Aid, Remittances, and the Informal Economy[*]

Santanu Chatterjee[a]
University of Georgia

Stephen J. Turnovsky[b]
University of Washington

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Abstract

Countries that are major recipients of foreign transfers such as aid and remittances are systematically associated with large informal sectors that absorb, on average, more than 40% of their labor force and produce more than 50% of GDP. Using a two-sector small open economy model that incorporates many of the structural features of these economies, we examine the impact of remittances and aid on the sectoral allocation of labor and output (formal vs informal), as well as the dynamics of the real exchange rate, the current account, and welfare. The dynamic adjustment to remittances is characterized by sharp intertemporal trade-offs: while these inflows generate a short-run economic expansion, the long-run is characterized by an increase in the size of the informal sector and an economic contraction. The effects of aid, on the other hand, are more ambiguous, depending on its internal allocation between public investment and general budget support: while tying more aid to public investment leads to an expansion of the formal sector and the aggregate economy, diverting aid away from investment has the opposite effect. Further, we find that while remittances generate a Dutch Disease effect, i.e., a real exchange rate appreciation leading to a contraction of the economy's formal sector and GDP, such an effect is not associated with foreign aid. The quantitative analysis is conducted using data for 40 aid and remittance recipient countries for the period 1999-2007.

Keywords: Remittances, foreign aid, informal sector, real exchange rate, Dutch Disease, labor mobility

JEL Classification: E2, E6, F2, F3, F4

1.   Introduction

Most developing countries are characterized by a large informal sector, populated mainly by small, unregistered firms having low productivity, and producing basic non-traded goods and services. As Schneider et al. (2010) and La Porta and Shleifer (2014) document, this sector absorbs a disproportionately large share of the labor force, with very limited mobility out of this sector, and has little or no access to credit and capital. At the same time, these countries are also often the recipients of large capital inflows, originating primarily from two distinct sources, namely foreign aid (Official Development Assistance) and remittances sent by migrant workers living and working abroad. These two sources of finance impinge on the economy in distinct ways. While ODA takes the form of an official transfer to the government, often with donor-imposed restrictions on how it can be spent, remittances are an unofficial direct transfer to private residents, often working in the informal sector. Since these two sets of recipients operate under different constraints and likely have contrasting objectives, it is important to compare the effects of aid and remittances on the evolution of the economy, particularly in the presence of a substantial informal sector. To do so is the central objective of this paper.

Table 1 reports the average share of the informal sector and foreign transfers in GDP for 40 developing countries, for the period 1999-2007. The average share of the informal sector's output in GDP was about 42%, while the average share of its employment varied between 49-54%. During this period these countries received, on average, about 12% of their GDP in the form of foreign transfers, with remittances accounting for the larger share (approx. 53%) of these inflows. Figure 1 depicts the intertemporal correlation between aggregate foreign transfers and the output share of the informal sector for a selected group of eight countries, all of which are characterized by large informal sectors and receive a significant share of their GDP in the form of foreign transfers. As can be seen from the figure, transfers and the size of the informal economy seem to be positively correlated during the sample period, with the shares of both increasing over time.

The data in Table 1 and the country-specific examples in Figure 1 suggest the importance of examining the relationship between these two key characteristics of developing countries. But a

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priori, it is unclear as to what the underlying mechanism driving this relationship might be. While both aid and remittances augment the resources of the recipient country, they do so in distinct ways that are likely to lead to different equilibrium outcomes. On the one hand, foreign aid contributes to the government's revenue, which it may then allocate to tax reduction (untied aid), or directly to some productivity enhancing activity (tied aid). In contrast, remittances augment households' resources, which they then may allocate to consumption or to saving, the latter impacting only indirectly on the productive capacity of the economy. Second, households in developing countries typically face binding borrowing constraints and intersectoral adjustment costs, especially with respect to labor mobility, and this may have important consequences for the absorption of both aid and remittances in the formal or informal sectors, and more generally for the macroeconomy.

In examining the relationship between foreign transfers and the informal sector, we seek to address an important, and seemingly neglected aspect of economic development. To our knowledge, research on the informal economy, and that on aid and remittances, have evolved independently of each other. For example, studies on the informal economy have focused either on (i) the measurement of its relative size to GDP (Schneider and Enste 2000, La Porta and Shleifer 2008, 2014, and Gomis-Porqueras et al. 2014), or on (ii) issues pertaining to tax policy and enforcement (Rauch 1991, Ihrig and Moe 2004, Turnovsky and Basher 2009, Prado 2011, and Ordonez 2014). On the other hand, the literature on foreign aid and remittances has focused mainly on economic growth and the macroeconomic adjustment of what can be characterized as being formally structured economies; See, for example, Burnside and Dollar (2000), Easterly (2003), Chatterjee and Turnovsky (2007), Giuliano and Luiz-Arranz (2009), Acosta et al. (2009), and Mandelman (2012). In either case, there has been no analysis linking the role of aid and remittances and the growth of the informal economy, despite the compelling evidence suggesting their co-movement over time. Embedding both the informal economy and foreign transfers in a general equilibrium model thus enables us to investigate this important, but neglected relationship.

We develop a two-sector model of an open economy that incorporates many of the stylized features of the informal sector, as reported in La Porta and Shleifer (2014). Specifically, the economy produces two goods: a traded good that can be used for consumption or investment,

manufactured in the formal sector, and a non-traded consumption good (such as services) produced in the informal sector. Both sectors use labor for production and benefit from government-provided infrastructure capital (through a spillover effect), with the key difference that while the formal sector employs private capital for production, the informal sector has no access to private capital. Thus, private capital is traded internationally, but is immobile internally, restricted only to the formal sector. Households consume both goods, allocate time to work in both sectors, invest in formal sector firms, and receive a flow of remittances from abroad. However, in apportioning their labor across the two sectors, households face convex mobility costs, which reflect the inflexibilities in labor markets characteristic of developing economies. The presence of these costs, in conjunction with gradual success in the job search process and the probability of job separation in the formal sector, generate sluggish migration of labor between the informal and formal sector, in line with the findings of La Porta and Shleifer (2014), as well as long-term unemployment. For the household, income (capital and labor) from the formal sector is subject to taxation by the government, but labor income derived in the informal sector manages to avoid being taxed.[1]

The government receives tax revenues from the formal sector (via household income), a flow of foreign aid from abroad, and provides a government consumption good as well as investment in infrastructure (public capital). The model is closed by assuming that both households and the government have access to an internationally traded bond that can be used to accumulate debt over time, in the event that current expenditures exceed income. The key feature here is that both agents face an endogenous borrowing cost, determined by a mark-up over the world interest rate, with the markup itself reflecting the economy's debt-servicing capacity.

The analytical model we develop is a complex one, yielding a high-order dynamic system, and requiring a numerical solution. We calibrate the model to yield a long-run equilibrium consistent with sample averages for 40 developing countries for the period 1999-2007. In doing so, we ensure that the macroeconomic equilibrium is representative of a developing economy with a large informal

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[1] Ihrig and Moe (2004) and Turnovsky and Basher (2009) focus on issues relating to auditing and penalties imposed on potential tax evaders in the informal sector. We do not address such issues here.

sector, long-run unemployment, and one that receives a substantial share of its GDP in the form of external transfers such as aid and remittances.

Our calibrated model suggests that on average about 35% of foreign aid is allocated to public investment. Given this composition of aid, our results indicate that an *increase in foreign aid* is associated with an increase in the size of the informal sector. However, if for a *given level of aid*, donors or recipient governments increase its allocation to public investment, the size of the informal sector declines over time, with both the share of the formal sector (output and employment), along with aggregate output, increasing during transition. In contrast, if aid is "fungible", i.e., the recipient government diverts aid away from public investment, then the size of the informal sector increases, and the overall economy contracts. Our welfare results suggest that while re-allocating aid to public investment does increase welfare, there are diminishing returns to this allocation, with the maximum welfare gains realized when about 30-40% of aid inflows are tied to public investment. Remittances, on the other hand, are characterized by sharp intertemporal trade-offs: while these inflows generate a short-run economic expansion, the long-run is characterized by an increase in the size of the informal sector and an economic contraction.

We also derive some interesting results regarding the relationship between external transfers and the real exchange rate. In the case of aid, the dynamics of the real exchange rate depend critically on its composition, i.e., the allocation between public investment and general budget support for the government. In general, we find that the dynamic adjustment of the real exchange rate is non-monotonic, with an instantaneous appreciation (depreciation) leading to a depreciation (appreciation), the eventual reversal of which depends on how aid is allocated in the government's budget. On the other hand, a remittance shock leads to a real appreciation of the exchange rate, along with a monotonic adjustment over time. An interesting aspect here is that remittances generate a Dutch Disease effect, with a real appreciation and a contraction in aggregate output and the share of the formal sector. In sharp contrast, foreign aid is not associated with such an effect.

Since the model is evaluated numerically, we conduct extensive sensitivity analysis with respect to a menu of key structural productivity and labor-market parameters. Our central results

regarding the effect of aid and remittances on the informal sector (employment and output shares) remain remarkably robust to variations in these parameters.

The rest of the paper is organized as follows. Section 2 outlines the theoretical model, Section 3 describes the macroeconomic equilibrium and fiscal sustainability, and Section 4 discusses the calibration of the benchmark equilibrium. Section 5 considers counterfactual increases in aid and remittances, as well as changes in the composition of aid. Section 6 reports summarizes the sensitivity analysis, and Section 7 concludes.

2.   Analytical Framework

We begin by outlining the analytical model. The description below is general, with the specific functional forms employed in the simulations being introduced in Section 4.

2.1.   Production

Production in the economy takes place in two sectors: a formal sector, which produces a traded good that can be used either for consumption or investment, and an informal sector, which produces a basic non-traded consumption good (e.g., services).[2] Both sectors use labor as an input in production, and a government-provided stock of infrastructure (henceforth referred to as public capital) generates positive productivity spillovers for the two technologies. The key difference between the two sectors lies in their relative usage or access to private capital: while the formal sector uses private capital as a factor complementary to labor in production, the informal sector employs labor as the only private factor of production. Since the informal sector is typically much more labor intensive, this polar case serves as a convenient benchmark.[3]

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[2] The general two-sector production structure is similar to those of previous studies, such as Ihrig and Moe (2004) and Turnovsky and Basher (2009). However, in contrast to our approach, those papers focused on a closed economy and abstract away from issues related to the provision and financing of public goods. In our context of an open economy, the two sectors produce distinct goods (traded and non-traded), generating an endogenously determined real exchange rate and raising issues associated with a small "dependent" economy.

[3] Several studies have documented that informal sector firms are characterized by extremely low capital-labor ratios; See, for example, Thomas (1992), de Paula and Scheinkman (2007), D-Giannatale et al. (2011), and also La Porta and Shleifer (2014). However, as Ordonez (2014) points out, whether the existence of low capital-labor ratios in the informal sector are a signal of credit constraints or self-financing is an open question. Our assumption of a zero capital-labor ratio in the informal sector, while agnostic to the underlying cause, maintains analytical tractability while remaining consistent with stylized facts.

The production technology of a representative firm in the formal sector, (denoted by the subscript f), is described by:

$$Y_f = A_f(K_G)F(K, L_f) \equiv Y_f(K, L_f, K_G) \tag{1a}$$

where Y_f is the flow of output, L_f is the employment of labor, and K is the total stock of private capital fully employed in the formal sector.[4] The production function (1a) has the usual neoclassical properties with respect to the two private factors of production. In addition, A_f is the sector-specific level of productivity, which is influenced by the stock of public capital, K_G, and subject to diminishing returns, i.e., $A_f'(.) > 0$ and $A_f''(.) < 0$.

The representative firm in the informal sector (denoted by the subscript s) uses only labor and public capital for production:[5]

$$Y_s = A_s(K_G)H(L_s) \equiv Y_s(L_s, K_G) \tag{1b}$$

where, A_s is the sector-specific level of productivity, and L_s is the labor employment. As in the formal sector, public capital is subject to diminishing returns, i.e., $A_s'(.) > 0$ and $A_s''(.) < 0$, while the productivity of labor is positive but diminishing.

The formal sector is assumed to include a well-defined factor market, so that profit maximization in the formal sector yields the usual first-order conditions

$$w_f = \frac{\partial Y_f}{\partial L_f} \equiv w_f(K, L_f, K_G); \quad r_K = \frac{\partial Y_f}{\partial K} \equiv r_K(K, L_f, K_G) \tag{2}$$

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[4] The idea that public capital may generate positive spillovers for private production has been the subject of a voluminous theoretical and empirical literature; Theoretical contributions date back to the seminal work of Arrow and Kurz (1970), in a neoclassical framework, and Barro (1990) for an endogenous growth model. More recent contributions include Futagami et al. (1993), Glomm and Ravikumar (1993), Turnovsky and Fisher (1998), Chatterjee et al. (2003), Rioja (2003), and Chatterjee and Turnovsky (2007, 2012), A comprehensive survey of this literature is provided by Agenor (2013). For surveys of the extensive empirical studies see Gramlich (1994), and more recently Bom and Lithgart (2013).

[5] The assumption that all capital is employed in the formal sector is also adopted by Ihrig and Moe (2004) and Turnovsky and Basher (2009). We can also interpret the production function (1b) as being of the form $Y_s = A(K_G)H(L_s, \bar{K}_s)$ where $\bar{K}_s$ is fixed and does not accumulate. Garcia-Penalosa and Turnovsky (2005) assume that capital is intersectorally mobile but that the informal sector has a lower capital-labor ratio. The qualitative results remain essentially unchanged from the present assumption that private capital is intersectorally immobile.

where w_f and r_K represent the real wage of labor and the rental rate for capital employed in the formal sector. In contrast, in the informal sector, with no capital, we assume that all income accrues to labor.

2.2.   Households

The economy is populated by an infinitely-lived representative household that maximizes utility:

$$\int_0^\infty U\left(C_f, C_S\right)e^{-\beta t}dt \tag{3}$$

where, C_f and C_s represent the consumption of goods produced in the formal and informal sectors, respectively, and β is the rate of time preference. The utility function has the standard curvature properties, i.e., both consumption goods yield positive but diminishing marginal utility. The household allocates part of its time to working in the formal and informal sectors, and earns a return on private capital lent out to the formal sector.[6] Households also accumulate debt (through an internationally traded bond) to finance any excess expenditure over earnings:

$$\dot{N} = rN + C_f + pC_s + \Omega(I, K) - (1-\tau)\left[r_K K + w_f L_f\right] - pY_s\left(L_s, K_G\right) + T_f - R \tag{4}$$

where N is the current stock of household debt, r is the borrowing interest rate, p is the relative price of the informal sector good, $\Omega(.)$ incorporates a convex adjustment cost associated with accumulating private capital and is homogeneous of degree one, I is the rate of private investment (in the formal sector), τ is the tax rate on income earned in the formal sector, T_f is a lump-sum tax, and R represents an inflow of remittances received from abroad. A key feature of the economy is that while household labor and capital income derived from the formal sector is subject to taxation by the government, labor income from the informal sector is outside the tax radar of the government, and hence escapes taxation.[7] Further, since the formal sector produces the economy's traded good

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[6] Since we do not model an endogenous labor-leisure choice in our model, time not spent working by the household is characterized by involuntary unemployment. We discuss this further in Section 2.3.

[7] This raises issues relating to the costs and benefits of auditing the informal sector and incentives to force compliance with the tax payments. These issues are addressed by Turnovsky and Basher (2009) but lie outside the scope of the present paper.

(taken as numeraire) and the informal sector produces the non-traded good, the relative price of the informal sector good, p, is also the economy's real exchange rate. As such, an increase (decrease) in p denotes a real appreciation (depreciation) of the exchange rate.

Private investment expenditures lead to the accumulation of physical capital, which as eq. (1a) indicates, is used as an input in the formal production sector:

$$\dot{K} = I - \delta_K K \tag{5}$$

where δ_K is the rate of depreciation of capital. All investment is denominated in terms of the formal (traded) good. Informal (non-traded) output is solely used for consumption, i.e., $Y_s = C_s$, for all t.

We assume that the borrowing rate on debt is a mark-up over the world interest rate, r^*, with the mark-up depending on the economy's aggregate stock of debt relative to its debt-servicing capacity, as measured by GDP:

$$r = r^* + \omega\left(\frac{N+B}{Y}\right), \quad \omega' > 0 \tag{6}$$

where $Y \equiv Y_f + pY_s$ denotes aggregate GDP, measured in terms of traded output, and $N+B$ is the *aggregate* national debt of the economy, comprising the sum of private (household) debt, N, and public (government) debt, B, with $\omega(.)$ representing the borrowing premium over the (fixed) world interest rate. Being atomistic, in allocating their resources, households treat the borrowing rate in (6) as given, although the equilibrium private borrowing rate is determined endogenously as a consequence of the collective private and public borrowing decisions.[8]

2.3. Labor Market

An important feature of the economy is the presence of labor market rigidity, which reflects the structural inefficiencies characteristic of developing economies, the result of which is to generate

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[8] A basic issue in modelling small open economies such as this is the closure of the financial market; see Turnovsky (1997) and Schmitt-Grohé and Uribe (2003) where several alternatives are detailed. These include introducing an endogenous borrowing premium, as in (6), which is most appropriate in the case of the developing economy being analyzed here. This approach originated with Bardhan (1967), and many variants can be identified in the literature; see also Eaton and Gersovitz (1981).

unemployment due to time spent on job search in moving from one sector to another. These inefficiencies arise from such factors as the absence of formal institutions to promote coordination between employer and employee, the reliance on social networks involving friends and relatives, and ethnic and religious associations to facilitate the job search.[9] As Turnovsky and Basher (2009) document, because of such structural inefficiencies, the job search period in developing countries in general is high, varying between one and four years. This contrasts with developed countries like the United States, for example, where the average job search period is about twelve to sixteen weeks.

Households are endowed with one unit of time that can be used to either working in the formal sector (L_f), the informal sector (L_s), or remaining unemployed (L_U). This implies the following time allocation[10]

$$L_f + L_s + L_U = 1 \qquad\qquad (7)$$

Suppose an agent seeks to increase his employment in the formal sector by reducing his employment in the informal sector at the rate u. In the process of this reallocation, we assume $(\chi/2)u^2$ amount of labor time is temporarily lost in job searching.[11] The parameter χ determines the rate of this loss and reflects the rigidity in the labor market. We also assume that at every point of time, a fraction σ of the current pool of unemployed gets re-employed, while on the other hand a fraction z of those employed in the formal sector experience job separation.[12] Thus the exodus out of the informal sector and the evolution of unemployment are described by

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[9] For example, more than 72 percent of those who work in the shadow economy and more than 52 percent of those who work in the formal sector rely on the social networks to move from one sector to another in Venezuela (Marquez and Ruiz-Tagle 2004). These networks pay off only when someone is already unemployed for a while (Marquez and Ruiz-Tagle 2004, Gong , van Soest, and Villagomez 2004, Serneels 2007).

[10] By assuming that labor is supplied inelastically we abstract from the labor-leisure choice. This assumption not only aids analytical tractability but is plausible for a developing economy. Given the low rates of consumption in such economies, it is unlikely that much leisure is consumed.

[11] This quadratic structure of the loss of labor implies that the migration of labor from one sector to another, in either direction, creates temporary unemployment. The use of the quadratic function to specify adjustment costs has a long tradition in economics, dating back to Holt et al. (1960). Most of the applications have been to *aggregate* factor adjustments; see e.g., Lucas (1967) and Abel and Blanchard (1983) for capital, Sargent (1979), Krugman (1991), and Georges (1995) for labor. The present application parallels that of Morshed and Turnovsky (2004) who impose quadratic adjustment costs on *intersectoral* capital movements.

[12] Since the status of workers employed in the informal sector is undocumented we cannot identify people losing employment in the informal sector.

$$\dot{L}_s = -u \tag{8a}$$

$$\dot{L}_U = \frac{\chi}{2}u^2 + zL_f - \sigma L_U \tag{8b}$$

Taking the time derivative of (7) and combining with (8a) and (8b), yields the rate at which employment in the formal sector is evolving

$$\dot{L}_f = u - \frac{\chi}{2}u^2 - zL_f + \sigma L_U \tag{9}$$

Thus the net rate of change of employment in the formal economy equals the net outflow from the informal economy, exclusive of job searchers, plus the inflow from the existing unemployed, less those terminated. The presence of labor mobility costs, as described in (8) generates sluggishness in the adjustment of sectoral labor supply, which implies that the sectoral labor allocation decisions L_f and L_s, represent investment decisions, analogous to those involving asset accumulation. Our specification of labor movements as a gradual process contrasts with that of earlier authors (e.g. Ihrig and Moe, 2005, García-Peñalosa and Turnovsky, 2005) who allow labor to move instantaneously, but is a more accurate description of the process in developing countries.[13]

2.4. The Public Sector and Current Account

The economy's stock of public capital evolves over time according to

$$\dot{K}_G = G_I - \delta_G K_G \tag{10}$$

where δ_G is the rate of depreciation of public capital and $G_I \equiv G_I^d + \lambda F$ represents the flow of public investment. The resources for financing public investment come from two sources: domestic financing, G_I^d, and foreign aid, F. The parameter λ, with $0 \le \lambda \le 1$, represents the extent to which the foreign aid is committed to public investment. Of the two polar extremes, $\lambda = 1$ denotes the case where aid is fully "tied" to public investment, while $\lambda = 0$ indicates that aid is "untied" and serves as a general source of revenue to the recipient government.

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[13] The instantaneous movement of labor across sectors is obtained by setting $\chi = z = 0$, $\sigma \to \infty$.

We assume that the government accumulates debt to finance excess public expenditures net of revenues

$$\dot{B} = rB + G_I^d + G_C^d - \tau\left(r_K K + w_f L_f\right) - T_f - (1-\lambda)F \tag{11}$$

where B is the current stock of government (public) debt, and G_C^d represents domestic government consumption spending on the traded good, with the government's borrowing cost being given by the interest rate specification in (6). The evolution of the economy's current account is obtained by combining the household and government budget constraints

$$\dot{V} = r(.)V + C_f + \Omega(.) + G_I^d + G_C^d - Y_f - R - (1-\lambda)F \tag{12}$$

where, $V \equiv N + B$ denotes the aggregate stock of debt of the economy. In deriving (12), the informal sector market clearing condition, $Y_s = C_s$, has been imposed. Eq. (12) also highlights how the allocation of aid determines its impact on national debt accumulation.

3.   Macroeconomic Equilibrium

The household maximizes (3), subject to (4), (5), (7), (8) and (9), while taking into account (2). Note that, in making its allocation decisions, the household takes the borrowing rate specified in (6) and government policy as given. The resulting optimality conditions are

$$\frac{\partial U(C_f, C_s)}{\partial C_f} = q_1 \tag{13a}$$

$$\frac{\partial U(C_f, C_s)}{\partial C_s} = pq_1 \tag{13b}$$

$$\Omega_I(I, K) = q_K \tag{13c}$$

$$u = \frac{1}{\chi}\left(\frac{q_f - q_s}{q_f}\right) \tag{13d}$$

$$\beta - \frac{\dot{q}_1}{q_1} = r \tag{13e}$$

$$\frac{\dot{q}_K}{q_K} + \frac{(1-\tau)r_K}{q_K} - \frac{\Omega_K(I,K)}{q_K} - \delta_K = r \qquad (13f)$$

$$\frac{\dot{q}_f}{q_f} + \frac{(1-\tau)w_f}{q_f} - (\sigma + z) = r \qquad (13g)$$

$$\frac{\dot{q}_s}{q_s} + \frac{\partial Y_s(L_s, K_G)/\partial L_s}{q_s} - \sigma\left(\frac{q_f}{q_s}\right) = r \qquad (13h)$$

where, q_1 is the shadow value of household debt (traded bonds) and q_K, q_f and q_s denote the shadow values of private capital, employment in the formal and informal sectors, respectively, with the latter three shadow values being normalized by q_1.[14] In addition, the following transversality conditions must hold

$$\lim_{t\to\infty} q_1 N e^{-\beta t} = \lim_{t\to\infty} q_K q_1 K e^{-\beta t} = \lim_{t\to\infty} q_f q_1 L_f e^{-\beta t} = \lim_{t\to\infty} q_s q_1 L_s e^{-\beta t} = 0 \qquad (14)$$

Eqs. (13a) and (13b) equate the marginal utility of consumption for the two consumption goods to the shadow price of household debt, while Eq. (13c) equates the marginal cost of private investment to the shadow price of capital. Eq. (13d) describes the rate at which labor moves from one sector to the other. This rate is determined by the difference in the shadow values in the two sectors, and the speed with which this occurs varies inversely with the rigidity in the labor market, as parameterized by χ.[15] Observe that $u \gtrless 0$, implying that agents may also move from the formal to the informal sector, depending upon the relative shadow values. The remaining four conditions, (13e)-(13h) are intertemporal efficiency conditions, equating the return on consumption, the return on capital, and the net returns on employment investment in the formal and informal sectors, respectively, to the marginal cost of borrowing.

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[14] That is, if we let μ_K denote the shadow (utility) value of capital, then $q_K \equiv \mu_K/q_1$ and similarly for q_f, q_s. Written in this way, the normalized prices become "asset prices" independent of utility units, and the optimality conditions (13g) and (13h) treat labor as an asset, analogous to capital.

[15] The parallels between (13d) and the corresponding relation in the pioneering Harris-Todaro (1970) migration model are apparent. That paper postulated the movement of labor between rural and urban areas to be proportional to the current wage differential between the two sectors. In contrast, we find that labor movement is proportional to the differential asset price, which upon integrating the arbitrage relationships (13g) and (13h) forward, is the discounted sum of expected future sectoral after-tax wage differentials.

3.1.   Equilibrium Dynamics

The internal equilibrium dynamics for the economy can be expressed in terms of the evolution of the following quantities: (i) private capital, (ii) employment in the two production sectors, (iii) national debt, and (iv) the shadow values of debt, private capital, and the sectoral employments. To the extent that foreign aid is tied to public investment, the equilibrium will also be driven by the accumulation of public capital, in accordance with (10).

To derive the equilibrium dynamics, we first solve the static first-order conditions (13a) and (13b) for the equilibrium sectoral consumption quantities

$$C_j = C_j(p, q_1), \quad j = f, s \tag{15a}$$

Using (15a) in conjunction with the market-clearing condition for the informal sector,

$$C_s(p, q_1) = Y_s(L_s, K_G) \tag{16}$$

we can derive the short-run equilibrium real exchange rate:

$$p = p(q_1, L_s, K_G) \tag{15b}$$

It is straightforward to show that $\partial p / \partial q_1 < 0$, $\partial p / \partial L_s < 0$ and $\partial p / \partial K_G < 0$. An increase in the shadow value of household debt, or an increase in employment in the informal sector or public investment each increase the excess supply of the informal (nontraded) good, causing the real exchange rate to depreciate in order for the informal goods market equilibrium to be maintained. Combining (15a) and (15b) we obtain

$$C_j = C_j[q_1, p(q_1, L_s, K_G)] \equiv C_j[q_1, L_s, K_G], \quad j = f, s \tag{15a'}$$

Next, solving (13c), yields

$$I = \psi(q_K)K \tag{15c}$$

enabling us to write $\Omega(I,K)=\Omega(\psi(q_K))K$ and $\Omega_K(I/K)=\Omega_K(\psi(q_K))$. In addition, from (6), (1a), (1b), and (15b) we obtain the reduced-form expression $r=r(L_f,L_s,K,K_G,q_1,V)$.

Using these relationships we can write

$$\dot{K}=\left(\psi(q_K)-\delta_K\right)K \tag{16a}$$

$$\dot{L}_f=\frac{q_f-q_s}{\chi q_f}-\frac{\chi}{2}\left(\frac{q_f-q_s}{\chi q_s}\right)^2-zL_f+\sigma\left(1-L_f-L_s\right) \tag{16b}$$

$$\dot{L}_s=-\left(\frac{q_f-q_s}{\chi q_f}\right) \tag{16c}$$

$$\dot{V}=r(.)V+C_f(q_1,L_s,K_G)+\Omega\left(\psi(q_K)\right)K+G_I^d+G_C^d$$
$$-Y_f(K,L_f,K_G)-R-(1-\lambda)F \tag{16d}$$

$$\dot{q}_1=\left(\beta-r\right)q_1 \tag{16e}$$

$$\dot{q}_K=(r(.)+\delta_K)q_K+\Omega_K\left(\psi(q_K)\right)-(1-\tau)r_K(K,L_f,K_G) \tag{16f}$$

$$\dot{q}_f=(r(.)+\sigma+z)q_f-(1-\tau)w_f(K,L_f,K_G) \tag{16g}$$

$$\dot{q}_s=r(.)q_s+\sigma q_f-\frac{\partial Y_s(L_s,K_G)}{\partial L_s}p(q_1,L_s,K_G) \tag{16h}$$

together with the accumulation of public capital, (10), yielding an autonomous macrodynamic equilibrium in $K,L_f,L_s,N,q_1,q_K,q_f,q_s,K_G$.

3.2.   Steady-State Equilibrium

The steady state (denoted by tildes) is attained when $\dot{K}=\dot{L}_f=\dot{L}_s=\dot{V}=\dot{q}_1=\dot{q}_K=\dot{q}_f=\dot{q}_s=\dot{K}_G=0.$

Imposing these restrictions on (16a)-(16h) and (10), yields

$$\psi(\tilde{q}_K)=\delta_K \tag{17a}$$

$$\sigma\left(1-\tilde{L}_f-\tilde{L}_s\right)=z\tilde{L}_f \tag{17b}$$

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$$\tilde{q}_f = \tilde{q}_s \qquad (17c)$$

$$\beta \tilde{V} + C_f(\tilde{q}_1, \tilde{L}_s, \tilde{K}_G) + \Omega\big(\psi(\tilde{q}_K)\big)\tilde{K} + G_I^d + G_C^d$$
$$-Y_f(\tilde{K}, \tilde{L}_f, \tilde{K}_G) - R - (1-\lambda)F = 0 \qquad (17d)$$

$$\tilde{r} = r(\tilde{L}_f, \tilde{L}_s, \tilde{K}, \tilde{K}_G, \tilde{q}_1, \tilde{V}) = \beta \qquad (17e)$$

$$(\beta + \delta_K)\tilde{q}_K + \Omega_K\big(\psi(\tilde{q}_K)\big) - (1-\tau)r_K(.) = 0 \qquad (17f)$$

$$(\beta + \sigma + z)\tilde{q}_f - (1-\tau)w_f(\tilde{K}, \tilde{L}_f, \tilde{K}_G) = 0 \qquad (17g)$$

$$\beta \tilde{q}_s + \sigma \tilde{q}_f - \frac{\partial Y_s(\tilde{L}_s, \tilde{K}_G)}{\partial \tilde{L}_s} p(\tilde{q}_1, \tilde{L}_s, \tilde{K}_G) = 0 \qquad (17h)$$

$$\delta \tilde{K}_G = G_I^d + \lambda F \qquad (17i)$$

Eqs.(17a)-(17i) yield 9 equations that can be solved for $\tilde{K}, \tilde{L}_f, \tilde{L}_s, \tilde{V}, \tilde{K}_G, \tilde{q}_1, \tilde{q}_K, \tilde{q}_f$ and $\tilde{q}_s$. In addition, in steady state the government's budget constraint is

$$\tilde{r}\tilde{B} + G_I^d + G_C^d = \tau Y_f(\tilde{K}, \tilde{L}_f, \tilde{K}_G) + T_f + (1-\lambda)F \qquad (18)$$

Given the government's policy choices G_I^d, G_C^d, τ, lump-sum taxes, T_f, and aid inflow F, along with the steady-state solutions, (17) can be solved for the steady-state level of public debt.[16] Finally, substituting the steady-state solutions obtained from (17) into eqs. (15) and the production functions (1), enables us to solve for $\tilde{p}$, $\tilde{I}$, the sectoral consumption levels, $\tilde{C}_f, \tilde{C}_s = \tilde{Y}_s$, and output of the formal sector, $\tilde{Y}_f$. This completes the solution for the economy's steady-state equilibrium.

In general, the steady state is too complex to enable us to determine analytically the long-run impact of foreign transfers on the economy. However, the following can be noted. First, the shadow value of capital depends only on the depreciation rate. This implies that the equilibrium rate of investment, I/K is independent of aid or remittances. Second, the long-run borrowing rate is equal

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[16] Writing the household budget constraint (4) as $\dot{N}(t) = r(t)N(t) + X(t) + T_f(t)$, the first transverality condition in (14) can be written as $N_0 + e^{\int_0^t r(\tau)d\tau}\int_0^t [X(\tau) + T_f(\tau)]e^{-\int_0^\tau r(s)ds} d\tau = 0$, which determines the path for lump sum taxes.

to the rate of time preference. Third, since the shadow values of employment in the two sectors are equalized at steady state, flows into and out of the informal sector cease. Finally, if $\lambda = 0$, so that all aid is untied, both aid and remittances enter the current account additively, as $R + F$, and are equivalent. On the other hand, this equivalence does not apply to incremental untied aid, if there is already some pre-existing tied aid.

From (17b) we see that in steady state, equilibrium unemployment is related to sectoral employment by:

$$\tilde{L}_U = \left(\frac{z}{\sigma + z} \right)\left(1 - \tilde{L}_s\right) = \frac{z}{\sigma}\tilde{L}_f \tag{19}$$

Thus unemployment in the steady-state arises solely due to job termination in the formal sector and not due to agents seeking to change jobs. Equation (19) further implies that $d\tilde{L}_s = -\left([\sigma + z]/z\right)d\tilde{L}_U$, $d\tilde{L}_f = \left(\sigma/z\right)d\tilde{L}_U$, so that any increase in unemployment is reflected by a more than proportionate reduction in employment in the informal sector, and accompanied by a smaller increase in employment in the formal sector. Further, the sectoral returns on employment are obtained by combining (17g) and (17h), while noting (17c), are related by:

$$(1 - \tau)\tilde{w}_f = \left(1 + \frac{z}{\beta + \sigma}\right)\tilde{p}\frac{\partial \tilde{Y}_s}{\partial \tilde{L}_s} \tag{20}$$

From (20) we see that in steady-state, workers in the formal sector earn an after-tax premium over the return to labor in the informal sector, with the wage premium being positively related to the job separation parameter, z, and inversely proportional to the job finding parameter, σ. When $z = 0$ or $\sigma \to \infty$, the steady-state after-tax wage rates are equalized across the two sectors, and there is no equilibrium unemployment; see (19).

4.   Calibration and Numerical Analysis

The macroeconomic equilibrium set out in Sections 3.1 and 3.2 is described by an 8-th order dynamic system having four state variables, $\left(K, L_f, L_s, \text{ and } V\right)$, and four co-state variables

$\left(q_1, q_K, q_f, \text{ and } q_s \right).$[17]   The high dimensionality and the complexity of the model's structure and dynamics renders an analytical solution intractable. We therefore proceed to analyze the model's local dynamic properties through a numerical calibration, by linearizing the equilibrium dynamics around the steady state equilibrium described in Section 3.2. Table 2 specifies the functional forms used for calibrating the model, and Table 3 describes the parameterization of the benchmark model specification. Our numerical simulations confirm the existence of a saddle-point equilibrium, characterized by four stable (negative) and four unstable (positive) eigenvalues.

The intertemporal elasticity of substitution for consumption in utility is given by $1/(1-\gamma)$. We set $\gamma = -1.5$ implying an elasticity of 0.4, consistent with the evidence provided by Guvenen (2006). The rate of time preference, β, is set at 0.06, slightly higher than the standard value of 0.04 used in the macro-growth literature, mainly to capture two features of a typical developing economy: relative impatience and higher mortality rates, both of which tend to raise the rate of time preference. $\theta = 0.5$ reflects the assumption that there is no bias toward either consumption good. The world interest rate, r^* and the borrowing premiums, ω, are set to yield an aggregate debt-output ratio that is consistent with our reference sample (to be described below). Further, $\beta > r^*$ ensures that the economy is a net debtor in equilibrium, running a current account deficit.

We normalize the productivity level in the informal sector to 1 and set the corresponding productivity level in the formal sector to be 50% higher. The share of private capital in the formal sector, α, is set to 0.4, which is a standard assumption in the literature. In the benchmark specification, we assume that the output elasticity of public capital is the same across the two sectors, and set $\varepsilon = \phi = 0.15$, consistent with the empirical evidence reported by Bom and Ligthart (2013). We will, however, examine the sensitivity of the model's results to variations in these parameters. The production function in the formal sector is assumed to be Cobb-Douglas, so that $s_f = 1/(1+\rho) = 1$ (this will also be subject to a sensitivity analysis below), and the adjustment cost parameter for investment, h, is consistent with Auerbach and Kotlikoff (1987) and Ortiguera and Santos (1997). The depreciation rates for private capital is set at 8% per year, consistent with

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[17] The stock of public capital, K_G, is an additional state variable that evolves exogenously, given the rate of public investment and the "tying" of aid to public investment.

empirical evidence for a developing country provided by Schündeln (2013). We set the depreciation rate for public capital to be slightly less than that for private capital at 7%. The labor share of employment in the informal sector, η is chosen to match the observed employment share in the informal sector in our sample, and is also consistent with Turnovsky and Basher (2009). The values for the search cost and job separation rate are chosen to yield an equilibrium unemployment rate that is consistent with our reference data (to be described below). The aid "tying" parameter, λ, is set based on the calculations in Chatterjee et al. (2012) from the OECD CRS and DAC databases, and suggests that 35% of all aid is tied by donors for investment in public capital. Finally, the income tax rate on formal sector output is backed out from the sample means of (i) share of tax revenues in GDP, and (ii) the share of the informal sector in GDP.

4.1. Benchmark Equilibrium

The benchmark steady-state equilibrium quantities are reported in Table 4. We compare these quantities to their corresponding annual estimates from a sample of 40 countries for the period 1997-2009. The choice of countries in the sample was dictated by the joint availability of data on informal employment *and* output (ILO and Schneider et al., 2010). Given the poor coverage for informal employment, we use the estimates for the latest year available for the period 1999-2007 from the ILO database. Data on the ratio of (efficiency-adjusted) public to private capital and the aggregate capital-output ratio are obtained from Gupta et al. (2014). The shares of private and public consumption, public and private debt, foreign aid, remittances, and tax revenues in GDP are obtained from the WDI. The mean real exchange rate in the sample is calculated from UNCTAD data, while the share of public investment in GDP is obtained from the GFS database. Finally, we use the calculations in Schneider et al. (2010) to get the average share of the informal sector in GDP.

As can be seen from Table 4, the benchmark equilibrium implied by our model specification matches closely the corresponding sample averages. The ratio of public to private capital implied by the model is about 0.64, while the consumption-output and capital-output ratios are about 0.81 and 1.3, respectively. The share of public debt in GDP is about 0.61, while that of private debt is 0.29. The formal sector accounts for about 59% of GDP, while employing 43% of the labor force. The

long-run unemployment rate is about 8.6%. All of these equilibrium quantities are very close to their corresponding empirical estimates, indicating that our prototype benchmark economy is a good representation of a developing country with a sizable informal sector. The policy and transfer variables in the model are parameterized to match their corresponding averages in the data. Consequently, the aid-to-GDP ratio is set to 5.7%, and that for remittances to 6.5%. Similarly, the shares of domestic public investment and consumption spending are set to 2.6%, and about 14% of GDP, respectively. Thus, the economy's total inflow of foreign transfers is about 12% of GDP, while total government spending is about 17% of GDP.

5. Transfer Shocks

In this section, we analyze the dynamic consequences of two types of permanent external shocks, namely increases in (i) foreign aid and (ii) remittances. We calibrate the shocks so that the ratio of each type of external transfer in GDP increases permanently by 1 percentage point from its benchmark specification. The plotted transition paths represent percentage deviations from the pre-shock steady-state equilibrium.

5.1. Increase in Foreign Aid

We begin with an analysis of the effects of a one percentage-point increase in the share of foreign aid in GDP, from its benchmark specification, with its composition (λ) remaining unchanged at its benchmark value, $\lambda = 0.35$. The steady-state outcomes are reported in Table 5 (first row), and the transition dynamics are illustrated in Figure 2.

As can be seen from the first row of Table 5, an aggregate increase in foreign aid (with its composition or "tying" remaining unchanged) is associated with a larger informal sector in the long run, with the shares of both output and employment in the formal sector declining over time. Given that about a third of aid inflows are tied to public investment (see the benchmark calibration in Table 4), this increases the productivity of private capital, leading a higher capital stock in the long-run. The real exchange rate appreciates, and coupled with the higher share of the informal sector, this

leads to a steady-state increase in aggregate output and consumption, together with a decline in equilibrium unemployment. Aggregate welfare improves by about 3.4%.

Figure 2 depicts the adjustment of the economy to the increase in aid. Since the stocks of private and public capital, as well as the sectoral labor allocations are instantaneously fixed, the higher aid flow is absorbed by an instantaneous appreciation of the real exchange rate, which consequently overshoots its long-run equilibrium. This instantaneous real appreciation increases the relative price of informal sector goods, thereby leading to a short-run increase in GDP and private consumption. Further, since the output of the formal sector cannot respond instantaneously (see eq. (1)), the instantaneous real appreciation of the exchange rate lowers its share in GDP.

The instantaneous appreciation of the real exchange rate, by increasing the relative price of informal sector goods, causes the real wage in the informal sector to increase. Consequently, labor flows into the informal sector, leading to a continuous decline in the employment share of the formal sector. On the other hand, the higher aid flow increases public investment and the stock of infrastructure, making labor and private capital more productive through its spillover effect on the sectoral production functions. This enables total output to increase in transition, and the formal sector to recover some of its lost share in total output. As labor transitions into the informal sector, diminishing returns set in, which leads to a transitional depreciation of the real exchange rate. The adjustment of the real exchange rate over time eventually restores equilibrium in the labor market, through a convergence of the sectoral shadow prices of employment. The net long-run real exchange rate appreciation, coupled with higher investment (public and private), as well as consumption, lead to an increase in the economy's indebtedness, thereby worsening the current account.

To summarize: in the long-run, a 1% permanent increase in the aid-to-GDP ratio leads to about a 1.6% real appreciation of the exchange rate, and a 3.3% increase in GDP. The stock of private capital and consumption increases approximately by 2.4% and 4.5%, respectively. The share of output and employment of the formal sector declines by 0.84% and 1.2%, respectively, while the aggregate unemployment rate declines by about 1.1%, with the informal sector absorbing the slack in the labor market.

5.2.   Increase in the Flow of Remittances

Table 5 (second row) and Figure 3 report the dynamic effects of a permanent increase in the flow of remittances from the rest of the world. This is an interesting case, since it helps us distinguish the effects of public sector transfers (aid) from private sector transfers.[18] We calibrate an increase in the remittance inflow of 1% of GDP from its benchmark level. In illustrating the dynamic response of the economy to this shock in Figure 3, we also plot the corresponding response to an increase in aid (from Figure 2), for comparison purposes. Therefore, Table 5 and Figure 3 compare two external shocks: one that accrues to the household budget (remittances) and the other that supplements the government's budget (aid), with an *a priori* restriction on its use, i.e., $\lambda = 0.35$.

Comparing the first and second rows of Table 5 and the dynamic time paths in Figure 3 (where the dashed plots represent the foreign aid shock), we see that the long-run aggregate effects of remittances differ both qualitatively and quantitatively from those generated by an aid shock, in some important dimensions. For example, the aggregate economy contracts in response to an inflow of remittances, with a long-run decline in GDP and capital stock, in sharp contrast to the expansionary effect of aid. The real exchange rate appreciates in the long-run, with a slight improvement in the economy's net debt position. Further, both the output and employment share of the formal sector declines, with the magnitudes being larger than those from the aid shock. Aggregate consumption and welfare also increases, albeit by smaller amounts relative to aid.

An interesting aspect to note here is the presence of a long-run *Dutch Disease* effect: a remittance inflow leads to a long-run a real appreciation of the exchange rate, a contraction of the shares of formal sector output and employment, and a decline in aggregate GDP. This is in sharp contrast to the case of foreign aid: a real exchange rate appreciation and the decline in the output and employment share of the formal sector does not lead to an overall contraction in GDP primarily because some aid is "tied" to public infrastructure which, by improving the productivity of labor and

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[18] Technically, an inflow of remittances is observationally equivalent to an increase in "untied" aid, i.e., when aid is not tied to public infrastructure. In such a case, both aid and remittances enter the current account as a lumpsum transfer from abroad, as in Brock (1996); Also see Chatterjee et al. (2003) and Chatterjee and Turnovsky (2007).

capital, helps offset for a Dutch Disease effect in the long-run.[19]

Figure 3 depicts the transitional adjustment of the economy to a remittance shock, with the dynamics of the foreign aid shock from Figure 2 also plotted for comparison. While remittances increase GDP in the short-run due to the instantaneous appreciation of the real exchange rate, this increase cannot be sustained over time, with GDP declining in a sustained fashion over time. This is due to the fact that the increase in the relative price of the informal sector good draws labor into the informal sector, thereby reducing the productivity of private capital in the formal sector. With no complementary investment in public infrastructure (as in the case of aid), this leads to a decumulation of private capital, which leads to more labor leaving the formal sector. In the long-run the lower overall productivity in the formal sector more than offsets the gains in the informal sector, and GDP contracts. Another interesting feature of the transitional adjustment is that it is much more persistent relative to the case of aid. Again, the gradual decline in the stock of private capital leads to a persistent decline in the employment share of the final sector which, given the intersectoral mobility costs for labor, slows down the dynamic adjustment. In the case of aid, the higher investment in public infrastructure enables private capital to increase, which in turn, increases the convergence speed to the long-run equilibrium.

5.3.   The Composition of Aid

In this section, we consider the effects of a permanent change in the *internal* allocation of aid, while its aggregate inflow remains unchanged. Specifically, we consider an increase in the aid allocation parameter λ from its benchmark level of 0.35 to 0.4, so that more existing aid is allocated to public investment. The results are reported in Table 6, with the dynamics illustrated in Figure 4.

When the allocation of aid to public investment increases, the long-run effects, especially with respect to the size of the informal sector, contrast sharply from those resulting from an increase in aggregate aid (Table 5). From Table 6 we see that the share of the formal sector in GDP increases, as does the share of employment in that sector. Therefore, changing the composition of

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[19] In fact, it can be shown that diverting more aid to public investment in response to an increase in remittance inflows can, indeed, help to offset the Dutch Disease effect. The numerical results of this counterfactual experiment are available on request.

aid in favor of public investment can actually lead to a shrinking of the informal sector. Overall, tying more aid to public investment expands the economy. The real exchange rate appreciates in the long run, reflecting the increase in private sector productivity as aid is added to productive public investment. The current account declines, due to both the economic expansion and the decline in use of aid resources for paying down the economy's outstanding debt. At the same time, tying more aid to public investment increases long-run unemployment slightly. This occurs because the expansion and the consequent increase in labor demand in the formal sector increases the cost of labor mobility, thereby creating more involuntary unemployment. The net effect of these adjustments associated with increasing λ from 0.35 to 0.40 is to raise long-run welfare by about 1%.

Figure 4 shows the dynamic adjustment of the economy to the increase in the "tying" composition of aid. Tying more of existing aid to public investment instantaneously appreciates the real exchange rate (due to the expectation of higher long-run productivity). This instantaneous response charts the future course of the economy. An interesting point to note here is the non-monotonic adjustment of the labor share of the formal sector, which leads to similar non-monotonic adjustments in the time paths of the real exchange rate and the output share of the formal sector. Specifically, tying more existing aid to public investment initially reduces the employment and output shares of the formal sector (due to the instantaneous real appreciation of the exchange rate). However, as public capital accumulates (due to the re-allocation of aid), the higher productivity gains in the formal sector draws labor back to that sector, thus generating a U-shaped adjustment for formal sector employment. In the long-run, both the employment and the output shares of the formal sector increase.

Overall, the results in Table 6 and Figure 4 suggest that a change in the composition of aid can have important consequences for the real exchange rate, output, and the informal sector. While tying more existing aid to public investment expands the economy and the formal sector in the long run, re-allocating aid away from public investment has exactly the opposite effect, resulting in a

long-run economic contraction, worsening of welfare, and an increase in the size of the informal sector.[20]

5.4.   Welfare Analysis

The finding that increasing λ from 0.35 to 0.40 is welfare improving raises questions of the optimal degree to which aid should be tied, given the mean inflow of remittances of 6.5% of GDP. As is well known, the optimal allocation of resources to public capital depends on its productive elasticity. In this case with two productive sectors, there are two such parameters, characterizing the productive elasticities of public capital, in the formal and informal sectors, respectively. Accordingly, Figure 5 illustrates the welfare implications of the composition of aid – i.e., the degree of "tying," aid λ – with respect to variations in ε, and ϕ. Specifically, we consider welfare along two dimensions:

i.   The sensitivity of the steady-state *level* of welfare to the allocation of a *fixed aggregate level* of foreign aid, i.e. as λ increases from 0 (fully untied aid) to 1 (fully tied aid), and how this level is affected by the relative sectoral elasticities of public capital. This is illustrated in Figure 5A.

ii.   The sensitivity of the *change* in intertemporal welfare to the allocation of an *increase* in foreign aid to variations in its underlying composition and how it is impacted by the relative sectoral elasticities of public capital. This is depicted in Figure 5B.[21]

From Figure 5A, we see that for a given values of sector-specific elasticities of public capital, steady-state welfare increases monotonically with λ, i.e., as more existing aid is diverted to public investment. However, there is evidence of diminishing returns to aid, as the welfare increments across steady states decline as λ approaches 1. On the other hand, for a given value of a sector-

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[20] Decreasing the tying of aid by 1% has approximately symmetric effects. There is a slight asymmetry in the effects of a change in the composition of aid is due to the fact that each of these shocks moves the economy in opposite directions with respect to its social optimum. Consequently, this has differential effects on the return to labor and capital, leading to the asymmetries reported in Table 6.

[21] In each case, we calculate steady-state and intertemporal welfare as the amount of private consumption that must be transferred across steady-states or over the transition path to make the representative household indifferent between two equilibrium allocations.

specific elasticity, an increase in the other sector's output elasticity reduces the level of steady-state welfare for any given internal allocation of aid, further underscoring the existence of diminishing returns. The fact that steady-state welfare increases monotonically with λ is a reflection of the fact that even when $\lambda = 1$, for the calibrated model only 5.7% of GDP is designated to productive investment. With the productive elasticities generally larger than that suggests that the optimum allocation is in fact to set $\lambda > 1$ and supplement the allocation with higher taxes or debt.

Figure 5B compares the intertemporal welfare changes from a given increase in aid, as its composition and the relative sectoral elasticity of public capital changes. Here we see that the larger the sectoral output elasticities for public capital, the more is the increase in intertemporal welfare, for any given internal allocation of aid. However, Figure 5B also suggests that these welfare changes are maximum for $\lambda \in (0.3, 0.4)$; interestingly, our benchmark specification for the internal allocation of aid is 0.35, which lies right in the middle of this range.

6. Sensitivity Analysis

In this section, we examine the sensitivity of the main results of the model to two sets of structural parameters, namely

    i.   productivity parameters, where we examine the sectoral output elasticity of public capital (ε and ϕ), the elasticity of substitution in production for the formal sector, $1/(1+\rho)$, and the output elasticity of labor in the informal sector, η, and

    ii.   labor market parameters, where we vary the cost of labor mobility between the formal and informal sectors, χ, the rate of job separation (z), and the rate of job finding (σ).

The results of our sensitivity analysis are reported graphically, in Figures 6-9. We consider the case of an aggregate foreign aid shock (with the benchmark specification for its composition, in Figures 6-7) and a remittance shock (Figures 8-9).

For the set of productivity parameters, we consider the following cases. For the sectoral output elasticity of public capital, we consider (i) $\varepsilon = 0.15, \phi = 0$ (no sectoral productivity benefits

from public capital in the informal sector), (ii) $\varepsilon = \phi = 0.15$ (benchmark case), and (iii) $\varepsilon = 0, \phi = 0.15$ (public capital not productive for the formal sector). For the elasticity of substitution in production in the formal sector, we consider cases where (i) $s_f = 0.75$ (low elasticity), (ii) $s_f = 1$ (Cobb-Douglas), and (iii) $s_f = 1.25$ (high elasticity). For the labor elasticity in the informal sector, we consider (i) $\eta = 0.05$, (ii) $\eta = 0.75$, and (iii) $\eta = 0.95$, indicating a large range for labor productivity (and share) in the informal sector. For the set of labor market parameters, we consider the following: for the intersectoral labor mobility cost, we consider (i) $\chi = 1$ (low mobility cost), (ii) $\chi = 15$ (benchmark specification), and (iii) $\chi = 30$ (high mobility cost). For the rate of job separation, we set the parameter z to 0, 0.01, and 0.05, respectively. For the rate of job finding, we consider $\sigma = 0.05$, 0.15, and 0.3.

We report the dynamic responses of four key macroeconomic variables for variations in each structural parameter, namely the employment share of the formal sector, the output share of the formal sector, aggregate output (GDP), and the real exchange rate. Qualitatively, Figures 6-9 suggest that the model's transitional dynamics with respect to both aid and remittance shocks are remarkably robust to variations in these key structural parameters.

7. Conclusions

Developing countries that receive a large share of their GDP in the form of external transfers such as aid and remittances are also typically associated with large informal sectors that absorb, on average, more than 40 percent of their labor force and account for more than 50% of GDP. We develop a two-sector open economy model that characterizes many of the features of these economies, such as costly (and sluggish) movement of labor from the informal to the formal sector, long-run structural unemployment, and the lack of capital usage and tax collection in the informal economy. On the other hand, productive public goods (which may be financed by aid, for example) generate spillovers for both the formal and the informal sectors, generating demand for private capital (in the formal sector) and labor demand (in both sectors). Further, remittances, by freeing up scarce private resources, may also affect the sectoral allocation of time to work.

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By embedding external transfers and the informal sector in a dynamic general equilibrium model, we bridge two important areas of research in development economics. On the one hand, the literature on the informal economy has focused mainly on the issues of size, measurement, and tax avoidance and enforcement, largely ignoring the issue of external transfers. On the other, the literature on foreign aid and remittances has dealt with their macroeconomic effects on the aggregate (or formal) economy, without reference to their implications for the informal sector. Our paper is the first systematic approach in bringing these two areas of work together.

Our results indicate that while remittances may lead to a short-run economic expansion, they are unambiguously associated with an economic contraction and more informality in the long run. On the other hand, the effects of aid are more ambiguous, depending critically on how aid is internally spent. If aid is allocated more towards public investment, the aggregate economy expands and informality declines over time. However, if aid is fungible, and is diverted away from investment into, say, general budget support, the economy contracts and informality increases. Aid and remittances also have interesting effects on the real exchange rate. Remittances generate a long-run Dutch Disease effect, with a real appreciation of the exchange rate, an increase in informality (non-traded sector), and an aggregate economic contraction. However, such a phenomenon is not associated with aid. Further, our welfare analysis suggests that while re-allocating aid to public investment increases welfare, there are diminishing returns to this allocation, with the maximum welfare gains realized when about 30-40% of aid inflows are tied to public investment.

Our analysis has, indeed, abstracted away from several other important features that may characterize the relationship between international transfers and the informal economy. These include (but are not restricted to) the skill composition of the labor force, formal entry barriers into the labor market, public-sector inefficiencies, borrowing constraints for households, and tax enforcement. These are undoubtedly important considerations, and we intend to pursue them in future research.

TABLE 1. Foreign Transfers and the Informal Economy, 1999-2007

	Mean	Median	Min	Max	Standard Deviation
Foreign Aid (% of GDP)	5.692	1.907	0.014	33.629	7.692
Remittances (% of GDP)	6.511	3.101	0.078	50.576	9.379
Informal sector output (% of GDP)	41.836	41.639	16.067	68.122	11.426
Self-employment (% of total employment)	48.816	46.033	8.289	91.3	22.792
Informal employment (% of total employment)*	53.689	59.6	6.1	83.5	20.319
Number of countries = 40					

Data Source: Schneider et al. (2010), OECD, WDI, ILO
*Informal Employment data is for the latest year available in the sample (ILO, 2012)

TABLE 2. Functional Forms

Description	Functional Form
Utility function	$U\left(C_f, C_s\right) = \left(C_f^\theta C_s^{1-\theta}\right)^\gamma / \gamma$
Production function-Formal sector	$Y_f = A_f\left[\alpha K^{-\rho} + (1-\alpha)L_f^{-\rho}\right]^{-1/\rho}$; $A_f = \bar{A}_f K_G^\varepsilon$
Production function-Informal Sector	$Y_s = A_s L_s^\eta$; $A_s = \bar{A}_s K_G^\phi$
Borrowing cost functions	$r = r^* + e^{\omega(N+B)/Y} - 1$
Adjustment cost for investment	$\Omega = I\left(1 + \dfrac{h}{2}\dfrac{I}{K}\right)$

TABLE 3. Parameterization of the Benchmark Model

A. Structural Parameters

Parameter	Description	Value
γ	Intertemporal elasticity of substitution in consumption	-1.5
β	Rate of time preference	0.06
θ	Relative weight of formal-sector good in utility	0.5
ω	Borrowing premium-Households	0.022
r^*	World interest rate	0.04
$\bar{A}_f$	Productivity level-formal sector	1.5
$\bar{A}_s$	Productivity level-informal sector	1
α	Share of private capital in formal sector	0.4
ε	Output elasticity of public capital-formal sector	0.15
ϕ	Output elasticity of public capital-informal sector	0.15
s_f	Elasticity of substitution in formal sector production	1
h	Adjustment cost for investment	15
δ_K	Depreciation rate for private capital (annual)	0.08
δ_G	Depreciation rate for public capital (annual)	0.07
η	Share of labor in informal sector production	0.75
z	Rate of job separation	0.01
σ	Rate of job finding	0.05
χ	Labor mobility cost	15
λ	Aid allocation to public investment	0.35
τ	Tax rate on formal sector output	0.3

TABLE 4. Benchmark Steady-State Equilibrium

Endogenous Variables	Description	Model	Data*	Data Source
K_G/K	Ratio of pubic to private capital	0.640	0.676	Gupta et al. (2014)
C/Y	Aggregate consumption-output ratio	0.813	0.803	WDI
K/Y	Aggregate capital-output ratio	1.279	1.163	Gupta et al. (2014)
B/Y	Public debt-output ratio	0.605	0.604	WDI
N/Y	Private debt-output ratio	0.295	0.299	WDI
Y_f/Y	Share of formal sector in GDP	0.593	0.582	Schneider et al. (2010)
L_f/L	Share of formal employment (in total employment)**	0.426	0.463	ILO
L_U	Unemployment rate	0.086	0.086	WDI
p	Real exchange rate	0.827	0.973	UNCTAD
Calibrated Variables	**Description**	**Model**	**Data***	**Data Source**
G_I/Y	Share of public investment in GDP	0.026	0.026	GFS
G_C/Y	Share of public consumption in GDP	0.143	0.143	WDI
F/Y	Foreign aid (share of GDP)	0.057	0.057	OECD (DRS)
R/Y	Remittances (share of GDP)	0.065	0.065	WDI

*Sample averages for 40 developing countries for the period 1999-2007.
**Employment share of the formal sector is for the latest year available in the ILO database (between 1999-2007).

TABLE 5. Aid versus Remittances: Steady-State Effects

	dY_f/Y	dL_f/L	dL_U	dK	dC	dY	dV	dp	Welfare Change
Foreign aid shock	-0.836	-1.176	-1.076	2.402	4.526	3.265	3.265	1.638	3.375
Remittance Shock	-0.924	-1.299	-1.189	-1.189	1.078	-0.267	-0.267	0.268	1.629

TABLE 6. Composition of Aid: Steady-State Effects

	dY_f/Y	dL_f/L	dL_U	dK	dC	dY	dV	dp	Welfare Change
More "tying" to public investment	0.139	0.197	0.179	2.549	2.199	2.407	2.407	0.898	1.024
Less "tying" to public investment	-0.138	-0.195	-0.178	-2.719	-2.389	-2.586	-2.586	-0.987	-1.113

All changes are reported as percentage deviations from the pre-shock steady state.
All shocks are calibrated to equal a one percentage-point increase from their benchmark specification.

FIGURE 1. Foreign Transfers and the Informal Economy
Selected Countries, 1999-2007
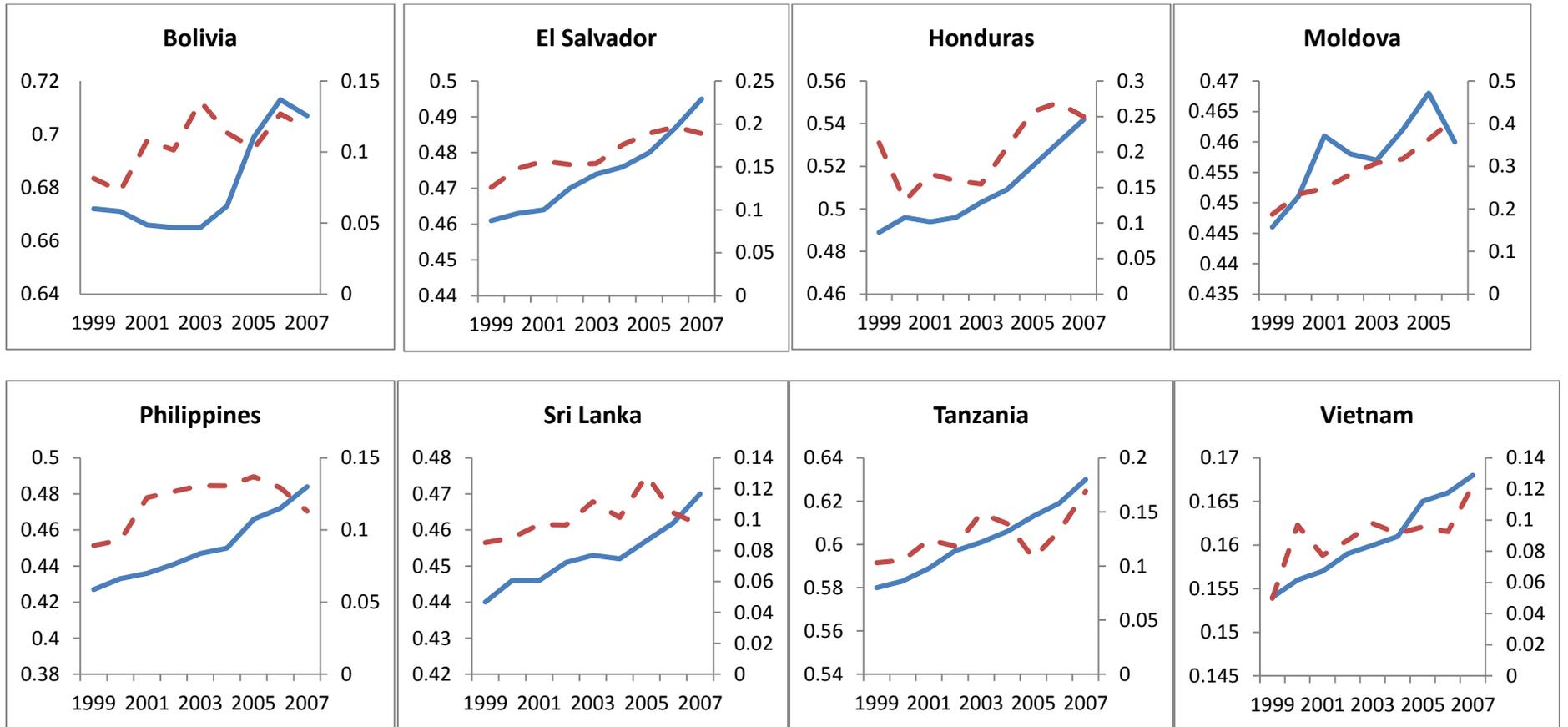
Share of informal sector in GDP (left vertical axis)

Share of foreign transfers (aid + remittances) in GDP (right vertical axis)

Data Sources: Schneider et al. (2010), OECD, WDI, and authors' calculations

FIGURE 2: Aggregate Foreign Aid Shock
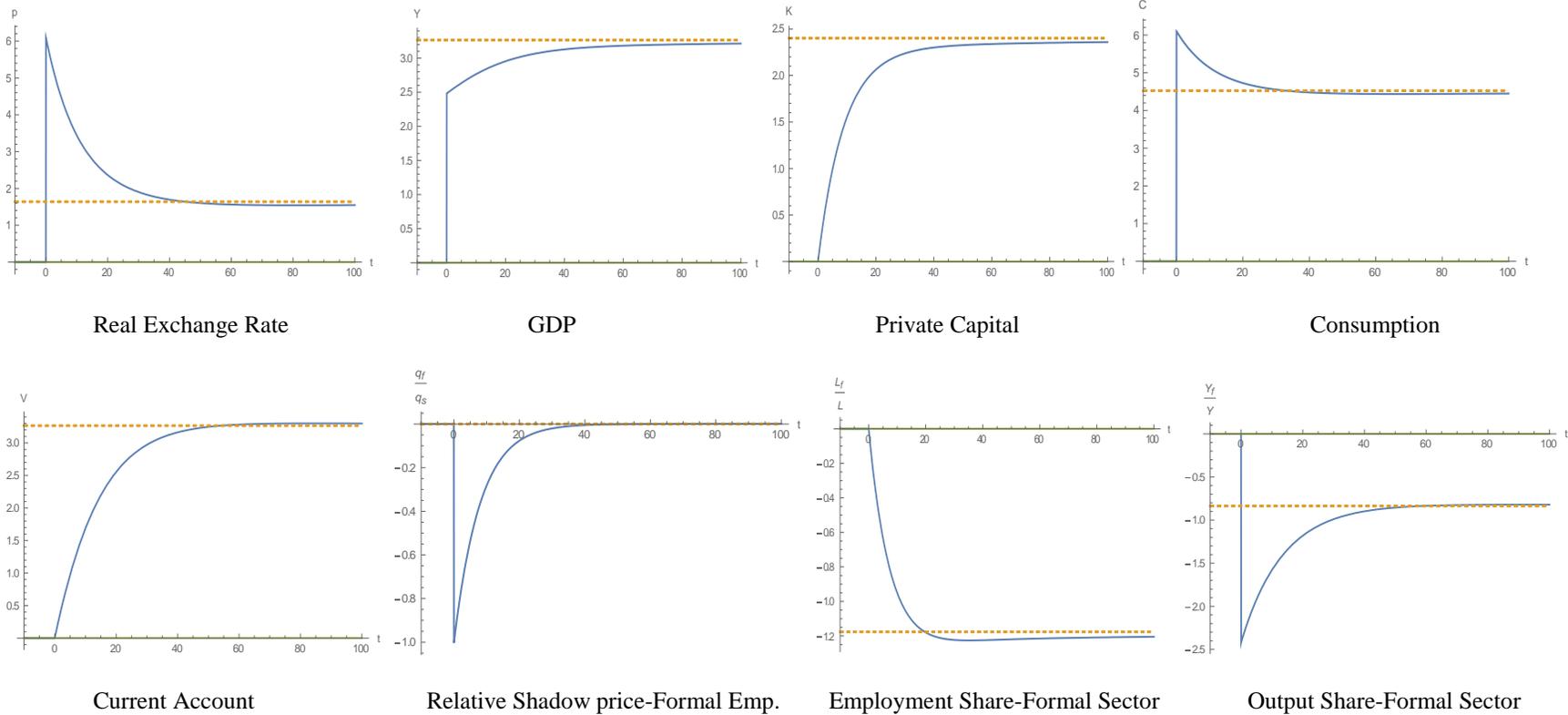
| Real Exchange Rate | GDP | Private Capital | Consumption |

| Current Account | Relative Shadow price-Formal Emp. | Employment Share-Formal Sector | Output Share-Formal Sector |

All variables are plotted as percentage deviations from their pre-shock steady state levels

FIGURE 3: Remittance Shock
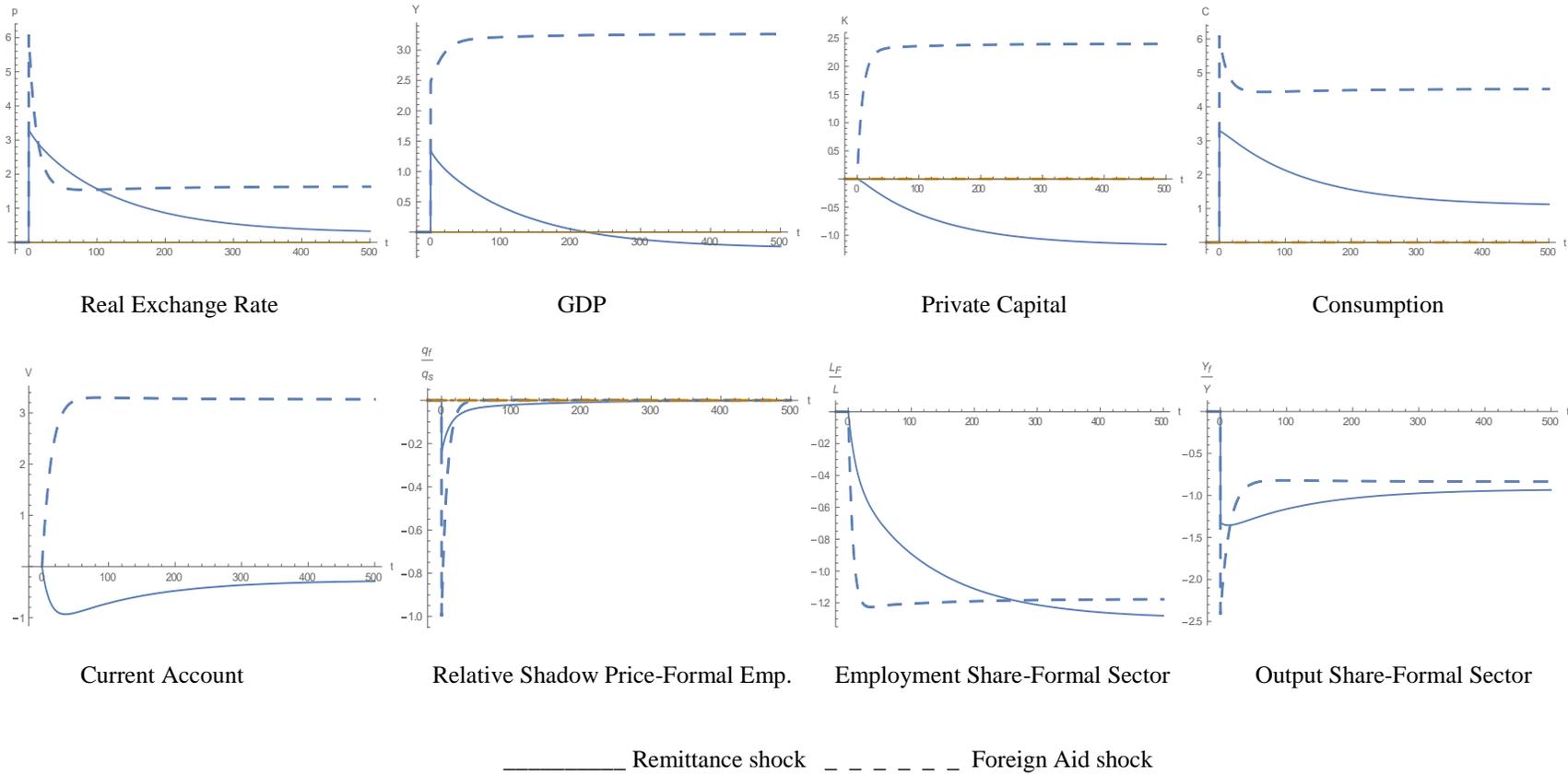
Real Exchange Rate   GDP   Private Capital   Consumption

Current Account   Relative Shadow Price-Formal Emp.   Employment Share-Formal Sector   Output Share-Formal Sector

_____ Remittance shock   _ _ _ _ _ Foreign Aid shock

All variables are plotted as percentage deviations from their pre-shock steady state levels

FIGURE 4: Change in the Composition of Foreign Aid
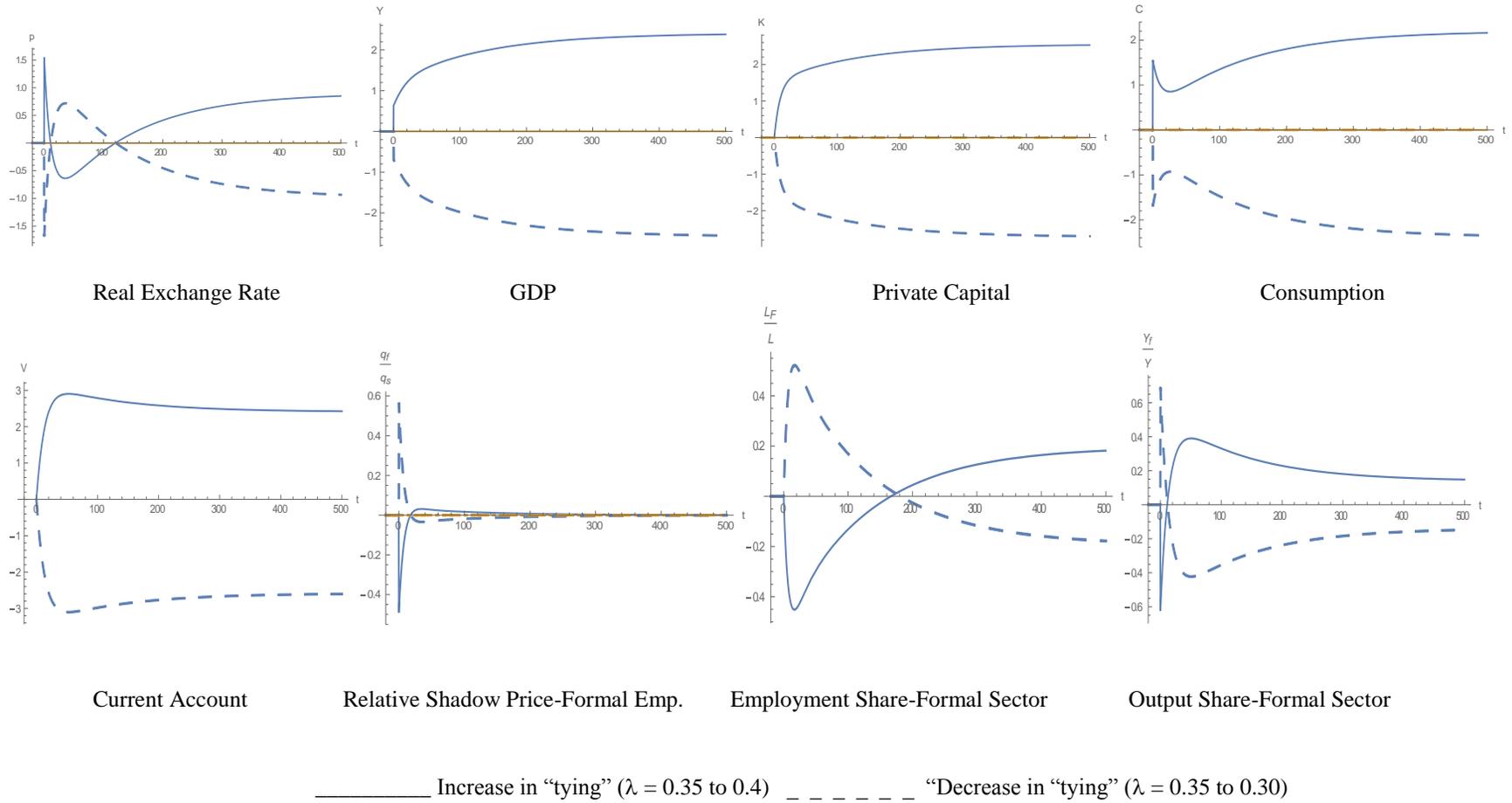
Real Exchange Rate

GDP

Private Capital

Consumption

Current Account

Relative Shadow Price-Formal Emp.

Employment Share-Formal Sector

Output Share-Formal Sector

_____ Increase in "tying" ($\lambda = 0.35$ to 0.4)   _ _ _ _ _ _ "Decrease in "tying" ($\lambda = 0.35$ to 0.30)

All variables are plotted as percentage deviations from their pre-shock steady state levels

FIGURE 5: The Sectoral Elasticity of Public Capital, the Composition of Aid, and Welfare

A. Steady-State Welfare Level
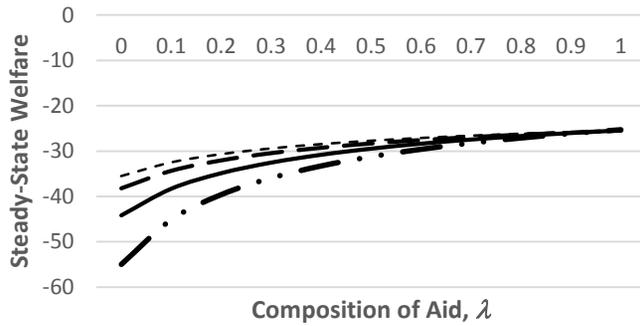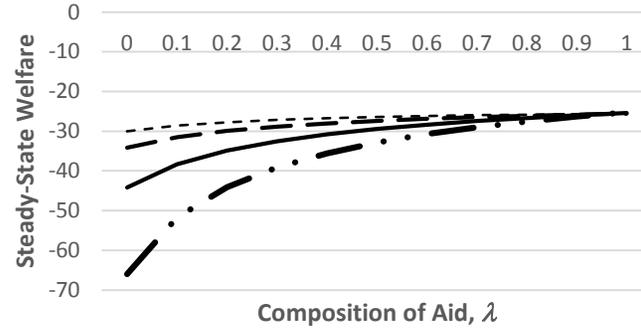
B. Intertemporal Welfare Changes
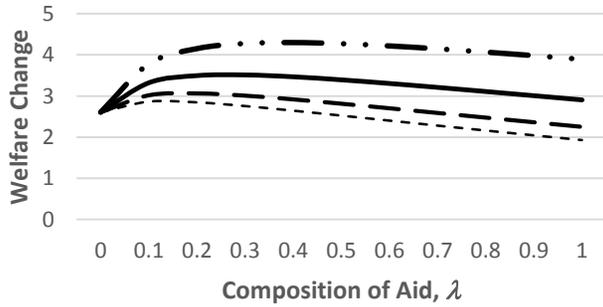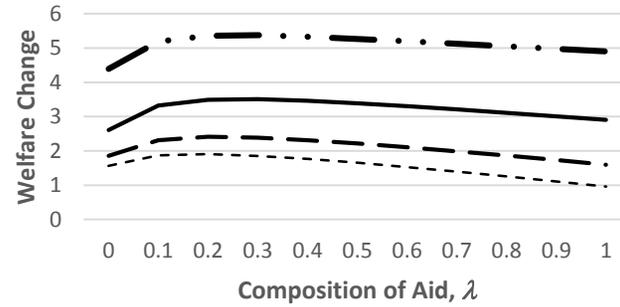
FIGURE 6. Foreign Aid Shock: Sensitivity Analysis-Productivity Parameters

A. Sectoral Output Elasticity of Public Capital (ε, φ)

$- - - - -$ ε = 0.15, φ = 0 _____ ε = φ = 0.15 $— — — —$ ε = 0, φ = 0.15

B. Elasticity of Substitution in Production-Formal Sector [$s_f = 1/(1+\rho)$]

$- - - - -$ $s_f = 0.75$ _____ $s_f = 1$ $— — —$ $s_f = 1.25$
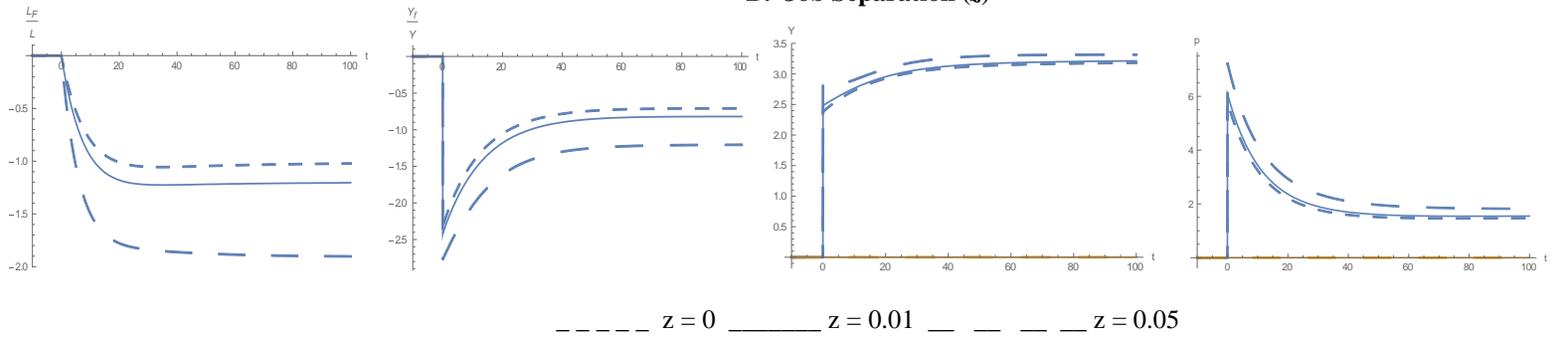
C. Output Elasticity of Labor-Informal Sector (η)

$- - - - -$ η = 0.05 _____ η = 0.75 $— — —$ η = 0.95

FIGURE 7. Foreign Aid Shock: Sensitivity Analysis-Labor Market Parameters

A. Mobility Cost (χ)

$- - - -\ \chi = 1 \quad \underline{\qquad}\ \chi = 15 \quad - \ - \ - \ -\ \chi = 30$

B. Job Separation (z)

$- - - -\ z = 0 \quad \underline{\qquad}\ z = 0.01 \quad - \ - \ - \ -\ z = 0.05$

C. Job Finding (σ)

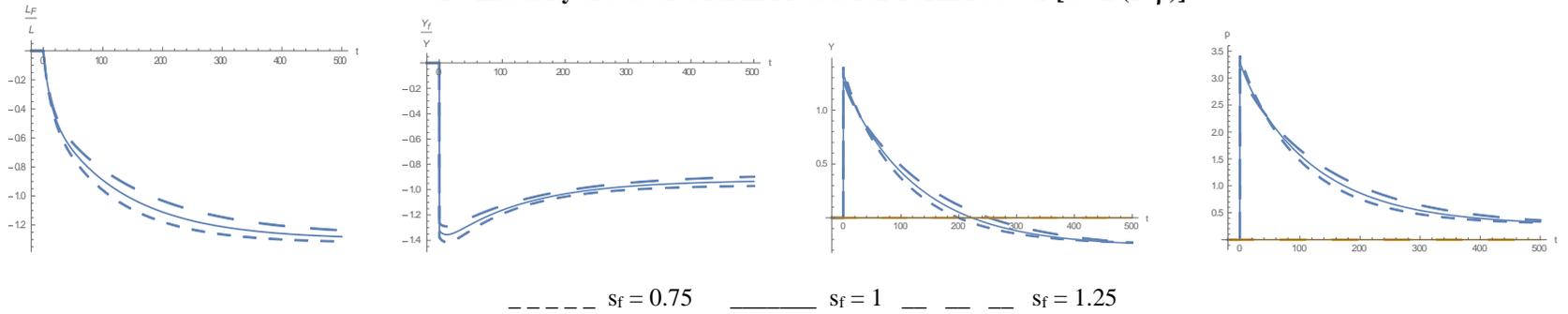$- - - -\ \sigma = 0.05 \quad \underline{\qquad}\ \sigma = 0.15 \quad - \ - \ - \ -\ \sigma = 0.3$
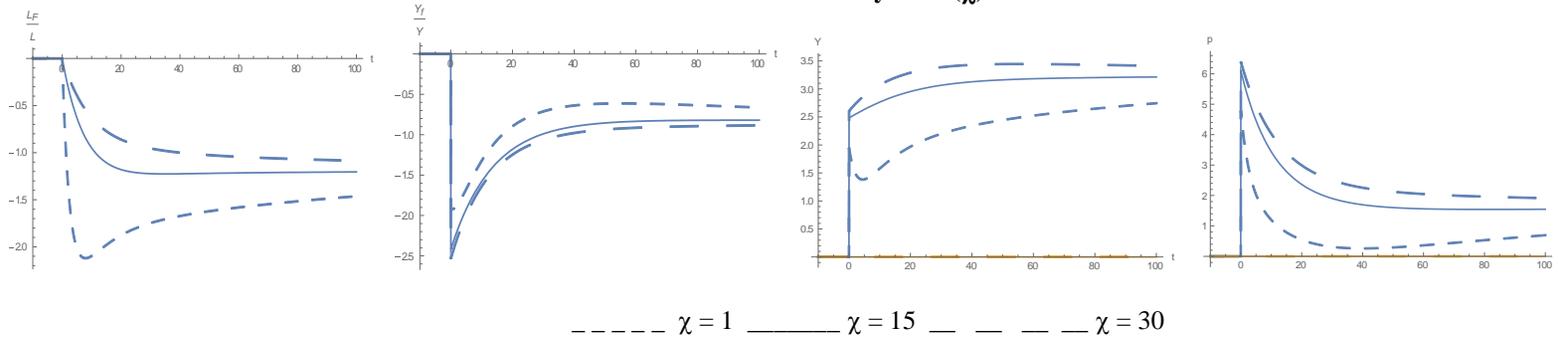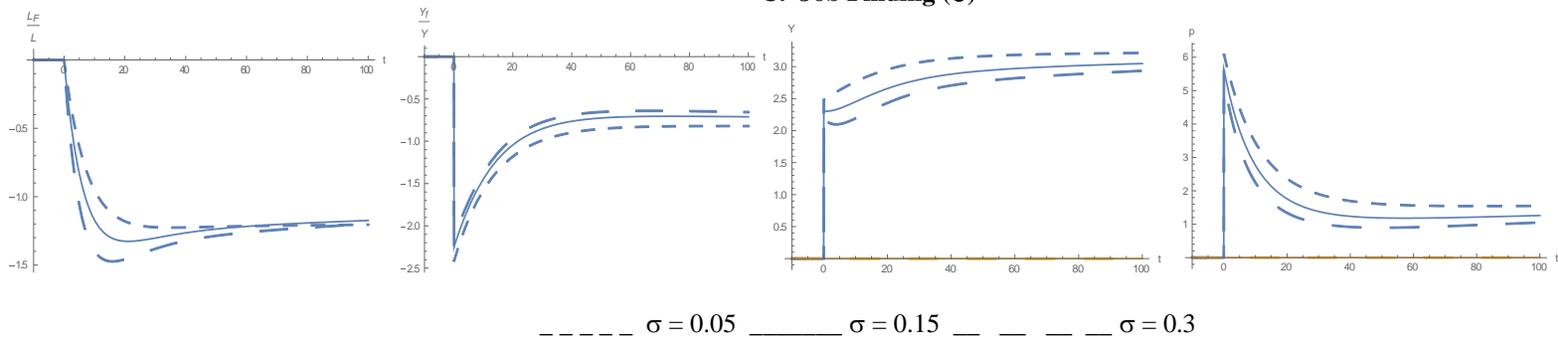
FIGURE 8. Remittance Shock: Sensitivity Analysis-Productivity Parameters

A. Sectoral Output Elasticity of Public Capital (ε, φ)

$- - - - - \varepsilon = 0.15, \phi = 0$ _____ $\varepsilon = \phi = 0.15$ $- \ - \ - \ - \ \varepsilon = 0, \phi = 0.15$

B. Elasticity of Substitution in Production-Formal Sector [$s_f = 1/(1+\rho)$]

$- - - - - s_f = 0.75$ _____ $s_f = 1$ $- \ - \ - \ s_f = 1.25$

C. Output Elasticity of Labor-Informal Sector (η)

$- - - - - \eta = 0.05$ _____ $\eta = 0.75$ $- \ - \ - \ \eta = 0.95$
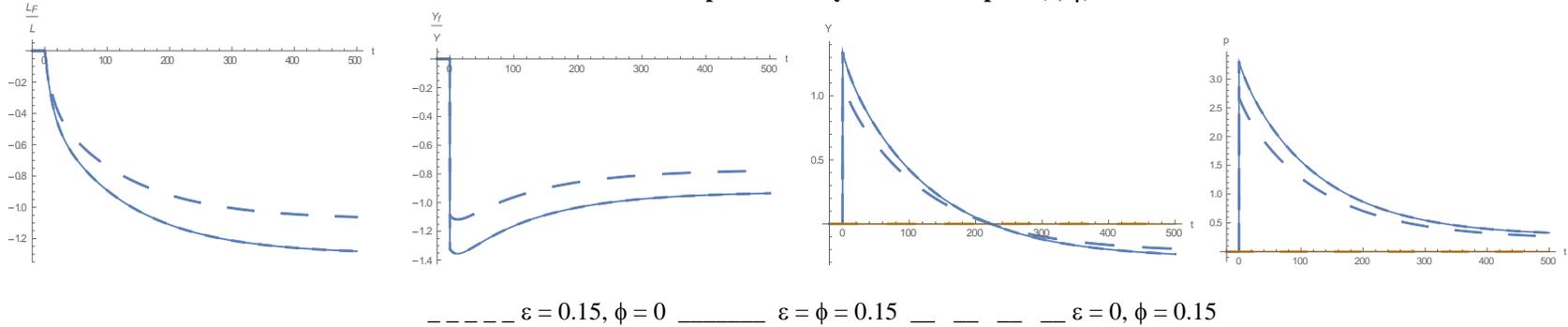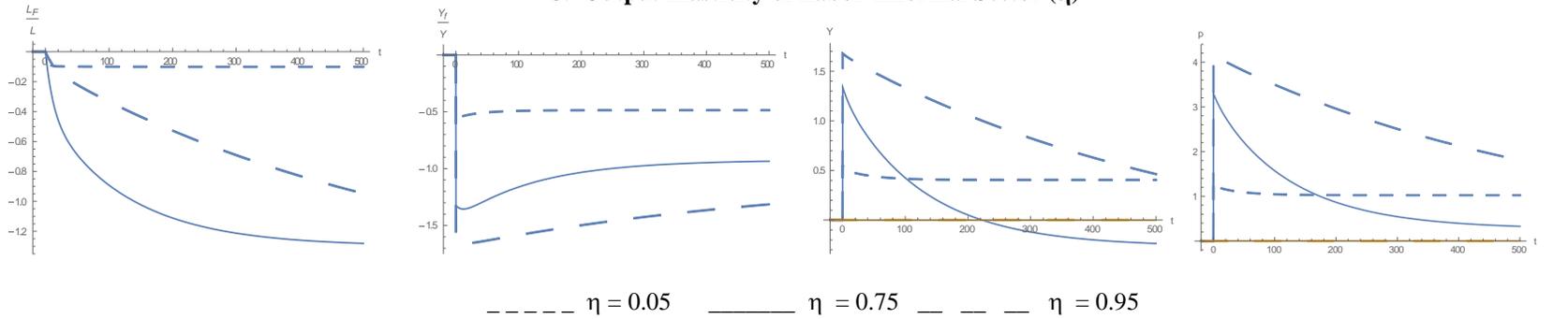
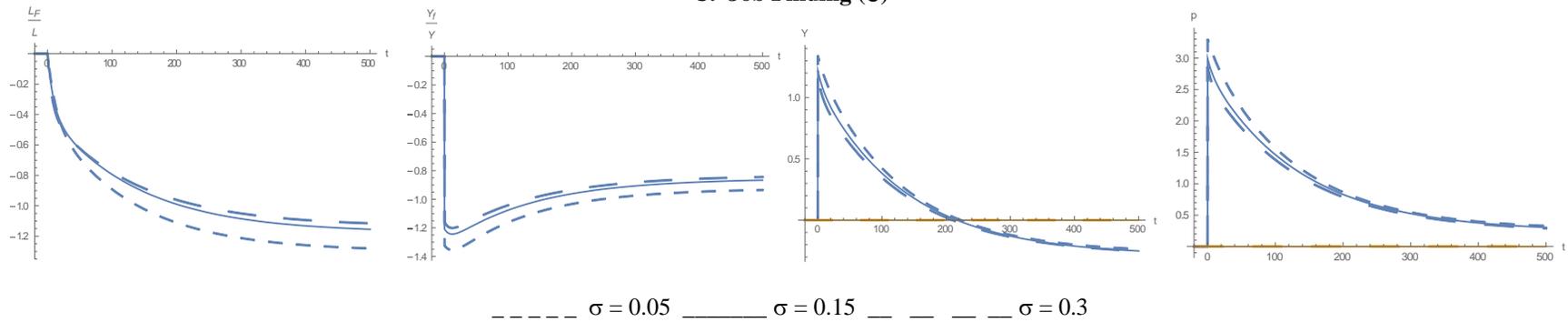## FIGURE 9. Remittance Shock: Sensitivity Analysis-Labor Market Parameters

A. Mobility Cost (χ)

$----$ $\chi = 1$ _____ $\chi = 15$ $--$ $--$ $--$ $\chi = 30$

B. Job Separation (z)

$----$ $z = 0$ _____ $z = 0.01$ $--$ $--$ $--$ $z = 0.05$

C. Job Finding (σ)

$----$ $\sigma = 0.05$ _____ $\sigma = 0.15$ $--$ $--$ $--$ $\sigma = 0.3$
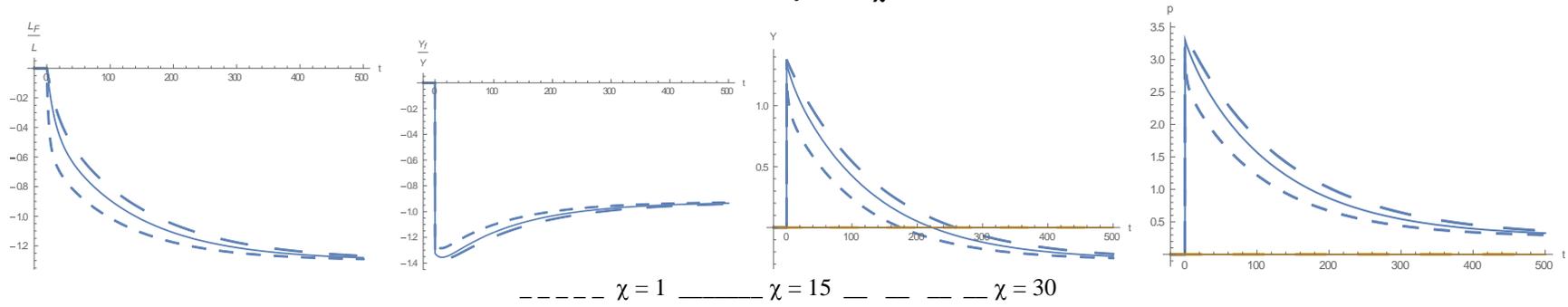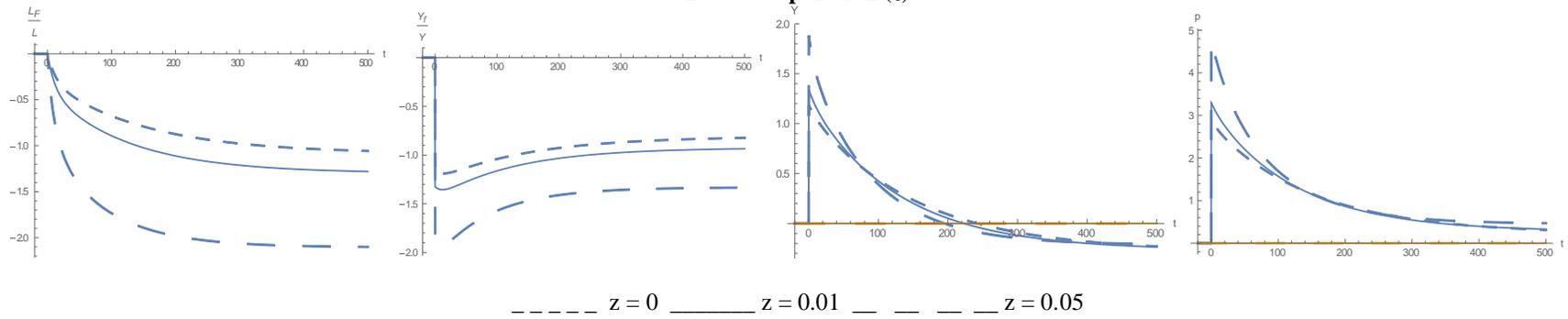
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