Monetary Policy, Capital Controls, and International Portfolios*

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November 27, 2017

Job Market Paper

Abstract

In the past two decades, there has been a large increase in cross-border holdings of financial assets, making currency movements important sources of capital gains and losses. In this context, monetary policy can enhance risk-sharing across countries by influencing exchange rates. Furthermore, the strength of this channel depends on the portfolio the country holds, giving rise to a potential rationale for capital controls. To shed light on these issues, I study an open economy model with nominal rigidities, incomplete markets, and assets denominated in home and foreign currency. I develop an approximation method that allows me to characterize the optimal policy explicitly. I show that optimal monetary policy is a weighted average of an inflation target and an insurance target and characterize the optimal weight sharply. Perhaps surprisingly, as insurance considerations become more important, home-currency positions become larger, and the excess-return volatility of home-currency assets actually decreases, rather than increases as one would expect with fixed ad hoc portfolios. In addition, I find that private portfolio decisions in small open economies are approximately efficient so that differential capital controls on foreign- vs. home-currency assets are not called for by the approximate solution. In my baseline calibration, the welfare gains from the optimal policy are 1.5 times larger than those from inflation-targeting.

JEL codes: E52, F31, F41

Keywords: Optimal Monetary Policy, Exchange Rate Volatility, Incomplete Markets, Portfolio Choice, Capital Controls, Macroprudential Policy

*I am very grateful to Iván Werning, Arnaud Costinot, and Ricardo Caballero for their continuous guidance and support. I would also like to thank Manuel Amador, Marios Angeletos, Anmol Bhandari, Vivek Bhattacharya, Javier Bianchi, José Fanelli, Daniel Greenwald, Jonathan Heathcote, Greg Howard, Anastasios Karantounias, Margarita Machelett, Fabrizio Perri, Alp Simsek, Ludwig Straub, Adrien Verdelhan, and Olivier Wang for very helpful comments. I also would like to thank seminar participants at the 2017 conference of the Society for Economic Dynamics, the Minneapolis Fed, MIT, and MIT Sloan. All errors are my own. *Email:* sfanelli@mit.edu.

1 Introduction

The size of international balance sheets has increased dramatically in the past two decades (Lane and Milesi-Ferretti, 2007). A recent literature has argued that this financial integration has significant implications for the dynamics of a country's net foreign asset position, as movements in asset prices create sizeable capital gains and losses. Today, these *valuation effects* are often of comparable magnitude to current-account fluctuations (Gourinchas and Rey, 2013; Lane and Milesi-Ferretti, 2007; Tille and van Wincoop, 2010). The goal of this paper is to study the implications of financial integration for optimal monetary policy and capital controls.

The analysis is motivated by two observations. The first observation is that monetary policy and capital controls can be used to influence exchange rate movements, which are one of the most important sources of asset price fluctuations in open economies (Lane and Shambaugh, 2010*b*). For example, tightening monetary policy and taxing savings typically leads to a stronger currency, increasing the real value of home-currency bonds. Thus, by increasing the returns of the country's international portfolio in bad times, and decreasing them in good times, central bank policies can improve the hedging properties of the portfolio; that is, they can play an *insurance* role.[1]

The second observation is that the country's international portfolio is a key determinant of the strength of the insurance channel. When agents have sizeable positions in home-currency bonds, exchange rate movements can be very powerful as a means of completing markets. This has two implications. First, there is a two-way feedback between monetary policy and portfolio choice, as positions depend on agents' expectations of monetary policy. Second, capital controls taxing the composition of international portfolios may be desirable, as agents do not internalize the effect of their portfolio choice on the ability of the central bank to provide insurance. Indeed, the presence of incomplete markets and nominal rigidities guarantees this will be the case (Geanokoplos and Polemarchakis, 1986, Farhi and Werning, 2016). However, there is little guidance as to how important these taxes may be.

The main contribution of this paper is to characterize optimal monetary policy and capital controls in a model that allows for the previous considerations. From an economic standpoint, this requires: (i) extending the typical open economy macroeconomic model used for optimal policy analysis, where either markets are complete and there is no insurance role, or there is a single asset and there is no role for portfolio choice;[2] and (ii) developing new tools to study optimal policy in these richer environments, where the standard linear-quadratic framework cannot immediately be applied due to the indeterminacy of the portfolio at the steady state. To this end, I extend a canonical open economy model by allowing the home country to trade multiple assets with the rest-of-the-world. I assume these assets are insufficient to span the whole state space (i.e., markets

[1] It is well-understood that monetary policy can play an insurance role in environments with incomplete markets via terms-of-trade manipulation (Obstfeld and Rogoff, 2002, Corsetti, Dedola and Leduc, 2010). I assume the terms-of-trade are exogenous to focus on the role of gross positions, which is less well-understood.

[2] An important exception is the work of Benigno (2009*a*), Benigno (2009*b*), and Senay and Sutherland (2017). There are important differences with these papers, which I discuss in detail at the end of this section.

are incomplete) and the return of some of these assets depends on monetary policy. I overcome the indeterminacy of the steady-state portfolio by showing how the perturbation approach in Judd and Guu (2001) employed in positive analysis can be used to extend the linear-quadratic normative framework in Benigno and Woodford (2012).[3] Using this new approximation method, I provide a sharp characterization of the solution and comparative statics.

The main results in this paper arise from the interaction between exchange rate management and international portfolio choice. To illustrate the forces at play in the simplest possible way, I start with a two-period open economy model where agents have an endowment of tradable goods and produce nontradable goods. In the first period, agents only trade financial assets. In the second period, the state of the world is realized, agents produce, honor their financial obligations, and consume. The model has two key ingredients. First, like in the canonical model, there are nominal rigidities (price stickiness). This ingredient gives rise to the traditional demand-management role for monetary policy, concerned with undoing the distortions associated with price stickiness. The second ingredient is the availability of home- and foreign-currency bonds that can be traded internationally.[4] This ingredient gives rise to the insurance channel discussed above, and a nontrivial portfolio problem.

In this environment, I study the problem of a planner that maximizes the utility of home households under commitment.[5] She has two set of tools: monetary policy and capital controls. Monetary policy is a state-contingent exchange rate rule. Capital controls are taxes on financial assets. Since there is no consumption in the first-period, this baseline model is essentially static so there is a single portfolio tax. In the dynamic model I discuss later, capital controls also include a savings tax.

My approximation method allows me to get closed form solutions for the optimal monetary policy, portfolio, and capital controls. These solutions are valid for small disturbances around the nonstochastic steady state. Monetary policy is characterized to first-order, the portfolio is characterized at the steady state ("zero-order"), and the portfolio tax is characterized to second-order.

In terms of monetary policy, I show that the optimal policy is a weighted average of two *targets*: a demand-management target and an insurance target. The former is the exchange rate that would be required to attain a zero output gap and no price dispersion. The latter is the exchange rate that would be required to replicate the transfer the planner would desire under complete markets. For example, if the home country experiences a negative endowment shock and has home-currency debt, the insurance target would imply a depreciation. The optimal weight depends on the outstanding portfolio. When positions are large in absolute value, providing insurance is cheap: only

[3]The approach is also related to the perturbation approach in Devereux and Sutherland (2011) and Tille and van Wincoop (2010), who use it to characterize the competitive equilibrium. I extend the analysis to optimal policy problems.

[4]In Appendix B.3, I consider a general asset structure with an arbitrary number of assets that are allowed to load arbitrarily on endogenous variables and shocks. All the results go through in this richer environment.

[5]In Appendix B.1, I study the problem from the point of view of a supranational authority that takes into account foreigners' welfare. I show that this increases the importance of the insurance motive, leading to more levered portfolios and a lower volatility of the exchange rate.

a small exchange rate movement is needed to replicate the desired transfer. By the same token, restoring production efficiency could be very costly, since the required exchange rate movement would create a large transfer of wealth that may be undesirable. As a result, the optimal weight on the insurance target increases with the size of the position. This captures a form of *fear-of-floating* due to currency mismatches.[6]

In terms of the portfolio, I show that its optimal sensitivity to the value of the home-currency is higher when the insurance motive is more relevant. This is a direct consequence of the fact that a larger position reduces the cost of providing insurance ex post and makes it harder to correct production inefficiencies, as discussed above. The relative importance of the insurance motive depends on both structural characteristics of the economy, such as risk aversion and the degree of price flexibility, and the stochastic properties of the shocks. For example, shocks to the terms-of-trade typically matter relatively more for the insurance motive compared to nontradable productivity shocks. If the former are more frequent, positions will be larger.

One perhaps surprising implication of the interaction between monetary policy and portfolio choice is that the optimal degree of exchange-rate volatility actually *decreases* with the insurance motive. By contrast, if the portfolio is constrained, volatility increases when insurance considerations are more important. To understand this result, consider first the case with a fixed portfolio. For the reasons described above, an increase in the importance of the insurance motive dampens the response aimed at correcting production inefficiencies but exacerbates exchange rate movements to provide insurance. If the constrained portfolio is optimal, this *composition effect* leads to higher exchange rate volatility. When the portfolios are endogenous, there is an opposing force: the insurance motive also induces a larger position, which not only lowers the incentives to correct production inefficiencies even further, but also decreases the required exchange rate movement to provide insurance. This last force dominates if home has home-currency liabilities, which is typically the case in the data.

In terms of capital controls, the model provides two potential rationales for taxing portfolios. First, if the foreign demand of home-currency bonds is not perfectly elastic there is a terms-of-trade externality: Agents overinsure on their own, so the planner finds it optimal to put a tax that pushes positions towards zero.[7] More interestingly, there is another motive related to the presence of incomplete markets and nominal rigidities. To isolate this second motive, suppose the foreign demand for home-currency bonds is perfectly elastic (i.e., a small open economy). In this environment, Farhi and Werning (2016) show that taxes are desirable to correct pecuniary and aggregate-demand externalities. Perhaps surprisingly, I show that taxes are zero in the approximate solution,

[6]In this paper, the relevant measure of the mismatches is the foreign-currency value of home-currency debt, different from the standard fear-of-floating, which refers to home-currency value of foreign-currency debt as in Lane and Shambaugh (2010*a*) and Bénétrix, Lane and Shambaugh (2015). The latter does not lead *per se* to a deviation from price stability in my model, which features a representative agent, exogenous terms-of-trade, and no borrowing constraints (unlike the work by Caballero and Krishnamurthy (2004) and Ottonello (2015), among others).

[7]The planner can manipulate the intratemporal price of consumption across states, i.e., the stochastic discount factor. The logic is the same as in Costinot, Lorenzoni and Werning (2014), except it manifests across states rather than time.

i.e., they are at most third-order. In other words, private portfolio decisions are approximately efficient. The key observation behind this result is that eliminating production inefficiencies in this economy is *feasible*, i.e., there is *divine coincidence*: output gaps and price dispersion can be closed simultaneously. As a result, the economy only experiences booms and recessions *because* the planner is trying to provide insurance. Formally, this implies that output gaps are, to first-order, proportional to social marginal utility. Furthermore, the wedge between social and private marginal utility is proportional to the output gap. These observations imply that social and private marginal utilities are proportional to one another, which is enough to establish the asymptotic optimality of the private portfolio decision as risk vanishes.

I then study a dynamic version of this economy. This extension serves three goals. First, I study the robustness of the results and show that they all generalize. The only qualification is that, if home-currency bonds are long-lived, results on volatility hold for the excess-return of the bond, rather than the exchange rate. Second, I characterize new features of the solution. Unlike the static model, where the planner had a single possibility to engineer an excess return (i.e., creating an output gap), now the planner has more options: she can promise either current or future output gaps and inflation and may try to manipulate tradable consumption over time. The planner then solves a cost-minimization problem among these tools. I show that capital controls on the total *size* of capital flows are desirable, although their *composition* is still efficient. In other words, the planner wants to tax financial assets, but not differentially so.[8] I also show their effectiveness is higher when bonds have a shorter duration. Indeed, if home-currency bonds are perpetuities and prices are perfectly rigid, optimal savings taxes are exactly zero. Third, I introduce ingredients that are important for the quantitative exercise. Of particular relevance are two shocks that play a key role in the calibrated model: (i) world-interest-rate shocks, which create a large demand for insurance, and (ii) liquidity shocks on home-currency bonds, which introduce noise in the return of the home-currency bond.

To conclude, I explore the quantitative implications of the optimal policy for monetary policy, observed portfolios, capital controls, and welfare. I compare it to the benchmark demand-management policy (strict inflation-targeting). I calibrate the model to Canada, a prototype small open economy. I find that the optimal policy increases external home-currency debt from 16% to 23%, which translates into an ex post weight on the insurance target of 8%.[9] This changes the variance decomposition of the excess-returns of home-currency bonds: the contribution of liquidity shocks decreases from around 80% to 70%, with a mirror increase in the contribution of interest-rate shocks. This, however, does not translate into noticeable changes in overall volatility. Concerning capital controls, I find a limited role for savings taxes and an important role for portfolio taxes, due to the limited size of the foreign-investor base in home-currency bond markets. This

[8]There are two main rationales for savings taxes. First, they can be used to move the exchange rate without creating an output gap. Second, they correct the misvaluation of tradable goods during booms and recessions.

[9]This result is sensitive to the cost of inflation. Reducing the average duration of prices from a year to three quarters leads to an insurance weight of 25%.

also implies a high value of cooperation, as I show in Section 5. Regarding welfare, I compute the gain in consumption-equivalents of moving from an economy without home-currency bonds to an economy with home-currency bonds and flexible prices (the first-best). Then, I compute how much of these welfare gains are achieved by each policy. I find that, while inflation-targeting attains 12% of the benefits, the optimal policy attains 17% - almost 1.5 times as much. Portfolio endogeneity is quantitatively important for this result: gains would only be 15% if home-currency debt were fixed at the calibrated value (15%).

Related literature This paper contributes to a large literature exploring deviations of optimal monetary policy from inflation-targeting in open economies, surveyed by Corsetti, Dedola and Leduc (2010). In particular, my analysis is related to the idea that, in incomplete markets, monetary policy plays an insurance role (Obstfeld and Rogoff, 2002; Corsetti, Dedola and Leduc, 2010). However, in most of the literature, insurance is improved via terms-of-trade manipulation and not through asset positions; formally, the solution is typically approximated around a symmetric steady state with a zero net-foreign-asset position. I abstract from this insurance channel by focusing on a small open economy that faces exogenous terms-of-trade. Instead, the insurance role in this paper is linked to the size of gross positions and is, therefore, closest to the work of Benigno (2009*a*), Benigno (2009*b*), and Senay and Sutherland (2017).[10] The first two papers study optimal monetary policy in a New Keynesian open economy model with home- and foreign-currency bonds. Importantly, in those papers the steady-state portfolio is exogenous. By contrast, in the present paper the portfolio is endogenous, which is key for my results. Senay and Sutherland (2017) present a rich two-country New Keynesian open economy model with two nominal bonds and equities in firms from both countries. They allow portfolios to be endogenous, but they do not study fully optimal policy. Instead, they focus on a limited set of policy rules, and optimize numerically over the parameters of such rule. By contrast, I study fully optimal monetary policy and characterize it analytically. None of these papers study capital controls.[11]

Farhi and Werning (2016) develop a general theory for the joint problem of optimal monetary policy and macroprudential policy and provide several applied examples. In one of their applications, they discuss a static small-open economy with home- and foreign-currency debt. They note that there is generally a trade-off between insurance and demand-management and provide a formula for portfolio taxes, pointing out that they are generally nonzero. My analysis and results confirms these observations, but also provides a sharper characterization of both monetary policy and macroprudential policy in this context. Indeed, using my approximation, I am able to show precisely how monetary policy is a weighted average of two targets, characterize the weight and the debt positions, and, somewhat surprisingly, show that while portfolio taxes are generally non-

[10] Devereux and Sutherland (2008) also stress the role of gross portfolio positions. However, they focus on a case where the optimal policy can replicate complete markets, so inflation-targeting is still optimal.

[11] Chang and Velasco (2006) also present an open economy model with home and foreign-currency bonds but they only compare two extreme exchange rate regimes (inflation-targeting and a fixed exchange rate).

zero, they are zero in an approximate sense around the steady state. My paper also extends the study of these issues to a dynamic setting to provide a quantitative analysis.

Another strand of literature studies monetary policy and portfolio choice between nominal and real debt in environments without commitment. In a seminal paper, Bohn (1988) demonstrates the optimality of issuing nominal debt to minimize distortionary taxation, despite the inflationary incentives created by lack of commitment. Bohn (1990) argues that foreign-currency debt may lower home-currency debt issuance, since unlike indexed debt it may have some desirable hedging properties. More recently, motivated by the increase in the share of government debt issued in home-currency in emerging markets during the past twenty years, a recent literature has revisited the trade-off between incentives (due to lack of commitment) and insurance in home-currency-bond issuance; see Du, Pflueger and Schreger (2017), Engel and Park (2017), and Ottonello and Perez (2017).[12] The present paper has a more normative focus and studies optimal monetary policy, assuming full commitment, as well as optimal capital controls, which these papers abstract from. This leads to two key results about optimal policy—lower volatility of the exchange rate and zero portfolio taxes, up to second-order—that have no counterparts in previous studies.

My paper is also related to a closed-economy literature that studies the potential of monetary policy to complete markets with nominal assets in environments with commitment; see Schmitt-Grohe and Uribe (2004), Siu (2006), Lustig, Sleet and Yeltekin (2008) and Sheedy (2014). In these papers, a similar trade-off between demand-management and insurance emerges, but insurance takes place between the government and the private sector, or between borrowers and savers. In addition, my analysis emphasizes the role of exchange rate movements, and the portfolio decision between home- and foreign-currency bonds, which is absent from these studies.

From a methodological perspective, this paper makes a contribution to the recent literature on portfolio choice within dynamic stochastic general equilibrium models (Devereux and Sutherland, 2011; Evans and Hnatkovska, 2012; Tille and van Wincoop, 2010). These papers are positive, aiming to approximate the competitive equilibrium given a policy rule. I extend these methods to tackle normative questions. Following the same steps in Benigno and Woodford (2012), I derive an approximate problem around an arbitrary steady-state portfolio that is linear-quadratic in all the remaining endogenous variables. Then, I use the perturbation approach in Judd and Guu (2001) on the first-order conditions of the nonlinear planning problem and show they coincide with the first-order conditions of the approximate problem, including the first-order conditions with respect

[12] Du, Pflueger and Schreger (2017) document that governments in countries where the home-currency bond has worse hedging properties issue a larger share of their debt in home-currency. They show that the interaction between limited commitment and risk-averse foreigners can rationalize their empirical findings. Engel and Park (2017) study an endowment small-open-economy model in which the government can borrow in both home- and foreign-currency subject to two commitment frictions: strategic default and debasement. They show that a higher cost of inflation leads to endogenously looser limits to borrowing in home-currency and that this can account for the surge in home-currency borrowing in recent decades. Ottonello and Perez (2017) study a small open economy model with tradable and nontradable goods where the government does not default but cannot commit to future policy. They show that lack of commitment can rationalize the currency-composition of government debt and the procyclicality of home-currency-debt shares for a sample of emerging markets.

to the steady-state portfolio.[13] The validity of the procedure depends on the availability of taxes. Otherwise, one needs an additional quadratic constraint.

Layout The paper is organized as follows. In Section 2, I present a static version of the model and derive the planning problem. In Section 3, I characterize the optimal policy in this setting. In Section 4, I extend the model to a dynamic setting and characterize the optimal policy in this context. In Section 5, I calibrate the model and explore the quantitative importance of the channels emphasized in the paper. Section 6 concludes. Appendix A contains all proofs and detailed derivations, Appendix B contains some additional extensions of the model, and Appendix C contains additional sensitivity exercises for the quantitative section.

2 Static model

In this section, I present a two-period version of the model. Henceforth, I refer to this version of the model as "static" because agents only trade financial assets in the first period. Section 2.1 presents the setup. Section 2.2 presents the planning problem and discusses the main trade-offs the planner faces. The optimal policy is analyzed in Section 3.

2.1 Set up

There are two periods, 0 and 1. At $t = 0$, agents trade financial assets. At $t = 1$, the state of the world is realized, agents produce, honor their financial obligations, and consume. There are two final goods (tradables and nontradables), and a continuum of varieties of intermediate inputs, which are used to produce nontradables.

Home households There is a continuum of households in the home country, maximizing a standard utility function

$$\mathbb{E}u(C_{Ts}, C_{Ns}, L_s; \xi_s) \tag{1}$$

where C_{Ts} is tradable consumption, C_{Ns} is nontradable consumption, L_s is labor, and ξ_s is a vector of shocks. The function u is assumed increasing and strictly concave in consumption and leisure $(-L)$.

Agents have access to two assets in period 0: a bond with a fixed payment in home-currency B and a bond with a fixed payment in foreign currency B^*. Since there is no consumption at $t = 0$, it is without loss of generality to normalize the exchange rate at $t = 0$ and the return of the foreign currency asset R^* to 1. I denote the return of the home-currency asset R. The budget constraint at $t = 0$ is

$$(1 + \tau_B)B + B^* = T_0,$$

[13] Du, Pflueger and Schreger (2017) employ an alternative route in the context of their application based on the approximation method of Campbell and Viceira (2002).

where τ_B is an ad-valorem tax on home bonds and T_0 is a lump-sum transfer from the central bank. I assume positions are bounded by a large constant $\bar{K}$, i.e., $|B| \leq \bar{K}$.

At $t = 1$, agents receive a tradable endowment $Y_T(\xi_s)$, which can be interpreted as the product of tradable output and the terms-of-trade as in Mendoza (1995). They also work, collect profits, honor their financial obligations, and consume both goods. The budget-constraint at $t = 1$ is

$$E_s C_{Ts} + P_{Ns} C_{Ns} = E_s Y_T(\xi_s) + (1 + \tau_L) W_s L_s + \Pi_{Ns} + \Pi_{Is} + RB + E_s B^* + T_s$$

where E_s is the nominal exchange rate, P_{Ns} is the price of nontradables, W_s is the nominal wage, τ_L is a labor subsidy, Π_{Ns} and Π_{Is} are profits from nontradable and intermediate good producers, respectively, and T_s are lump-sum transfers from the central bank. I use the convention that a higher exchange rate means a more depreciated currency and normalize the international price of the tradable good to 1. Optimization over labor and tradable and nontradable consumption yields

$$u_N(s)/u_T(s) = P_{Ns}/E_s. \quad (2)$$

$$(-u_L(s))/u_T(s) = (1 + \tau_L) W_s / E_s. \quad (3)$$

where $u_N(s)$, $u_T(s)$, and $u_L(s)$ are the first-derivatives with respect to nontradables, tradables, and labor, respectively. Asset optimization yields a no arbitrage condition,

$$\mathbb{E}\left[((1 + \tau_B)^{-1} R E_s^{-1} - 1) u_T(s)\right] = 0. \quad (4)$$

Nominal rigidities I introduce nominal rigidities in the form of sticky prices in the production of nontradable intermediate goods. There is a continuum of varieties $i \in [0, 1]$, which can be aggregated into a composite Y_I that can be used for production,

$$Y_{Is} = \left(\int_0^1 Y_{Is}(i)^{\frac{\eta-1}{\eta}} di\right)^{\frac{\eta}{\eta-1}}. \quad (5)$$

Production of the intermediate input is linear in labor,

$$Y_{Is}(i) = L_s(i). \quad (6)$$

In each product market i prices are set by a monopolistically competitive firm who faces a demand given by

$$Y_{Is}(i) = (P_{Is}(i)/P_{Is})^{-\eta} Y_{Is}. \quad (7)$$

A share ϕ of firms has their nominal price fixed at $\bar{P}_I$ and supply any amount of output that is required, while a share $1 - \phi$ can optimize its price state by state.[14] Firms that are able to optimize

[14] I could allow the former group to set the average price. This would only complicate the exposition, without changing the results. In the dynamic model of section 4 I show all the results are robust to standard Calvo price-setting.

set a constant mark-up over the marginal cost (the wage),

$$P_{Is}(i) = \frac{\eta}{\eta - 1} W_s. \tag{8}$$

Combining (3) and (8), one can write the ideal intermediate input price index P_{Is} as

$$P_{Is} = \left(\phi \bar{P}_I^{1-\eta} + (1-\phi)\left(\frac{\eta}{\eta-1}\frac{1}{1+\tau_L}\frac{-u_L(s)}{u_T(s)}\right)^{1-\eta} E_s^{1-\eta} \right)^{\frac{1}{1-\eta}}. \tag{9}$$

Foreign households A measure $m \in \mathbb{R}_+ \cup \infty$ of foreign households may participate in home-currency asset markets. Each household is endowed with $\{Y^*(\xi_s)\}$ units of the tradable good. Using asset market clearing conditions, this leads to a no-arbitrage condition given by

$$\mathbb{E}\left[(RE_s^{-1} - 1) u^{*\prime}(Y^*(\xi_s) - \frac{1}{m}(RE_s^{-1} - 1)B) \right] = 0. \tag{10}$$

When $m = \infty$, the small open economy takes the stochastic discount factor as given. In this case, I say there is perfect financial integration in home-currency markets. When $m < \infty$, there is limited participation and the home economy has market power in home-currency bond markets. Alternatively, one may interpret the case $m < \infty$ as a large economy whose actions affect the world's stochastic discount factor.

Final production Firms have access to an increasing and concave production function $F(Y_{Is}; \xi_s)$. They maximize profits, which are given by $\Pi = P_{Ns} F(Y_{Is}; \xi_s) - P_{Is} Y_{Is}$. Firm optimization implies

$$P_{Ns} F_Y(Y_{Is}; \xi_s) = P_{Is}. \tag{11}$$

where F_Y is the derivative with respect to intermediate inputs.

Central bank The central bank in the economy has three tools: monetary policy, capital controls, and the labor subsidy. Monetary policy is a state-contingent exchange rate policy rule $\{E_s\}_s$. Since some prices are fixed in home currency, this instrument allows the central bank to determine the equilibrium price of intermediate inputs in foreign-currency, which in turn affects the level of employment.[15] Capital controls in this model are represented by the portfolio tax τ_B. This instrument allows the central bank to effectively control the balance sheet of the country vis-a-vis the

[15] I could have also stated monetary policy as a money-supply rule. I follow the literature and consider a cashless economy, avoiding an explicit modelling of money demand. This allows me to focus on the demand-management motive, which is more relevant for most economies due to low inflation.

rest-of-the-world.[16] The proceeds are then rebated to home households through lump-sum taxes,

$$T_0 = \tau_B B. \tag{12}$$

Finally, the labor subsidy allows the planner to obtain production efficiency at the steady-state. These subsidies are also rebated lump-sum to households,[17]

$$\tau_L W_s L_s + T_s = 0. \tag{13}$$

The monetary authority announces the monetary and tax policies at the beginning of time, *before* agents engage in bond trading, and is assumed to be perfectly credible.

Goods and labor market clearing Replacing profits, labor income and the $t = 0$ budget constraint into the $t = 1$ budget constraint, I obtain

$$C_{Ts} = Y_{Ts} + (RE_s^{-1} - 1)B. \tag{14}$$

The market clearing condition for nontradables and labor are given by

$$C_{Ns} = F(Y_s^I, \xi_s) \tag{15}$$

$$L_s = \int_0^1 L_s(i)di \tag{16}$$

Next, I formally define a competitive equilibrium in this economy.

Definition 1. Given a policy $\{E_s\tau_B, T_0, \tau_L, T_s\}$, an allocation$\{C_{Ts}, C_{Ns}, L_s, L_s(i), Y_s(i), Y_s^I, B\}$ together with prices $\{P_{Ns}, P_{Is}, P_{Is}(i), W_s\}$ and a home-currency bond return R is a ***competitive equilibrium*** if and only if they solve (2)-(16).

2.2 Planning problem

The planner in the economy is the central bank, who chooses a state-contingent exchange rate and capital controls to maximize the utility of home households. To simplify the exposition, we set $\tau_L = \frac{1}{\eta-1}$, which is optimal at the steady state (i.e., when $\xi_s \equiv 0$), and do not discuss it any further as part of the policy mix.[18] Next, I simplify the problem to obtain a reduced set of implementability conditions that describe the set of equilibria that can be attained through different policies

[16] Equivalently, I could have the government be the only one allowed to borrow in foreign markets in home-currency bonds, which reflects the situation is many emerging economies (Du and Schreger (2015)). However, capital controls may still be desirable in these countries to control positions in other assets that are traded by the private sector and are exposed to the stance of monetary policy, such as equity and foreign direct investment (see Appendix B.3).

[17] Note that assuming the central bank balances its budget period-by-period is without loss of generality, since agents have access to the same set of assets (i.e., there is Ricardian equivalence).

[18] Given that there is price dispersion, optimizing over τ_L would introduce an additional constraint. This is without loss of generality for the approximate solution discussed in Section 3.

$\{E_s, T_0, \tau_B\}$. Given C_{Ts} and E_s, one can find $\{C_{Ns}, L_s, L_s(i), Y_{Is}(i), Y_s, W_s, P_{Is}(i), P_{Is}, P_{Ns}\}$ that solve (2), (3), (5) - (9), (11), (15) and (16).[19] Furthermore, the planner may use τ_B to satisfy (4) and T_0 to satisfy (12). Using these observations, I obtain the following implementability result.

Lemma 1. *An allocation for tradable consumption $\{C_{Ts}\}$, an exchange rate policy $\{E_s\}$, a home-currency position B and a home-currency bond return R form part of an equilibrium if and only if they solve (10) and (14).*

As in Farhi and Werning (2016), I define the following indirect utility function:

$$V(C_{Ts}, E_s; \xi_s) = \max_{\{C_{Ns}, L_s, L_s(i), Y_{Is}(i), Y_s, W_s, P_{Is}(i), P_{Is}, P_{Ns}\}} u(C_{Ts}, C_{Ns}, \int_0^1 L_s(i)di; \xi_s) \tag{17}$$
$$\text{subject to} (2), (3), (5) - (9), (11), (15), \text{ and } (16)$$

Using this definition, the planner's problem can be formulated as follows.

Problem 1. The planner's problem is to choose $\{C_{Ts}, E_s, B\}$ to maximize

$$\mathbb{E} V(C_{Ts}, E_s; \xi_s)$$

subject to

$$C_{Ts} = Y_T(\xi_s) + (RE_s^{-1} - 1)B$$
$$\mathbb{E}\left[(RE_s^{-1} - 1)u^{*\prime}(Y^*(\xi_s) - \frac{1}{m}(RE_s^{-1} - 1)B)\right] = 0$$

Before tackling this problem, it is useful to study a simpler benchmark with complete markets.

Problem 2. When markets are complete, the planner's problem is to choose $\{\mathcal{T}_s, E_s\}$ to maximize

$$\mathbb{E} V(Y_T(\xi_s) + \mathcal{T}_s, E_s; \xi_s)$$

subject to

$$\mathbb{E}\left[\mathcal{T}_s u^{*\prime}(Y^*(\xi_s) - \frac{1}{m}\mathcal{T}_s)\right] = 0$$

Under complete markets the transfer of wealth in each state of the world $\mathcal{T}_s$ is decoupled from monetary policy E_s. This implies the exchange-rate has a single role in this economy: closing the output gap and ensuring there is no price dispersion. This is the traditional *demand-management* role of monetary policy. Regarding transfers, the solution replicates the laissez-faire competitive equilibrium with flexible prices when $m = \infty$. When $m < \infty$, the solution is not the laissez-faire

[19] This system of equations can be reduced state by state to a system of two equations in $(Y_{Is}^{flex}, Y_{Is}^{fix})$, which I assume has at least one solution. The latter can be guaranteed in a neighborhood of the steady state using the implicit function theorem.

competitive equilibrium due to the presence of terms-of-trade externalities. Although the country is a price-taker in tradable good markets, it can influence the stochastic-discount-factor of the foreigners that participate in home-bond markets (i.e., the state prices). The solution follows from a reinterpretation of the results in Costinot, Lorenzoni and Werning (2014) across states instead of over time: The planner wants the state price to be high when it is relatively rich. This implies she wants foreigners to be relatively poor in those states, i.e., she wants less than full insurance. In Section 3.4 we provide an explicit solution for the tax, which reflects this motive.

The key difference between the complete-markets benchmark and the full problem is that in the latter the exchange rate $\{E_s\}$ and the transfers $\{\mathcal{T}_s\}$ are linked by the relationship $\mathcal{T}_s = (RE_s^{-1} - 1)B$. As a result, the exchange rate plays an additional *insurance* role, given by the desire to replicate some transfers $\{\mathcal{T}_s\}$. When prices are flexible, the planner can perfectly replicate the transfers from the complete markets solution since the exchange rate plays no demand-management role. When prices are sticky, there is a trade-off between both objectives of monetary policy. In the solution, the planner balances both forces so many states of the world feature nonzero output gaps and deviations from the complete markets transfer. Thus, portfolio taxes are generically nonzero even if $m = \infty$: the presence of ex post output gaps and pecuniary externalities due to incomplete markets implies agents fail to internalize the proper value of a unit of tradable goods (Farhi and Werning (2016)).

3 Optimal Policy

In this section, I study the optimal policy in the static model. I start with a brief description of the solution method in Section 3.1, which may be skipped without loss of continuity. I then present the main theoretical results. First, I characterize the optimal monetary policy and the optimal portfolio (Section 3.2). Second, I derive the implications for exchange rate volatility (Section 3.3). Beyond being interesting in its own right, this result illustrates the importance of portfolio endogeneity. Third, I characterize the optimal capital controls, i.e., the optimal portfolio tax (Section 3.4). Section 3.5 concludes with a simple example that illustrates the results.

Appendix B contains extensions of the model. Appendix B.1 revisits the problem from the point of view of a supranational authority that internalizes the effect of the optimal policy on foreigners' welfare. Appendix B.2 considers an environment with equity on nontradable firms, instead of a nominal asset. This alternative asset structure illustrates the differences between a nominal asset and a real asset with returns that are sensitive to monetary policy. Appendix B.3 generalizes all the results in this section to an environment with a multiple assets that load arbitrarily on endogenous variables and shocks. Appendix B.5 presents the problem without capital controls. Proofs for the results in the main text can be found in Appendix A.

3.1 An approximation for small risks

The standard linear-quadratic framework (Benigno and Woodford (2012)) cannot be used in environments with portfolio choice. The problem is that at the riskless steady-state all assets are perfect substitutes, which makes the portfolio indeterminate and prevents the application of the implicit function theorem. However, it is well-known that a perturbation approach can still be used to find approximate solutions to the first-order conditions (Judd and Guu (2001); Devereux and Sutherland (2011); Tille and van Wincoop (2010)). These approaches rely either on a bifurcation theorem, or on a higher-order approximation of the no-arbitrage equations.[20] In this paper, I show that deriving a linear-quadratic approximation around an arbitrary steady-state portfolio, and then maximizing over the approximation point, is correct when the planner has access to a portfolio tax, which is the case in the current setting.[21] Formally, the first-order conditions of the approximate problem coincide with the result of the perturbation approach in Judd and Guu (2001) to the original nonlinear first-order conditions. The main added benefit of using the linear-quadratic framework rather than just a perturbation approach is that it allows one not only to check locally the second-order conditions, but also to pick the best local solution when there is more than one. This is important because the solution of equilibrium objects such as the exchange rate is typically nonlinear in the steady-state portfolio, which in turn implies there are multiple portfolios that satisfy the first-order conditions. I prove a more general version of this result that holds in a dynamic setting with forward and backward-looking constraints and multiple assets, as in Benigno and Woodford (2012), in my companion paper (Fanelli (2017)).

Let ϵ denote the amount of risk in the economy, i.e., $\xi_s = \epsilon u_s$ where u_s is a random variable with compact support. I am interested in the limit $\epsilon \to 0$. The next lemma provides a "purely quadratic" second-order expansion of the objective function around a steady state with some arbitrary steady-state portfolio $\bar{B}$. This expression is obtained by combining a second-order expansion of the objective, the budget constraint, and the foreign Euler equation.[22] Lowercase letters denote log-deviations from the steady state.

Lemma 2. *When $\epsilon \to 0$,*

$$\mathbb{E}V(\{e_s, \bar{B}\}) = -k_0\mathbb{E}[\frac{1}{2}(\bar{B}e_s + \mathcal{T}_s)^2 + \frac{1}{2}\chi((1+\mu\bar{B})e_s - e_s^{dm}(0))^2] + t.i.p. + O(\epsilon^3) \tag{18}$$

where $k_0 > 0$, $\chi \geq 0$ and μ are constants, and t.i.p. stands for "terms independent of policy". $\mathcal{T}_s$ is the transfer of tradable goods the planner would desire if prices were flexible and $e_s^{dm}(0)$ is the exchange rate that would close the output gap if $\bar{B} = 0$. The objects $e_s^{dm}(0)$, $\mathcal{T}_s$, k_0, μ and χ are specified in appendix A.1.1.

[20] Both approaches are equivalent to the order of approximation I work in this paper.

[21] Without a tax, the problem has two degrees of indeterminacy at the steady state: the Lagrange multiplier on the home no-arbitrage condition and the portfolio. This implies that one needs to consider an additional quadratic constraint in the approximate problem (see Appendix B.5).

[22] It is important to remember not to drop terms that involve only the steady-state portfolio and no other endogenous variables; i.e., terms one would call "independent of policy" in the original Benigno and Woodford (2012) setup.

The next result shows that the solution to this problem yields a bifurcation point for the portfolio and a local linear approximation to the behavior of all remaining endogenous variables and is, thus, equivalent to a perturbation approach. Importantly, this result relies on the availability of the portfolio tax. Without it, one would need to add an additional quadratic constraint.[23]

Proposition 1. *(LQ equivalence to perturbation) Suppose u, u^* and F are locally analytic functions around the steady state.*[24] *Then, maximizing (18) with respect to ($\{e_s\}$ and $\bar{B}$) yields a linear approximation of a solution to the first-order conditions of problem 1 around ($\bar{B}, \epsilon = 0$) for $\{e_s\}$ and a bifurcation point of the system $\bar{B}$.*

Remark 1. The term $\frac{\partial B}{\partial \epsilon}$ in the welfare loss function V is irrelevant for welfare to second-order.

One important feature of the loss function derived in lemma 2 is that being able to change the portfolio as risk increases does not matter for welfare to a second-order of approximation - a result pointed out by Samuelson (1970) in the context of a standard portfolio problem. Intuitively, this is a consequence of the fact that assets are perfect substitutes not only in the steady state, but also to first-order, since agents behave as if they were risk neutral. This implies that one does not need to know how the portfolio varies with risk to characterize the optimal exchange rate to first-order. It also implies one can only pin down the steady-state portfolio to this order of approximation.

3.2 Optimal monetary policy and optimal portfolio

In this section, I characterize the optimal monetary policy and the optimal portfolio. I do so in two steps. First, I solve the *inner problem* (i.e., the solution conditional on a steady-state portfolio) in Section 3.2.1. Second, I use the results from the inner problem to solve the *outer problem* (i.e., the optimal portfolio) in Section 3.2.2.

3.2.1 The inner problem: Optimal monetary policy conditional on a portfolio

In this log-linear framework, the two objectives of monetary policy discussed in section 2.2 (insurance and demand-management) can be described by exchange-rate targets. The insurance target e_s^{in} replicates the desired transfers of tradable goods $\mathcal{T}_s$,

$$e_s^{in}(\bar{B}) = -\frac{1}{\bar{B}}\mathcal{T}_s + O(\epsilon^2)$$

while the demand-management target $e_s^{dm}(\bar{B})$ closes the output gap (and price dispersion),

$$e_s^{dm}(\bar{B}) = \frac{1}{1+\mu\bar{B}} e_s^{dm}(0) + O(\epsilon^2). \tag{19}$$

[23] See Appendix B.5 for an example where the additional constraint is needed.

[24] By analytic I mean they are infinitely differentiable, and their infinite Taylor expansion coincides with the actual function in a neighborhood of the approximation point. This is required to apply the bifurcation theorem in Judd and Guu (2001).

Note that to compute the solution to the approximate problem, one only needs the first-order behavior of $\mathcal{T}_s$ and $e_s^{dm}(0)$. These are complicated linear functions of the shocks, explicitly solved for in appendix A.1.1. In Section 3.5, I present an example where they take a simple form.

One important feature of these exchange-rate targets is that they depend on the outstanding portfolio. Consider first the insurance target. Any exchange rate movement of size ε creates a transfer of tradable goods of $-\varepsilon\bar{B}$. Thus, a more sizeable position requires a smaller exchange rate movement to replicate a desired transfer. Next, consider the demand-management target. The key parameter in this case is μ, which captures the effect of wealth on the demand-management exchange rate: When agents have an additional wealth of ε, the exchange rate must move $\mu\varepsilon$ to restore production efficiency. Typically $\mu < 0$,[25] which implies home-currency debt dampens exchange rate volatility under demand-management targeting. To see this, suppose the exchange rate would depreciate if $\bar{B} = 0$, i.e., $e_s^{dm}(0) > 0$. When home agents are holding home currency debt ($\bar{B} < 0$), the depreciation makes them richer. This typically leads to an increase in nontradable demand, which mitigates the required depreciation to restore production efficiency. The opposite is true if home agents are holding home-currency assets.[26]

Proposition 2. *(Optimal monetary policy) Consider an economy with small risks, i.e., $\epsilon \to 0$. Then,*

$$e_s^{op}(\bar{B}) = \frac{\chi}{\chi + f(\bar{B})^2} e_s^{dm}(\bar{B}) + \frac{f(\bar{B})^2}{\chi + f(\bar{B})^2} e_s^{in}(\bar{B}) + O(\epsilon^2). \tag{20}$$

where $f(\bar{B}) = \frac{\bar{B}}{1+\mu\bar{B}}$. The parameter χ satisfies the following properties:

1. (Price flexibility) If a marginal exchange rate depreciation leads to a positive output gap,[26] then there exists $\bar{\phi} \in (0, 1]$ such that for $\phi < \bar{\phi}$ χ increases with ϕ while for $\phi > \bar{\phi}$ χ decreases with ϕ. When $\phi = 0$, $\chi = 0$. Furthermore, χ is increasing and $\bar{\phi}$ is weakly decreasing in the elasticity of substitution across varieties η when $\phi > 0$.

2. (Risk aversion) It is decreasing in the planner's absolute risk-aversion to movements in tradable-good consumption $\tilde{\gamma}$ (computed in Appendix A.1.1), increasing in the measure of foreigners m, and, provided $m < \infty$, decreasing in absolute foreign risk aversion γ^*.

Proposition 2 shows that the optimal monetary policy conditional on the portfolio is a weighted average of both exchange rate targets. The weight has two main components: an exogenous one, controlled by the parameter χ, and an endogenous one $f(B)$, which depends on the portfolio. First, consider the exogenous component χ. One of its key determinants is the degree of price flexibility. When more firms optimize their price (i.e., when ϕ decreases), there are two opposing effects. On the one hand, more firms are able to reduce any production inefficiencies that an exchange rate movement may create, i.e., the sensitivity of the output gap to monetary policy decreases. On the other hand, higher flexibility increases price dispersion if most firms cannot adjust their prices.

[25] If u is Greenwood–Hercowitz–Huffman (GHH) or separable in labor, and the composite between tradables and nontradables is CES, $\mu \leq 0$ and devaluations are expansionary.

[26] As long as $B\mu > -1$. A sufficient condition for this case is provided in proposition 3.

Proposition 2 shows the first force dominates when price flexibility is high (i.e., χ decreases with price flexibility) while the latter sometimes dominates when price flexibility is low (high ϕ). The second region is more likely to exist when the elasticity of substitution across varieties is high, since this increases the cost of price dispersion. This result on price flexibility relies on the fact that the asset with returns that depend on monetary policy is a nominal asset. In Appendix B.2, I study a case where agents may sell equity in nontradable firms, instead of nominal bonds. Expression (20) still holds, but χ now increases with price flexibility. The intuition is simple: when prices are completely flexible, the return on equity is independent from monetary policy. It is only because there are nominal rigidities that the planner may manipulate the return on equity.

The second natural determinant of χ is risk aversion. As one would expect, if the home economy is more risk-averse to movements in tradable goods, the planner should place a larger weight on the insurance motive. This is captured by a parameter $\tilde{\gamma}$, computed in Appendix A.1.1, which depends not only on overall risk-aversion but also on the ability of the economy to substitute a lack of tradable goods with nontradable consumption and production. More interestingly, when foreign capital becomes more scarce (m is lower or foreign risk aversion γ^* is higher), the planner realizes that any additional volatility in the excess returns of home-currency bonds is penalized by foreigners with a lower relative price on the country's liabilities. As a result, the planner is less willing to let the exchange rate float freely to accommodate any changes in aggregate demand and, instead, prefers to dampen exchange rate volatility.

Corollary 1. *The weight on insurance $\omega(\bar{B}) = f(\bar{B})^2/(\chi + f(\bar{B})^2)$ increases with the size of the portfolio if and only if $B < -\mu^{-1}$.*

The other key determinant of the weight is the outstanding portfolio, which is endogenous. Consider first the case where the output gap is independent from tradable consumption ($\mu = 0$). When positions are small, providing insurance is very costly: any given transfer requires large movements in the exchange rate, which may significantly distort production efficiency. Conversely, attempting to use the exchange rate to close output gaps and achieve no price dispersion when positions are large generates sizeable transfers of tradable goods, which may be undesirable from an insurance perspective. In sum, the optimal weight on the insurance motive increases with the size of the position.

When $\mu \neq 0$, the size of the position is not conceptually the most adequate measure of sensitivity to monetary policy. To see why that is, suppose the planner was willing to create a "deviation" consisting of a 1% depreciation with respect to the demand-management target. If $B < 0$, such a deviation would also make agents richer. As discussed above, this would in turn appreciate the demand-management target, implying that the actual exchange rate movement, and resulting transfer, would be smaller than 1%. Indeed, the transfer would be $\bar{B}(1 + \mu\bar{B})^{-1}$%. This notion of "sensitivity to monetary policy" is robust to other asset market structures. In Appendix B.3 I show that with multiple assets that load arbitrarily on endogenous variables and shocks, one may still define a sufficient statistic that plays the same role of $f(\bar{B})$.

In Section 4 I show equation (20) generalizes to the dynamic model. Thus, once I have calibrated that model in Section 5, I can compute the weights in proposition 2 to measure the importance of the insurance motive vis-a-vis the demand-management motive in the data.

3.2.2 The outer problem: Optimal portfolio

The exchange rate targets depend on the outstanding portfolio. Choosing the portfolio optimally gives the planner a tool to mitigate the trade-off between insurance and demand-management as much as possible. Indeed, the linearity of the model implies that the planner could perfectly align the targets if there were a single shock by choosing:

$$f(\bar{B}) = \frac{\bar{B}}{1+\mu\bar{B}} = -\frac{\sigma_{\mathcal{T}e_s^{dm}(0)}}{\sigma^2_{e_s^{dm}(0)}} = -\frac{\mathcal{T}_s}{e_s^{dm}(0)}.$$

The sensitivity of the portfolio to monetary policy $f(\bar{B})$ depends on the size of the required transfer, and its sign depends on the covariance. As long as μ is not strong enough when the covariance between e and $\mathcal{T}$ is positive (so that $\bar{B} < -\mu^{-1}$), one may derive similar implications for the actual portfolio. Note that in this case, markets are "locally complete", i.e., the planner replicates the desired transfers to first-order. This is no longer the case when markets are locally incomplete. The next lemma solves the optimal portfolio in closed-form for the general case.

Lemma 3. *(Optimal policy portfolio) The optimal portfolio is given by*

$$B_{op} = -\frac{1}{2}\frac{\sigma^2_{\mathcal{T}} - \chi\sigma^2_{e^{dm}(0)} - 2\chi\mu\sigma_{\mathcal{T}e^{dm}(0)} + \sqrt{(\sigma^2_{\mathcal{T}} - \chi\sigma^2_{e^{dm}(0)})^2 + 4\chi(\sigma_{\mathcal{T}e^{dm}(0)})^2}}{(1-\mu^2\chi)\sigma_{\mathcal{T}e^{dm}(0)} + \mu(\sigma^2_{\mathcal{T}} - \chi\sigma^2_{e^{dm}(0)})} \tag{21}$$

Using the solution for the optimal portfolio, the next proposition derives comparative statics results.

Proposition 3. *Suppose that either $\mu\sigma_{\mathcal{T}e^{dm}(0)} > 0$ or $\mu\sigma_{\mathcal{T}e^{dm}(0)} < 0$ and the following holds,*

$$(1-\mu^2\chi)\sigma_{\mathcal{T}e^{dm}(0)} + \mu(\sigma^2_{\mathcal{T}} - \chi\sigma^2_{e^{dm}(0)}) > 0. \tag{22}$$

Then, under the optimal policy B_{op} satisfies $B_{op} < -\mu^{-1}$, and:

*(i) positions become **larger** (in absolute value) when the insurance motive becomes more important (i.e., when $\sigma^2_{\mathcal{T}}/\sigma^2_{e^{dm}(0)}$ increases or χ decreases)*

*(ii) a decrease in the covariance between the insurance and the demand-management targets - $|\sigma_{\mathcal{T}e^{dm}(0)}|/\sigma^2_{e^{dm}(0)}$ - makes positions **smaller** (in absolute value) if and only if the demand-management motive is more important than the insurance motive, i.e., if $\chi\sigma^2_{e^{dm}(0)} > \sigma^2_{\mathcal{T}}$. Conversely, i.e., if $\chi\sigma^2_{e^{dm}(0)} < \sigma^2_{\mathcal{T}}$, it makes positions **larger** (in absolute value)*

*(iii) positions have the **opposite sign** of $\sigma_{\mathcal{T}e^{dm}(0)}$;*

*(iv) positions become **smaller** (in absolute value) when m decreases*

If condition (22) does not hold, the result still holds in terms of the "sensitivity to monetary policy", i.e., $f(B_{op}) = \frac{B_{op}}{1+\mu B_{op}}$.

Proposition 3 shows how the planner resolves the trade-off when there are multiple shocks. The main result is that the sensitivity of the portfolio to monetary policy, which typically maps to a larger position,[27] increases with the importance of insurance considerations. Thus, if the economy faces shocks that create relatively large insurance needs ($\sigma^2_{\mathcal{T}}/\sigma^2_{e^{dm}(0)}$), or creating transfers is not very costly (low χ), gross positions are large (part i). This reflects that a more sizeable position reduces the cost of providing insurance ex post but increases the cost of using the exchange rate for demand-management. Furthermore, when correlation is imperfect the planner must prioritize one objective (part ii): a lower correlation pushes the planner towards larger positions (if insurance dominates) or smaller ones (if demand-management dominates). The alignment of the targets is reflected by part (iii): if the economy needs insurance when the exchange rate is weak ($\sigma_{\mathcal{T}e^{dm}(0)} > 0$), then she borrows in home-currency ($B < 0$). Finally, part (iv) shows that when there is a small foreign-investor base in home-currency markets, the planner avoids large positions, which are very expensive.

Replacing the optimal portfolio in (20) completes the characterization of the optimal monetary policy. Using the full solution, the next corollary provides some comparative statics.

Corollary 2. *The optimal insurance weight $\omega(B_{op})$ increases with the importance of the insurance motive (i.e., when $\sigma^2_{\mathcal{T}}/\sigma^2_{e^{dm}(0)}$ increases or χ decreases). It also increases with the measure of foreigners (m).*

A smaller χ leads to a larger insurance weight overall, since it leads to both a larger insurance weight conditional on the portfolio (proposition 2), and a larger portfolio (proposition 3). This suggests that portfolio endogeneity may be important quantitatively, as it amplifies the effect of structural features that affect χ, such as price flexibility and risk-aversion. Furthermore, in contrast to economies without a portfolio problem, risks now matter to first-order for the optimal monetary policy through their effect on the portfolio. For example, if shocks create a large demand for insurance, positions are sizeable, and the ex post weight on the insurance target increases. Finally, note that considering the portfolio endogeneity is also key to get the right sign for some comparative statics. For example, while a smaller measure of foreigners leads to a larger weight on insurance for a fixed portfolio, insurance is *less* important once the portfolio adjusts. This last result is also useful to think about the cooperative solution. In Appendix B.1, I show that the solution under cooperation is isomorphic to the decentralized solution with twice as many foreigners. Thus, an immediate implication of corollary 2 is that cooperation increases the size of the portfolio and the weight on the insurance motive.

[27] The mapping from the sensitivity of monetary policy $f(\bar{B})$ to actual portfolios $\bar{B}$ depends qualititatively on whether the condition (22) holds. The "reversal" case may only arise if the feedback effect μ is strong, the correlation between $e^{dm}_s(0)$ and $\mathcal{T}_s$ is small and negative, and shocks create a large demand for insurance. This case is especially unlikely to arise in the dynamic model, where the feedback effect μ is significantly weaker (see Section 4).

3.3 Implications for exchange rate volatility

It is often argued that using nominal assets to complete markets would imply excessive volatility in nominal quantities (Siu (2006)). In this section, I show that while this intuition is justified in an environment with exogenous portfolios, optimal portfolio choice may endogenously lead to environments with high leverage and low volatility in nominal quantities. To clarify the importance of portfolio endogeneity, I compare the baseline economy to an economy in which the planner cannot optimize its position because it is already at its upper bound, i.e., $|\bar{B}| = \bar{K}$.

Proposition 4. *(Optimal exchange rate volatility). Consider an economy with small risks ($\epsilon \to 0$).*

(i) Suppose the portfolio decision is constrained and, as a result, is unresponsive to marginal changes in risks or parameter values (i.e., $|\bar{B}| = \bar{K}$). Furthermore, assume $\bar{B}\mu > -1$. Then, exchange rate volatility $\sigma_e^2/\sigma^2_{e^{dm}(0)}$ ***increases*** *with the importance of the insurance motive (i.e., χ decreases or $\sigma^2_{\mathcal{T}}/\sigma^2_{e^{dm}(0)}$ increases, keeping $\sigma_{\mathcal{T}e^{dm}(0)}$constant)*

(ii) Suppose $\mu\bar{B} \geq 0$ and the optimum $\bar{B}$ is interior. Then, exchange rate volatility $\sigma_e^2/\sigma^2_{e^{dm}(0)}$ ***decreases*** *with the importance of the insurance motive (i.e., χ decreases or $\sigma^2_{\mathcal{T}}/\sigma^2_{e^{dm}(0)}$ increases, keeping $\sigma_{\mathcal{T}e^{dm}(0)}$ constant). If $\mu\bar{B} < 0$, the result is ambiguous.*

There are two main forces at play that shape the optimal degree of exchange rate volatility. First, there is a composition effect conditional on the portfolio. When the importance of insurance increases, the planner responds by increasing exchange rate volatility after shocks that create a demand for insurance. This is the standard channel considered in the closed-economy literature, which naturally leads to the conclusion that nominal quantities should become volatile. On the other hand, the planner exhibits more "fear-of-floating": afraid of the transfers of wealth a freely-floating exchange-rate regime may create, she decides to dampen exchange rate volatility. This effect is typically absent in the closed-economy literature, where a constant inflation rate is taken as the benchmark for proper demand management. Despite these two opposing effects, part (i) of proposition 4 establishes that overall exchange rate volatility increases if the portfolio is not locally endogenous, i.e., if it is constrained at the current level $\bar{B}$.[28]

Second, there is a novel effect through the portfolio. Consider first the case with $\mu = 0$. When insurance becomes more important, the planner chooses higher leverage. This implies that smaller exchange rate movements create larger transfers, which both lower the cost of providing insurance and increase the cost of letting the exchange rate float to stabilize aggregate demand. This force tends to lower exchange rate volatility. When $\mu < 0$ and $\bar{B} < 0$, depreciations coincide with positive transfers, which dampens the volatility of the demand-management target, adding another force towards lower overall volatility. In contrast, if $\bar{B} > 0$, depreciations coincide with negative

[28]For the result with respect to χ it is important that the portfolio is optimal (subject to the bound), and that it is in the "regular" region where wealth effects have a limited strength ($\bar{B}\mu > -1$). If the portfolio were entirely suboptimal this may not be true. For example, if there were only nontradable productivity shocks in the context of the example economy of section 3.5 and $B \neq 0$, volatility would decrease with the importance of insurance since only the dampening effect is present.

transfers, which increase the volatility of the demand-management target. Part (ii) of proposition 4 states that as long as the feedback through μ is not destabilizing (i.e., $\mu\bar{B} \leq 0$), the dampening effect through the portfolio is strong enough to overturn the composition effect described above.

3.4 Optimal capital controls

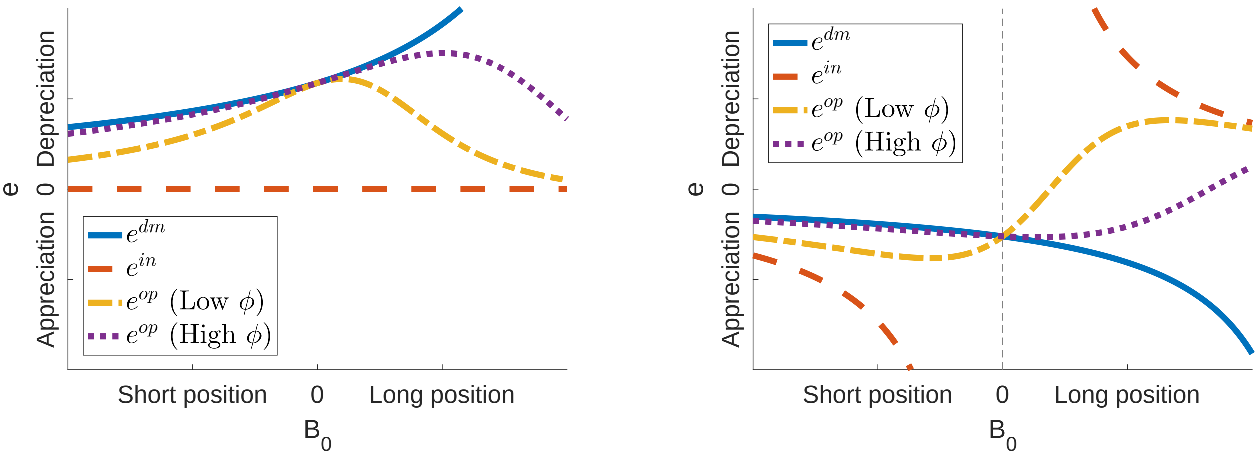
The previous sections have shown that portfolio choice endogeneity is key to alleviate the trade-offs involved in monetary policy design. A natural question is whether the private sector on its own would pick the right portfolio. Farhi and Werning (2016) showed that the answer is negative: Since there is imperfect stabilization of aggregate demand in some states of the world, and pecuniary externalities due to incomplete markets, agents do not properly internalize the value of a unit of tradable goods, which in turn implies they choose the wrong portfolio. The next proposition provides a qualification in the context of the present model.

Proposition 5. *(Asymptotic portfolio taxes) Consider an economy with small risks ($\epsilon \to 0$). Then, optimal portfolio taxes τ_B are given by*

$$\tau_B = \gamma^*_{ss} m^{-1} \bar{B} \mathbb{E} e_s^2 + O(\epsilon^3). \tag{23}$$

When $m \to \infty$, the only externalities in the economy are aggregate demand externalities (due to nominal rigidities) and pecuniary externalities (due to incomplete markets). Proposition 5 shows that in this case taxes are asymptotically of third-order. In other words, portfolio decisions are asymptotically efficient. What is the intuition behind this result? Even to first-order, private marginal utility and social marginal utility differ in every state. Indeed, in this environment, one can show that the gap between the social and the private marginal utility is proportional to the output gap, as agents do not internalize the effect of their choice on the planner's ability to create insurance. The key observation is that output gaps are themselves deviations generated with the sole intent and purpose of creating insurance. In other words, output gaps are proportional to social marginal utility under the optimal policy. As a result, private marginal utility and social marginal utility are proportional to one another and, since the planner sets the covariance of the exchange rate and social marginal utility to zero, the covariance of the exchange rate and private marginal utility is also zero. The key assumption that makes this result hold is the fact that *divine coincidence* holds in this model: If the planner wanted to she could always close the output gap and price dispersion at the same time. In contrast, if there were more than one output gap (i.e., an export and domestic sector), or there were markup shocks, taxes would be nonzero to second-order. This last effect, however, is unrelated to market incompleteness per se: In those environments the planner would like to put portfolio taxes even with complete markets. Furthermore, the result is *not* driven by the simple asset market structure with two bonds (I show it generalizes to much more general environments in Appendix B.3) nor by the absence of dynamics (I show it generalizes to a dynamic setting in Section 4).[29] The only difference in those complicated environments is that

[29] It also generalizes to a multi-asset dynamic model (see Appendix B.4).



(a) Nontradable productivity shock

(b) Tradable endowment shock

Figure 1: Exchange rate response after a positive nontradable productivity shock (panel a) and a positive endowment shock (panel b) under a demand-management policy (solid-blue line), an insurance policy (dashed-red), the optimal policy when prices are relatively flexible $\phi = 0.1$ (dotted-dashed-yellow), and the optimal policy when prices are relatively rigid $\phi = 0.9$ (dotted-purple line) for different home-currency positions. I set $\alpha = 0.55$, $\psi = 1$, $\eta = 6$, $\gamma^* = 1$, and $m = \infty$.

there are more wedges between private and social marginal utility (i.e., in addition to the output gap). However, the value of *each* wedge is proportional to the value of the insurance they create (to first-order). As a result, they are all proportional to social marginal utility and the result, once again, follows. Note that the fact that *portfolio decisions* are asymptotically efficient does not mean *savings decisions* are, as I will show in Section 4.

As discussed in Section 2.2, the model has an additional rationale for taxes when $m < \infty$. In this case, the home economy behaves as a monopolist in its home-currency bond market and has an incentive to distort the stochastic discount factor to increase its price. This implies taxing home-currency debt if $\bar{B} < 0$ or taxing home-currency assets if $\bar{B} > 0$. By curbing the demand of home agents, the planner manages to increase the value of the country's international investment position. Unlike aggregate demand externalities, which increase the size of the pie, this motive is clearly mercantilistic, i.e., it implies a redistribution from foreigners to home agents. In Appendix B.1, I show that this tax would be zero in the cooperative solution. Finally, note that this motive is also unrelated to market incompleteness, as discussed in Section 2.2.

3.5 An example

I illustrate the results of the previous section with a simple example. I set $u = \ln(C_{Ts}^{\alpha} C_{Ns}^{1-\alpha} - \frac{1-\alpha}{1+\psi} L_s^{1+\psi})$ and $F = Z_s Y_{Is}$, where Z_s is a productivity shock. The parameter α is an index of openness in this economy and I calibrate steady state values accordingly: $Y_{Tss} = \alpha$ and $Z_{ss} = 1 - \alpha$. Beyond standard productivity shocks, the example economy is also affected by endowment shocks $\{Y_{Ts}\}$, which capture in reduced form innovations to the "terms-of-trade" of a country, i.e., shocks to the world prices of the country's exports. These shocks are often considered as one of the key drivers of business cycles in open economies (Fernández, Schmitt-Grohé and Uribe (2017)). In the quantitative model in Section 5 I also consider other shocks that have been stressed by the literature, such as interest-rate shocks and shocks to the uncovered interest rate parity (UIP) condition. To make the figures, I set $\alpha = 0.55$, $\psi = 2$, $\eta = 6$, and $m = \infty$.

3.5.1 Optimal monetary policy and optimal portfolio

Inner problem I begin by computing the exchange rate targets. Figure 1 shows the response of the economy after a positive nontradable productivity shock (left panel) and a positive endowment shock (right panel). Consider first the insurance target (dashed-red line). A simplifying feature of this parametrization is that in equilibrium agents want to consume the same amount of tradables regardless of their consumption of nontradables.[30] As a result, the planner does not desire any transfer of tradables after Z shocks. In contrast, because they are risk-averse, home agents dislike volatility in their tradable consumption. Hence, when the endowment is low, the planner depreciates if home agents have home-currrency debt and viceversa,

$$\mathcal{T}_s = -\frac{\alpha}{1+2m^{-1}\tilde{\gamma}^{-1}\gamma^*}y_{Ts} + O(\epsilon^2) \Rightarrow e_s^{in}(\bar{B}) = (\frac{\alpha}{1+2m^{-1}\tilde{\gamma}^{-1}\gamma^*})\frac{1}{\bar{B}}y_{Ts} + O(\epsilon^2) \tag{24}$$

where $\tilde{\gamma} = 1/\alpha$, which is absolute risk aversion in the tradable good. Away from unitary elasticity between tradable and nontradables, $\tilde{\gamma}$ would depend on the elasticity of substitution, increasing with the degree of complementarity.

Next, consider the demand-management target (solid-blue line) and suppose $\bar{B} = 0$. Under flexible prices, a positive nontradable productivity shock would require an increase in employment and a decrease in the foreign-currency price of intermediate inputs. To replicate this price movement, the planner needs to depreciate the exchange rate. When $\bar{B} \neq 0$, this is not the end of the story: the depreciation triggers a negative transfer of tradables (if $\bar{B} > 0$). Agents would then be more poor, which implies that replicating the flexible price allocation requires a further depreciation of the currency. In other words, $\bar{B} > 0$ triggers a destabilizing wealth effect. Conversely, when $\bar{B} < 0$, this wealth effect is stabilizing. This effect is captured by μ,

$$\mu = -\frac{1}{\alpha}\frac{\psi}{\psi+\alpha}.$$

Its strength depends on openness (a more open economy faces a smaller proportional change in tradable consumption), and on the disutility of labor (if it were linear then increasing production would not require higher intermediate input prices). Note that, since an endowment shock is in fact a wealth transfer, its effect on the demand-management target is also given by μ. In sum, the

[30] This case is usually studied with separable utility. It can be shown that with the proposed steady state parametrization the result is also true for the Greenwood–Hercowitz–Huffman (GHH) utility I specified.

demand-management target is given by

$$e_s^{dm}(\bar{B}) = \frac{1}{1 - \frac{1}{\alpha}\frac{\psi}{\psi+\alpha}\bar{B}}(\frac{1-\alpha}{\psi+\alpha}z_s - \frac{\psi}{\psi+\alpha}y_{Ts}) + O(\epsilon^2) \tag{25}$$

To complete the characterization of the inner problem, I now compute the insurance weight $\omega(\bar{B})$ described in Section 3.2.1. To do so, I first need to compute the parameter χ,

$$\chi = \frac{1}{1+2m^{-1}\tilde{\gamma}^{-1}\gamma^*}\frac{\alpha\phi(1-\alpha)(\alpha+\psi)(\phi+(\alpha+\psi)(1-\phi)\eta)}{(\alpha+\psi+(1-\alpha-\psi)\phi)^2},$$

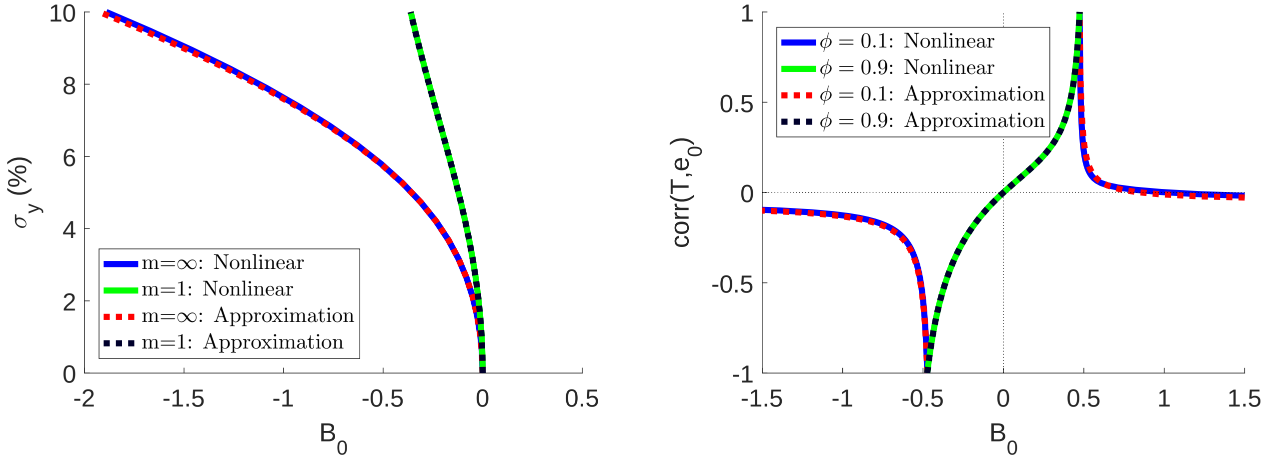
which is monotonic in ϕ if $\eta < 2$ and hump-shaped otherwise with a critical threshold of $\bar{\phi} = \frac{(\alpha+\psi)\eta}{(1+\alpha+\psi)\eta-2}$. Furthermore, note that χ becomes zero when the economy is completely closed or open ($\alpha \in \{0,1\}$). Since I assumed production in the tradable sector was not affected by the price rigidity, the output gap becomes irrelevant when $\alpha = 1$. On the other hand, when $\alpha \to 0$ exchange rate movements create transfers which, in proportion to the steady state level of tradable consumption, are very large. Note this does not imply the insurance weight goes to 1 under the optimal policy, since desired transfers also scale with openness so the portfolio is also affected. Figure 1 computes the optimal exchange rate *conditional* on a portfolio $\bar{B}$ for a case relatively flexible ($\phi = 0.1$, dotted-dashed-yellow line) and relatively sticky ($\phi = 0.9$, dotted-purple line) prices. As expected from proposition 2, the optimal policy lies closer to the demand-management target when positions are closer to zero and when the country issues home-currency debt, due to the endogenous feedback effect through μ. Note that in the depicted example, the "low" ϕ policy is sufficiently low such that high price flexibility actually implies a larger weight on the insurance motive.

Outer problem To find the optimal portfolio, I need to compute the covariance structure of the targets in an economy without home bonds. This yields,

$$\begin{aligned}
\sigma^2_{\mathcal{T}} &= (\frac{\alpha}{1+m^{-1}\tilde{\gamma}^{-1}\gamma^*})^2\sigma_y^2 \\
\sigma^2_{e^{dm}(0)} &= \frac{1}{(\psi+\alpha)^2}((1-\alpha)^2\sigma_z^2 + \psi^2\sigma_y^2 - (1-\alpha)\psi\sigma_{yz}) \\
\sigma_{\mathcal{T}e^{dm}(0)} &= \frac{1}{\psi+\alpha}\frac{1}{1+2m^{-1}\tilde{\gamma}^{-1}\gamma^*}(\psi\alpha\sigma_y^2 - \alpha(1-\alpha)\sigma_{yz}).
\end{aligned}$$

First, consider the case of uncorrelated shocks ($\sigma_{yz} = 0$). Panel (a) in Figure 2 illustrates how the optimal portfolio (x axis) varies as I increase the standard deviation of y from 0% to 10% (y axis) while keeping the volatility of nontradable productivity shocks at 10%.[31] I consider the cases with perfect integration in home-currency-bond markets ($m = \infty$, red and blue lines) and with

[31] Note that the volatility level only matters for the nonlinear model; for the approximation only relative volatilies matter. I present the values of the portfolios to assess the accuracy of the approximation, but the value themselves do not have a meaningful interpretation since this static model is too stylized to take to the data. For a quantitative analysis, see section 5.



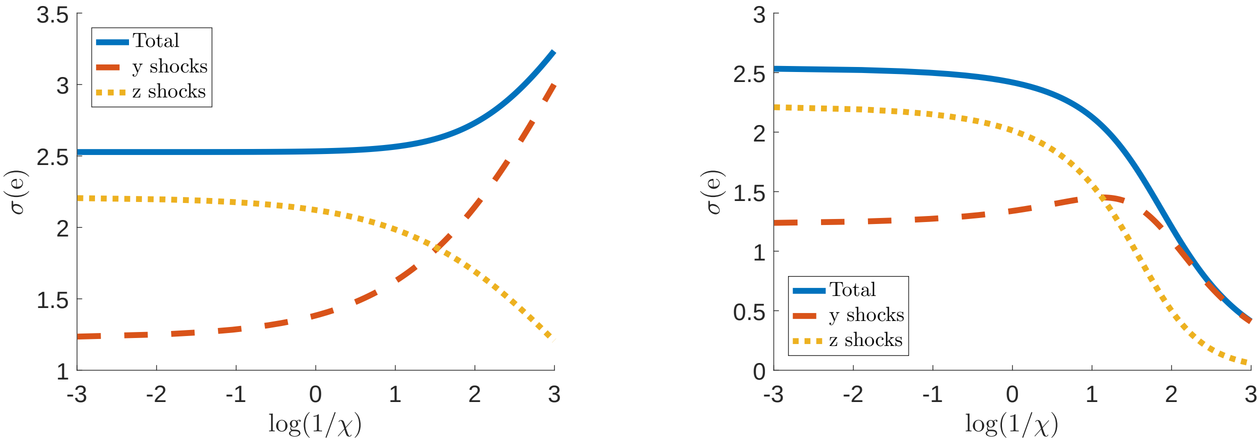
(a) Varying σ_y **(b)** Varying σ_{yz}

Figure 2: Optimal portfolio. On the left, I plot the optimal portfolio as I vary the volatility of tradable endowment shocks from 0 to 10% with the volatility of nontradable productivity shocks fixed at 10% in the approximated model (dotted lines) and in the original nonlinear model (solid lines) when $m = \infty$ (blue and red) and when $m = 1$ (green and black). On the right, I fix $\sigma_y = 2.5\%$ and $\sigma_z = 10\%$ and vary the covariance between y and z from -1 to 1 in the approximated model (dotted lines) and the original nonlinear model (solid lines) when the share of firms optimizing is $\phi = 0.1$ (blue and red) and $\phi = 0.9$ (green and black).

limited participation ($m = 1$, green and black lines). Since the exchange rate depreciates when the economy has a low endowment of tradables ($\sigma_{\mathcal{T}e^{dm}(0)} > 0$), the planner chooses a short position in home-currency bonds. The larger the endowment shocks, the shorter the position. This allows the planner to provide insurance more cheaply, i.e., by using smaller exchange rate movements. On the other hand, a smaller home-currency market (low m) induces the planner to choose smaller positions and insure less overall to avoid paying a large premium on its debt. In either case, the approximation (red and black dotted lines) is very close to the true solution in this model (blue and green lines).[32]

Next, suppose y and z are correlated. When they are perfectly negatively correlated, the exchange rate depreciates when the tradable endowment is low. Thus, the optimal portfolio is short home-currency bonds. In contrast, when they are perfectly positively correlated, there are opposing effects on the exchange rate. I assume y shocks are four times less volatile than z shocks ($\sigma_y = 2.5\%$), which implies the exchange rate depreciates when the tradable endowment is high. As a result, the optimal portfolio is long home-currency bonds. When correlation is imperfect, the response of the optimal portfolio depends on the relative importance of the insurance vis-a-vis the demand-management motive. Figure 2 shows how the optimal portfolio varies as I increase the correlation between y and z shocks from -1 to 1 in a case with low flexibility ($\phi = 0.9$, green and black lines) and in a case of high flexibility ($\phi = 0.1$, red and blue lines). To make the plot easier

[32] I observe the largest difference when the model yields large positions, since the objective function then becomes very flat. Even then, the difference is small; the approximation yields $B = -1.92$ vs $B = -1.89$ in the nonlinear model. I found similar results for other parameter values.



(a) Constrained $\bar{B}$

(b) Unconstrained $\bar{B}$

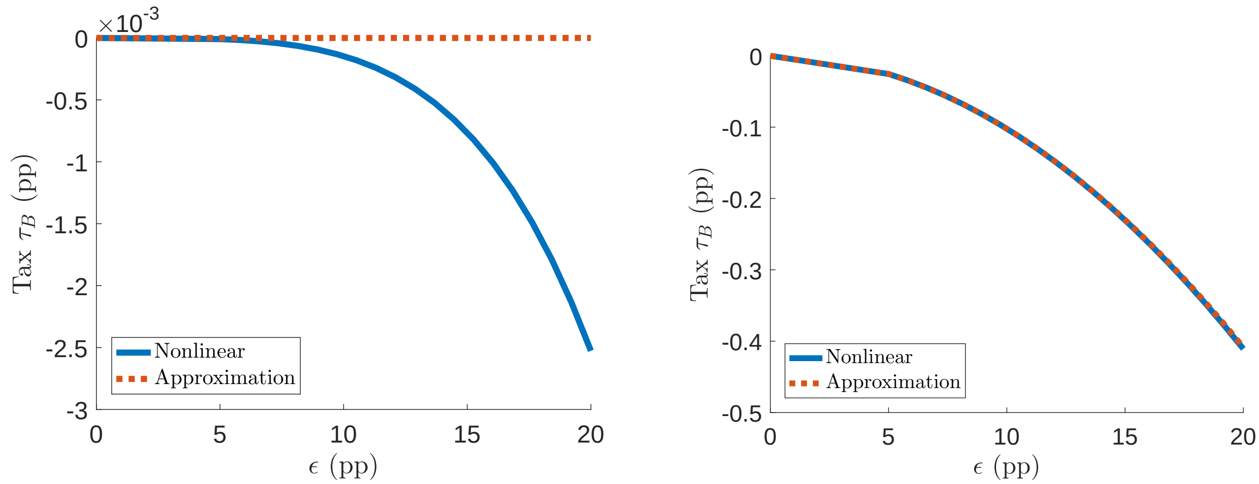
Figure 3: Exchange rate volatility. I decompose the total standard deviation of the exchange rate (solid-blue line) into the contribution of y shocks (red-dashed line) and z shocks (dotted-yellow line) as the parameter χ varies. Shocks are uncorrelated. On the left, the portflio is constrained at some level $\bar{K}$, which is suboptimally low for the plotted values χ. On the right, the portfolio is at its optimal value. The standard deviation is in percentage points. These figures were computed using the approximation. Results are very similar in the nonlinear model.

to interpret, I show the implied correlation between the desired transfers $\mathcal{T}$ and the demand-management exchange rate $e^{dm}(0)$ in the y axis rather than the correlation between fundamentals (they are one-to-one in the example). When flexibility is low (red and blue lines), the parametrization lies in the region where the demand-management objective dominates ($\chi\sigma^2_{e^{dm}(0)} > \sigma^2_{\mathcal{T}}$) and the planner decides to reduce leverage. In contrast, when flexibility is high (green and black lines), the parametrization lies in the region where the insurance objective dominates ($\chi\sigma^2_{e^{dm}(0)} < \sigma^2_{\mathcal{T}}$), and the planner increases leverage to reduce the cost of providing insurance.[33]

3.5.2 Implications for exchange rate volatility

Figure 3 illustrates how exchange rate volatility changes with the importance of insurance (χ) in the example economy (I keep $\sigma_z = 10\%$ and $\sigma_y = 2.5\%$). In panel (a), I keep the portfolio fixed at a level that would be optimal with the highest plotted χ (≈ 20). This case illustrates the composition effect: when the planner places a higher weight on the insurance target, it dampens the response after nontradable productivity shocks (dotted-yellow line) and takes a more active stance against endowment shocks (dashed-red line). Overall volatility (solid-blue line) increases, driven by the large response to endowment shocks. Panel (b) plots the response when the portfolio decision is unconstrained. Nontradable productivity shocks are now dampened even further, due to the

[33]When the correlation becomes zero, the problem becomes bang-bang. When demand-management dominates, the planner prefers to remain in autarky ($\bar{B} = 0$). In contrast, when insurance dominates the planner chooses the other corner, which is given by $\bar{B} = -\mu^{-1}$. More rigorously, I approach this value as the correlation goes to 0, since when $\bar{B} = \mu^{-1}$, the equilibrium is only well-defined if the exchange rate is fixed. Furthermore, when correlation is low and positive, the economy is in a "reversal" region not covered by proposition 2 and $\bar{B} > -\mu^{-1}$, which in turn implies demand-management responses change signs.



(a) Full integration ($m = \infty$)

(b) Limited participation ($m = 1$)

Figure 4: Optimal tax on home-currency assets (τ_B) as a function of risk (ϵ) using the approximation (dashed-red line) and the nonlinear model (solid-blue line). Both quantities are expressed in percentage points. On the left panel, I set $m = 0$ while on the right panel I set $m = 1$. I set $\sigma_y = \epsilon/4$ and $\sigma_z = \epsilon$.

larger portfolio. For endowment shocks, the effect is now ambiguous: Since the portfolio is larger, smaller exchange-rate movements are necessary to achieve the same amount of insurance and, in addition, the demand-management target becomes more stable. In this parametrization, I find a hump-shaped response, with volatility due to y shocks increasing when portfolios are relatively small and decreasing thereafter. Furthermore, since $\mu\bar{B} \geq 0$, overall volatility decreases with the strength of the insurance motive.

3.5.3 Optimal capital controls

Figure 4 plots the optimal tax in the example economy when there is perfect integration ($m = \infty$) and when there is limited participation ($m = 1$). I keep the covariance structure as in the previous section and vary the overall amount of risk ϵ in the economy from 0 to 20%. I compare the full nonlinear solution (solid-blue line) with the approximated solution (dashed-red line) given by equation (23). When $\epsilon \to 0$, the tax becomes zero in every case, since assets are perfect substitutes in the limit. When $m = \infty$, the approximation predicts a zero tax while the nonlinear model predicts a very small subsidy in home current assets. In contrast, when $m = 1$, the implied subsidy is two orders of magnitude larger.[34] Finally, note that the approximation tracks very closely the true nonlinear solution.

[34] The underlying portfolio is $\bar{B} = -0.6$.

4 Dynamic model

In this section, I generalize the model to a dynamic setting. I present the model in Section 4.1. Section 4.2 characterizes the optimal policy and is divided into two parts. Section 4.2.1 establishes the robustness of all the results in terms of the excess returns of home-currency bonds. Section 4.2.2 studies the implementation of these excess returns in terms of monetary policy and savings taxes, which allows me to deliver new results. Appendix B.4 shows the robustness of the results in an extension with multiple assets that load on endogenous variables and shocks.

4.1 Setup

Home agents solve

$$\max \mathbb{E}_{-1}\{\sum_{t=0}^{\infty} \beta^t u(C_{Tt}, C_{Nt}, L_t; \xi_t)\} \tag{26}$$

subject to

$$\begin{aligned} C_{Tt} + E_t^{-1} P_{Nt} C_{Nt} + (1+\tau_t^*) NFA_t + \tau_t^B B_t = Y_{Tt} + E_t^{-1}\Pi_{It} + E_t^{-1}\Pi_{Nt} + (1+\tau_L) W_t L_t \\ + R_{t-1}^* NFA_{t-1} + RR_t B_{t-1} + T_t \end{aligned} \tag{27}$$

where $NFA_t = B_t^* + B_t$ is the *net* foreign asset position of the country (with both asset positions measured in foreign currency), τ_t^* is a savings tax, and RR_t are the realized excess returns of the home-currency bond. I allow home-currency bonds to have a long duration by assuming they pay an initial coupon of δ units of home currency that decays at rate $1-\delta$, as in Hatchondo and Martinez (2009). Excess returns on home-currency bonds are given by

$$RR_t = \{(1+\psi_{t-1}) R_{t-1} (\delta E_t^{-1} E_{t-1} + (1-\delta) E_t^{-1} E_{t-1} R_t^{-1}) - R_{t-1}^*\}$$

where ψ is a shock to the liquidity service of home-currency bonds, similar to Lahiri and Végh (2003).[35] This shock introduces noise in the return of home-currency bonds, and is meant to capture in reduced form disturbances to the arbitrage between bonds in the spirit of portfolio balance models a la Kouri (1976). It will play an important role in the quantitative section, where they will allow the model to match the volatility of the nominal exchange rate. I assume these shocks are symmetric across agents, i.e., they do not reflect heterogeneous beliefs.[36] Furthermore, note that while this shock is, indeed, an uncovered interest rate parity (UIP) shock, it is different from the traditional one considered in the literature with a single foreign-currency bond (see, for example, Kollmann (2001)), which is captured in the framework by an interest-rate shock R_t^*. Crucially, the R_t^* shock affects the *savings* decision while the ψ_t shock affects the *portfolio* decision.

[35] Equivalently, I could have introduced a taste shock on the holdings of home bonds in the utility function (with a normalization such that taste shocks are still symmetric across agents).

[36] The framework cannot accommodate first-order differences in beliefs, since then positions would become unbounded as $\epsilon \to 0$.

I assume foreign agents that access home-currency bond markets are marginally indifferent between saving and consuming if they do not participate,

$$u^{*\prime}(Y_t^*) = \beta^* R_t^* \mathbb{E}_t u^{*\prime}(Y_{t+1}^*), \tag{28}$$

where R^* is unaffected by the decision of participating foreigners.[37] Optimization yields the no-arbitrage condition,

$$\mathbb{E}_{t-1} RR_t u'(Y_t^* + R_{t-1}^* B_{t-1}^{f*} - B_t^{f*} - RR_t m^{-1} B_{t-1}) = 0$$

where B^{f*} is the savings decision of foreigners participating in home-currency markets.

Finally, staggered price-setting is modeled by making the identity of the $1-\phi$ share of re-optimizing firms stochastic (i.e., Calvo price-setting). The rest of the model is the same as before.

4.2 Optimal policy

4.2.1 Characterization in terms of excess returns

The key economic quantity to characterize the optimal policy with long bonds is the realized *excess return* of home-currency bonds. This is not a property of the dynamic model per se; rather, it is a consequence of breaking the perfect correlation between excess returns and contemporaneous exchange rate movements that arises with short-bonds.[38] Define, by analogy to the static model, the demand-management target as the excess return that is consistent with a zero output gap and no savings taxes,

$$rr_t^{dm}(\bar{B}_{t-1}) = \frac{1}{1+\mu\bar{B}_{t-1}} rr_t^{dm}(0) + O(\epsilon^2), \tag{29}$$

where $rr_t^{dm}(0)$ is the excess return in an economy without home-currency bonds but *with* foreign-currency bonds. Similarly, define the insurance excess return as the one that creates the transfer of goods the planner would desire if prices were flexible ($\mathcal{T}_t$),

$$rr_t^{in}(\bar{B}_{t-1}) = \frac{1}{\bar{B}_{t-1}} \mathcal{T}_t + O(\epsilon^2). \tag{30}$$

The next lemma shows that welfare can still be written as a function of deviations from these two targets.

Lemma 4. *In the limit $\epsilon \to 0$, the optimal excess returns of home-currency bonds at t and the optimal*

[37] For this reason, the case with $m < \infty$ can no longer be interpreted as a large economy. Relaxing this assumption would imply the economy has an additional terms-of-trade manipulation motive as it would try to influence the world interest rate in its favor, exactly as in Costinot, Lorenzoni and Werning (2014).

[38] In Appendix B.2, I show an example in the context of the static model. In that example, agents can trade equity in the nontradable sector. There, too, the important economic quantity is the realized excess return.

portfolio at $t-1$ solve

$$\mathcal{W} = \max_{\{rr_t, \bar{B}_{t-1}\}} -k_0 \mathbb{E}_{t-1}\{\frac{1}{2}\bar{B}_{t-1}^2(rr_t - rr_t^{in}(\bar{B}_{t-1}))^2 + \frac{1}{2}\chi(1+\mu\bar{B}_{t-1})^2(rr_t - rr_t^{dm}(\bar{B}_{t-1}))^2\}$$
$$+ t.i.p. + O(\epsilon^3) \tag{31}$$

The objects $rr_t^{dm}(0)$, $\mathcal{T}_t$, k_0, μ and χ are specified in Appendix A.2.

Lemma 4 shows that the problem of finding the optimal first-order behavior of the excess return of home-currency bonds and the steady-state optimal portfolio is essentially a static problem that is isomorphic to the one in Section 3, stated in terms of the excess returns of home-currency bonds, rather than the exchange rate. The portfolio problem is separable over time: future portfolios do not affect the path of expected variables to first-order. This implies that it is without loss of generality to shut-down uncertainty from $t+1$ onwards (a manifestation of certainty equivalence) to study the optimal distribution of endogenous variables at t and the portfolio chosen at $t-1$.

Lemma 4 immediately implies that the following results from Section 4.2 generalize to the dynamic environment: (i) the optimal excess returns are a weighted average of the targets implied by the two motives - equations (29) and (30), (ii) the optimal portfolio satisfies an equation analogous to (21), and, (iii) the volatility of excess returns decreases with the importance of insurance if the portfolio is endogenous but it increases if it is exogenous (when $\mu\bar{B} \geq 0$). Furthermore, proposition 3.4 shows the optimal portfolio tax result also generalizes to the dynamic model. In other words, no portfolio taxes are necessary in the absence of terms-of-trade manipulation ($m = \infty$), or with cooperation. The intuition is the same as before: excess return deviations are proportional to the value of insurance.[39]

Proposition 6. *Propositions 2, 3, and 4 generalize to the dynamic environment in terms of excess returns rr_s (the static model is a special case after using $rr_s = -e_s$). The portfolio tax formula generalizes to*[40]

$$\tau_{Bt} = (1-\beta)\gamma_{ss}^* m^{-1} B \mathbb{E}_{t-1} rr_t^2 + O(\epsilon^3).$$

Finally, note that while the reduced form representation is the same, χ is now a more complicated object. Whereas in the static model there is a single way of creating an excess return, in the dynamic model the planner has more tools, such as promises of future exchange rate movements. χ now contains the information on how to choose among the many instruments the planner has to deliver any given excess returns rr_t at a minimum cost, a problem I study in Section 4.2.2. Another important difference with the static model is that now agents only consume a share $1-\beta$

[39] In Appendix B.4 I show that all these results also generalize to an economy with one asset sensitive to monetary policy and multiple exogenous assets. If there are multiple assets that are sensitive to monetary policy, I can show that portfolio taxes are still zero if $m = \infty$ and monetary policy also has a similar weighted average representation. However, unlike the static model, the portfolio can no longer be solved in closed form and propositions 3 and 4 do not seem to carry over. A simple algorithm is provided to solve this case.

[40] The $1-\beta$ reflects the fact that foreigners can also borrow in their own currency, smoothing bad shocks. If they lived for only period, then the formula would hold with a 1 instead of $1-\beta$, exactly the same as in the static model.

of any additional transfer. This has two important consequences. First, the wealth effect on the demand-management target is now much weaker: μ is now multiplied by $\beta^{-1}(1-\beta)$. Second, it significantly reduces the impact of a transfer on marginal utility. This is clearly seen in the extreme with rigid prices ($\phi = 1$) and short-bonds. In this case, χ is divided by $\beta^{-1}(1-\beta)$. Naturally, to the extent that shocks may be close to permanent, insurance desirability ($\mathcal{T}$) also increases, which offsets the smaller effect of a transfer by increasing the size of the desired transfer and the optimal portfolio.

Remark 2. The feedback effect from transfers to equilibrium demand-management exchange rates is much weaker: $\mu_{dynamic} = \beta^{-1}(1-\beta)\mu_{static}$. In addition, if prices are rigid $\phi = 1$ and bonds are short, $\chi_{dynamic} = \beta(1-\beta)^{-1}\chi_{static}$.[41]

4.2.2 Implementation of excess returns

In this section, I study the "continuation" problem, which involves finding the optimal value of endogenous variables at t and the path for expected endogenous variables from $t+1$ onwards after the state of the world at t is realized. In other words, I take the promised excess return rr_0 as given and ask: What is the optimal path that delivers this excess return? By certainty equivalence, this is a deterministic problem. The answer to this question pins down the cost of replicating rr_0, i.e., it pins down χ. I first describe the optimal path for monetary policy and then the implications for the path of savings taxes.

Monetary Policy The implementation in terms of monetary policy is now more complicated: the planner needs to specify a full path for the exchange rate.[42] To streamline the discussion, consider the response of the economy after an innovation at time $t = 0$ and suppose the economy was at a steady-state at -1. Combining the firms' first-order condition and consumers' demand for nontradables I obtain:

$$\Delta e_t = \pi_{It} + \beta(1-\beta)^{-1}\mu\Delta c_{Tt} + k'_{e\xi}\Delta\xi_t + k_{ex}\Delta x_t + O(\epsilon^2) \tag{32}$$

where x_t is the output gap, π_{It} is intermediate-input inflation, and k_{ex} and $k'_{e\xi}$ are constants ($k'_{e\xi}$ is a vector). To find the demand-management *exchange rate* target, set $x_t = \pi_{It} = 0$ and plug in the solution for c_{Tt} (from the consumer's Euler and the budget constraint). This yields a path $\{e_t^{dm}(\bar{B})\}$. By contrast, the insurance exchange target consistent with rr_0^{in} is not uniquely defined if bonds are long. Indeed, any $\{\Delta e_t\}$ path that satisfies

$$\beta rr_0^{in} = -e_0 - \sum_{t=1}^{\infty} \beta^t (1-\delta)^t (\Delta e_t + r^*_{t-1} + \psi_{t-1}) + O(\epsilon^2)$$

[41] If I had a share ϕ of "flexible" firms and the rest setting prices one period in advance, then this result would be still hold with short bonds.

[42] Since the trilemma holds in the economy, monetary policy can also be restated in terms of an interest rate path and a long-run level of the exchange rate.

is admissible, as long as one can find a path $\{x_t, \pi_{It}, c_{Tt}\}$ that satisfies equation (32), the budget constraint, a transversality condition for bonds, and the dynamic Phillips curve

$$\pi_{It} = \kappa x_t + \beta \mathbb{E}_t \pi_{It+1} + O(\epsilon^2)$$

where $\kappa > 0$ is a constant. Among these many potential paths, I define the insurance exchange rate target as the path $\{e_t^{in}\}$ that delivers rr_0^{in} at the minimum cost. This implies that, to find such path, the planner needs to solve a cost-minimization problem, choosing the optimal paths of $\{c_{Tt}, x_t, \pi_{It}, e_t\}$ that minimize consumption distortions and production inefficiency. To characterize the solution to this problem it will be convenient to define the *costly exchange rate deviations* $\{\tilde{e}_t\}$ associated with an exchange rate path $\{e_t\}$ and an excess return rr_0 as:

$$\tilde{e}_t(B) = e_t(B) - \{e_t^{dm}(0) + \mu \bar{B} rr_0\}.$$

Whenever $\tilde{e} \neq 0$, the planner must either distort consumption (i.e., put a savings tax) or production (i.e., have a nonzero output gap). Naturally, the demand-management policy has no deviations: $\tilde{e}_t^{dm} \equiv 0$. In contrast, when $\bar{B} rr_0^{dm} \neq \mathcal{T}_0$, the planner needs to create deviations ($\tilde{e}_t \neq 0$) to provide the desired insurance. The next proposition characterizes the optimal behavior of these deviations.

Proposition 7. *(Optimal monetary policy II) Let $\tilde{e}_t(B) = e_t(B) - \{e_t^{dm}(0) + \mu \bar{B} rr_0\}$ denote the costly exchange rate deviations.*

1. If prices are rigid, then:

$$\tilde{e}_t = -\bar{k}_{e1}(1-\delta)^t \{(1+\mu\bar{B}) rr_0 - rr_0^{dm}(0)\} + \bar{k}_{e2}\{(1+\mu\bar{B}) rr_0 - rr_0^{dm}(0)\} + O(\epsilon^2)$$

where $\bar{k}_{e1} > 0$ and $\bar{k}_{e2} \geq 0$. Furthermore, $sign(\tilde{e}_0) = sign(-\{(1+\mu\bar{B}) rr_0 - rr_0^{dm}(0)\})$. If $\mu = 0$, then $\bar{k}_{e2} = 0$.

2. Suppose $k_{ex} > 0$ (expansionary devaluations) and suppose inflation is relatively more costly than the output gap: $\kappa \lambda_\pi k_{ex} - \lambda_x > 0$, where λ_x and λ_π are the costs of (squared) output gaps and inflation, respectively.[43] *If bonds are short, then:*

$$\Delta \tilde{e}_t = \mathcal{P}(k_{ex}) R_\pi^{t-1} \{(1+\mu\bar{B}) rr_0 - rr_0^{dm}(0)\} + O(\epsilon^2) \ \forall t \geq 2$$
$$\Delta \tilde{e}_1 = \bar{k}_{e3}\{(1+\mu\bar{B}) rr_0 - rr_0^{dm}(0)\} + O(\epsilon^2)$$
$$\tilde{e}_0 = -\bar{k}_{e4}\{(1+\mu\bar{B}) rr_0 - rr_0^{dm}(0)\} + O(\epsilon^2)$$

where R_π is the optimal decay rate of inflation, $\bar{k}_{e3} > 0$ and $\bar{k}_{e4} > 0$ are constants and $\mathcal{P}(k_{ex})$ is a continuous and monotonic function of k_{ex} with $\mathcal{P}(0) > 0$ and $\lim_{k_{ex}\to\infty} \mathcal{P}(k_{ex}) < 0$.

3. In either of the cases above, setting $rr_0 = rr_0^{in}$ yields the deviations corresponding to the insurance

[43] If u is GHH or separable, the composite between tradables and nontradable is CES, and $F = Y_{Is}^{\alpha_F}$, then $k_{ex} = \frac{\alpha_F}{1-\alpha_F+\rho^{-1}\alpha_F} > 0$ so devaluations are expansionary.

target. Furthermore, once the targets have been defined, the optimal exchange rate has the same weighted average representation as before (even when $\delta < 1$ and $\phi < 1$), i.e.,

$$e_t^{op} = \frac{\bar{B}^2}{\chi(1+\mu\bar{B})^2 + \bar{B}^2} e_t^{in}(B) + \frac{\chi(1+\mu\bar{B})^2}{\chi(1+\mu\bar{B})^2 + \bar{B}^2} e_t^{dm}(\bar{B}) + O(\epsilon^2)$$

Proposition 7 shows that the optimal exchange rate has the same weighted average representation as excess returns if one defines the insurance target as the one that minimizes the cost of providing the transfer. More interestingly, it characterizes the deviations $\{\tilde{e}_t\}$ in two special cases: with rigid intermediate-input prices, and with short bonds. When prices are rigid, the planner spreads the adjustment in proportion to the amount of debt maturing in each period: If she wants to deliver a positive excess return, she creates a persistent overvaluation of the exchange rate, which decays at the same rate as bonds if $\mu = 0$. If $\mu < 0$ the planner brings consumption forward, which implies the exchange rate eventually settles at a more depreciated level. In other words, the exchange rate appreciates on impact and then slowly returns back to its demand-management level (given the new long-run net foreign asset position). When bonds are short, the contemporaneous exchange rate is immediately determined by the desire to create an excess return. When devaluations are expansionary, the planner induces a recession and deflation today, followed by a boom and inflation tomorrow.[44] This immediately implies an appreciation at $t = 0$ followed by a depreciation at $t = 1$. After $t = 1$, the boom and inflation slowly subside, with opposing implications on the exchange rate path. Thus, depending on the sensitivity of the output gap to exchange rate movements, the $t = 1$ exchange rate may overshoot (if k_{ex} is large) or converge monotonically (if k_{ex} is small) to its new long-run level. While the general case can also be written in closed form, it is easy to see from these examples how a variety of cases may arise.

Savings taxes Proposition 8 characterizes savings taxes in the cases with either rigid prices or short bonds.

Proposition 8. *(Savings taxes) Suppose the planner wants to create an excess return of rr_0 at $t = 0$ and suppose there are no further shocks. Then:*

1. If prices are rigid ($\phi = 1$), savings taxes decay at rate $1 - \delta$

$$\tau_{B^*t} = -\bar{K}_0(\delta)(\delta\mu - \bar{K}_1 k_{ex}^{-1} k_{ux})(1-\delta)^t \{(1+\mu\bar{B})rr_0 - rr_0^{dm}(0)\} \tag{33}$$

where $\bar{K}_0 > 0$, $\bar{K}_1 > 0$ are constants, k_{ux} captures the reaction of private marginal utility to the output gap ($k_{ux} > 0$ implies agents overvalue tradable goods in booms).[45] When $\delta = 0$, $\bar{K}_0(\delta) = 0$.

[44] The latter depends on inflation being relatively more costly than the output gap. (Otherwise the planner may promise deflation in the continuation, see Appendix A.2.4)

[45] If u is GHH, the composite between tradables and nontradable is CES, and $F = Y_{Is}^{\alpha_F}$, then $k_{ux} = \frac{1-\alpha}{\rho} > 0$. If u is separable, $k_{ux} = (\gamma - \rho^{-1})(1-\alpha)\alpha_F$, so it depends on whether tradables and nontradables are Edgeworth complements or substitutes (i.e., if $\gamma\rho > 1$, they are substitutes and $k_{ux} > 0$).

2. If bonds are short ($\delta = 1$), then saving taxes from $t \geq 1$ are given by

$$\tau_{B^*t} = -\bar{k}_1 k_{ux} R_\pi^{t-1} \pi_1$$

where R_π is the optimal decay rate of inflation after $t = 1$ and $\bar{k}_1 > 0$. At $t = 0$,

$$\tau_{B^*0} = -\bar{k}_2 \mu \{(1 + \beta^{-1}(1 - \beta)k_{ec})rr_0 - rr_0^{dm}(0)\} + k_{ux}\Delta x_1$$

where $\bar{k}_2 > 0$. If $k_{ex} > 0$ and $\kappa \lambda_\pi k_{ex} - \lambda_x > 0$, then $\Delta x_1 > 0$ and $\pi_1 > 0$.

Like the exchange rate, savings taxes decay at the same rate as bonds when prices are rigid. However, since savings taxes are only useful to the extent that the adjustment is unevenly split across periods, the *level* of the tax is very small if debt is very long. Indeed, if debt is a perpetuity, the optimal tax is zero. The two terms inside the first parenthesis in equation (33) reflect the two reasons why savings taxes are useful. The first term reflects that when $\mu \neq 0$ manipulating tradable consumption allows the planner to move the exchange rate at no cost in terms of production efficiency. For example, suppose the planner wants to create a positive excess return on home-currency bonds. Then, if $\mu < 0$, the planner taxes savings, which boosts consumption and appreciates the exchange rate. The second term reflects that agents do not value tradable goods properly. Although state-by-state this valuation mistake is proportional to the value of insurance if $m = \infty$ and, hence, no portfolio taxes are required, the strength of the externality varies across periods. If agents undervalue tradable goods in recessions ($k_{ux} > 0$), the planner has an incentive to boost savings in booms and tax them in recessions. In the example discussed above, where the planner creates a persistent yet declining overvaluation, the planner has an additional incentive to tax savings.

With staggered prices and short bonds, the only incentive to tax savings after $t = 1$ is the aggregate demand externality. Consider the case with $k_{ex} > 0$ and suppose the planner wants to increase the return on home assets. Recall from the earlier discussion this induced a recession at $t = 0$ followed by a boom from $t = 1$ onwards. When agents undervalue tradables in recessions ($k_{ux} > 0$), the planner wants to tax savings at $t = 0$ and subsidize them at $t = 1$ to correct the aggregate demand externality. Furthermore, at $t = 0$ she also wants to tax savings to increase consumption, which has the added benefit of appreciating the exchange rate (if $\mu < 0$).

5 Quantitative Analysis

In this section, I evaluate the quantitative importance of the theoretical results presented in Sections 3 and 4. It is divided into three parts. Section 5.1 briefly describes the calibration strategy. Section 5.2 presents the implications of the optimal policy for the statistics that played a key role in the analysis and welfare. As a benchmark, I also present the results under a demand-management-targeting policy, which is equivalent to strict (intermediate-input) inflation targeting. Section 5.3

Table 1: Parameter values and shocks

Parameter	Description	Value	Parameter	Description	Value
		A. Structural parameters			
β	Discount factor	0.99	ϕ	Probability of not adjusting prices	0.75
γ	Home risk aversion	2	η	Elasticity of substitution (varieties)	6
γ^*	Foreign risk aversion	2	δ	Bond depreciation	0.042
α	Tradable share	0.55	m	Measure of foreigners	0.18
ν^{-1}	Frisch elasticity	0.5	ϕ_π	Reaction to CPI inflation	1.5
ρ	Elasticity of substitution (T/NT)	0.74	ρ_i	Smoothing coefficient	0.84
		B. Shocks			
σ_z	Productivity s.d.	0.47%	ρ_ψ	Liquidity service persistence	0.79
σ_{p*}	Terms-of-trade s.d.	0.2%	$corr(\epsilon_t^z, \epsilon_t^{p*})$	Correlation: z and p^*	0.26
σ_{r*}	World interest-rate s.d.	0.23%	$corr(\epsilon_t^z, \epsilon_t^{r*})$	Correlation: z and r^*	−0.13
σ_{y*}	Foreigners' output s.d.	0.53%	$corr(\epsilon_t^z, \epsilon_t^{y*})$	Correlation: z and y^*	0.41
σ_ψ	Liquidity service s.d.	0.92%	$corr(\epsilon_t^{p*}, \epsilon_t^{r*})$	Correlation : p^* and r^*	−0.51
ρ_z	Productivity persistence	0.81	$corr(\epsilon_t^{p*}, \epsilon_t^{y*})$	Correlation: p^* and y^*	0.36
ρ_{p*}	Terms-of-trade persistence	0.74	$corr(\epsilon_t^{r*}, \epsilon_t^{y*})$	Correlation: r^* and y^*	−0.15
ρ_{r*}	World interest-rate persistence	0.87	$corr(\epsilon_t^{\psi}, \epsilon_t^{x})$	Correlation: ψ and others	0
ρ_{y^*}	World output persistence	0.88			

studies the sensitivity of the results to the presence of liquidity shocks and the cost of inflation, and shows that they play an important role.

Appendix C provides additional sensitivity analysis and shows how the results change when savings taxes are not available.[46]

5.1 Calibration

Table 1 lists the parameter values and stochastic processes used in the baseline calibration. A period in the model is one quarter. Flow utility is assumed to take a standard separable form,[47]

$$u = \frac{1}{1-\gamma} C_t^{1-\gamma} - \frac{1-\alpha}{1+\nu} N_t^{1+\nu}$$
$$C_t = (\alpha^{\frac{1}{\rho}} C_{Tt}^{\frac{\rho-1}{\rho}} + (1-\alpha)^{\frac{1}{\rho}} C_{Nt}^{\frac{\rho-1}{\rho}})^{\frac{\rho}{\rho-1}}.$$

I adopt standard values for the discount factor (0.99), risk aversion (2), and the Frisch elasticity of labor supply ($\frac{1}{2}$). For simplicity, I assume nontradable production is linear in intermediate inputs and set the elasticity of substitution between tradable and nontradable goods at $\rho = 0.74$, following Mendoza (1992), who estimates it in a sample of 13 industrial countries. I assume that intermediate

[46] I provide additional sensitivity exercises with respect to the elasticity of substitution between tradable and nontradable goods, risk-aversion, the degree of openness, the duration of home-currency bonds, and the discount factor.

[47] I also explored the predictions of a model with GHH utility, as in Section 3. Results with respect to the outcomes of interest are similar but the performance worsens in other dimensions: it predicts too high consumption and output volatility.

good producers do not reoptimize each period with probability 0.75, and set the elasticity across varieties η to 6, as in Gali and Monacelli (2005). For the remaining parameters and stochastic processes, I use data from Canada, which I take as a prototype small open economy. I assume that the monetary authority follows a simple Taylor rule,

$$i_t = (1-\rho_i)(\beta^{-1}-1) + \rho_i i_{t-1} + (1-\rho_i)\phi_\pi \pi_t$$

where π_t is CPI inflation, $\phi_\pi = 1.5$ - a standard value - and $\rho_i = 0.84$, which is the estimated persistence of the 3-month Canadian treasury bill rate over the sample period 1997:1-2016:4. I classify as nontradable sectors those with a very low export share: construction and services related to real estate services, public administration, education, health services and professional and scientific services. This leads to a share of tradables in output (α) of 55%. Furthermore, the net foreign asset position is balanced (i.e, $NFA_{ss} = 0$).

I assume productivity shocks are perfectly correlated across sectors due to lack of reliable data on sectoral output at a quarterly frequency (i.e., $Y_{Tt} = Z_t$ and $Y_{Nt} = Z_t Y_{It}$). I also allow for terms-of-trade shocks, implying total tradable income is $P_t^* Y_{Tt}$ (leaving the foreign-currency price of imports fixed at 1). In order to calibrate the stochastic properties of the exogenous driving forces, I fit AR(1) processes to (log) labor productivity (z), the (log) terms of trade in Canada (p^*), (log) U.S. real seasonally-adjusted output (y^*) and the U.S. 3 month treasury bill rate deflated by the U.S. CPI (r^*), using quarterly HP-filtered data (except for r^*) over the sample period 1997:1-2016:4.[48] As is well known, calibrating these shocks alone would predict too little exchange rate volatility. For this reason, I add an AR(1) liquidity shock ψ_t which, as explained in the previous section, adds noise to the return of holding home-currency bonds. I calibrate it to match the exchange rate volatility in the period ($\sigma_{\Delta e} = 0.036$), while keeping the persistence of the 3-month Canadian treasury bill rate unaffected. This yields $\sigma_\psi = 0.0092$ and $\rho_\psi = 0.79$. This shock also moves the model closer to the data in other respects, generating a negative Fama coefficient and a low R^2 in a Fama regression. I pick the measure of intermediaries m to match the size of home-currency debt liabilities in 2012 (-15%), the last year I have access to data from Bénétrix, Lane and Shambaugh (2015). Finally, since I lack data on the duration of home-currency external debt, I choose δ to match a duration of 4.85 years, which corresponds to the average duration of Canadian government debt between 1997 and 2010.[49]

Table 2: Results in baseline model.

	Calibrated Taylor rule	Demand Management	Optimal	Optimal: fixed B	Optimal: Cooperation
A. Home-currency bond positions and excess returns					
ω		0%	7.75%	3.57%	22.1%
$\bar{B}$	−15.0%	−16.0%	−22.8%	−15.0%	−58.0%
$\sigma(rr)$:total	6.12%	3.79%	3.72%	3.82%	3.42%
$\sigma(rr) : r^*$	2.75%	1.58%	1.99%	1.91%	2.26%
$\sigma(rr) : \psi$	5.72%	3.44%	3.15%	3.32%	2.56%
$\sigma(rr) : y^*$	0%	0%	0.35%	0.24%	0.70%
B. Policy instruments					
τ_B/risk premium		−81.1%	−104%		0%
$\sigma(\tau^*)$		0%	0.03%	0.02%	0.06%
$\sigma(e)$: total	3.59%	1.48%	1.60%	1.58%	1.70%
$\sigma(e) : r^*$	1.85%	1.48%	1.60%	1.59%	1.64%
$\sigma(e) : \psi$	3.34%	0.06%	0.19%	0.11%	0.49%
$\sigma(e) : y^*$	0%	0%	0.15%	0.10%	0.29%
C. Welfare gains (% of first-best)					
Gains		11.9%	16.9%	15.0%	41.3%

Note: In column 4, the portfolio is fixed at −15% while the remaining columns it is optimally chosen by the planner. The portfolio is normalized by annual gdp. Every other variable is expressed in quarterly units. The portfolio tax τ_B is normalized by the second-order risk-premium on home-currency bonds, which is positive. Welfare gains are measured by how much of the welfare gap between the first-best (a model with flexible prices) and an economy without home bonds ($\bar{B} = 0$) economy is achieved by each policy: $\frac{welf(policy) - welfare(\bar{B}=0)}{welfare(firstbest) - welfare(\bar{B}=0)}\%$.

5.2 Baseline model

5.2.1 Implications for key statistics

In this section, I explore the importance of the theoretical results presented in the previous sections. The first result stated that optimal monetary policy is a weighted average of a demand-management target and an insurance target. The first row in panel A of Table 2 shows that the planner places a nontrivial weight on the insurance motive of around 8% (column 3), although demand-management is still the most important consideration for monetary policy. The second result stated that the optimal portfolio is larger under the optimal policy than under demand-management. Despite the modest weight on the insurance motive, row 2 in Table 2 shows the portfolio is indeed significantly more sizeable: home-currency debt under the optimal policy is 23%, compared to only 16% under demand-management targeting (column 2). The third result

[48] Note that the autocovariance of y^* is irrelevant in the model; what matters is the interest rate. Implicitly, in writing equation (28) I am assuming there are discount factor shocks $\{\beta_t^*\}$ that make the foreign Euler equation hold.

[49] The data is from OECD statistics and the series was discontinued after 2010. In a sample of emerging markets, Du and Schreger (2015) report an average McCauley duration of home-currency government debt held by foreigners of 5 years.

stated that excess returns are less volatile under the optimal policy. While true, this result does not seem to be quantitatively relevant (row 3). This, however, masks an important composition effect. Rows 4-6 compute the volatility of excess returns driven by interest-rate r^* (row 4), liquidity service ψ (row 5), and world output y^* (row 6) shocks.[50] The optimal policy *shifts* the volatility: it smooths the response after ψ shocks and increases it after interest-rate r^* and y^* shocks. Lowering the exposure to ψ shocks, which are essentially noise for agents, allows them to take large positions at a low cost. This increases the effectiveness of policy after the shocks that create the majority of the demand for insurance: interest-rate shocks (for home agents) and world output shocks (for foreigners).

Panel B shows the implications for the policy instruments. Consider first the portfolio tax τ_B (row 1). Since home agents hold home-currency debt, theory implies that the planner should put a subsidy on home-currency assets (i.e., $\tau_B < 0$). In fact, the subsidy is quantitatively relevant: It is even larger than the steady-state risk-premium.[51] This large subsidy reflects that the model predicts a low penetration of foreigners into home-currency bond markets to match the observed portfolios. Next, consider saving taxes τ^* (row 2). These taxes are very small, with a standard deviation of around 3 basis points. In fact, I show in Appendix C that solving the optimal policy *without* the savings taxes yields quantitatively very similar results.[52]

One recurring theme in this paper is that the endogeneity of portfolios matters. To illustrate this, column 4 in Table 2 presents the results if the portfolio were fixed at the calibrated 15% of home-currency debt over GDP. In this case, the policy deviates noticeably less from demand-management. Indeed, the weight on the insurance target is more than halved.

Finally, column 5 in Table 2 shows the results from the point of view of a supranational authority. Theory indicates that when there is limited foreign participation in home-currency markets (i.e., with $m < \infty$) coordination increases the importance of the insurance motive, since the cooperative planner internalizes the hedging benefits accrued to foreigners (see Appendix B.1). Given that the model requires very limited foreign access to home-currency bond markets to match observed positions, the difference between the decentralized and the cooperative solution is substantial: the weight on the insurance target and the portfolio increase by almost a factor of three, which in turn imply a noticeable decrease in the volatility of home-currency excess returns.

5.2.2 Welfare gains

How effective is the optimal policy in completing markets? To answer this question, I compute the welfare gains (in consumption equivalents) of moving from an economy without bonds to an economy with these bonds and flexible prices (i.e., the first best). I then compute what share

[50] The remaining shocks explain a tiny amount of the volatility. Furthermore, note that since shocks are correlated this is not a strict variance decomposition.

[51] The risk premium is a second-order phenomenon, like the portfolio tax.

[52] This result is explained by the long calibrated duration of the bonds. In Appendix C I show that savings taxes are important when bonds have a duration of one year.

Table 3: Sensitivity analysis

	Baseline	$\psi_t \equiv 0$	$\eta = 11$	$\phi = .8$	$\phi = 2/3$
$\bar{B}$: Demand-management	-16.0%	-10.1%	-16.0%	-15.6%	-16.5%
$\bar{B}$: Optimal Policy	-22.8%	-10.8%	-19.6%	-19.4%	-34.4%
ω	7.75%	14.8%	3.60%	4.1%	24.5%
Welfare: Demand-management	11.9%	67.7%	11.9%	12.2%	11.5%
Welfare: Optimal Policy	16.9%	72.2%	14.5%	15.2%	23.6%

Note: Column (2) turns off ψ shocks, column (3) increases the elasticity of substitution across varieties, and columns (4) and (5) change the frequency of price adjustment. In every case except (2), m and σ_ψ are re-calibrated to match the exchange volatility and a portfolio of -15% over annual GDP. In (2), only m is re-calibrated to match the portfolio. Welfare gains are measured as the steady-state-consumption-equivalent gains under such a policy with respect to the demand-management $\bar{B} = 0$ economy as a share of the total potential gains under the first-best: $\frac{welf(policy) - welfare(\bar{B}=0)}{welfare(firstbest) - welfare(\bar{B}=0)}\%$.

of these gains are attained in the economy with sticky prices under different policies. Panel C shows the results. The optimal policy improves significantly over the demand-management policy, attaining 17% instead of 12%, a gain almost 1.5 times as large.[53] Portfolio endogeneity is again quantitatively important for this result: If the portfolio were fixed at 15%, gains would only be 15%. Finally, the cooperative solution attains a much larger share of the gains, reflecting the limited participation of foreigners in home-currency markets.

5.3 Sensitivity analysis

5.3.1 No liquidity shocks

In this section, I turn off liquidity shocks and recalibrate the measure of foreigners to match observed portfolios. Column 2 in Table 3 shows the results. Column 1 reproduces the results in the baseline model to facilitate the comparison. The most important difference is that the trade-off between demand-management and insurance is significantly weakened. This can be seen by computing the correlation between the desired transfers ($\mathcal{T}$) and the excess-returns when $\bar{B} = 0$. While the correlation is -0.35 with liquidity shocks, without them the correlation is much higher at -0.82 (not shown in table). One reason why the exchange rate targets are so correlated is that the model significantly underestimates the volatility of exchange rates under the calibrated Taylor rule, with the remainder tightly linked to fundamentals.[54] The high correlation implies that portfolios under demand-management targeting and the optimal policy are very close to one another, differing by less than a percentage point. By the same token, the demand-management policy already achieves 67.7% of the potential gains of completing markets, with the optimal policy only increasing this number to 72.2%.

[53] The *absolute* size of the gains is tiny: the first-best gains are only 0.067% steady-state consumption equivalents. This is a manifestation of the well-known fact that the welfare gains from completing markets in standard macroeconomic models are very small (Lucas (1987)), which is exacerbated in this context by limited participation by foreigners.

[54] Exchange rate volatility in this model is essentially explained by r^*.

Although policies are on average similar, the insurance weight is actually higher (15%) since foreign participation in home-currency markets is even more restricted, which implies the planner is now heavily penalized with a low home-bond price if excess returns are volatile. To understand why this is the case, note that given that the model underpredicts the volatility of excess returns in the absence of liquidity shocks, agents would like to choose a very large position if there were perfect intermediation ($m = \infty$). As a result, very limited foreign participation in home-currency markets is necessary to match the observed portfolios. Although the planner would be willing to dampen returns in order to lower costs, the very high correlation implies this trade-off hardly ever materializes: the demand-management target and the insurance target are often aligned.

5.3.2 Cost of inflation

Next, I explore the sensitivity of the results to the cost of inflation. First, I lower the mark-up in intermediate inputs to 10% ($\eta = 11$), which increases the cost of price dispersion. Column 3 in Table 3 shows the results. Clearly, this parameter is important for the quantitative results. Doubling the equilibrium mark-up of intermediate good producers essentially halves the importance of insurance, as measured by its implications for the portfolio, the weight, and welfare.

Second, I vary the share of firms that are able to update their prices. The results suggest this is also a critical parameter to determine the relative importance of the insurance motive. Assuming that firms update their prices on average every 3 quarters implies the portfolio more than doubles in size compared to demand-management targeting, implying an insurance weight of almost 25%. The planner is also able to attain a much larger share of the welfare gains of completing markets. Conversely, assuming firms update their prices every 5 quarters halves the importance of the insurance motive.

6 Conclusion

I developed a framework to study optimal monetary policy and capital controls in open economies with incomplete markets and portfolio choice. I presented three main results. First, I showed that monetary policy can be described by an exchange rate rule that is a weighted average of two targets: a demand-management/inflation target, concerned with the traditional role of "undoing" nominal rigidities, and an insurance target, which is the exchange rate that would be required to replicate the transfer the planner would desire under complete markets. Second, I showed that positions in home-currency become larger when insurance considerations are more important, which in turn leads to a higher ex post weight on the insurance target. To lower the cost of such large positions, the planner strongly dampens the volatility of the exchange rate when insurance is not needed (an example of "fear-of-floating"). Perhaps surprisingly, when portfolios are endogenous this effect is so strong that *overall* volatility under the optimal policy is lower than under inflation-targeting. By contrast, with constrained portfolios the effect is weaker and overall volatility would actually be higher than under inflation-targeting, illustrating the importance of modeling portfolio choice. Finally, I showed that in a small open economy portfolio decisions are approximately efficient despite the presence of aggregate demand externalities (due to nominal rigidities) and pecuniary externalities (due to incomplete markets), so no capital controls on the composition of capital flows are necessary in the approximate solution.

In this paper, I focused on the trade-off between insurance and demand-management, abstracting from other relevant macroeconomic forces. However, the methodology I develop is widely applicable and can be used to explore optimal policy in other environments with portfolio choice and incomplete markets. It is also simple computationally, even with multiple assets (see Appendix B.4), so it could be applied to richer macroeconomic models than the one presented in this paper. Doing so could deliver new interesting results on capital controls, as well as a reappraisal of the quantitative importance of the insurance channel of monetary policy.

References

Akinci, Özge. 2011. "A Note on the Estimation of the Atemporal Elasticity of Substitution Between Tradable and Nontradable Goods." *Mimeo*, 1–17.

Bénétrix, Agustin S, Philip R Lane, and Jay C Shambaugh. 2015. "International currency exposures, valuation effects and the global financial crisis." *Journal of International Economics*, 96(S1): S98—-S109.

Benigno, Pierpaolo. 2009*a*. "Are valuation effects desirable from a global perspective?" *Journal of Development Economics*, 89(2): 170–180.

Benigno, Pierpaolo. 2009*b*. "Price Stability with Imperfect Financial Integration." *Journal of Money, Credit and Banking*, 41(1).

Benigno, Pierpaolo, and Michael Woodford. 2012. "Linear-quadratic approximation of optimal policy problems." *Journal of Economic Theory*, 147(1): 1–42.

Bohn, Henning. 1988. "Why do we have nominal government debt?" *Journal of Monetary Economics*, 21: 127–140.

Bohn, Henning. 1990. "A positive theory of foreign currency debt." *Journal of International Economics*, 29: 273–292.

Caballero, Ricardo J, and Arvind Krishnamurthy. 2004. "Inflation targeting and sudden stops." *The Inflation-Tergeting Debate*, I(December): 423–446.

Campbell, John Y., and Luis M. Viceira. 2002. *Strategic asset allocation: Portfolio choice for long-term investors.* Oxford University Press.

Chang, Roberto, and Andrés Velasco. 2006. "Currency mismatches and monetary policy: A tale of two equilibria." *Journal of International Economics*, 69(1 SPEC. ISS.): 150–175.

Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc. 2010. "Optimal monetary policy in open economies." In *Handbook of International Economics, Vol. III*.

Costinot, Arnaud, Guido Lorenzoni, and Iván Werning. 2014. "A Theory of Capital Controls as Dynamic Terms-of-Trade Manipulation." *Journal of Political Economy*, 122(1): 77–128.

Devereux, Michael B., and Alan Sutherland. 2008. "Financial globalization and monetary policy." *Journal of Monetary Economics*, 55(8): 1363–1375.

Devereux, Michael B., and Alan Sutherland. 2011. "Country portfolios in open economy macro-models." *Journal of the European Economic Association*, 9(2): 337–369.

Du, Wenxin, and Jesse Schreger. 2015. "Sovereign Risk , Currency Risk , and Corporate Balance Sheets." *Working Paper*.

Du, Wenxin, Carolin E Pflueger, and Jesse Schreger. 2017. "Sovereign Debt Portfolios, Bond Risks, and the Credibility of Monetary Policy." *Working paper*.

Engel, Charles, and Jungjae Park. 2017. "Debauchery and Original Sin: The Currency Composition of Sovereign Debt." *Working paper*.

Evans, Martin D D, and Viktoria Hnatkovska. 2012. "A method for solving general equilibrium models with incomplete markets and many financial assets." *Journal of Economic Dynamics and Control*, 36(12): 1909–1930.

Fanelli, Sebastián. 2017. "An almost linear-quadratic approximation of optimal policy problems with portfolio choice." *Working Paper*.

Farhi, Emmanuel, and Iván Werning. 2016. "A Theory of Macroprudential Policies in the Presence of Nominal Rigidities." *Econometrica*, 84(5): 1645–1704.

Fernández, Andrés, Stephanie Schmitt-Grohé, and Martín Uribe. 2017. "World shocks, world prices, and business cycles: An empirical investigation." *Journal of International Economics*, 108.

Gali, Jordi, and Tommaso Monacelli. 2005. "Monetary Policy and Exchange Rate Volatility in a Small Open Economy." *The Review of Economic Studies*, 72(3): 707–734.

Geanokoplos, John D., and Heraklis M. Polemarchakis. 1986. "Existence, regularity, and constrained suboptimality of competitive allocations when the asset market is incomplete."

Gourinchas, Pierre-olivier, and Hélène Rey. 2013. "External Adjustment, Global Imbalances and Valuation Effects." In *Handbook of International Economics*. 585–640.

Hatchondo, Juan Carlos, and Leonardo Martinez. 2009. "Long-duration bonds and sovereign defaults." *Journal of International Economics*, 79(1): 117–125.

Judd, Kenneth L., and Sy-Ming Guu. 2001. "Economic Theory." *Economic Theory*, 18(1): 127–157.

Kollmann, Robert. 2001. "The exchange rate in a dynamic-optimizing business cycle model with nominal rigidities: A quantitative investigation." *Journal of International Economics*, 55(2): 243–262.

Kouri, Pentti J K. 1976. "The Exchange Rate and the Balance of Payments in the Short Run and in the Long Run: A Monetary Approach." *The Scandinavian Journal of Economics*, 78(2): 280–304.

Lahiri, Amartya, and Carlos A Végh. 2003. "Delaying the Inevitable : Interest Rate Defense and Balance of Payments Crises Delaying the Inevitable : Interest Rate Defense and Balance of Payments Crises Amartya Lahiri Carlos A . Ve." *Journal of Political Economy*, 111(2): 404–424.

Lane, Philip R., and Gian Maria Milesi-Ferretti. 2007. "The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities, 1970-2004." *Journal of International Economics*, 73(2): 223–250.

Lane, Philip R, and Jay C Shambaugh. 2010*a*. "Financial Exchange Rates and International Currency Exposures." *The American Economic Review*, 100(1): 518–540.

Lane, Philip R., and Jay C. Shambaugh. 2010*b*. "The long or short of it: Determinants of foreign currency exposure in external balance sheets." *Journal of International Economics*, 80(1): 33–44.

Lucas, Robert E. 1987. *Models of business cycles.* Basil Blackwell Oxford.

Lustig, Hanno, Christopher Sleet, and Sevin Yeltekin. 2008. "Fiscal hedging with nominal assets." *Journal of Monetary Economics*, 55(4): 710–727.

Mehra, Rajnish, and Edward C. Prescott. 1985. "The equity premium: A puzzle." *Journal of Monetary Economics*, 15: 145–161.

Mendoza, Enrique G. 1992. "The effects of macroeconomic shocks in a basic equilibrium f." *Staff Papers (International Monetary Fund)*, 39(4): 855–889.

Mendoza, Enrique G. 1995. "The terms of trade, the real exchange rate, and economic fluctuations." *International Economic Review*, 36(1): 101–137.

Obstfeld, Maurice, and Kenneth Rogoff. 2002. "Global Implications of Self-Oriented National Monetary Rules." *The Quarterly Journal of Economics*, 117(2): 503–535.

Ottonello, Pablo. 2015. "Optimal Exchange-Rate Policy Under Collateral Constraints and Wage Rigidity." *Mimeo*, 1–56.

Ottonello, Pablo, and Diego J Perez. 2017. "The currency composition of sovereign debt." *Working paper*.

Samuelson, Paul A. 1970. "The fundamental approximation theorem of portfolio analysis in terms of means, variances and higher moments." *Review fo Economic Studies*, 37: 537–542.

Schmitt-Grohe, Stephanie, and Martín Uribe. 2004. "Optimal fiscal and monetary policy with sticky wages and sticky prices." *Journal of Economic Theory*, , (114): 198–230.

Senay, Ozge, and Alan Sutherland. 2017. "Optimal Monetary Policy, Exchange Rate Misalignments and Incomplete Financial Markets." *Working paper*.

Sheedy, Kevin D. 2014. "Debt and Incomplete Financial Markets: A Case for Nominal GDP Targeting." *Brookings Papers on Economic Activity*, 2014(1): 301–373.

Siu, Henry E. 2006. "Optimal fiscal and monetary policy with sticky wages and sticky prices." *Review of Economic Dynamics*, 9(4): 683–714.

Tille, Cédric, and Eric van Wincoop. 2010. "International capital flows." *Journal of International Economics*, 80(2): 157–175.

Woodford, Michael. 2003. *Interest and prices: Foundations of a theory of monetary policy.* Princeton university press.

A Appendix: Proofs and derivations

A.1 Proofs for section 3

A.1.1 Lemma 2

First, I derive a log-linear approximation to the output gap in this economy. Second, I derive an equation linking the exchange rate to the output gap, tradable consumption and shocks. Third, I compute the static Phillips curve linking the output gap and intermediate-input prices. Finally, I derive a second-order approximation to welfare around the riskless steady state $(\bar{B}, \epsilon = 0)$. I use undercapitalized letters for log deviations from the steady state for all variables except B. I use $\bar{B}$ to denote the steady-state value of the home-currency position and B_ϵ to denote $\frac{\partial B}{\partial \epsilon}$. For simplicity, I only consider the following shocks: a tradable endowment shock $\{Y_{Ts}\}$, a nontradable productivity shock $\{Z_s\}$, a general taste shock $\{\iota_s\}$ that premultiplies flow utility, and a shock to foreigners' endowment $\{Y^*_{Ts}\}$.

Output gap I compute the output gap between the actual output and the output that would arise if prices were flexible, conditional on having the same level of tradable consumption. First, I need to compute the level of employment at the flexible price allocation l_s^{flex}, which solves $u_N F_Y + u_L =$

0 (set $\phi = 0$ in equation (9)). Let Z denote the shock to the final nontradable production function, i.e $F(Y_s^I, Z_s)$. To a first-order approximation,

$$L_{ss} l_s^{flex} = -\frac{F_Y u_{NN} F_Z + u_N F_{YZ} + u_{NL} F_Z}{F_Y^2 u_{NN} + 2F_Y u_{NL} + u_N F_{YY} + u_{LL}} Z_{ss} z_s - \frac{F_Y u_{TN} + u_{TL}}{F_Y^2 u_{NN} + 2F_Y u_{NL} + u_N F_{YY} + u_{LL}} C_{Tss} c_{Ts}.$$

Define the output gap as $x_s = F^{-1} F_Y (y_{Is} - y_{Is}^{flex})$ and let $\Delta \equiv \int_0^1 (\frac{P_{Is}(i)}{P_{Is}})^{-\eta} di - 1$ denote intermediate input price dispersion deviations. Since production at the steady state is efficient, Δ is 0 to first-order. Thus,

$$\begin{aligned} x_s = & \frac{F^{-1} F_Y}{F_Y^2 u_{NN} + 2F_Y u_{NL} + u_N F_{YY} + u_{LL}} \{ (F_Y^2 u_{NN} + 2F_Y u_{NL} + u_N F_{YY} + u_{LL}) L_{ss} l_s \\ & + (F_Y u_{NN} F_Z + u_N F_{YZ} + u_{LN} F_Z) Z_{ss} z_s + (F_Y u_{TN} + u_{TL}) C_{Tss} c_{Ts} \} \end{aligned} \tag{34}$$

Exchange rate sensitivity to the output gap Log-linearizing the equation

$$u_N(C_{Ts}, F(Y_{Is}, Z_s), L_s) F_Y(Y_{Is}, Z_s) / u_T(C_{Ts}, F(Y_{Is}, Z_s), L_s) = P_{Is}/E_s,$$

which comes from the first-order condition of nontradable good producers and consumer optimization, yields

$$\tilde{\phi}_c C_{Tss} c_{Ts} + \tilde{\phi}_z Z_{ss} z_s + \tilde{\phi}_l L_{ss} l_s = p_{Is} - e_s$$

where

$$\begin{aligned} \tilde{\phi}_c &= u_N^{-1} u_{TN} - u_T^{-1} u_{TT} \\ \tilde{\phi}_z &= u_N^{-1} u_{NN} F_Z + F_Y^{-1} F_{YZ} - u_T^{-1} u_{TN} F_Z \\ \tilde{\phi}_l &= -u_T^{-1} u_{TN} F_Y - u_T^{-1} u_{TL} + u_N^{-1} u_{NN} F_Y + u_N^{-1} u_{NL} + F_Y^{-1} F_{YY} \end{aligned}$$

Or, in terms of the output gap,

$$e_s = p_s^i + k_{ec} C_{Tss} c_{Ts} + k_{ez} Z_{ss} z_s + k_{ex} x_s \tag{35}$$

where

$$\begin{aligned} k_{ex} &= -F F_Y^{-1} \tilde{\phi}_l \\ k_{ec} &= -\tilde{\phi}_c + \tilde{\phi}_l \frac{F_Y u_{TN} + u_{TL}}{F_Y^2 u_{NN} + 2F_Y u_{NL} + u_N F_{YY} + u_{LL}} \\ k_{ez} &= -\tilde{\phi}_z + \tilde{\phi}_l \frac{F_Y u_{NN} F_Z + u_N F_{YZ} + u_{LN} F_Z}{F_Y^2 u_{NN} + 2F_Y u_{NL} + u_N F_{YY} + u_{LL}} \end{aligned}$$

In general, $k_{ex} > 0$, $k_{ec} < 0$, $k_{ez} > 0$, although it depends on the application (and, more generally, on the source of nominal rigidities).

Static Phillips curve Let P_{Is}^{flex} denote the price of firms who optimize. To first-order,

$$p_{Is}^{flex} = e_s - u_T^{-1}u_{TT}C_{Tss}c_{Ts} - u_T^{-1}u_{TN}(F_Y L_{ss}l_s + F_Z Z_{ss}z_s) - u_T^{-1}u_{TL}L_{ss}l_s \\ + u_L^{-1}u_{TL}C_{Tss}c_{Ts} + u_L^{-1}u_{NL}(F_Y L_{ss}l_s + F_Z Z_{ss}z_s) + u_L^{-1}u_{LL}L_{ss}l_s \tag{36}$$

Log-linearizing (9),

$$p_{Is} = \frac{1-\phi}{\phi}(p_{Is}^{flex} - p_{Is}) \tag{37}$$

Using (36) and (35), I obtain

$$p_{It} = \kappa x_t \tag{38}$$

where

$$\kappa = \frac{1-\phi}{\phi}\lambda_x F^{-1}u_N^{-1} > 0$$
$$\lambda_x = -\frac{F_Y^2 u_{NN} + F_Y u_{NL} + u_N F_{YY} + u_{NL}F_Y + u_{LL}}{F^{-2}F_Y^2} > 0$$

Welfare loss around ($\bar{B}$,0) Following steps analogous to those in Gali and Monacelli (2005), I find that to second-order price-dispersion deviations (Δ) are given by

$$\Delta_s = \frac{\eta}{2}[\phi(-p_{Is})^2 + (1-\phi)(p_{Is}^{flex} - p_{Is})^2] + O(\epsilon^3).$$

Using (37), I can rewrite this as

$$\Delta_s = \frac{\eta}{2}\frac{\phi}{1-\phi}p_{Is}^2 + O(\epsilon^3). \tag{39}$$

To simplify, I assume taste shocks take the form $\exp(\iota_s)u(C_{Ts}, C_{Ns}, L_s)$. Using (39), I find that a second-order approximation to the utility flow yields

$$u_s = u_T C_{Tss}c_{Ts} + \frac{1}{2}(u_{TT}C_{Tss}^2 + u_T C_{Tss})c_{Ts}^2 + u_{TN}F_Z Z_{ss}z_s c_{Ts} + (u_{TN}F_Y + u_{TL})C_{Tss}L_{ss}c_{Ts}l_s + \iota_s u_T C_{ss}c_{Ts} \\ + (u_N F_{YZ} + u_{NN}F_Y F_Z + u_{NL}F_Z)Z_{ss}L_{ss}z_s l_s + \frac{1}{2}(u_N F_{YY} + u_{NN}F_Y^2 + 2u_{NL}F_Y + u_{LL})L_{ss}^2 l_s^2 \\ + u_L L_{ss}\frac{\eta}{2}\frac{\phi}{1-\phi}p_{Is}^2 + t.i.p. + O(\epsilon^3) \tag{40}$$

Define:

$$\lambda_\pi \equiv -u_L L_{ss}\eta\frac{\phi}{1-\phi} > 0$$
$$V_{11} \equiv u_{TT} - F^2 F_Y^{-2}\lambda_x^{-1}(F_Y u_{TN} + u_{TL})^2 < 0$$
$$V_{1Z} \equiv u_{TN}F_Z - F^2 F_Y^{-2}\lambda_x^{-1}(F_Y u_{NN}F_Z + u_N F_{YZ} + u_{NL}F_Z)(F_Y u_{TN} + u_{TL}).$$

Then, using (34) I can rewrite welfare as

$$u_s = u_T C_{Tss}c_{Ts} + \iota_s u_T C_{Tss}c_{Ts} + \frac{1}{2}(V_{11}C_{Tss}^2 + u_T C_{Tss})c_{Ts}^2 + V_{1Z}Z_{ss}z_s c_{Ts} - \frac{1}{2}\lambda_x x_s^2 - \frac{1}{2}\lambda_\pi p_{Is}^2 + t.i.p. + O(\epsilon^3)$$

A second-order approximation to the budget constraint yields

$$C_{Tss}c_{Ts} + \frac{1}{2}C_{Tss}c_{Ts}^2 = Y_{Tss}y_s + \frac{1}{2}Y_{Tss}y_s^2 + \bar{B}(r-e_s) + \frac{1}{2}\bar{B}(r-e_s)^2 + (r-e_s)B_\epsilon + O(\epsilon^3)$$

A first-order approximation of the foreign Euler equation yields

$$\mathbb{E}(r-e_s) + O(\epsilon^2) = 0$$

while a second-order approximation yields

$$\mathbb{E}(r-e_s) + \frac{1}{2}\mathbb{E}(r-e_s)^2 - \mathbb{E}\gamma_{ss}^*(r-e_s)(y_s^* - \frac{1}{m}\bar{B}(r-e_s)) + O(\epsilon^3) = 0,$$

where I assumed $Y_{Tss}^* = 1$. Noting that I only need the know the first order behavior of $r - e_s$ to compute $(r-e_s)B_\epsilon$ and that B_ϵ is determined ex ante, I get $\mathbb{E}(r-e_s)B_\epsilon = 0$ so this term vanishes from the approximated welfare function. Combining the second-order approximation to the budget constraint with the second-order approximation to the foreign Euler equation (58) I obtain

$$\begin{aligned}\mathbb{E}V = \mathbb{E}[&u_T\bar{B}(r-e_s)\gamma_{ss}^*y_s^* - u_T(\gamma_{ss}^*/m)\bar{B}^2(r-e_s)^2 + u_T\iota_s C_{ss}c_{Ts}\\ &+ \frac{1}{2}V_{11}C_{Tss}^2c_{Ts}^2 + V_{1Z}Z_{ss}C_{Tss}z_sc_{Ts} - \frac{1}{2}\lambda_x x_s^2 - \frac{1}{2}\lambda_\pi p_{Is}^2] + t.i.p. + O(\epsilon^3)\end{aligned}$$

Note that in the absence of shocks, $e_s = 0$. Since shocks are mean zero, I obtain using the foreign Euler that $r = 0$. Thus, using a first-order approximation to the budget constraint and to the foreigners' Euler equation, the previous expression simplifies to

$$\begin{aligned}\mathbb{E}V = \mathbb{E}[&u_T\bar{B}e_s\gamma_{ss}^*y_s^* - u_T(\gamma_{ss}^*/m)\bar{B}^2e_s^2 + \frac{1}{2}V_{11}(Y_{Tss}y_s - \bar{B}e_s)^2\\ &+ (V_{1Z}Z_{ss}z_s + u_T\iota_s)(Y_{Tss}y_s - \bar{B}e_s) - \frac{1}{2}\lambda_x x_s^2 - \frac{1}{2}\lambda_\pi p_{Is}^2] + t.i.p. + O(\epsilon^3)\end{aligned} \tag{41}$$

Define the "desired transfer" $\mathcal{T}_s$ as

$$\mathcal{T}_s = \frac{1}{1 - 2m^{-1}u_TV_{11}^{-1}\gamma_{ss}^*}\{-Y_{Tss}y_s - V_{11}^{-1}V_{1Z}Z_{ss}z_s - V_{11}^{-1}u_T\iota_s + V_{11}^{-1}u_T\gamma_{ss}^*y_s^*\}.$$

Then, I can rewrite (41) as

$$\mathbb{E}V = \mathbb{E}[\frac{1}{2}(1 - 2m^{-1}V_{11}^{-1}\gamma_{ss}^*)V_{11}(\bar{B}e_s + \mathcal{T}_s)^2 - \frac{1}{2}\lambda_x x_s^2 - \frac{1}{2}\lambda_\pi p_{Is}^2] + O(\epsilon^3) + t.i.p. \tag{42}$$

Next, I define the "demand-management" exchange rate $e_s^{dm}(B)$, which closes the output gap with no price dispersion efficiency-loss when the portfolio is $\bar{B}$,

$$e_s^{dm} = \frac{1}{1 + k_{ec}\bar{B}}(k_{ec}Y_{Tss}y_s + k_{ez}Z_{ss}z_s).$$

This implies

$$e_s - e_s^m(\bar{B}) = \frac{1}{1 + k_{ec}\bar{B}}(k_{ex}x_s + p_{Is})$$

Using this relationship, the static Phillips curve (38) and the exchange rate function (35), I can rewrite (42) as

$$\mathbb{E}V = \mathbb{E}[\frac{1}{2}(1-2u_T m^{-1}V_{11}^{-1}\gamma_{ss}^*)V_{11}(\bar{B}e_s+\mathcal{T}_s)^2 - \frac{1}{2}(1+k_{ec}\bar{B})^2(\frac{\lambda_x+\kappa^2\lambda_\pi}{(\kappa+k_{ex})^2})(e_s-e_{Bs}^m)^2] + t.i.p. + O(\epsilon^3)$$

Defining

$$\begin{aligned}
e_s^{in}(\bar{B}) &= -\frac{1}{\bar{B}}\mathcal{T}_s \\
\tilde{\gamma}_{ss} &= -u_T^{-1}V_{11} > 0 \\
\chi &= \frac{u_T^{-1}}{\tilde{\gamma}_{ss}+2m^{-1}\gamma_{ss}^*}(\frac{\lambda_x+\kappa^2\lambda_\pi}{(\kappa+k_{ex})^2}) > 0 \\
k_0 &= u_T(\tilde{\gamma}_{ss}+2m^{-1}\gamma_{ss}^*) > 0
\end{aligned}$$

I obtain the expression (18) in lemma 2.

A.1.2 Proposition 1

I first find a perturbation of the first-order conditions using a bifurcation theorem. Then, I derive our linear quadratic approximation and show it implies the same FOC for $\bar{B}$. Equivalence conditional on $\bar{B}$ is a well-known result so I do not prove it (Benigno and Woodford (2012)).

I show a more general version of the result here. To define a useful class of problems, I isolate some endogenous variables from the rest: the excess return on an asset $rr_s^j \in \mathbb{R}^S$, its expected return $R^j \in \mathbb{R}_+$ and the portfolio $\theta \in R^{J+1}$, where j indexes the asset with 0 being a reference asset. The root of the indeterminacy at the steady state comes from two conditions, which I will assume hold in our problem: First, I assume the portfolio by itself has **no direct effect** on utility or the constraints: its only effect is indirect through the transfers it creates $rr_s'\theta$. Second, there is a **no-arbitrage** constraint on each asset that implies that in the steady state $rr_s^j = 0\,\forall j, s$. Let π denote the objective function, F denote other constraints on the problem, X^j denote the determinants of the equilibrium excess returns of asset j and M determine the stochastic discount factor of the agent that prices the asset. All functions are assumed to be locally analytic, which is necessary to apply the bifurcation theorem in Judd and Guu (2001). The class of problems I consider take the following form:

$$\begin{aligned}
\max \mathbb{E}\pi(y_s, rr_s'\theta, \xi_s) &\text{ subject to} \\
F(y_s, \xi_s, rr_s'\theta) &= 0 \\
R^j X^j(y_s, \xi_s, \mathcal{T}_s) - X^0(y_s, \xi_s, rr_s'\theta) - rr_s^j &= 0\,\forall j \\
\mathbb{E}_0 rr_s^j M(y_s, \xi_s, rr_s\theta) &= 0\,\forall j.
\end{aligned}$$

where y_s are other endogenous variables, and ξ_s are shocks,

$$\xi_s = \epsilon u_s$$

where $\{u_s\}$ is a random variable with compact support. For example, in our problem above, $\pi = \iota_s V(C_{Ts}, E_s^{-1})$, $y_s = \{C_{Ts}, E_s^{-1}\}$, $\xi_s = \{\iota_s, Z_s, Y_s, Y_s^*\}$,$\theta = B$, $X^0 = 1$, $X^1 = E_s^{-1}$, $F = Y_{Ts} - C_{Ts} + Brr_s$,

$M = u'^*(Y_s^* - Brr_s)$. Naturally, I could also have written the problem including more equilibrium objects and more constraints F.

Let λ_s^k denote the multiplier on constraint F^k, $\mu\theta$ denote the multiplier on the definition of excess returns, and $\varphi\theta$ denote the multiplier on the no-arbitrage conditions. Following Benigno and Woodford (2012), I derive the following quadratic expansion to the objective with respect to ϵ around an arbitrary steady-state portfolio θ_0

$$\begin{aligned}\mathbb{E}_0\pi(y_s,\xi_s,rr_s\theta) = {} & \frac{1}{2}\mathbb{E}_0\sum\lambda_s^k\{\tilde{y}_s' D_{yy}F\cdot\tilde{y}_s + 2\tilde{\xi}_s' D_{y\xi}F\cdot\tilde{y}_s + 2\theta_0\tilde{rr}_s D_{y\mathcal{T}}F\cdot\tilde{y}_s \\ & + 2\theta_0\tilde{rr}_s D_{\xi\mathcal{T}}F\cdot\tilde{\xi}_s + \theta_0'\tilde{rr}_s D_{\mathcal{T}\mathcal{T}}F\cdot\tilde{rr}_s'\theta_0\} \\ & + \frac{1}{2}\theta_0\varphi\mathbb{E}_0\{\tilde{rr}_s D_y M\cdot\tilde{y}_s + \tilde{rr}_s D_\xi M\cdot\tilde{\xi}_s + 2\theta_0\tilde{rr}_s D_{\mathcal{T}}M\cdot\tilde{rr}_s'\} \\ & + \frac{1}{2}\mathbb{E}_0\{\tilde{y}_s' D_{yy}\pi\cdot\tilde{y}_s + 2\tilde{\xi}_s' D_{y\xi}\pi\cdot\tilde{y}_s + 2\theta_0'\tilde{rr}_s D_{y\mathcal{T}}\pi\cdot\tilde{y}_s \\ & + 2\theta_0'\tilde{rr}_s D_{\xi\mathcal{T}}\pi\cdot\tilde{\xi}_s + \theta_0'\tilde{rr}_s D_{\mathcal{T}\mathcal{T}}\cdot\pi\cdot\tilde{rr}_s'\theta_0\} + t.i.p. + O(\epsilon^3)\end{aligned} \tag{43}$$

and a linear approximation to the constraints,

$$D_y F\cdot\tilde{y}_s + D_\xi F\cdot\tilde{\xi}_s + \theta_0' D_{\mathcal{T}}F\tilde{rr}_s = O(\epsilon^2) \tag{44}$$

$$\mathbb{E}_0\tilde{rr}_s^j = O(\epsilon^2) \tag{45}$$

$$R^j D_y X\cdot\tilde{y} + R^j D_{\tilde{\xi}} X\cdot\tilde{\xi}_s + \theta_0^j R^j D_{\mathcal{T}}X\cdot\tilde{rr}_s^j - \tilde{rr}_s^j = O(\epsilon^2) \tag{46}$$

where $\tilde{x} = x - \bar{x}$ with bars denoting steady-state values. In the context of our particular model, applying this procedure and substituting in the constraints yields (18). The next theorem states that solving this problem yields a correct linear-quadratic approximation, in the sense that it yields a linear-approximation to optimal policy in terms of (y_s, R, rr_s) and a bifurcation point θ_0. To prove the result, I show that using the first-order conditions of the nonlinear problem and then using a bifurcation theorem yields the same answer.

Using bifurcation theorem The problem is

$$\max\mathbb{E}_0\pi(y_s,\xi_s,rr_s\theta)$$

subject to:

$$\begin{aligned}F(y_s,\xi_s,rr_s\theta) &= 0 \\ RX(y_s,\xi_s,rr_s\theta) - 1 - rr_s &= 0 \\ \mathbb{E}_0 rr_s g(y_s,\xi_s,rr_s\theta) &= 0\end{aligned}$$

Let $\varphi\theta$ denote the multiplier on the last equation and $\mu_s\theta$ denote the mulitplier on the second equation. The FOC yield

$$\begin{aligned}
D_y \pi(y_s, \xi_s, rr_s\theta) + \lambda_s' D_y F(y_s, \xi_s, rr_s\theta) + \theta\varphi rr_s D_y g(y_s, \xi_s, rr_s\theta)& \\
-R\mu_s D_y X(y_s, \xi_s, rr_s\theta) &= 0 \\
\mathbb{E}_0 \theta\mu_s X(y_s, \xi_s, rr_s\theta) &= 0 \\
\{D_{\mathcal{T}} \pi(y_s, \xi_s, rr_s\theta) + \lambda_s' D_{\mathcal{T}} F(y_s, \xi_s, rr_s\theta) + \theta\varphi rr_s D_{\mathcal{T}} g(y_s, \xi_s, rr_s\theta)& \\
+R\theta\mu_s D_{\mathcal{T}} X(y_s, \xi_s, rr_s\theta)\} + \mu_s + \varphi g(y_s, \xi_s, rr_s\theta) &= 0 \\
F(y_s, \xi_s, rr_s\theta) &= 0 \\
rr_s - (RX(y_s, \xi_s, rr_s\theta) - 1) &= 0 \\
\mathbb{E}_0 rr_s g(y_s, \xi_s, rr_s\theta) &= 0 \\
\mathbb{E}_0[rr_s'\{D_{\mathcal{T}} \pi(y_s, \xi_s, rr_s\theta) + \lambda_s' D_{\mathcal{T}} F(y_s, \xi_s, rr_s\theta) + R\theta\mu_s D_{\mathcal{T}} X& \\
+\varphi\theta rr_s D_{\mathcal{T}} g(y_s, \xi_s, rr_s\theta)\}] &= 0
\end{aligned}$$

Clearly at the steady state $rr = 0$ and $\mu = 0$. This implies that at the steady state,

$$\{D_{\mathcal{T}} \pi(y_s, \xi_s, rr_s\theta) + \lambda_s' D_{\mathcal{T}} F(y_s, \xi_s, rr_s\theta)\} = -\varphi g(y_s, \xi_s, rr_s\theta).$$

I can rewrite the last equation as

$$\mathbb{E}_0[rr_s'\{D_{\mathcal{T}} \pi(y_s, \xi_s, rr_s\theta) + \lambda_s' D_{\mathcal{T}} F(y_s, \xi_s, rr_s\theta) + \varphi\theta rr_s D_{\mathcal{T}} g(y_s, \xi_s, rr_s\theta) + R\theta\mu_s rr_s D_{\mathcal{T}} X + \varphi g(y_s, \xi_s, rr_s\theta)\}] = 0.$$

Furthermore, I can apply the IFT on the first five equations to obtain $y_s(\theta, \epsilon)$, $x(\theta, \epsilon)$, $rr_s(\theta, \epsilon)$, $\lambda(\theta, \epsilon)$, $\varphi(\theta, \epsilon)$. It is easy to check the first derivative of these objects with respect to θ is zero (i.e. the steady state values of the other variables does not depend on θ).

Let

$$\begin{aligned}
H(\theta, \epsilon) \equiv \mathbb{E}_0[rr_s\{&D_{\mathcal{T}} \pi(y_s(\theta, \epsilon), \xi_s(\epsilon), rr_s(\theta, \epsilon)\theta) + \lambda_s'(\theta, \epsilon) D_{\mathcal{T}} F(y_s(\theta, \epsilon), \xi_s(\epsilon), rr_s(\theta, \epsilon)\theta) \\
&+ \varphi(\theta, \epsilon)\theta rr_s(\theta, \epsilon) D_{\mathcal{T}} g(y_s(\theta, \epsilon), \xi_s(\epsilon), rr_s(\theta, \epsilon)\theta) \\
&+ \theta R(\theta, \epsilon)\mu_s(\theta, \epsilon) D_{\mathcal{T}} X + \varphi(\theta, \epsilon) g(y_s(\theta, \epsilon), \xi_s(\epsilon), rr_s(\theta, \epsilon)\theta)\}]
\end{aligned}$$

First, I show there is a singularity. Note:

$$\begin{aligned}
\frac{\partial H}{\partial \theta} &= \mathbb{E}_0(\frac{\partial rr_s}{\partial \theta})\{D_{\mathcal{T}} \pi + \lambda_s' D_{\mathcal{T}} F + \varphi\theta rr_s D_{\mathcal{T}} g + R\theta\mu_s D_{\mathcal{T}} X + \varphi g\} \\
&+ rr_s\{(D_{\mathcal{T}y} + \lambda_s' D_{\mathcal{T}y} F + \varphi\theta rr_s D_{\mathcal{T}y} g + R\theta\mu_s D_{\mathcal{T}y} X + \varphi D_y g)\frac{\partial y}{\partial \theta} + \theta\mu_s D_{\mathcal{T}y} X \frac{\partial R}{\partial \theta} \\
&+ (\theta D_{\mathcal{T}\mathcal{T}} + \theta\lambda_s' D_{\mathcal{T}\mathcal{T}} F + \varphi\theta^2 rr_s D_{\mathcal{T}\mathcal{T}} g + \varphi\theta D_{\mathcal{T}} g + \theta R\mu_s D_{\mathcal{T}\mathcal{T}} X + \theta\varphi D_{\mathcal{T}} g)\frac{\partial rr_s}{\partial \theta} \\
&+ \frac{\partial \lambda_s'}{\partial \theta} D_{\mathcal{T}} F + \varphi rr_s D_{\mathcal{T}} g + \mu_s rr_s D_{\mathcal{T}} X + (rr_s D_{\mathcal{T}} g + g)\frac{\partial \varphi}{\partial \theta} + R D_{\mathcal{T}} X \frac{\partial \mu_s}{\partial \theta}\}
\end{aligned}$$

Ignoring terms preceded by $\{D_{\mathcal{T}} \pi + \lambda_s' D_{\mathcal{T}} F + \varphi\theta rr_s D_{\mathcal{T}} g + R\theta\mu_s rr_s D_{\mathcal{T}} X + \varphi g\}$, μ_s and rr_s, which will be zero anyway at the steady state. and using that the derivatives wrt θ are also zero, this immediately yields $\frac{\partial^2 H}{\partial\theta\partial\epsilon} = 0$ so I have a singularity.

I use the "dividing by ϵ trick" to solve the singularity,

$$\hat{H}(\theta,\epsilon) = \begin{Bmatrix} \frac{H(\theta,\epsilon)}{\epsilon} \text{ if } \epsilon \neq 0 \\ \frac{\partial H}{\partial \epsilon} \text{ if } \epsilon = 0 \end{Bmatrix}$$

Since $H(\theta, 0) = 0 \; \forall \theta$, $H = \epsilon \hat{H}$. Then I can rewrite the old FOC equation as

$$\mathbb{E}_0 \hat{H}(\theta,\epsilon) = 0.$$

To solve this, I use the following bifurcation theorem in Judd and Guu (2001),

Theorem 1. *(Bifurcation Theorem). Suppose $H : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, H is analytic for (x,ϵ) in a neighborhood of $(x_0, 0)$, and $H(x, 0) = 0 \; \forall x \in \mathbb{R}$. Furthermore, suppose that*

$$H_x(x_0, 0) = 0 = H_\epsilon(x_0, 0), H_{x\epsilon} \neq 0.$$

Then $(x_0, 0)$ is a bifurcation point and there is an open neighborhood $\mathcal{N}$ of $(x_0, 0)$ and a function $h(\epsilon)$, $h(\epsilon) \neq 0$ for $\epsilon \neq 0$, such that h is analytic and $H(h(\epsilon),\epsilon) = 0$ for $(h(\epsilon),\epsilon) \in \mathcal{N}$.

Let's check the conditions for the bifurcation theorem. First, note that to compute $\frac{\partial \hat{H}}{\partial B}$ at $\epsilon = 0$, I need to compute $\frac{\partial^2 H}{\partial \epsilon \partial B}$, which I already showed is zero. In addition, it is clear that if I take another derivative wrt ϵ the answer will not be zero. I just need to find the new bifurcation points, which I find by solving $\frac{\partial \hat{H}}{\partial \epsilon}|_0 = \frac{\partial^2 H}{\partial \epsilon^2}|_0$:

$$\begin{aligned}
\frac{\partial H}{\partial \epsilon} &= \mathbb{E}_0(\frac{\partial rr_s}{\partial \epsilon})\{D_\mathcal{T}\pi + \lambda_s' D_\mathcal{T} F + \varphi\theta rr_s D_\mathcal{T} g + R\theta\mu_s D_\mathcal{T} X + \varphi g\} \\
&+ rr_s\{(D_{\mathcal{T}y} + \lambda_s' D_{\mathcal{T}y} F + \varphi\theta rr_s D_{\mathcal{T}y} g + R\theta\mu_s D_{\mathcal{T}y} X + \varphi D_y g)\frac{\partial y}{\partial \epsilon} + \mu_s D_{\mathcal{T}y} X \frac{\partial R}{\partial \epsilon} \\
&+ (D_{\mathcal{T}\xi} + \lambda_s' D_{\mathcal{T}\xi} F + \varphi\theta rr_s D_{\mathcal{T}\xi} g + R\theta\mu_s D_{\mathcal{T}\xi} X + \varphi D_\xi g)\frac{\partial \xi}{\partial \epsilon} \\
&+ \theta(D_{\mathcal{T}\mathcal{T}} + \lambda_s' D_{\mathcal{T}\mathcal{T}} F + \varphi\theta rr_s D_{\mathcal{T}\mathcal{T}} g + \varphi D_\mathcal{T} g + \theta R\mu_s D_{\mathcal{T}\mathcal{T}} X + \varphi D_\mathcal{T} g)\frac{\partial rr_s}{\partial \epsilon} \\
&+ D_\mathcal{T} F \frac{\partial \lambda_s'}{\partial \epsilon} + (rr_s D_\mathcal{T} g + g)\frac{\partial \varphi}{\partial \epsilon} + R D_\mathcal{T} X \frac{\partial \mu_s}{\partial \epsilon}\}
\end{aligned}$$

Ignoring the terms preceded by rr_s and $\{D_\mathcal{T}\pi + \lambda_s' D_\mathcal{T} F + \varphi\theta rr_s D_\mathcal{T} g + \varphi g\}$ which are zero at the steady state, I compute $\frac{\partial^2 H}{\partial \epsilon^2}$,

$$\begin{aligned}
\frac{\partial^2 H}{\partial \epsilon^2} &= 2\mathbb{E}_0[\frac{\partial rr_s}{\partial \epsilon}\{(D_{\mathcal{T}y} + \lambda_s' D_{\mathcal{T}y} F + \varphi\theta rr_s D_{\mathcal{T}y} g + R\theta\mu_s D_{\mathcal{T}y} X + \varphi D_y g)\frac{\partial y}{\partial \epsilon} + \mu_s D_{\mathcal{T}y} X \frac{\partial R}{\partial \epsilon} \\
&+ (D_{\mathcal{T}\xi} + \lambda_s' D_{\mathcal{T}\xi} F + \varphi\theta rr_s D_{\mathcal{T}\xi} g + R\theta\mu_s D_{\mathcal{T}\xi} X + \varphi D_\xi g)\frac{\partial \xi}{\partial \epsilon} \\
&+ \theta(D_{\mathcal{T}\mathcal{T}} + \lambda_s' D_{\mathcal{T}\mathcal{T}} F + \varphi\theta rr_s D_{\mathcal{T}\mathcal{T}} g + \varphi D_\mathcal{T} g + R\mu_s D_\mathcal{T} X + \theta R\mu_s D_{\mathcal{T}\mathcal{T}} X + \varphi D_\mathcal{T} g)\frac{\partial rr_s}{\partial \epsilon} \\
&+ D_\mathcal{T} F \frac{\partial \lambda_s'}{\partial \epsilon} + (rr_s D_\mathcal{T} g + g)\frac{\partial \varphi}{\partial \epsilon} + R D_\mathcal{T} X \frac{\partial \mu_s}{\partial \epsilon}\}
\end{aligned}$$

Noting that $rr_s = 0$ and $\mu_s = 0$ at the steady state,

$$\begin{aligned}\frac{\partial^2 H}{\partial \epsilon^2} &= 2\mathbb{E}_0[\frac{\partial rr_s}{\partial \epsilon}\{(D_{\mathcal{T}y} + \lambda_s' D_{\mathcal{T}y}F + \varphi D_y g)\frac{\partial y}{\partial \epsilon} \\ &+ (D_{\mathcal{T}\xi} + \lambda_s' D_{\mathcal{T}\xi}F + \varphi D_\xi g)\frac{\partial \xi}{\partial \epsilon} \\ &+ \theta(D_{\mathcal{T}\mathcal{T}} + \lambda_s' D_{\mathcal{T}\mathcal{T}}F + 2\varphi D_{\mathcal{T}} g)\frac{\partial rr_s}{\partial \epsilon} \\ &+ D_{\mathcal{T}}F\frac{\partial \lambda_s'}{\partial \epsilon} + g\frac{\partial \varphi}{\partial \epsilon} + RD_{\mathcal{T}}X\frac{\partial \mu_s}{\partial \epsilon}\}\end{aligned}$$

Welfare expansion approach Starting with the objective function,

$$\begin{aligned}\mathbb{E}_0\pi(y_s, \xi_s, rr_s\theta) &= \bar{\pi} + \mathbb{E}_0\{D_y\pi \cdot \tilde{y}_s + D_\xi \pi \cdot \tilde{\xi}_s + \theta D_{\mathcal{T}} \cdot \pi \cdot \tilde{r}r_s + rr_s D_{\mathcal{T}}\pi \cdot \tilde{\theta} \\ &+ \frac{1}{2}\mathbb{E}_0\{\tilde{y}_s' D_{yy}\pi \cdot \tilde{y}_s + 2\tilde{\xi}_s' D_{y\xi}\pi \cdot \tilde{y}_s + 2\theta \tilde{r}r_s D_{y\mathcal{T}}\pi \cdot \tilde{y}_s + 2rr_s\tilde{\theta}D_{y\mathcal{T}} \cdot \tilde{y}_s \\ &+ \tilde{\xi}_s' D_\xi \pi \cdot \tilde{\xi}_s + 2\theta \tilde{r}r_s D_{\xi\mathcal{T}}\pi \cdot \tilde{\xi}_s + 2rr_s\tilde{\theta}D_{\mathcal{T}} \cdot \pi D_{\xi\mathcal{T}}\pi \cdot \tilde{\xi}_s \\ &+ \theta^2 \tilde{r}r_s D_{\mathcal{T}\mathcal{T}} \cdot \pi \cdot \tilde{r}r_s + 2\tilde{\theta}D_{\mathcal{T}} \cdot \pi \cdot \tilde{r}r_s + \tilde{\theta}rr_s D_{\mathcal{T}\mathcal{T}} \cdot \pi \cdot rr_s\tilde{\theta}\} + O(\epsilon^3)\end{aligned}$$

Using that $rr_s = 0$,

$$\begin{aligned}\mathbb{E}_0\pi(y_s, \xi_s, rr_s\theta) &= \mathbb{E}_0\{D_y\pi \cdot \tilde{y}_s + \theta D_{\mathcal{T}} \cdot \pi \cdot \tilde{r}r_s \\ &+ \frac{1}{2}\mathbb{E}_0\{\tilde{y}_s' D_{yy}\pi \cdot \tilde{y}_s + 2\tilde{\xi}_s' D_{y\xi}\pi \cdot \tilde{y}_s + 2\theta \tilde{r}r_s D_{y\mathcal{T}}\pi \cdot \tilde{y}_s \\ &+ 2\theta \tilde{r}r_s D_{\xi\mathcal{T}}\pi \cdot \tilde{\xi}_s + \theta^2 \tilde{r}r_s D_{\mathcal{T}\mathcal{T}} \cdot \pi \cdot \tilde{r}r_s + 2\tilde{\theta}D_{\mathcal{T}} \cdot \pi \cdot \tilde{r}r_s\} + O(\epsilon^3) + tip\end{aligned}$$

Next, I approximate the F constraints,

$$\begin{aligned}\mathbb{E}_0 F(y_s, \xi_s, rr_s\theta) &= \mathbb{E}_0\{D_y F \cdot \tilde{y}_s + D_\xi F \cdot \tilde{\xi}_s + \theta D_{\mathcal{T}}F \cdot \tilde{r}r_s \\ &+ \frac{1}{2}\mathbb{E}_0\{\tilde{y}_s' D_{yy}F \cdot \tilde{y}_s + 2\tilde{\xi}_s' D_{y\xi}F \cdot \tilde{y}_s + 2\theta \tilde{r}r_s D_{y\mathcal{T}}F \cdot \tilde{y}_s \\ &+ \tilde{\xi}_s' D_\xi F \cdot \tilde{\xi}_s + 2\theta \tilde{r}r_s D_{\xi\mathcal{T}}F \cdot \tilde{\xi}_s \\ &+ \theta^2 \tilde{r}r_s D_{\mathcal{T}\mathcal{T}}F \cdot \tilde{r}r_s + 2\tilde{\theta}D_{\mathcal{T}}F \cdot \tilde{r}r_s\} + O(\epsilon^3)\end{aligned}$$

and the g constraint,

$$\mathbb{E}_0 rr_s g(y_s, \xi_s, rr_s\theta) = \mathbb{E}_0\{g(y_s, \xi_s, rr_s\theta)\tilde{r}r_s + \mathbb{E}_0\{\tilde{r}r_s D_y g \cdot \tilde{y}_s + \tilde{r}r_s D_\xi g \cdot \tilde{\xi}_s + \theta \tilde{r}r_s D_{\mathcal{T}}g \cdot \tilde{r}r_s\} + O(\epsilon^3)$$

Recall that at the steady state,

$$\begin{aligned}D_y\pi(y_s, \xi_s, rr_s\theta) + \lambda_s' D_y F(y_s, \xi_s, rr_s\theta) &= 0 \\ D_{\mathcal{T}}\pi(y_s, \xi_s, rr_s\theta) + \lambda_s' D_{\mathcal{T}}F(y_s, \xi_s, rr_s\theta) + \varphi g(y_s, \xi_s, rr_s\theta) &= 0\end{aligned}$$

Then,

$$D_y\pi \cdot \tilde{y}_s + \theta D_{\mathcal{T}} \cdot \pi \cdot \tilde{r}r_s = -\lambda_s' D_y F \cdot \tilde{y}_s - \theta D_{\mathcal{T}}\pi \cdot \tilde{r}r_s - \theta\lambda_s' D_{\mathcal{T}}F \cdot \tilde{r}r_s - \theta\varphi g \cdot \tilde{r}r_s$$

Using this I can get rid of the linear terms,

$$\begin{aligned}\mathbb{E}_0\pi(y_s,\xi_s,rr_s\theta) = &\frac{1}{2}\mathbb{E}_0\sum\lambda_s^k\{\tilde{y}_s'D_{yy}F\cdot\tilde{y}_s + 2\tilde{\xi}_s'D_{y\xi}F\cdot\tilde{y}_s + 2\theta\tilde{r}r_sD_{y\mathcal{T}}F\cdot\tilde{y}_s \\ &+ \tilde{\xi}_s'D_{\xi}F\cdot\tilde{\xi}_s + 2\theta\tilde{r}r_sD_{\xi\mathcal{T}}F\cdot\tilde{\xi}_s \\ &+ \theta^2\tilde{r}r_sD_{\mathcal{T}\mathcal{T}}F\cdot\tilde{r}r_s + 2\tilde{\theta}D_{\mathcal{T}}F\cdot\tilde{r}r_s\} \\ &+ \frac{1}{2}\theta\varphi\mathbb{E}_0\{\tilde{r}r_sD_yg\cdot\tilde{y}_s + \tilde{r}r_sD_{\xi}g\cdot\tilde{\xi}_s + 2\theta\tilde{r}r_sD_{\mathcal{T}}g\cdot\tilde{r}r_s\} \\ &+ \frac{1}{2}\mathbb{E}_0\{\tilde{y}_s'D_{yy}\pi\cdot\tilde{y}_s + 2\tilde{\xi}_s'D_{y\xi}\pi\cdot\tilde{y}_s + 2\theta\tilde{r}r_sD_{y\mathcal{T}}\pi\cdot\tilde{y}_s \\ &+ 2\theta\tilde{r}r_sD_{\xi\mathcal{T}}\pi\cdot\tilde{\xi}_s + \theta^2\tilde{r}r_sD_{\mathcal{T}\mathcal{T}}\cdot\pi\cdot\tilde{r}r_s + 2\tilde{\theta}D_{\mathcal{T}}\cdot\pi\cdot\tilde{r}r_s\} + tip\end{aligned}$$

Using the g constraint to first order yields $\mathbb{E}\tilde{r}r_s = 0$ and since $\tilde{\theta}$ is predetermined,

$$\begin{aligned}\mathbb{E}_0\pi(y_s,\xi_s,rr_s\theta) = &\frac{1}{2}\mathbb{E}_0\sum\lambda_s^k\{\tilde{y}_s'D_{yy}F\cdot\tilde{y}_s + 2\tilde{\xi}_s'D_{y\xi}F\cdot\tilde{y}_s + 2\theta\tilde{r}r_sD_{y\mathcal{T}}F\cdot\tilde{y}_s \\ &+ 2\theta\tilde{r}r_sD_{\xi\mathcal{T}}F\cdot\tilde{\xi}_s \\ &+ \theta^2\tilde{r}r_sD_{\mathcal{T}\mathcal{T}}F\cdot\tilde{r}r_s\} \\ &+ \frac{1}{2}\theta\varphi\mathbb{E}_0\{\tilde{r}r_sD_yg\cdot\tilde{y}_s + \tilde{r}r_sD_{\xi}g\cdot\tilde{\xi}_s + 2\theta\tilde{r}r_sD_{\mathcal{T}}g\cdot\tilde{r}r_s\} \\ &+ \frac{1}{2}\mathbb{E}_0\{\tilde{y}_s'D_{yy}\pi\cdot\tilde{y}_s + 2\tilde{\xi}_s'D_{y\xi}\pi\cdot\tilde{y}_s + 2\theta\tilde{r}r_sD_{y\mathcal{T}}\pi\cdot\tilde{y}_s \\ &+ 2\theta\tilde{r}r_sD_{\xi\mathcal{T}}\pi\cdot\tilde{\xi}_s + \theta^2\tilde{r}r_sD_{\mathcal{T}\mathcal{T}}\cdot\pi\cdot\tilde{r}r_s\} + tip\end{aligned}$$

The objective is to maximize this then subject to the first-order constraints,

$$\begin{aligned}D_yF\cdot\tilde{y}_s + D_{\xi}F\cdot\tilde{\xi}_s + \theta D_{\mathcal{T}}F\tilde{r}r_s &= 0 \\ \mathbb{E}_0\tilde{r}r_s &= 0 \\ RD_yX\cdot\tilde{y} + RD_{\tilde{\xi}}X\cdot\xi + \theta RD_{\mathcal{T}}X\cdot\tilde{r}r_s - \tilde{r}r_s &= 0\end{aligned}$$

The FOC wrt θ yields

$$\begin{aligned}FOC = \mathbb{E}_0[&\sum\lambda_s^k\{\tilde{r}r_sD_{\xi\mathcal{T}}F\cdot\tilde{\xi}_s + \theta\tilde{r}r_sD_{\mathcal{T}\mathcal{T}}F\cdot\tilde{r}r_s + \tilde{r}r_sD_{y\mathcal{T}}F\cdot\tilde{y}_s\} \\ &+ \varphi\{\tilde{r}r_sD_yg\cdot\tilde{y}_s + \tilde{r}r_sD_{\xi}g\cdot\tilde{\xi}_s + 2\theta\tilde{r}r_sD_{\mathcal{T}}g\cdot\tilde{r}r_s\} \\ &+ \{\tilde{r}r_sD_{y\mathcal{T}}\pi\cdot\tilde{y}_s + \tilde{r}r_sD_{\xi\mathcal{T}}\pi\cdot\tilde{\xi}_s + \theta\tilde{r}r_sD_{\mathcal{T}\mathcal{T}}\cdot\pi\cdot\tilde{r}r_s\} \\ &+ \tilde{\lambda}_sD_{\mathcal{T}}F\cdot\tilde{r}r_s + \tilde{\mu}_sD_{\mathcal{T}}X\cdot\tilde{r}r_s\}]\end{aligned}$$

Rearranging,

$$\begin{aligned}FOC = &\mathbb{E}_0\tilde{r}r_s\{(D_{y\mathcal{T}}\pi + \sum\lambda_s^jD_{y\mathcal{T}}F^k + D_yg)\cdot\tilde{y}_s \\ &+ (D_{\xi\mathcal{T}}\pi + \sum\lambda_s^jD_{\xi\mathcal{T}}F^k + D_{\xi}g)\cdot\tilde{\xi}_s \\ &+ \theta(\sum\lambda_s^kD_{\mathcal{T}\mathcal{T}}F + 2\varphi D_{\mathcal{T}}g + D_{\mathcal{T}\mathcal{T}}\cdot\pi + \tilde{\lambda}_sD_{\mathcal{T}}F + R\tilde{\mu}_sD_{\mathcal{T}}X)\tilde{r}r_s\},\end{aligned}$$

which coincides with the other method.

A.1.3 Proposition 2

Differentiating the expression in equation (18) with respect to e_s yields

$$\bar{B}(\bar{B}e_s + \mathcal{T}_s) + \chi(1 + \bar{B}\mu)((1 + \bar{B}\mu)e_s - e_s(0)) = 0$$

In terms of $e_s^{in}(B)$ and $e_s^{dm}(B)$,

$$(\bar{B}^2 + \chi(1 + \bar{B}\mu)^2)e_s - \bar{B}^2 e_s^{in}(\bar{B}) - \chi(1 + \bar{B}\mu)^2 e_s^{dm}(\bar{B}) = 0.$$

Rearranging yields the desired expression.

To show part (i), note

$$\kappa^2 \lambda_\pi = \tilde{k}_0 \eta \frac{1 - \phi}{\phi}$$
$$\kappa = \tilde{k}_1 \frac{1 - \phi}{\phi}$$

for some positive constants $\tilde{k}_0$ and $\tilde{k}_1$. This allows us to write

$$\chi = k'(\frac{\lambda_x + \tilde{k}_0 \eta \tilde{\phi}}{(\tilde{k}_1 \tilde{\phi} + k_{ex})^2})$$

for some positive constant k'. Differentiating χ with respect to $\tilde{\phi}$,

$$\chi = \frac{k'}{(\tilde{k}_1 \tilde{\phi} + k_{ex})^3}(-2\tilde{k}_1(\lambda_x + \tilde{k}_0 \eta \tilde{\phi}) + \tilde{k}_0 \eta (\tilde{k}_1 \tilde{\phi} + k_{ex}))$$

Since depreciations are assumed expansionary, $k_{ex} > 0$, the derivative is positive iff

$$-2\tilde{k}_1(\lambda_x + \tilde{k}_0 \eta \tilde{\phi}) + \tilde{k}_0 \eta (\tilde{k}_1 \tilde{\phi} + k_{ex}) \geq 0$$

Solving,

$$\tilde{\phi} \leq \frac{\tilde{k}_0 k_{ex} \eta - 2\tilde{k}_1 \lambda_x}{\tilde{k}_1 \tilde{k}_0 \eta}.$$

This immediately translates into a threshold in terms of $\bar{\phi}$ such that χ is increasing in ϕ if $\phi \leq \bar{\phi}$. Since $\tilde{\phi}$ increases with η, $\bar{\phi}$ decreases with η. For $\bar{\phi}$ to be meaningful, I need $\bar{\phi} < 1$ or

$$\tilde{\phi} > 0 \Leftrightarrow \eta > \tilde{k}_0^{-1} k_{ex}^{-1} 2\tilde{k}_1 \lambda_x.$$

Part (ii) follows from differentiating χ with respect to m, γ^* and $\tilde{\gamma}$.

A.1.4 Lemma 3

Let $\mathcal{W}(B) = V(B; \{e_s^{op}(B)\})$ where $e_s^{op}(B)$ is the optimal exchange rate policy. Replacing the optimal exchange rate inside $\mathbb{E}V$ and simplifying, we obtain

$$\mathcal{W}(\bar{B}) = \frac{1}{2}\frac{k_0}{\chi(1+\mu\bar{B})^2 + \bar{B}^2}\{(1+\mu\bar{B})^2\chi^2\sigma^2_{e^{dm}(0)} + \bar{B}^2\sigma^2_{\mathcal{T}} - 2\bar{B}\chi(1+\mu\bar{B})\sigma_{\mathcal{T}e^{dm}(0)}\} + t.i.p. + O(\epsilon^3) \tag{47}$$

The first derivative with respect to $\bar{B}$ yields

$$\begin{aligned}\frac{\partial\mathcal{W}(B)}{\partial\bar{B}} = \frac{k_0\chi}{(\chi(1+\mu\bar{B})^2+\bar{B}^2)^2}\{&(\sigma_{\mathcal{T}e^{dm}(0)} + \mu(\sigma^2_{\mathcal{T}} - \chi\sigma^2_{e^{dm}(0)} - \chi\mu\sigma_{\mathcal{T}e^{dm}(0)}))B^2 \\ &+ (\sigma^2_{\mathcal{T}} - \chi\sigma^2_{e^{dm}(0)} - 2\chi\mu\sigma_{\mathcal{T}e^{dm}(0)})B - \chi\sigma_{\mathcal{T}e^{dm}(0)}\}\end{aligned} \tag{48}$$

Solving the quadratic and picking the local maximum yields the result.

A.1.5 Proposition 3

It is convenient to prove these results to work directly with the expression in equation (48). For expositional reasons, I prove part (iii) first together with the remark that $B_{op}\mu > -1$.

(iii + $B < -\mu^{-1}$) First, doing algebra it can be shown that the determinant of the quadratic inside the bracket is given by $(\sigma^2_{\mathcal{T}} - \chi\sigma^2_{e^{dm}(0)})^2 + 4\chi\sigma^2_{\mathcal{T}e^{dm}(0)} > 0$.

I prove the case of $\mu < 0$. The case $\mu > 0$ is analogous. I need to consider several subcases. Consider first the case $\sigma_{\mathcal{T}e^{dm}(0)} > 0$. If $\sigma^{-2}_{e^{dm}(0)}\sigma^2_{\mathcal{T}} < \chi + (\chi\mu - \mu^{-1})\sigma^{-2}_{e^{dm}(0)}\sigma_{\mathcal{T}e^{dm}(0)}$, the quadratic is convex. Since the intercept is negative, this implies $B_{op} < 0$. Next, suppose $\sigma^{-2}_{e^{dm}(0)}\sigma^2_{\mathcal{T}} > \chi + (\chi\mu - \mu^{-1})\sigma^{-2}_{e^{dm}(0)}\sigma_{\mathcal{T}e^{dm}(0)}$. Since $\mu < 0$, $\sigma^{-2}_{e^{dm}(0)}\sigma^2_{\mathcal{T}} > \chi + 2\chi\mu\sigma^{-2}_{e^{dm}(0)}\sigma_{\mathcal{T}e^{dm}(0)}$. Thus, the quadratic is concave and increasing at $B = 0$. Since the intercept is negative, this implies $B_{op} > 0$. I call this the "reversal" case since $sign(\sigma_{\mathcal{T}e^{dm}(0)}) = sign(B_{op})$. The condition in (22) rules this case out.

Next, consider the case $\sigma_{\mathcal{T}e^{dm}(0)} < 0$. If $\sigma^{-2}_{e^{dm}(0)}\sigma^2_{\mathcal{T}} > \chi + (\chi\mu - \mu^{-1})\sigma^{-2}_{e^{dm}(0)}\sigma_{\mathcal{T}e^{dm}(0)}$, then the quadratic is concave. Since the intercept is positive, this implies $B_{op} > 0$. Next, suppose $\sigma^{-2}_{e^{dm}(0)}\sigma^2_{\mathcal{T}} < \chi + (\chi\mu - \mu^{-1})\sigma^{-2}_{e^{dm}(0)}\sigma_{\mathcal{T}e^{dm}(0)}$. Since $\sigma_{\mathcal{T}e^{dm}(0)} < 0$ and $\mu < 0$, $\sigma^{-2}_{e^{dm}(0)}\sigma^2_{\mathcal{T}} < \chi + 2\chi\mu\sigma^{-2}_{e^{dm}(0)}\sigma_{\mathcal{T}e^{dm}(0)}$. Thus, the quadratic is convex and decreasing at $B = 0$. Since the intercept is positive, $B_{op} > 0$. Note there is no "reversal" case when $sign(\mu) = sign(\sigma_{\mathcal{T}e^{dm}(0)})$.

Finally, I show that $1 + \mu B_{op} > 0$ under the condition in (22). I prove the case of $\mu < 0$; the case $\mu > 0$ is analogous. I only need to show this for the case $\sigma_{\mathcal{T}e^{dm}(0)} < 0$, since $B_{op} < 0$ in the admissible range for $\sigma_{\mathcal{T}e^{dm}(0)} > 0$ under the condition in (22). Evaluating (48) at $B = -\mu^{-1}$ yields

$$\mathcal{W}'(-\mu^{-1}) = (-V_{11})\chi\mu^2\sigma_{\mathcal{T}e^{dm}(0)} < 0$$

Since the intercept is positive, it follows that $B_{op} \in (0, -\mu^{-1})$.

(i) I apply the implicit function theorem. Note:

$$\frac{\partial B}{\partial\sigma^2_{\mathcal{T}}\sigma^{-2}_{e^{dm}(0)}} \propto \frac{(\mu B_{op} + 1)B_{op}}{(-\mathcal{W}''(B))}$$

Since under the condition in (22), $sign(-\sigma_{\mathcal{T}e^{dm}(0)}) = sign(B)$ and $1 + \mu B_{op} > 0$, $|B_{op}|$ increases with

$\sigma^2_{\mathcal{T}}\sigma^{-2}_{e^{dm}(0)}$.

With respect to χ,

$$\frac{\partial B}{\partial \chi} \propto \frac{-(1+\mu B_{op})B_{op} + (1+\mu B_{op})^2 \sigma_{\mathcal{T}e^{dm}(0)}}{(-\mathcal{W}''(B))}.$$

Since under the condition in (22), $sign(-\sigma_{\mathcal{T}^\infty e_{B=0}}) = sign(B)$ and $1+\mu B_{op} > 0$, $|B_{op}|$ decreases with χ.

(ii) By the implicit function theorem, and given the condition in (22) I can write

$$\frac{\partial |B|}{\partial |\sigma_{\mathcal{T}e^{dm}(0)}|} \propto -\frac{(1+\mu B_{op})|B_{op}|}{(-\mathcal{W}''(B))}(\sigma^{-2}_{e^{dm}(0)}\sigma^2_{\mathcal{T}} - \chi)$$

Thus $|B_{op}|$ increases (decreases) with $|\sigma_{\mathcal{T}e^{dm}(0)}|$ if $\sigma^{-2}_{e^{dm}(0)}\sigma^2_{\mathcal{T}} < (>)\chi$.

(iv) Define

$$\mathcal{T}^\infty_s = (1+2m^{-1}\tilde{\gamma}^{-1}\gamma^*_{ss})\mathcal{T}_s,$$
$$\chi^\infty = (1+2m^{-1}\tilde{\gamma}^{-1}\gamma^*_{ss})\chi,$$

Then

$$\begin{aligned}\mathcal{W}'(B) \propto \{&((1+2m^{-1}\tilde{\gamma}^{-1}\gamma^*_{ss})\sigma_{\mathcal{T}^\infty e^{dm}(0)} + \mu(\sigma^2_{\mathcal{T}^\infty} - (1+2m^{-1}\tilde{\gamma}^{-1}\gamma^*_{ss})\chi^\infty\sigma^2_{e^{dm}(0)} - (\chi^\infty\mu\sigma_{\mathcal{T}^\infty e^{dm}(0)})))B^2_{op}\\ &+ (\sigma^2_{\mathcal{T}^\infty} - (1+2m^{-1}\tilde{\gamma}^{-1}\gamma^*_{ss})\chi^\infty\sigma^2_{e^{dm}(0)} - 2\chi^\infty\mu\sigma_{\mathcal{T}^\infty e^{dm}(0)})B - \chi^\infty\sigma_{\mathcal{T}^\infty e^{dm}(0)}\}\end{aligned}$$

Then, using the implicit function theorem and

$$\frac{\partial B}{\partial m} \propto \frac{-\sigma_{\mathcal{T}^\infty e^{dm}(0)}B^2_{op} + (1+B_{op}\mu)B_{op}\chi^\infty_s\sigma^2_{e^{dm}(0)}}{(-\mathcal{W}''(B_{op}))}$$

Since under the condition in (22), $sign(-\sigma_{\mathcal{T}^\infty e^{dm}(0)}) = sign(B_{op})$ and $1+\mu B_{op} > 0$, $\frac{\partial B}{\partial m}$ decreases with m if $B_{op} < 0$ and increases with m when $B_{op} > 0$.

Finally, we show that if the condition in equation (22) does not hold, the result still holds in terms of the sensitivity $f(\bar{B})$. To see this, note that welfare can be written as

$$\mathcal{W}(f(\bar{B})) = \frac{1}{2}\frac{k_0}{\chi + f(\bar{B})^2}\{\chi^2\sigma^2_{e^{dm}(0)} + f(\bar{B})^2\sigma^2_{\mathcal{T}} - 2f(\bar{B})\chi\sigma_{\mathcal{T}e^{dm}(0)}\} + t.i.p. + O(\epsilon^3) \tag{49}$$

This is isomorphic to the previous problem with $\mu = 0$, which satisfies the condition in (22).

A.1.6 Corollary 1

The weight clearly decreases with χ conditional on $f(\bar{B})$. Furthermore, proposition 3 implies $f(\bar{B})$ decreases with χ and increases with $\sigma^2_{\mathcal{T}}/\sigma^2_{e^{dm}(0)}$ and ω increases in $f(\bar{B})$. These observations immediately imply the first two results.

The last observation regarding m requires more work. Using $\chi = h(m)\chi^\infty$ as in the proof of proposition 3, one may write the insurance weight as

$$\omega = \frac{h(m)^{-1}f(B)^2}{h(m)^{-1}f(B)^2 + \chi^\infty} = \frac{\tilde{B}^2}{\tilde{B}^2 + \chi^\infty},$$

where $\tilde{B} = (\sqrt{h(m)})^{-1} f(B)$. Using $\mathcal{T} = h(m)\mathcal{T}^{\infty}$, the first derivative with respect to the sensitivity $f(B)$ yields

$$\frac{\partial \mathcal{W}(f(B))}{\partial f(B)} = \frac{k_0 \chi h(m)^2}{(\chi + f(B)^2)^2} \{\sigma_{\mathcal{T}^{\infty} e^{dm}(0)} \tilde{B}^2 + (h(m)^{\frac{1}{2}} \sigma_{\mathcal{T}}^2 - h(m)^{-\frac{1}{2}} \chi^{\infty} \sigma_{e^{dm}(0)}^2) \tilde{B} - \chi^{\infty} \sigma_{\mathcal{T}^{\infty} e^{dm}(0)}\}$$

Thus,

$$\frac{\partial \mathcal{W}(f(B))}{\partial m} = \frac{1}{2} h'(m) \tilde{B} \{h(m)^{-\frac{1}{2}} \sigma_{\mathcal{T}}^2 + 3\chi^{\infty} \sigma_{e^{dm}(0)}^2\}.$$

Recall $h'(m) > 0$, so by the implicit function theorem: $\frac{\partial \tilde{B}}{\partial m} > 0$ if $\tilde{B} > 0$ and $\frac{\partial \tilde{B}}{\partial m} < 0$ if $\tilde{B} < 0$. Thus, $\frac{\partial \omega}{\partial m} > 0$.

A.1.7 Proposition 4

i. Using (20), I see that the variance of the optimal exchange rate is given by

$$\sigma_e^2 = \frac{1}{(\chi(1 + \mu B_{op})^2 + B_{op}^2)^2} (\chi^2 (1 + \mu B_{op})^4 \sigma_{e^{dm}(B_{op})}^2 + B_{op}^2 \sigma_{\mathcal{T}}^2 - 2B_{op} \chi (1 + \mu B_{op})^2 \sigma_{\mathcal{T} e^{dm}(B_{op})}).$$

Note that $e^{dm}(B_{op})$ is the exchange rate policy under a demand-management policy if agents are holding $\bar{B} = B_{op}$ (rather than the optimal portfolio corresponding to the demand-management policy $\bar{B} = B_{dm}$). Using that $e^{dm}(\bar{B}) = (1 + \mu \bar{B})^{-1} e^{dm}(0)$, I can rewrite this as

$$\sigma_e^2 / \sigma_{e^{dm}(0)}^2 = \frac{1}{(\chi(1 + \mu B_{op})^2 + B_{op}^2)^2} (\chi^2 (1 + \mu B_{op})^2 + B_{op}^2 (\sigma_{\mathcal{T}}^2 / \sigma_{e^{dm}(0)}^2) - 2B_{op} \chi (1 + \mu B_{op}) (\sigma_{\mathcal{T} e^{dm}(0)} / \sigma_{e^{dm}(0)}^2))$$

Clearly, if $B_{op} = \bar{K}$, an increase in $\sigma_{\mathcal{T}}^2 / \sigma_{e_{B=0}}^2$ leads to higher volatility, $\sigma_e^2 / \sigma_{e^{dm}(0)}^2$.

The derivative with respect to χ yields

$$\frac{\sigma_e^2}{\sigma_{e^{dm}(0)}^2} = -\frac{2B_{op}(1 + \mu B_{op})}{(\chi(1 + \mu B_{op})^2 + B_{op}^2)^3} \{B_{op}^2 (\sigma_{\mathcal{T} e^{dm}(0)} / \sigma_{e^{dm}(0)}^2) + B_{op}(1 + \mu B_{op})((\sigma_{\mathcal{T}}^2 / \sigma_{e^{dm}(0)}^2) - \chi) \\ - \chi (1 + \mu B_{op})^2 (\sigma_{\mathcal{T} e^{dm}(0)} / \sigma_{e^{dm}(0)}^2)\}$$

Since B_{op} is optimal it must be that if B_{op} is positive, the term in brackets is positive and, conversely, if B_{op} is negative, the term in brackets is negative (it has the same sign as the FOC). Under the condition in (22), $1 + \mu B_{op} > 0$, so $\sigma_e^2 / \sigma_{e^{dm}(0)}^2$ decreases with χ.

ii and iii) In an interior optimum,

$$B_{op}^2 \sigma_{\mathcal{T} e^{dm}(0)} + B_{op}(1 + \mu B_{op})(\sigma_{\mathcal{T}}^2 / \sigma_{e^{dm}(0)}^2 - \chi) - \chi (1 + \mu B_{op})^2 (\sigma_{\mathcal{T} e^{dm}(0)} / \sigma_{e^{dm}(0)}^2) = 0$$

$$\frac{(1 + \mu B_{op})^2}{(\chi(1 + \mu B_{op})^2 + B_{op}^2)^2} (\chi^2 (1 + \mu B_{op})^2 + B_{op}^2 (\sigma_{\mathcal{T}}^2 / \sigma_{e^{dm}(0)}^2) - 2B_{op} \chi (1 + \mu B_{op}) (\sigma_{\mathcal{T} e^{dm}(0)} / \sigma_{e^{dm}(0)}^2)) = \sigma_e^2 / \sigma_{e^{dm}(B_{op})}^2$$

One can rewrite this as

$$(\frac{B_{op}}{1+\mu B_{op}})^2\sigma_{\mathcal{T}e^{dm}(0)} + \frac{B_{op}}{1+\mu B_{op}}(\sigma^2_{\mathcal{T}}/\sigma^2_{e^{dm}(0)} - \chi) - \chi(\sigma_{\mathcal{T}e^{dm}(0)}/\sigma^2_{e^{dm}(0)}) = 0$$
$$\frac{1}{(\chi+(1+\mu B_{op})^{-2}B^2_{op})^2}(\chi^2 + (\frac{B_{op}}{1+\mu B_{op}})^2(\sigma^2_{\mathcal{T}}/\sigma^2_{e^{dm}(0)}) - 2\frac{B_{op}}{1+\mu B_{op}}\chi(\sigma_{\mathcal{T}e^{dm}(0)}/\sigma^2_{e^{dm}(0)})) = \sigma^2_e/\sigma^2_{e^{dm}(B_{op})}$$

Letting $\tilde{B} = f(B) = (1+\mu B)^{-1}B$, one may rewrite this as

$$\frac{\sigma_{\mathcal{T}e^{dm}(0)}}{\sigma^2_{e^{dm}(0)}}\tilde{B}^2 + (\frac{\sigma^2_{\mathcal{T}}}{\sigma^2_{e^{dm}(0)}} - \chi)\tilde{B} - \frac{\sigma_{\mathcal{T}e^{dm}(0)}}{\sigma^2_{e^{dm}(0)}} = 0$$
$$\frac{1}{(\chi+\tilde{B}^2)^2}(\chi^2 + \tilde{B}^2\frac{\sigma^2_{\mathcal{T}}}{\sigma^2_{e^{dm}(0)}} - 2\chi\tilde{B}\frac{\sigma_{\mathcal{T}e^{dm}(0)}}{\sigma^2_{e^{dm}(0)}}) = \sigma^2_e/\sigma^2_{e^{dm}(B_{op})}$$

It can be verified using Mathematica that plugging in the correct root from the first equation into the second equation and computing the derivative with respect to $\frac{\sigma^2_{\mathcal{T}}}{\sigma^2_{e^{dm}(0)}}$ and χ yields

$$\partial\frac{\sigma^2_e}{\sigma^2_{e^{dm}(B_{op})}}/\partial\frac{\sigma^2_{\mathcal{T}}}{\sigma^2_{e^{dm}(0)}} = -\frac{(\frac{\sigma^2_{\mathcal{T}}}{\sigma^2_{e^{dm}(0)}}+\chi)(\frac{\sigma_{\mathcal{T}e^{dm}(0)}}{\sigma^2_{e^{dm}(0)}})^2}{\left((\frac{\sigma^2_{\mathcal{T}}}{\sigma^2_{e^{dm}(0)}})^2 - 2\frac{\sigma^2_{\mathcal{T}}}{\sigma^2_{e^{dm}(0)}}\chi + 4(\frac{\sigma_{\mathcal{T}e^{dm}(0)}}{\sigma^2_{e^{dm}(0)}})^2\chi + \chi^2\right)^{3/2}} < 0, \tag{50}$$

and

$$\partial\frac{\sigma^2_e}{\sigma^2_{e^{dm}(B_{op})}}/\partial\chi = \frac{2(\frac{\sigma^2_{\mathcal{T}}\sigma^2_{e^{dm}(0)}}{\sigma_{\mathcal{T}e^{dm}(0)}} - 1)}{\left((\frac{\sigma^2_{\mathcal{T}}}{\sigma^2_{e^{dm}(0)}})^2 - 2\frac{\sigma^2_{\mathcal{T}}}{\sigma^2_{e^{dm}(0)}}\chi + 4(\frac{\sigma_{\mathcal{T}e^{dm}(0)}}{\sigma^2_{e^{dm}(0)}})^2\chi + \chi^2\right)^{3/2}} > 0, \tag{51}$$

Finally, note:

$$\frac{\sigma^2_e}{\sigma^2_{e^{dm}(0)}} = \frac{\sigma^2_e}{\sigma^2_{e^{dm}(B_{op})}}\frac{\sigma^2_{e^{dm}(B_{op})}}{\sigma^2_{e^{dm}(0)}} = \frac{\sigma^2_e}{\sigma^2_{e^{dm}(B_{op})}}(\frac{1}{1+\mu B_{op}})^2.$$

I know the first term decreases by the results in (50) and (51). In addition, I know that B_{op} becomes larger in absolute value with a larger importance of the insurance motive ($\uparrow \sigma^2_{\mathcal{T}}/\sigma^2_{e^{dm}(0)}$ or $\downarrow \chi$). Thus, if μB_{op} is nonnegative, a larger portfolio $|B_{op}|$ makes the second term weakly smaller, which in turn shows that overall volatility strictly decreases. In contrast, if $\mu B_{op} < 0$ the effect on volatility is ambiguous.

A.1.8 Proposition 5

I start by deriving an approximation to the consumers' Euler equation. Expanding the first-order condition of the consumer with respect to tradable consumption yields

$$u_T\iota_s + u_{TT}C_{Tss}c_{Ts} + u_{TN}F_Z Z_{ss}z_s + (u_{TN}F_Y + u_{TL})L_{ss}l_s = u_T\lambda_s + O(\epsilon^2),$$

where λ_s is the first-order expansion of the multiplier on the budget constraint. Rewriting this in terms of the output gap,

$$u_T\iota_s + V_{11}C_{Tss}c_{Ts} + V_{1Z}Z_{ss}z_s + u_T\iota_s + FF_Y^{-1}(u_{TN}F_Y + u_{TL})x_s = u_T\lambda_s + O(\epsilon^2) \tag{52}$$

Next, note that to first-order τ^B must be zero. Otherwise, portfolio positions would become unbounded. Expanding the no-arbitrage condition of home agents to second order,

$$\mathbb{E}[r - e_s + (r - e_s)^2 + \lambda_s(r - e_s)] = \tau^B + O(\epsilon^3)$$

and expanding the no-arbitrage condition of foreigners to second-order,

$$\mathbb{E}[(r - e_s) + (r - e_s)^2 - \gamma_{ss}^*(y_s^* - \frac{1}{m}\bar{B}(r - e_s))(r - e_s)] + O(\epsilon^3) = 0$$

where I used $C_{ss}^* = Y_{ss}^* = 1$. Then,

$$\mathbb{E}[(\lambda_s + \gamma_{ss}^* y_s^* - m^{-1}\bar{B}(r - e_s))(r - e_s)] = \tau^B + O(\epsilon^3). \tag{53}$$

For future reference, note that the planner's objective can be rewritten as

$$\mathbb{E}V(\{e_s, \bar{B}\}) = -k_0'\mathbb{E}[\frac{1}{2}(\bar{B}e_s + \mathcal{T}_s^\infty)^2 + \frac{1}{2}\chi^\infty((1+\mu\bar{B})e_s - e_s^{dm}(0))^2 - u_T\gamma_{ss}^* m^{-1}V_{11}^{-1}\bar{B}^2 e_s^2] + t.i.p + O(\epsilon^3)$$

where I used the definitions of $\mathcal{T}_s^\infty$ and χ^∞ given in the proof of part (iv) of proposition 3.

Next, note that equation (52) can be rewritten as

$$-V_{11}(\bar{B}e_s + \mathcal{T}_s^\infty) + FF_Y^{-1}(u_{TN}F_Y + u_{TL})x_s = u_T(\lambda_s + \gamma_{ss}^* y_s^*) + O(\epsilon^2)$$

Using (35),

$$x_s = k_{ex}^{-1}(1 + \mu\bar{B})\{e_s - (1 + \mu\bar{B})^{-1}e_s^{dm}(0)\} + O(\epsilon^2)$$

I can write

$$-V_{11}(\bar{B}e_s + \mathcal{T}_s^\infty) + \tilde{k}\{(1 + \mu\bar{B})e_s - e_s^{dm}(0)\} = u_T(\lambda_s + \gamma_{ss}^* y_s^*) + O(\epsilon^2)$$

for some constant $\tilde{k}$. I also know that the planner picks e_s to solve

$$-\bar{B}V_{11}(\bar{B}e_s + \mathcal{T}_s^\infty) + 2m^{-1}\gamma_{ss}^* r r_s \bar{B}^2 + (1 + \mu\bar{B})V_{11}\chi_s^\infty((1 + \mu\bar{B})e_s - e_s(0)) = O(\epsilon^2)$$

Then,

$$\tilde{k}'\left(-V_{11}(\bar{B}e_s + \mathcal{T}_s^\infty) - 2u_T m^{-1}\gamma_{ss}^* e_s\bar{B}\right) = u_T(\lambda_s + \gamma_{ss}^* y_s^* - 2m^{-1}\gamma_{ss}^* e_s\bar{B}) + O(\epsilon^2)$$

The planners' FOC implies

$$\mathbb{E}e_s\{V_{11}(\bar{B}e_s - \mathcal{T}_s) + 2m^{-1}\gamma_{ss}^*\bar{B}e_s\} = O(\epsilon^3)$$

Thus,

$$\mathbb{E}e_s(\lambda_s + \gamma_{ss}^* y_s^* - 2m^{-1}\gamma_{ss}^* e_s\bar{B}) = O(\epsilon^3)$$

Comparing this to equation (53) implies the result.

A.2 Proofs for section 4

I first present some preliminary computations required for the proofs in Section A.2.1. I then use the results to prove lemma 4 in Section A.2.2, both for the case with savings taxes covered in the main text, and the case without them, which I discuss in Appendix C. Section A.2.3 presents the generalization of the results in the static model (proposition 6). Section A.2.4 proves the exchange rate result (proposition 7) and Section 8 proves the results for savings taxes (proposition 8)

A.2.1 Model solution and preliminary computations

First, I derive the Phillips curve. Second, I derive a second-order approximation to the welfare loss around the riskless steady state around some portfolio $\{B_t, \epsilon = 0\}$. Note that given a path $\{B_t\}$ - I have a standard linear-quadratic problem, which I can solve using the certainty equivalent property. Third, I solved the relaxed (continuation) problem under flexible prices. Fourth, I show how this relaxed problem allows me to write the welfare function in a familiar form. Fifth, I will rewrite the bond-pricing equation in a way that makes it easier to solve the planning problem. Sixth, I derive the excess returns of home-currency bonds in a savings-only ($B = 0$) economy.

Deriving the Phillips' curve The FOC with respect to $P_{It}(i)$ for intermediate-good producer i yields

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\phi)^s \iota_s Y_{It+s} \left\{ E_s^{-1} u_{Tt+s} (1-\eta) (\frac{P_{It}(i)}{P_{It+s}})^{-\eta} - \eta(1-\tau) P_{It}(i)^{-1} (\frac{P_{It}(i)}{P_{It+s}})^{-\eta} u_{Lt+s} \right\} = 0$$

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\phi)^s \iota_s u_{Tt+s} (\frac{P_{It}(i)}{P_{It+s}})^{-\eta} Y_{It+s} \left\{ \frac{P_{It}(i)}{E_{t+s}} + \frac{u_{Lt+s}}{u_{Tt+s}} \right\} = 0$$

Log-linearizing around steady state and rearranging,

$$p_{It}^* - (1-\beta\phi)\{e_t - u_T^{-1} u_{TT} Y_{Tss} c_{Tt} - u_T^{-1} u_{TN} (F_Y L_{ss} l_t + F_Z Z_{ss} z_t) - u_T^{-1} u_{TL} L_{ss} l_t$$
$$+ u_L^{-1} u_{TL} C_{Tss} c_{Tt} + u_L^{-1} u_{NL} (F_Y L_{ss} l_t + F_Z Z_{ss} z_t) + u_L^{-1} u_{LL} L_{ss} l_t\} + \beta\phi \mathbb{E}_t p_{It+1}^* = 0 \tag{54}$$

where p_{It}^* is the wage optimizers at t set. The evolution of the aggregate intermediate-input price index is given by

$$P_{It} = [\phi P_{It-1}^{1-\eta} + (1-\phi) P_{It}^{*1-\eta}]^{1/(1-\eta)}$$

Log-linearize around zero (intermediate-input-price) inflation,

$$\pi_{It} = \frac{1-\phi}{\phi}(p_{It}^* - p_{It})$$

Replacing in (54),

$$\pi_{It} = \frac{1-\phi}{\phi}(1-\beta\phi)(e_t - p_{It} - u_T^{-1} u_{TT} Y_{Tss} c_{Tt} - u_T^{-1} u_{TN} (F_Y L_{ss} l_t + F_Z Z_{ss} z_t) - u_T^{-1} u_{TL} L_{ss} l_t$$
$$+ u_L^{-1} u_{TL} Y_{Tss} c_{Tt} + u_L^{-1} u_{NL} (F_Y L_{ss} l_t + F_Z Z_{ss} z_t) + u_L^{-1} u_{LL} L_{ss} n_{t+s}) + \beta \mathbb{E}_t \pi_{It+1} \tag{55}$$

The firms' FOC is now given by

$$k_{ec}C_{Tss}c_{Tt} + k_{ez}Z_{ss}z_t + k_{ex}x_t = e_t - p_{It}.$$

Replacing in (55) and simplifying,

$$\pi_{It} = \kappa x_t + \beta \mathbb{E}_t \pi_{It+1}$$

where

$$\kappa = \frac{1-\phi}{\phi}(1-\beta\phi)\lambda_x F^{-1} u_N^{-1} > 0.$$

Welfare loss around steady-state path $\{B_t\}$ Proceeding as in the static model and using the result in Woodford (2003) (ch.6),

$$\sum \beta^t Var_i(\hat{p}_{It}(i)) = \frac{\phi}{(1-\phi)(1-\beta\phi)} \sum \beta^t \pi_{It}^2$$

where $\hat{p}_{It}(i) = p_{It}(i) - p_{It}$, I obtain

$$U = \mathbb{E}_0 \sum \beta^t \{u_T C_{Tss} c_{Tt} + \frac{1}{2}(V_{11}C_{Tss}^2 - u_T C_{Tss})c_{Tt}^2 + (V_{1Z}Z_{ss}z_t + u_T \xi_t)C_{ss}c_{Tt} \tag{56}$$
$$- \frac{1}{2}\lambda_x x_t^2 - \frac{1}{2}\lambda_\pi \pi_{It}^2\} + t.i.p. + O(\epsilon^3)$$

where

$$\lambda_\pi \equiv -\frac{\eta\phi}{(1-\beta\phi)(1-\phi)} u_L.$$

The constants V_{11}, V_{1Z}, u_T and λ_x are the same as in the static model.

A second-order approximation to the budget constraint - equation (27) - yields

$$C_{Tss}c_{Tt} + \frac{1}{2}C_{Tss}c_{Tt}^2 + b_t^* - (\beta^{-1}b_{t-1}^* + \beta^{-1}B_{ss}^* r_{t-1}^* + \frac{1}{2}\beta^{-1}B_{ss}^* r_{t-1}^{*2} + \beta^{-1}b_{t-1}^* r_{t-1}^*) =$$
$$Y_{Tss}y_{Tt} + \frac{1}{2}Y_{Tss}y_{Tt}^2 + \bar{B}_{t-1}rr_t + B_{\epsilon t-1}rr_t + O(\epsilon^3). \tag{57}$$

A second order approximation to the foreign no-arbitrage condition yields

$$\mathbb{E}_{t-1}\{rr_t - \gamma_{ss}^* rr_t(y_t^* + \beta^{-1}b_{t-1}^{*f} - b_t^{*f} - \frac{1}{m}\bar{B}rr_t) + O(\epsilon^3) = 0,$$

Furthermore, since for foreigners r^* and y^* shocks are exactly compensated by β^* shocks, they always save a constant share of their income,

$$b_t^{*f} = b_{t-1}^{*f} - \beta m^{-1}\bar{B}_{t-1}rr_t,$$

where I used that at the steady state $\beta = \beta^*$. Let $\tilde{x}_t = x_t - \mathbb{E}_{t-1}x_t$ denote the innovation for a generic variable x. Then,

$$\mathbb{E}_{t-1}\{rr_t - \gamma_{ss}^* rr_t(\tilde{y}_t^* - \frac{1-\beta}{m}\bar{B}_{t-1}rr_t) + O(\epsilon^3) = 0, \tag{58}$$

Combining (57) and (58), and replacing in (56),

$$U = \mathbb{E}_0[\sum \beta^t \{\beta^{-1} u_T b^*_{t-1} - b^*_t) + u_T \bar{B}_{t-1} rr_t \gamma^*_{ss} \tilde{y}^*_t - u_T \bar{B}^2_{t-1} \frac{1-\beta}{m} \gamma^*_{ss} rr^2_t + u_T \beta^{-1} b^*_{t-1} r^*_{t-1}$$
$$+ \frac{1}{2} V_{11} Y^2_{Tss} c^2_{Tt} + (V_{1Z} Z_{ss} z_t + u_T \xi_t) C_{Tss} c_{Tt} - \frac{1}{2} \lambda_x x^2_t - \frac{1}{2} \lambda_\pi \pi^2_{It}\}] + t.i.p. + O(\epsilon^3)$$

Using the transversality condition for bonds,

$$U = \beta^{-1} u_T b^*_{-1} + \mathbb{E}_0[\sum \beta^t \{u_T \bar{B}_{t-1} rr_{Bt} \gamma^*_{ss} \tilde{y}^*_t - u_T \bar{B}^2_{t-1} \frac{1-\beta}{m} \gamma^*_{ss} rr^2_t + u_T \beta^{-1} b^*_{t-1} r^*_{t-1}$$
$$+ \frac{1}{2} V_{11} Y^2_{Tss} c^2_t + (V_{1Z} Z_{ss} Y_{Tss} z_t + u_T \xi_t) c_{Tt} - \frac{1}{2} \lambda_x x^2_t - \frac{1}{2} \lambda_\pi \pi^2_{It}\}] + t.i.p. + O(\epsilon^3)$$

This is already purely quadratic. Using a first-order approximation to the budget constraint,

$$\begin{aligned} U = {} & u_T b^*_{t-1} + \mathbb{E}_0[\sum \beta^t \{u_T \bar{B}_{t-1} rr_t \gamma^*_{ss} \tilde{y}^*_t - u_T \bar{B}^2_{t-1} \frac{1-\beta}{m} \gamma^*_{ss} rr^2_t + u_T \beta^{-1} b^*_{t-1} r^*_{t-1} \\ & + \frac{1}{2} V_{11} (Y_{Tss} y_t + \bar{B}_{t-1} rr_t + \beta^{-1} B^*_{ss} r^*_{t-1} - b^*_t + \beta^{-1} b^*_{t-1})^2 \\ & + (V_{1Z} Z_{ss} z_t + u_T \xi_t)(Y_{Tss} y_t + \bar{B}_{t-1} rr_t + \beta^{-1} B^*_{ss} r^*_{t-1} - b^*_t + \beta^{-1} b^*_{t-1}) - \frac{1}{2} \lambda_x x^2_t - \frac{1}{2} \lambda_\pi \pi^2_{It}\}] + t.i.p. + O(\epsilon^3) \end{aligned} \tag{59}$$

The approximate problem (given $\{\bar{B}_t\}$) is to maximize (59) subject to

$$\kappa x_t + \beta \mathbb{E}_t \pi_{It+1} = \pi_{It}$$
$$\beta^{-1}(r_{t-1} - \Delta e_t - \beta(1-\delta) r_t - r^*_{t-1}) = rr_t$$

By the certainty equivalent property (conditional on $\{\bar{B}_t\}$), it is without loss of generality to consider the case in which the economy is at a steady state at $t = -1$, receives a shock at $t = 0$ and no further shocks from $t = 1$ onwards. In other words, from $t \geq 1$ onwards the second equation is a bond pricing equation,

$$r_{t-1} = \Delta e_t + \beta(1-\delta) r_t - r^*_{t-1}.$$

A relaxed problem: Flexible prices Consider the relaxed problem

$$\begin{aligned} \mathcal{W}_{R1} = {} & \max_{\{\hat{b}^*_t\}^\infty_{t=0}} [u_T \bar{B} rr_0 \gamma^*_{ss} \tilde{y}^*_0 - u_T B^2_{t-1} \frac{1-\beta}{m} \gamma^*_{ss} rr^2_0 \\ & + \frac{1}{2} V_{11} (Y_{Tss} y_0 + \bar{B} rr_0 - b^*_0)^2 + (u_T \iota_0 + V_{1Z} Z_{ss} z_0)(Y_{Tss} y_0 + \bar{B} rr_0 - b^*_0)] \\ & + \sum_{t=1}^{\infty} \beta^t \{\frac{1}{2} V_{11} (Y_{Tss} y_t + \beta^{-1} B^*_{ss} r^*_{t-1} - b^*_t + \beta^{-1} b^*_{t-1})^2 + u_T b^*_t r^*_t \\ & + (V_{1Z} Z_{ss} z_t + u_T \iota_t)(Y_{Tss} y_t + \beta^{-1} B^*_{ss} r^*_{t-1} - b^*_t + \beta^{-1} b^*_{t-1})\} + t.i.p. + O(\epsilon^3) \end{aligned}$$

with $\bar{B}$ and rr_0 given. Note that this is the problem the planner would solve if prices were flexible.

For $t \geq 1$, the FOC with respect to b^*_t yields

$$k_\xi \xi_{t+1} + V_{11} b^{*R}_{t+1} = (1 + \beta^{-1}) V_{11} b^{*R}_t - \beta^{-1} b^*_{t-1} + k_{L \cdot \xi} \xi_t$$

where

$$k_{\xi} = [-V_{11}Y_{Tss}, -V_{1Z}Z_{ss}, -V_{11}\beta^{-1}\rho_{r^*}^{-1}B_{ss}^*, -u_T, 0]$$
$$k_{L\cdot\xi} = [-V_{11}Y_{Tss}, -V_{1Z}Z_{ss}, -V_{11}\beta^{-1}\rho_{r^*}^{-1}B_{ss}^* + u_T, -u_T, 0]$$

and $\xi = [y_T, z, r^*, \iota, \psi]$. Assuming shocks follow an VAR1, $\xi_{t+1} = V_{\xi}\xi_t$, with the world interest rate following an AR1 for simplicity. The solution is given by

$$b_t^{*R} = b_{t-1}^{*R} - \beta(k_{L\xi} - k_{\xi}\boldsymbol{V}_{\xi})(\boldsymbol{I}_{dim(\xi)} - \beta\boldsymbol{V}_{\xi})^{-1}V_{11}^{-1}\xi_t$$

At $t = 0$,

$$u_T r_0^* - V_{11}(\beta\bar{B}rr_0 + Y_{Tss}y_0 - b_0^*) - V_{1Z}Z_{ss}z_0 - u_T\iota_0$$
$$+V_{11}(Y_{Tss}y_1 + \rho_{r^*}^{-1}\beta^{-1}B_{ss}^*r_1^* - b_1^* + \beta^{-1}b_0^*) + V_{1Z}Z_{ss}z_1 + u_T\iota_1 = 0,$$

i.e., the same except that $\beta^{-1}(\rho_{r^*}^{-1}B_{ss}^*r_0^*)$ is missing. Since $\rho_{r^*}^{-1}B_{ss}^*r_0^*$enters like b_{-1}^*, I get

$$b_0^{*R} = \beta\bar{B}rr_0 - \rho_{r^*}^{-1}B_{ss}^*r_0^* - \beta(k_{L\xi} - k_{\xi}\boldsymbol{V}_{\xi})(\boldsymbol{I}_{dim(\xi)} - \beta\boldsymbol{V}_{\xi})^{-1}V_{11}^{-1}\xi_0$$

If all shocks are AR1 with no cross-lag terms,

$$b_0^{*R} = \beta\bar{B}rr_0 + \frac{\beta(1-\rho_z)}{(1-\beta\rho_z)}\frac{V_{1Z}}{V_{11}}Z_{ss}z_0 + \frac{\beta(1-\rho_{\xi})}{(1-\beta\rho_{\xi})}\frac{u_T}{V_{11}}\iota_0 + \frac{\beta(1-\rho_y)}{(1-\beta\rho_y)}Y_{Tss}y_0 - \frac{\beta}{1-\beta\rho_q}\frac{u_T}{V_{11}}r_0^* - \frac{1-\beta}{1-\beta\rho_{r^*}}B_{ss}^*r_0^*. \tag{60}$$

Back to the original problem Let $b_0^{*dm}(0)$ denote the optimum in a savings-only economy (demand-management with no savings taxes), and $\tilde{b}_0 = b_0^* - b_0^{*R} = b_0^* - b_0^*(0) - \beta\bar{B}rr_0$. Using the solution for $b_0^{*dm}(0)$ and imposing a transversality condition on $\{\tilde{b}_t\}$, I can rewrite the welfare loss function as

$$U = \frac{1}{2}(1-\beta)V_{11}(\bar{B}rr_0 - \mathcal{T}_0)^2 + \frac{1}{2}V_{11}\tilde{b}_0^{*2} - \frac{1}{2}\lambda_x x_0^2 - \frac{1}{2}\lambda_\pi\pi_{I0}^2$$
$$+\sum\beta^t\{-\frac{1}{2}\lambda_x x_t^2 - \frac{1}{2}\lambda_\pi\pi_{It}^2 + V_{11}(-\tilde{b}_t + \beta^{-1}\tilde{b}_{t-1})^2\} + t.i.p. + O(\epsilon^3)$$

where

$$\mathcal{T}_0 = -\frac{1}{1-\beta}\frac{1}{1-2m^{-1}u_T V_{11}^{-1}\gamma_{ss}^*}\{Y_{Tss}\tilde{y}_0 + V_{11}^{-1}V_{1Z}Z_{ss}\tilde{z}_0 + V_{11}^{-1}u_T\tilde{\iota}_0 + V_{11}^{-1}u_T\gamma_{ss}^*\tilde{y}_0^* - b_0^*(0)\}$$

If all shocks are AR1, then after replacing $b_0^*(0)$ we obtain

$$\mathcal{T}_0 = -\frac{1}{1-\beta}\frac{1}{1-2m^{-1}u_T V_{11}^{-1}\gamma_{ss}^*}\{\frac{1-\beta}{1-\beta\rho_y}Y_{Tss}\tilde{y}_0 + \frac{1-\beta}{1-\beta\rho_z}V_{11}^{-1}V_{1Z}Z_{ss}\tilde{z}_0 + \frac{1-\beta}{1-\beta\rho_{\xi}}V_{11}^{-1}u_T\tilde{\iota}_0$$
$$+ V_{11}^{-1}u_T\gamma_{ss}^*\tilde{y}_0^* + \frac{\beta}{1-\beta\rho_r}V_{11}^{-1}u_T\tilde{r}_0^* + \frac{1-\beta}{1-\beta\rho_{r^*}}B_{ss}^*\tilde{r}_0^*\}$$

Note this is the transfer that would be required to complete markets, starting from an economy that has access to the foreign bond B^*. I wrote it in terms of the innovations $\tilde{x}$ to emphasize that I only obtained the levels because I assumed the economy was at the steady state at $t = -1$; what requires insurance between -1 and 0 is obviously only the **innovations**.

Rewriting bond pricing constraint After replacing the exchange rate depreciation using equation (35), the bond-pricing constraint ($t \geq 1$) and excess returns constraint ($t = 0$) are given by

$$\beta^{-1}(-k_{ex}x_0 - \pi_{I0} - k_{ez}Z_{ss}z_0 - k_{ec}(Y_{Tss}y_0 + \bar{B}rr_0 - b_0^*) - \beta(1-\delta)r_0) = rr_0$$
$$k_{ex}\Delta x_t + \pi_{It} + k_{ez}Z_{ss}\Delta z_t + k_{ec}\Delta c_{Tt} + \beta(1-\delta)r_t + r_{t-1}^* - \psi_{t-1} = r_{t-1}$$

Note the second equation at $t = 1$ becomes

$$k_{ex}\Delta x_1 + \pi_{I1} + k_{ez}Z_{ss}\Delta z_1 + k_{ec}(\Delta y_1 + \beta^{-1}b_0^* - \bar{B}rr_0 + \beta^{-1}B_{ss}^* r_0^* - b_1 + b_0) + \beta(1-\delta)r_1 + r_0^* - \psi_0 = r_0$$

and for $t \geq 2$,

$$k_{ex}\Delta x_t + \pi_{It} + k_{ez}Z_{ss}\Delta z_t - k_{ec}(\Delta y_{Tt} + \beta^{-1}\Delta b_{t-1}^* + \beta^{-1}B_{ss}^*\Delta r_{t-1}^* - \Delta b_t^*) + \beta(1-\delta)r_t + r_{t-1}^* - \psi_{t-1} = r_t$$

Making $\tilde{b}$ appear and writing in terms of the savings-only economy (using $b_t^{*dm}(0) = b_t^R - \beta\bar{B}rr_0$)

$$\beta^{-1}(-k_{ex}x_0 - \pi_{I0} - k_{ez}Z_{ss}z_0 - k_{ec}(Y_{Tss}y_0 + (1-\beta)\bar{B}rr_0 - \tilde{b}_0^* - b_0^{*dm}(0)) - \beta(1-\delta)r_0) = rr_0 \quad (61)$$
$$k_{ex}\Delta x_1 + \pi_{I1} + k_{ez}Z_{ss}\Delta z_1 + k_{ec}(\Delta y_1 + \beta^{-1}(\tilde{b}_0^* + b_0^{*dm}(0)) +$$
$$\beta^{-1}B_{ss}^* r_0^* - (\tilde{b}_1 - \tilde{b}_0) - (b_1^{*dm}(0) - b_0^{*dm}(0)) + \beta(1-\delta)r_1 + r_0^* - \psi_0 = r_0 \quad (62)$$
$$k_{ex}\Delta x_t + \pi_{It} + k_{ez}Z_{ss}\Delta z_t - k_{ec}(\Delta y_{Tt} + \beta^{-1}(\Delta\tilde{b}_{t-1}^* + \Delta b_{t-1}^{*dm}(0)) +$$
$$\beta^{-1}B_{ss}^*\Delta r_{t-1}^* - (\Delta\tilde{b}_t^* + \Delta b_t^{*dm}(0))) + \beta(1-\delta)r_t + r_{t-1}^* - \psi_{t-1} = r_{t-1} \quad (63)$$

Next let $rr_0^{dm}(0)$ and $\{r_t^{dm}(0)\}$ solve

$$\beta^{-1}(-k_{ez}Z_{ss}z_0 - k_{ec}(Y_{Tss}y_0 - b_0^{*dm}(0)) - \beta(1-\delta)r_0) = rr_0^{dm}(0)$$
$$k_{ez}Z_{ss}\Delta z_1 + k_{ec}(\Delta y_1 + \beta^{-1}b_0^{*dm}(0) + \beta^{-1}B_{ss}^* r_0^* - (b_1^{*dm}(0) - b_0^{*dm}(0)) + \beta(1-\delta)r_1^{dm}(0) + r_0^* - \psi_0 = r_0^{dm}(0)$$
$$k_{ez}Z_{ss}\Delta z_t - k_{ec}(\Delta y_{Tt} + \beta^{-1}\Delta b_{t-1}^{*dm}(0) + \beta^{-1}B_{ss}^*\Delta r_{t-1}^* - \Delta b_t^{*dm}(0)) + \beta(1-\delta)r_t^{dm}(0) + r_{t-1}^* - \psi_{t-1} = r_{t-1}^{dm}(0)$$

Then, I can rewrite (61)-(63) as

$$\beta^{-1}(-k_{ex}x_0 - \pi_{I0} + k_{ec}\tilde{b}_0^* - \beta(1-\delta)(r_0 - r_0^{dm}(0))) = (1 + \beta^{-1}(1-\beta)k_{ec})rr_0 - rr_0^{dm}(0)$$
$$k_{ex}\Delta x_1 + \pi_{I1} + k_{ec}(\beta^{-1}\tilde{b}_0^* - (\tilde{b}_1 - \tilde{b}_0)) + \beta(1-\delta)(r_1 - r_1^{dm}(0)) = r_0 - r_0^{dm}(0)$$
$$k_{ex}\Delta x_t + \pi_{It} - k_{ec}(\beta^{-1}\Delta\tilde{b}_{t-1}^* - \Delta\tilde{b}_t^*) + \beta(1-\delta)(r_t - r_t^{dm}(0)) = r_{t-1} - r_{t-1}^{dm}(0)$$

Defining

$$\tilde{r}_0 = r_0 - r_0^{dm}(0) + k_{ex}x_0 - \beta^{-1}k_{ec}\tilde{b}_0^*$$
$$\tilde{r}_t = r_t - r_t^{dm}(0) + k_{ex}x_t - \beta^{-1}k_{ec}\Delta\tilde{b}_t$$

I get

$$\beta^{-1}(-(1-\beta(1-\delta))k_{ex}x_0 - \pi_{I0} - \delta k_{ec}\tilde{b}_0^* - \beta(1-\delta)\tilde{r}_0) = (1+\beta^{-1}(1-\beta)k_{ec})rr_0 - rr_0^{dm}(0)$$
$$k_{ex}(1-\beta(1-\delta))x_t + \pi_{It} + \beta(1-\delta)\tilde{r}_t - \delta k_{ec}\Delta\tilde{b}_t^* = \tilde{r}_{t-1}$$

Rewriting Euler equation constraint (for the case without capital controls) The Euler equation is given by

$$FF_Y^{-1}(u_{TN}F_Y + u_{TL})\Delta x_{t+1} + V_{11}C_{Tss}\Delta c_{Tt+1} + u_T\Delta\iota_{t+1} + V_{1Z}Z_{ss}\Delta z_{t+1} - r_t^* = 0$$

Using the budget constraint,

$$FF_Y^{-1}(u_{TN}F_Y + u_{TL})\Delta x_1 + V_{11}(Y_{Tss}\Delta y_1 + \beta^{-1}B_{ss}^* r_0^* + \beta^{-1}b_0^* + \bar{B}rr_0 - \Delta b_1^*) + u_T\Delta\iota_1 + V_{1Z}Z_{ss}\Delta z_1 - r_0^* = 0$$
$$FF_Y^{-1}(u_{TN}F_Y + u_{TL})\Delta x_{t+1} + V_{11}(Y_{Tss}\Delta y_{t+1} + \beta^{-1}B_{ss}^*\Delta r_t^* + \beta^{-1}\Delta b_{t+1}^* - \Delta b_t^*) + u_T\Delta\iota_{t+1} + V_{1Z}Z_{ss}\Delta z_{t+1} - r_t^* = 0 \ \forall t \geq 1$$

The relaxed b^R defined above solves

$$V_{11}(Y_{Tss}\Delta y_1 + \beta^{-1}B_{ss}^* r_0^* - \Delta b_1^R + \beta^{-1}b_0^R + rr_0) + u_T\Delta\iota_1 + V_{1Z}Z_{ss}\Delta z_1 - r_0^* = 0$$
$$V_{11}(Y_{Tss}\Delta y_{t+1} + \beta^{-1}B_{ss}^*\Delta r_{t+s}^* - \Delta b_{t+s+1}^R + \beta^{-1}b_{t+s}^R) + u_T\Delta\iota_1 + V_{1Z}Z_{ss}\Delta z_1 - r_{t+s}^* = 0 \ \forall s \geq 1.$$

The reason this is true is, essentially, that if prices were flexible no capital controls would be required. Substracting,

$$FF_Y^{-1}(u_{TN}F_Y + u_{TL})\Delta x_1 + V_{11}(-\Delta\tilde{b}_1^* + \beta^{-1}\tilde{b}_0^*) = 0$$
$$FF_Y^{-1}(u_{TN}F_Y + u_{TL})\Delta x_{t+1} + V_{11}(-\Delta\tilde{b}_{t+1}^* + \beta^{-1}\Delta\tilde{b}_t^*) = 0 \ \forall s \geq 1$$

Implied excess returns on home-currency bonds in $\bar{B} = 0$ economy Solving forward,

$$\beta rr_0(\mathbf{0}) = \left(k_\xi^B - \beta(1-\delta)k_{L\cdot\xi}^B\right)\left(\boldsymbol{I}_{dim(\xi)} - \beta(1-\delta)V_\xi\right)^{-1}\tilde{\xi}_0 + k_{ec}\rho_{r^*}^{-1}\beta^{-1}B_{ss}^*\tilde{r}_0^* + k_{ec}b_0^{*dm}(0) + k_{ec}\beta(1-\delta)((k_{L\xi} - k_\xi\boldsymbol{V}_\xi)V_{11}^{-1}(\boldsymbol{I}_{dim(\xi)} - \beta(1-\delta)V_\xi))^{-1}\tilde{\xi}_0$$

where

$$k_\xi^B = [-k_{ec}Y_{Tss}, -k_{ez}Z_{ss}, -k_{ec}\rho_{r^*}^{-1}\beta^{-1}B_{ss}^*]$$
$$k_{L\cdot\xi}^B = [-k_{ec}Y_{Tss}, -k_{ez}Z_{ss}, -k_{ec}\rho_{r^*}^{-1}\beta^{-1}B_{ss}^* + 1, 1]$$

A.2.2 Lemma 4

With capital controls If capital controls are available, the problem is

$$\mathcal{W}_{t\geq 1} = \max_{\{\tilde{b}_t, x_t, \pi_t\}} \{\frac{1}{2}\frac{(1-\beta)V_{11}}{1-2m^{-1}u_T V_{11}^{-1}\gamma_{ss}^*}(\bar{B}rr_0 - \mathcal{T}_s)^2 + \frac{1}{2}V_{11}\tilde{b}_0^2 - \frac{1}{2}\lambda_x x_0^2 - \frac{1}{2}\lambda_\pi \pi_{I0}^2$$
$$+ \sum \beta^t \{-\frac{1}{2}\lambda_x x_t^2 - \frac{1}{2}\lambda_\pi \pi_{It}^2 + \frac{1}{2}V_{11}(-\tilde{b}_t + \beta^{-1}\tilde{b}_{t-1})^2\} + t.i.p. + O(\epsilon^3)$$

s.t.

$$\kappa x_t + \beta \pi_{It+1} = \pi_{It}$$
$$\beta^{-1}(-(1-\beta(1-\delta))k_{ex}x_0 - \pi_{I0} + \delta k_{ec}\tilde{b}_0^* - \beta(1-\delta)\tilde{r}_0) = (1+\beta^{-1}(1-\beta)k_{ec})rr_0 - rr_0^{dm}(0)$$
$$(1-\beta(1-\delta))k_{ex}x_t + \pi_{It} - \delta k_{ec}(\tilde{b}_t - \tilde{b}_{t-1}) + \beta(1-\delta)\tilde{r}_t = \tilde{r}_{t-1}.$$

I will first solve the problem from $t \geq 1$ onwards,

$$\mathcal{W}_{t\geq 1} = \max \sum \beta^t \{-\frac{1}{2}\lambda_x x_t^2 - \frac{1}{2}\lambda_\pi \pi_{It}^2 + \frac{1}{2}V_{11}(-\tilde{b}_t + \beta^{-1}\tilde{b}_{t-1})^2\} + t.i.p. + O(\epsilon^3)$$

s.t.

$$\kappa x_{t+s} + \beta \pi_{It+s+1} = \pi_{It+s}$$
$$(1-\beta(1-\delta))k_{ex}x_{t+s} - \delta k_{ec}(\tilde{b}_t - \tilde{b}_{t-1}) + \pi_{It+s} + \beta(1-\delta)\tilde{r}_{t+s} = \tilde{r}_{t+s-1}$$

where $\tilde{b}_0^*$, $\tilde{r}_0$ and π_{I1} are given. Let ϕ_{1t} denote the multiplier on the first constraint and ϕ_{2t} denote the multiplier on the second constraint. FOC:

$$-\lambda_x x_t = -\kappa\phi_{1t} + k_{ex}(1-\beta(1-\delta))\phi_{2t}$$
$$\lambda_\pi \pi_{It+1} + \phi_{1t+1} + \phi_{2t+1} = \phi_{1t}$$
$$(1-\delta)\phi_{2t} = \phi_{2t+1}$$
$$V_{11}\tilde{b}_{t+1} + \beta k_{ec}\delta\phi_{2t+1} = (1+\beta^{-1})V_{11}\tilde{b}_t - V_{11}\beta^{-1}\tilde{b}_{t-1} + \delta\phi_{2t}$$

plus the constraints. There are two unit roots inside the unit circle, a unit root root and three outside, which reflects that the bond position has a unit root. I pick ϕ_{11}, ϕ_{21} and $\tilde{b}_1^*$ to kill the exploding roots. Finally, I write

$$\mathcal{W}_{t\geq 1} = \frac{1}{2}W_\pi \pi_{I1}^2 + \frac{1}{2}W_r \tilde{r}_0^2 + \frac{1}{2}W_b \tilde{b}_0^2 + W_{r\pi}\pi_{I1}\tilde{r}_0 + W_{rb}\tilde{r}_0\tilde{b}_0 + W_{b\pi}\pi_{I1}\tilde{b}_0$$

and compute these constants by using the envelope theorem and then matching coefficients. It can be shown that $W_{b\pi} = W_{rb} = 0$ and $W_b = \beta^{-2}V_{11}(1-\beta)$. This reflects that whether bonds are initially higher or lower, they do not make it more or less costly to satisfy the constraint since they have a unit root. The planner does use, however, deviations to lower the cost of the promise (by tilting $\tilde{b}$ path). The remaining constants are explicit but very complicated.[55]

I can then use these expressions as the continuation values at $t = 0$. The problem at $t = 0$ is

[55] I find them using the symbolic toolbox in MATLAB.

given by

$$\mathcal{W} = \max\{\frac{1}{2}\frac{(1-\beta)V_{11}}{1-2m^{-1}u_T V_{11}^{-1}\gamma_{ss}^*}(\bar{B}rr_0 - \mathcal{T})^2 + \frac{1}{2}V_{11}\tilde{b}_0^{*2} - \frac{1}{2}\lambda_x x_0^2 - \frac{1}{2}\lambda_\pi \pi_{I0}^2$$
$$+ \frac{1}{2}\beta W_b \tilde{b}_0^{*2} + \frac{1}{2}\beta W_\pi \pi_{I1}^2 + \beta W_{\pi r}\pi_{I1}\tilde{r}_0 + \frac{1}{2}\beta W_r \tilde{r}_0^2\} + t.i.p. + O(\epsilon^3)$$

s.t.

$$\kappa x_0 + \beta \pi_{I1} = \pi_{I0}$$
$$-(1-\beta(1-\delta))k_{ex}x_0 - \pi_{I0} + \delta k_{ec}\tilde{b}_0^* - \beta(1-\delta)\tilde{r}_0 = \beta\{(1+k_{ec}\beta^{-1}(1-\beta)\bar{B})rr_0 - rr_0^{dm}(0)\}$$

The FOC are

$$-\lambda_x x_0 + \kappa\phi_{10} - (1-\beta(1-\delta)k_{ex}\phi_{20} = 0$$
$$-\lambda_\pi \pi_{I0} - \phi_{10} - \phi_{20} = 0$$
$$W_{\pi\pi}\pi_{I1} + W_{\pi r}\tilde{r}_0 + \phi_{10} = 0$$
$$W_{\pi r}\pi_{I1} + W_{rr}\tilde{r}_0 + (1-\delta)\phi_{20} = 0$$
$$(V_{11} + \beta W_{bb})\tilde{b}_0 + \delta k_{ec}\phi_{20} = 0$$

plus the constraints. This is just a linear system that is easy to solve. Note that the multiplier on the second constraint contains the information on how costly it is to deviate from a demand-management policy. Solving and using the envelope theorem, I may write

$$\mathcal{W} = \{\frac{1}{2}\frac{(1-\beta)V_{11}}{1-2m^{-1}u_T V_{11}^{-1}\gamma_{ss}^*}(\bar{B}rr_0 - \mathcal{T})^2 - \frac{1}{2}\beta^2\tilde{\phi}((1+k_{ec}\beta^{-1}(1-\beta)\bar{B})rr_0 - rr_0^{dm}(0))^2\} + t.i.p. + O(\epsilon^3)$$

Thus, defining $\chi = -\frac{1-2m^{-1}u_T V_{11}^{-1}\gamma_{ss}^*}{(1-\beta)V_{11}}\beta^2\tilde{\phi} > 0$ and $\mu = \beta^{-1}k_{ec}(1-\beta)$, I can write the welfare function as desired,

$$\mathcal{W} = \frac{(1-\beta)V_{11}}{1-2m^{-1}u_T V_{11}^{-1}\gamma_{ss}^*}\{\frac{1}{2}(\bar{B}rr_0 - \mathcal{T})^2 + \frac{1}{2}\chi((1+\mu\bar{B})rr_0 - rr_0^{dm}(0))^2 + t.i.p. + O(\epsilon^3)$$

Without savings taxes (with portfolio taxes) Let $\tilde{c}_t = \beta^{-1}\tilde{b}_t - \tilde{b}_{t-1}$. I have

$$\mathcal{W} = \max\{\frac{1}{2}\frac{(1-\beta)V_{11}}{1-2m^{-1}u_T V_{11}^{-1}\gamma_{ss}^*}(\bar{B}rr_0 - \mathcal{T})^2 + \frac{1}{2}V_{11}\tilde{b}_0^2 - \frac{1}{2}\lambda_x x_0^2 - \frac{1}{2}\lambda_\pi \pi_{I0}^2$$
$$+ \sum \beta^t\{-\frac{1}{2}\lambda_x x_t^2 - \frac{1}{2}\lambda_\pi \pi_{It}^2 + \frac{1}{2}V_{11}\tilde{c}_{Tt}^2\} + t.i.p. + O(\epsilon^3)$$

s.t.

$$\begin{aligned}\kappa x_t + \beta\pi_{It+1} &= \pi_{It}\\ \beta^{-1}(-k_{ex}x_0 - \pi_{I0} - k_{ec}\tilde{c}_{Tt} + \beta(1-\delta)(r_0 - r_0^{dm}(0)) &= (1+\mu\bar{B})rr_0 - rr_0^{dm}(0)\\ k_{ex}\Delta x_t + \pi_{It} + k_{ec}\Delta\tilde{c}_{Tt} + \beta(1-\delta)(r_t - r_t^{dm}(0)) &= (r_{t-1} - r_{t-1}^{dm}(0))\\ k_{ux}\Delta x_{t+1} + k_{uc}\Delta\tilde{c}_{Tt+1} &= 0\end{aligned}$$

where

$$\begin{aligned}k_{ux} &\equiv FF_Y^{-1}(u_{TN}F_Y + u_{TL})\\ k_{uc} &\equiv V_{11}C_{Tss}.\end{aligned}$$

Note the last constraint implies

$$k_{uc}\tilde{c}_{Tt} + k_{ux}x_t = \nu \ \forall t$$

where ν is a choice variable. Using this insight, I rewrite the problem as

$$\begin{aligned}\mathcal{W} = \max\{&\frac{1}{2}\frac{(1-\beta)V_{11}}{1-2m^{-1}u_T V_{11}^{-1}\gamma_{ss}^*}(\bar{B}rr_0 - \mathcal{T})^2 + \frac{1}{2}V_{11}\tilde{b}_0^2 - \frac{1}{2}\lambda_x x_0^2 - \frac{1}{2}\lambda_\pi\pi_{I0}^2\\ &+ \sum\beta^t\{-\frac{1}{2}\lambda_x x_t^2 - \frac{1}{2}\lambda_\pi\pi_{It}^2 + \frac{1}{2}V_{11}k_{uc}^{-2}(\nu - k_{ux}x_t)^2\} + t.i.p. + O(\epsilon^3)\end{aligned}$$

s.t.

$$\begin{aligned}\kappa x_t + \beta\pi_{It+1} &= \pi_{It}\\ \beta^{-1}(-(k_{ex} - k_{ec}k_{uc}^{-1}k_{ux})x_0 - \pi_{I0} - k_{ec}k_{uc}^{-1}\nu - \beta(1-\delta)(r_0 - r_0^{dm}(0))) &= (1+\mu\bar{B})rr_0 - rr_0^{dm}(0)\\ (k_{ex} - k_{ec}k_{uc}^{-1}k_{ux})\Delta x_t + \pi_{It} + \beta(1-\delta)(r_t - r_t^{dm}(0)) &= r_{t-1} - r_{t-1}^{dm}(0)\end{aligned}$$

Next define $\tilde{r}_t = r_t - r_t^{dm}(0) - (k_{ex} - k_{ec}k_{uc}^{-1}k_{ux})x_t$. Thus,

$$\begin{aligned}\kappa x_t + \beta\pi_{It+1} &= \pi_{It}\\ \beta^{-1}(-(1-\beta(1-\delta))(k_{ex} - k_{ec}k_{uc}^{-1}k_{ux})x_0 - \pi_{I0} - k_{ec}k_{uc}^{-1}\nu - \beta(1-\delta)\tilde{r}_0) &= (1+\mu\bar{B})rr_0 - rr_0^{dm}(0)\\ (1-\beta(1-\delta))(k_{ex} - k_{ec}k_{uc}^{-1}k_{ux})x_t + \pi_{It} + \beta(1-\delta)\tilde{r}_t &= \tilde{r}_{t-1}\end{aligned}$$

The continuation problem is given by

$$\mathcal{W}_{t\geq 1} = \max\{\sum\beta^t\{-\frac{1}{2}\lambda_x x_t^2 - \frac{1}{2}\lambda_\pi\pi_{It}^2 + \frac{1}{2}V_{11}k_{uc}^{-2}(\nu - k_{ux}x_t)^2\} + t.i.p. + O(\epsilon^3)$$

s.t.

$$\begin{aligned}\kappa x_t + \beta\pi_{It+1} &= \pi_{It}\\ (1-\beta(1-\delta))(k_{ex} - k_{ec}k_{uc}^{-1}k_{ux})x_t + \pi_{It} + \beta(1-\delta)\tilde{r}_t &= \tilde{r}_{t-1}\\ \nu_{t+1} &= \nu_t\end{aligned}$$

with $(\pi_{It}, \tilde{r}_{t-1}, \nu_t)$ given. The FOC yield

$$-(\lambda_x - V_{11}k_{uc}^{-2}k_{ux}^2)x_t - V_{11}k_{uc}^{-2}k_{ux}\nu_t + \kappa\phi_{1t} - (1-\beta(1-\delta))(k_{ex} + k_{ec}k_{uc}^{-1}k_{ux})\phi_{2t} = 0$$
$$\lambda_\pi \pi_{It+1} + \phi_{1t} - \phi_{1t+1} - \phi_{2t+1} = 0$$
$$(1-\delta)\phi_{2t} - \phi_{2t+1} = 0$$

I solve by picking (ϕ_{10}, ϕ_{20}) to kill the exploding roots. Finally, I write

$$\mathcal{W}_{t\geq 1} = \frac{1}{2}W_{\pi\pi}(\pi_{I1})^2 + W_{\pi\nu}\pi_{I1}\nu + \frac{1}{2}W_{\nu\nu}\nu^2.$$

To find these constants, let $s_t = [\pi_t; \nu_t]$ and write $\boldsymbol{x}_t = [x_t; \pi_{It}; \nu_t] = k[\pi_{It}; \tilde{r}_{t-1}; \nu_t]$ and $s_{t+1} = V_s s_t$.

$$s_1' \mathcal{W}_{t\geq 1} s_1 = \sum \beta^t s_1'(V_s^t)' k' A_w k V_s^t s_1$$

where A_w is a negative definite matrix that represents the previous loss function. Solving,

$$vec(\mathcal{W}_{t\geq 1}) = (I - \beta(V_s)' \otimes (V_s)')^{-1} vec(k' A_w k).$$

Next, note that using the relation

$$b_t = \beta^{-1} b_{t-1}^* + k_{uc}^{-1}k_{ux}x_t - k_{uc}^{-1}\nu_t$$

I find b_0^* that is consistent with the promised π_0, ν_0 and the transversality condition. To do so, write this as a system,

$$[b_t; \pi_{It+1}; \tilde{r}_t; \nu_{t+1}] = K[b_{t-1}^*; \pi_{It}; \tilde{r}_{t-1}; \nu_t].$$

and find b_0^* to kill the exploding root. This yields

$$b_0^* = k_{br}\tilde{r}_0 + k_{b\nu}\nu_1 + k_{b\pi}\pi_{I1}.$$

At $t = 0$,

$$\mathcal{W} = \max\{\frac{1}{2}\frac{(1-\beta)V_{11}}{1 - 2m^{-1}u_T V_{11}^{-1}\gamma_{ss}^*}(\bar{B}rr_0 - \mathcal{T})^2 + \frac{1}{2}V_{11}\tilde{b}_0^2 - \frac{1}{2}\lambda_x x_0^2 - \frac{1}{2}\lambda_\pi \pi_{I0}^2$$
$$+ \frac{1}{2}\beta W_{\pi\pi}\pi_{I1}^2 + \beta W_{\pi r}\pi_{I1}\tilde{r}_0 + \beta W_{\pi\nu}\pi_{I1}\nu + \frac{1}{2}\beta W_{rr}\tilde{r}_0^2 + \beta W_{r\nu}\tilde{r}_0\nu + \frac{1}{2}\beta W_{\nu\nu}\nu^2 + t.i.p. + O(\epsilon^3)$$

s.t.

$$\kappa x_0 + \beta\pi_{I1} = \pi_{I0}$$
$$\beta^{-1}\{-(1-\beta(1-\delta))(k_{ex} - k_{ec}k_{uc}^{-1}k_{ux})x_0 - \pi_{I0} - k_{ec}k_{uc}^{-1}\nu - \beta(1-\delta)\tilde{r}_0\} = (1+\mu\bar{B})rr_0 - rr_0^{dm}(0)$$
$$-k_{uc}\tilde{b}_0 + k_{ux}x_0 = \nu$$
$$k_{br}\tilde{r}_0 + k_{b\nu}\nu_1 + k_{b\pi}\pi_{I1} = b_0^*$$

FOC,

$$
\begin{aligned}
-\lambda_x x_0 + \kappa\phi_1 - (1-\beta(1-\delta))(k_{ex} - k_{ec}k_{uc}^{-1}k_{ux})\phi_2 + k_{ux}\phi_3 &= 0 \\
-\lambda_\pi \pi_{I0} - \phi_1 - \phi_2 &= 0 \\
W_{\pi\pi}\pi_{I1} + W_{\pi r}\tilde{r}_0 + W_{\pi\nu}\nu + \phi_1 + k_{b\pi}\phi_4 &= 0 \\
W_{\pi r}\pi_{I1} + W_{rr}\tilde{r}_0 + W_{r\nu}\nu + (1-\delta)\phi_2 + k_{bq}\phi_4 &= 0 \\
W_{\pi\nu}\pi_{I1} + W_{r\nu}\tilde{r}_0 + W_{\nu\nu}\nu - \beta^{-1}k_{ec}k_{uc}^{-1}\phi_2 - \beta^{-1}\phi_3 + k_{b\nu}\phi_4 &= 0 \\
V_{11}\tilde{b}_0 - k_{uc}\phi_3 - \beta^{-1}\phi_4 &= 0
\end{aligned}
$$

This is just a linear system that is easy to solve. Note that the multiplier on the second constraint contains the information on how costly it is to deviate from a demand-management policy. Solving and using the envelope theorem, I may write

$$\mathcal{W} = \{\frac{1}{2}(1-\beta)V_{11}(\bar{B}rr_0 - \mathcal{T})^2 - \frac{1}{2}\beta^2\tilde{\phi}((1+k_{ec}\beta^{-1}(1-\beta)\bar{B})rr_0 - rr_0^{dm}(0))^2\} + t.i.p. + O(\epsilon^3)$$

Thus, defining $\chi = -\frac{1-2m^{-1}u_T V_{11}^{-1}\gamma_{ss}^*}{(1-\beta)V_{11}}\beta^2\tilde{\phi} > 0$ and $\mu = \beta^{-1}k_{ec}(1-\beta)$, I can write the welfare function as desired,

$$\mathcal{W} = \frac{(1-\beta)V_{11}}{1-2m^{-1}u_T V_{11}^{-1}\gamma_{ss}^*}\mathbb{E}\{\frac{1}{2}(\bar{B}rr_0 - \mathcal{T})^2 + \frac{1}{2}\chi((1+\mu\bar{B})rr_0 - rr_0^{dm}(0))^2 + t.i.p. + O(\epsilon^3)$$

Note the only difference is that the muliplier $\tilde{\phi}$ will now be larger because it is a more restricted problem.

A.2.3 Proposition 6

Propositions 2 and 4 are trivial given Lemma 4. Next, I prove the generalization of proposition 5.

Combining the home and foreign Eulers to second order,

$$\mathbb{E}[(\lambda_0 + \gamma_{ss}^* y_0^* - (1-\beta)m^{-1}\bar{B}rr_0)rr_0] = \tau^B + O(\epsilon^3). \tag{64}$$

where λ_0 is still the multiplier on the budget constraint.

In the dynamic model private marginal utility still satisfies

$$u_T\iota_0 + V_{11}C_{Tss}c_{T0} + V_{1Z}Z_{ss}z_0 + FF_Y^{-1}(u_{TN}F_Y + u_{TL})x_0 = u_T\lambda_0 + O(\epsilon^2)$$

Using the budget constraint,

$$u_T\iota_0 + V_{11}(Y_{Tss}y_0 + \bar{B}rr_0 - b_0^*) + V_{1Z}Z_{ss}z_0 + FF_Y^{-1}(u_{TN}F_Y + u_{TL})x_0 = u_T\lambda_0 + O(\epsilon^2)$$

Using the solution from the "relaxed" problem discussed above, I can rewrite this as

$$u_T\iota_0 + V_{11}(Y_{Tss}y_0 + (1-\beta)\bar{B}rr_0 - b_0^{*R}) - V_{11}\tilde{b}_0 + FF_Y^{-1}(u_{TN}F_Y + u_{TL})x_0 = u_T\lambda_0 + O(\epsilon^2),$$

Using the solution for b_0^{*R}, I obtain,

$$(1-\beta)V_{11}(\bar{B}rr_0 - \mathcal{T}_0^\infty) - V_{11}\tilde{b}_0 + FF_Y^{-1}(u_{TN}F_Y + u_{TL})x_0 = u_T\lambda_0 + u_T\gamma_{ss}^* y_0^* + O(\epsilon^2)$$

Regardless of whether there are capital controls or not, I know from solving the problem in proposition 4, that I can write

$$\tilde{b}_0 = \tilde{k}_b\{(1+\mu\bar{B})rr_s - rr_s^{dm}(0)\} + O(\epsilon^2)$$
$$x_0 = \tilde{k}_x\{(1+\mu\bar{B})rr_s - rr_s^{dm}(0)\} + O(\epsilon^2)$$

for some constants $\tilde{k}_b$ and $\tilde{k}_x$, so

$$(1-\beta)V_{11}(\bar{B}rr_0 - \mathcal{T}_0^\infty) + \tilde{k}_{rr}\{(1+\mu\bar{B})rr_s - rr_s^{dm}(0)\} = u_T\lambda_0 + u_T\gamma_{ss}^* y_0^* + O(\epsilon^2)$$

for some constant $\tilde{k}_{rr}$. I also know that the planner picks rr_s to solve

$$-\bar{B}V_{11}(\bar{B}rr_s - \mathcal{T}_s^\infty) + 2m^{-1}\gamma_{ss}^* rr_s\bar{B}^2 + (1+\mu\bar{B})V_{11}\chi_s^\infty((1+\mu\bar{B})e_s - e_s(0)) = O(\epsilon^2)$$

Then,

$$\tilde{k}'\left(-V_{11}(\bar{B}e_s + \mathcal{T}_s^\infty) - 2u_T m^{-1}\gamma_{ss}^* e_s\bar{B}\right) = u_T(\lambda_s + \gamma_{ss}^* y_s^* - 2(1-\beta)m^{-1}\gamma_{ss}^* rr_s\bar{B}) + O(\epsilon^2)$$

for some constant $\tilde{k}'$. The planners' FOC implies

$$\mathbb{E}rr_s\{V_{11}(\bar{B}e_s - \mathcal{T}_s) + 2m^{-1}\gamma_{ss}^*\bar{B}rr_s\} = O(\epsilon^3)$$

so

$$\tau^B = (1-\beta)m^{-1}\gamma_{ss}^*\bar{B}\mathbb{E}rr_0^2 + O(\epsilon^3).$$

Note that if foreigners were "short-lived" the $1-\beta$ would vanish. In that case, their Euler equation would look exactly the same as in the static model. The fact that they can save lowers their risk aversion by $(1-\beta)^{-1}$.

A.2.4 Proposition 7

Rigid prices and long bonds With rigid prices, the continuation problem is:

$$\mathcal{W} = \max_{\{\tilde{b}_t, x_t, \pi_{It}\}}\{\frac{1}{2}\frac{(1-\beta)V_{11}}{1-2m^{-1}u_1V_{11}^{-1}\gamma_{ss}^*}(\bar{B}rr_0 - \mathcal{T}_s)^2 + \frac{1}{2}V_{11}\tilde{b}_0^2 - \frac{1}{2}\lambda_x x_0^2$$
$$+\sum\beta^t\{-\frac{1}{2}\lambda_x x_t^2 + \frac{1}{2}V_{11}(-\tilde{b}_t + \beta^{-1}\tilde{b}_{t-1})^2\} + t.i.p. + O(\epsilon^3)$$

s.t.

$$\beta^{-1}(-(1-\beta(1-\delta))k_{ex}x_0 + \delta k_{ec}\tilde{b}_0^* - \beta(1-\delta)\tilde{r}_0) = (1+\beta^{-1}(1-\beta)k_{ec})rr_0 - rr_0^{dm}(0)$$
$$(1-\beta(1-\delta))k_{ex}x_t - \delta k_{ec}(\tilde{b}_t - \tilde{b}_{t-1}) + \beta(1-\delta)\tilde{r}_t = \tilde{r}_{t-1}.$$

From $t \geq 1$ onwards,

$$\mathcal{W} = \max_{\{\tilde{b}_t, x_t, \pi_t\}}\{\sum\beta^t\{-\frac{1}{2}\lambda_x x_t^2 + \frac{1}{2}V_{11}(-\tilde{b}_t + \beta^{-1}\tilde{b}_{t-1})^2\} + t.i.p. + O(\epsilon^3)$$

The solution is

$$\begin{aligned}\tilde{b}_t &= \tilde{b}_{t-1} - \beta\delta k_{ec} V_{11}^{-1} \bar{K}_0 \tilde{r}_{t-1} \\ \phi_1 &= \bar{K}_0 \tilde{r}_{t-1} \\ x_t &= -\lambda_x^{-1}(1-\beta(1-\delta)) k_{ex} \bar{K}_0 \tilde{r}_{t-1} \\ \tilde{r}_t &= (1-\delta)\tilde{r}_{t-1}\end{aligned}$$

where

$$\bar{K}_0 = k_{ex}^{-2}\lambda_x \frac{(1-\beta(1-\delta)^2)}{(1-\beta(1-\delta))^2 - \lambda_x k_{ex}^{-2} V_{11}^{-1}\beta\delta^2 k_{ec}^2} > 0.$$

Since there is no inflation, one may write, for $t \geq 1$

$$\tilde{e}_t = k_{ex} x_t + k_{ec}(\beta^{-1}\tilde{b}_{t-1} - \tilde{b}_t) + O(\epsilon^2)$$

Replacing,

$$\tilde{e}_t = (\lambda_x^{-1} k_{ex}^2 - k_{ec}^2 V_{11}^{-1})(1-\beta(1-\delta))\bar{K}_0 \tilde{r}_{t-1} + (1-\beta) k_{ec}^2 V_{11}^{-1} \bar{K}_0 \tilde{r}_0.$$

Noting the problem at $t = 0$ is the same with $rr_0 - \frac{1}{1+\mu\bar{B}} rr_0^{dm}(0)$ instead of $-\tilde{r}_0$,

$$\begin{aligned}\tilde{e}_t = &-(\lambda_x^{-1} k_{ex}^2 - k_{ec}^2 V_{11}^{-1})(1-\beta(1-\delta))\bar{K}_0 (1-\delta)^t \{(1+\mu\bar{B}) rr_0 - rr_0^{dm}(0)\} \\ &-(1-\beta) k_{ec}^2 V_{11}^{-1} \bar{K}_0 \{(1+\mu\bar{B}) rr_0 - rr_0^{dm}(0)\}.\end{aligned}$$

Defining $\bar{k}_{e1} = (\lambda_x^{-1} k_{ex}^2 - k_{ec}^2 V_{11}^{-1})(1-\beta(1-\delta))\bar{K}_0 > 0$ and $\bar{k}_{e2} = -(1-\beta) k_{ec}^2 V_{11}^{-1} \bar{K}_0 \geq 0$. Note $\mu = 0$ if and only if $k_{ec} = 0$. Finally, note at $t = 0$, $\tilde{e}_t$ can be rewritten as

$$\tilde{e}_0 = -(\lambda_x^{-1} k_{ex}(1-\beta(1-\delta)) - \beta\delta k_{ec}^2 V_{11}^{-1})\bar{K}_0 \{(1+\mu\bar{B}) rr_0 - rr_0^{dm}(0)\} + O(\epsilon^2),$$

so $\tilde{e}_0$ has the same sign as $-\{(1+\mu\bar{B}) rr_0 - rr_0^{dm}(0)\}$.

Calvo and short bonds The problem is

$$\begin{aligned}\mathcal{W} = \max_{\{\tilde{b}_t, x_t, \pi_t\}} &\{\frac{1}{2}\frac{(1-\beta)V_{11}}{1-2m^{-1}u_1 V_{11}^{-1}\gamma_{ss}^*}(\bar{B} rr_0 - \mathcal{T}_s)^2 + \frac{1}{2} V_{11}\tilde{b}_0^2 - \frac{1}{2}\lambda_x x_0^2 - \frac{1}{2}\lambda_\pi \pi_{I0}^2 \\ &+ \sum \beta^t \{-\frac{1}{2}\lambda_x x_t^2 - \frac{1}{2}\lambda_\pi \pi_{It}^2 + \frac{1}{2} V_{11}(-\tilde{b}_t + \beta^{-1}\tilde{b}_{t-1})^2\} + t.i.p. + O(\epsilon^3)\end{aligned}$$

s.t.

$$\begin{aligned}\beta^{-1}(-k_{ex} x_0 + k_{ec}\tilde{b}_0^*) &= (1 + \beta^{-1}(1-\beta) k_{ec}) rr_0 - rr_0^{dm}(0) \\ \beta\pi_{It+1} + \kappa x_t &= \pi_{It}\end{aligned}$$

The continuation problem is a standard disinflation problem (clearly $\tilde{b}_t = \tilde{b}_{t-1}$ is the solution for savings part):

$$\mathcal{W}_{t\geq 1} = \sum \beta^t \{-\frac{1}{2}\lambda_x x_t^2 - \frac{1}{2}\lambda_\pi \pi_{It}^2\} + t.i.p. + O(\epsilon^3)$$

s.t.

$$\beta \pi_{It+1} + \kappa x_t = \pi_{It}$$

Solving this,

$$\begin{aligned} x_t &= \kappa \lambda_x^{-1} \tilde{\phi} \pi_{It} \\ \pi_{t+1} &= \frac{1}{\beta}(1 - \kappa^2 \lambda_x^{-1} \tilde{\phi}) \pi_{It} \end{aligned}$$

Note $1 - \kappa^2 \lambda_x^{-1} \tilde{\phi}_0 < \beta$. Thus, from $t \geq 2$ onwards,

$$\begin{aligned} \Delta \tilde{e}_t &= \pi_{It} + k_{ex} \Delta x_t \\ &= \mathcal{P}(k_{ex}) \pi_{It-1}. \end{aligned}$$

where

$$\mathcal{P}(k_{ex}) = (1 - k_{ex} \kappa \lambda_x^{-1} \tilde{\phi}) \frac{1}{\beta}(1 - \kappa^2 \lambda_x^{-1} \tilde{\phi}) + k_{ex} \kappa \lambda_x^{-1} \tilde{\phi}$$

This is clearly increasing in k_{ex} with $\mathcal{P}(0) = 1$ and $\mathcal{P}(\infty) < 0$.

The problem at 0 is given by

$$\mathcal{W} = \max_{\{\tilde{b}_t, x_t, \pi_t\}} \{\frac{1}{2} \frac{(1-\beta) V_{11}}{1 - 2m^{-1} u_1 V_{11}^{-1} \gamma_{ss}^*} (\bar{B} rr_0 - \mathcal{T}_s)^2 + \frac{1}{2} \beta^{-1} V_{11} \tilde{b}_0^2 - \frac{1}{2} \lambda_x x_0^2 - \frac{1}{2} \lambda_\pi \pi_{I0}^2 - \frac{1}{2} \beta \tilde{\phi} \pi_{I1}^2\} + t.i.p. + O(\epsilon^3)$$

s.t.

$$\begin{aligned} \beta \pi_{I1} + \kappa x_0 &= \pi_{I0} \\ (-k_{ex} x_0 - \pi_{I0} + k_{ec} \tilde{b}_0^*) &= \beta\{(1 + \beta^{-1}(1-\beta) k_{ec}) rr_0 - rr_0^{dm}(0)\} \end{aligned}$$

Let ν denote the multiplier on the promise-keeping constraint (i.e., the second one). The solution is:

$$\begin{aligned} \pi_{I1} &= \tilde{\phi}^{-1} \frac{\kappa \lambda_x^{-1} k_{ex} - \lambda_\pi^{-1}}{\beta \tilde{\phi}^{-1} + \kappa^2 \lambda_x^{-1} + \lambda_\pi^{-1}} \nu_0 \\ x_0 &= -\frac{\kappa \lambda_\pi^{-1} + k_{ex}(\beta \tilde{\phi}^{-1} + \lambda_\pi^{-1})}{\lambda_x(\beta \tilde{\phi}^{-1} + \kappa^2 \lambda_x^{-1} + \lambda_\pi^{-1})} \nu_0 \\ \pi_{I0} &= -\frac{(\kappa \lambda_x^{-1} k_{ex} + \beta \tilde{\phi}^{-1} + \kappa^2 \lambda_x^{-1})}{\lambda_\pi(\beta \tilde{\phi}^{-1} + \kappa^2 \lambda_x^{-1} + \lambda_\pi^{-1})} \nu_0 \end{aligned}$$

Note at $t = 0$, if $\nu > 0$, I get $x_0 < 0$ and $\pi_{I0} < 0$ unambiguously when $k_{ex} > 0$. This implies $\tilde{e}_0 < 0$. Depending on how large k_{ex} is, the continuation may feature inflation o deflation.

At $t = 1$,

$$\Delta \tilde{e}_1 = \pi_{I1} + k_{ex} \Delta x_1$$

If k_{ex} is large, $\pi_{I1} > 0$ and $x_1 > 0$. Thus, $\Delta\tilde{e}_1 > 0$. If k_{ex} is small, $\pi_{I0} < 0$ and $x_1 < 0$.

A.2.5 Proposition 8

Rigid prices and long bonds Savings taxes from $t \geq 1$, are

$$\begin{aligned}\tau_{B^*t} &= -V_{11}(\Delta\tilde{b}_{t+1} - \Delta\tilde{b}_t) + k_{ux}\Delta x_{t+1} \\ &= -\beta\delta k_{ec}\bar{K}_0(\Delta\tilde{r}_t) - k_{ux}\lambda_x^{-1}(1-\beta(1-\delta))k_{ex}\bar{K}_0(\Delta\tilde{r}_t) \\ &= (\beta\delta k_{ec} - \lambda_x^{-1}k_{ux}(1-\beta(1-\delta))k_{ex})\bar{K}_0\delta\tilde{r}_{t-1}\end{aligned}$$

Thus, it decays at rate $1-\delta$. Since, $\tilde{r}_t = (1-\delta)\tilde{r}_{t-1}$,

$$\tau_{B^*t} = (1-\delta)^t\tau_{B1}$$

At $t=0$ it's the same (there's no asymmetry). Thus,

$$\tau_{B^*0} = -\bar{K}_0\beta\delta(\beta\delta k_{ec} - k_{ex}k_{ux}\lambda_x^{-1})\{(1+\mu)rr_0 - rr_0^{dm}(0)\}$$

Since $\bar{K}_0 < 0$, and usually $k_{ex} > 0$, $k_{ux} > 0$, $k_{ec} < 0$, it has the opposite sign of $\{(1+\mu)rr_0 - rr_0^{dm}(0)\}$, i.e, if she creates a positive excess return, she taxes savings.

Calvo pricing and short bonds Savings tax ($t \geq 1$):

$$\begin{aligned}k_{ux}\Delta x_{t+1} &= \tau_{B^*t} \\ k_{ux}(\frac{1}{\beta}(1-\kappa^2\lambda_x^{-1}\tilde{\phi}) - 1)\pi_{It} &= \tau_{B^*t}\end{aligned}$$

So, if the economy is experiencing disinflation ($\pi_{It} > 0$), $\Delta x < 0$ and $\tau_{B^*} < 0$ (subsidize savings).
At $t=0$, the savings tax is given by

$$\tau_{B^*} = V_{11}\tilde{b}_0^* + k_{ux}\Delta x_{t+1}$$

and

$$\tilde{b}_0^* = -\beta V_{11}^{-1}\nu_0 k_{ec},$$

which has the same sign as μ.

B Appendix: Extensions

B.1 Cooperation

In the previous sections, the utility of the home household was the relevant welfare metric. As a result, the planner found it optimal to manipulate the stochastic discount factor of the foreigners that participate in home-currency bond markets to redistribute wealth to home agents. In this section, I revisit the optimal policy from the point of view of a supranational authority, which would also consider foreigners' welfare. More specifically, the welfare metric is now

$$\mathbb{E}\left(\lambda^H u(C_{Ts}, C_{Ns}, L_s, \xi_s) + m\lambda^F u^*(C_{Ts}^*, \xi_s)\right)$$

where λ^H and λ^F are fixed at their steady state values, implying the supranational authority does not seek to redistribute wealth ex ante.

Proposition 9. *(Cooperation) Let $\chi^{nc}(m)$ and $\mathcal{T}^{nc}(m)$ denote the parameter χ and the desired transfers in the noncooperative solution when the measure of foreigners is m. Then, in the cooperative solution,*

$$\chi^{coop}(m) = \chi^{nc}(2m)$$
$$\mathcal{T}^{coop}(m) = \mathcal{T}^{nc}(2m).$$

In other words, it is ***as if*** *the model had twice the number of foreigners. Furthermore, the optimal capital control (i.e., the portfolio tax) is zero to second-order,*

$$\tau_B = O(\epsilon^3).$$

Proposition 9 shows the connection with the decentralized solution. Since the planner now internalizes the benefits of insuring foreigners, she will desire larger transfers $\mathcal{T}$. In addition, since she realizes that the additional risk-premium from insurance increases foreigners welfare, she will be less afraid to allow the exchange rate to float given a position B, so χ increases. Noting that the cooperative solution is analogous to increasing m, proposition 3 and corollary 2 immediately imply that positions $|B|$ and the insurance weight ω are larger in the cooperative solution. Finally, since the global planner is not trying to redistribute wealth in expectation, there is no need for capital controls.

Corollary 3. *Under the same conditions as in proposition 3, the portfolio $|B|$ and the insurance weight ω are larger under cooperation (if $m < \infty$).*

B.1.1 Proof of Proposition B.1

I prove the result for the static model to simplify the exposition, but it is immediate that the result extends to the dynamic model. Expanding the welfare loss function, we obtain the following new term,

$$\text{new term} = \lambda_F m \left(u_1^* C^* c_s^* + \frac{1}{2} u_{11}^* C^{*2} c_s^{*2} + u_1^* C_{ss}^* c_s^{*2} \right) + O(\epsilon^3)$$

A second-order approximation to foreigners' budget constraint yields

$$C^* c_s^* + u_1^* C c_s^{*2} = Y_{ss}^* y_s^* + \frac{1}{2} Y_{ss}^{*2} y_s^{*2} - m^{-1} \bar{B} rr_s - m^{-1} \bar{B}^2 rr_s^2 - m^{-1} B_\epsilon rr_s$$

Replacing,

$$\mathbb{E}\text{new term} = m u_T \mathbb{E} \left(Y_{ss}^* y_s^* + \frac{1}{2} Y_{ss}^{*2} y_s^{*2} - m^{-1} \bar{B} rr_s - m^{-1} \bar{B}^2 rr_s^2 - \frac{1}{2} \gamma_{ss}^* C_{ss}^{*2} c_s^{*2} \right) + O(\epsilon^3)$$

where $\gamma_{ss}^* \equiv -u_1^{*-1} u_{11}^*$. Using a first-order approximation of foreigners' budget constraint to get rid of c_s^{*2},

$$\mathbb{E}\text{new term} = u_T \mathbb{E} \left(-\bar{B} rr_s - \bar{B}^2 rr_s^2 + \gamma_{ss}^* Y_{ss}^* y_s^* \bar{B} rr_s - \frac{1}{2} \gamma_{ss}^* m^{-1} \bar{B}^2 rr_s^2 \right) + O(\epsilon^3)$$

Thus, the new welfare loss function is given by

$$\mathbb{E}V = \mathbb{E}[u_T\bar{B}(r-e_s)\gamma^*_{ss}y^*_s - \frac{1}{2}u_T(\gamma^*_{ss}/m)\bar{B}^2(r-e_s)^2 + u_T\iota_s C_{ss}c_{Ts} \\ + \frac{1}{2}V_{11}C^2_{Tss}c^2_{Ts} + V_{1Z}Z_{ss}C_{Tss}z_s c_{Ts} - \frac{1}{2}\lambda_x x^2_s - \frac{1}{2}\lambda_\pi p^2_{Is}] + t.i.p. + O(\epsilon^3)$$

The only difference is the $\frac{1}{2}$ in front of the term $u_T(\gamma^*_{ss}/m)\bar{B}^2(r-e_s)^2$. Thus, the model is isomorphic to a model with having twice as many foreigners in the decentralized solution. There is one difference, however: the private utility still has the original m. It follows immediately from the proof of propositions 5 and 6 that $\tau_B = O(\epsilon^3)$ regardless of m.

B.2 Model with equity in nontradables

In this Appendix, I consider an extension of the example economy in Section (3.5) that allows for non-unitary elasticity of substitution between tradables and nontradables, as well as decreasing returns to scale in nontradable production:

$$u(s) = \ln((\alpha^{\frac{1}{\rho}}C_{Ts}^{\frac{\rho-1}{\rho}} + (1-\alpha)^{\frac{1}{\rho}}C_{Ns}^{\frac{\rho-1}{\rho}})^{\frac{\rho}{\rho-1}} - \frac{1-\alpha}{1+\psi}N_s^{1+\psi})$$

$$F = Z_s Y_{Is}^{\alpha_F}$$

Dividends on nontradable firms are then given by,

$$D_s = (1-\alpha_F)E_s^{-1}P_{Ns}Z_s Y_{Is}^{\alpha_F}$$

To a first-order approximation,

$$div_s = (1-\rho^{-1})x_s + (z_s + \alpha_F y_{Is}^{flex} + p_{Ns}^{*flex})$$

The elasticity of substitution is the key parameter governing the dependence of equity on monetary policy. On the one hand, a depreciation boosts employment, which increases profits. On the other hand, it also increases the supply of nontradables goods, depressing their price. If tradables and nontradables are complements, the price decreases significantly, and profits (in foreign currency) decrease, as in Ottonello (2015). If they are substitutes, the opposite is true: profits increase with a depreciation. In the knife-edge of $\rho = 1$, equity is independent from monetary policy and thus, absent nominal bonds, the optimal policy would be demand-management (i.e., inflation targeting). Note that, given that countries are short their own equity, and typically are debtors in home-currency, the total exposure will be larger with $\rho < 1$ since in that case, the return on both assets moves in the same direction with monetary policy.

Replacing the flexible price allocation y_{Is}^{flex},

$$div_s = (1-\rho^{-1})x_s + \frac{1}{\alpha_F\alpha + \rho(\psi+1-\alpha_F)}\{(\rho-1)(\psi+1)z_s + (\alpha_F\alpha + (\psi+1-\alpha_F))c_{Ts}\}$$

A similar argument that the one given for monetary policy explains that the effect of z_s on profits depends on ρ. Regardless of ρ, higher tradable consumption pushes up demand for nontradable goods, unambiguously increasing profits.

Assume the return on equity is the dividend plus noise,

$$rr_s^{eq} = div_s + \nu_s$$

and write

$$rr_s^{eq} = k_{rrx}^{eq} x_s - \mu^{eq} c_{Ts} + k_{rr\xi}^{eq} \xi_s$$

where $\xi_s = [z_s, \nu_s]$ collects the shocks.[56]

The welfare function is still given by

$$\mathbb{E}V = \mathbb{E}_0[\frac{1}{2}(1 - 2m^{-1}V_{11}^{-1}\gamma_{ss}^*)V_{11}(\bar{B}^{eq}rr^{eq} + \mathcal{T}_s)^2 - \frac{1}{2}(\lambda_x + \lambda_\pi\kappa^2)x_s^2] + O(\epsilon^3) + t.i.p. \quad (65)$$

Replacing,

$$\mathbb{E}V = -k_0\mathbb{E}_0[\frac{1}{2}(Brr_s - \mathcal{T}_s)^2 + \chi(1 + \mu^{eq}B)^2(rr_s - \frac{1}{1 + \mu^{eq}B}rr_s^{dm}(0))^2] + O(\epsilon^3) + t.i.p.$$

where

$$\chi = \frac{\lambda_x + \kappa^2\lambda_\pi}{k_{rrx}^2 V_{11}} = \frac{\lambda_x + \kappa^2\lambda_\pi}{(1 - \rho^{-1})^2 V_{11}}.$$

Using that $\kappa = \tilde{k}(\frac{1-\phi}{\phi})$ for some positive constant $\tilde{k} > 0$, it is immediate that price flexibility decreases the ability of monetary policy to provide insurance when the economy features real assets (indeed, $\chi \to \infty$ when $\phi \to 0$). By contrast, with nominal bonds, we had $k_{rrx} = -k_{ex} - \kappa$. The κ^2 in the denominator that appears in that case is what formally drives the difference in the results. The next remark summarizes this "real-asset" case.

Remark 3. In an economy where no asset loads on nominal quantities, i.e., in an economy with only real assets, higher price flexibility decreases the importance of the insurance motive (i.e., χ increases).

B.3 General asset structure: Static model

The reader may wonder at this point whether the results are driven by the very special asset structure considered in the previous sections. To address this concern, I now consider a general asset structure. To save notation, I assume there is still a risk-free asset in foreign-currency with a return normalized to one. Let $\Theta \in \mathbb{R}^N$ denote the remaining N assets in the economy. I allow these assets' returns to depend on any of the equilibrium variables and the shocks. After log-linearizing, the excess-return can be written in reduced form as

$$rr_s = k_{rrx}x_s - \mu c_{Ts} + k_{rr\xi}\xi_s + O(\epsilon^2)$$

where $k_{rrx}, \mu \in \mathbb{R}^N$ and $k_{rr\xi} \in \mathbb{R}^{N+S}$. I assume $N < S$ so that markets are still locally incomplete. Premultiplying by Θ', assuming $\Theta' k_{rrx} \neq 0$, and solving yields

$$x_s = \frac{1 + \Theta'\mu}{\Theta' k_{rrx}}(\Theta' rr_s - \frac{\Theta'}{1 + \Theta'\mu}rr_s^{dm}(0)). \quad (66)$$

[56]Noise is introduced to prevent the planner from approximating the first-best arbitrarily closely when $\rho = 1$.

Although there are many assets, the planner still has a single way of deviating from demand-management - by creating output gaps. This suggests this economy will still have a "sufficient statistic" for the sensitivity to monetary policy. Indeed, this sufficient statistic is given by

$$B \equiv \frac{\Theta' k_{rrx}}{1 + \Theta' \mu}. \tag{67}$$

Lemma 5. *Suppose $B \neq 0$.*[57] *Then, the objective function can be written as*

$$\mathbb{E}V(e_s, \Theta) = -k_0 \mathbb{E}_0[\frac{1}{2}(\Theta' rr_s - \mathcal{T}_s)^2 + \frac{1}{2}\chi B^{-2}(\Theta' rr_s - \frac{\Theta'}{1+\Theta'\mu} rr_s^B(\mathbf{0}))^2] + t.i.p + O(\epsilon^3)$$

Taking the first-order condition with respect to $\bar{rr}_s = \Theta' rr_s$, I find that the total return of the portfolio is still a weighted average of the "demand-management" target and the "insurance target",

$$\bar{rr}_s = \frac{B^2}{B^2 + \chi}(\Theta' \bar{rr}_s)^{in} + \frac{\chi}{\chi + B^2}(\Theta' \bar{rr}_s)^{dm} + O(\epsilon^2) \tag{68}$$

where

$$(\Theta' \bar{rr}_s)^{in} = \mathcal{T}_s$$
$$(\Theta' \bar{rr}_s)^{dm} = \frac{1}{1 + \Theta' \mu}\Theta' rr_s(\mathbf{0})$$

and $rr_s(\mathbf{0})$ is the return in financial autarky. Replacing back, I obtain

$$\mathbb{E}V(e_s, \tilde{\Theta}, B) = -\frac{1}{2}\frac{k_0 \chi}{B^2 + \chi}\mathbb{E}_0[\mathcal{T}_s^2 + (\tilde{\Theta}' rr_s^{dm}(\mathbf{0}))^2 - 2\mathcal{T}_s \tilde{\Theta}' rr_s^{dm}(\mathbf{0})] + t.i.p. + O(\epsilon^3) \tag{69}$$

where $\tilde{\Theta}' \equiv (1 + \mu' \Theta)^{-1}\Theta'$ is a convenient rotation to cancel out the wealth effect.

Proposition 10. *The optimal portfolio $\tilde{\Theta}$ conditional on some sensitivity to monetary policy B is given by*

$$\tilde{\Theta} = k_{\Theta 0} + k_{\Theta B} B + O(\epsilon) \tag{70}$$

where

$$k_{\Theta 0} = \left(\Sigma^{-1}_{rr^{dm}(\mathbf{0})} - \Sigma^{-1}_{rr^{dm}(\mathbf{0})} k_{rrx}(k'_{rrx}\Sigma^{-1}_{rr^{dm}(\mathbf{0})} k_{rrx})^{-1} k'_{rrx}\Sigma^{-1}_{rr^{dm}(\mathbf{0})}\right)\sigma_{rr^{dm}(\mathbf{0})\mathcal{T}}$$
$$k_{\Theta B} = \Sigma^{-1}_{rr^{dm}(\mathbf{0})} k_{rrx}(k'_{rrx}\Sigma^{-1}_{rr^{dm}(\mathbf{0})} k_{rrx})^{-1}.$$

Note that the necessary transfer may now be smaller than before: Even without any exposure to monetary policy, the planner may choose the portfolio to diversify away some risk. For example, if she can sell claims to the tradable endowment, then she does not need to use monetary policy, which is a costly source of insurance, against these shocks. This is captured by $k_{\Theta 0}$. Furthermore, even if assets are uncorrelated with the desired transfers, they may still be useful to hedge the "undesirable transfers" created by the nominal asset. This last effect is captured by $k_{\Theta B}$. Define the

[57] If $B = 0$, then feasibility implies $\Theta' rr_s - \frac{\Theta'}{1+\Theta'\mu} rr_s^B(\mathbf{0})$. Thus, this objective function is continuous in B at $B = 0$.

realized excess-return of the portfolio that is *sensitive to monetary policy* as

$$rr_s^B = k'_{\Theta B} rr_s. \tag{71}$$

Then, (68) can be expressed in a more familiar form,

$$rr_s^B = \frac{B^2}{B^2+\chi} rr_s^{B,in} + \frac{\chi}{\chi+B^2} rr_s^{B,dm} \tag{72}$$

with

$$\begin{aligned} rr_s^{B,in} &= B^{-1}(\mathcal{T}_s - k'_{\Theta 0} rr_s(\mathbf{0})) \\ rr_{ss}^{B,dm} &= k'_{\Theta B} rr_s^{dm}(\mathbf{0}) \end{aligned}$$

Note that the linearity of the model implies *all endogenous variables* are weighted averages of their target values. In other words, the exchange rate is also a weighted average,

$$e_s^B = \frac{B^2}{B^2+\chi} e_s^{B,in} + \frac{\chi}{\chi+B^2} e_s^{B,dm}.$$

Replacing (70),

$$\mathcal{W}(B) = \frac{1}{2}\frac{k_0}{B^2+\chi}\mathbb{E}_0[B^2\sigma^2_{\tilde{\mathcal{T}}} + \chi^2\sigma^2_{rr^B(\mathbf{0})} + 2B\chi\sigma_{\tilde{\mathcal{T}}rr^B(\mathbf{0})}] + t.i.p. + O(\epsilon^3) \tag{73}$$

where $\tilde{\mathcal{T}}_s = \mathcal{T}_s - k'_{\Theta 0} rr_s^{dm}(0)$. It is immediate from (72) and (73) that propositions 3 and 4 hold in this environment in terms of the exposure to monetary policy and volatility of the returns that are sensitive to it. If there is a single "exposed" asset, then it is immediate that the volatility of the excess-returns of such an asset, which may be called "home-currency" asset, is lower under the optimal policy. Finally, I show that the generality of the asset markets still does not justify a tax if $m < \infty$ or countries cooperate. Even though incomplete markets introduce pecuniary externalities, these are still proportional to the value of social utility asset-by-asset under the optimal policy, implying portfolio taxes are still not required.

Proposition 11. *Propositions 3 and 4 hold in terms of the "sensitivity" to monetary policy B defined in (67) and its return defined in (71). In addition, if $m = \infty$ or countries cooperate, the optimal capital controls policy is*

$$\tau^j = O(\epsilon^3)\ \forall j.$$

B.3.1 Proof of lemma 5

Following the same steps as in the proof of lemma 2 I obtain,

$$\mathbb{E}V = \mathbb{E}[\frac{1}{2}V_{11}(\bar{\Theta}' rr_s - \mathcal{T}_s)^2 - \frac{1}{2}\tilde{\lambda}_x x_s^2] + t.i.p. + O(\epsilon^3)$$

where $\tilde{\lambda}_x = \lambda_x + \kappa^{-2}\lambda_\pi$. Using equations (66) and (67) and replacing I obtain the desired expression.

B.3.2 Proof of lemma 10

To prove this one needs to maximize (69) subject to $B = k'_{rrx}\tilde{\Theta}$. The FOC yields,

$$\Sigma_{rr}\tilde{\Theta} - \sigma_{rr\mathcal{T}} - \lambda k_{rrx} = 0$$
$$k'_{rrx}\tilde{\Theta} = B$$

Solving this using a formula for the inverse of a block matrix yields the desired result.

B.3.3 Proof of proposition 11

I prove here the result on taxes. The remainder follow from the isomorphism of equations (72) and (73). To prove the no-tax result, take a few steps back and note that the planner maximizes

$$U = \mathbb{E}_0[u_T \sum_j \bar{\Theta}^j rr^j_s \gamma^*_{ss} y^*_s + u_T \iota_s C_{ss} c_{Ts} + \frac{1}{2} V_{11} C^2_{ss} c^2_{Ts} + V_{1Z} Z_{ss} Y_{ss} z_s c_{Ts} - \frac{1}{2}\tilde{\lambda}_x x^2_s] + t.i.p. + O(\epsilon^3)$$

subject to

$$c_{Ts} = y_{Ts} + \bar{\Theta}' \bar{r}r_s$$
$$rr^j_s = k^j_{rrx} x_s - \mu^j (c_{Ts} - c_{Ts}(\mathbf{0})) + k^j_{rr\xi} \xi_s$$

The FOC yield

$$u_T \iota_s + V_{11} C_{ss} c_{Ts} + V_{1Z} Z_{ss} Y_{ss} z_s + \sum_j k^j_{rrc} \lambda^j_{rrs} = u_1 \lambda_s$$
$$u_T \Theta^j (\lambda_s + \gamma^*_{ss} y^*_s) = \lambda^j_{rrs}$$
$$\tilde{\lambda}_x x_s = \sum k^j_{rrx} \lambda^j_{rrs}$$
$$\mathbb{E} rr^j_s (\lambda_s + \gamma^*_{ss} y^*_s) = 0$$

Recall that private marginal utility is given by

$$u_T \iota_s + V_{11} C_{ss} c_{Ts} + V_{1Z} Z_{ss} Y_{ss} z_s + k_{ux} x_s = u_1 \lambda^{priv}_s$$

Note here the two sources of externalities: $\sum_j k^j_{rrc} \lambda^j_{rr}$ captures pecuniary externalities due to incomplete markets,[58] while $k_{ux} x_s$ captures the aggregate-demand externality due to nominal rigidities. However: (i) the second FOC implies the pecuniary externality is proportional to the social value of insurance asset-by-asset; (ii) the aggregate demand externality is **purely endogenous** and is thus only there to relax pecuniary externalities. (i) and (ii) together imply that the strength of the externalities must be related to their social value $\lambda_s + \gamma^*_{ss} y^*_s$. Thus, optimality of the portfolio implies

$$\mathbb{E} rr^j_s x_s = 0 \forall \text{j}$$
$$\mathbb{E} rr^j_s \lambda^j_{rr} = 0, \forall \text{j}$$

[58] They were still there in the two asset case, but they did not show up because I had directly substituted out consumption.

which in turn implies

$$\mathbb{E}rr_s^j(\lambda_s^{priv} + \gamma_{ss}^* y_s^*) = 0\forall \text{j}$$

so $\tau^j = O(\epsilon^3)$ ∀j.

B.4 General asset structure: Dynamic model

In this section, I study an economy with multiple assets. Noting the "relaxed" continuation problem is the same as before, I obtain:

$$\mathcal{W} = \max_{\{\tilde{b}_t, x_t, \pi_t\}} \{\frac{1}{2}\frac{(1-\beta)V_{11}}{1-2m^{-1}u_1 V_{11}^{-1}\gamma_{ss}^*}(\bar{\Theta}' rr_0 - \mathcal{T}_s)^2 + \frac{1}{2}V_{11}\tilde{b}_0^2 - \frac{1}{2}\lambda_x x_0^2 - \frac{1}{2}\lambda_\pi \pi_{I0}^2 \tag{74}$$
$$+ \sum \beta^t \{-\frac{1}{2}\lambda_x x_t^2 - \frac{1}{2}\lambda_\pi \pi_{It}^2 + \frac{1}{2}V_{11}(-\tilde{b}_t + \beta^{-1}\tilde{b}_{t-1})^2\} + t.i.p. + O(\epsilon^3)$$

I divide assets into two classes: "nominal" assets, which have a stationary price in home-currency such as a nominal bond, and "real" assets, which have a stationary price in foreign-currency. I rule out "mixed" assets, which would not have a stationary price in either currency. For tractability, I assume dividends of asset j decay at rate $\delta^j \leq 1$. For simplicity, I assume there is a risk-free asset in foreign currency, although this is not necessary.

The return on real assets is assumed to satisfy

$$r_{t-1}^j = (1-\beta(1-\delta^j))\left(k_{rrx}^j x_t + k_{rrc}^j c_{Tt} + k_{rr\xi}^j \xi_t\right) + k_{rr\pi}^j \pi_{It} + \beta(1-\delta^j) r_t^j - r_{t-1}^*.$$

For these assets, define

$$\tilde{r}_t^j = r_t^j + k_{rrc}^j\{(1-\beta)\bar{\Theta}' rr_0 + \beta^{-1}(1-\beta)\tilde{b}_t\} - r_0^{j,dm}(\mathbf{0}).$$

Then,

$$\beta^{-1}\left\{(1-\beta(1-\delta^j))k_{rrx}^j x_0 + k_{rr\pi}^j \pi_{I0} - \delta^j k_{rrc}^j \tilde{b}_0^* - \beta(1-\delta^j)\tilde{r}_0^j\right\} = rr_0^j - \beta^{-1}(1-\beta)k_{rrc}^j(\Theta' rr_0) - rr_0^{j,dm}(\mathbf{0})$$
$$-(1-\beta(1-\delta^j))k_{rrx}^j x_t + k_{rr\pi}^j \pi_{It} + \delta^j k_{rrc}^j \Delta\tilde{b}_t^* + \beta(1-\delta^j)\tilde{r}_t^j = \tilde{r}_{t-1}^j$$

The return on nominal assets is assumed to satisfy

$$r_{t-1}^j = \Delta e_t + \beta(1-\delta^j) r_t^j - \psi_{t-1}^j - r_{t-1}^*.$$

Define like before,

$$\tilde{r}_0^j = r_0^j - r_0^{j,dm}(\mathbf{0}) + k_{ex}x_0 - \beta^{-1}k_{ec}\tilde{b}_0^*$$
$$\tilde{r}_t^j = r_t^j - r_t^{j,dm}(\mathbf{0}) + k_{ex}x_t - \beta^{-1}k_{ec}\Delta\tilde{b}_t$$

where $r_0^{dm}(\mathbf{0})$ is again the return when the agent only holds risk-free short foreign-currency bonds B^*. Then,

$$\beta^{-1}\left\{-(1-\beta(1-\delta^j))k_{ex}x_0 - \pi_{I0} - \delta k_{ec}\tilde{b}_0^* - \beta(1-\delta)\tilde{r}_0\right\} = rr_0^j + \beta^{-1}(1-\beta)k_{ec}(\bar{\Theta}' rr_0) - rr_0^{j,dm}(0)$$
$$k_{ex}(1-\beta(1-\delta^j))x_t + \pi_{It} - \delta k_{ec}\Delta\tilde{b}_t^* + \beta(1-\delta)\tilde{r}_t = \tilde{r}_{t-1}$$

Define $k_{rrx} = -k_{ex}$, $k_{rr\pi} = -1$, $k_{rrc} = -k_{ec}$ for nominal assets. Note that, within real assets, there are two classes of assets: those sensitive to monetary policy and savings taxes (i.e., either $k^j_{rrc} \neq 0$, $k^j_{rrx} \neq 0$, or $k^j_{rrc} \neq 0$) and those that are not ($k^j_{rrc} = k^j_{rrx} = k^j_{rrc} = 0$). All nominal assets are clearly sensitive. Let J_1 denote the set of "sensitive" assets, and J_2 denote the set of "insensitive" assets. For all $j \in J_2$, it must be that $\tilde{r}^j_t = 0$.

The next proposition shows that taxes are still zero in this environment. However, unless there is only one asset sensitive to monetary policy (i.e., $\#J_1 = 1$), there is no longer one "sufficient statistic" so propositions (3) and (4) do not carry over. While the solution is not explicit when $\#J_1 > 1$, the solution solves a polynomial ensuring all possible candidates are taken into account, providing an algorithm for this case.

Proposition 12. *Let Θ^{J_1} denote the positions in assets in J_1 (i.e., sensitive to monetary policy) and Θ^{J_2} denote the positions in assets in J_2 (i..e, insensitive to monetary policy). Furthermore, let Σ_{J_i} denote the excess-returns of assets in J_i in a savings-only economy, $\Sigma_{J_1 J_2}$ their covariance, and $\Sigma_{J\mathcal{T}}$ the covariance with the desired transfers. Also, define*

$$\tilde{\Theta}' \equiv \Theta'(I_{J_1 \times J_1} + \mu \Theta'^{J_1})^{-1}.$$

The optimal weight on the insurance motive is

$$\omega = \frac{\tilde{\Theta}^{J_1\prime} \chi^{-1} \tilde{\Theta}^{J_1}}{1 + \tilde{\Theta}^{J_1\prime} \chi^{-1} \tilde{\Theta}^{J_1}}$$

where χ is now a symmetric positive definite matrix $\#J_1 \times \#J_1$. The optimal portfolio on assets that are insensitive to monetary policy Θ^{J_2} is given by:

$$\tilde{\Theta}^{J_2} = -\Sigma_{J_2}^{-1} \Sigma'_{J_2 J_1} \tilde{\Theta}^{J_1} + \Sigma_{J_2}^{-1} \Sigma'_{\mathcal{T} J_2}.$$

The optimal portfolio on assets that are sensitive to monetary policy $\tilde{\Theta}^{J_1}$ solves

$$\mathcal{W} = -\frac{1}{2} \frac{k_0}{1 + \tilde{\Theta}^{J_1\prime} \chi^{-1} \tilde{\Theta}^{J_1}} [\tilde{\sigma}^2_{\mathcal{T}} + \tilde{\Theta}^{J_1\prime} \tilde{\Sigma}_{J_1} \tilde{\Theta}^{J_1} - 2 \tilde{\Sigma}_{\mathcal{T} J_1} \tilde{\Theta}^{J_1}] + t.i.p + O(\epsilon^3)$$

where χ is now a symmetric positive definite matrix $\#J_1 \times \#J_1$ and

$$\tilde{\sigma}^2_{\mathcal{T}} = \sigma^2_{\mathcal{T}} - \Sigma_{\mathcal{T} J_2} \Sigma_{J_1}^{-1} \Sigma'_{\mathcal{T} J_2}$$
$$\tilde{\Sigma}_{J_1} = \Sigma_{J_1} - \Sigma_{J_1 J_2} \Sigma_{J_2}^{-1} \Sigma'_{J_1 J_2}$$
$$\tilde{\Sigma}_{\mathcal{T} J_1} = \Sigma_{\mathcal{T} J_1} - \Sigma_{\mathcal{T} J_2} \Sigma_{J_1}^{-1} \Sigma'_{J_1 J_2}.$$

If there is only one asset sensitive to monetary policy (i.e., if $\#J_1 = 1$), then propositions (3) and (4) carry over to this environment. Regardless of $\#J_1$, if $m = \infty$ optimal portfolio taxes are given by

$$\tau^j = O(\epsilon^3).$$

B.4.1 Proof & algorithm for solving $\tilde{\Theta}$

The continuation problem is to maximize (74) subject to

$$\kappa x_t + \beta \pi_{It+1} = \pi_{It}$$
$$\beta^{-1}\left\{(1-\beta(1-\delta^j))k^j_{rrx}x_0 + k^j_{rr\pi}\pi_{I0} - \delta^j k^j_{rrc}\tilde{b}^*_0 - \beta(1-\delta^j)\tilde{r}^j_0\right\} = rr^j_0 - \beta^{-1}(1-\beta)k^j_{rrc}(\Theta' rr_0) - rr^{j,dm}_0(\mathbf{0})\forall j \in J_1$$
$$-(1-\beta(1-\delta^j))k^j_{rrx}x_t + \delta^j k^j_{rrc}\Delta\tilde{b}^*_t + \beta(1-\delta^j)\tilde{r}^j_t = \tilde{r}^j_{t-1}\forall j \in J_1$$

I solve this problem the same way as before. First, I take the $t \geq 1$ problem with $\tilde{b}^*_0$, $\tilde{r}_0$ and π_{It} taken as given. Then, I use the solution to solve the $t = 0$ problem.

$$\mathcal{W} = -\frac{1}{2}k_0(\bar{\Theta}' rr_0 - \mathcal{T}_s)^2 - \frac{1}{2}(rr^{J_1}_0 + \mu\Theta' rr_0 - rr^{J_1,dm}_0(\mathbf{0}))'\chi(rr^{J_1}_0 + \mu\Theta' rr_0 - rr^{J_1 dm}_0(\mathbf{0})) + t.i.p + O(\epsilon^3)$$

where χ is now a $J_1 \times J_1$ symmetric positive-definite matrix and $\mu = -\beta^{-1}(1-\beta)k^{J_1}_{rrc}$. This can be rewritten as

$$\mathcal{W} = -\frac{1}{2}k_0\left\{(\bar{\Theta}' rr_0 - \mathcal{T}_s)^2 + (rr^{J_1}_0 - \tilde{rr}^{J_1}_0)'(I + \mu\Theta^{J_1\prime})'\chi(I + \mu\Theta^{J_1\prime})(rr^{J_1}_0 - \tilde{rr}^{J_1}_0)\right\} + t.i.p + O(\epsilon^3)$$

where

$$\tilde{rr}^{J_1}_0 = (I + \mu\Theta^{J_1\prime})^{-1}(-\mu\Theta^{J_2\prime}rr^{J_2}_0 + rr^{J_1,dm}_0(\mathbf{0})).$$

where I assume $(I + \mu\Theta^{J_1\prime})^{-1}$ is invertible, which is necessary for the equilibrium to be well-defined. (This is equivalent to $B \neq -\mu^{-1}$ in the two-asset model). Next, we choose $rr^{J_1}_0$ to minimize this expression subject to $\Theta' rr_0 = \bar{T}$. Let ν_0 denote the multiplier. The FOC yields

$$(I + \mu\Theta^{J_1\prime})'\chi(I + \mu\Theta^{J_1\prime})(rr^{J_1}_0 - \tilde{rr}^{J_1}_0) + \nu_0\Theta^{J_1} = 0$$

Solving for the multiplier,

$$\nu_0 = -\frac{1}{\tilde{\Theta}^{J_1\prime}\chi^{-1}\tilde{\Theta}^{J_1}}(\Theta^{J_1}rr^{J_1}_0 - \tilde{\Theta}^{J_1\prime}rr^{J_1,dm}_0(\mathbf{0}) + \mu\tilde{\Theta}^{J_2\prime}rr^{J_2}_0)$$

so

$$(I + \mu\Theta'^{J_1})(rr^{J_1}_0 - \tilde{rr}^{J_1}_0) = \frac{\chi^{-1}\tilde{\Theta}^{J_1}}{\tilde{\Theta}^{J_1\prime}\chi^{-1}\tilde{\Theta}^{J_1}}(\Theta^{J_1}rr^{J_1}_0 - \tilde{\Theta}^{J_1\prime}rr^{J_1,dm}_0(\mathbf{0}) + \mu\tilde{\Theta}^{J_2\prime}rr^{J_2}_0)$$

Replacing in the objective and simplifying,

$$\mathcal{W} = -\frac{1}{2}k_0\left\{(\bar{T} - \mathcal{T}_s)^2 + \frac{1}{\tilde{\Theta}^{J_1\prime}\chi^{-1}\tilde{\Theta}^{J_1}}(\bar{T} - \tilde{\Theta}' rr^{dm}_0(\mathbf{0}))^2\right\} + t.i.p + O(\epsilon^3)$$

The FOC with respect to $\bar{T}$ yields

$$\bar{T} = \frac{\tilde{\Theta}^{J_1\prime}\chi^{-1}\tilde{\Theta}^{J_1}}{1 + \tilde{\Theta}^{J_1\prime}\chi^{-1}\tilde{\Theta}^{J_1}}\mathcal{T}_s + \frac{1}{1 + \tilde{\Theta}^{J_1\prime}\chi^{-1}\tilde{\Theta}^{J_1}}\tilde{\Theta}' rr^{dm}_0(\mathbf{0}).$$

Note that using the same arguments as in the two-asset dynamic model, we see that every endogenous variable can be written as a weighted average, i.e.,

$$e_t = \frac{\tilde{\Theta}^{J_1\prime}\chi^{-1}\tilde{\Theta}^{J_1}}{1+\tilde{\Theta}^{J_1\prime}\chi^{-1}\tilde{\Theta}^{J_1}} e_t^{in} + \frac{1}{1+\tilde{\Theta}^{J_1\prime}\chi^{-1}\tilde{\Theta}^{J_1}} e_t^{dm}$$

where once again e_t^{in} is the one that minimizes the cost. The deviation can be computed from the paths for x_t and π_{It} computed above:

$$\tilde{e}_t^{in} = e_t^{in} - \{e_t^{dm}(\mathbf{0}) + k_{ec}\mathcal{T}_s\}.$$

Replacing back,

$$\mathcal{W} = -\frac{1}{2}\frac{k_0\chi}{\tilde{\Theta}^{J_1\prime}\chi^{-1}\tilde{\Theta}^{J_1}+\chi}\mathbb{E}_0[\mathcal{T}_s^2 + (rr_s^{dm}(0)'\tilde{\Theta})^2 - \mathcal{T}_s rr_s^{dm}(\mathbf{0})'\tilde{\Theta}] + t.i.p. + O(\epsilon^3)$$

Solving for the optimal "insensitive" assets yields

$$\tilde{\Theta}^{J_2} = -\Sigma_{J_2}^{-1}\Sigma_{J_2J_1}'\tilde{\Theta}^{J_1} + \Sigma_{J_2}^{-1}\Sigma_{\mathcal{T}J_2}'.$$

Replacing back,

$$\mathcal{W} = -\frac{1}{2}\frac{k_0\chi}{\tilde{\Theta}^{J_1\prime}\chi^{-1}\tilde{\Theta}^{J_1}+\chi}[\tilde{\sigma}_{\mathcal{T}}^2 + \tilde{\Theta}^{J_1\prime}\tilde{\Sigma}_{J_1}\tilde{\Theta}^{J_1} - 2\tilde{\Sigma}_{\mathcal{T}J_1}\tilde{\Theta}^{J_1}] + t.i.p + O(\epsilon^3)$$

where

$$\begin{aligned}\tilde{\sigma}_{\mathcal{T}}^2 &= \sigma_{\mathcal{T}}^2 - \Sigma_{\mathcal{T}J_2}\Sigma_{J_1}^{-1}\Sigma_{\mathcal{T}J_2}' \\ \tilde{\Sigma}_{J_1} &= \Sigma_{J_1} - \Sigma_{J_1J_2}\Sigma_{J_2}^{-1}\Sigma_{J_1J_2}' \\ \tilde{\Sigma}_{\mathcal{T}J_1} &= \Sigma_{\mathcal{T}J_1} - \Sigma_{\mathcal{T}J_2}\Sigma_{J_1}^{-1}\Sigma_{J_1J_2}'.\end{aligned}$$

This does not have a closed-form solution if $\#J_1 > 1$. However, there are finite solutions that can be compared. To see this, define $B^2 = \tilde{\Theta}^{J_1\prime}\chi^{-1}\tilde{\Theta}^{J_1}$ and solve the problem conditional on a "sensitivity" B. This yields

$$\begin{aligned}\tilde{\Sigma}_{J_1}\tilde{\Theta}^{J_1} - \tilde{\Sigma}_{\mathcal{T}J_1} - \lambda\chi^{-1}\tilde{\Theta}^{J_1} &= 0 \\ \tilde{\Theta}^{J_1\prime}\chi^{-1}\tilde{\Theta}^{J_1} &= B^2.\end{aligned}$$

Solve for $\tilde{\Theta}$,

$$\tilde{\Theta}^{J_1\prime} = (\tilde{\Sigma}_{J_1} - \lambda\chi^{-1})^{-1}\tilde{\Sigma}_{\mathcal{T}J_1}$$

and replace to obtain an equation in λ,

$$\tilde{\Sigma}_{\mathcal{T}J_1}'(\tilde{\Sigma}_{J_1} - \lambda\chi^{-1})^{-1}\chi^{-1}(\tilde{\Sigma}_{J_1} - \lambda\chi^{-1})^{-1}\tilde{\Sigma}_{\mathcal{T}J_1} = B^2.$$

Note this can be written as

$$\frac{\mathcal{P}_1(\lambda)}{(\mathcal{P}_2(\lambda))^2} = B^2$$

where $\mathcal{P}_1(\lambda)$ is a polynomial of degree $(\#(J_1)-1)^2$ and $\mathcal{P}_2(\lambda)$ is a polynomial degree $\#(J_1)$. Thus, there are at most $\max\{(\#J_1-1)^2, \#J_1\}$ solutions which need to be checked. Using this and then maximizing over B^2 one can compute the optimal portfolios. Unfortunately, $\tilde{\Theta}$ is nonlinear in B if $\#J_1 > 1$, so propositions (3) and (4) do not carry over.

No portfolio tax Next, I show that the result on taxes does carry over. I have:

$$\mathcal{W} = \max\{\frac{1}{2}(1-\beta)V_{11}(\Theta' rr_0 - \mathcal{T})^2 + \frac{1}{2}V_{11}\tilde{b}_0^2 - \frac{1}{2}\lambda_x x_0^2 - \frac{1}{2}\lambda_\pi \pi_{I0}^2 + \sum_{t\geq 1}\beta^t\{-\frac{1}{2}\lambda_x x_t^2 - \frac{1}{2}\lambda_\pi \pi_{It}^2 + \frac{1}{2}V_{11}\tilde{c}_{Tt}^2\} + t.i.p. + O(\epsilon^3)$$

s.t.

$$\kappa x_t + \beta \pi_{It+1} = \pi_{It}$$
$$\beta^{-1}(k_{rrx}x_0 + k_{rr\pi}\pi_{I0} + k_{rrc}\tilde{c}_{Tt} + \beta(1-\delta)(r_0 - r_0^{dm}(0)) = rr_0^j + k_{rrc}^j \Theta' rr_0 - rr_0^{dm}(0)$$
$$k_{ex}\Delta x_t + \pi_{It} + k_{ec}\Delta\tilde{c}_{Tt} + \beta(1-\delta)(r_t - r_t^{dm}(0)) = (r_{t-1} - r_{t-1}^{dm}(0))$$

The first-order condition with respect Θ^j yields

$$\mathbb{E}rr_0^j\left(V_{11}(\Theta' rr_0 - \mathcal{T}_s) + \sum_{m\in J_1} k_{rrc}^m \phi^m\right) = 0.$$

The first-order condition with respect to rr^j yields

$$\phi^j + \Theta^j \sum_{m\in J_1} k_{rrc}^m \phi^m = -\Theta^j V_{11}(\Theta' rr_0 - \mathcal{T}).$$

Note:

$$\phi^j + \Theta^j \sum_{m\in J_1} k_{rrc}^m \phi^m = -\Theta^j V_{11}(\Theta' rr_0 - \mathcal{T})$$
$$k_{rrc}^j\phi^j + k_{rrc}^j\Theta^j \sum_{m\in J_1} k_{rrc}^m \phi^m = -k_{rrc}^j\Theta^j V_{11}(\Theta' rr_0 - \mathcal{T})$$
$$\sum_{m\in J_1} k_{rrc}^j\phi^j + \sum_{m\in J_1} k_{rrc}^j\Theta^j \sum_{m\in J_1} k_{rrc}^m \phi^m = -\sum_{m\in J_1} k_{rrc}^j\Theta^j V_{11}(\Theta' rr_0 - \mathcal{T})$$
$$\sum_{m\in J_1} k_{rrc}^m \phi^m = -\frac{\sum_{m\in J_1} k_{rrc}^j\Theta^j}{1+\sum_{m\in J_1} k_{rrc}^j\Theta^j} V_{11}(\Theta' rr_0 - \mathcal{T})$$

Thus,

$$\frac{1}{1+\sum k_{rrc}^j \Theta^j}\mathbb{E}rr_0^j\left(\Theta' rr_0 - \mathcal{T}\right) = 0.$$

Recalling that

$$rr_0^{J_1} + \mu\Theta' rr_0 - rr_0^{J_1,dm}(\mathbf{0}) \propto \Theta' rr_0 - \tilde{\Theta}' rr_0^{dm}(\mathbf{0}) \propto \Theta' rr_0 - \mathcal{T}$$

implies that the two sources of externalities introduced by the planner are once more proportional to the value of insurance,

$$x_t \propto \Theta' r r_0 - \mathcal{T}$$
$$\tilde{b}_t \propto \Theta' r r_0 - \mathcal{T}$$

so the result still follows.

B.5 No capital controls

I will show how the results change in the static model. Clearly, the availability of portfolio taxes only matters when $m < \infty$. The dynamic model can be solved analogously (the continuation problem is the same). Without capital controls, one needs to keep track of the home no-arbitrage equation. Proceeding as in the proof of proposition 5, one obtains

$$\mathbb{E}\left[\bar{B}e_s\left((\bar{B}e_s + \mathcal{T}_s^{priv}) + k_{ux}\chi((1+\mu\bar{B})e_s - e_s^{dm}(0))\right)\right] = O(\epsilon^3) \tag{75}$$

where

$$\mathcal{T}_s^{priv} = \frac{1 - u_1 V_{11}^{-1} 2\frac{\gamma_{ss}^*}{m}}{1 - u_1 V_{11}^{-1}\frac{\gamma_{ss}^*}{m}}\mathcal{T}_s = \tilde{k}(m)\mathcal{T}_s$$

is the transfer desired by home agents.

The problem without portfolio taxes is to maximize

$$\mathbb{E}V(\{e_s, \bar{B}\}) = -k_0\mathbb{E}[\frac{1}{2}(\bar{B}e_s + \mathcal{T}_s)^2 + \frac{1}{2}\chi((1+\mu\bar{B})e_s - e_s^{dm}(0))^2] + t.i.p. + O(\epsilon^3) \tag{76}$$

subject to (75), which is an additional quadratic constraint. Ignoring this constraint would lead to a solution $\bar{B}$ that is not feasible as $\epsilon \to 0$. Formally, one may see that the multiplier on the Home no-Arbitrage condition is also indeterminate at the steady state. The planner's FOC with respect to the portfolio and the home-no arbitrage condition are the two set of conditions that together allow one to pin down these values at the steady state. Let $\bar{\eta}$ denote the multiplier. The Lagrangian is given by,

$$\begin{aligned}\mathbb{E}V(\{e_s, \bar{B}\}) = -k_0\mathbb{E}[&\frac{1}{2}(\bar{B}e_s + \mathcal{T}_s)^2 + \frac{1}{2}\chi((1+\mu\bar{B})e_s - e_s^{dm}(0))^2 \\ &- \bar{\eta}\bar{B}e_s\left((\bar{B}e_s + \tilde{k}(m)\mathcal{T}_s) + k_{ux}\chi((1+\mu\bar{B})e_s - e_s^{dm}(0))\right)] + t.i.p. + O(\epsilon^3)\end{aligned}$$

Conditional on the multiplier $\bar{\eta}$ and the position $\bar{B}$, one may still write the optimal exchange rate as a weighted average,

$$e_s = \frac{(1+2\bar{\eta})\bar{B}^2}{(1+2\bar{\eta})\bar{B}^2 + \left(1 + 2\frac{\bar{\eta}\bar{B}}{1+\mu\bar{B}}k_{ux}\right)\chi} e_s^{in}(\bar{\eta}, \bar{B}) + \frac{\left(1 + 2\frac{\bar{\eta}\bar{B}}{1+\mu\bar{B}}k_{ux}\right)\chi}{(1+2\bar{\eta})\bar{B}^2 + \left(1 + 2\frac{\bar{\eta}\bar{B}}{1+\mu\bar{B}}k_{ux}\right)\chi} e_s^{dm}(\bar{\eta}, \bar{B})$$

where

$$e_s^{in}(\bar{\eta}, \bar{B}) = B^{-1}\frac{1+\tilde{k}(m)\bar{\eta}}{1+2\bar{\eta}}\mathcal{T}_s$$

$$e_s^{dm}(\bar{\eta}, \bar{B}) = (1+\mu\bar{B})^{-1}\left(\frac{1+\frac{\bar{\eta}\bar{B}}{1+\mu\bar{B}}k_{ux}}{1+2\frac{\bar{\eta}\bar{B}}{1+\mu\bar{B}}k_{ux}}\right)e_s^{dm}(0).$$

Note that the targets depend on the multiplier: the planner now internalizes that his monetary policy may lead agents to pick undesirable positions. One can then replace back in (76), maximize over $\bar{B}$, get $\bar{B}(\bar{\eta})$ and then look for a fixed point of the home no-arbitrage equation (75) to find $\bar{\eta}$. If there is more than one solution for $\bar{\eta}$, one may then compare them using the welfare function. It is also important to check the second-order conditions of the inner problem with respect to the exchange rate.

Clearly, this problem is significantly less tractable than the case with capital controls when $m < \infty$. Perhaps surprisingly, one can show that when $\chi \to 0$ (i.e., flexible prices), the solution converges to the cooperative solution.[59]

C Appendix: Quantitative analysis

In this section, I provide additional sensitivity analysis and show how the results change with the availability of savings taxes.

C.1 Savings taxes and bond duration

In all the previous experiments, I found that the taxes were very small. This suggests their availability is not very important for the optimal policy. Table 3 confirms these results: portfolios, weights, and welfare are in general quite similar if they are not available. The analysis in Section 4 suggests this could be tightly related to the duration of the bonds. Column 7 in Table 3 shows the results when I assume home-currency bonds have a duration of 1 year rather than 4.85 years. Savings taxes are now much more important: While the insurance weight without them is only 11%, once savings taxes are allowed, the weight increases almost four-fold to 43%. Accordingly, agents expand their positions from 34% to 58% and the planner achieves 52% of the potential welfare gains from completing markets, compared to only 37% without savings taxes. Finally, note that welfare gains under demand-management targeting are now larger, which is a result of the higher correlation between returns and transfers implied by short bonds (they are less sensitive to liquidity shocks).

C.2 Other sensitivity analysis

I start by varying the complementarity between tradable and nontradable goods (columns 1 and 2 in table 4). I adopt two values, which correspond to the bounds on the estimates in the literature

[59] One must be careful with randomization in these environments without capital controls. While the cooperative solution solves the approximate problem when prices are flexible, I found that with CRRA the planner may approximate the solution with capital controls arbitrarily closely by randomizing and putting a vanishing probability on $c = 0$. (The argument relies on higher order derivatives, so it does not show up if one approximates the problem first, i.e., with quadratic utility randomization is not optimal).

Table 4: Sensitivity analysis.

	$\rho = 0.4$	$\rho = 1.5$	$\alpha = 0.4$	$\alpha = 0.7$	$\gamma = 10$	$\beta = 0.98$	$1y$ bond
$\bar{B}$: Inflation-targeting	−16.8%	−15.2%	−16.6%	−15.6%	−20.4%	−19.3%	−33.5%
$\bar{B}$: Optimal Policy (savings tax)	−25.0%	−21.3%	−22.2%	−26.3%	−33.8%	−32.7%	−58.4%
$\bar{B}$: Optimal Policy (no savings tax)	−23.0%	−20.9%	−20.9%	−25.4%	−31.8%	−31.5%	−41.0%
ω: savings tax	9.79%	6.36%	6.19%	13.4%	12.1%	11.3%	43.3%
ω: no tax	6.93%	5.92%	4.57%	12.0%	9.83%	9.97%	11.3%
Welfare: Inflation-targeting	12.9%	10.5%	12.6%	11.2%	11.2%	9.93%	30.7%
Welfare: Optimal Policy (savings tax)	18.9%	14.7%	16.8%	18.8%	18.1%	16.4%	52.2%
Welfare: Optimal Policy (no savings tax)	17.4%	14.4%	15.9%	18.1%	17.0%	15.8%	37.3%

Note: Columns (1) and (2) change the elasticity of substitution between tradable and nontradable goods, columns (3) and (4) change the share of tradables in consumption, column (5) changes risk-aversion, column (6) changes the discount factor, and column (7) modifies the duration of bonds to 1 year. In every case I re-calibrated m and σ_ψ to match the exchange volatility and a portfolio of −15% over annual GDP. Welfare gains are measured as the steady-state-consumption-equivalent gains under such a policy with respect to the demand-management $\bar{B} = 0$ economy as a share of the total potential gains under the first-best: $\frac{welf(policy)-welfare(B=0)}{welfare(firstbest)-welfare(\bar{B}=0)}\%$.

(see Akinci (2011) for a survey): $\rho = 0.4$ and $\rho = 1.5$. A lower elasticity of substitution decreases the pass-through of the exchange rate to the output gap, which lowers the cost of providing insurance. In this range, however, the effects are modest: portfolios, insurance weights, and welfare gains vary only a few percentage points from one extreme to the other. More interestingly, a low elasticity of substitution also increases the effects of savings' manipulation on bond returns since shifts in tradable consumption create large movements in demand for nontradable goods. As a result, the lower the elasticity, the more effective capital controls are.

Next, I vary openness increasing and decreasing it by $15pp$ (columns 3 and 4). In very open economies, the inefficiencies affect a smaller share of the economy in our model. Put differently, deviating from demand-management to provide insurance is less costly from a welfare perspective because the planner cares less about the output gap and pricee dispersion compared to smoothing tradable consumption. As a result, the weight on insurance increases. Indeed, when tradables represent 70% of the economy, the insurance weight almost doubles, reaching over 13%. Accordingly, the planner achieves a larger share of the insurance gains of being able to issue home-currency debt.[60]

Next, I vary risk aversion (column 5). I set $\gamma = 10$ - the upper bound of the range considered by Mehra and Prescott (1985). A higher risk aversion naturally makes insurance more important. However, while it increases the weight to around 12%, the realized share of the gains compared to demand-management is only slightly larger.

Finally, I change the discount factor (column 6). For illustrative purposes, I set β=0.98, which is very low for a model at the quarterly frequency. Ceteris paribus the shocks, a higher discount factor implies transfers become more valuable, as their present value increases. It has a similar effect to risk aversion: while it increases the weight to 11%, the realized share of the gains is only slightly larger compared to demand-management.

[60] For this it becomes important where one introduces the price-rigidity. This result is likely to be less sensitive to openness if there is also stickiness in the tradable sector.