

[1] See Bae et al. (2005) for the derivation of this results.

France, Germany, Italy, the Netherlands, Sweden, and Switzerland the Money Market rate of return is used, and for Greece, Ireland, Japan, and Norway we use the return from Demand Deposits.

Each country's time period is reported in Table 2. Except for Australia, Austria, and France, all the data runs through the last quarter of 2004. The starting dates vary by country with the earliest being the first quarter of 1957 (Belgium, Canada, France, Germany, Japan, UK, US) and the latest starting at the beginning of 1979 (Norway).

To determine which of the cases in Table 2 the seventeen countries satisfy, we employ Nielsen's (2004) multivariate maximum likelihood estimator and likelihood ratio test to jointly estimate and test the values of d_π and d_R. The Nielsen MLE is a semiparametric estimator that ignores the series's short-run behavior, which has no bearing on the long-run Fisher effect hypothesis, and models only the long-run dynamics of the multivariate series.[2]

As a multivariate estimator, Nielsen's estimator and likelihood based test statistic overcomes the lack of power and the inefficiencies that univariate estimators of the fractional orders of integration and classical unit-root tests suffer by ignoring the correlation between the series. It is well established that the unit-root test suffers low power when the true data generating process is a fractionally integrated processes (see Sowell, 1990; and Diebold and Rudebusch, 1991). With Nielsen's MLE we replace the knife-edge approach of the classical unit-root test of $d = 1$ against $d = 0$, with a range of stationary $(-1/2 < d < 1)$ and nonstationary $(1 \leq d < \infty)$ possibilities. This continuum of values for d and the asymptotic chi-square distribution of the Nielsen estimator's likelihood ratio test makes carrying out inference concerning the relative value of d_π and d_R easy and straight forward.

We report the bivariate estimates of the fractional orders of integration, d_π and d_R, in Table 2, where the second and third columns of the table contain d_π and d_R, respectively. The table's last six columns list the likelihood ratio test statistics (LR-stat) and their corresponding p-values for the null hypothesis; $H_0 : d_\pi = 1$; i.e., the simple hypothesis that inflation is a unit-root process,[3] $H_0 : d_\pi = 1$ and $d_R = 1$; i.e., inflation and the nominal interest rate both follow a unit-root process, and $H_0 : d_\pi = d_R$; i.e., the orders of integration for inflation and the nominal interest rate are the same value.

[2] By using a semiparametric estimator, we also avoid the pitfalls associated with a misspecified fully parameterized autoregressive, fractionally integrated, moving average model of inflation and the nominal interest rate.

[3] Because the Nielsen estimator and its test statistics are likelihood based, the simple unit-root hypothesis test for inflation requires evaluating the likelihood function not only under the null but also under the univariate estimate of d_π. This requires a univariate estimate of d_π. Applying the Nielsen MLE to only inflation, we find the estimate of d_π to be nearly identical to the bivariate estimates. As a result, we choose not to report them.

The estimates of d_π suggest that a permanent change to inflation did not take place in any of the countries during the time period listed. The estimated differencing parameters range from 0.15 to 0.64, with the inflation rate in Canada, France, Italy, Norway, and the US reporting a fractional order of integration greater than 1/2. Because all of these country's d_π are larger than zero but still less than one, the effect of an unexpected monetary shock on inflation will be long-lived but not permanent. Even in those countries where the inflation rates's order of integration is largest (France, Italy, and the US, where d_π equals 0.64), in time the effect of the shock wears off. Hence, all seventeen countries fall under Case (i).

The simple and joint unit-root hypothesis test statistics found in Table 2 support this Case (i) classification. All seventeen of the simple unit-root tests for inflation in the fourth column of Table 2 clearly reject the presence of a unit-root. From the p-values listed in the fifth column, the unit-root null is rejected at significance levels nearly indistinguishable from zero.

The US's LR-stat found in the sixth column of Table 2 for the joint unit-root hypothesis tests provides a clear example where inflation does not follow a unit-root process. With a LR-stat of 29.19, the joint unit-root null is easily rejected at conservative significance levels. However, this rejection of the joint unit-root hypothesis could be caused either by d_π being significantly less than one, or d_R being significantly different from one. Notice that the estimated order of integration for the nominal interest rate in the US is closer to one ($d_R = 0.989$) than inflation's order of integration ($d_\pi = 0.64$). So rejection of the joint unit-root null follows from the order of integration in US inflation being significantly less than one. Hence, we again conclude that the US falls under Case (i).

The other country's joint unit-root LR-stat have a similar interpretation to that of the US, the difference being that in most of these countries the nominal interest rate's order of integration, d_R, is greater than one (Denmark, the Netherlands, Sweden, and the US are the only countries where d_R is found to be less than one). Thus, the interest rate in these countries is nonstationary. However, in these countries the absolute difference between the order of integration for inflation and one is larger than the difference between d_R and one. In other words, rejection of the joint unit-root null is either because d_π and d_R are both significantly different from one, or, given the simple unit-root hypothesis for inflation, because d_π is significantly less than one. Not because d_R is only different from one. We again conclude that the long-run Fisher effect hypothesis cannot be tested for in any of the seventeen countries.

Country	d_π	d_R	$H_0: d_\pi = 1$ LR-Stat	p-value	$H_0: d_\pi = 1, d_R = 1$ LR-stat	p-value	$H_0: d_\pi = d_R$ LR-stat	p-value
Australia (1969:3-2002.2)	0.4995	1.0402	43.67	0	45.84	0	35.60	0
Austria (1967:1-1998.4)	0.3649	1.0980	70.20	0	71.76	0	59.58	0
Belgium (1957:1-2004:4)	0.1561	1.0622	114.86	0	116.30	0	102.72	0
Canada (1957:1-2004:4)	0.5333	1.0733	48.40	0	54.27	0	44.19	0
Denmark (1957:1-2004:4)	0.4230	0.8041	45.56	0	55.90	0	21.86	0
France (1957:1-1999:1)	0.6370	1.1521	20.29	0	24.42	0	23.18	0
Germany (1957:1-2004:4)	0.3723	1.1838	74.38	0	82.22	0	77.07	0
Greece (1961:1-2004:4)	0.2969	1.2075	107.37	0	122.26	0	121.71	0
Ireland (1962:1-2004:4)	0.4490	1.1207	62.73	0	65.98	0	57.48	0
Italy (1971.1-2004.4)	0.6380	1.0840	16.80	0	40.07	0	36.86	0
Japan (1957:1-2004:4)	0.3915	1.1373	80.58	0	93.98	0	87.18	0
Netherlands (1960:1-2004:4)	0.3355	0.8803	77.57	0	79.34	0	37.69	0
Norway (1979.1-2004.4)	0.5070	1.1213	30.78	0	33.31	0	30.29	0
Sweden (1963.1-2004.4)	0.4282	0.9651	61.60	0	61.60	0	38.25	0
Switzerland (1975.4-2004.4)	0.3926	1.0452	44.76	0	47.11	0	36.75	0
UK (1957:1-2004:4)	0.4426	1.1463	70.54	0	74.47	0	62.20	0
US (1957:1-2004:4)	0.6397	0.9893	26.78	0	29.19	0	21.60	0
US (1957:1-1979:3)	0.7770	1.0682	4.44	0.04	7.79	0.02	6.75	0.03
(1979:4-1982:3)	0.6688	0.7863	0.86	0.35	0.10	0.60	0.38	0.40
(1982:4-2004:4)	0.5437	1.0994	15.77	0	21.81	0	19.28	0

Table 2: Maximum likelihood estimates of a bivariate fractionally integrated model of the log change in quarterly Consumer Price Index, π, and nominal short-term interest rate, R, along with a simple test of the unit-root hypothesis, $d_\pi = 1$, a joint test of the hypothesis that π and R follow a unit-root process ($d_\pi = 1$ and $d_R = 1$), and the hypothesis $d_\pi = d_R$. Under the null, the LR-stat has the standard asymptotic chi-square distribution.

3.1 Robustness to monetary regimes

Some have found long-memory behavior to be a spurious result associated with a long, but infrequent, monetary regime (see Diebold and Inoue, 2001; and Jensen and Liu, forthcoming). To test the robustness of the US's Case (i) classification to different monetary regimes, we split the original sample into three subperiods, 1957:1 to 1979:3, 1979:4 to 1982:3, and 1982:4 to 2004:4. We choose these time periods because the relationship between inflation and nominal interest rates is believed to have been altered as a result of changes in the Fed's monetary policy on these dates (see Clarida and Friedman, 1984; and Huizinga and Mishkin, 1986).[4] The post-October 1979 period is also the only time period Rose (1988)

[4]These three regimes also closely match up with the structural breaks in inflation found by Evans and Lewis (1995).

finds any evidence of nonstationary behavior in US inflation. Estimates from these time periods should help us determine if post-war inflation's fractional stationary behavior is spurious or not.

The bivariate MLE of d_π and d_R for the three time periods are found at the bottom of Table 2. Because the second time period from 1979:4 to 1982:3 contains only 12 observations, any inferences made concerning this period's parameter estimates will not stand up to the asymptotic scrutiny required for the estimator. Hence, we make no inference about testing the long-run Fisher effect during this middle time period. Instead, we focus our attention on the results of the relative fractional order of integration for the pre-1979 and post-1982 time periods.

The post-1982 estimate of $d_\pi = 0.54$ provides strong evidence against any permanent change having occurred in US inflation. During this time period d_π is smaller than the estimate from using the entire time series. The post-1982 estimate of d_π also contradicts Rose (1988), who found inflation to be an I(1) series over the post-1979 period. Our estimate of d_π, however, does support Mishkin's (1992) conclusion that inflation did not exhibit a stochastic trend over this time period.

Because the pre-1979 estimate of $d_\pi = 0.78$ is larger than the post-1982 estimate, monetary shocks have a more lasting impact on inflation during the pre-1979 period. However, d_π is still significantly less than one so the impact will not be permanent. The nominal interest rate's order of integration is also slightly larger over the pre-1979 period than it was during the entire post-war period. The nominal interest rates d_R is now greater than one at 1.07 and d_π has increased to 0.78 from 0.64. These larger orders of integration impact the significance level of both the simple and joint unit-root null hypothesis by raising their p-values from zero for the entire time series to a p-value of 0.04 and 0.02, for the pre-1979 periods simple and joint unit-root hypothesis, respectively. Although these significance levels are larger, rejection at the fourth percentile is still good enough to conclude that during the pre-1979 period inflation in the US was not permanently affect by a monetary shock.

3.2 Monthly data

To lend further support to the mean-reverting, long-memory behavior of inflation and the Case (i) classification of testing the long-run Fisher effect, we estimate d_π and d_R using monthly measures of inflation and the nominal short-term interest rate. Chambers (1997) and Souza (2005) show that a long-memory series observed at two different frequencies, for example, monthly and quarterly observations, will have the same fractional order of integration. If inflation and nominal interest rates are long-memory, fractionally integrated series, we would thus expect to find the estimates of d_π and d_R with monthly data to be similar to those found with quarterly data.

Out of the original 17 developed countries, we estimate the order of integration in the 13 countries found in Table 3. These countries were selected because their monthly data series is available for the same time period as their quarterly series. The monthly estimates of d_π, d_R, and the relevant test statistics are found in Table 3.

In each of the 13 countries, the monthly estimate of d_π is smaller than its corresponding quarterly estimate. The monthly estimates of d_π range between 0.1 to 0.2 units smaller than their quarterly estimates. Part of this difference can be attributed to the monthly series having four times as many observations as the quarterly measurements of inflation and the short-memory type of noise inherent in monthly data.

A similar difference is also found in the monthly estimates of d_R. However, unlike all the monthly estimate of d_π being greater than their quarterly estimate, some of the monthly estimates of d_R are smaller (Austria, Denmark, Japan, the Netherlands, Sweden, and Switzerland) , while for others it is larger (Belgium, Canada, Italy, the UK, and the US). In only three countries (Germany, Japan, and Switzerland) does the dynamics of the series change from nonstationary behavior in the quarterly nominal interest rate to stationary behavior in the monthly series.

These smaller monthly estimates of d_π provide more evidence that inflation in these countries follow a stationary process and that our Case (i) classification is correct. Because a smaller d_π leads to a larger LR-stat, the LR-stats in Table 3 are larger than the test statistics found in Table 2. We, thus, again conclude that the long-run Fisher effect cannot be tested even with higher frequency monthly data.

3.3 GDP deflator and long-term interest rates

For those countries with post-war, quarterly, GDP price deflator and long-term interest rate series long enough to reveal their long-run behavior, we estimate d_π and d_R using the log-differenced GDP deflator and a long-term nominal interest rate. Out of the original seventeen countries, there are six that qualify; Canada, France, Germany, Japan, the UK, and the US. Each country's long-term nominal interest rate is set equal to the government bond yield reported in the International Financial Series database.

Rapach (2003) also tested for the long-run Fisher effect using the quarterly GDP price deflator and long-term nominal interest rate in three of the six countries; Canada, the UK, and the US. The data for France was unavailable, and Rapach excluded Germany and Japan because his earlier results with annual data failed to find a unit-root in these

Country	d_π	d_R	$H_0 : d_\pi = 1$ LR-Stat	p-value	$H_0 : d_\pi = 1, d_R = 1$ LR-stat	p-value	$H_0 : d_\pi = d_R$ LR-stat	p-value
Austria (1967:1-1998.12)	0.2761	0.9727	176.52	0	176.04	0	135.66	0
Belgium (1957:1-2004:3)	0.0885	1.0955	366.61	0	374.22	0	347.31	0
Canada (1957:1-2004:3)	0.3263	1.1722	269.96	0	291.93	0	276.20	0
Denmark (1972:1-2000:12)	0.2942	0.6990	167.85	0	206.69	0	58.72	0
France (1957:1-2002:9)	0.4887	1.0882	117.34	0	122.48	0	107.67	0
Germany (1960:1-2004:3)	0.3078	0.7714	221.68	0	262.30	0	94.36	0
Italy (1971.1-2004.2)	0.4775	1.2010	119.76	0	149.90	0	147.69	0
Japan (1957.1-2004.2)	0.2075	0.9015	305.07	0	313.82	0	204.00	0
Netherlands (1960.1-1998.12)	0.1041	0.8795	301.57	0	309.65	0	190.43	0
Sweden (1962.12-2001.10)	0.2781	0.9334	231.42	0	233.52	0	157.97	0
Switzerland (1975.9-2004.3)	0.2457	0.7177	189.96	0	228.59	0	74.72	0
UK (1964.1-2004.2)	0.3482	1.2501	194.81	0	231.69	0	228.70	0
US (1964.1-2004.3)	0.4556	1.0861	139.25	0	147.51	0	127.46	0

Table 3: Maximum likelihood estimates of a bivariate fractionally integrated model of the log change in the monthly Consumer Price Index, π, and the nominal short-term interest rate, R, along with a simple test of the unit-root hypothesis, $d_\pi = 1$, a joint test of the hypothesis that π and R follow a unit-root process ($d_\pi = 1$ and $d_R = 1$), and the hypothesis $d_\pi = d_R$. Under the null, the LR-stat has the standard asymptotic chi-square distribution.

country's inflation rates. Rapach rejected the long-run Fisher effect hypothesis for Canada and the UK, but was unable to reject the hypothesis for the US.

Our estimates of the fractional order of integration with the quarterly deflator and long-term nominal interest rate are found in Table 4. The estimates of d_π suggest that in addition to excluding Germany and Japan from the Fisher effect test, Rapach should have also excluded Canada, the UK, and the US. In each of these countries, d_π is significantly less than one and smaller than the estimates found using quarterly CPI. In the extreme case, the inflation rate for Japan is now negatively integrated with $d_\pi = -0.1$; i.e., Japan's inflation rate is anti-persistent.

Except for the US, the estimates of d_R are smaller than the estimates found with the short-term interest rate. Our Case (i) classification for these six countries is thus robust to the price index measure and the maturity length of the fixed income security used in calculating the inflation and nominal interest rate series.

Country	d_π	d_R	$H_0: d_\pi = 1$ LR-Stat	p-value	$H_0: d_\pi = 1, d_R = 1$ LR-stat	p-value	$H_0: d_\pi = d_R$ LR-stat	p-value
Canada (1957:1-2004:4)	0.5091	1.0482	52.92	0	56.37	0	47.79	0
France (1970:1-2004:2)	0.2060	1.0933	115.98	0	118.36	0	105.71	0
Germany (1960:1-2004:.2)	0.2465	1.1444	124.60	0	128.84	0	111.91	0
Japan (1966:4-2004.3)	-0.1021	1.0360	138.22	0	144.74	0	134.14	0
UK (1957.1-2004.2)	0.3203	1.0944	89.46	0	91.15	0	82.63	0
US (1957.1-2004.4)	0.6378	1.1018	32.78	0	40.25	0	36.08	0

Table 4: Maximum likelihood estimates of a bivariate fractionally integrated model of the log change in the quarterly GDP price deflator, π, and the long-term interest rate, R, along with a simple test of the unit-root hypothesis, $d_\pi = 1$, a joint test of the hypothesis that π and R follow a unit-root process ($d_\pi = 1$ and $d_R = 1$), and the hypothesis $d_\pi = d_R$. Under the null, the LR-stat has the standard asymptotic chi-square distribution.

4 Conclusion

This paper has studied the long-run Fisher effect in seventeen industrialized countries over the post-war period. Using a bivariate estimator of inflation and nominal interest rate's fractional order of integration, and quarterly and monthly measures of inflation and short and long-term nominal interest rates, we have found inflation to be a slow, mean-reverting, fractionally integrated process in all seventeen countries. One important implication is that, because inflation never experiences a permanent change, the long-run Fisher effect cannot be tested with the reduced form approach. Thus, this paper provides a reason why past reduced form tests of the long-run Fisher effect have been unable to find support for the hypothesis. They have simply been applying a test that is not valid given the stationary behavior of the inflation series.

The prevalence of this slow mean-reversion in inflation causes us to wonder what the mechanisms are between monetary policy and the inflation rate and whether inflation is actually a monetary phenomenon. Although monetary shocks do not permanently affect inflation, the long-memory behavior of inflation suggests that monetary policy can have a lasting impact on inflation. How big, or how long a fractionally integrated inflation series will be affected by a change in monetary policy is an important, but, unanswered question.

The presence of fractional integration in the inflation rate of so many industrialized economies also causes us to wonder if this long-memory behavior is the result of some common economic institution found in these economies. Perhaps it is a universal feature found in their banking systems, such as a common monetary rule, or a statistical artifact of aggregating up prices of different goods and services into a price index. These and other questions related to the fractional nature of inflation will surely be of interest to monetary economists.

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