# Sovereign Default: The Role of Expectations. 

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- Argentina defaulted at the end of 2001, after a 3 year recession.
- During the previous 8-year currency board period:

1. Inflation was like in the US.
2. Average rate on dollar denominated bonds was $10 \%=3 \%+7 \%$.
3. GDP per capita grew at about $4 \%$ a year.
4. Average debt to GDP ratio was $40 \%$. Deficit never above $2 \%$.
5. Satisfied all conditions of Maastricht treaty, since 1991 to 2001, except for...
.....the interest rate on government bonds. (Market determined)

- The excess rate ( $7 \%$ ) implied, over the 8 years, an additional payment of almost $20 \%$ of GDP, half of the debt.
- The IMF withdrew support in Sept 2001 (7b U\$).
- Would Argentina had defaulted had the rates been $3 \%$ ?


## Recent European Experience

## European Bond Spreads

Basis points, 10-year bond spread to German bonds


Source: Global Financial Data

- How can a country avoid a sovereign default crisis?
- By growing.
- The US grew out of its debt after the war.
- "Once you start thinking about it, it is hard to think about anything else"
- By policy intervention?
- Did the ECB save Europe?
- "Once you start thinking about it,....."
- Caveat: German Constitutional Court ruling.
- Can a country be trapped in an equilibrium, where interest rates are high because default probabilities are high, but default probabilities are high because interest rates are high?
- Calvo (1988), more recently Lorenzoni and Werning (2013).
- Eaton-Gersovitz (1981), Aguiar and Gopinath (2006), Arellano (2008), compute a single low interest rate equilibrium.
- The main point of this paper: The standard quantitative model of sovereign default can have multiple equilibrium interest rates.
- We take a model similar to the one in Aguiar and Gopinath (2006) and Arellano (2008), and show that multiple equilibria arise with minor changes in modelling choices concerning

1. the timing of moves by debtors and creditors or
2. the actions that they may take.

- The change in modelling choices are minor because there is no direct evidence to discriminate across them.

Plan

- Two period model, standard distributions for endowment: Multiple equilibria with high and low rates.
- High rate equilibria are fragile. High rate equilibria with good and bad times (long stagnation) have good properties.
- Role for policy.
- Other timing, and action assumptions. Literature.
- Dynamic model with sunspot variable that shifts schedules. Numerical exercise.

The two-period small open economy model

- The endowment is 1 in the first period.
- In the second period it is $y \in[1, Y]$, with density $f(y)$ and $\operatorname{cdf} F(y)$.
- The initial level of debt is zero. The agent can borrow in a noncontingent bond. Cannot commit to repay.
- There is a penalty for defaulting: consumption at $t=2$ is equal to 1 .
- Foreign lenders are risk neutral.
- The expected return, considering default, $R$, has to be equal to the risk-free rate, $R^{*}$.
- The timing of moves. First period:
- Creditor $i \in[0,1]$ offers limited funds at gross interest rate $R_{i}$.
- The borrower moves next and borrows from the low rate creditors $b=\int_{0}^{1} b_{i} d i$.
- In equilibrium, $R_{i}=R$. Let $b_{i}=b$.
- Second period: The borrower defaults if

$$
y^{\prime}-b R<1
$$

so the probability of default is given by $F\left[1+b^{\prime} R\right]$.

- At period 1, the borrower chooses $b$ to maximize utility, subject to some maximum level of debt.
- At an interior solution, the first order condition

$$
U^{\prime}(1+b)=R \beta \int_{1+b R}^{Y} U^{\prime}(y-b R) f(y) d y
$$

defines a demand function.

## A supply curve

- Arbitrage condition for the creditors:

$$
R^{*}=R[1-F(1+b R)] \equiv h(R ; b)
$$

- For a given level of debt, if $R=0, h(0 ; b)=0$. With a bounded support, for $R$ high enough, $h(R ; b)=0$.
- For standard distributions, the function $h(R ; b)$ is concave.

- $h(R ; b)=R[1-F(1+b R)]$. An increase in $b$ shifts the curve down.

- Increasing and decreasing schedule.


## Equilibrium

- Supply

$$
R^{*}=R[1-F(1+b R)]
$$

- Demand

$$
U^{\prime}(1+b)=R \beta \int_{1+b R}^{Y} U^{\prime}(y-b R) f(y) d y
$$



## Fragility

- Consider a small - SW - perturbation of the decreasing schedule.



## Good and bad times: A bimodal distribution.

- Let

$$
y^{1} \backsim N\left(\mu^{1}, \sigma\right), \text { and } y^{2} \backsim N\left(\mu^{2}, \sigma\right), \text { with } \mu^{1}<\mu^{2}
$$

- The endowment is $y^{1}$ with probability $p$.
- If $\mu^{1}$ and $\mu^{2}$, are sufficiently apart, the arbitrage condition has four solutions.
- Interpretation: risk of stagnation.

- $R^{*}=R[1-F(1+b R)]$. The number of solutions depends on how large is $b$.
- Thus, the possible schedules are



## Adding the demand



## Policy

- Imagine a foreign investor with deep pockets lends to the country, at $R^{P}$, any amount up to $b^{P}$.
- If $\left(R^{P}, b^{P}\right)$ properly chosen, points on the high rate schedule are not equilibria.
- At the rate $R^{P}$ there would be profits.
- The amount borrowed from the large lender is zero.
- Why after the 2008-09 financial crisis?
- Average accumulation for advanced economies between 2008 and 2011 (3 deficits) was $25 \%$.
- For Portugal $72 \%$ to $108 \%$ in 2011 , Spain $40 \%$ to $70 \%$, Italy $106 \%$ to $120 \%$.
- Long period of stagnation for the three countries.



## Alternative action

- Does it matter (for multiplicity) whether the choice for the borrower is $b$ or $a=R b$ ?
- Calvo (1988), Lorenzoni and Werning (2013). Schedule in $b$, multiple equilibria.
- Aguiar and Gopinath (2006), Arellano (2008). Schedule in $a$, single equilibrium.
- Let $q \equiv \frac{1}{R}$ and $a=R b$, and write demand and supply as

$$
\begin{gathered}
R^{*}=R[1-F(1+b R)] \\
U^{\prime}(1+b)=R \beta \int_{1+b R}^{Y} U^{\prime}(y-b R) f(y) d y
\end{gathered}
$$

or

$$
\begin{aligned}
R^{*} & =\frac{1}{q}[1-F(1+a)] \\
U^{\prime}(1+q a) & =\frac{1}{q} \beta \int_{1+a}^{Y} U^{\prime}(y-a) f(y) d y
\end{aligned}
$$

- They are the same two equations in $R=\frac{1}{q}$ and $a=R b$.


## Alternative timing/action

- The borrower moves first and chooses $b$ or $a=b R$.
- If the choice is $b$ or $a$, creditors move next and offer schedules $R(b)$ or $q(a)=\frac{1}{R(a)}$.
- If the borrower chooses $b$, the schedule is

$$
R^{*}=R(b)[1-F(1+R(b) b)]
$$

- If the borrower chooses $a$, the schedule is

$$
R^{*}=R(a)[1-F(1+a)]=\frac{1}{q(a)}[1-F(1+a)]
$$



- Picking $a$ is like picking the probability of default, or $R$.
- Default probabilities will be on the low rate schedule.
- This is the assumption in Aguiar and Gopinath (2006) and Arellano (2008).

Lorenzoni-Werning (2013) (closer to Calvo (1988)).

- The borrower is a government with exogenous deficits or surpluses. If the surplus is high enough the debt is repaid. Otherwise there is default.
- Argument against choice of $a$ : Game without commitment within period.
- The government cannot commit not to reissue.
- In the limit, government is a price taker (durable good monopoly).
- LW focus on equilibria with debt dilution. Longer maturity.


## The infinite period model: Numerical exploration

- The endowment $y$ has bounded support, given by $\left[y_{\min }, y^{\max }\right] \subset \mathbb{R}_{+}$ and follows a Markov process with distribution $F\left(y^{\prime} \mid y\right)$.
- We let the value after default be $V^{a u t}=\frac{U\left(y^{d}\right)}{1-\beta}$.
- In here we explore interest rate schedules for $R$ the depend on $b$. (Calvo)
- They will also depend on $y$, and $s$, which is a sunspot variable that selects one of the multiple schedules for the interest rate.
- Our exploration will be based on the bimodal distribution we studied above. (at most two increasing solutions?)
- Thus, $s=1,2$, with transition probabilities, $p_{11}=p_{22}=p$.
- The value for the representative agent, after deciding not to default, is given by value functions $V$, and schedules $R$, satisfying

$$
\begin{aligned}
& V(\omega, y, 1)= \max _{c, b, \omega^{\prime}}\left\{U(c)+\beta \mathbb{E}_{y^{\prime}}\left[\begin{array}{c}
p \max \left\{V\left(\omega^{\prime}, y^{\prime}, 1\right), V^{\text {aut }}\right\} \\
+(1-p) \max \left\{V\left(\omega^{\prime}, y^{\prime}, 2\right), V^{\text {aut }}\right\} \mid y
\end{array}\right]\right\} \\
& \text { subject to } \\
& c \leq \omega+b \\
& \omega^{\prime}= y^{\prime}-b R(b, y, 1) \\
& b \leq \underline{b}
\end{aligned}
$$

and

$$
\begin{aligned}
& V(\omega, y, 2)= \max _{c, b, \omega^{\prime}}\left\{U(c)+\beta \mathbb{E}_{y^{\prime}}\left[\begin{array}{c}
p \max \left\{V\left(\omega^{\prime}, y^{\prime}, 2\right), V^{\text {aut }}\right\} \\
+(1-p) \max \left\{V\left(\omega^{\prime}, y^{\prime}, 1\right), V^{\text {aut }}\right\} \mid y
\end{array}\right]\right\} \\
& \text { subject to } \\
& c \leq \omega+b \\
& \omega^{\prime}= y^{\prime}-R\left(b^{\prime}, y, 2\right) \\
& b \leq \underline{b}
\end{aligned}
$$

## Conditions the schedules must satisfy

- In state 1 , investors offer the schedule $R(b, y, 1)$.
- The following period the state is 1 with probability $p$, and 2 with probability $1-p$.
- If in state $s$, the threshold for default is defined by

$$
V^{a u t}=V(\omega, \underline{y}(\omega, y, s), s)
$$

Then

$$
R^{*}=R(b, y, 1)\left[p\left[1-F\left(\underline{y}\left(\omega^{\prime}, y^{\prime}, 1\right) \mid y\right)\right]+(1-p)\left[1-F\left(\underline{y}\left(\omega^{\prime}, y^{\prime}, 2\right) \mid y\right)\right]\right]
$$

- Similarly, in state 2 , investors offer the schedule $R\left(b^{\prime}, y, 2\right)$.
- Then, the arbitrage condition must be written as

$$
R^{*}=R\left(b^{\prime}, y, 2\right)\left[p\left[1-F\left(\underline{y}\left(\omega^{\prime}, y^{\prime}, 2\right) \mid y\right)\right]+(1-p)\left[1-F\left(\underline{y}\left(\omega^{\prime}, y^{\prime}, 1\right) \mid y\right)\right]\right]
$$

## Equilibrium

An equilibrium is given by functions

$$
V(\omega, y, s), c(\omega, y, s), b(\omega, y, s), R(b(\omega, y, s), y, s), \underline{y}(\omega, y, s)
$$

such that,

1. given $V(\omega, y, s), \underline{y}(\omega, y, s)$ solves the threshold equation.
2. given $R(b(\omega, y, s), y, s), V(\omega, y, s), c(\omega, y, s), b(\omega, y, s)$ solve the DP problems.
3. The arbitrage conditions and the law of motion for $\omega$ are satisfied.

## Conclusions:

- The condition on interest rates in the good state

$$
R^{*}=R\left(b^{\prime}, y, 2\right)\left[p\left[1-F\left(\underline{y}\left(\omega^{\prime}, y^{\prime}, 2\right) \mid y\right)\right]+(1-p)\left[1-F\left(\underline{y}\left(\omega^{\prime}, y^{\prime}, 1\right) \mid y\right)\right]\right]
$$

- Following 2008:

1. Learned that stagnation was a possibility (Greece?)
2. That possibility was there for Italy, Portugal, Spain
3. Large increases in public debt.

- Draghi or German Constitutional Court?

