Financial Intermediation and Capital Reallocation

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Abstract

To understand the link between financial intermediation activities and the real economy, we put forward a general equilibrium model where agency frictions in the financial sector affect the efficiency of capital reallocation across firms and generate aggregate economic fluctuations. We develop a recursive policy iteration approach to fully characterize the nonlinear equilibrium dynamics and the off-steady state crisis behavior. In our model, adverse shocks to agency frictions exacerbate capital misallocation and manifest themselves as variations in total factor productivity at the aggregate level. Our model endogenous generate counter-cyclical volatility in aggregate time series and counter-cyclical dispersion of marginal product of capital and asset returns in the cross-section.

Keywords: Financial Intermediation, Capital Misallocation, Volatility, Crisis, Limited enforcement

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I Introduction

The purpose of this paper is to study the mechanism through which financial intermediation affects macroeconomic fluctuations and asset prices. We present a general equilibrium model to link intermediation activities in the financial sector to capital reallocation across non-financial firms in the real sector. We show that shocks originated from the financial sector can account for a significant fraction of macroeconomic fluctuations.

Two main features distinguish our approach from the previous literature. The first is the emphasis on capital reallocation across firms with heterogenous productivity. The second is the recursive policy function iteration approach which allows us to obtain global solutions of a general equilibrium model with occasionally binding incentive compatibility constraints.

We focus on a heterogenous firm setup for two reasons. In aggregate, the U.S. corporate sector is almost never constrained: it typically has more cash flow than what is needed to finance investment. As shown by Chari (2015), a typical feature of models with agency frictions is that firms do not pay dividend whenever they are financially constrained. However, the net dividend payment of the U.S. corporate sector as a whole is almost always positive, and significantly so most of the time. To understand why some firms are constrained in downturns while others are not, it is necessary to have a model with heterogenous firms.

From a quantitative point of view, models with capital reallocation allow financial frictions to play a significant role in generating large economic fluctuations. In representative firm models, financial frictions affect the efficiency of intertemporal investment. Previous researchers (for example, Kocherlakota (2000)) have argued that this mechanism alone is unlikely to cause large economic fluctuations because investment is only a small fraction of the total capital stock of the economy.[1] In contrast, recent study on capital misallocation, for example, Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), found that large efficiency gains can be achieved by improving capital misallocation, on the order of $30-50\%$.

We develop a recursive policy function iteration approach to fully account for the dynamics of the occasionally binding constraints of our model. A prominent feature of major financial crisis is elevated volatility at the aggregate level and sudden increases in the cross-sectional dispersions in prices and quantities. The majority of previous literature with financial frictions are solved using local approximation methods, which typically cannot capture the time variation of volatility implied by the model. The recursive policy function iteration

[1] In standard RBC models, annual investment is about ten percent of capital stock and capital contributes to roughly one third of total output. According this calculation, the maximum effect of investment on output is about 3.3%.

method allows us to characterize the variation of the tightness of the incentive compatibility constraints across time and across firms, which is the key feature of our model.

To formalize the link between financial intermediation and capital reallocation, we develop a model of financial intermediation where firms are subject to idiosyncratic productivity shocks and credit transactions must be intermediated. Due to the heterogeneity in productivity, reallocating capital across firms improves efficiency in production, but requires high productivity firms to borrow from the rest of the economy. In addition, because of the limited enforcement of lending contracts, the accumulation of intermediaries' debt or declines in their net worth increase their incentive to default and limit their borrowing capacity. These features of our model have two implications. In the time series, adverse shocks to intermediary net worth weaken their borrowing capacity and slow down the formation of new capital. In the cross section, intermediaries who finance for high productivity firms are more likely to be affected, because they need to borrow more from the rest of the economy and have a higher incentive to default. The later mechanism amplifies negative primitive shocks by lowering the efficiency of the reallocation of the existing capital stock.

We consider two versions of our model in calibration: one with total factor productivity (TFP) shocks and another with financial shocks. We calibrate the volatility of the primitive shocks to match the volatility of output in the U.S. data and evaluate the quantitative importance of financial frictions in both specifications. In our model with TFP shocks, the amplification effect from agency frictions accounts for about 10% of the total volatility of output and is fairly temporary. The magnitude of amplification is modest because of the well-known difficulty for real business cycle (RBC) models to generate large volatilities in asset prices. Because productivity shocks are not associated with significant variations in asset prices and intermediary net worth, they induce only a limited amount of amplification from financial frictions.

Motivated by the lack of volatility in asset prices in the model with TFP shocks and the finding in the asset pricing literature that a large fraction of asset price variations can be attributed to discount rate shocks, our second calibration models financial shocks as exogenous variations in bank managers' discount rate.[2] Two features distinguish this model from the one with TFP shocks: persistence and asymmetry. A temporary shock to banks' net worth lowers their borrowing capacity and reduces the efficiency of capital reallocation in the subsequent period. Elevated capital misallocation depresses output and triggers another round

[2]This is much smaller than the variation in discount rates typically found in the asset pricing literature, for example, Campbell and Shiller (1988), and more recently, Lettau and Ludvigson (forthcoming).

of drop in bank net worth. This effect propagates over time and has a long-lasting impact on future economic growth. In addition, negative shocks tighten banks' financing constraints and make the economy more vulnerable to future shocks, whereas positive shocks relax these constraints and have a smaller impact on capital misallocation. In the extreme case, continued negative shocks deplete banking sector net worth, lower the borrowing capacity of all banks to suboptimal levels, and send the economy into a financial crisis marked by heightened macroeconomic volatility, large and persistent drops in output and asset prices, and sharp increases in interest rate spreads.

In our benchmark calibration, the standard deviation of banker's discount rate is about 2.3% at the annual level. Nevertheless, the model matches well the macroeconomic moments in the U.S. and produces a volatility of aggregate output of 3.6% from the capital reallocation channel. More importantly, it endogenously generates a counter-cyclical volatility in the time series of aggregate output and consumption, a counter-cyclical dispersion in the cross section of firm output and stock returns, and a counter-cyclical efficiency of capital reallocation and capital utilization as in the data.

Our paper belongs to the literature on macroeconomic models with a financial intermediary sector.[3] The papers that are most related to our are Gertler and Kiyotaki (2010), He and Krishnamurthy (2014), and Rampini and Viswanathan (2014). The nature of agency frictions in our model is the same as that in Gertler and Kiyotaki (2010). Different from the papers, we allow heterogeneity in firms' productivity and evaluate the quantitative importance of the capital reallocation channel.

Several other papers also emphasize the importance of capital reallocation in understanding credit market frictions. For example, Eisfeldt and Rampini (2006), Eisfeldt and Rampini (2008), Shourideh and Zetlin-Jones (2012), Kurlat (2013), Chen and Song (2013), Fuchs et al. (2013), Brunnermeier and Sannikov (2014), Chari (2014), Li and Whited (2014), and Midrigan and Xu (2014). Eisfeldt and Rampini (2006) provide empirical evidence that the amount of capital reallocation is procyclical and the benefit of capital reallocation is countercyclical. They also present a model where the cost of capital reallocation is correlated with TFP shocks to rationalize these facts. Eisfeldt and Rampini (2008), Kurlat (2013), Fuchs

[3]There is a vast literature on macro models with credit market frictions, which we do not attempt to summarize here. A partial list includes Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), Kiyotaki and Moore (2005), Bernanke et al. (1999), Krishnamurthy (2003), Kiyotaki and Moore (2008), Mendoza (2010), Gertler and Karadi (2011), Jermann and Quadrini (2012), He and Krishnamurthy (2012), He and Krishnamurthy (2013), Li (2013), and Bianchi and Bigio (2014). Quadrini (2011) and Brunnermeier et al. (2012) provide comprehensive reviews of this literature.

et al. (2013), and Li and Whited (2014) study adverse selection problems, while we focus on limited enforcement of financial contracts. From the modeling perspective, we differ from the above papers by explicitly allowing for a financial intermediary sector in our model and by using empirical evidence on bank loans and interest rate spreads to discipline our calibration. Quantitatively, we show that relatively small shocks to agency frictions are able to generate quantitatively large macroeconomic fluctuations. Finally, none of the above papers link the countercyclical volatility in aggregate time series to countercyclical dispersion in the cross section in a unified general equilibrium framework.

The idea that shocks may originate directly from the financial sector and affect economic activities is related to the setup of Jermann and Quadrini (2012). Different from Jermann and Quadrini (2012), our paper focus on financial intermediation and capital reallocation and their connections with the macroeconomy.

Our paper is also related to the literature in economics and finance that emphasize the importance of counter-cyclical volatility in understanding the macroeconomy and asset markets. Many authors have documented a strong countercyclical relationship between real activity and uncertainty as proxies by stock market volatility and/or dispersion in firm level earnings and productivity, for example, Bloom (2009), Bloom et al. (2012), Bachmann et al. (2013), and Jurado et al. (2015), among others. A large literature in asset pricing emphasizes the importance of counter-cyclical volatility in understanding stock market returns, for example Bansal and Yaron (2004), Bansal et al. (2012), and Campbell et al. (2013). Our model generates countercyclical volatility as an endogenous equilibrium outcome even though the primitive shocks are homoscedastic.

Our computational approach is related to recent development in using global methods to solve macro models with financial frictions. Brunnermeier and Sannikov (2014), He and Krishnamurthy (2012, 2014), and Maggiori (2013) use continuous time methods to obtain global solutions. Their models all have a single state variable and equilibrium conditions can be reduced to ordinary differential equations, whereas our model involves multiple state variables in order to quantitatively capture a rich set of macroeconomic moments. Mendoza and Smith (2006) study small open economies with margin requirements and use value function iteration to solve their model. We use a policy function iteration approach which greatly improves the numerical efficiency in our general equilibrium setup because it does not involve multiple recursive operators and it uses first order conditions to reduce optimization problems to solving nonlinear equations. Our method can potentially be applied to many other models in this literature, which are often solved using local approximation methods.

The rest of the paper is organized as follows. We provide a summary of some stylized facts that motivate the development of our model in Section II. We describe the model setup in Section III. In Section IV, we discuss the construction of the Markov equilibrium of our model and the recursive policy function iteration approach. In Section V, we analyze a deterministic version of our model to illustrate qualitatively the link between financial intermediation and capital reallocation. We calibrate our model and evaluate its quantitative implications on macroeconomic quantities and asset prices in Section VI. Section VII concludes.

II Stylized Facts

Below we present several stylized facts that motivate our interest in studying the link between financial intermediation and capital reallocation. We first show that measured TFP is highly correlated with measured efficiency of capital reallocation.

1. Measured total factor productivity (TFP) is highly correlated with a measure of the efficiency of capital reallocation and the rate of capital utilization.[4]

   In Figure 1, we plot the time series of log TFP (dashed line), measured efficiency of capital reallocation (solid line) and log capital utilization rates (dash-dotted line) in the U.S., where all series are HP filtered. We follow a similar procedure as Hsieh and Klenow (2009) and measure capital misallocation by the variance of the cross-sectional distribution of log marginal product of capital within narrowly defined industries (classified by the four-digit standard industry classification code) and translate this measure into log TFP units.[5] The measured efficiency of capital reallocation tracks the time series log TFP remarkably closely, indicating that the efficiency of capital reallocation may account for a significant fraction of variations in measured TFP. The same pattern is true for capital utilization rates: economic downturns are typically also associated with sharp declines in capital utilization rates.

2. The total volume of bank loans is procyclical. It is negatively correlated with measures of volatility and capital misallocation.

[4] Capital under-utilization can be interpreted as a special form of misallocation.

[5] We use the formula in Hsieh and Klenow (2009) to translate the variance of log marginal product of capital into a measure of the efficiency of capital reallocation. In Appendix A, we show that this is equivalent to a first order approximation of the efficiency of capital reallocation measured in log TFP units. We detail the data construction in Appendix B.

The above fact is what motivates our theory of financial intermediation and its connection with capital reallocation. We calculate the total volume of bank loans of the non-financial corporate sector in the U.S. from the Flow of Funds Table. Total bank loans are calculated as the difference between total corporate credits and corporate bond issuance. The details of the data construction can be found in Appendix B.

We plot the annual changes in the total volume of bank loans and the GDP growth rate of the U.S. economy in Figure 2. The shaded areas indicate NBER defined recessions. It is clear that the total volume of bank loans is strongly procyclical. The correlation between the two series is 0.42 at the annual level.

In Figure 3, we plot the annual changes in the total volume of bank loans and the measured cross-sectional dispersion in the marginal product of capital from the COMPUSTAT data set. We provide the details of the construction of the dispersion measure in Appendix B. Clearly, the innovations of the total volume of bank loans are strongly negatively correlated with our measure of capital misallocation — the correlation of the two series is -0.43 at the annual frequency. This is consistent with the key mechanism of our model: when banks are constrained, the total volume of bank loans decreases, and capital reallocation is less efficient.

We plot the annual changes in the total volume of bank loans and aggregate stock market volatility in Figure 4. Stock market volatility is calculated by aggregating realized variance of monthly returns. The correlation between the two time series is about -0.25 at the annual level. We also plot the cross-sectional dispersion of firm profit in Figure 5. It is clear that changes in the total volume of bank loans is strongly negatively correlated with both measures of volatility.

The rest of the stylized facts are well-known. We therefore do not provide detailed discussion here but refer to the relevant literature. The second fact is about the business cycle properties of capital reallocation. This is documented in Eisfeldt and Rampini (2006).

3. The amount of capital reallocation is procyclical and the cross-sectional dispersion of marginal product of capital is countercyclical.

The third, fourth and fifth facts are about the cyclical properties of the volatility of macroeconomic quantities and asset returns and are well-known in the macroeconomics literature and the asset pricing literature, for example, Bloom (2009), Bansal et al.

(2012) and Campbell et al. (2001).

4. The volatility of macroeconomic quantities, including consumption, investment, and aggregate output is countercyclical.

5. The volatility of aggregate stock market return is also countercyclical. Equity premium and interest rate spreads are countercyclical.

6. The volatility of idiosyncratic returns on the stock market is countercyclical.

In the following sections, we setup and analyze a general equilibrium model with financial intermediation and capital reallocation to provide a theoretical and quantitative framework to interpret the above facts.

III Model Setup

In this section, we describe a general equilibrium model with heterogenous firms and with agency frictions in the financial intermediation sector.

A Non-financial Firms

There are three types of non-financial firms in our model, intermediate goods producers, final goods producers and capital goods producers. Because non-financial firms do not make intertemporal decisions in our model, we suppress the dependence of prices and quantities on state variables in this subsection.

The specification of the production technology of intermediate goods and final goods follows the standard monopolistic competition setup in the capital misallocation literature, for example, Hsieh and Klenow (2009). Final goods are produced by a representative firm on a perfectly competitive market using a continuum of intermediate inputs. We normalize the price of final goods to one and write the profit maximization problem of the final goods producer as:

$$\max \left\{ Y - \int_{[0,1]} p_j y_j dj \right\}$$

$$Y = \left[\int_{[0,1]} y_j^{\frac{\eta-1}{\eta}} dj \right]^{\frac{\eta}{\eta-1}}, \tag{1}$$

where p_j and y_j are the price and quantity of input j produced on island j, respectively. Y stands for the total output of final goods. The parameter η is the elasticity of substitution across input varieties. The constant return to scale technology and the fact that the final goods market is perfectly competitive imply that final goods producers earn zero profit in equilibrium. In this case, final goods producer's demand function for input variety j can be written as:

$$p_j = \left[\frac{y_j}{Y}\right]^{-\frac{1}{\eta}}. \tag{2}$$

There is continuum of monopolistically competitive intermediate goods producers indexed by $j \in [0,1]$, each producing a different variety on a separate island.[6] We use j as the index for both the intermediate input and the island on which it is produced. The profit maximization problem for the producer on island j is given by:

$$\begin{aligned} D_F(j) &= \max\left\{p_j y_j - MPK_j \cdot k_j - MPL \cdot l_j\right\} \\ \textit{subject to}: \quad & p_j = [y_j/Y]^{-\frac{1}{\eta}} \\ & y(j) = \bar{A} a_j k_j^{\alpha} l_j^{1-\alpha}. \end{aligned} \tag{3}$$

Here, the production of variety j requires two factors, capital k_j and labor l_j. $\bar{A}$ is the aggregate productivity common across all firms. a_j is island j-specific idiosyncratic productivity shock, which we assume to be i.i.d. over time. MPK_j is the rental price of capital on island j and MPL is the *economy wide* wage rate. Because our focus is on capital reallocation across islands with different idiosyncratic productivity shocks, we allow the rental price of capital to be island specific, but assume frictionless labor market across the whole economy. We use $D_F(j)$ to denote the total profit of firm j, which is paid to households as dividend.

We assume, for simplicity, that there are only two possible realizations of idiosyncratic productivity shocks, a_H and a_L. We denote

$$\Pr ob\,(a = a_H) = \pi; \quad \Pr ob\,(a = a_L) = 1 - \pi. \tag{4}$$

We adopt a convenient normalization,

$$\pi a_H^{1-\eta} + (1-\pi)\, a_L^{1-\eta} = 1. \tag{5}$$

[6] We use the terminology "island" to emphasize that capital cannot move freely among producers of different input varieties. The details of capital market frictions is introduced in Section C.

As will become clear later, the above condition implies that the average idiosyncratic productivity is one and total output is given by the standard Cobb-Douglas production function, $\bar{A}K^{\alpha}N^{1-\alpha}$ in the absence of misallocation.

In addition to the standard monopolistic competition setup, we specify a risk-free storage technology, which endogenizes variable capital utilization in our model. More importantly, it allows us to capture the effect of "fly-to-safety" or "fly-to-quality": when financial intermediaries are constrained, volatility spikes, and capital moves into the risk-free storage technology despite the high productivity of intermediate goods producers. We assume that current period capital can be used for two purposes: producing output and storage. The capital goods producers maximize profit by operating the following storage technology:

$$D_K = \max_{K_S} \left\{ G\left(K_S, K\right) - QK_S \right\}, \tag{6}$$

where K_S is the total amount of current period capital used in the storage technology, $H\left(K_{S,}K\right)$ is a concave and constant return to scale production technology. We use D_K to denote the profit of capital goods producers, which is paid back to household as dividend and Q to denote the market price of capital.

We assume that capital depreciation at a constant rate δ if used for production. Therefore the law of motion of next period capital is

$$K' = G\left(K_S, K\right) + \left(1-\delta\right) K_U + I, \tag{7}$$

where I is the total amount of new investment in the current period. Without loss of generality, we denote $u = \frac{K_u}{K}$ and

$$G\left(K_S, K\right) = g\left(\frac{K_S}{K}\right) K$$

for some concave function $g\left(\cdot\right)$. Using the resource constraint,

$$K_U + K_S = K,$$

equation (7) can be simplified to:

$$K' = \left[g\left(1-u\right) + \left(1-\delta\right) u\right] K + I.^{7} \tag{8}$$

[7]It is more common in the literature to assume total depreciation to depend on u and write $K' =$

B Household

There is a representative household with log preferences, and is endowed with one unit of labor in every period which it supplies inelastically to firms. The representative household owns the ultimate claims of all assets in the economy. To make the intermediation problem non-trivial and to prevent the model from collapsing into a single representative agent setup, as in Gertler and Kiyotaki (2010), we have assumed incomplete market between the household and the intermediary. That is, the only financial contract allowed between the household and the financial intermediary is a risk-free deposit account. The household does not have access to markets that trade aggregate state-contingent payoffs, but instead must delegate its investment decisions in capital markets to financial intermediaries. The household starts the current period with total amount of disposable wealth W, and decides the allocation of wealth between consumption and investment in the risk-free account with banks.

We assume (and later verify) that the household's utility maximization problem can be written in a recursive fashion:

$$V(\mathbf{Z}, W) = \max_{C, B_f} \ln C + \beta E\left[V(\mathbf{Z}', W')\right]$$

$$C + B_f = W$$

$$W' = B_f R_f(\mathbf{Z}) + \int D_F(j)(\mathbf{Z}')\,dj + \int D_B(j)(\mathbf{Z}')\,dj + D_K(\mathbf{Z}') + MPL(\mathbf{Z}'). \quad (9)$$

In the above maximization problem, we assume that there exists a vector of Markov state variables $\mathbf{Z}$, the law of motion of which will be specified later, that completely summarizes the history of the economy.[8] Taking the equilibrium interest rate $R_f(\mathbf{Z})$, the dividend payments from intermediate goods producers, $\{D_F(j)(\mathbf{Z}')\}_{j\in[0,1]}$, from the capital goods producers, $D_K(\mathbf{Z}')$, and from the banks, $\{D_B(j)(\mathbf{Z}')\}_{j\in[0,1]}$ as given, the household makes its optimal consumption C and saving decisions B_f given its initial amount of disposable wealth, W. Household income includes total savings in the bank account, $B_f R_f(\mathbf{Z})$, total dividends (monopolistic rents) from intermediate goods producers, $\int D_F(j)(\mathbf{Z}')\,dj$, total dividend payment from banks, $\int D_B(j)(\mathbf{Z}')\,dj$, total dividend payment from capital goods producers, $D_K(\mathbf{Z}')$, and total labor income, $MPL(\mathbf{Z}')$.

$(1-\delta(u))K + I$. Our parameterization implies that utilized capital depreciate at a constant rate, which simplifies our numerical analysis. At the same time, it allows us to capiture the same dynamics as the variable capital utilization literature.

[8] In another words, we will focus on Markov equilibria with state variable $\mathbf{Z}$. We do not explicitly specify $\mathbf{Z}$ here. We construct the Markov equilibrium with the state variable $\mathbf{Z}$ in Section IV of the paper.

C Financial Intermediaries

There is one financial intermediary on each island.[9] Financial intermediaries or bankers are the only agents in the economy who have access to the capital markets.

Consider a bank who enters into a period with initial net worth N. It chooses the total amount of borrowing from the household, B_f, amount of borrowing from peer banks, B_I, and the total amount of capital stock for the next period K'. Because there is no capital adjustment cost, the price of capital is one, and banks' budget constraint is:

$$K' = N + B_f + B_I.^{10} \tag{10}$$

In our model, the total amount of capital for the next period, K' is determined at the end of the current period before the realization of shocks of the next period. That is, we assume one period time to plan as in standard RBC models. However, different from the standard representative firm setup, capital can be reallocated across firms after idiosyncratic productivity shocks are realized, which we describe in detail.

Figure 6 illustrates the time of events in period t and period $t+1$. At the end of period t, the household has total disposable income W and the total net worth of the intermediary sector is N. The household wealth is allocated between consumption in the current period, C and a risk-free deposit with the banks, B_f. From the bank's perspective, the total net worth and the total consumer loans, B_f are used to purchase capital. At the end of period t, a typical bank purchased K' amount of capital for period $t+1$ production before the realization of the productivity shocks in $t+1$.

Period $t+1$ is divided into four subperiods. In the first subperiod, the aggregate productivity shock A' and the idiosyncratic productivity shock, a' are realized and the capital reallocation market opens. Banks on the high (idiosyncratic) productivity islands have an incentive to purchase more capital on the reallocation market and banks on the low productivity islands have an incentive to sell. Note that transactions on the capital reallocation market must be done by issuing interbank credit, because at this point production has not begun and banks have not received payment from firms yet. Production happens in the second subperiod, and firms pay back the cost of capital to local banks at the end of the

[9] Because financial intermediaries on each island face competitive capital markets, one should interpret our model as having a continuum of identical financial intermediaries on each island.

[10] With a slight abuse of notation, we use B_f as both the amount of saving of the household and the amount of borrowing of the bank. We do so to save notation, because market clearing requires that the demand and supply of bank loans must equal.

second subperiod.

In the third subperiod, banks payback their interbank loans and household deposits. Importantly, after banks receive payment from local firms and before they pay back loans to creditors, banks have an opportunity to default. Upon default, bankers can abscond with a fraction of their assets, and set up a new bank to operate on some other island.

In the last subperiod, bankers clear their interbank transactions and consumers receive dividend payments from banks and firms, risk-free returns from bank deposits and make their consumption and saving decisions. At this point, bank net worth is allowed to move freely across islands.[11]

We now describe in detail the bank's problem in the third subperiod. We use $RA_j(\mathbf{Z}')$ to denote the total amount of capital purchased on the reallocation market by intermediary j in state $\mathbf{Z}'$.[12] Let $Q(\mathbf{Z}')$ denote the price of capital on the capital reallocation market in state $\mathbf{Z}'$, and let $Q_j(\mathbf{Z}')$ denote the price of capital on an island with idiosyncratic productivity shock a_j for $j = H, L$, in aggregate state $\mathbf{Z}'$. Here we allow $Q(\mathbf{Z}')$, $Q_H(\mathbf{Z}')$ and $Q_L(\mathbf{Z}')$ to be potentially different because limited commitment of financial contracts may prevent the marginal product of capital from being equalized to the price of capital on the reallocation market when the constraint is binding. We note that no arbitrage on the capital markets within an island implies that

$$Q_j(\mathbf{Z}') = MPK_j(\mathbf{Z}') + 1 - \delta. \tag{11}$$

The interpretation is that one unit of capital on island j produces an additional current period output $MPK_j(\mathbf{Z}')$ in the current period and depreciates at rate δ after production. In a frictionless market the above condition and the fact $Q_j(\mathbf{Z}') = Q(\mathbf{Z}')$ for all j guarantees that the marginal product of capital must be equalized across all islands. In our model, misallocation may happen in equilibrium due to limited enforcement of financial contracts.

[11] As in Gertler and Kiyotaki (2010), the assumption that bank net worth moves freely at the end of every period is made for tractability. It implies that the expected return on all islands are equalized and therefore the ratio of bank net worth to capital must be equalized across all islands. As a result, the decision problems for banks on all islands are identical at the end of the last subperiod. This allows us to use the optimal decision problem of the representative bank to construct the equilibrium. Without this assumption, bank net worth depends on the history of the realization of idiosyncratic productivity shocks and the distribution of bank net worth across islands becomes a state variable in the construction of Markov equilibria. In our setup, the heterogeneity in the realization of idiosyncratic productivity shocks at the beginning of a period motivates the need for capital reallocation. At the same time, the possibility of moving bank net worth across islands at the end of a period avoids the need to keep track of the the distribution of bank net worth across islands.

[12] We allow $RA(\mathbf{Z}', a')$ to be negative.

The total net worth of intermediary j at the end of the next period after the repayment of household loan and interbank borrowing is:

$$N_j' = Q_j\left(\mathbf{Z}'\right)\left[K' + RA_j\left(\mathbf{Z}'\right)\right] - Q\left(\mathbf{Z}'\right) RA_j\left(\mathbf{Z}'\right) - R_f\left(\mathbf{Z}\right) B_f - R_I\left(\mathbf{Z}\right) B_I, \tag{12}$$

where $Q_j\left(\mathbf{Z}'\right)\left[K' + RA_j\left(\mathbf{Z}'\right)\right]$ is the total value of capital on island j, including the capital purchased in the current period, K', and the capital obtained on the reallocation market, $RA_j\left(\mathbf{Z}'\right)$. The intermediary also needs to pay back the cost of capital obtained on the reallocation market, $Q\left(\mathbf{Z}'\right) RA_j\left(\mathbf{Z}'\right)$, and one-period risk-free loans borrowed from the household and other banks, B_f, and B_I.

After banks receive payment from local firms and before they pay back loans to creditors, banks have an opportunity to default. Upon default, bankers take away all of the capital on the island, but they can only sell a fraction θ of them on the market and the remaining fraction can be viewed as the deadweight loss associated with bankruptcy. Therefore, upon default, the total receipt of bankers on island j is $\theta Q_j\left(\mathbf{Z}'\right)\left[K' + RA_j\left(\mathbf{Z}'\right)\right]$. In addition, similar to Gertler and Kiyotaki (2010), we assume that bankers have a better technology to enforce contracts than households. This is captured by the parameter $\omega \in [0, \theta]$. The interpretation is that in the event of default, a fraction ω of interbank borrowing can be recovered. The case $\omega = 0$ means banks are no better than households in enforcing contracts, and $\omega = 1$ corresponds to the case of a frictionless interbank market. Thus the amount of assets bankers can abscond with upon default is:

$$\theta Q_j\left(\mathbf{Z}'\right)\left[K' + RA_j\left(\mathbf{Z}'\right)\right] - \omega\left[Q\left(\mathbf{Z}\right) RA_j\left(\mathbf{Z}'\right) + R_I\left(\mathbf{Z}\right) B_I\right]. \tag{13}$$

The possibility of default implies that the contracting between borrowing and lending banks must respect the following limited enforcement constraint:

$$N_j' \geq \theta Q_j\left(\mathbf{Z}'\right)\left[K' + RA_j\left(\mathbf{Z}'\right)\right] - \omega\left[Q\left(\mathbf{Z}\right) RA_j\left(\mathbf{Z}'\right) + R_I\left(\mathbf{Z}\right) B_I\right], \quad \forall\ \mathbf{Z}' \textit{ and } \forall\ j, \tag{14}$$

where N' is given by equation (12). Inequality (14) is the incentive compatibility constraint for banks. It implies that anticipating the possibility of default, lending banks will make sure that the borrowing bankers do not have the incentive to default on loans in all possible states of the world.

We assume that the representative household is divided into bankers and workers, and there is perfect consumption insurance between bankers and workers within the household.

Under this assumption, banks evaluate future cash flows using the "stochastic discount factor", M', implied by the marginal utility of the household.[13] Let $C(\mathbf{Z})$ denote the consumption policy that is consistent with household optimality in subsection (B).[14] Under the assumption of log utility, the stochastic discount factor takes a simple form:

$$M' = \beta \left(\frac{C(\mathbf{Z}')}{C(\mathbf{Z})} \right)^{-1}. \tag{15}$$

As is standard in the dynamic agency literature, for example, DeMarzo and Sannikov (2006) and DeMarzo and Fishman (2007), we assume that bank managers are less patient than households and use Λ to denote the ratio of bankers' discount rate relative to that of the households. Equivalently, with probability $1 - \Lambda$, bankers' net worth is liquidated and paid back to the household as dividends. With probability Λ, where $\Lambda \in (0, 1)$, bankers survive to the next period. This assumption is a parsimonious way to capture the idea that the managers of banks have a shorter investment horizon than the representative household and is a necessary condition for agency frictions to persist in the long-run.

Because banks' objective function is linear and the constraints (10), (12), and (14) are homogenous, the value function of banks, taking equilibrium prices as given, must be linear in bank net worth N. In addition, since bank net worth can be freely moved across islands at the end of every period, the marginal value of bank networth must be equalized across all islands at the end of every period. This feature of the model greatly simplifies our analysis, because it implies that banks on different islands are just scaled versions of each other after redistribution of bank net worth. We denote the value function of banks as $\mu(\mathbf{Z}) N$. A typical bank maximizes:

$$\mu(\mathbf{Z}) N = \max_{B_f, B_I, K', \{RA_j(\mathbf{Z}')\}_{\mathbf{Z}', j}} E\left[M' \left\{ (1 - \Lambda(\mathbf{Z}')) N' + \Lambda(\mathbf{Z}') \mu(\mathbf{Z}') N' \right\} \middle| \mathbf{Z} \right]$$

by choosing total capital stock for the next period, K', total borrowing from households, B_f, total borrowing from peer banks, B_I, and a state-contingent plan for capital reallocation, $RA_j(\mathbf{Z}')$ for all possible realizations of $\mathbf{Z}'$ and j, subject to constraints (10), (12), and (14).

[13] See Gertler and Kiyotaki (2010) for details.

[14] The policy functions of the dynamic programming problem have two state variables, which we can denote as $\hat{C}(\mathbf{Z}, W)$. Rational expectation requires that $C(\mathbf{Z}) = \hat{C}(\mathbf{Z}, \mathbf{W})$ when $\mathbf{W}$ is interpreted as total wealth of all households. In Section IV, we show that in our construction of the Markov equilibrium, $\mathbf{W}$ is a function of $\mathbf{Z}$: $\mathbf{W} = W(\mathbf{Z})$, and the rational expectation condition holds because the equilibrium consumption policy function satisfies $C(\mathbf{Z}) = \hat{C}(\mathbf{Z}, \mathbf{W}(\mathbf{Z}))$.

In our model with financial shocks, we assume that the discount rate, Λ, follows a Markov process. The macro-asset pricing literature found large discount rate variations in the data. One way of interpreting our specification of financial shocks is that we explore the implications of discount rate variations on agency frictions. We show in our calibration that relatively small variations of the discount rate, Λ, can be amplified by agency frictions and generate large fluctuations in measured total factor productivity and output.

D Market Clearing

Because market clearing conditions have to hold in every period, we suppress the dependence of all quantities on time and state variables in this section to save notation. We list the resource constraints and market clearing conditions below.

First, the total amount of capital utilized on island j is $K + RA_j, \quad for\ j = H, L$. The resource constraint requires that the amount of capital used for production on all islands must sum up to uK, which is the total amount of utilized capital in the economy:

$$\pi \left(K + RA_H\right) + \left(1 - \pi\right)\left(K + RA_L\right) = uK. \tag{16}$$

Second, the total amount of interbank borrowing in the economy must be zero. Because banks are ex ante identical before the realization of idiosyncratic productivity shocks, and because interbank borrowing is determined before the realization of these shocks,

$$B_I = 0. \tag{17}$$

The possibility of interbank bank borrowing on the intertemporal bank loan market does not affect allocation but determines the interbank borrowing rate, an object that can be measured empirically and used to discipline our quantitative exercise.

Third, the total net worth of the banking sector equals the sum of bank net worth across all islands:

$$N = \pi N_H + \left(1 - \pi\right) N_L. \tag{18}$$

Fourth, labor market clearing requires labor input $l_j, \quad for\ j = H, L$ to satisfy

$$\pi l_H + \left(1 - \pi\right) l_L = 1, \tag{19}$$

because we assumed inelastic labor supply and normalized total labor endowment to one.

Finally, market clearing for final goods requires that total consumption and investment sum up to total output:

$$C + K = Y, \tag{20}$$

where Y is the total output of final goods defined in equation (1).

Note that market clearing implies that the sum of the household's disposable wealth, W and the total net worth of the banking sector, N must equal to the total financial wealth of the economy. We do not list this condition here because it is redundant given all other market clearing conditions due to Walras' law.

IV Construction of the Markov Equilibrium

A Markov equilibrium consists of i) a set of equilibrium prices and quantities as functions of the state variable $\mathbf{Z}$, and ii) the law of motion of the state variable $\mathbf{Z}$, such that households maximize utility, non-financial firms and financial intermediaries maximize their profit and all markets clear. We follow the following procedure to construct the Markov equilibrium. First, we assume, but do not explicitly specify, the existence of a vector of Markov state variables $\mathbf{Z}$, and derive a set of equilibrium conditions from optimality and market clearing conditions. Second, we explicitly identify the state variables $\mathbf{Z}$ and use equilibrium conditions to construct the law of motion of $\mathbf{Z}$ as well as the equilibrium functions (equilibrium prices and allocations as functions of $\mathbf{Z}$). Finally, we verify that given the construction of the state variable $\mathbf{Z}$, our proposed pricing functions and quantities constitute a Markov equilibrium. Because our construction of the Markov equilibrium is a recursive procedure, it leads naturally to an iterative procedure to numerically solve the model. We describe our solution method in subsection C.

We define the capital allocation ratio in our model as the ratio of capital employed on high productivity islands relative to that on low productivity islands and denote it as ϕ:

$$\phi = \frac{K + RA_H}{K + RA_L}. \tag{21}$$

It is straightforward to show that the first best level of capital allocation ratio, which we will denote as $\hat{\phi}$ is:

$$\hat{\phi} = \left(\frac{a_H}{a_L}\right)^{\eta-1}.$$

Intuitively, it is optimal to allocate more capital to high productivity islands and less to low productivity islands. The optimal capital allocation ratio is increasing in the elasticity of substitution of output across varieties. The absence of reallocation implies $RA_H = RA_L = 0$ and $\phi = 1$. In general, in our model, $\phi \in \left[1, \hat{\phi}\right]$ and $u \in [0,1]$ summarizes the severity of capital misallocation.

We make one more assumption on the aggregate productivity $\bar{A}$. We assume $\bar{A}_t = A_t K_t^{1-\alpha}$, where A_t is a Markov process of exogenous productivity shocks. This specification follows Frankel (1962) and Romer (1986) and is a parsimonious way to inject endogenous long-run growth into the model. From a technical point of view, this allows us to explore homogeneity and reduce one state variable in the construction of the Markov equilibrium. In particular, equilibrium quantities are homogenous of degree one in K and equilibrium prices do not depend on K. It is therefore convenient to work with normalized quantities. We define:

$$c = \frac{C}{K},\ i = \frac{I}{K},\ n = \frac{N}{K},\ b_f = \frac{B_f}{K}. \tag{22}$$

Using the above notation, equation (8) can be written as:

$$\frac{K'}{K} = g\left(1-u\right) + \left(1-\delta\right)u + i. \tag{23}$$

The homogeneity property implies that, while K must be one of the state variables in the construction of the Markov equilibrium, normalized equilibrium quantities do not depend on K and only depend on $\mathbf{z}$. We denote $\mathbf{Z} = (\mathbf{z}, K)$, where $\mathbf{z}$ is a vector of state variables to be specified later.

A Equilibrium Conditions

In this section, we analyze the optimality conditions of firms and banks. We provide an aggregation result in Proposition 1, where we show that total output and marginal product of capital can all be represented as functions of capital reallocation ratio, ϕ, and capital utilization u. Our key result in this section is Proposition 2, which provides a characterization of the nature of binding constraints as a function of state variables.

Product Market Optimality The product market in our model is the standard monopolistic competition setup (see for example, Meltiz 2003). The total output and the marginal product of capital can be represented as functions of u and ϕ, which we summarize in the following proposition.

Proposition 1 *(Aggregation of the Product Market)*

The total output of the economy is given by

$$Y = Auf(\phi)K,$$

where the function $f : \left[1, \hat{\phi}\right] \to [0,1]$ *is defined as:*

$$f(\phi) = \frac{\left(\pi\hat{\phi}^{1-\xi}\phi^{\xi} + 1 - \pi\right)^{\frac{\alpha}{\xi}}}{(\pi\phi + 1 - \pi)^{\alpha}\left(\pi\hat{\phi} + 1 - \pi\right)^{\frac{\alpha}{\xi} - \alpha}} \tag{24}$$

The marginal product of capital on low productivity islands, denoted MPK_L*, and the marginal product of capital on high productivity island, denoted* MPK_H*, can be written as:*

$$MPK_L(A,\phi) = \alpha\left(1 - \frac{1}{\eta}\right)Af(\phi)\frac{\pi\phi + 1 - \pi}{\pi\hat{\phi}^{1-\xi}\phi^{\xi} + 1 - \pi}, \tag{25}$$

$$MPK_H(A,\phi) = MPK_L(A,\phi)\left(\frac{\hat{\phi}}{\phi}\right)^{1-\xi}, \tag{26}$$

where the parameter $\xi \in (0,1)$ *is defined as* $\xi = \frac{\alpha\eta - \alpha}{\alpha\theta - \alpha + 1}$.
Proof. *See Appendix A.* ■

Note that the function $uf(\phi)$ is a measure of misallocation. It is straightforward to show that f is strictly increasing with $f\left(\hat{\phi}\right) = 1$. In general, $f(\phi) \leq 1$ and misallocation happens when strict inequality holds. Variations in capital misallocation affect $uf(\phi)$ and act like TFP shocks in our model.

The first order condition for capital goods producing firm implies: $Q(\mathbf{z}) = g'(1 - u(\mathbf{z}))$. We use this condition to define Q as a function of u:

$$Q(u) = g'(1-u). \tag{27}$$

Optimality of Banks' Problem We first simplify the limited enforcement constraints for banks. Combining equations (13) and (14), the limited enforcement constraint can be written as:

$$(1-\theta)Q_H(\mathbf{z}')K' - [(1-\omega)Q(\mathbf{z}') - (1-\theta)Q_H(\mathbf{z}')]RA_H(\mathbf{z}') \geq R_f(\mathbf{z})B_f(\mathbf{z}), \tag{28}$$

for banks on high productivity islands and

$$(1-\theta)\,Q_L\left(\mathbf{z}'\right)K' - \left[(1-\omega)\,Q\left(\mathbf{z}'\right) - (1-\theta)\,Q_L\left(\mathbf{z}'\right)\right]RA_L\left(\mathbf{z}'\right) \geq R_f\left(\mathbf{z}\right)B_f\left(\mathbf{z}\right), \tag{29}$$

for banks on low productivity islands. We observe that equation (16) and the definition of ϕ and u jointly imply

$$\frac{RA_H}{K} = \frac{u\phi}{\pi\phi+1-\pi} - 1, \quad \frac{RA_L}{K} = \frac{u}{\pi\phi+1-\pi} - 1. \tag{30}$$

Note also that the no arbitrage condition (11) and equations (25, 26) on the marginal products of capital imply that $Q_H(\mathbf{z})$ and $Q_L(\mathbf{z})$ depend on state variables only through (A,ϕ). With a slight abuse of notation, we define

$$Q_H(A,\phi) = MPK_H(A,\phi) + 1 - \delta, \tag{31}$$

$$Q_L(A,\phi) = MPK_L(A,\phi) + 1 - \delta. \tag{32}$$

If we divide both sides of equation (28) by K' and use equation (30), we can show that $Q_H(A,\phi)$ must satisfy:

$$(1-\theta)\,Q_H(A',\phi') - \left[(1-\omega)\,Q(u') - (1-\theta)\,Q_H(A',\phi')\right]\left(\frac{u'\phi'}{\pi\phi'+1-\pi} - 1\right) \geq s', \tag{33}$$

where we denote

$$s' = \frac{R_f b_f}{g(1-u) + (1-\delta)\,u + i}. \tag{34}$$

Similarly, equation (29) implies that $Q_L(A,\phi)$ must satisfy:

$$(1-\theta)\,Q_L(A',\phi') - \left[(1-\omega)\,Q(u') - (1-\theta)\,Q_L(A',\phi')\right]\left(\frac{u'}{\pi\phi'+1-\pi} - 1\right) \geq s'. \tag{35}$$

Let ζ_H and ζ_L denote the Lagrangian multipliers on the limited enforcement constraint (14). The first order conditions with respect to $RA(\mathbf{Z}')$ can be used to derive a relationship between Lagrangian multipliers and the prices of capital on high and low productivity islands.

We use this relationship to define:

$$\zeta_H(A',\phi',u') = \frac{\pi\left[Q_H(A',\phi') - Q(u')\right]}{(1-\omega)Q(u') - (1-\theta)Q_H(A',\phi')} \geq 0, \tag{36}$$

$$\zeta_L(A',\phi',u') = \frac{(1-\pi)\left[Q_L(A',\phi') - Q(u')\right]}{(1-\omega)Q(u') - (1-\theta)Q_L(A',\phi')} \geq 0. \tag{37}$$

Note that if both of the limited enforcement constraints (33) and (35) hold with equality, then they jointly determine ϕ' and u' as functions of (A', s'). If none of constraints (33) and (35) is binding, then $\zeta_H(A',\phi',u') = \zeta_L(A',\phi',u') = 0$ imply $Q_H(A',\phi') = Q_L(A',\phi') = Q(u')$. Again, ϕ' and u' can be determined as functions of (A', s'). In general, equations (33), (35), (36), (37) and the complementary slackness condition determine ϕ' and u' as functions of (A', s'), which we will denote as $\phi(A', s')$ and $u(A', s')$. The following proposition builds on this observation and characterizes the nature of the binding constraints.

Proposition 2 *(Characterization of Binding Constraints)*
There exist functions $\hat{s}(A)$, $\bar{s}(A)$ and $s^*(A)$, such that $\hat{s}(A') < \bar{s}(A') < s^*(A')$

1. *If $s' \leq \hat{s}(A')$, then none of the limited commitment constraints bind, and $\phi(A', s')$ and $u(A', s')$ are determined by (36) and (37) in equality.*

2. *If $\hat{s}(A') < s' \leq \bar{s}(A')$, then the limited commitment constraint for banks on high productivity islands binds, and $\phi(A', s')$ and $u(A', s')$ are determined by (34) in equality and (37).*

3. *If $\bar{s}(A') < s' \leq s^*(A')$, then the limited commitment constraint for all banks bind, and $\phi(A', s')$ and $u(A', s')$ are determined by (34) and (35) in equality .*

4. *The cutoff levels, $\hat{s}(A')$ and $\bar{s}(A')$ are all increasing functions of A'.*

Proof. *See Appendix C.* ■

The result of the above proposition is intuitive. s' is the total amount of liability that banks need to pay back to households (normalized by capital stock). When s' is below $\hat{s}(A')$, debt level is low enough and the limited enforcement constraints never bind. As debt level increases, when $\hat{s}(A') < s' \leq \bar{s}(A')$, the limited enforcement constraint bind only if the island receives a high productivity shock. Efficiency of capital reallocation requires that banks on high productivity islands borrow more than those on low productivity islands. Therefore, the limited enforcement constraint is more likely to bind for banks on high productivity islands. In the region where $s' > \bar{s}(A')$, the banking sector accumulated too much debt and the limited enforcement constants bind for all realizations of idiosyncratic productivity shocks. Note that the cutoff levels depends on next period aggregate productivity shock, A'. According to the final part of the above proposition, both $\hat{s}(A')$ and $\bar{s}(A')$ are increasing functions of A'; therefore, the limited enforcement constraints are more likely to bind in states where aggregate productivity is low.

The above proposition has two important implications. First, in the cross-section, the limited enforcement constraint is more likely to bind for intermediaries on high productivity islands. This is the mechanism for misallocation in our model: when banks are constrained, more productive projects cannot be financed and measured TFP drops.

Second, in time series, the limited enforcement constraint is more likely to bind when bank net worth is low and/or when aggregate productivity drops. This is the amplification mechanism in our model. Adverse shocks to TFP and bank net worth are amplified because they tighten the limited enforcement constraints and exacerbate capital misallocation.

Other Optimality Conditions Given our definition of the Lagrangian multipliers in equations (36) and (37), we can use other first order conditions to characterize the equilibrium policy functions. Here we use the property that equilibrium prices depend only on **z** but not on K to simplify notation. First, the first order condition for households' optimal investment decision, together with equations (8) and (20), leads to the usual intertemporal Euler equation,

$$E\left[M\left(z, z'\right)\right] R_f\left(z\right) = 1, \tag{38}$$

where $M\left(z, z'\right)$, the stochastic discount factor of households, satisfies:

$$M\left(z, z'\right) = \frac{\beta\left[Au\left(z\right) f\left(\phi\left(z\right)\right) - i\left(z\right)\right]}{c\left(z'\right)\left[g\left(1 - u\left(z\right)\right) + \left(1 - \delta\right) u\left(z\right) + i\left(z\right)\right]}. \tag{39}$$

Second, banks' optimal choice for intertemporal investment implies

$$\mu\left(z\right) = E\left[\widetilde{M}\left(z, z'\right)\left\{1 + \left(1 - \omega\right)\left(\zeta_H\left(A', \phi\left(z'\right), u\left(z'\right)\right) + \zeta_L\left(A', \phi\left(z'\right), u\left(z'\right)\right)\right)\right\} Q\left(u'\right)\right]. \tag{40}$$

where $\widetilde{M}\left(z, z'\right)$ is defined as

$$\widetilde{M}\left(z, z'\right) = M\left(z, z'\right)\left\{1 - \Lambda' + \Lambda' \mu\left(z'\right)\right\}. \tag{41}$$

Third, banks' optimal choice for interbank loan implies

$$\frac{R_I(z)}{R_f(z)} = \frac{E_t\left[\widetilde{M}(z,z')\left\{1+\zeta_H(A',\phi(z'),u(z'))+\zeta_L(A',\phi(z'),u(z'))\right\}\right]}{E_t\left[\widetilde{M}(z,z')\left\{1+(1-\omega)\left(\zeta_H(A',\phi(z'),u(z'))+\zeta_L(A',\phi(z'),u(z'))\right)\right\}\right]}. \quad (42)$$

Fourth, the envelope condition on banks' optimization problem is

$$\mu(z) = E\left[\widetilde{M}(z,z')\left\{1+\zeta_H(A',\phi(z'),u(z'))+\zeta_L(A',\phi(z'),u(z'))\right\}\right]R_f(z). \quad (43)$$

Finally, we note that the resource constraint requires

$$c(z)+i(z) = Au(z)f(\phi(z)). \quad (44)$$

Note that the four unknown equilibrium functions, $c(z)$, $i(z)$, $\mu(z)$, and $R_f(z)$ can be determined by the four functional equations (38), (40), (43), and (44). Given the equilibrium functions, $c(z)$, $i(z)$, $\mu(z)$, and $R_f(z)$, interbank interest rate $R_I(z)$ can be determined by equation (42).

B Construction of the Markov Equilibrium

Subject to some technical details, the four functional equations can be used to determine the four equilibrium functions, $\{c(z), i(z), \mu(z), R_f(z)\}$ once the law of motion of the state variables are specified. Proposition 2 suggests that it is convenient to include $s' = \frac{R_f b_f}{g(1-u)+(1-\delta)u+i}$ to be one of the state variables. Motivated by this observation, we denote $x = (\Lambda, A)$ to be the vector of exogenous shocks. We conjecture and then verify that a Markov equilibrium can be constructed with $z = (x, s)$ as the state variables. In the rest of this section, we detail the construction of the Markov equilibrium of our model as the fixed point of an appropriate recursive operator.

Because x is an exogenous Markov process, we only need to specify the law of motion of the endogenous state variable, s. Using the law of motion of bank net worth on high and low productivity islands, equation (12), and the definition of total bank net worth, equation (18), we can derive the law of motion of normalized bank net worth, $n = \frac{N}{K}$:

$$n' = \Lambda'\left\{\alpha\left(1-\frac{1}{\eta}\right)A'u'f(\phi') + (1-u')MPK(u') + (1-\delta) - s'\right\}. \quad (45)$$

Divide both sides of the bank budget constraint (10) by K to obtain:

$$g(1-u)+(1-\delta)u+i=n+b_f. \tag{46}$$

By the definition of s, we have:

$$s'=\frac{R_f b_f}{g(1-u)+(1-\delta)u+i}=\frac{g(1-u)+(1-\delta)u+i-n}{g(1-u)+(1-\delta)u+i}R_f.$$

Now we can replace n in the above equation using equation (45) to obtain the law of motion of s:

$$s'=R_f(z)\left\{1-\frac{\Lambda\left\{\alpha\left(1-\frac{1}{\eta}\right)Au(z)f(\phi(z))+(1-u(z))MPK(u(z))+(1-\delta)-s\right\}}{g(1-u(z))+(1-\delta)u(z)+i(z)}\right\}. \tag{47}$$

Our construction of the Markov equilibrium is formally summarized by the following proposition:

Proposition 3 *(Markov Equilibrium)*

Suppose there exists a set of equilibrium functions, $\{c(z),i(z),\mu(z),R_f(z)\}_z$ *such that with the law of motion of* s *given by equation (47),* $\{c(z),i(z),\mu(z),R_f(z)\}_z$ *satisfy the functional equations (38), (40), (43), and (44), then* $\{c(z),i(z),\mu(z),R_f(z)\}_z$ *constitutes a Markov equilibrium.*

Proof. *See Appendix D.* ■

C Recursive Policy Function Iteration

In this section, we describe an operator that maps the space of equilibrium functionals into itself such that if a fixed point for the operator exists, it constitutes a Markov equilibrium described in section B. There are potentially many such operators. Because the construction of the operator leads naturally to iterative numerical procedures to compute the equilibrium functionals, our construction is aimed toward numerical efficiency.

First, we observe that Proposition 2 allows us to determine the policy functions $\phi(z)$ and $u(z)$ without any iteration. Second, given an initial guess of next period consumption, $c(z)$ and the value of bank net worth, $\mu(z)$, we can use the intertemporal Euler equation (40) to determine the current period consumption and investment policies and use the envelope condition (43) to determine the current period value of bank net worth. At the same time, we need to verify that the policy functions and the law of motion of the state variable, equation (47) are consistent with each other. Because both equations (40) and (43) are discounting relationships, it is reasonable to expect that if we iterate this procedure, the policy functions, $c(z)$ and $\mu(z)$ will converge.

Note that our approach makes full use of the first order optimality conditions to improve numerical efficiency. In fact, In Appendix E, we show that the iterative procedure boils down to solving a nonlinear equation for each point in the state space in each step of the iteration. Therefore, our computation algorithm is very similar to that used in a standard RBC model with productivity shocks. Thanks to the simplification of Proposition 2, the dependence of policy functions on the occasionally binding limited enforcement constraints is fully determined before any iteration. Below are the details of our approach.

1. Use Proposition 2 to construct the policy functions $\phi(z)$ and $u(z)$.

2. Start from an initial guess of the equilibrium functionals $\{c^0(z), \mu^0(z)\}$.

3. Given a set of equilibrium functionals, $\{c^n(z), \mu^n(z)\}$, let $c(z') = c^n(z')$ and $\mu(z') = \mu^n(z')$ in the definition of $M(z, z')$ and $\widetilde{M}(z, z')$ in equation (39) and (41), respectively. For each z in the state space, solve the four unknows $c(z)$, $\mu(z)$, $i(z)$, $R_f(z)$ from the equations, (38), (40), (43), and (44), with s' defined by (47).

   This is a key step in our iterative procedure. It involves solving four nonlinear equations for four unknowns for each point z in the state space. In appendix E we show that the computation in this step can be reduced to solving a single nonlinear equation for each point z in the state space.

4. Update the equilibrium functionals:

$$c^{n+1}(z) = c(z), \quad \mu^{n+1}(z) = \mu(z),$$

   where $c(z)$ and $\mu(z)$ are the solutions obtained in step 3.

5. Iterate on step 3 and 4 until the error is smaller than a preset convergence criteria, ε:

$$\sup_z \left|c^{n+1}(z) - c^n(z)\right| + \sup_z \left|\mu^{n+1}(z) - \mu^n(z)\right| < \varepsilon.$$

Finally, we note that although (x, s) is a convenient choice of state variable that simplifies our construction of the equilibrium and allows for efficient numerical methods to solve the model, any one-to-one function of (x, s) can be used as state variables as well. From an economics point of view, it is more intuitive to use bank net worth as a state variable. Equation (45) defines that mapping between (x, s) and (x, n). We will discuss the implications of our model using (x, n) as the state variable in the rest of the paper.

V Deterministic Dyanmics

In this section, we use the policy functions of the deterministic version of our model to illustrate the mechanism through which bank net worth affects capital misallocation and economic fluctuations. There are no stochastic shocks to x in the deterministic model, and all equilibrium prices and normalized quantities are functions of the normalized net worth n.

A Output, Consumption and Investment

In Figure 7, we plot output (top panel), consumption (middle panel) and investment (bottom panel) as functions of bank net worth n. In the figure, $\hat{n}$ is the level of bank net worth above which the limited enforcement constraints do not bind for any bank and there is no capital misallocation (that is, $\hat{n}$ is the level of net worth corresponds to the $\hat{s}$ defined in Proposition 2). Further increases in bank net worth n do not affect output, consumption or investment because productivity is constant and capital reallocation stays at its first best level. As n decreases towards $\bar{n}$, only the limited commitment constraint for high productivity islands, equation (28), binds. In this case, as n declines, capital misallocation between high productivity and low productivity islands deteriorates but capital utilization is fully efficient. As n drops below $\bar{n}$, which is the level of net worth corresponding to $\bar{s}$ defined in Proposition 2, the limited enforcement constraint for both islands bind, and output, consumption, and investment drop sharply.

Figure 7 illustrates two key features of our model that continue to hold in the stochastic version. First, total output increases with bank net worth. Note that even in the absence of productivity shocks, when bank net worth is low, the limited commitment constraint (28) and/or (29) binds and limits the efficiency of capital reallocation. As a result, output drops even if factor inputs do not, as if the economy is hit by a negative productivity shock.

Second, the limited commitment constraints are more likely to bind and capital reallocation is less efficient when bank net worth is low. For $n \geq \hat{n}$, output does not depend on bank net worth. In this region of the state space, our model behaves like the frictionless RBC model. Productivity shocks (if any) are the only reason for output fluctuations. For $\hat{n} < n \leq \bar{n}$, the limited enforcement constraint on high productivity islands starts to bind and bank net worth affects aggregate output. In the stochastic version of our model, amplification occurs in this region. Negative productivity shocks not only lower output directly through the production function, but also indirectly by reducing bank net worth and the efficiency of capital reallocation. The state space where $n < \bar{n}$ can be intuitively interpreted as the "crisis region", in which the limited enforcement constraints on both types of banks bind. Capital is not only misallocated across high and low productivity firms, but also under-utilized.

B Prices

In Figure 8, we plot the price of assets as functions of bank net worth. The top panel shows the market price of capital on the reallocation markets, determined by the unconstrained firms who equalize their marginal product of capital to its market price. As bank net worth shrinks, the efficiency of capital reallocation deteriorates, and the marginal product of capital of the unconstrained firms drops. At the same time, asset markets are depressed, as we show in the second panel, where we plot the price of an asset that pays aggregate consumption as a dividend.[15] In our model bank net worth affect asset prices for two reasons. First, drops in bank net worth affect the efficiency of real production, and as a result, firms cut dividend payments. Second, banks are constrained, and are under pressure to sell. In equilibrium, the market clearing condition implies that asset prices have to decline. In our stochastic model, the two forces reinforce each other to generate large recessions and financial market crisis.

The fact that lower levels of net worth tighten banks' borrowing constraint also manifests itself on the interbank lending market. We plot the spread between the interbank interest rate, calculated from equation (42), and the household deposit rate in the bottom panel of Figure 8. Interest rate spread widens when the expected return on capital is high, household deposit rate is low, and banks are constrained because of their low net worth. At the same

[15]The price to consumption claim is calculated as:

$$p_c(z) = \frac{g(1-\rho(z)) + (1-\delta)\rho(z) + i(z)}{\mu(z)} E\left[\widetilde{M}(z,z')\{1+(1-\omega)(\zeta_H(z')+\zeta_H(z'))\}\{p_c(z')+c(z')\}\right].$$

time, banks are less constrained on the interbank market because peer banks have better contract enforcement technologies. As a result, banks race to the interbank lending market and drive up the interest rate, R_I. This effect is particularly pronounced in the crisis region where $n < \bar{n}$, and all banks are constrained.

C Capital Reallocation

As shown in Eisfeldt and Rampini (2006), the amount of capital reallocation is procyclical and the benefit to capital reallocation is countercyclical. Our model is consist with this fact. We plot the dispersion in the marginal product of capital (top panel), the total amount of capital reallocation (second panel), the percentage of capacity utilization (third panel), and the marginal value of bank net worth (bottom panel) as functions of bank net worth in Figure 9. As shown in the top panel of the figure, in the region $n \geq \hat{n}$, capital reallocation is fully efficient, and the marginal products of capital equalize across all islands. As bank net worth decreases to $\bar{n}$, the marginal products of capital on high and low productivity islands diverge, but the allocation of capital between low productivity islands and the storage technology is fully efficient. As bank net worth drops further, low productivity islands become constrained as well, and capital "fly to safety", i.e. they are invested in the risk-free storage technology despite its low marginal product. Clearly, the benefit of capital reallocation increases as bank net worth declines.

The divergence of the marginal product of capital is echoed by reductions in the total amount of capital reallocation (second panel) and decreases in the capital utilization rate (third panel). Again, drops in capital reallocation and capital utilization are much more pronounced in the crisis region where the limited enforcement constraints bind for all banks. Finally, we plot the marginal value of bank net worth in the bottom panel of the figure. By the envelope condition (43), the more likely the bank will be constrained in the next period, the higher is the marginal value of bank net worth today. As a result, the marginal value of bank net worth is a decreasing function of n.

D Bank Leverage

We plot the total amount of bank debt (top panel) and bank leverage (middle panel) as a function of bank net worth in Figure 10.

In Figure 11, we plot the next period net worth as a function of current period net worth as the dotted line. The dotted line intersects the 45 degree line, which is the solid line in the figure, only once where its slope is below 45 degree, indicating there is a unique stationary steady state in the model. Bank net worth converges to n_{SSE} in the long run. The solid line is very close to the 45 degree line, especially in the region where both banks are constrained, indicating convergence to steady state is slow and shocks to bank net worth have persistent effects.

In the deterministic model, the economy converges to the steady state n_{SSE} with probability one and stays at the steady state afterwards. In a stochastic model, shocks to TFP and/or discount rates constantly push the system away from the steady state and generate nontrivial economic fluctuations. It is natural to expect the stochastic model to have the following properties. First, negative shocks depress bank net worth and lower the efficiency of capital reallocation. Second, volatility of the economy spikes in the crisis region because output is much more sensitive to shocks that affect bank net worth in this region. Third, capital "flies" to the risk-free storage technology and the interest rate spread widens in the crisis region as the limited enforcement constraints bind for all banks. The stock market is depressed not only because expected cash flow drops, but also because intermediaries are constrained and under pressure to sell. We evaluate these effects quantitatively in the next section.

VI Quantitative Results

In this section, we consider two specifications of our model, a specification with TFP shocks only and a specification with shocks to agency frictions only, and evaluate quantitatively the impact of financial frictions. In the model with TFP shocks only, productivity shocks are the only source of primitive shocks and are amplified by financial frictions. Consistent with previous literature (for example, Kocherlakota (2000) and Chen and Song (2013)), we find financial frictions do amplify TFP shocks, but the effect is quantitatively small. Amplification accounts for about 11% of the macroeconomic fluctuations in the model with TFP shocks. In addition, the economy almost never runs into the crisis region where the limited enforcement constraint binds for all banks, because TFP shocks do not generate large enough variations in bank net worth.

Our preferred calibration is the model with financial shocks, or shocks to agency frictions. In this specification of the model, we introduce stochastic shocks to bankers' discount rate. We show that relatively small shocks generate large fluctuations in capital misallocation and can account for most of the macroeconomic fluctuations in the U.S. economy. We show that this model endogenously generates countercyclical volatility at the aggregate level and countercyclical dispersion in the cross-section. In addition, this version of the model captures several salient features of the recent financial crisis, such as spikes in macroeconomic volatility, sharp drops in capital reallocation and capital utilization, and sudden increases in interest rate spreads.

To facilitate comparison, we choose the same parameters, except the volatility of exogenous shocks, for both specifications of our model. This approach guarantees that both the model with productivity shocks and that with financial shocks have the same deterministic steady state. We then calibrate the volatility of the exogenous shocks in both models to match the volatility of aggregate output in the data and evaluate the model's implications on the dynamics of macroeconomic quantities and asset prices.

A Calibration

We calibrate our model at the quarterly frequency. The calibrated parameter values are listed in Table 1. We choose the standard preference and technology parameters to be consistent with the real business cycle literature. We set the quarterly discount rate $\beta = 0.999$ and the quarterly depreciation rate $\delta = 2\%$. We choose capital share $\alpha = 0.333$ and the elasticity of substitution across varieties to be $\eta = 4$, which is consistent with the value used in Hsieh and Klenow (2009).

The second group of technology parameters are specific to our model and we calibrate them to jointly match relevant moments in the data. We first choose the model parameters, except the volatility of exogenous shocks to match the first moment of various aspects of the U.S. economy during the period of 1929-2010. We calibrate our model at the quarterly frequency and simulate the model to compute annual moments. We choose the capital storage technology to be of the CES form:

$$g(x) = a_0 + \frac{b_0}{\nu} x^{\nu}.$$

The parameters of the storage technology a_0 and b_0 jointly determine the steady-state total capital depreciation rate and capital utilization rate. We set $a_0 = -0.0118$ and $b_0 = 0.982$ to target a depreciation rate of 2% and a capital utilization rate of 81%, which is the average capital utilization rate in our sample. We choose the elasticity parameter $\nu = 0.98$ so that the volatility of capital utilization rate in our model with financial shocks matches that in the data, 4.08% per year.[16] We choose the rest of the six parameters, the ratio of productivity across firms, $\frac{a_H}{a_L}$, the fraction of high productivity firms, π, the average productivity, $E[A]$, the fraction of assets bankers can divert, θ, the recovery rate of interbank loans upon default, ω, the mean of bankers' discount rate, Λ, to match the following six moments in the data. We choose $\frac{a_H}{a_L}$ =1.9402 and π =0.2578 to match the average capital reallocation rate of 57% reported in Eisfeldt and Rampini (2006), and to set output by high productivity firms to be one half.[17] We choose $E[A] = 0.1645$ to match mean aggregate growth rate of 0.5%, consistent with the calibration of Gertler and Kiyotaki (2010). We set banker discount rate $E[\Lambda] = 0.9612$ to match investment-output ratio of 20%. We set $\theta = 0.3026$ to yield a steady leverage ratio of the banking sector of 3.67, consistent with Gertler and Kiyotaki (2010). We set $\omega = 0.0772$, so that the steady interbank interest rate spread in our model matches the historical average of the TED spread (the spread between T-bills and the LIBOR) of 0.16% per year.

In the model with productivity shocks, we calibrate $\ln A$ to be an i.i.d. process and set the standard deviation process to match the volatility of aggregate output in the data. In the model with the model with financial shocks, we set $\Lambda_t = \frac{\exp\{\lambda_t\}}{\exp\{\lambda_t\}+\exp\{-\lambda_t\}}$. This specification allows us to specify λ_t as an i.i.d. process and grantees that Λ is a valid discount factor for all values of λ_t. We calibrate the standard deviation of the AR(1) process of λ to match the volatility of output in the data.

B Impulse Response

To understand the different implications of TFP shocks and discount rate shocks on financial frictions, we use the policy function iteration method introduced in Section IV to numerically solve the model. We then plot the impulse functions for shocks to $\ln A$ in Figure 12 and those for shocks to λ in Figure 13, where the solid lines indicate positive shocks and connected dotted lines stand for negative shocks. To emphasize the endogenous persistence generated from our model, we assume all shocks are purely transitory when plotting the impulse response functions. For example, we inject a positive shock into $\ln A$ for one period,

[16]The elasticity ν is the only technology parameter that is pin down by a second moment in the data. We choose ν so that the volatility of capital utilization matches our preferred model, which is the one with financial shocks.

[17]The simplicity of our model does not allow us to match a rich set of moments TFPR dispersion in the data. Hsieh and Klenow (2009) report that the ratio of the 75th to 25th percentiles of TFPR is 1.7 and that of the 90th to 10th percentiles is 3.3 in the U.S. in 1997. The ratio of the high productivity to low productivity in our model, a_H/a_L is within this range.

and assume that $\ln A$ returns to its steady state value immediately after that, even though $\ln A$ is autocorrelated in our calibration.

We make several observations. First, shocks to λ have much more persistent effects than shocks to productivity A, even though both shocks occur for one period and return to steady state immediately afterwards. Immediately after a positive productivity shock, bank net worth increases; however, the increase in current period net worth is accompanied by an increase in debt (b_f) of a similar magnitude, this is because high productivity triggers high consumption and high investment at the same time, and as a result, banks must borrow more to finance the additional investment. Because productivity returns to steady state immediately, so does return to capital. In this case, the initial increase in bank net worth is offset by the increases in interest payment. In fact, as we see in the impulse response functions, banks' own net worth, after service to debt holders, drops below steady state. Therefore, TFP shocks do affect bank net worth; however, the effect completely disappears after one period.

The model with λ shocks are completely different in this respect, because increases in bank net worth are accompanied by a change in bank debt in the opposite direction, and the two effects reinforce each other, generating long-lasting impact on the economy. An increase in banker's discount rate reduces dividend payment and increases bank net worth immediately. Because the increase in net worth is not accompanied by increases in productivity, the income effect raises consumption immediately and investment drops due to the resource constraint. As a result, banks borrow less from the households. This effect relaxes the limited enforcement constraint going forward, improves capital reallocation, boosts production, and generates a new round of increase in bank net worth. As a result, the initial shock to bank net worth creates a self-reinforcing loop and generates extremely persistent impact. Eventually, it dies off and all quantities converge to steady state. However, the effect is so persistent, that the system is still far from convergence after twenty quarters.

Second, the effect of productivity shocks is largely symmetric: positive and negative shocks in productivity result in changes in quantities and prices of similar magnitude. Qualitatively, as we have seen in the policy functions in the deterministic case, negative shocks to net worth have larger impact on capital misallocation than positive ones, especially in the "crisis" region. Quantitatively, however, productivity shocks induce very modest changes in bank net worth due to the offsetting effect of bank debt. Although asymmetry and countercyclical volatility are present in this case, they are quantitatively small.

In contrast, the asymmetry in the impulse responses of quantity and prices with respect to shocks to agency frictions is apparent in Figure 13. A positive shock to λ relaxes the limited enforcement constraint and reduces the effect of future shocks. A negative shock to λ tightens the limited enforcement constraint, making the system more sensitive to additional disturbances. As a result, negative shocks are amplified and positive shocks are dampened, leading to endogenous counter-cyclical volatility in our model.

Third, a positive productivity shock is associated with an improvement in capital utilization (i.e., increases in u) but a deterioration in capital reallocation (i.e., drops in ϕ). Upon impact, increases in A attract more capital from the storage technology into the productive sector and raises capital utilization rate, u. However, more capital goes into the low productivity firms because limited enforcement constraint for them is not binding. As a result, the efficiency of capital reallocation among productive firms deteriorates even though more capital is deployed in the productive sector. Overall, the efficiency of capital reallocation as measured by $uf(\phi)$ improves but the effect is quantitatively small. Because bank net worth quickly drops back to the steady state level, so does the efficiency in capital reallocation.

A positive innovation in financial shocks, on the other hand, improves the efficiency of capital reallocation and capital utilization at the same time. The two effects reinforce each other, leading to pronounced and persistent changes in total output. At the same time, the Lagrangian multipliers on the limited enforcement constraints shrink and interbank interest spread declines.

C Simulation

To understand the quantitative implications of the model, we simulate the model for 800 quarters and discard the first 400 quarters, aggregate the quarterly quantities in the remaining part of the simulation into annual quantities, and compute moments for annualized quantities. We report moments of macroeconomic quantities in the data and in our models in Table 2. Both specifications of our model are calibrated to match the mean, the volatility of output growth, and the average level of interest rate spread in the data. All other moments are endogenously generated from the model. Both versions of our model are consistent with the basic features of the data in terms of the relatively low volatility of consumption growth, the high volatility of investment, and the comovement between consumption and investment. The level of risk-free interest is too high in both versions of the model — this is the risk-free rate puzzle in production economies, which we do not attempt to address in this paper.[18]

[18]Ai et al. (2013) show that this issue can be resolved by using a recursive utility with high intertemporal elasticity of substitution.

Consistent with the data, both versions of our model produce fairly mild volatility of the interbank interest rate spread.

Our model with financial shocks produces a strong countercyclical volatility in aggregate time series, while the model with TFP shocks does not. In Table 2, the notation $Corr[\Delta lnY, Vol(\Delta lnY)]$ stands for the correlation between current period output growth and the realized variance of future output growth. For each year, we compute the realized variance of future output growth in the data as the realized variance of the growth rates of quarterly industrial production during the next two years. In the model, we compute it as the realized variance of output growth for the next eight quarters in our simulation. As in the data, the correlation between output growth and realized variance of future output growth is strongly negative in our model with financial shocks. However, the same correlation is negligible in the model with TFP shocks. This phenomenon is also evident in the impulse functions we plot for shocks to $\ln A$ (Figure 12) and shocks to λ (Figure 13). As we explain previously, symmetric shocks in $\ln A$ produce roughly symmetric responses in total output, consumption and investment, as in standard neoclassical models, while negative shocks to λ produce a significantly larger effect on total output than positive shocks.

Note that the measured log TFP in our model equals $\ln \bar{A}_t + \ln u_t f(\phi_t)$, where the component $\ln u_t f(\phi_t)$ depends on the efficiency of capital reallocation. In the last row of Table 2, we report the fraction of the realized variance of TFP growth that comes from variations in capital misallocation in out models:

$$\frac{Var\left[\ln\left(u_{t+1} f\left(\phi_{t+1}\right)\right) - \ln\left(u_t f\left(\phi_t\right)\right)\right]}{Var\left[\ln \bar{A}_{t+1} - \ln \bar{A}_t + \ln\left(u_{t+1} f\left(\phi_{t+1}\right)\right) - \ln\left(u_t f\left(\phi_t\right)\right)\right]}.$$

In the model with TFP shocks, the efficiency of capital reallocation accounts for 11% of total variation in TFP. Therefore, amplification is present in this version of the model, but is quantitatively small. In the model with financial shocks only, the efficiency of capital reallocation accounts for virtually all of the macroeconomic fluctuations.

We document the statistics related to the quantity and benefit of capital reallocation in Table 3. Both versions of our model are consistent with the empirical evidence of procyclical capital reallocation and procyclical capital utilization. However, consistent with small magnitude of amplification, the variations in capital reallocation and capital utilization in the model with TFP shocks are much smaller compared to the data and compared to our model with financial shocks. In addition, in the model with TFP shocks, the cross-sectional dispersion of the marginal product of capital is positively correlated with measured TFP, while this correlation is negative both in our model with financial shocks and in the data (see also the empirical evidence in Eisfeldt and Rampini (2006)). The reason that TFP shocks generate procyclical benefit of capital reallocation is that positive TFP shocks move more capital from the risk-free storage technology to the productive sector, but most of the capital goes to the less productive firms whose limited enforcement constraint does not bind. As a result, although positive TFP shocks improve capital utilization, they also elevate the cross-sectional dispersion of the marginal product of capital, as shown in Figure 12.

To further understand the implications of our model on volatility dynamics and economic recessions, we report the moments of macroeconomic quantities and interest rate spreads in the data and those in our model for recession periods and for non-recession periods separately. For simplicity, we use a "rule of thumb" classification and define recession as two consecutive quarters of declines in real GDP both in the data and in the model. Our definition yields very similar results as the NBER definition of recession, and results in about 20% of the sample being classified as recession both in the data and the model simulation.

Clearly, the volatility of consumption and output are strongly countercyclical in our model, as in the data. Interestingly, there is no significant difference between the volatility of investment in recession periods and that in non-recession periods both in the data and in our model. Spikes in the volatility of output do not lead to significant increases in the volatility of investment in our model, because shocks to λ are stationary. In recessions, bank net worth is low and expected return is high. Negative shocks to bank net worth reduces total output, but they also raise expected return. The two effects offset each other and do not lead to significant increases in the volatility of investment. Overall, our model with financial shocks is consistent with the pattern of capital utilization and interest rate spread in the data. In recessions, the level of capital utilization drops, but the volatility of capital utilization rates rises. The spread between interbank lending rate and household deposit rate widens, and so does the volatility of the spread. All the above features are the endogenous outcomes of the financial frictions in the model.

VII Conclusion

We presented a general equilibrium model with financial intermediary and capital reallocation. Our model emphasizes the role of financial intermediary in reallocating capital across firms with heterogenous productivity. We show that shocks to financial frictions alone may account for a large fraction of the fluctuations of measured TFP and aggregate output.

Our calibrated model is consistent with the salient features of business cycle variations in macroeconomic quantities and asset prices. In particular, our model successfully generates countercyclical volatility in aggregate consumption and output, and countercyclical dispersion in the cross-section.

An important next step is to infer or impute shocks to financial frictions from the data and investigate whether our model can account for the realized variations in macroeconomic quantities and asset prices once these shocks are fed into the model. One possible way is to infer financial frictions from the dispersion in the marginal product of capital in the data. The close link between the dispersion measure and TFP in Figure 1 suggests that our model holds promises. A stronger discipline may be imposed on the model if we can infer shocks to θ directly from banks' balance sheet variables. We leave these for future research.

VIII Appendix

A Misallocation and Aggregation on the Product Market

Aggregation

We first derive an aggregation result that is similar to Hsieh and Klenow (2009) and Hopenhayn and Neumeyer (2008). In fact, the product market of our model is a special case of Hsieh and Klenow (2009) and Hopenhayn and Neumeyer (2008) without labor market distortions.

Consider the maximization problem in equation (3); first order conditions with respect to $k(j)$ and $l(j)$ imply:

$$(1-\alpha)\left(1-\frac{1}{\eta}\right)p_j y_j = MPL \cdot l_j \tag{48}$$

$$\alpha\left(1-\frac{1}{\eta}\right)p_j y_j = MPK_j \cdot k_j. \tag{49}$$

Together, the above imply:

$$\frac{k_j}{l_j} = \frac{MPL}{MPK_j}\frac{\alpha}{1-\alpha}. \tag{50}$$

To save notation, we denote $A_j = Aa(j)$ in this section. Note also, total output of firm j can be written as:

$$y_j = A_j k_j^{\alpha} l_j^{1-\alpha} = A_j \left[\frac{k_j}{l_j}\right]^{\alpha} l_j \tag{51}$$

$$= A_j \left[\frac{l_j}{k_j}\right]^{1-\alpha} k_j. \tag{52}$$

Using equations (50) and (51), we can write l_j as a function of y_j:

$$l_j = \frac{y_j}{A_j}\left[\frac{\alpha MPL}{(1-\alpha)MPK_j}\right]^{-\alpha}. \tag{53}$$

Similarly, equations (50) and (52) together implies

$$k_j = \frac{y_j}{A_j}\left[\frac{\alpha MPL}{(1-\alpha)MPK_j}\right]^{1-\alpha}. \tag{54}$$

Using the demand function $p_j = \left[\frac{y_j}{Y}\right]^{-\frac{1}{\eta}}$, we can replace y_j in the above equations by $p_j^{-\eta}Y$, and integrate across all j, we have:

$$\bar{K} = \int \frac{p_j^{-\eta}}{A_j}\left[\frac{1}{MPK_j}\right]^{1-\alpha} dj \left[\frac{\alpha MPL}{1-\alpha}\right]^{1-\alpha} Y \tag{55}$$

$$\bar{L} = \int \frac{p_j^{-\eta}}{A_j}\left[\frac{1}{MPK_j}\right]^{-\alpha} dj \left[\frac{\alpha MPL}{1-\alpha}\right]^{-\alpha} Y, \tag{56}$$

where $\bar{K}$ and $\bar{L}$ stands for the total capital and total labor employed for production, respectively. Together, equations (55) and (56) imply

$$Y = \frac{\bar{K}^{\alpha}\bar{L}^{1-\alpha}}{\left[\int \frac{p_j^{-\eta}}{A_j}\left[\frac{1}{MPK_j}\right]^{1-\alpha} dj\right]^{\alpha}\left[\int \frac{p_j^{-\eta}}{A_j}\left[\frac{1}{MPK_j}\right]^{-\alpha} dj\right]^{1-\alpha}}. \tag{57}$$

We can express p_j in equation (57) by functions of productivity and prices. Note that equations (48) and (49) imply

$$MPK_j \cdot k_j + MPL \cdot l_j = \left(1 - \frac{1}{\eta}\right) p_j y_j. \tag{58}$$

Using equations (53) and (54), we have:

$$MPK_j \cdot k_j + MPL \cdot l_j = \frac{y_j}{A_j}\left[\frac{MPL}{(1-\alpha)}\right]^{1-\alpha}\left[\frac{MPK_j}{\alpha}\right]^{\alpha}. \tag{59}$$

Combining (58) and (59), we have:

$$p_j = \frac{\eta}{\eta-1}\frac{1}{A_j}\left[\frac{MPL}{(1-\alpha)}\right]^{1-\alpha}\left[\frac{MPK_j}{\alpha}\right]^{\alpha}. \tag{60}$$

Note that the normalization of price we choose in (2) implies $\int p_j dj = 1$. Integrating equation (60) over j, we have:

$$\frac{\eta}{\eta-1}\left[\frac{MPL}{(1-\alpha)}\right]^{1-\alpha} = \left\{\int \frac{1}{A_j}\left[\frac{MPK_j}{\alpha}\right]^{\alpha} dj\right\}^{-1}. \tag{61}$$

Together, equations (60) and (61) imply

$$p_j = \frac{\frac{1}{A_j}\left[\frac{MPK_j}{\alpha}\right]^{\alpha}}{\int \frac{1}{A_j}\left[\frac{MPK_j}{\alpha}\right]^{\alpha} dj}. \tag{62}$$

Replacing p_j in equation (57) with equation (62), and using $A_j = A^{1-\alpha} a\left(j\right)$, we can write $Y = TFP\bar{K}^{\alpha}\bar{L}^{1-\alpha}$, where

$$TFP = A\frac{\left\{\int \left(\frac{a_j}{MPK_j^{\alpha}}\right)^{\eta-1} di\right\}^{\frac{\eta}{\eta-1}+\alpha-1}}{\left\{\int \left(\frac{a_j}{MPK_j^{\alpha}}\right)^{\eta-1} \frac{1}{MPK_j} di\right\}^{\alpha}}. \tag{63}$$

Under the assumption (**??**), it is straightforward to show that $TFP = A$ if $MPK_j = MPK$ for all j. We define

$$EF = \frac{\left\{\int \left(\frac{a_j}{MPK_j^{\alpha}}\right)^{\eta-1} di\right\}^{\frac{\eta}{\eta-1}+\alpha-1}}{\left\{\int \left(\frac{a_j}{MPK_j^{\alpha}}\right)^{\eta-1} \frac{1}{MPK_j} di\right\}^{\alpha}} \tag{64}$$

to be the efficiency measure of capital reallocation. Under the assumption $\ln \alpha_j$ and $\ln MPK_j$ are jointly normally distributed, we can show that

$$\ln EF = -\frac{1}{2}\left[\alpha\left(\eta-1\right)+1\right]\alpha\sigma^2, \tag{65}$$

where σ^2 is the cross-sectional variance of marginal product of capital. Note also, equation (65) is approximately true for arbitrary distributions as long as the deviation of $\ln \alpha_j$ and $\ln MPK_j$ from there mean is small. Therefore, equation (65) can be viewed as a first order Taylor approximation that maps the cross-sectional variance of marginal product of capital into TFP losses due to misallocation.

Proof of Proposition 1

In the special case where a_j takes on only two values, a_H and a_L as in equation (21), we define $\phi = \frac{K_H}{K_L}$ to be the ratio of capital employed on islands with high productivity shock with respect to that employed on islands with low productivity shock, as in equation (21). Note that

$$MPK_j = \alpha A a_j \left(\frac{l_j}{k_j}\right)^{1-\alpha}; \quad MPL = \left(1-a\right) A a_j \left(\frac{k_j}{l_j}\right)^{\alpha}.$$

Note that because labor market is perfectly mobile, MPL must equalize across all islands. Using the labor market clearing condition, equation (19) and assumption (5), we can prove conditions (25) and (26). Using there conditions to replace MPK_j in equation (64), the efficiency measure in equation (64) can be written as equation (24). This completes the proof of Proposition 1.

B Data Construction

B.1 Misallocation and TFP

In Figure 1, we plot the measure of capital misallocation and total factor productivity. We measure the cross-sectional dispersion of TFPR following Hsieh and Klenow (2009). In the context of our model, equation (49) implies

$$MPK_j = \alpha \left(1 - \frac{1}{\eta}\right) \frac{p_j y_j}{k_j}.$$

Following Chen and Song (2013), we measure MPK_j by the ratio of Operating Income before Depreciation (OIBDP) to one-year-lag net Plant, Property and Equipment (PPENT). As in Hsieh and Klenow (2009), we focus on the manufacturing sector and compute the cross-sectional dispersion measure within narrowly defined industries (as classified by the 4-digit standard industry classification code). Specifically, for firm j in industry i, we compute

$$\frac{MPK_{i,j}}{MPK_i} = \frac{\alpha \left(1 - \frac{1}{\eta}\right) \frac{p_{i,j} y_{i,j}}{k_{i,j}}}{\alpha \left(1 - \frac{1}{\eta}\right) \frac{p_j y_j}{k_j}} = \frac{\frac{p_{i,j} y_{i,j}}{k_{i,j}}}{\frac{p_j y_j}{k_j}},$$

where $\frac{p_j y_j}{k_j}$ is measured at the industry level. We then compute the variance of $\frac{MPK_{i,j}}{MPK_i}$ for each year. This is our empirical measure of σ^2 in equation (65). We use the first order approximation in equation (65) to construct the time series of the misallocation measure, which is the solid line in Figure 1. The measure of total factor productivity is directly taken from the published TFP series on the Federal Reserve Bank of St Louis website. Both series are HP filtered.

B.2 Total Volume of Bank Loans

We measure the total volume of bank loans of non-financial corporate sector through the aggregate balance sheet of nonfinancial corporate business (Table B.102) as reported in the U.S. Flow of Funds Table. In particular, the bank loan is calculated as the difference between total credit market liability (Line 23) and corporate bond (Line 26). Under this construction, bank loans consist of the following credit market liability items: commercial paper (Line 24), municipal securities (Line 25), depository institution loans (Line 27), other loans and advances (Line 28) and mortgages (Line 29).

C Characterization of Binding Constraints

We first introduce some notations. We define the function $\hat{Q}(\cdot)$ and $u(\cdot)$ as:

$$\hat{Q}(A) = \alpha \left(1 - \frac{1}{\eta}\right) A + 1 - \delta,$$

and

$$u(Q) = 1 - \left(\frac{b_0}{Q}\right)^{\frac{1}{1-\nu}}.$$

Intuitively, $\hat{Q}(A)$ is the price of capital in the first best case, which is the sum of the marginal product of capital in the first best case, and the value of capital after depreciation. The function $u(\cdot)$ is the inverse function of $g'(\cdot)$. That is, $u(Q)$ is the optimal capital utilization rate that satisfies the first order condition of capital goods producer's optimization problem in (6) with the price of capital given by Q. Using the above notation, the cutoff value $\hat{s}(A)$ in Proposition 2 is given by:

$$\hat{s}(A) = \frac{\hat{Q}(A)}{\pi\hat{\phi} + 1 - \pi} \left\{ \left[(1-\omega) - (\theta - \omega) u\left(\hat{Q}(A)\right)\right] \hat{\phi} - (1-\omega)(1-\pi)\left(\hat{\phi} - 1\right) \right\}.$$

Let $\bar{\phi}(A)$ be the unique solution to the following equation on ϕ:

$$MPK_L(A, \phi) \left[(1-\omega)(\phi - 1) - (1-\theta)\left(\hat{\phi}^{1-\xi}\phi^{\xi} - 1\right)\right] + (1-\delta)(\theta - \omega)(\phi - 1) = 0.$$

Define the function $D_1(A,\phi)$ as

$$D_1(A,\phi) = \frac{MPK_L(A,\phi)}{\pi\phi+1-\pi}\left\{(1-\theta)\,u_L(A,\phi)\,\hat{\phi}^{1-\xi}\phi^{\xi}-(1-\omega)\left[(\phi-1)(1-\pi)-\phi\left(1-u_L(A,\phi)\right)\right]\right\}$$
$$+\frac{1-\delta}{\pi\phi+1-\pi}\left\{(1-\theta)\,u_L(A,\phi)\,\phi-(1-\omega)\left[(\phi-1)(1-\pi)-\phi\left(1-u_L(A,\phi)\right)\right]\right\},$$

where we denote $u_L(A,\phi)=u\left(MPK_L(A,\phi)+1-\delta\right)$ to simplify notation. The cutoff value $\bar{s}(A)$ in Proposition 2 is given by:

$$\bar{s}(A)=D_1\left(A,\bar{\phi}(A)\right).$$

Finally, we define the functions $Q(A,\phi)$ and $D_2(A,\phi)$ as

$$Q(A,\phi)=(1-\delta)+\frac{1}{1-\omega}\left[(1-\theta)\,MPK_L(A,\phi)\,\frac{\hat{\phi}^{1-\xi}\phi^{\xi}-1}{\phi-1}-(\theta-\omega)(1-\delta)\right],$$

$$D_2(A,\phi) = (1-\theta)\left[MPK_L(A,\phi)+1-\delta\right]$$
$$-\left[(1-\omega)\,Q(A,\phi)-(1-\theta)\left(MPK_L(A,\phi)+1-\delta\right)\right]\left(\frac{u\left(Q(A,\phi)\right)}{\pi\phi+1-\pi}-1\right).$$

Let $\phi^*(A)$ be the solution to the equation on ϕ

$$(1-\theta)\left[MPK_L(A,\phi)+1-\delta\right]+\left[(1-\omega)\,Q(A,\phi)-(1-\theta)\left(MPK_L(A,\phi)+1-\delta\right)\right]=0.$$

The cutoff value $s^*(A)$ in Proposition 2 is given by:

$$s^*(A)=D_2\left(A,\phi^*(A)\right).$$

The following lemma provides details of the policy functions $\phi(x',s')$, $u(x',s')$ and the prices of capital $Q_H(x',s')$, $Q_L(x',s')$, and $Q(x',s')$ in the state space.

Lemma 1 *The optimal policy functions, $\phi(x',s')$, $u(x',s')$ and the prices of capital $Q_H(x',s')$, $Q_L(x',s')$, and $Q(x',s')$ are given by the following.*

1. *For all (x',s') such that $s'<\hat{s}(A')$, the following prices and quantities jointly satisfy*

the equilibrium conditions with $\zeta_H() = \xi_L() = 0$.

$$\phi(x', s') = \hat{\phi}$$

$$Q_H(x', s') = Q_L(x's') = Q(x', s') = \hat{Q}(A'),$$
$$u(x', s') = u\left(\hat{Q}(A')\right).$$

2. *For all* (x', s') *such that* $\hat{s}(A) \le s' < \bar{s}(A')$, *the following prices and quantities jointly satisfy the equilibrium conditions with* $\zeta_H() > 0$ *and* $\xi_L() = 0$.

   (a) *The policy* $\phi(x', s')$ *is implicitly defined by the solution to the following equation:*

   $$D_1(A', \phi) = s'.$$

   (b) *The equilibrium prices of capital are given by:*

   $$Q_H(x', s') = MPK_H(A', \phi(x', s')) + 1 - \delta,$$

   $$Q_L(x', s') = Q(x', s') = MPK_L(A', \phi(x', s')) + 1 - \delta.$$

   (c) *The policy function* $u(x', s')$ *is given by:*

   $$u(x', s') = u_L(A', \phi(x', s')).$$

3. *For all* (x', s') *such that* $\bar{s}(A) \le s' \le s^*(A')$, *the following prices and quantities jointly satisfy the equilibrium conditions with* $\zeta_H() > 0$ *and* $\xi_L() = 0$.

   (a) *The policy* $\phi(x', s')$ *is implicitly defined by the solution to the following equation:*

   $$D_2(A', \phi) = s'$$

   (b) *The equilibrium prices of capital are given by:*

   $$\begin{aligned} Q_H(x', s') &= MPK_H(A', \phi(x', s')) + 1 - \delta \\ Q_L(x', s') &= MPK_L(A', \phi(x', s')) + 1 - \delta \\ Q(x', s') &= Q(A', \phi(x', s')). \end{aligned}$$

(c) *The policy function* $u(x', s')$ *is given by:*

$$u(x', s') = 1 - \left(\frac{b_0}{Q(x', s')}\right)^{\frac{1}{1-\nu}}.$$

Proof. Substituting equations (31) and (32) into two limited commitment constraints (34) and (35), we write the two limited commitment constraints as:

$$(1-\theta)\left[MPK_H(A', \phi') + (1-\delta)\right] - \left[\begin{array}{c}(1-\omega)\,MPK(u') \\ -(1-\theta)\,MPK_H(A', \phi') + (\theta-\omega)(1-\delta)\end{array}\right]\left(\frac{u'\phi'}{\pi\phi' + (1-\pi)} - 1\right) \geq s', \tag{66}$$

$$(1-\theta)\left[MPK_L(A', \phi') + (1-\delta)\right] \tag{67}$$

$$-\left[\begin{array}{c}(1-\omega)\,MPK(u') \\ -(1-\theta)\,MPK_L(A', \phi') + (\theta-\omega)(1-\delta)\end{array}\right]\left(\frac{u'}{\pi\phi' + (1-\pi)} - 1\right) \geq s'. \tag{68}$$

Similarly we substitute equations (31) and (32) into Lagrangian multipliers (36) and (37), and define:

$$\zeta_H(A', \phi', u') = \frac{\pi\left[MPK_H(A', \phi') - MPK(u')\right]}{\left[(1-\omega)\,MPK(u') - (1-\theta)\,MPK_H(A', \phi')\right] + (\theta-\omega)(1-\delta)}, \tag{69}$$

$$\zeta_L(A', \phi', u') = \frac{(1-\pi)\left[MPK_L(A', \phi') - MPK(u')\right]}{\left[(1-\omega)\,MPK(u') - (1-\theta)\,MPK_L(A', \phi')\right] + (\theta-\omega)(1-\delta)}. \tag{70}$$

The Kuhn-Tucker conditions imply:

$$\zeta_H(A', \phi', u') \;\geq\; 0, \;\; > 0 \implies (66) \textit{ holds with } =, \tag{71}$$

$$\zeta_L(A', \phi', u') \;\geq\; 0,, \;\; > 0 \implies (68) \textit{ holds with } =. \tag{72}$$

Note the policy functions $\phi(A, s)$ and $u(A, s)$ are determined by conditions (66), (68), (71) and (72). To simplify notation, here and after in this section, we use current period state variable Z instead of Z'. We also define the LHS of constraint (68) as a function of (A, ϕ, u):

$$\Psi(A,\phi,u) = (1-\theta)\left[MPK_H(A,\phi) + (1-\delta)\right]$$
$$-\left\{(1-\omega)MPK(u) - (1-\theta)MPK_H(A,\phi) + (\theta-\omega)(1-\delta)\right\}\left(\frac{u\phi}{\pi\phi + (1-\pi)} - 1\right).$$

To study the nature of the binding constraint, it is convenient to define Δ as:

$$\begin{aligned}\Delta(A,\phi,u) &= (1-\theta)MPK_L(A,\phi) - (1-\theta)\phi MPK_H(A,\phi)\\ &\quad + (1-\omega)(\phi-1)MPK(u) + (\theta-\omega)(\phi-1)(1-\delta).\end{aligned}$$

Throughout, we maintain the assumption $\theta > \omega$. Note that we have three cases: ■

- Only constraint (66) binds $\Longrightarrow \Delta > 0$.
- Both constraints (68) and (68) binds $\Longrightarrow \Delta = 0$.
- Only constraint (68) binds $\Longrightarrow \Delta < 0$.

First best case, no constraint binds: We define

$$\hat{s}(A) = \Psi\left(A, \hat{\phi}, u\left(\hat{Q}(A)\right)\right).$$

and simplify the expression as

$$\Psi\left(A,\hat{\phi},u\left(\hat{Q}(A)\right)\right) = \frac{\hat{Q}(A)}{\pi\hat{\phi} + 1 - \pi}\left\{\left[(1-\omega) - (\theta-\omega)\hat{u}(A)\right]\hat{\phi} - (1-\omega)(1-\pi)\left(\hat{\phi}-1\right)\right\}.$$

Claim 1 *If $s \leq \hat{s}(A)$ then the optimal policy is given by:*

$$\phi(A,s) = \widehat{\phi}, \quad u(A,s) = u\left(\hat{Q}(A)\right). \tag{73}$$

Proof. We need to show that (66), (68), (71) and (72) are satisfied with above choices of the Lagrangian multipliers. Under the proposed policies and prices, the LHS of (66) is

$$\Psi\left(A,\hat{\phi},u\left(\hat{Q}(A)\right)\right) = \hat{s}(A) \geq s.$$

Also,

$$\begin{aligned}\Delta &= (1-\theta)\, MPK_L(A,\phi) - (1-\theta)\,\phi MPK_H(A,\phi) + (1-\omega)\,(\phi-1) MPK(u) \\ &= (\theta-\omega)\,(\phi-1)\, MPK_H(A,s) > 0\end{aligned}$$

Therefore, both (66) and (68) are satisfied. Finally, note that (73) implies that $MPK_H(A,\phi(A,s)) = MPK_L(A,\phi(A,s)) = MPK(A,s) = \alpha\left(1-\frac{1}{\eta}\right)A$, and therefore $\xi_H(A,s) = \xi_L(A,s) = 0$. As a result, the Kuhn-Tucker conditions (71) and (72) are satisfied with $\xi_H(A,s) = \xi_L(A,s) = 0$. ■

Only the constraint on high productivity islands binds: Define $u_L(A,\phi)$ as

$$u_L(A,\phi) = 1 - \left(\frac{b_0}{MPK_L(A,\phi)+1-\delta}\right)^{\frac{1}{1-\nu}}.$$[19]

Let $\bar{\phi}(A)$ be the unique solution to

$$\Delta(A,\phi,u_L(A,\phi)) = 0,$$

and let $\bar{s}(A)$ be

$$\bar{s}(A) = \Psi\left(A,\bar{\phi}(A),u_L\left(A,\bar{\phi}(A)\right)\right).$$

Given the definition of $u_L(A,\phi)$, we can show that

$$\begin{aligned}&\Psi(A,\phi,u_L(A,\phi)) \\ =\; &\frac{MPK_L(A,\phi)}{\pi\phi+1-\pi}\left\{(1-\theta)\,u_L(A,\phi)\,\hat{\phi}^{1-\xi}\phi^{\xi} - (1-\omega)\left[(\phi-1)(1-\pi) - \phi\,(1-u_L(A,\phi))\right]\right\} \\ &+\frac{1-\delta}{\pi\phi+1-\pi}\left\{(1-\theta)\,u_L(A,\phi)\,\phi - (1-\omega)\left[(\phi-1)(1-\pi) - \phi\,(1-u_L(A,\phi))\right]\right\},\end{aligned}$$

$$\Delta(A,\phi,u_L(A,\phi)) = MPK_L(A,\phi)\left[(1-\omega)(\phi-1) - (1-\theta)\left(\hat{\phi}^{1-\xi}\phi^{\xi}-1\right)\right] + (1-\delta)(\theta-\omega)(\phi-1).$$

Using the above expressions, we can prove that $\Psi(A,\phi,u_L(A,\phi))$ is strictly decreasing in ϕ and $\Delta(A,\phi,u_L(A,\phi))$ is strictly increasing functions of ϕ. As a result, i) $\phi \geq \bar{\phi}(A)$ if and only if $\Delta(A,\phi,u_L(A,\phi)) \geq 0$; ii) $\phi \geq \bar{\phi}(A)$ if and only if $\Psi(A,\phi,u_L(A,\phi)) \leq \Psi\left(A,\bar{\phi}(A),u_L\left(A,\bar{\phi}(A)\right)\right)$.

[19] That is, $u_L(A,\phi) = u(MPK_L(A,\phi)+1-\delta)$.

Claim 2 *If $\hat{s}(A) \leq s \leq \bar{s}(A)$ then the optimal policy $\phi(A,s)$ is implicitly defined by the unique solution to*

$$\Psi(A, \phi, u_L(A, \phi)) = s. \tag{74}$$

Given $\phi(A,s)$, the optimal policy $u(A,s)$ is given by

$$u(A,s) = u_L(A, \phi(A,s)). \tag{75}$$

Proof. *First, by construction, $\Psi(A, \phi, u_L(A,\phi)) = s$ and (66) holds with equality. Also, the assumption that $s \leq \bar{s}(A)$ implies $\phi \geq \bar{\phi}(A)$ and $\Delta(A, \phi, u_L(A,\phi)) \geq 0$; therefore, (68) is satisfied. Finally, condition (75) implies $MPK(A,s) = MPK_L(A, \phi(A,s))$ and $\xi_L(A,s) = 0$; therefore, the Kuhn-Tucker condition (72) is satisfied.* ■

Both constraints bind: Define

$$s^*(A) = \max_{\phi, u} \Psi(A, \phi, u)|_{\Delta(A,\phi,u)=0}. \tag{76}$$

Claim 3 *If $s^*(A) \leq s < \bar{s}(A)$ then the optimal policy $\{\phi(A,s), u(A,s)\}$ are jointly determined by:*

$$\Psi(A, \phi, u) = s, \qquad \Delta(A, \phi, u) = 0. \tag{77}$$

Proof. *Clearly, by construction, both (66) and (68) holds with equality. Also, we can show that $u(\phi, s) < u_L(A, \phi(A,s))$; therefore, $MPK(A,s) < MPK_H(A, \phi(A,s))$, $MPK_L(A, \phi(A,s))$ and $\xi_H(A,s)$, $\xi_L(A,s) > 0$. As a result, the Kuhn-Tucker conditions (71) and (72) are satisfied.* ■

Note that we can simplify further by using $\Delta(A, \phi, u) = 0$ to define $Q(A, \phi)$ as the price of capital such that $\Delta(A, \phi, u) = 0$:

$$Q(A, \phi) = (1-\delta) + \frac{1}{1-\omega}\left[(1-\theta) MPK_L(A,\phi) \frac{\hat{\phi}^{1-\xi}\phi^{\xi} - 1}{\phi - 1} - (\theta - \omega)(1-\delta)\right].$$

With this notation, we can substitute u and write $\Psi(A, \phi, u)$ as

$$\Psi(A, \phi, u(Q(A,\phi))) = (1-\theta)\left[MPK_L(A,\phi) + 1 - \delta\right]$$
$$- \left[(1-\omega) Q(A,\phi) - (1-\theta)\left(MPK_L(A,\phi) + 1 - \delta\right)\right]\left(\frac{u(Q(A,\phi))}{\pi\phi + 1 - \pi} - 1\right).$$

Equation (77) becomes $\Psi(A,\phi,u(Q(A,\phi))) = s$, and (76) becomes

$$s^*(A) = \max_{\phi} \Psi(A,\phi,u(Q(A,\phi))).$$

D Construction of the Markov Equilibrium

We first use the result of Appendix C to construct the policy functions $\phi(A,s)$ and $u(A,s)$ and the associated prices and Lagrangian multipliers. We then characterize other equilibrium conditions. The FOC (40) and (43) imply:

$$E\left[\widetilde{M}'\{1+\zeta_H(A',s')+\zeta_L(A',s')\}\right]R_f(Z) = E\left[\widetilde{M}'\{1+(1-\omega)[\zeta_H(A',s')+\zeta_L(A',s')]\}Q(A',s')\right]. \tag{78}$$

Suppose we start with an initial guess of policy functions of normalized consumption and marginal value of net worth, $c(A',s')$ and $\mu(x',s')$, the SDF can be written as:

$$M(Z,Z') = \beta\frac{C(Z)}{C(Z')} = \frac{\beta[Auf(\phi)-i]}{c(A',s')}\frac{K}{K'} = \frac{\beta[m(A,s)-i]}{c(A',s')[1-\delta(A,s)+i]},$$

where we denote

$$\begin{aligned} m(A,s) &= Au(A,s)f(\phi(A,s)), \\ 1-\delta(A,s) &= h(1-u(A,s))+(1-\delta)u(A,s). \end{aligned}$$

Because the risk-free interest rate $R_f(Z)$ satisfies

$$R_f(Z) = \frac{1}{E[M(Z,Z')|Z]}, \tag{79}$$

we have:

$$R_f(Z) = \frac{1}{E\left[\frac{\beta[m(A,s)-i]}{c(A',s')[1-\delta(A,s)+i]}\right]}, \tag{80}$$

thus the LHS of equation (78) is now written as:

$$\begin{aligned}
&\frac{E\left[\frac{\beta[m(A,s)-i]}{c(A',s')[1-\delta(A,s)+i]}\left\{(1-\Lambda')+\Lambda'\mu\left(x',s'\right)\right\}\left\{1+\zeta_H\left(A',s'\right)+\zeta_L\left(A',s'\right)\right\}\right]}{E\left[\frac{\beta[m(A,s)-i]}{c(A',s')[1-\delta(A,s)+i]}\right]} \\
= \ &\frac{E\left[\frac{\{(1-\Lambda')+\Lambda'\mu(x',s')\}}{c(A',s')}\left\{1+\zeta_H\left(A',s'\right)+\zeta_L\left(A',s'\right)\right\}\right]}{E\left[\frac{1}{c(A',s')}\right]}.
\end{aligned}$$

The RHS of equation (78) is

$$E\left[\frac{\beta\left[m\left(A,s\right)-i\right]\left\{(1-\Lambda')+\Lambda'\mu\left(x',s'\right)\right\}}{c\left(A',s'\right)\left[1-\delta\left(A,s\right)+i\right]}\left\{1+(1-\omega)\left[\zeta_H\left(A',s'\right)+\zeta_L\left(A',s'\right)\right]\right\}Q\left(A',s'\right)\right]$$

Therefore, equation (78) is can be written as:

$$\begin{aligned}
&E\left[\frac{\beta\left[m\left(A,s\right)-i\right]\left\{(1-\Lambda')+\Lambda'\mu\left(x',s'\right)\right\}}{c\left(A',s'\right)\left[1-\delta\left(A,s\right)+i\right]}\left\{1+(1-\omega)\left[\zeta_H\left(A',s'\right)+\zeta_L\left(A',s'\right)\right]\right\}Q\left(A',s'\right)\right] \\
= \ &\frac{E\left[\frac{\{(1-\Lambda')+\Lambda'\mu(x',s')\}}{c(A',s')}\left\{1+\zeta_H\left(A',s'\right)+\zeta_L\left(A',s'\right)\right\}\right]}{E\left[\frac{1}{c(A',s')}\right]}, \qquad (81)
\end{aligned}$$

or

$$\frac{m\left(A,s\right)-i}{\left[1-\delta\left(A,s\right)+i\right]}=\frac{E\left[\frac{\{(1-\Lambda')+\Lambda'\mu(x',s')\}}{c(A',s')}\left\{1+\zeta_H\left(A',s'\right)+\zeta_L\left(A',s'\right)\right\}\right]}{\beta E\left[\frac{\{(1-\Lambda')+\Lambda'\mu(x',s')\}}{c(A',s')}\left\{1+(1-\omega)\left[\zeta_H\left(A',s'\right)+\zeta_L\left(A',s'\right)\right]\right\}Q\left(A',s'\right)\right]\times E\left[\frac{1}{c(A',s')}\right]}. \qquad (82)$$

Next we describe the law of motion of the state variable. Let N denote the aggregate net worth of the banking sector in the current period, N'_H, and N'_L denote the total net worth of the bank in the next period in the case of high productivity and low productivity, respectively. Because only a fraction Λ of the banks survive to the next period, total bank net worth in the period, N' is:

$$N'=\Lambda'\left\{\pi N'_H+(1-\pi)\,N'_L\right\}.$$

Using equation (12),

$$\pi N'_H + (1-\pi) N'_L = \pi Q_H(Z')[K' + RA_H(Z')] + (1-\pi) Q_L(Z')[K' + RA_L(Z')] - Q(Z')[\pi RA_H(Z') + (1-\pi) RA_L(Z')] - R_f(Z)B_f \quad (83)$$

Denoting $K'_H = K' + RA_H(Z')$ and $K'_L = K' + RA_L(Z')$, and using $\pi MPK_H(Z') K'_H + (1-\pi) MPK_L(Z') K'_L = \alpha\left(1 - \frac{1}{\eta}\right) Y'$, together with $\pi K'_H + (1-\pi) K'_L = u'K'$, we write the first two terms of the above equation as

$$\begin{aligned} & \pi Q_H(Z') K'_H + (1-\pi) Q_L(Z') K'_L \\ = {} & \pi MPK_H(Z') K'_H + (1-\pi) MPK_L(Z') K'_L + (1-\delta)[\pi K'_H + (1-\pi) K'_L]. \\ = {} & \alpha\left(1 - \frac{1}{\eta}\right) Y' + (1-\delta) u'K'. \end{aligned} \quad (84)$$

By the resource constraint (16), we have $\pi RA_H(Z') + (1-\pi) RA_L(Z') = (u'-1) K'$. In addition, $Q(Z') = MPK(Z') K' + (1-\delta)$, we can combine the first three terms in (83) as:

$$\begin{aligned} & \pi Q_H(Z') K'_H + (1-\pi) Q_L(Z') K'_L - Q(Z')[\pi RA_H(Z') + (1-\pi) RA_L(Z')] \\ = {} & \alpha\left(1 - \frac{1}{\eta}\right) Y' + (1-\delta) u'K' - Q(Z')(u'-1) K' \quad (85) \\ = {} & \alpha\left(1 - \frac{1}{\eta}\right) Y' + (1-u') MPK(Z') K' + (1-\delta) K'. \quad (86) \end{aligned}$$

Therefore, (83) can be written as:

$$\pi N'_H + (1-\pi) N'_L = \alpha\left(1 - \frac{1}{\eta}\right) Y' + (1-u') MPK(Z') K' + (1-\delta) K' - R_f(Z) B_f.$$

We have:

$$N' = \Lambda' \left[\alpha\left(1 - \frac{1}{\eta}\right) Y' + (1-u') MPK(Z') K' + (1-\delta) K' - R_f(Z) B_f\right]. \quad (87)$$

Next we derive the law of motion for state variable s. Using banks' budget constraint (10), we derive

$$s' = \frac{R_f(Z) B_f}{K'} = \frac{R_f(Z)(K' - N)}{K'} = R_f(Z)\left[1 - \frac{N}{K'}\right] = R_f(Z) \frac{K}{K'}\left[\frac{K'}{K} - \frac{N}{K}\right]. \quad (88)$$

We can express all the terms on the RHS of equation (87) as functions of the state variables (A, s):

$$Y = Am(A,s)K, \quad u = u(A,s), \quad R_f(Z_{-1})B_{f,-1} = sK.$$

Therefore, we can express $\frac{N}{K}$ as a function of the state variable (A, s):

$$\frac{N}{K} = \Lambda\left[\alpha\left(1-\frac{1}{\eta}\right)Am(A,s) + (1-u(A,s))MPK(A,s) + (1-\delta) - s\right] \tag{89}$$

Now we can combine equations (88) and (89) and use (80) to derive the law of motion of the state variable s:

$$s' = \frac{\left\{[1-\delta(A,s)+i] - \Lambda\left[\alpha\left(1-\frac{1}{\eta}\right)Am(A,s) + (1-u(A,s))MPK(A,s) + (1-\delta) - s\right]\right\}}{\beta[m(A,s)-i]E\left[\frac{1}{c(A',s')}\right]} \tag{90}$$

Now we can combined equations (82) and (90) to solve for the two unknowns, i and s'.

E Computation Details

E.1 Recursive policy function iteration approach

We use the result of Appendix C to construct the policy functions $\phi(A,s)$ and $u(A,s)$ and the associated prices and Lagrangian multipliers. Let $\{c^n(x,s), \mu^n(x,s)\}$ be an initial guess of an equilibrium functional. Use equations (82) and (90) to construct the policy function $i(A,s)$ and law of motion $s'(A,s)$. This involves for each (A,s) solving the two nonlinear equations (82) and (90). Use the envelop condition (43) and the resource constraint to update $\{c^n(x,s), \mu^n(x,s)\}$:

$$\mu^{n+1}(x,s) = \frac{E\left[\frac{\{(1-\Lambda')+\Lambda'\mu^n(x',s')\}}{c^n(x',s')}\{1+\zeta_H(A',s')+\zeta_L(A',s')\}\middle| x\right]}{E\left[\frac{1}{c^n(x',s')}\middle| x\right]},$$

$$c^{n+1}(x,s) = m(A,s) - i(x,s).$$

We then iterate until convergence.

E.2 Solving for the deterministic steady state equilibrium (SSE)

We first consider the deterministic SSE assuming that the SSE is in case 2, that is, only one constraint in binding.

1. Using equation (36) to define ξ_H as function of ϕ:

$$\zeta_H(\phi) = \frac{\pi MPK_L(A,\phi)\left[\left(\frac{\hat{\phi}}{\phi}\right)^{1-\xi} - 1\right]}{(\theta-\omega)\left[MPK_L(A,\phi) + (1-\delta)\right]}. \tag{91}$$

2. Given $\xi(\phi)$, equation (82) can be used to define investment as a function ϕ:

$$1 - \delta + i = \frac{1 + (1-\omega)\,\xi_H(\phi)}{1 + \xi(\phi)}\left[MPK_L(A,\phi) + 1 - \delta\right]. \tag{92}$$

   Note that here, we are using the fact that $\delta(A,s) = \delta$ in the steady state.

3. We can now use the law of motion of s to express SSE s as a function of ϕ:

$$(\beta - \Lambda)\,s = (1-\delta)(1-\Lambda) + i(\phi) - \Lambda\left\{\alpha\left(1 - \frac{1}{\eta}\right)Au_L(A,\phi)\,f(\phi) + (1 - u_L(A,\phi))\,MPK_L(A,\phi)\right\}.$$

4. Finally, we use (74) to solve for the SSE ϕ:

$$\begin{aligned} s(\phi) &= \frac{MPK_L(A,\phi)}{\pi\phi + 1 - \pi}\left\{(1-\theta)\,u_L(A,\phi)\,\hat{\phi}^{1-\xi}\phi^{\xi} - (1-\omega)\left[(\phi-1)(1-\pi) - \phi(1 - u_L(A,\phi))\right]\right\} \\ &\quad + \frac{1-\delta}{\pi\phi + 1 - \pi}\left\{(1-\theta)\,u_L(A,\phi)\,\phi - (1-\omega)\left[(\phi-1)(1-\pi) - \phi(1 - u_L(A,\phi))\right]\right\}. \end{aligned}$$

After solving the SSE level of ϕ, we can derive all other SSE quantities using equilibrium conditions. Note that in the above calculation, if we assume SSE levels of depreciation rate δ_{sse} and capital utilization rate, u_{sse}, the steady state is determined as a function of the following five primitive parameters of the model: A_{sse}, $\hat{\phi}$, θ, ω, and Λ_{sse}.

E.3 Setting moments

1. Given a SSE capital depreciation $\delta = 0.02$, we can use the growth rate to set investment-to-capital ratio:

$$gro_{DATA} = 1 - \delta_{DATA} + i_{DATA}.$$

Note that we have used a moment to set an endogenous variable, not a parameter. However, equations (91) and (92) jointly put a restriction on other parameter values with this choice of i: $i(\phi) = i_{DATA}$, where ϕ is a function of the parameters $\left(A_{sse}, \hat{\phi}, \theta, \omega, \pi\right)$ from equations (91) and (92).

2. Note that SSE capital reallocation-to-investment ratio is

$$\left.\frac{RA}{I}\right|_{DATA} = \frac{\pi\left[(1-\pi)(\phi-1)+(1-u_{sse})\phi\right]}{(\pi\phi+1-\pi)\,i}.$$

   the ratio of output from high relative to low productivity firms is

$$\left.\frac{\pi TR_H}{(1-\pi)\,TR_L}\right|_{DATA} = \frac{\pi}{1-\pi}\hat{\phi}^{1-\xi}\phi^{\xi}.$$

   Note that SSE ϕ is a function of parameters of the model, $\left(A_{sse}, \hat{\phi}, \theta, \omega, \Lambda_{sse}, \pi\right)$. Therefore, by setting $\frac{RA}{I} = 57.3\%$ and $\frac{\pi TR_H}{(1-\pi)TR_L} = 1$, we have two more conditions to put restrictions on the parameters (assuming a steady-state u_{sse}).

3. Given u_{DATA}, and given i_{DATA}, the investment-to-output ratio is:

$$\left.\frac{I}{Y}\right|_{DATA} = \frac{i_{DATA}}{Auf(\phi)}.$$

   Note that ϕ is a function of all parameter values of the model. This constitutes another equation that can be used to set parameter values of the model.

4. Bank leverage is another moment in the data that we can use to set parameters values of the model.

$$Leverage = \frac{1-\delta_{sse}+i}{n} = \frac{1-\delta_{sse}+i}{n}$$

   Note that $s' = \frac{R_f B_f}{K'} = R_f \frac{K'-N}{K'} = R_f\left(1-\frac{n}{1-\delta_{sse}+i}\right)$. Therefore, $\frac{n}{1-\delta_{sse}+i} = 1-\frac{s'}{R_f}$. Using $R_f = \frac{1-\delta_{sse}+i}{\beta}$, we have

$$Leverage = 1+\frac{\beta s}{1-\delta_{sse}+i-\beta s}.$$

5. Also the SSE interbank interest rate is given by:

$$R_I = MPK_L\left(A, \phi\right) + 1 - \delta.$$

Therefore, the interbank interest spread is:

$$spread = \ln\left(\frac{1 - \delta + i}{\beta}\right) - \ln\left(MPK_L\left(A, \phi\right) + 1 - \delta\right).$$

Figure 1: **Log TFP and Capital Misallocation Measured in Log TFP Units**

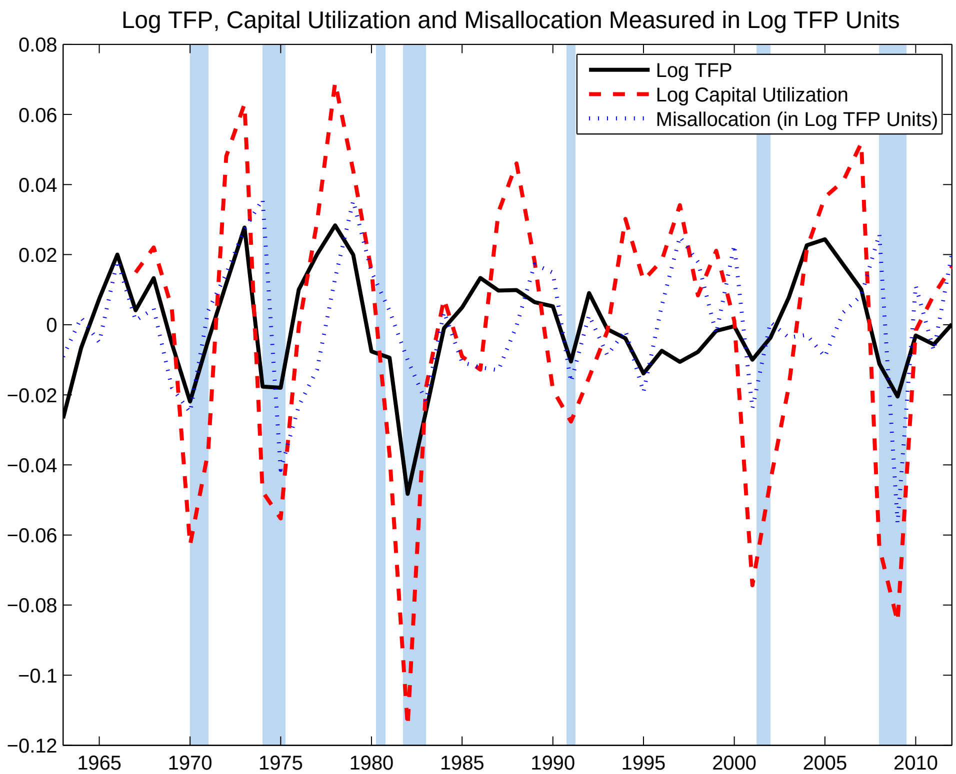

Figure 1 plots the time series of total factor productivity (dashed line) in the U.S. and the measure of capital misallocation (solid line) in the period 1963-2012. The construction of the misallocation measure follows Hsieh and Klenow (2009). We provide the details of the construction in Appendix B. We use the first order Taylor expansion in equation (65) to translate the misallocation measure into log TFP units. Both series are HP filtered.

Figure 2: **Business Cycle Variations of the Total Volume of Bank Loan**

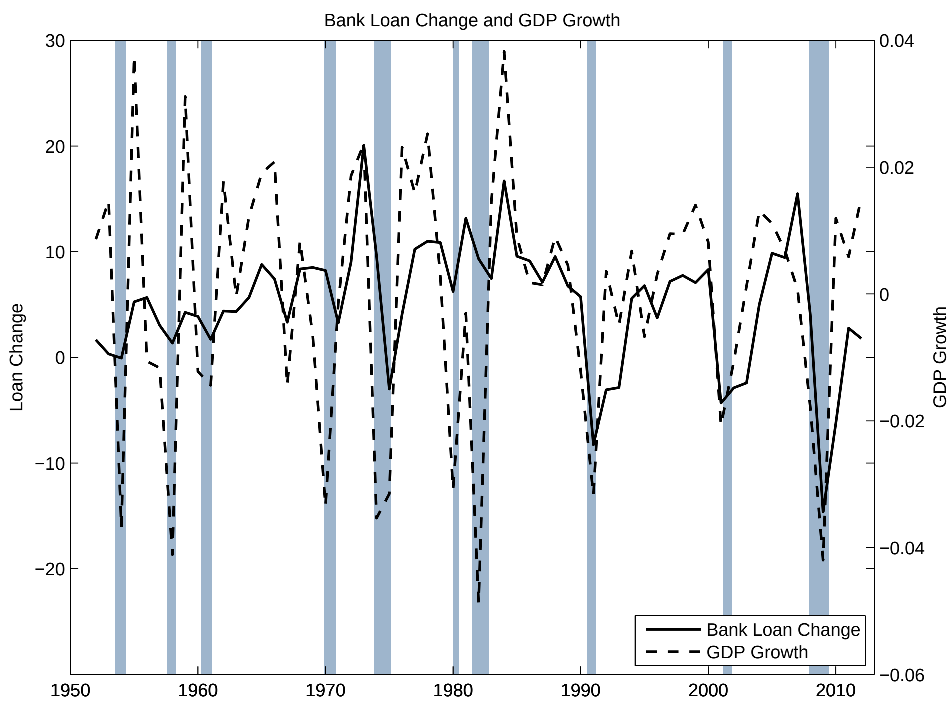

Figure 2 plots the business cycle variations of the total volume of bank loans for all non-financial firms in the US corporate sector. The solid line is the changes in the total volume of bank loans and the dashed line is GDP growth. Shaded areas stand for NBER classified recessions.

Figure 3: **Total Volume of Bank Loan and Capital Misallocation**

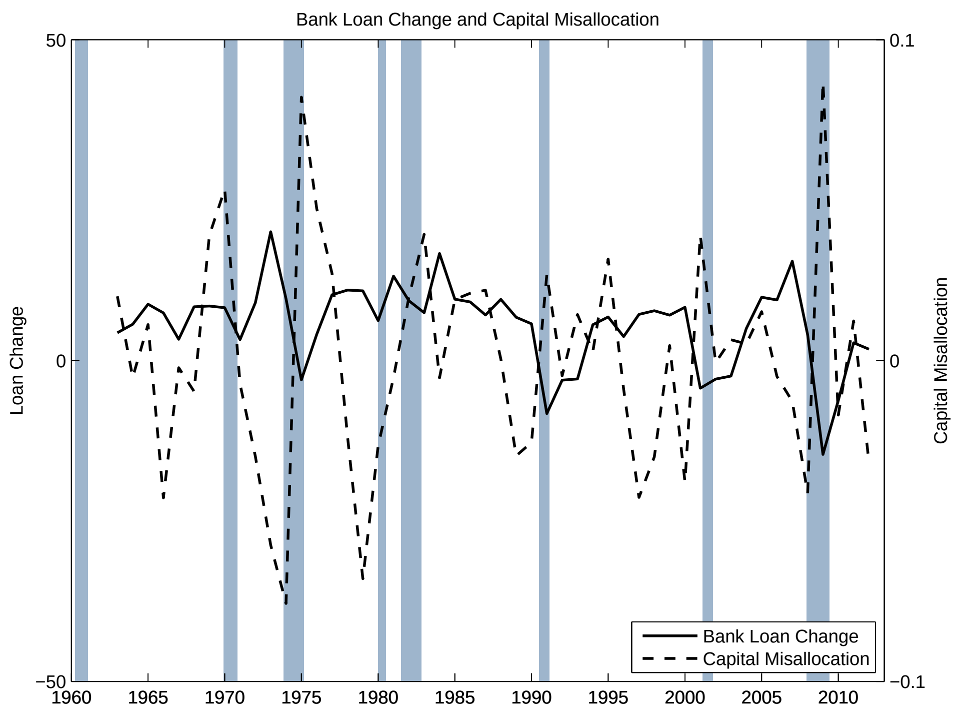

Figure 3 plots the net increases in the total volume of bank loan and our measure of capital misallocation constructed from COMPUSTAT firms during the period 1958-2012.

Figure 4: **Total Volume of Bank Loan and Aggregate Volatility**

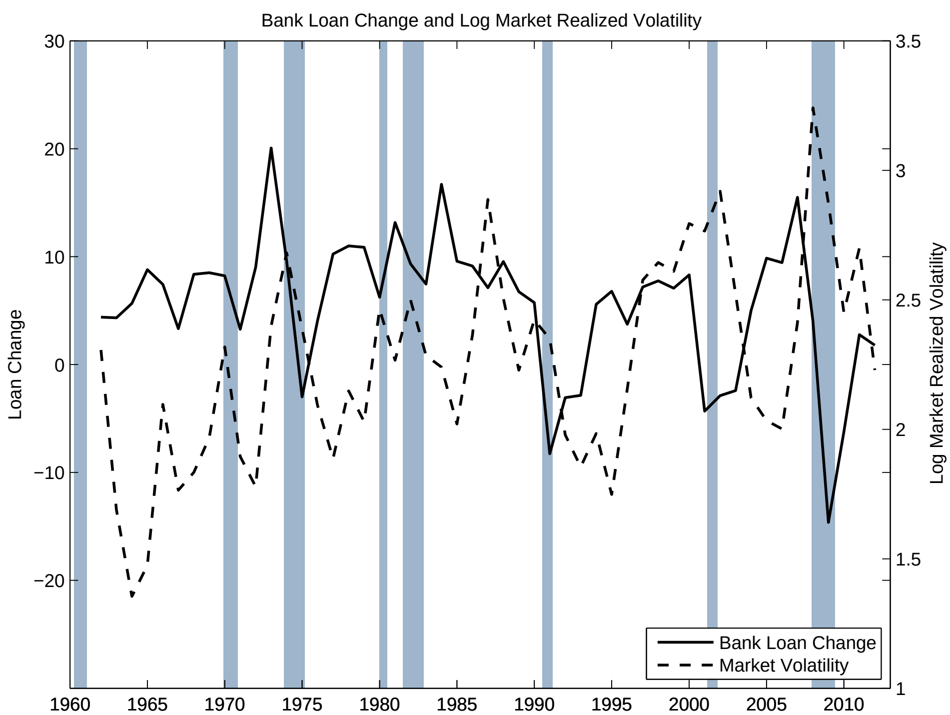

Figure 4 plots the net increases in the total volume of bank loan and stock market volatility in the U.S. during the period 1958-2012.

Figure 5: **Total Volume of Bank Loan and Idiosyncratic Volatility**

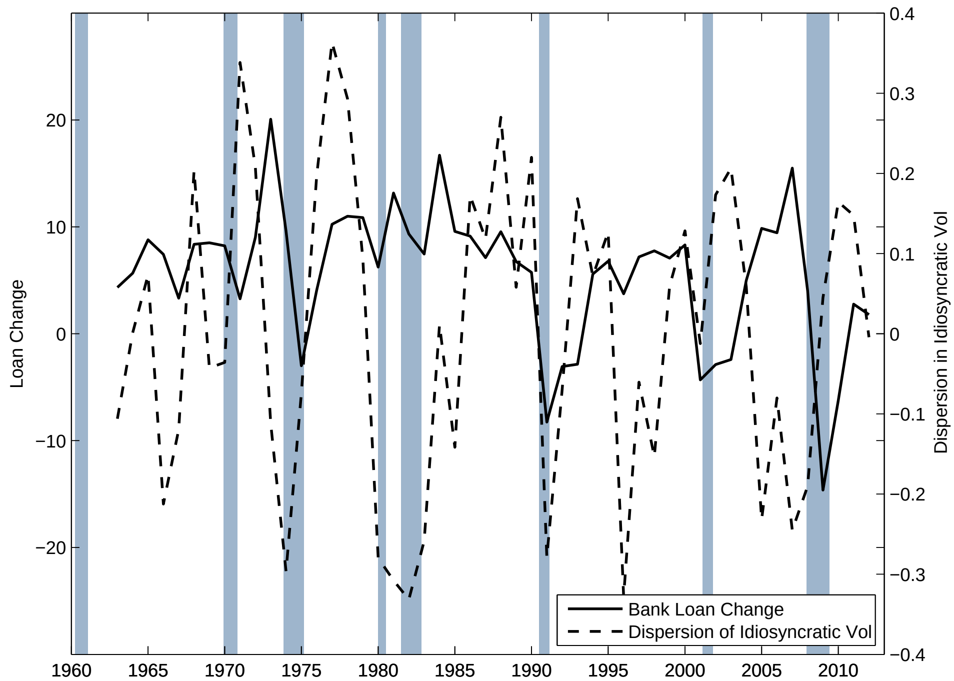

Figure 5 plots the net increases in the total volume of bank loan and the cross-sectional dispersion of firm profit for COMPUSTAT firms.

Figure 6: **Timing of Events**

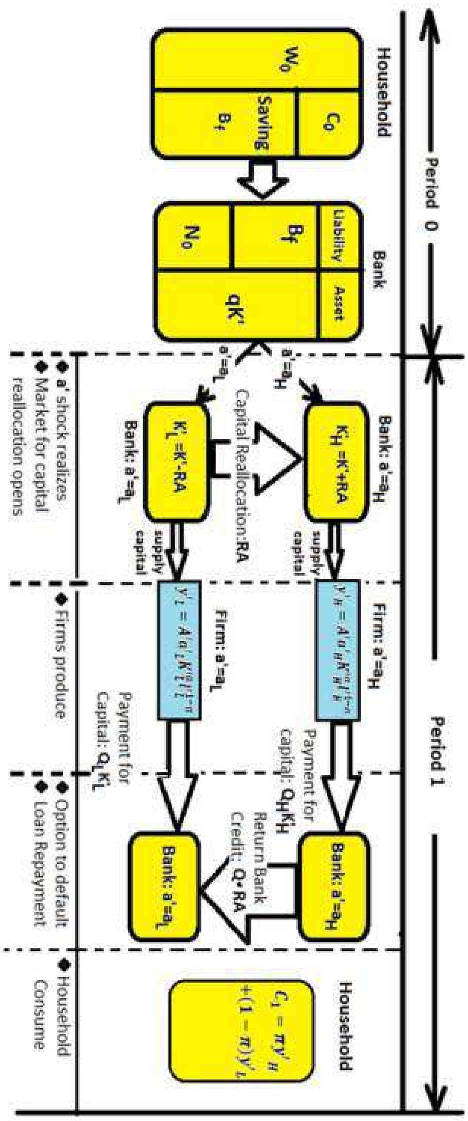

Figure 6 illustrates the timing of event from period t to period $t+1$ in the infinite horizon model.

Figure 7: **Macroeconomic Quantities and Bank Net Worth**

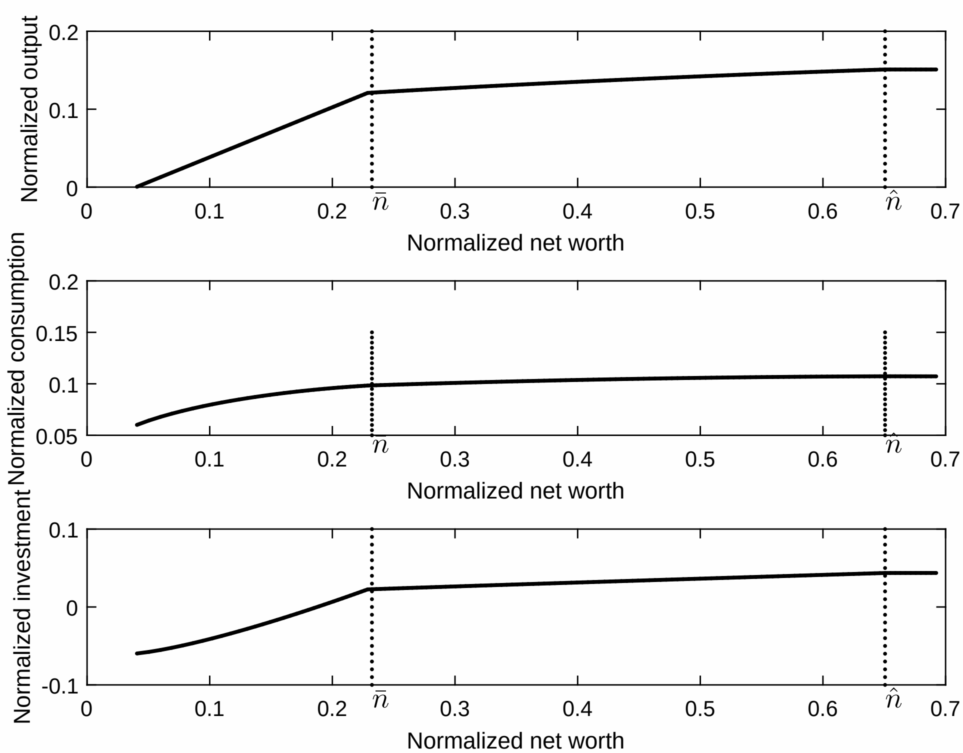

Figure 7 plots the normalized output (top panel), normalized consumption (middle panel) and the normalized investment (bottom panel) as functions of bank net worth. $\hat{n}$ is the cutoff value of bank net worth below which the limited enforcement constraints start to bind for some banks, and $\bar{n}$ is the cutoff value below which the limited commitment constraint bind for all banks.

Figure 8: **Asset Prices and Bank Net Worth**

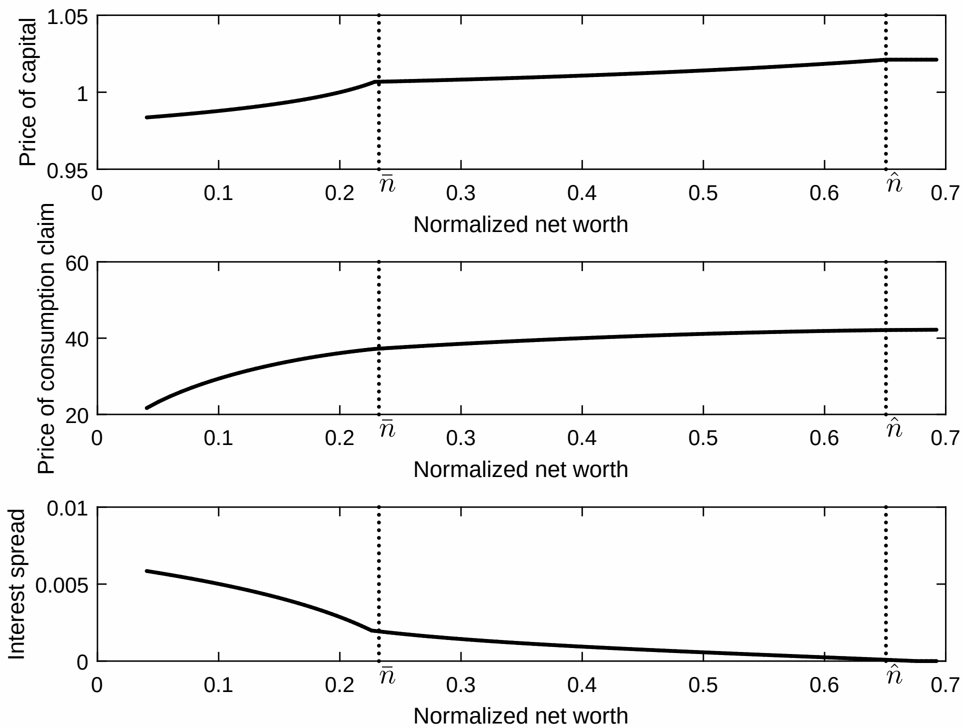

Figure 8 plots the price of capital on the reallocation market (top panel), the price of consumption claim (middle panel), and the spread between the interbank lending rate and the household deposit rate (bottom panel) as functions of bank net worth. $\hat{n}$ is the cutoff value of bank net worth below which the limited enforcement constraints start to bind for some banks, and $\bar{n}$ is the cutoff value below which the limited commitment constraints bind for all banks.

Figure 9: **Capital Reallocation and Bank Net Worth**

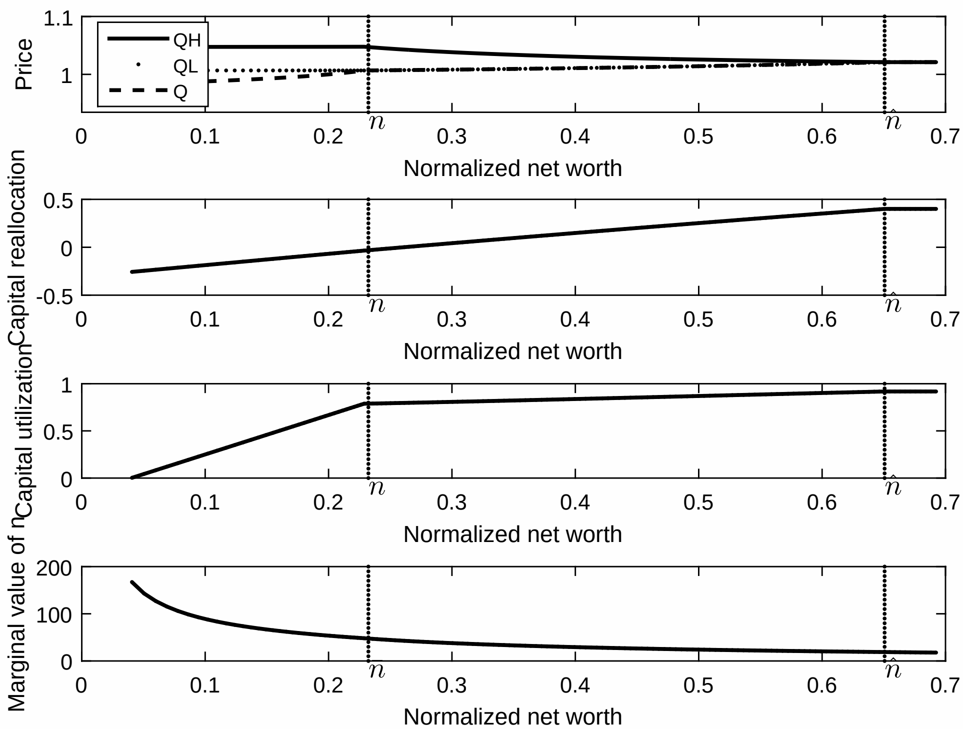

Figure 9 plots the marginal product of capital (top panel), total amount of capital reallocation (second panel), rate of capital utilization (third panel), and the marginal value of bank net worth (bottom panel) as functions of bank net worth. In the top panel, the solid line is marginal product of capital on high productivity islands, the dotted line is that on low productivity islands, and the dashed line is the marginal product of capital in the risk-free storage technology.

Figure 10: **Leverage and Bank Net Worth**

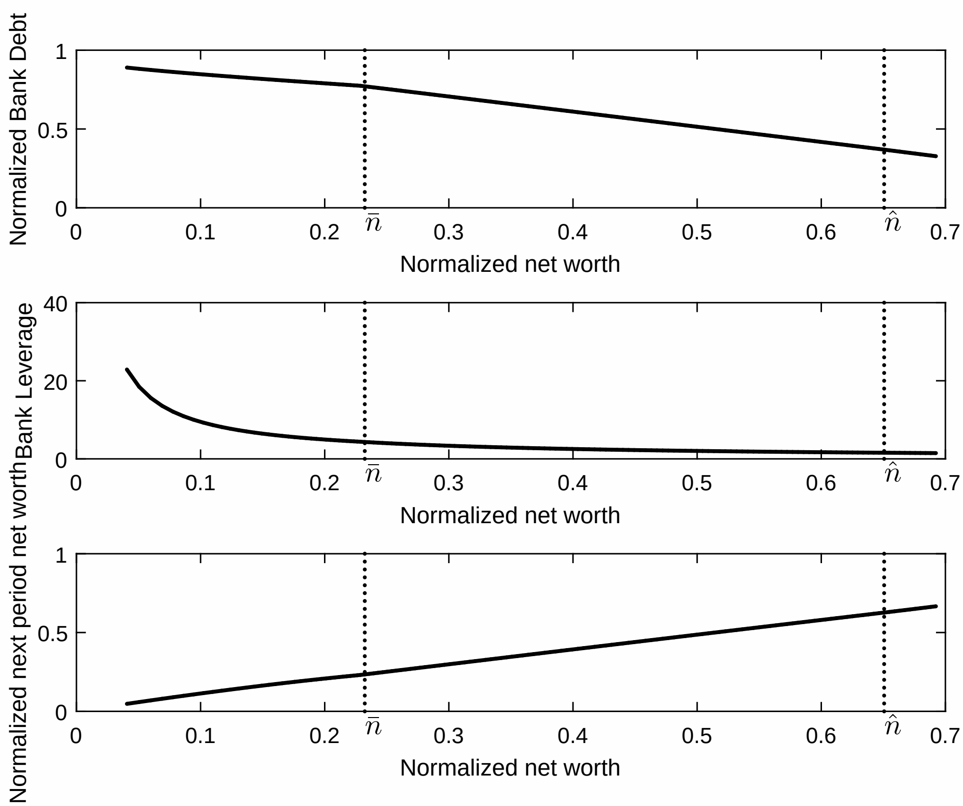

Figure 10 plots the normalized bank debt (top panel), bank leverage (middle panel) and next period net worth (bottom panel) as functions of current period net worth.

Figure 11: **Dynamics of Bank Net Worth**

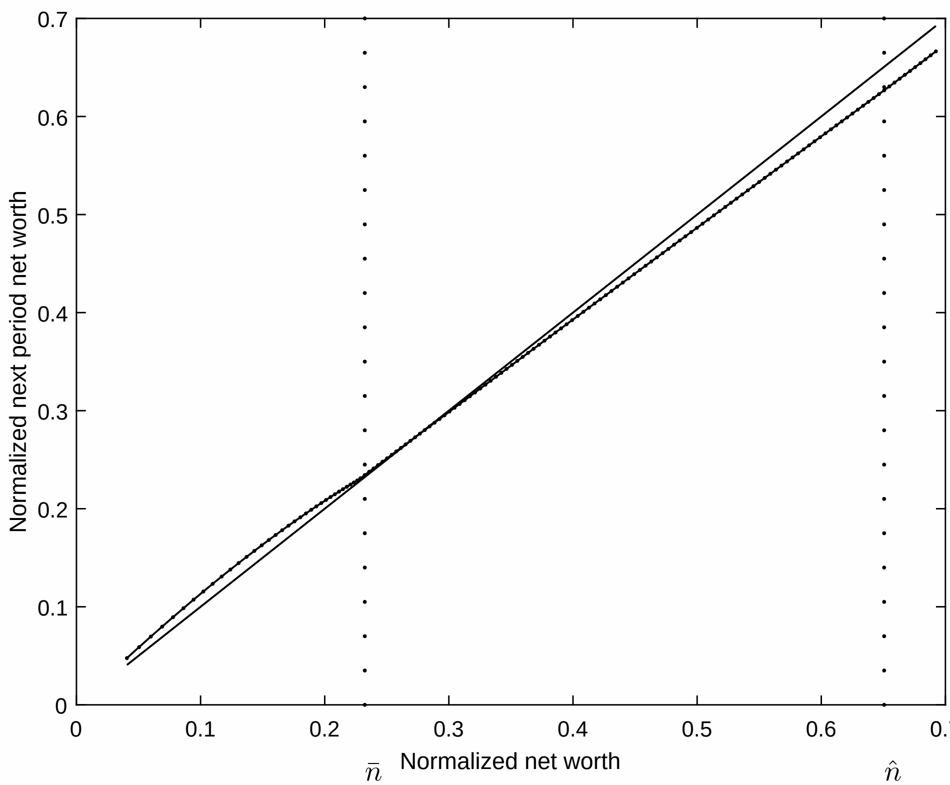

Figure 11 plots next period net worth as a function of current bank net worth (dotted line) and the 45 degree line (solid line). The intersection is the steady state level of bank net worth (n_{SSE}).

Figure 12: **Impulse Responses to Productivity Shocks**

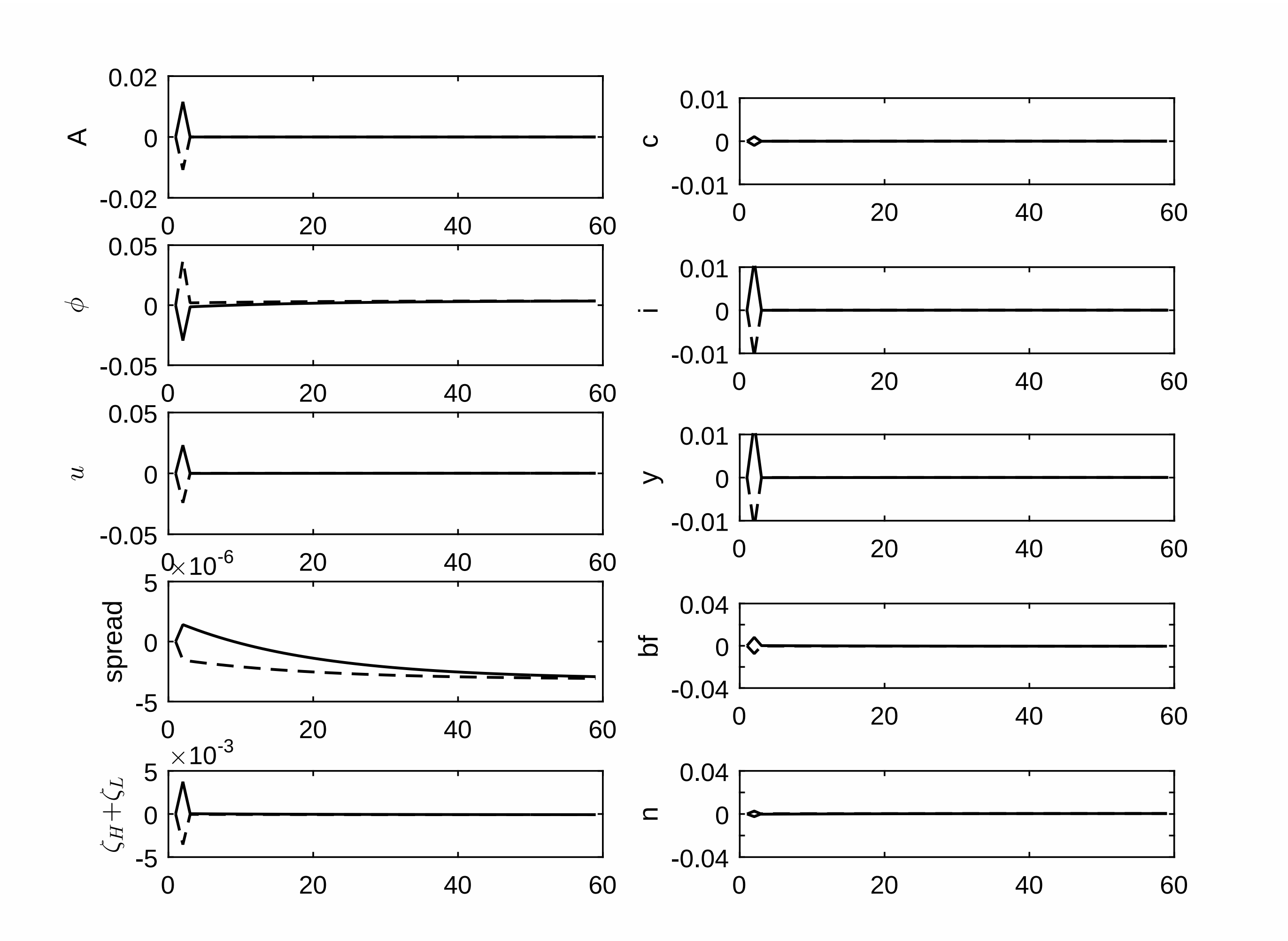

Figure 12 plots the impulse response functions of quantities and prices with respect to a positive innovation in productivity shocks (solid line) and those with respect to a negative innovation in productivity shocks (connected dotted line). We assume shocks are purely transitory and all impulse responses are plotted as deviations from the steady state.

Figure 13: **Impulse Responses to Financial Shocks**

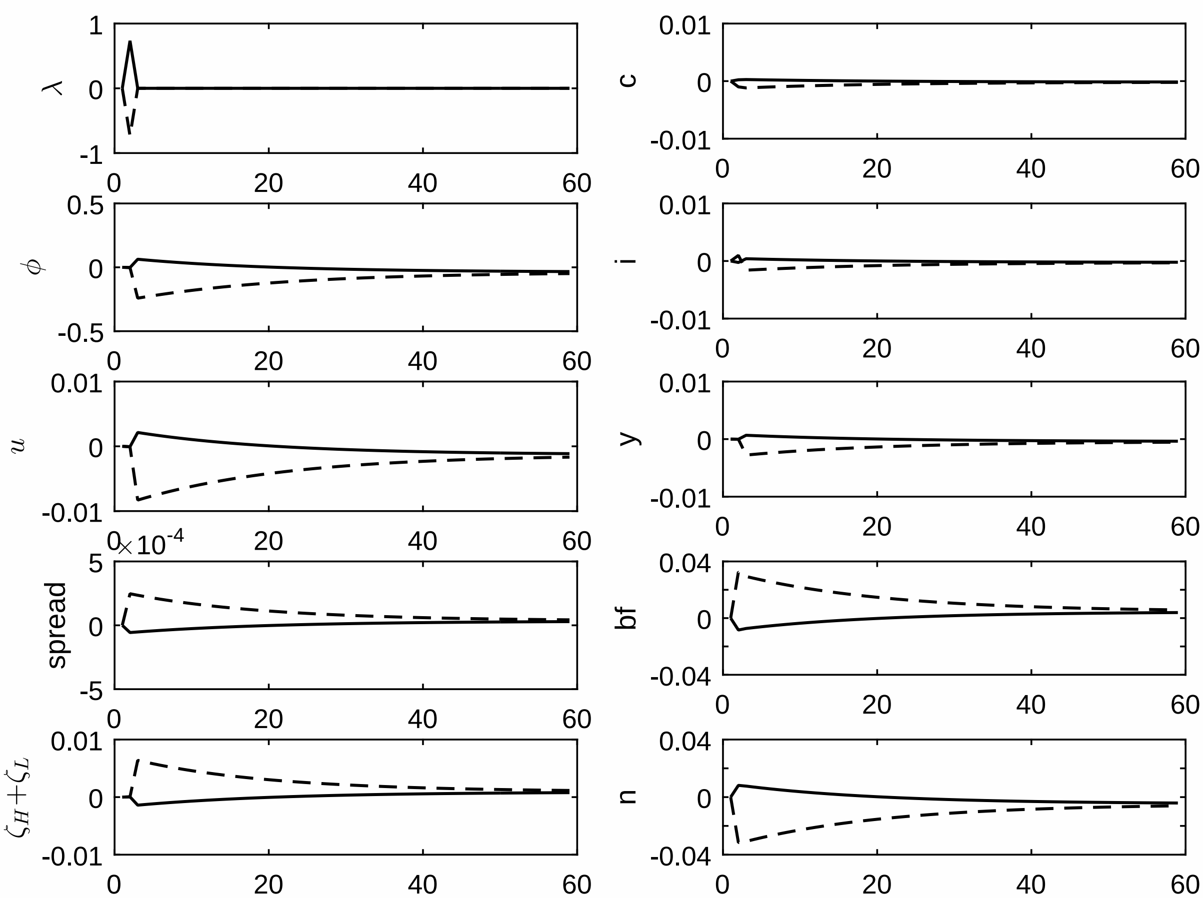

Figure 13 plots the impulse response functions of quantities and prices with respect to a positive innovation in discount rate shocks (solid line) and those with respect to a negative innovation in discount rate shocks (connected dotted line). We assume shocks are purely transitory and all impulse responses are plotted as deviations from the steady state.

Table 1: Calibrated Parameters and Targeted Moments (quarterly)

Parameter		Value	Targeted Moment	Data
β	discount rate	0.999	———	–
α	capital share	0.333	Kydland and Prescott (1982)	–
$\frac{\eta}{\eta-1}$	markup	1.25	Hsieh and Klenow (2009)	–
δ	depreciation	2.4%	Kydland and Prescott (1982)	–
$\frac{a_H}{a_L}$	productivity	1.9402	mean capital reallocation	57.23%
π	prob. of a_H	0.2578	output by high prod. firms	50%
$E[A]$	agg. productivity	0.1645	mean aggregate growth rate	0.5%
Λ	banker discount rate	0.9612	investment-output ratio	20%
θ	banker outside option	0.3026	bank leverage	3.67
ω	interbank friction	0.0423	mean TED spread	0.16%
a_0	storage technology	−0.0118	average capital utilization	81%
b_0	storage technology	0.9819	average depreciation	2%
ν	storage technology	0.98	—	—-

Table 1 lists the parameter values we use in our model and the macroeconomic moments used to calibrate these parameter values.

Table 2: Macroeconomic Moments

Moments	Data	TFP Shocks	Financial Shocks
$E[\Delta \ln Y]$	1.8%	1.85%	1.77%
$Vol[\Delta \ln Y]$	3.49%	3.40%	3.57%
$Vol[\Delta \ln C]$	2.53%	2.45%	1.81%
$Vol[\Delta \ln I]$	13.51%	7.13%	20.42%
$Corr[\Delta \ln C, \Delta \ln I]$	39.7%	92%	28.6%
$AC[\Delta \ln C]$	49%	0.45%	50%
$E[\ln R_f]$	0.86%	4.76%	4%
$Vol[\ln R_f]$	0.97%	0.53%	1.73%
$E[R_I] - E[R_f]$	0.64%	0.74%	0.93%
$Vol[\ln R_I - \ln R_f]$	0.88%	0.04%	0.48%
$Corr[\Delta \ln Y, Var(\Delta \ln Y)]$	-0.15	-0.03	-0.44
$Var[\Delta uf(\phi)]/Var[\Delta TFP]$	$-$	11%	96%

Table 2 documents moments of macroeconomic quantities and interest rates in the U.S. (1930-2009), those generated by our model with TFP shocks, and those generated by our model with financial shocks. $Corr[\Delta \ln Y, Var(\Delta \ln Y)]$ stands for the correlation of output growth and the variance of future output growth. The latter is calculated as the realized variance of quarterly output growth for the next two years. $Var[\Delta uf(\phi)]/Var[\Delta TFP]$ stands for the fraction of variance in output that can be accounted for by changes in the efficiency of capital reallocation.

Table 3: Capital Reallocation and Capital Utilization

Moments	Data	TFP Shocks	Financial Shocks
$E\left[RA/I\right]$	25%	41%	33%
$E\left[Var\left(\ln MPK\right)\right]$	70%	5.13%	5.10%
$Vol\left[Var\left(\ln MPK\right)\right]/E\left[Var\left(\ln MPK\right)\right]$	24%	3.1%	32.3%
$E\left[u\right]$	80%	79.7%	78.3%
$Vol\left[u\right]$	4.08%	1.50%	4.04%
$Corr\left[\Delta \ln \widetilde{TFP}, \Delta \ln RA\right]$	0.24	0.64	0.32
$Corr\left[\Delta \ln \widetilde{TFP}, Var\left(\ln MPK\right)\right]$	-0.14	0.37	-0.48
$Corr\left[\Delta \ln \widetilde{TFP}, \ln u\right]$	0.30	0.96	0.91

Table 3 documents the moments of capital reallocation and capital utilization in the data, and those generated by our models. Our construction of capital reallocation series follows Eisfeldt and Rampini (2006). Details of the calculation of the cross-section dispersion in log marginal product of capital ($\ln MPK$) can be found in Appendix A of the paper. The capacity utilization rate (u) is published by Federal Reserve Bank of St. Louis.

Table 4: Crisis Dynamics

Moments	Non-Recession Periods		Recession Periods	
	Data	Model	Data	Model
$Vol\left[\Delta\ln Y\right]$	3.53%	3.2%	4.18%	5.96%
$Vol\left[\Delta\ln C\right]$	2.32%	1.28%	3.42%	2.28%
$Vol\left[\Delta\ln I\right]$	9.07%	14.08%	9.86%	13.92%
$E\left[u\right]$	81.1%	81.12%	78.41%	73.23%
$Vol\left[u\right]$	3.87%	2.87%	5.27%	4.8%
$E\left[\ln R_I - \ln R_f\right]$	0.56%	0.84%	0.98%	1.62%
$Vol\left[\ln R_I - \ln R_f\right]$	0.36%	0.17%	0.59%	0.23%

Table 4 documents the first and second moments of macroeconomic quantities and interest rates in recession and non-recession periods in the data and in our model with financial shocks. Both in the model and in the data, recession is classified as two consecutive quarters of decline in real GDP.

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