Trading Liquidity and Funding Liquidity in Fixed Income Markets: Implications of Market Microstructure Invariance

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Presentation for the Federal Reserve Bank of Atlanta
2016 Financial Markets Conference
Getting a Grip on Liquidity: Markets, Institutions, and Central Banks
Amelia Island, Florida
May 2, 2016

Main Idea: Back-of-Envelope Extrapolation

Apply market microstructure invariance to fixed income markets:

- ▶ The invariance-implied illiquidity $1/L$ incorporates "business time," measured by "bet arrival rate" γ, and therefore consistently adjusts for time.

- ▶ Trading liquidity: Bets (meta-orders) are executed at business-time scales.

- ▶ Funding liquidity: Marking to market, collateral disputes, forced liquidations take place in business time.

- ▶ Illiquidity measure $1/L$ from market microstructure measures both trading liquidity and funding liquidity.

- ▶ Extrapolate empirical estimates of bet size and transaction costs from stocks to Treasury and corporate bonds.

- ▶ Intuitive check whether results seem economically sensible.

Results

- ▶ Portfolio Transitions (Kyle-Obizhaeva-2016): Typical stock has average bet size of \$470,000, average transaction cost of $1/L = 43$ basis points, with $\gamma = 85$ bets per day.

- ▶ Treasury bonds have average bet size of \$20 million, average transaction cost of $1/L = 1$ basis point, with 8,900 bets per day.

- ▶ Corporate bonds have average bets size of \$400,000, average transaction cost of $1/L \approx 55$ basis points, with 3 bets per day.

- ▶ Business time in Treasury market is faster than corporate bond market by a factor of $55^2 \approx 3,000$.

- ▶ Business time in commercial banks operates in slow motion.

- ▶ Flash crash and flash rally both probably resulted from trading large quantities ultra-rapidly.

Market Microstructure Invariance

$$\text{Illiquidity} = \frac{1}{L_{jt}} = \left(\frac{C \cdot \sigma_{jt}^2}{m^2 \cdot P_{jt} \cdot V_{jt}} \right)^{1/3} \tag{1}$$

$$\text{Business Time} = \gamma_{jt} = \frac{\sigma_{jt}^2 \cdot L_{jt}^2}{m^2} = \left(\frac{P_{jt} \cdot V_{jt} \cdot \sigma_{jt}}{m \cdot C} \right)^{2/3} \text{bets per day} \tag{2}$$

- $P_{jt} \cdot V_{jt} = $ dollar volume, $\sigma_{jt} = $ volatility
- $C = $ (dollars) and $m^2 = $ (dimensionless) are scaling constants.

Properties of $1/L_{jt}$ and γ_{jt}

- $1/L_{jt}$ is dimensionless, leverage invariant, measure of illiquidity, measures expected transaction cost of a bet (basis points).
- γ_{jt} measure per unit of time, measures number of bets per day.
- C scaled to measure expected dollar transaction cost of a bet.
- m^2 scaled to satisfy $P_{jt} \cdot \bar{Q}_{jt} = C \cdot L_{jt}$.
- Market microstructure invariance says C and m^2 are invariant constants, same for all assets.

Trading Liquidity: Results from Stocks

Invariance implies transactions cost (basis points) has form

$$C^{\%}(Z_{jt}) = \frac{1}{L_{jt}} \cdot f(Z_{jt}), \qquad \text{where} \qquad Z_{jt} = \frac{P_{jt} \cdot Q_{jt}}{C \cdot L_{jt}} \qquad (3)$$

Kyle-Obizhaeva-2016 use stock portfolio transactions to estimate

$$C \approx \$2,000, \qquad m^2 \approx 1/4, \qquad (4)$$

$$f(Z) = 0.15 + 0.0576 \cdot |Z|, \quad \log(|\tilde{Z}|) \sim N(-\eta^2/2, \eta^2), \quad \eta^2 = 2.53. \qquad (5)$$

For "benchmark stock" with $P_{jt} \cdot V_{jt} = \$40$ million per day and $\sigma_{jt} = 2\%$ per day$^{1/2}$:

$$\frac{1}{L_{jt}} \approx 0.0043, \quad P_{jt} \cdot \bar{Q}_{jt} \approx \$470,000, \quad P_{jt} \cdot Q_{jt,median} \approx \$133,000. \qquad (6)$$

Results consistent with Angel, Harris, Spatt (2015).

Funding Liquidity: Incorrect Theory

Funding liquidity is measured by haircut needed to make a repo transaction very safe.

If asset is infinitely liquid, ΔT is mark-to-market interval at which collateral is posted, S is number of standard deviations of protection, then (incorrect!) haircut is

$$\text{Haircut} = S \cdot \sigma_{jt} \cdot \Delta T^{1/2}. \tag{7}$$

- ▶ This theory fails to take account of how time interacts with funding liquidity.
- ▶ It takes time to value collateral, mark it to market, resolve valuation disputes, sell defaulted collateral.

Funding Liquidity: Invariance

Funding liquidity takes place in business time γ_{jt}, different for different assets. Volatility per bet:

$$\text{Volatility per Bet} = \frac{\sigma_{jt}}{\gamma_{jt}^{1/2}} = \frac{m}{L_{jt}}. \qquad (8)$$

If valuation of collateral, marking to market, resolving disputes, liquidating defaulted collateral take place in business time proportional to rate at which bets arrive, then standard deviation of horizon of H bets is $m/L_{jt} \cdot H^{1/2}$.

Fire Sales: Urgent Trades

Kyle, Obizhaeva, Wang (2016) suggest temporary price impact proportional to speed of buying or selling. Suggests functional form

$$f(Z, H) = \kappa + \lambda \cdot \frac{h(Z)}{H} \cdot |Z|, \tag{9}$$

where H denotes horizon of execution (in bets) and $h(Z)$ is the "normal" bet horizon.

This analysis suggests haircut proportional to $1/L$, not σ:

$$\text{Haircut} = \frac{1}{L_{jt}} \cdot \left(S \cdot m \cdot H^{1/2} + \lambda \cdot \frac{h(Z)}{H} \cdot |Z| \right). \tag{10}$$

Treasury Trading Activity

Add together duration-weighted trading volumes to obtain volume for composite "on-the-run" Treasury markets:

$$\text{Treasury Trading Activity} = P_{10} \cdot V_{10} \cdot \sigma_{10} + P_5 \cdot V_5 \cdot \sigma_2 + P_2 \cdot V_2 \cdot \sigma_2 \tag{11}$$

Volatilities σ proportional to durations of 2, 5, 10 years.

Estimates from Joint Staff Report on Flash Rally

Table: Daily Treasury Bond Trading Volume

Cash or Futures	Maturity (Years)	Daily Volume ($ billion)	Weight	Weighted Volume ($ billion)
Futures	10	80	1.00	80
Futures	5	40	0.50	20
Futures	2	16	0.20	8
Cash	10	40	1.00	40
Cash	5	40	0.50	20
Cash	2	20	0.20	4
Sum		232		168

10-Year Treasury Liquidity

Using $\sigma_{UST} = 50$ basis points and $P_{UST} \cdot V_{UST} = \$168$ billion, obtain

$$\frac{1}{L_{UST}} \approx 1 \text{ basis point.} \tag{12}$$

Many implications:

- Average bet size: \$20 million
- Number of bets: 8900 per day. More than 100 times faster than typical stock.
- Average transaction cost: 1 basis point per bet

Implied Treasury Transactions Costs

Table: Implied Probability Distribution of U.S. Treasury 10-Year Bet Sizes

Standard Deviation	Scaled Size	Size ($ Million)	Probability Larger	T-Cost (bp)
$+0 \cdot \eta = $ med	0.28	6	0.50	0.17
$+0.7953 \cdot \eta = $ avg	1.00	20	0.21	0.21
$+1 \cdot \eta$	1.38	28	0.16	0.23
$+2 \cdot \eta$	6.79	136	0.023	0.54
$+3 \cdot \eta$	33.34	667	0.0013	2.07
$+4 \cdot \eta$	163.59	3271	0.000032	9.57
$+4.6113 \cdot \eta$	432.56	8651	0.0000020	25.04

Flash Crash and Flash Rally

- ▶ Flash Crash: \$4+ billion of sales ($Z \approx 300$, $15X$ faster than "normal") over 20 minutes drove S&P E-mini price down and up about 5%.

- ▶ Flash Rally: 10-year Treasury prices rose and fell about 1.20% in 12 minute "round trip."

- ▶ Invariance-implied "pseudo-flash-crash" in Treasuries: \$5.6 billion in purchases over 48 minutes would drive prices up and down by 277 basis points. Flash crash was "bigger" than flash rally.

- ▶ Invariance-implied match for Treasuries, assuming "natural" bet horizon proportional to bet size: \$2 billion purchased over 20 minutes ($20X$ normal speed) would drive prices up and down by 130 basis points. Similar to aggregate purchases by asset managers, broker-dealers, and PTFs (HFTs) in Joint Staff figure 3.4, p. 61.

Funding Liquidity in Treasuries

- ▶ Minimizing haircuts would imply marking to market many times per day, with tiny haircuts.
- ▶ Treasury market funding liquidity practice does not fit predictions of invariance.
- ▶ Probably more institutionally convenient to mark to market at calendar intervals of one day.
- ▶ Defaulted collateral could be liquidated in minutes unless position size is many billions of dollars.
- ▶ Regulatory leverage ratio probably interferes greatly with Treasury market arbitrage (on-the-run versus off-the-run, Treasuries versus swaps).

Corporate Bonds: Trading Liquidity

Assume 3 bets per day and volatility $\sigma_{CB} = 50$ basis points per day. Then (rounding error)

$$\frac{1}{L_{CB}} \approx 0.0058 = 58 \approx 55 \text{ basis points.} \qquad (13)$$

- Average bet is $\approx \$342,000$.
- Average transactions cost is about 55 basis points.
- Results for institutional size similar to Harris (2015). Implied costs of small trades (5 basis points) much smaller than actual costs. Corporate bond market does not serve small investors well.

Corporate Bonds: Funding Liquidity

- "Fire sale" might have double transaction cost of $55 \times 2 = 110$ basis points.
- Takes place over say 25 business days with $\sigma_{CB} \cdot 25^{1/2} = 250$ basis point standard deviation.
- Need haircut of $2 \cdot 55 + 3 \cdot 250 = 860$ basis points for 3-standard-deviation cushion.

Funding Liquidity Episodes: Illiquid Collateral

- ▶ BSAM Hedge Funds 2007: Bear Stearns took possession of collateral after time passed.
- ▶ LTCM 1998: "Crisis" unfolded over several months.
- ▶ London Whale: Trades by longs and shorts influenced prices in illiquid markets, making risk management and funding liquidity issues difficult.

Invariance, Short-Selling, and Derivatives

- ▶ Invariance implies collateral more concentrated in less actively traded issues, especially with low turnover.
- ▶ Therefore, short sellers have difficultly borrowing collateral to short.
- ▶ This amplifies funding liquidity issues with illiquid collateral.
- ▶ Pay-as-you go swaps substitute for short sales.

Banking

- Banking is like corporate bond market in slow motion.
- Value-at-risk and other metrics should recognize time by using $1/L$, not σ.
- Bank equity becomes illiquid during stress due to leverage and resulting volatility when capital buffer depleted.
- Stress tests should assume long horizon during which portfolio is frozen and no equity issued. Longer than 3 years.

Conclusion

► Invariance implies dramatic differences in liquidity across fixed income markets.

► Illiquidity measure $1/L$ takes speed into account.

► Maybe $55X$ difference in liquidity from on-the-run Treasury to off-the-run corporate bonds is reasonable.

► Need more research connecting liquidity and time.