Asset Co-movements: Features and Challenges

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Abstract

This paper documents and characterizes the time-varying structure of U.S. and international asset co-movements. It draws attention to the fact that while some of the time variation could be genuine, the sampling uncertainty and time series properties of the series can distort significantly the underlying signal dynamics. Proper transformation of the asset prices prior to extraction of common components is crucial for the validity and the robustness of the analysis. Frequency of the data also plays a role in the effectiveness of risk identification. We discuss some recent examples that illustrate the pitfalls from drawing conclusions from local trends of asset prices. On a more constructive side, we find that the U.S. main asset classes and the major international stock indices share a factor that is closely related to the business cycle. At even lower frequency, the common asset co-movement appears to be driven by demographic trends.

Keywords: Cross-asset, within-asset and international asset co-movements; Rolling correlation; Time-variability; Persistence; Higher moments; Tail and asymmetric dependence; Risk factors; Sampling frequency.

JEL Classification : G13, G14, G17

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1. Introduction

Identifying and quantifying the underlying co-movements in financial data have important implications for asset allocation, portfolio valuation, regulatory enforcement, and policy analysis. Since the 2008 financial crisis, there have been several episodes when the correlations between different asset classes were elevated but dropped off quickly afterwards and exhibited substantial variability. For example, the prevailing view in the last few years is of an increased correlation across asset classes; that is, the "risk-on/risk-off" view of markets. Is this due to fundamental or transient (technical) factors? Can we reliably isolate, in real time, the underlying source of risk from data and estimation noise? Is the observed time-variability of these co-movements a true property of the data-generating process or a symptom of misspecification that arises from omitted common factors and higher-moment distributional features?

To answer these questions, it is desirable first to document and characterize the main regularities in the asset co-movements across asset classes over time. While the scope of our empirical analysis is limited to some frequently studied co-movements within and across asset classes, we still attempt to provide some arguments about possible sources of common variation in asset returns, emphasize some pitfalls in commonly used measures of these co-movements, and discuss the challenges of measuring statistically the shifts of the distribution at various frequencies. The underlying sources of common asset variations can be crudely classified into long-run, transient, and spurious factors. The long-run (fundamental) sources of risk include structural macroeconomic and demographic factors at business-cycle or even lower frequency. The transient factors are roughly interpreted as fluctuations prompted by macro, political, and market-structure (technical) events that can force medium-term asset reallocations and rotations and induce movements in the asset risk premia. Finally, the short-term co-movements can arise from unanticipated (oil supply, for example) shocks or data noise. The data noise and finite samples can generate possible spurious co-movements and observational equivalence between the different factors, which renders a sharp distinction between the underlying sources of risk almost infeasible. Despite these limitations, it is often tempting to ascribe "fundamental" structure to some of the observed short-term movements as illustrated by some examples discussed in this paper.

The discussion in the paper will be focused around several general observations. First, if there is robust evidence that the asset co-movements are genuinely time-varying, then it would be useful to identify the source of this common variation. Market participants have suggested several sources of common variation: regulatory changes and reduced market liquidity in some markets, correlated arbitrage, growth of passive investment funds, particular investment strategies, and algorithmic trading, among others. The heightened interdependence within and between asset classes can have important implications for policymakers and can make the diversification and hedging implementation increasingly challenging. On the one hand, the strengthened asset co-movements open up the possibility of systematic market risk (such as concerns about the destabilizing effects of the "taper tantrum" in 2013) and the proper calibration of monetary policy to economic conditions. On the other hand, if monetary policy is a driver of the correlation change, then it is important to understand how monetary policy decisions (and especially surprises) will affect markets; for example, how quantitative easing could influence long-term bond and other asset prices. Identifying the sources of this increased dependence in the joint distribution of asset returns may shed light on how to adapt to this shifting and potentially unstable landscape.

Second, it should be acknowledged that another possibility for the increased time variation, and potential instability, in the correlations across asset classes could be purely statistical due to the limitations of the modeling framework and the use of second-moment measures.[1] We argue in the paper that particular attention should be paid to the sampling frequency and estimation uncertainty associated with computing moments of the distribution, the effect of persistence on correlation-type measures of co-movement, and differences in the correlation measures across different parts of the distribution. While most drawbacks of the standard measures of co-movements are thoroughly discussed and documented in the literature, they are often downplayed, due to their computational and interpretational convenience, by practitioners. In this respect, it is surprising how underappreciated the sampling and estimation uncertainty could be in forming investment and policy decisions based on nonrobust measures of second-moment variation. As a result, there is a tendency to over-analyze some high-frequency movements and attribute them to a particular underlying signal without taking fully into account the data noise and estimation uncertainty. Unlike high-frequency co-movements that are largely elusive and fairly transitory, low-frequency analysis provides a more robust and reliable way to evaluate these dependencies and suggests that the asset common variation appears

[1] There is a large and well-developed literature in financial econometrics on estimating dynamics conditional correlation models (see Engle, 2002, for example). To avoid technical details, we do not discuss this literature here. However, some of the remarks that we provide for time-varying correlations apply to this literature as well. For some caveats about dynamics conditional correlation models, see Caporin and McAleer (2013).

to exhibit more stable relationships with macroeconomic and slowly moving demographic factors at business-cycle or longer duration.

Third, the increased time variation in the correlations across asset classes could well be statistical and arise purely from limitations of the modeling and conceptual framework and the use of second-moment measures. There is now overwhelming evidence that higher moments of asset returns exhibit features that cannot be reconciled with the dominating asset-pricing and portfolio-allocation models in the academic literature, and the difference between the model-implied statistics and their empirical counterparts is "anomalously" or "puzzlingly" large. For example, if sources of risk exhibit differential impacts on the shape of the asset distribution over time (either by fattening the tail risk or shifting the tail risk from one side of the distribution to the other), the information reflected in the second moments will be incomplete and potentially misleading as it may exhibit substantial time variability even when the underlying population correlations are constant. More broadly, if we seek to understand how investors assess risk and reallocate their portfolios in response to the changing distribution of asset returns and how their actions may themselves help to shape this distribution, more general dependence measures—that reflect and summarize the information in the whole distribution—are therefore necessary for identifying the underlying risk factors. The modern portfolio and asset-pricing theory can then be modified accordingly to reflect this higher-order moment information.[2]

In light of these remarks, it is prudent to approach the analysis of asset co-movements by explicitly acknowledging the model and estimation uncertainty surrounding all investment and asset-allocation decisions. Since all financial models are constructed to approximate a complex reality, they are inherently misspecified. This is often done intentionally, as parsimonious models draw only partial or incomplete maps of the latent objects of interest either to emphasize particular aspects or because the underlying structure is completely unknown. How large is the effect of this model misspecification on the quantity of interest (for example, portfolio weights, stochastic discount factor) is an empirical question. It appears that most of the anomalous and puzzling results in economics and finance tend to diminish in importance when the prevalent analytical framework, based on linearity and multivariate normality, is relaxed. Furthermore, while there is suggestive evidence of an increased asset-class interdependence and a shifting market landscape, the exact source of this structural change (correlated arbitrage, interdependencies, trading strategies, passive investment, algorithmic trading) is difficult to

[2] Embrechts et al. (2002) provide a comprehensive discussion on the properties and pitfalls of correlation for measuring general dependence.

pin down due to short samples, noisy data, and estimation uncertainty. Finally, since the object of interest is often the joint distribution of all assets under consideration, more general and robust measures of dependence, that embed information in the shape (asymmetry, tails) of this distribution, are highly desirable, even though they suffer from similar drawbacks (noisiness and large estimation uncertainty) as the correlation measure.

We approach this problem from an empirical perspective since economic theory provides only limited guidance on the fundamental sources of time-varying co-movements across asset classes. There is a well-established literature that studies the large directional swings in the covariance between U.S. bond and stock returns in the post-World War II period. For instance, it is a stylized fact that this correlation was largely positive between 1953 and the late 1990s but then turned negative, especially during the 2001 and 2007–09 recessions, with bonds providing a hedge against equity and macroeconomic risks. Baele at al. (2010), Campbell et al. (2015), David and Veronesi (2016), and Song (2017), among others, propose models that better fit the observed dynamics of the correlation between bond and stock returns. However, developing a unifying framework for modeling jointly several major asset classes (stocks, government bonds, corporate bonds, commodities, currencies) proves to be prohibitively difficult. Cochrane (2015) identified the question "Once we find the factor structure in bonds, what is the factor structure of expected returns across asset classes?" as one of the "bigger" questions facing financial economics. On that note, dimension reduction techniques such as factor analysis prove to be useful tools for summarizing the common variation across asset markets. Identifying the main sources of risk via the estimated factors is of fundamental importance not only for guiding investment decisions but also for monitoring financial stability, stress testing, and more.

While distinguishing among fundamental, transitory, and spurious sources of risk proves empirically difficult, we find clear evidence of long-run co-movements at business-cycle or even lower frequency. There is a reasonable basis for tying these co-movements to macroeconomic fundamentals and demographic factors. While there is also some evidence of co-movement at the shorter end, one needs to be careful in analyzing this evidence as there are several sources of possible error that make the competing hypotheses observationally equivalent. These short-term co-movements, which force asset reallocations and induce temporary co-movements in the asset-risk premia, tend to be more unstable. Nevertheless, there is a tendency to ascribe more fundamental structure to these variations, which warns against overreliance on various nonrobust measures of co-movement that are routinely used for portfolio allocation, evaluation of asset–pricing models, and factor investing.

The rest of the paper is organized as follows. Section 2 provides a motivating example that exposes the potential challenges and biases when analyzing co-movements based on recent local-trend, high-frequency data. Section 3 discusses a few statistical problems that could affect the reliability of rolling second-moment measures of dependence. In particular, we discuss the effects of the persistence of the individual series, the estimation and sampling uncertainty, and the sensitivity of these measures to changes in the higher moments of the distribution. Section 4 attempts to extract the common price variation across asset classes and relates them to business cycle fluctuations and longer-run demographic trends. This allows us to draw some tentative conclusions about these co-movements based on future projections of these business and demographic cycles. Section 5 offers some concluding remarks.

2. An Illustrative Example: Oil Price Co-movements

From the beginning of 2014 to the middle of 2016, the price of crude oil dropped by more than 50 percent, a drop that was accompanied by a substantially elevated correlation between oil price and several other asset classes. This unusually high correlation attracted the attention of market analysts for at least two main reasons. First, this co-movement significantly restricted the set of diversification and hedging opportunities in the market. Second, oil prices as the driving factor behind the asset co-movements on a more sustained basis may pose risks to the stability of the financial system given the highly volatile nature of commodity prices, with their larger exposure to geopolitical risk, supply disruptions, and more.[3]

Figure 1 plots the dynamics of several asset prices—5-year, 5-year forward breakeven inflation from Treasury Inflation-Protected Securities (TIPS) and Treasury prices, U.S. dollar index (DAX), and one-year changes in the S&P 500 index, and Barclays high yields index—versus the log oil price from the beginning of 2014 to the end of February 2017. Ending in May 2016, the series (or some transformations of them) are plotted as solid lines to illustrate the seemingly tight relationship between the oil price and other asset prices that was widely documented and discussed by pundits and the media at that time. In contrast, the dashed lines after May 2016 tend to suggest that this relationship has substantially weakened.

[3] The median values of the option-implied volatility indices OVX and VIX (for oil prices and S&P 500, respectively) since the beginning of 2014 (2015) are 37.78 (42.28) and 14.12 (14.48).

Figure 1. Oil price dynamics against (1) 5-year, 5-year forward breakeven inflation; (2) U.S. dollar index (DXY); (3) percent changes in S&P 500 index from a year ago; and (4) Barclays high yield index. The solid lines denote the dynamics of each series from January 2014 to May 2016, and the dashed lines represent the period June 2016 to February 2017.

Was this co-movement due to fundamental common factor/information driving all these variables (global demand shock, for example), or was it caused by transitory (market-specific) factors reflected only in the asset risk premia? While seeking answers to these questions, we want to emphasize some statistical features—such as the possibility of spurious time variability due to persistence (trending behavior) of the variables over this particular period or sensitivity to movements in other parts (skewness, tails) of the joint or marginal distributions—that are often overlooked by practitioners.

To gain a better understanding of these issues, we focus on the relationship between breakeven inflation (BEI), embedded in nominal and inflation-protected securities, and oil prices from the beginning of 2014 to the middle of 2016. During this particular period, both oil prices and BEI exhibited a downward trend, and this strong correlation was largely attributed to the effect of oil prices on inflation expectations. This interpretation was challenged on several grounds (see, for example, Gospodinov, Tkac, and Wei, 2016). First, BEI is not a clean measure of inflation expectations as it also contains other, unobservable components such as liquidity and risk premium. Second, it is somewhat counterintuitive for the day-to-day (transitory) variations in oil prices to affect inflation expectations 5 to 10 years out. Finally, both variables are highly persistent and the sample correlation may provide a spurious signal of a relationship between the two variables.

A proper decomposition of the observable BEI into its latent components (see Gospodinov and Wei, 2015) reveals the following. Despite the wide variations in BEI, long-term inflation expectations appear to be stable and uncorrelated with oil prices. A large portion of the low-frequency variation in BEI can be attributed to the inflation risk premium. But most of the high-frequency variation in the recent dynamics of BEI can be attributed to a "liquidity" premium. This "liquidity" factor captures a wide range of market-related factors such as seasonal carry, deflation floor, limits to arbitrage, tenor-specific liquidity, redemptions, reallocations, and hedging in the TIPS market following an oil price drop or global financial turbulence. For example, an event that drives flight-to-quality into nominal Treasuries and/or forced sales of TIPS leads to a lower BEI without any change in inflation expectations.

Figure 2 shows that almost all of the recent correlation between BEI and oil prices is being picked up by the "liquidity" premium. As shown above, other asset classes (stock prices, high-yield bonds, municipal bonds, exchange rate) have also exhibited an elevated correlation with oil prices during this period. If genuine, it is useful to determine if this is due to a global demand driver, correlated risk, or market specificities (such as forced de-risking and liquidations, reallocations, flight-to-safety, or covering hedges).

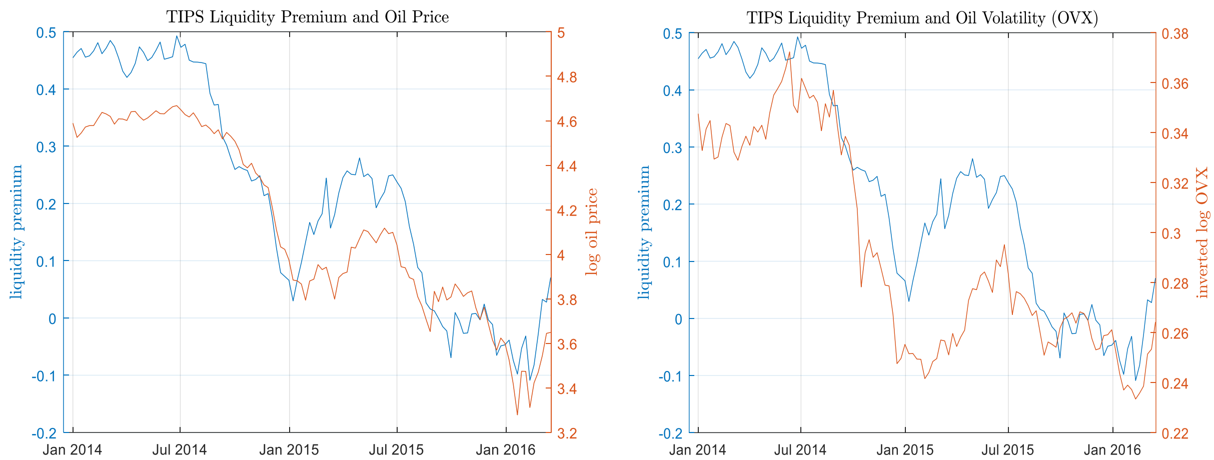

Figure 2. Time plot of weekly TIPS liquidity premium with oil prices and oil option-implied (OVX) volatility

For example, numerous studies have documented the existence of persistent mispricing in various asset markets. It is conjectured that common factors could drive this mispricing and the resulting arbitrage. Brunnermeier and Pedersen (2009) argue that availability of funding may have liquidity effects on asset prices. Also, if capital returns slowly to the fixed-income funds following a period of flat performance, then the arbitrage in various fixed-income markets (corporate, CDS, Treasury, TIPS) can exhibit commonalities (Fleckenstein, Longstaff, and Lustig, 2014). Finally, the macroeconomic environment after the financial crisis may also have contributed to the strengthened asset co-movements. For instance, Datta et al. (2017) provide evidence that oil and equity returns have become more responsive to macroeconomic news during the zero lower bound period.[4]

While this and other (more direct or anecdotal) evidence on the 2014–16 episode suggests that the co-movement of various asset prices and oil price can be attributed to market-structure factors, the next section attempts to raise the awareness that a part of this co-movement can be purely coincidental or unreliable. In particular, we explore the role of persistence of the series under considerations, estimation, and sampling uncertainty, as well as the potential limitations of second-moment based measures of co-movement to represent accurately changes in the joint distribution of interest.

[4] There is also evidence that the transmission mechanism of propagating the oil shocks through the US economic system has changed due to the increased role of domestic oil production (Baumeister and Kilian, 2016).

3. Statistical Challenges for Measuring Asset Co-movements

3.1 Persistence and Measures of Co-movement

As illustrated above, the co-movement across asset classes is often analyzed using (commodity, for example) prices and yields. While commodity prices and bond yields are usually modeled as mean-reverting processes, they are strongly persistent with a very slow mean reversion. Because the high persistence generates local trends, these local trends can induce spurious correlation even when the underlying processes are completely unrelated to each other. Furthermore, even when the processes are genuinely related, the serial correlation present in prices or yields can obfuscate the underlying relationship if the serial correlation is not properly taken into account.

To illustrate the latter point, we examine the leverage effect (the negative relationship between stock prices and volatility) that occupies a prominent place in financial economics and econometrics. Figure 3 plots the time series dynamics of the S&P 500 (SPX) and VIX indices (implied by options written on SPX) from January 2000 to February 2017.

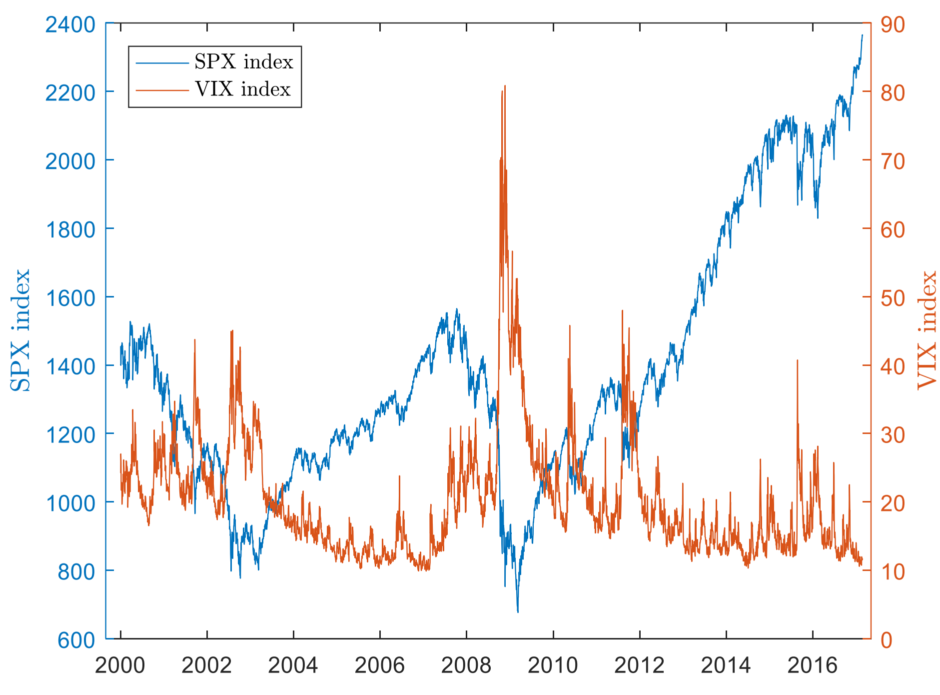

Figure 3. Time series dynamics of SPX and VIX indices

While the negative relationship is evident during several episodes of sharp stock declines, there are periods (the beginning of the sample as well as the 2012–16 period) when any possible co-movement may be masked by the trending behavior of the SPX index.

Figure 4 below suggests that the 120-day sample correlation between the SPX and VIX (blue line) is highly volatile with even positive values over some sub-samples. It is tempting to conjecture that removing the local trends in the SPX index by transforming it to SPX log returns may stabilize and uncover better the leverage effect. The red line in figure 4 reveals that this transformation reduces the variability of the sample correlation coefficient, but it leads to a severely biased measure that underestimates significantly the negative co-movement between the SPX and VIX. This is due to the fact that the SPX returns and the VIX index are highly unbalanced in terms of their statistical properties, since the former is an uncorrelated (or only weakly correlated) process and the latter is a highly persistent and bounded variable.

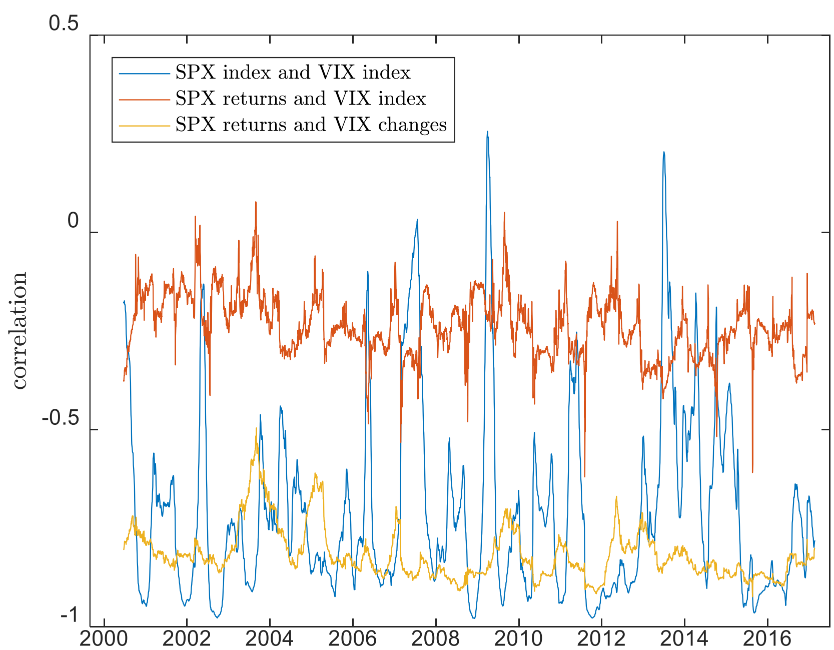

Figure 4. 120-day correlation between SPX and VIX

To render the statistical properties of both series similar, we compute the log changes of the VIX index. Figure 4 plots the 120-day rolling correlation[5] of SPX returns and VIX changes in yellow. In fact, this is akin to the way the leverage effect is modeled in financial econometrics. The correlation now is much more stable and averages around -0.8, which is the magnitude of the leverage effect that is typically estimated in the empirical literature. Note that some of the variability in the rolling correlation coefficient can be attributed to overlapping estimation uncertainty as discussed below.

To illustrate the point that the observed correlation may be spurious, we generate data from two uncorrelated, near-unit root processes (that match the persistence in bond yields and oil prices) and plot in figure 5 one realized sample path of the two processes. The unconditional correlation between the two series is 0.36 with an even stronger correlation over sub-samples that arises from "common" local trending behavior. Any observed commonality is completely spurious, as the two series are generated as independent of each other.

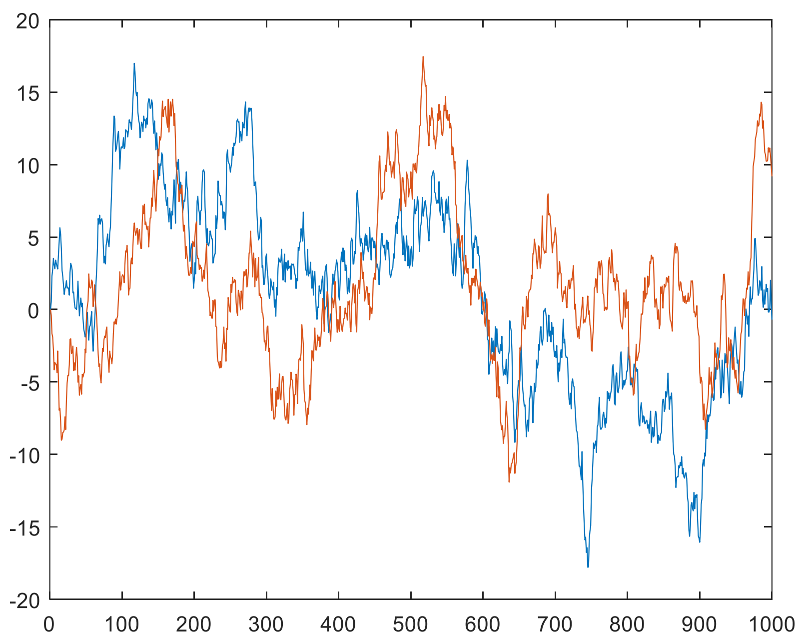

Figure 5. Time plot of two independent highly persistent processes

[5] In this paper, we use Spearman's (rank) correlation instead of the standard Pearson's correlation coefficient. This choice is dictated by the robustness properties of the rank correlation. Also, when appropriate, we computed the correlation coefficient using standardized returns using a GARCH(1,1) model for estimating the conditional variance of returns. The results were very similar and are not reported, unless specified otherwise.

This should serve as a warning sign in analyzing and justifying observed co-movements of highly persistent processes. It also suggests that one should avoid computing correlations in levels and analyze instead the transformed series after the persistence is removed. This is the approach that we follow in the rest of the paper.

3.2 Time Variability and Estimation Uncertainty

The last decade has witnessed a proliferation of passive investment funds. Since 2005, the share of S&P 500 ownership of passive mutual funds and exchange-traded funds (ETF) increased from 4.6 percent to 11.6 percent (*Wall Street Journal*, October 2016) while the share of active mutual funds and ETFs remained flat around 17 percent. This same period has been characterized by an increased correlation among U.S. equities due to convergence of equity betas of individual stocks. Passive investing is believed to have benefited and possibly contributed to this empirical regularity, although this relationship is confounded by other factors that include proliferation of electronic and algorithmic trading, globalization, unconventional monetary policy, and reduced market liquidity arising from regulatory changes in the developed countries. This type of observation is typically made based on computing some time-varying correlation coefficient and following its dynamics over time.

Figure 6 presents the 60-day and 120-day rolling correlations between large-cap (S&P 100) and small-cap (S&P 600) returns from January 2000 to February 2017. The figure reveals substantial variability in these correlation coefficients with occasional sharp upward and downward turns (for example, the one that occurred in the beginning of this year) that could affect directly the inflow or outflow of funds and performance of passive and active investment strategies. What is less discussed, however, is the substantial estimation uncertainty surrounding these estimates. There are two main sources of this uncertainty. First, it is the length of the sample and the sampling frequency, as short samples and high frequency data may be preferred to ensure that the recent momentum in the series is followed nearly in real time. Second, rolling correlations involve overlapping observations and a large estimation error can be accumulated and amplified and, combined with the short sample size, can persist.[6]

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[6] Rolling estimators can also be interpreted as nonparametric estimators (Ang and Kristensen, 2012; Adrian et al., 2015) with a slow rate of convergence and larger variability.

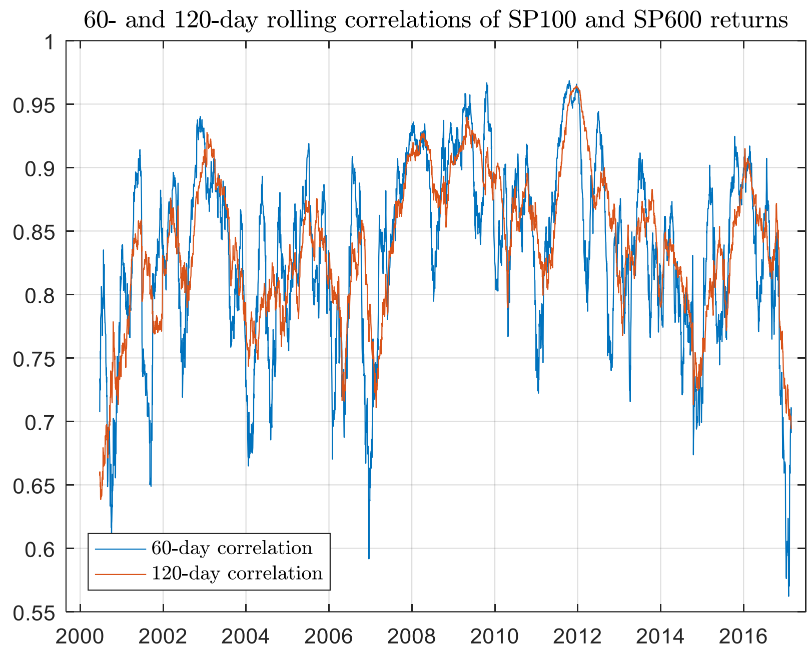

Figure 6. Rolling correlations of S&P 100 and S&P 600 (small cap) returns

It is instructive to investigate further how much of the time variation of the rolling correlation coefficient can be attributed to estimation and sampling uncertainty. For this purpose, we generate artificial data that is calibrated to the actual data by matching the sample size, estimated means, unconditional covariance, and conditional variances of the S&P 100 and S&P 600 returns. The standardized returns are drawn from a bivariate *t*-distribution with varying tail thickness (a degree of freedom parameter that is drawn from a uniform distribution on the interval [4, 13]) but a constant correlation parameter. Figure 7 plots the 60-day and 120-day rolling correlations for the simulated data. Interestingly, these correlations exhibit similar time variation as the correlations in figure 6 even though the true correlation is constant. Part of this variation is due to changes in the tails of the distribution, arising from the time-specific degrees of freedom parameter of the *t*-distribution, but most of the variation is a result of overlapping estimation uncertainty. While this is only suggestive, it warns against overreliance (without properly taking into account the estimation uncertainty) on rolling estimation that is routinely used for portfolio allocation, evaluation of asset-pricing models, and factor investing (for a recent example, see Asness et al., 2017).

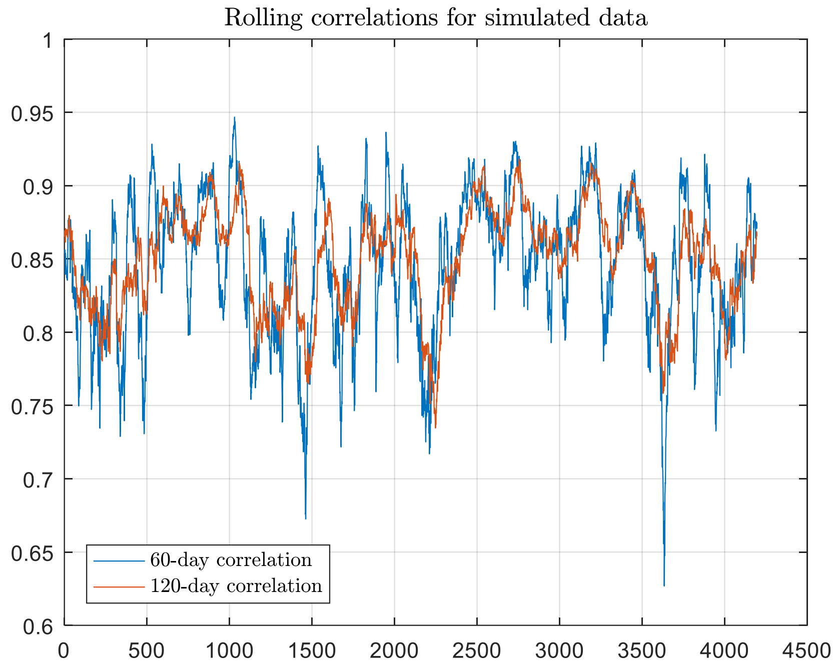

Figure 7. Rolling correlations for simulated returns data

DeMiguel et al. (2009) have demonstrated that the estimation imprecision may adversely affect asset allocation decisions. We mimic their argument by computing out-of-sample Sharpe ratios from an optimal mean-variance portfolio with estimated weights[7] and the naïve portfolio with equal (1/N, where N is the number of assets) weights. The asset returns, which are also used later in this paper, are for four major U.S. asset classes—— S&P 500 index (SPX) returns, Bloomberg Barclays Treasury total returns, Goldman Sachs commodity index (GSCI) returns, and USD index (DXY) returns—as well as five international equity indices (converted in USD)—S&P 500 (SPX), FTSE 100 (UKX), Nikkei 225 (NKY), DAX, and MSCI emergent markets (MXEF) index. The data are daily from January 2000 to February 2017. We use a rolling window sample of 500 daily observations for estimation of the weights for the mean-variance problem. These estimated weights, as well as the fixed

[7] The recent literature has suggested various ways to sharpen the estimation of the portfolio weights under the mean-variance risk measure. Another way is to replace the *expectation bounded risk* (such as the mean-variance risk measure) with a more robust, which takes explicitly into account the tails of the distribution, *coherent risk measure* such as the conditional value-at-risk (Assa and Gospodinov, 2014). Coherent risk measures are tightly linked to the Choquet expected utility, which can distort the probability of different events by assigning, for example, larger weights to less favorable events and smaller weights to more favorable ones (see Bassett et al., 2004).

1/N weights, are then interacted with the next-day return and these out-of-sample returns are used to compute the corresponding Sharpe ratio. The Sharpe ratios for the U.S. across-asset portfolio are 0.205 for the equal-weight and 0.160 for the mean-variance portfolio, with the difference being statistically significant at the 5 percent significance level (*p*-value of 0.028). For the international portfolio, the Sharpe ratios are 0.274 for the U.S. across-asset portfolio and 0.261 for the international portfolio, with the difference between the Sharpe ratios being statistically insignificant. This is in line with the results in DeMiguel et al. (2009) showing the effects of estimation error on optimal portfolio allocation.[8]

Summarizing the time variation of the correlation structure of more than two asset returns, which is the case for the two portfolios considered above, is not as trivial as the bivariate correlation coefficient. To do this, we construct a distance that measures similarities between two correlation matrices over different time periods. Let Σ_P denote the correlation matrix of asset returns for the period t to $t+P$, and Σ_R denote the correlation matrix of asset returns for the period $t+P$ to $t+P+R$. Then, we compute the correlation matrix distance (Herdin et al., 2005) as

$$d(\Sigma_P, \Sigma_R) = 1 - \frac{tr(\Sigma_P \Sigma_R)}{\|\Sigma_P\| \|\Sigma_R\|},$$

where $tr(\cdot)$ is the trace operator and $\|\cdot\|$ denotes the Frobenius norm. This distance metric takes values between zero (when the two correlation matrices are equal) and one. In what follows, we set $P = 240$ and $R = 120$.

The left panel of figure 8 plots the time variability of the correlation matrix for the four U.S. asset returns. While there was a change in the correlation matrix during the 2007–09 recession, the largest jump occurred in 2013. For the sake of comparison, the right panel of figure 8 presents the correlation matrix distance computed from 2-, 5-, 10- and 30-year bond yield changes (intentionally plotted on the same scale as the right graph). Due to the tight restrictions imposed by the term structure of interest rates, the bond correlation matrix exhibits very little time variation.

[8] It may be useful to remind the readers that any asset return can be decomposed (by identity) as the product of its sign and its absolute value (Anatolyev and Gospodinov, 2010, 2015). If the commonality is primarily in the sign (directional) changes across assets but not in the volatility, then this could be another reason why the fixed-weight portfolio may dominate the mean-variance optimization.

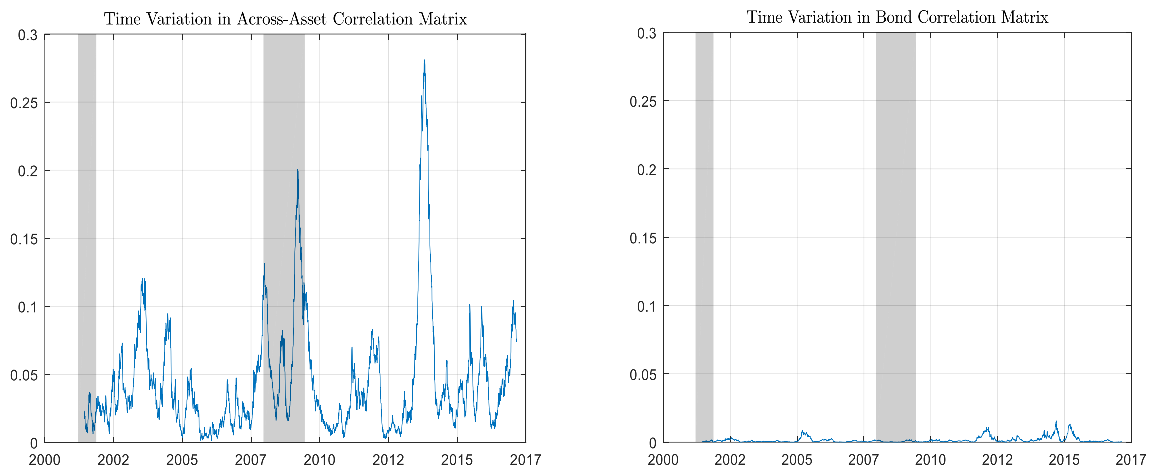

Figure 8. Time-varying changes in the correlation matrix distance measure of U.S. asset classes (left) and bond returns (right)

In summary, one should exercise caution when the analysis is based on rolling correlations with a relatively short sample window, as seemingly pronounced time variability can be due to estimation and data noise.

3.3 Robustness and Sensitivity to Time-Variability in Higher Moments

It is well known that the unconditional marginal and joint distributions of financial asset returns have pronounced non-Gaussian features. As we argued in the previous section, part of the observed time variability in the second-moment statistics (covariance, correlation) can be attributed to movements in the higher moments of the distribution if the latter are ignored and not adequately modeled. Later in this section, we discuss a more holistic (information-theoretic) approach to studying dependence across assets that incorporates information in the whole distribution.

Before we do this, we adopt a simpler, more descriptive method for characterizing the dependence in different parts of the distribution. It has been documented (Ang and Chen, 2002; Longin and Solnik, 2001; Karolyi and Stulz, 1996; among others) that the correlations increase, often dramatically, during extreme downward market movements. These increased correlations diminish the benefits of portfolio diversification and hedging, especially in situations when they are needed the most.[9] More generally, we would like to compute robust dependence measures along the quantiles of the distribution, including the tails. Let $r_{x,t}$ and $r_{y,t}$ $(t=1,\ldots,T)$ denote the ranks of the pair of (standardized) asset returns x and y. Using this notation, Spearman's rank correlation is defined as $\rho = Corr(r_{x,t}, r_{y,t})$. The sample quantile dependence at quantile α is constructed as (Patton, 2013)

$$\rho_\alpha = \begin{cases} \frac{1}{\alpha T}\sum_{t=1}^{T} I\{r_{x,t} \le q, r_{y,t} \le q\}, & 0 < q \le 0.5, \\ \frac{1}{(1-\alpha)T}\sum_{t=1}^{T} I\{r_{x,t} > q, r_{y,t} > q\}, & 0.5 < q < 1, \end{cases}$$

where I is the indicator function. This definition can be easily modified to quantile or exceedance correlations based on the Spearman's rank correlation.

Figure 9 plots the quantile dependence between four international stock returns (FTSE, NKY, DAX and MSCI emerging markets) and the S&P 500. The computation is based on standardized daily returns for the period January 2000–February 2017. For these stock returns, there is no pronounced dependence asymmetry in the tails of the distribution. The dependence is largest in the middle of the distribution and drops off in the tails. If the interest lies in a particular quantile of the distribution, the quantile dependence can be computed over a rolling window and supplement the information for the time-varying pair-wise correlation coefficients. The quantile (exceedance) correlation is characterized by similar level and pattern.

[9] For some drawbacks and biases in evaluating and testing the change in the correlation during market turmoil, which is accompanied with elevated asset volatility, see Boyer et al. (1999), Campbell et al. (2008), and Forbes and Rigobon (2002).

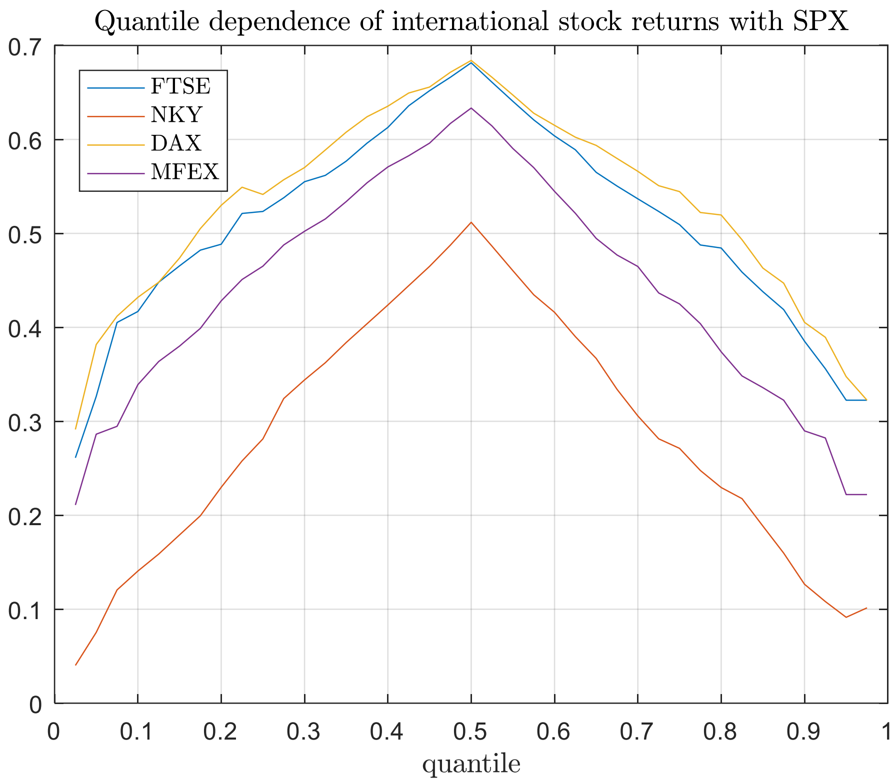

Figure 9. Quantile dependence between international stock returns (FTSE, NKY, DAX and MSCI emerging markets (MFEX)) and the S&P500 for the period January 2000 – February 2017

Computing general dependence measures, instead of correlation coefficients, could help us to gain some understanding of the level of dependence and the stability of the relationship between the different asset classes. This has implications for the approach to asset allocation and evaluation of systemic risk. In what follows, we focus on pair-wise or bivariate dependence and co-movement. One approach to studying the dependence between two random variables x and y is to resort to Sklar's theorem, which states that there exists a unique copula C with dependence parameter ρ such that the joint distribution $F(x, y)$ of the two variables can be expressed as

$$F(x, y) = C(G(x), H(y), \rho),$$

where $G(x)$ and $H(y)$ are continuous marginal distributions. When $C_\Phi = \Phi_2(\Phi^{-1}(x), \Phi^{-1}(y), \rho)$, where Φ and Φ_2 denote the marginal and bivariate Gaussian cumulative distribution functions, then ρ becomes the standard correlation coefficient. Other choices of a copula function are also possible.

A more general approach to modeling dependence is based on the generalized contrast or maximum entropy principle. Let P and Q be two probability measures; for example, two probability measures associated with two asset returns or the physical and risk-neutral probability measures. One way to measure the divergence between the two measures (Csiszár, 1972) is to solve the following optimization problem

$$D_{\phi}(P,Q) = \int \phi\left(\frac{dP}{dQ}\right) dQ,$$

where $\phi : \mathbb{R} \to [0,+\infty)$ is a convex, continuously differentiable function such that $\phi(1) = \phi'(1) = 0$. The measure D_{ϕ} is nonnegative and $D_{\phi}(P,Q) = 0$ if and only if the two measures coincide, $P = Q$. The function ϕ is often assumed to belong to the Cressie-Read (1984) divergence family

$$\phi(a) = \frac{a^{\pi+1} - 1}{\pi(\pi+1)} - \frac{1}{\pi} a + \frac{1}{\pi}.$$

One celebrated member of this divergence family is the Kullback-Leibler divergence, which is obtained by setting $\pi \to 0$ and takes the form

$$\phi(a) = a\ln(a) - a + 1 \text{ for } a \geq 0.$$

To illustrate the usefulness of this approach, suppose that R_i^e denotes the excess return on the risky asset i ($i = 1,\ldots,N$), P signifies the data-generating measure, and Q is the risk-neutral measure. The risk-neutral measure Q with minimal entropy relative to the physical measure P can be obtained as the solution to the problem (Stutzer, 1995)

$$Q^* = \underset{Q}{\operatorname{argmin}}\ \mathbb{E}^Q\left[\ln\left(\frac{dQ}{dP}\right)\right],$$

subject to the no-arbitrage restriction

$$\mathbb{E}^Q[R_i^e] \equiv \int R_i^e dQ = 0,\ i = 1,\ldots,N.$$

The solution Q^* gives rise to the following density

$$\frac{dQ^*}{dP} = \frac{\exp(\sum_{i=1}^N w_i^* R_i^e)}{\mathbb{E}[\exp(\sum_{i=1}^N w_i^* R_i^e)]},$$

where the density parameters (portfolio weights) $w^* = (w_1^*,\ldots,w_N^*)$ are the solution to the problem

$$w^* = \underset{w=(w_1,\ldots,w_N)}{\operatorname{argmin}}\ \ln \mathbb{E}[\exp(\sum_{i=1}^N w_i^* R_i^e)].$$

One interesting observation is that $\ln \mathbb{E}[\exp(\sum_{i=1}^{N} w_i R_i^e)]$ is the cumulant-generating function of $\sum_{i=1}^{N} w_i R_i^e$, which characterizes all the information in the distribution of the excess returns. When excess returns are assumed to be multivariate normal, all cumulants beyond the first two cumulants are zero and the above optimization problem collapses to the usual mean-variance portfolio problem

$$w^* = \underset{w=(w_1,\ldots,w_N)}{\operatorname{argmin}} \ \mathbb{E}[R^e]' w + 0.5 w' Cov[R^e] w,$$

which admits the closed-form solution $w^* = -Cov[R^e]^{-1}\mathbb{E}[R^e]$. Substituting for w^*, the relative entropy bound collapses to the scaled Hansen-Jagannathan (1991) bound $0.5\mathbb{E}[R^e]' Cov[R^e]^{-1}\mathbb{E}[R^e]$ which in the case of one asset becomes the squared Sharpe ratio of this asset return. Some numerical calculations suggest that a large part of the entropy is accounted for by the higher-than-second cumulants that arise from the non-Gaussianity of the excess return data. Thus, ignoring these higher moments in measuring entropy and dependence will result in a significant misspecification and spurious dynamics in the first two moments. Once higher moments and more general loss/risk functions are allowed for, most "anomalies" and "puzzles" tend to diminish in terms of magnitude and economic significance.

A more convenient measure of general dependence between two asset returns is based on the Hellinger distance measure obtained (up to a scale) from the Cressie-Read family of divergence measures above by setting $\pi = -1/2$.[10] Let $f(x,y)$ be the joint density and $g(x)\cdot h(y)$ be the product of the marginal densities of the asset returns x and y. In this case, the Hellinger dependence measure is given by

$$D_h = \frac{1}{2}\int\int\left(\sqrt{f(x,y)} - \sqrt{g(x)\cdot h(y)}\right)^2 dxdy.$$

If x and y are independent, $f(x,y) = g(x)\cdot h(y)$ and $D_h = 0$; otherwise $D_h > 0$. Conveniently, when x and y are bivariate normally distributed, $D_h = 0$ if $\rho = 0$ and $D_h = 1$ if $|\rho| = 1$, where ρ again denotes the correlation coefficient. Finally, the Hellinger distance is directly related to the copula (Granger, Maasoumi, and Racine, 2004)

$$D_h = \int\int [1 - c^{1/2}(u,v)] dudv,$$

[10] Unlike the other measures in the Cressie-Read divergence family, the Hellinger distance is a proper measure of distance since it is positive and symmetric, and it satisfies the triangular inequality.

where $c^{1/2}(u,v)=\partial^2 C(u,v)/\partial u \partial v$ and $C(\cdot)$ is the copula function introduced above.

This dependency measure can be easily accommodated to account for time variation and asymmetric dependence (Jiang, Wu, and Zhou, 2016). While we do not provide empirical results based on the Hellinger measure in this paper, we recommend that the statistics discussed in this section be used to augment the standard procedures based on second-moment information. We refer the interested reader to Gospodinov and Maasoumi (2017) for an application of this general approach to aggregating information from potentially misspecified asset-pricing models.

4. Risk Factor Extraction

It is widely believed that financial asset returns contain a small low-frequency, persistent component; see, for example, Bansal and Yaron (2004) for the role of long-run risks in explaining the equity premium puzzle. Despite its theoretical appeal, the empirical evidence on the existence of such long-run components is rather weak, as the observed asset returns do not appear to exhibit any persistence. The potential explanation for this tension between theory and empirics is that the underlying slow-moving component is overwhelmed by higher frequency noise and volatility, and quantifying its impact and empirical detection prove challenging using only time series data on intrinsically more volatile asset returns. However, provided that this trend component is common across assets, one could use a cross-section of asset returns and estimate their common variation by the method of principal components. Thus, cross-sectional information can be useful to extract more precise signals about common variation and factors.

Unlike the correlation and dependence measures discussed in Section 3 that apply largely to pair-wise relationships and can exhibit some idiosyncrasies, principal components and factor analysis are a convenient tool to summarize the co-movements in a large cross-section of asset returns. Another advantage of this approach is that the precision of the estimation increases with the dimension of the asset returns included in the analysis. Below, we attempt to isolate common low-frequency movements in asset returns and relate them to some macroeconomic and demographic factors.

4.1 Business Cycle Co-movements

Here, we follow Bai and Ng (2004) by estimating the common factor from returns and then integrate the process to obtain the common stochastic trend component.[11] This method guards against the possibility of spurious trends and is particularly effective when the number of assets is large. An alternative approach is to model the common variation in asset prices through cointegrating vectors. Given the differences in the persistence of the individual series in levels and the lack of robustness of the co-integration approach to deviations from the exact unit root for each process (Elliott, 1998), we have adopted the principal component estimation of Bai and Ng (2004).

We first analyze the factor structure in the four U.S. asset classes considered in Section 3: S&P 500 index (SPX), Bloomberg Barclays Treasury total return index, Goldman Sachs commodity index (GSCI), and USD index (DXY). All series are daily returns for the period January 2000–February 2017. The common factor is estimated from these returns by the method of principal components, and figure 10 plots the integrated and linearly detrended process.

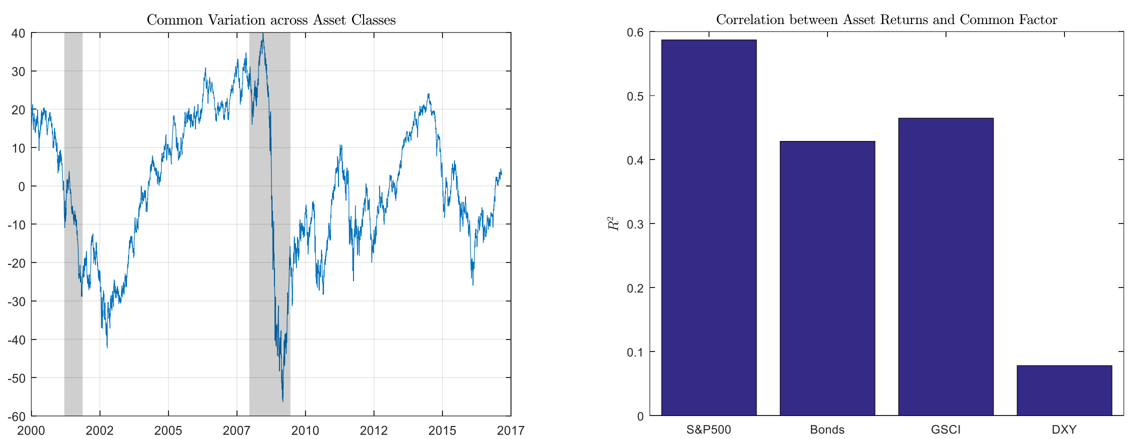

Figure 10. Left: Common variation across asset classes with shaded areas representing NBER-dated recessions. Right: R^2 from projecting individual asset returns on the common factor

The estimated factor exhibits sharp decreases during recessions, with the drop during the 2007–09 recession being particularly pronounced. The factor also appears to exhibit some higher-frequency cycles, but they are more difficult to identify. The right panel in figure 10 reports the R^2 from projecting the individual asset returns on the common factor. The largest loading on the factor is for the S&P 500 index, although bonds and commodity prices also contribute significantly to the common factor variation.

[11] The integrated process is linearly detrended and demeaned.

It would be interesting to see if this common variation is specific to the United States or if it reflects some global factor as well. For this purpose, we compute the common factor from international stock returns on S&P 500, FTSE in UK, Nikkei in Japan (NKY), DAX in Germany, and MSCI emerging markets index (MXEF) in both USD and local currency. Figure 11 plots the common factors.

Interestingly, these returns share very similar dynamics with the U.S. common factor. Moreover, the largest contributors to these dynamics are FTSE, DAX, and MXEF and not S&P 500. Consistent with figure 9, Nikkei appears to have a large idiosyncratic component. Also in line with figure 10 where DXY exhibits little correlation with the common factor, the common variation in USD and local currency is fairly similar. The results in figures 9 and 11 suggest a substantial integration of the international equity markets in the post-2000 period that is accompanied with a decline in the benefits from global diversification (see also Cotter et al., 2016).[12]

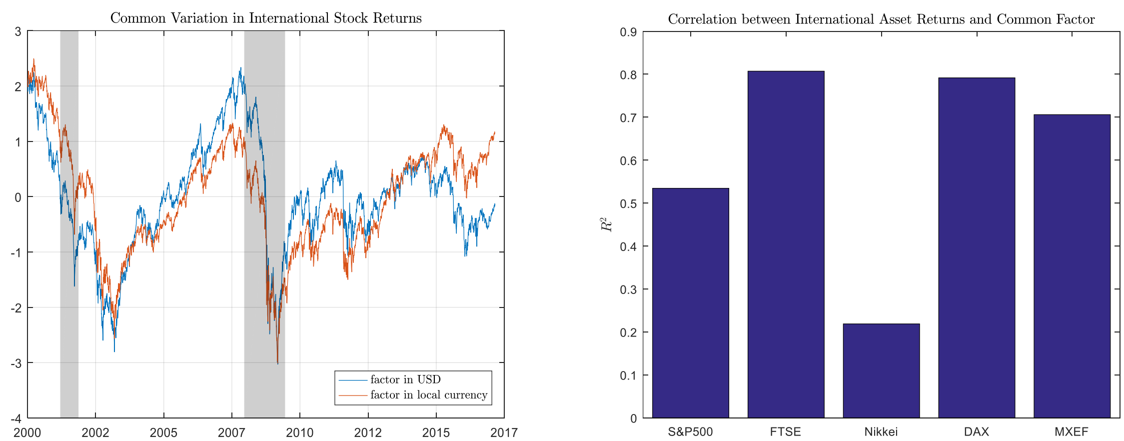

Figure 11. Left: Common variation in international stock returns with shaded areas representing NBER-dated recessions. Right: R^2 from projecting individual stock returns on the common factor

Despite the fairly short time span of the data, it would be instructive to relate more convincingly this common variation with some underlying macroeconomic signal or risk factor. To provide some suggestive evidence, figure 12 presents the Atlanta Fed/New York Fed turning point indicator of the labor market,[13] along with the Chicago Fed national activity index (CFNAI). The labor market indicator is computed from vintage data and is available "almost" in real time. It is particularly useful

[12] Pukthuanthong and Roll (2009) provide an insightful analysis on the flaws of correlation as a measure of global market integration. Instead, they recommend the use of global factor exposure for gauging financial market integration.

[13] This is based on ongoing joint work with Richard Crump and Ayşegül Şahin from the Federal Reserve Bank of New York.

for monitoring since it is a one-sided filter with no distortionary effects on the cyclical dynamics. The CFNAI series is relatively noisy and is constructed with data that are released with a delay. While the smoothing in the Atlanta Fed/New York Fed indicator induces a phase shift in the series, it provides a much improved estimate of the turning points and the local trends in the underlying signal.

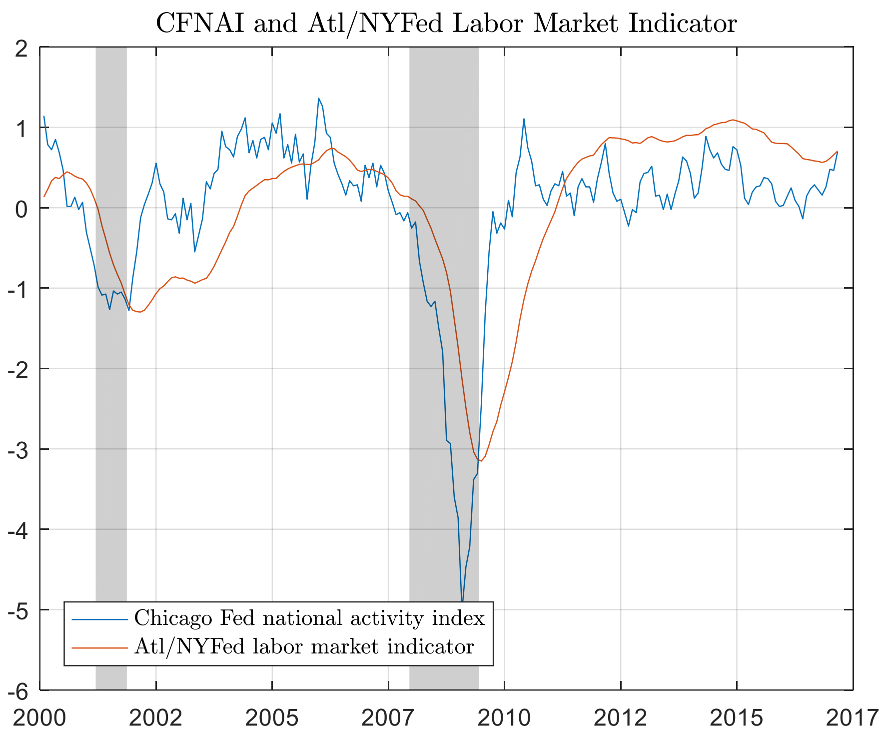

Figure 12. Chicago Fed national activity index (CFNAI) and AtlFed/NYFed labor market indicator. Shaded areas are NBER-dated recessions

There are a couple of observations that warrant some remarks. First, the smoothed labor market indicator identifies the turning points of the business cycle in advance, dating the recessions and the expansions, which would be of great value to policymakers. The noisy monthly reports in the data-dependent policy or the volatile asset dynamics in asset allocation can be supplemented and validated with this lower-frequency, local-trend information. Moreover, it is striking how closely the smoothed labor indicator underlies the dynamics of the common variation in asset returns, suggesting the presence of a strong business cycle component in both domestic and global financial asset prices.

Pinning down these business cycle components in asset prices has some important implications for medium-term investment and policy decisions. For example, this information can be used in preparing supervisory scenarios for annual stress tests by the Federal Reserve. More specifically, a factor-augmented vector autoregressive model, with an asset-pricing factor estimated as above, can be used to generate conditional forecast paths (see Waggoner and Zha, 1999), under various scenarios, for the variables of interest. Incorporating this asset-pricing factor, using both domestic and international data, should further improve the fit of the model and the accuracy of the conditional forecasts.

4.2 Long Cycles: Low Frequency Information in Demographic Variables

In this section, we investigate if there is any other common variation in asset prices—and stock prices in particular—at even lower frequency than that of the business cycle. Recent literature has established the usefulness of low-frequency demographic variables for long-horizon stock market returns. This is because savings rates, and possibly risk preferences, vary substantially over the life cycle, with savings rates peaking in middle age and then being drawn down in old age. These savings directly impact the pool of funds available for investment in the stock market. In fact, they may explain and predict some of the very persistent, low-frequency movements in stock market valuation ratios, such as the dividend or earnings price ratios.

To try to address this issue, we use annual data for the period 1946–2015 for the S&P 500 returns, returns on long-term and intermediate-term U.S. government bonds, changes in S&P 500 price-dividend ratio, changes in S&P 500 dividend yield, and changes in S&P 500 earnings-price ratio.[14] The data for the government bond returns are from Ibbotson Associates and the stock price data are from Robert Shiller's and Amit Goyal's websites.

We also collect annual demographic data from the Census Bureau and construct the variable middle-young (MY) ratio as the ratio of middle-aged (40–49) and young (20–29) cohorts. In addition to the historical data, the Census Bureau provides projections until 2060. Figure 13 plots the smoothed common factor estimated from the asset returns, as described in the previous section, along with the middle-young ratio.

[14] More specifically, dividends and earnings are 12-month moving sums. The dividend-price and earnings-price ratios are constructed as the difference between the log of dividends (earnings) and log of prices. The dividend yield is the difference between the log of dividends and log of lagged prices. Due to the near-unit root behavior of these series, we work with their first differences.

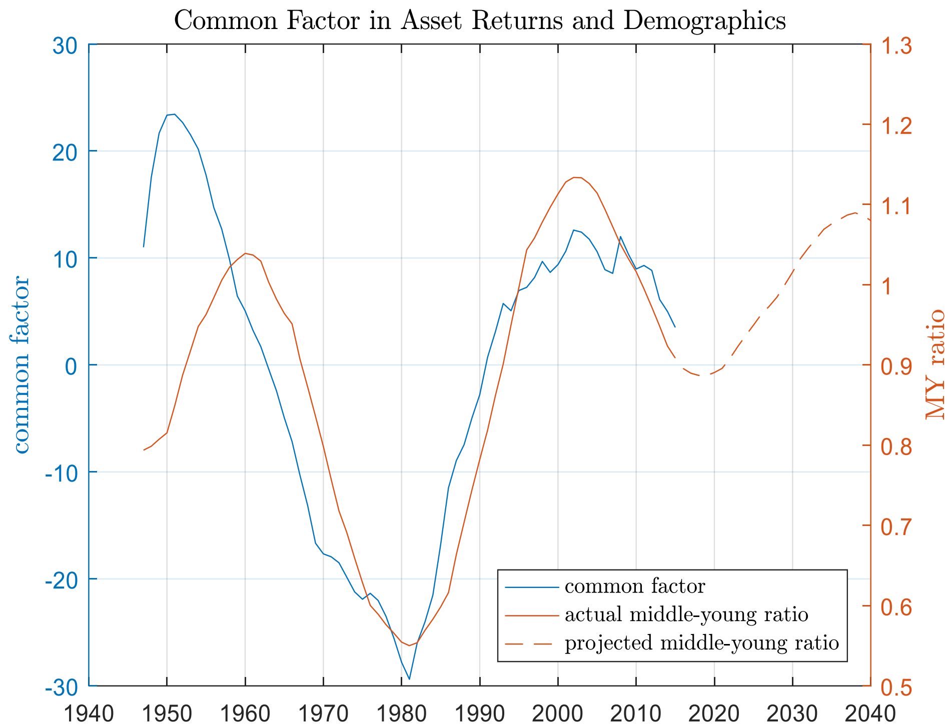

Figure 13. Common factor in asset returns and middle-young ratio

During the 1946–2015 period, the long-cycle factor in asset returns seems to co-move closely with the demographic trend, with a common trough in the early 1980s and a common peak in the early 2000s. This is consistent with other results in the literature on the relationship of the demographic variables with stock valuation ratios (Favero et al., 2011) and interest rates (Favero et al., 2016).

A unique feature of the demographic variables is that their low frequency trends can be projected with reasonable accuracy decades into the future. The projections of the MY ratio by the Census Bureau until 2040 (represented on the graph with a dashed line) thus facilitate the forecasting of both the market valuations and the small low-frequency component of market returns. The MY ratio is projected to fall until 2020, which implies a downward pressure on the stock valuation ratios and interest rates due to demographic factors. After 2020, the MY ratio starts to increase again until 2040, when it reaches a new peak. It is expected that during this period, the downward pressure on valuation ratios and interest rates from demographics is diminished and even reversed.

It is instructive to discuss further the implications of the demographic-trend projections on long-horizon forecasting for stock returns and valuation ratios.[15] The literature has explored many predictors for asset returns, with varying and often debated success (for a comprehensive review, see Goyal and Welch, 2008). However, the future trajectory of most predictors is itself highly uncertain, making them less useful for longer-term forecasts. Also, the innovations from the predictive regression for stock returns and the dynamic model for the predictor are often strongly correlated. For example, when the lagged dividend-price (*dp*) ratio is used as a predictor, this correlation is -0.95 implying that a fall (rise) in the *dp* ratio is associated with positive (negative) returns. But this strong contemporaneous relationship between stock returns and *dp* cannot be exploited for prediction without knowledge of the future values of *dp*.

Demographic variables allow us to address both of these shortcomings. With the Census Bureau projections for the MY ratio, we can construct recursive forecasts for *dp* that could be included in the predictive regression for future stock returns. Some tentative findings from this exercise can be summarized as follows. The model suggests that near-term returns are expected to be low by historical standards until 2020 due to the projected fall of the MY ratio. After 2020, the MY ratio rises again, with more middle-aged savers putting upward pressure on stock prices. Thus, after 2020, the long-term stock returns are projected to increase again but settle at a level that is lower than the historical average over the last 70 years.

While these projections reflect pure demographic information[16] and are only suggestive about the future long-term path of U.S. stock returns, they conform to a broader set of arguments put forward in the literature. A consensus is now emerging that the changing demographic structure in the developed economies has contributed to the recent decline in the equilibrium real interest rates (Gagnon et al., 2016). Historically, periods of low real interest rates[17] are associated with lower asset returns in the next five years (Dimson et al., 2013). These lower expected returns then pose a direct risk to institutional investors with long-term commitments.

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[15] This is based on ongoing work with Alex Maynard (University of Guelph) and Elena Pesavento (Emory University).

[16] We acknowledge that the middle-young ratio could be just a convenient proxy for other slowly moving socioeconomic and political factors such as safety-net development and political polarization. Also, the demographic projections are based on assumptions about future immigration dynamics, which are influenced by policy decisions.

[17] It should be noted that most of the previous episodes of low real interest rates were due to above-average inflation instead of low nominal interest rates, as in the years after the 2007–09 recession.

5. Concluding Remarks

This paper discussed and summarized some issues arising in the analysis of asset co-movements for the purposes of asset allocation and portfolio diversification, as well as macroeconomic and macro-prudential policy. One of the main findings that emerges from this review is that a judiciously performed factor analysis appears to identify a common variation across domestic and international asset classes at business and longer cycles. While the reported evidence is only tentative, a more comprehensive empirical analysis with a larger cross-section of U.S. and international asset returns would shed further light on the important sources of risks across asset classes, the integration of the global markets, and their implications for diversification, re-allocation and policy analysis. Attaching a risk factor interpretation to short- and medium-term co-movements appears to be more difficult due to some statistical challenges in analyzing the data. Explicitly acknowledging the estimation and model uncertainty as well as shifting the focus to more general measures of dependence would robustify the decision-making process and reduce the risk of false positives/negatives in signal extraction and performance evaluation.

One interesting topic that was omitted from the discussion is the increased importance of factor ("smart beta") investing. Studying more formally the dependence structure of these investment factors and evaluating their performance using rigorous statistical criteria is an area of ongoing research. While most of this research has focused on equities, constructing factors across divergent asset classes enhances the ability of capturing multiple sources of systematic risk that are difficult to identify and estimate statistically. Thus, a sufficient heterogeneity of spread factors across asset classes could potentially span the underlying factor space (Roll, 2013) and contribute beneficially to risk diversification and financial system stability.

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