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Redistribution and Fiscal Policy

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Abstract: This paper studies the optimal behavior of a democratic government in its use of fiscal policies to redistribute income. First, I characterize the optimal Ramsey allocation in heterogeneous agents' version of the neoclassical growth model. Second, I show that if I follow the empirical evidence in Storesletten, Telmer, and Yaron (2002) and assume labor income inequality to be countercyclical, fiscal policy is also highly countercyclical, challenging the Chari, Christiano, and Kehoe (1994) result that labor taxes should be smooth over the cycle.

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Redistribution and Fiscal Policy

1. Introduction

This paper examines the optimal behavior of a democratic government in its use of fiscal policies to redistribute income over the business cycle.

On the theoretical side, one set of models study optimal redistribution under either no or idiosyncratic uncertainty; for example, Perotti (1993) and Persson, and Tabellini (1994) study the effects of income inequality on growth. In addition, Krusell, Rios-Rull, and Quadrini (1997) and Krusell, and Rios-Rull (1999) have worked on the effects of inequality on optimal fiscal policies in a recursive framework. All these papers conclude that the higher the inequality the higher the income taxes and the optimal income redistribution. Another set of models incorporate aggregate uncertainty and study optimal taxation in a representative agent environment: Chari, Christiano, and Kehoe (1994) study optimal taxation in a neoclassical growth model and Gorostiaga (2002) extend their analysis to one in which labor markets are not competitive. Both papers find that labor income taxes should be roughly constant over the cycle to minimize distortions.

On the empirical side, Storesletten, Telmer, and Yaron (2002) have provided evidence that income inequality is countercyclical; increasing in recessions and decreasing in expansions. Therefore, the combination of this empirical fact with the theoretical results from optimal redistribution suggest that if we consider a model with heterogeneous agents and countercyclical income inequality the smoothing labor income taxes result may be challenged. In this paper, we study this possibility using a heterogeneous agents version of the neoclassical growth model.

The environment is a stochastic dynamic general equilibrium model with three agents: two infinitely lived consumers with different skill levels and a government that maximizes the median voter utility.[1] We assume that the fraction of each consumer type is constant over time and that the fraction of low skilled households is bigger than the fraction of high skilled. Consumers supply labor elastically and are able to borrow and lend in a complete markets environment. The government sets labor income taxes and transfers. Technology is linear and separable on agent's labor and there is an aggregate productivity shock that affects households skill level. A Ramsey problem is defined where the low skilled household plays the role of the planner and he/she optimizes over all possible sequences.

[1] We will see that we do not need to speak about the median voter, since we will assume one of the types of consumers is majoritarian. However, I will call this one the median voter in order to compare our results with Persson and Tabellini (1994).

We consider two versions of the aggregate productivity shock: First, we assume that the cycle affects both agents symmetrically, leaving the skill level and the labor income inequality constant over the business cycle. This version is called the **Symmetric Shock Model**. Second, we take a more realistic approach and follow the empirical evidence in Storesletten, Telmer, and Yaron (2002) and assume labor income inequality to be countercyclical. The productivity shock only affects the lower skill level and labor income inequality increases in recessions and decreases in expansion. This case is labeled as the **Asymmetric Shock Model**.

The main result is the following: If labor income inequality is countercyclical Chari, Christiano, and Kehoe (1994) result does not hold and the optimal labor income tax is also countercyclical. We think this result is important because it challenges standard optimal smoothing labor tax results.

In addition, we study two other policy questions. As Hall (1988) has shown, permanent and non-permanent labor income shocks have very different intertemporal implications. The fact that we characterize the solution to the Ramsey problem allows us to test whether it is also the case for optimal redistribution. We explore the implications for the optimal redistribution policy of the **Symmetric** or the **Asymmetric Shock Model** faced with permanent and non-permanent labor income shocks. In either model, the optimal redistribution policy is independent of the persistence of the aggregate shock regardless of the considered labor income process.

The last part of this paper studies optimal redistribution consequences of labor income inequality convergence. We consider several **Asymmetric** economies with different starting values for the labor income ratio that converge to the same ratio and ask whether the optimal income tax policy is affected by the initial inequality conditions. The answer is yes. While facing the same labor inequality today the higher the initial inequality the higher today's redistribution. Therefore, economies with identical fundamentals but different initial labor income inequality deliver different optimal income taxes and redistribution policies in the long run.

Finally, a technical contribution of the paper should be highlighted: In order to solve for the optimal labor income tax policy function, we characterize the optimal fiscal policy that solves the above described Ramsey problem. We are not aware of any paper that had done this before in a heterogeneous agents model like the one presented here. Garcia-Milà et.al. (2001) work in a similar framework but they do not solve for the optimal Ramsey allocation. They characterize the set of competitive equilibrium and choose the one that matches some features of the data better. Here, we go one step further and pick the competitive equilibrium that maximizes government's problem.

The rest of the paper is organized as follows. Section 2 presents the two versions of the model. There we will define and characterize both the equilibrium and the Ramsey problem. Section 3 presents the results, and Section 4 the final remarks.

2. The Model

The rest of this section is as follows. First, we introduce the **Symmetric Shock Model** and both the equilibrium and the Ramsey problem are defined and characterized. Second, the same is done for the **Asymmetric Shock Model**. Finally, we highlight the differences between the two models.

2.1. The Symmetric Shock Model

We study a dynamic stochastic general equilibrium model with two types of households, household type "h" and household type "l", and a government that maximizes the utility of the median voter. Household type h has measure γ and household type l has measure $1-\gamma$, where $\gamma \in (0, 0.5)$. This fact implies that a type l household is the median voter. We consider and economy without capital and with a single final good, y_t, that is produced using elastically supplied labor in the following way

$$y_t = (\gamma(1-x_{h,t})\phi_h + (1-\gamma)(1-x_{l,t})\phi_l)\theta_t, \tag{1}$$

where $(1-x_{i,t})$ is the amount of labor supplied by a household of type $i \in I \equiv \{h, l\}$, $\theta_t\phi_i$ is its marginal product and θ_t is a aggregate productivity shock following a Markov process

$$\ln\theta_t = \rho\ln\theta_{t-1} + \varepsilon_t, \qquad |\rho| < 1, \qquad \varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right).$$

As the reader can observe, in this environment the aggregate productivity shocks, θ_t, have not effect on the ratio between household type l and household h marginal products, $\frac{\theta_t\phi_l}{\theta_t\phi_h}$. As we will see, this fact implies that labor income inequality is constant over the cycle. This is why we call this set up the **Symmetric Shock Model**.

Households Consumers derive utility from consumption and leisure. The household's type $i \in I$ objective function is

$$E_0\sum_{t=0}^{\infty}\beta^t U(c_{i,t}, x_{i,t}), \tag{2}$$

where U is strictly increasing and concave on its two arguments. Consumer type i is endowed with one unit of time which is devoted to work and leisure. Besides, the household can lent to or borrow from other households or the government using a full array of one period bonds that complete the markets. Thus, a type i household faces the following budget constraint every period

$$c_{i,t} + \int p_t(\theta) b_{i,t}(\theta) d\theta = (1 - \tau_t)\omega_{i,t}(1 - x_{i,t}) + b_{i,t-1}(\theta_t) + T_t, \tag{3}$$

taking as given θ_0 and $b_{i,-1}$. Where $c_{i,t}$ denotes the consumption level of the type i consumer at t, $\omega_{i,t}$ denotes the hourly wage rate of household type i at t, $x_{i,t}$ denotes leisure of household type i at t, $p_t(\theta)$ is the price at t of a bond that pays a unit of the final good at $t+1$ if the aggregate productivity shock is θ, $b_{i,t}(\theta)$ is the type i consumer demand at t for bonds that pay a unit of the final good at $t+1$ if the aggregate productivity shock is θ, T_t is the level of transfers fixed by the government at t, τ_t is the level of labor taxes fixed by the government at t. In addition, there are upper and lower bounds for $b_{i,t}$ large enough not to bind in equilibrium but finite to avoid Ponzi games.

Government Government maximizes median voter's utility, i.e. consumer's of type l utility, subject to the following sequence of budget constraints

$$T_t + b_{t-1}(\theta_t) = \tau_t(\gamma(1 - x_{h,t})\omega_{h,t} + (1 - \gamma)(1 - x_{l,t})\omega_{l,t}) + \int p_t(\theta) b_t(\theta) d\theta, \tag{4}$$

taking as given θ_0 and b_{-1}. Where $b_t(\theta)$ is the government demand at t for bonds that pay a unit of the final good at $t+1$ if the aggregate productivity shock is θ. In addition, there are upper and lower bounds for b_t large enough not to bind in equilibrium but finite to avoid Ponzi games.

Market clearing conditions The clearing condition in the bond market is

$$(1 - \gamma) b_{l,t}(\theta) + \gamma b_{h,t}(\theta) = b_t(\theta), \tag{5}$$

and this same condition for $t = -1$ implies that

$$(1 - \gamma) b_{l,-1} + \gamma b_{h,-1} = b_{-1}. \tag{6}$$

Since there is not capital, the final good clearing market condition is

$$(1 - \gamma) c_{l,t} + \gamma c_{h,t} = y_t. \tag{7}$$

Therefore, given the production function (1) and the final good clearing market condition (7), we can write the economy resource constraint

$$\gamma c_{h,t} + (1-\gamma)c_{l,t} = (\gamma(1-x_{h,t})\phi_h + (1-\gamma)(1-x_{l,t})\phi_l)\theta_t. \tag{8}$$

2.1.1. Competitive Equilibrium

In this section we first describe which is the households' problem and then define a competitive equilibrium.

Household's type i problem is to choose $\{c_{i,t}, x_{i,t}, b_{i,t}(\theta)\}$ that maximizes the objective function (2) subject to the sequence of budget constraints (3) and taking the sequence of wages, taxes, transferences, prices $\{\omega_{i,t}, \tau_t, T_t, p_t(\theta)\}$, the initial stock of bonds and shock $b_{i,-1}$ and θ_0 as given. The first order conditions with respect to bonds holdings and leisure require

$$p_t(\theta) = \beta \frac{U_{c,i,t+1}(\theta)}{U_{c,i,t}} \Pr(\theta_{t+1} = \theta/\theta_t), \tag{9}$$

and

$$\frac{U_{x,i,t}}{U_{c,i,t}} = (1-\tau_t)\omega_t, \tag{10}$$

where $U_{c,i,t}$ and $U_{x,i,t}$ are the marginal utilities with respect to consumption and labor respectively.

In this environment a competitive equilibrium is defined as follows:

Definition 1 (Competitive Equilibrium Definition). *Given* $\{\theta_0, b_{l,-1}, b_{h,-1}, b_{-1}\}$ *such that* (6) *holds, a competitive equilibrium is a process for allocations* $\{(c_{i,t})_{i\in I}, (x_{i,t})_{i\in I}, (b_{i,t}(\theta))_{i\in I}, b_t(\theta)\}$, *taxes and transfers* $\{\tau_t, T_t\}$ *and prices* $\{p_t(\theta), (\omega_{i,t})_{i\in I}\}$ *such that:*

1. *For each* $i \in I$, $\{c_{i,t}, x_{i,t}, b_{i,t}(\theta)\}$ *maximizes household's utility function* (2) *subject to the budget constraint* (3) *given* $\{\tau_t, T_t\}$, $\{p_t(\theta), \omega_{i,t}\}$, $b_{i,-1}$ *and* θ_0.

2. *For each* $i \in I$
$$\omega_{i,t} = \phi_i\theta_t. \tag{11}$$

3. *The government budget constraint* (4), *the bonds market clearing condition* (5) *and economy resource constraint* (8) *hold.*

We assume competitive labor markets, hence, in any competitive equilibrium $\omega_{i,t} = \phi_i\theta_t$. Therefore, in the **Symmetric Model** the wage ratio is equal to ϕ_l/ϕ_h and constant over the cycle. We will refer to this ratio as the skill ratio.

2.1.2. The Ramsey Problem

As mentioned before, government maximizes consumer of type l's utility. The government is aware of consumers answer to policy announcements and takes this reaction into account when its solve its maximization problem. This is what has been called a Ramsey problem. Hence, the Ramsey problem consists on choosing taxes and transfers that maximize the utility of a household type l over the set of competitive equilibriums defined above. When doing so, the government faces various trade-offs. Consumers of type l derive utility from higher transfers but those transfers has to be finance throughout taxes or government debt. Taxes affect both consumers symmetrically and they distort labor supply decisions. Higher government debt today increases taxes tomorrow. Since the government solves an intertemporal problem it will try to smooth taxes throughout time and states of nature.

As widely noticed in the literature, this problem is not time consistent. To avoid dealing with this issue, we assume that the government has some device such it can commit itself to the Ramsey outcome.

Technically the Ramsey problem consists of maximizing consumer of type l's utility over the set of competitive equilibria. This is equivalent to choose the allocations, taxes, transfers and prices that maximizes consumer of type l's utility over the set of allocations, taxes, transfers and prices that define a competitive equilibria. In general this problem can be very complicated. Thus, the next step it is to define the minimal set of equations that characterize the set of allocations, taxes, transfers and prices that define a competitive equilibria. We do this in the next proposition.

Proposition 1 (Competitive Equilibrium Charaterization). *Given* $\{\theta_0, b_{l,-1}, b_{h,-1}, b_{-1}\}$ *such that* (6) *holds, if the equilibrium is interior and unique, then the equilibrium process for* $\left\{(c_{i,t})_{i\in I}, (x_{i,t})_{i\in I}, (b_{i,t}(\theta))_{i\in I}, b_t(\theta)\right\}$, $\{\tau_t, T_t\}$ *and* $\left\{p_t(\theta), (\omega_{i,t})_{i\in I}\right\}$ *is uniquely determined by the following conditions:*

- $\exists\ \lambda$ *such that*

$$\frac{U_{c,l,t}}{U_{c,h,t}} = \lambda, \tag{12}$$

  and

$$\frac{U_{x,l,t}\phi_h}{U_{x,h,t}\phi_l} = \lambda, \tag{13}$$

  holds.

- *The next restriction is satisfied*

$$b_{h,-1} - b_{-1} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_{c,h,t}}{U_{c,h,0}} (\Phi_{h,t} - \Phi_t), \tag{14}$$

where

$$\Phi_{i,t} = c_{i,t} - (1 - x_{i,t})\frac{U_{x,i,t}}{U_{c,i,t}} \qquad \forall i \in I,$$

and

$$\Phi_t = (1 - \frac{U_{x,h,t}}{U_{c,h,t}\theta_t\phi_h})(\gamma(1 - x_{h,t})\phi_h + (1-\gamma)(1 - x_{l,t})\phi_l)\theta_t.$$

- *The economy resource constraint* (8) *holds.*

Proof. The proof will be as follows. First, we are going to show that given a sequence for consumption and leisure allocations for both types of consumers $\{(c_{i,t})_{i\in I}, (x_{i,t})_{i\in I}\}$ such that the resource constraint (8), the restrictions (12) and (13) and (14) hold for some λ, we can find a sequence for bonds $\{(b_{i,t}(\theta))_{i\in I}, b_t(\theta)\}$, taxes and transfers $\{\tau_t, T_t\}$ and prices $\{p_t(\theta), (\omega_{i,t})_{i\in I}\}$ such that consumers' budget constraint (3) for $\forall i \in I$, the equilibrium wage (11) for $\forall i \in I$, the government budget constraint (4), the bonds market clearing condition (5), the resource constraint (8) and the households' first order conditions (9) and (10) $\forall i \in I$ hold.

At this point, it is important to notice that with concave utility function, and if the equilibrium is interior and unique (as assumed), the solution to the maximization problem of the consumer is uniquely determined by the consumer's budget constraint (3) and the first order conditions (9) and (10), so that the consumer's budget constraint (3) for $\forall i \in I$, the equilibrium wage (11) for $\forall i \in I$, the government budget constraint (4), the bonds market clearing condition (5), the resource constraint (8) and the households' first order conditions (9) and (10) $\forall i \in I$ are necessary and sufficient for competitive equilibrium.

Assume that $\{(c_{i,t})_{i\in I}, (x_{i,t})_{i\in I}\}$ and λ are such the resource constraint (8), the restrictions (12) and (13) and (14) hold. Now, we are going to find $\{p_t(\theta)\}_{t=0}^{\infty}$ such that the first order condition (9) holds for $\forall i \in I$.

First, define wages as $\omega_{i,t} = \phi_i\theta_t$ $\forall i \in I$ and $\forall t$, which is (11) for $\forall i \in I$.

Define $\{p_t(\theta)\}_{t=0}^{\infty}$ as

$$p_t(\theta) = \beta\frac{U_{c,l,t+1}(\theta)}{U_{c,l,t}}\Pr(\theta_{t+1} = \theta/\theta_t) = \beta\frac{U_{c,h,t+1}(\theta)}{U_{c,h,t}}\Pr(\theta_{t+1} = \theta/\theta_t), \tag{15}$$

(which is (9) for $i = l$) then, (12) implies (9) for $i = h$.

Let us now probe that exists $\{\tau_t\}_{t=0}^{\infty}$ such that the first order condition (10) holds for $\forall i \in I$.

Define $\{\tau_t\}_{t=0}^{\infty}$ as

$$1 - \tau_t = \frac{U_{x,l,t}}{U_{c,l,t}\theta_t\phi_l} = \frac{U_{x,h,t}}{U_{c,h,t}\theta_t\phi_h}, \tag{16}$$

then, using (11) for $\forall i \in I$ we can write

$$\frac{U_{x,l,t}}{U_{c,l,t}\theta_t\phi_l} = \frac{U_{x,h,t}}{U_{c,h,t}\theta_t\phi_h} = \omega_{i,t}\left(1-\tau_t\right),$$

(which is (10) for $i = l$) then (12) and (13) imply that is equal to (10) for $i = h$.

Define $\{T_t\}_{t=0}^{\infty}$ as

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_{c,h,t}}{U_{c,h,0}} T_t = -b_{-1} + E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_{c,h,t}}{U_{c,h,0}} \Phi_t. \tag{17}$$

Then, restriction (14) implies

$$b_{h,-1} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_{c,h,t}}{U_{c,h,0}} (\Phi_{h,t} - T_t), \tag{18}$$

and the initial bonds market clearing condition, (6), implies

$$b_{l,-1} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_{c,h,t}}{U_{c,h,0}} (\Phi_{l,t} - T_t). \tag{19}$$

Define household type h demand for bonds $\{b_{h,t}\}_{t=0}^{\infty}$ as

$$b_{h,t} = E_{t+1} \sum_{j=0}^{\infty} \beta^j \frac{U_{c,h,t+1+j}}{U_{c,h,t+1}} \left(\Phi_{h,t+j+1} - T_{t+j+1}\right), \tag{20}$$

household type l demand for bonds $\{b_{l,t}\}_{t=0}^{\infty}$ as

$$b_{l,t} = E_{t+1} \sum_{j=0}^{\infty} \beta^j \frac{U_{c,h,t+1+j}}{U_{c,h,t+1}} \left(\Phi_{l,t+j+1} - T_{t+j+1}\right), \tag{21}$$

and government demand for bonds $\{b_t\}_{t=0}^{\infty}$ as

$$b_t = E_{t+1} \sum_{j=0}^{\infty} \beta^j \frac{U_{c,h,t+1+j}}{U_{c,h,t+1}} \left(\Phi_{t+j+1} - T_{t+j+1}\right). \tag{22}$$

Notice that the three bonds demand definitions (20), (21) and (22) are such that bonds market clearing condition (5) holds. Now, using the definition of prices (15) and taxes (16), the condition (18) and the definition of bonds demand (20) we can write

$$b_{h,t} = w_{h,t+1} + E_{t+1}\left[\beta\frac{U_{c,h,t+2}}{U_{c,h,t+1}}E_{t+2}\sum_{t=1}^{\infty}\beta^{t-1}\frac{U_{c,h,t+2+j}}{U_{c,h,t+2}}\Phi_{h,t+j+2}\right] = w_{h,t+1} + \int p_{t+1}(\theta)b_{h,t+1}(\theta)\,d\theta,$$

for $t \geq -1$, what it means that the household's type h budget constraint (3) (the sequence of budget constraints for consumer type h) holds. Using a similar procedure with (17) and (19) we can show that the household's type l budget constraint (3) for $i = l$ (the sequence of budget constraints for consumer type l) and the government budget constraint (4) (the sequence of government budget constraints) also hold.

Second, we are going to probe that given a sequence for consumption, leisure and bonds $\{(c_{i,t})_{i\in I}, (x_{i,t})_{i\in I}, (b_{i,t}(\theta))_{i\in I}, b_t(\theta)\}$, taxes and transfers $\{\tau_t, T_t\}$ and prices $\{p_t(\theta), (\omega_{i,t})_{i\in I}\}$ such that consumers' budget constraint (3) for $\forall i \in I$, the equilibrium wage (11) for $\forall i \in I$, the government budget constraint (4), the bonds market clearing condition (5), the resource constraint (8) and the households' first order conditions (9) and (10) $\forall i \in I$ hold we can find a λ such that the restrictions (12), (13) and (14) hold.

From (9) for $\forall i \in I$, we obtain that

$$\frac{U_{c,l,t}}{U_{c,h,t}} = \frac{U_{c,l,t+1}(\theta)}{U_{c,h,t+1}(\theta)} \quad \forall t, \theta,$$

i.e. the ratio marginal utilities of consumption at t is equal to the ratio at $t+1$ with probability one. By induction

$$\frac{U_{c,l,0}}{U_{c,h,0}} = \frac{U_{c,l,t}}{U_{c,h,t}} \;\forall t,$$

but, since $b_{h,-1}$, $b_{l,-1}$ and θ_0 are given, we can define λ to be

$$\lambda \equiv \frac{U_{c,l,0}}{U_{c,h,0}},$$

such that (12) holds.

Then, note that from (10) and (11) for $\forall i \in I$ we have

$$\frac{U_{x,l,t}\phi_h}{U_{x,h,t}\phi_l} = \frac{U_{c,l,t}}{U_{c,h,t}},$$

that together with (12) imply (13).

Finally, using (9) and (11) for $i = h$ in the consumer type h budget constraint (3) and in the government budget constraint (4) and solving recursively both restrictions we get

$$b_{h,-1} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_{c,h,t}}{U_{c,h,0}} (c_{h,t} - (1-\tau_t)(1-x_{h,t})\phi_h \theta_t - T_t),$$

and

$$b_{-1} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_{c,h,t}}{U_{c,h,0}} (\tau_t \theta_t (\gamma(1-x_{h,t})\phi_h + (1-\gamma)(1-x_{l,t})\phi_l) - T_t).$$

If we combine these two equations with (10) for $i = h$ we get (14). ■

Proposition 1 implies two important features of a competitive equilibrium. First, if we assume separable between consumption and leisure utility function,[2] (12) and (13) imply that both consumption and hours worked ratios between the two types of households are constant over time and realizations of the productivity shock. Second, λ, and, consequently, the two mentioned ratios, depend on the whole productivity shock sequence, and not only its actual realization. As we will see in the numerical exercise to be presented in the next section, this implies the initial conditions, i.e. θ_0, b_{-1}, $b_{l,-1}$ and $b_{h,-1}$, are going to very important on today's households consumption and hours worked optimal choices and, therefore, on the optimal fiscal policy.

But the most important implication of proposition 1 is the following: The economy resource constraint (8) and the restrictions (12), (13) and (14) are necessary and sufficient for *competitive equilibrium*. Hence, for each sequence of $\{(c_{i,t})_{i\in I}, (x_{i,t})_{i\in I}\}$ and λ, such that (8) and the restrictions (12), (13) and (14) hold, there exists allocations, taxes, transfers and prices such that they define a competitive equilibria. Therefore, the Ramsey Problem will be to choose $\{(c_{i,t})_{i\in I}, (x_{i,t})_{i\in I}\}$ and λ such that maximize type l consumer's utility subject to (8), (12), (13) and (14).

Formally, the Ramsey problem becomes

$$\underset{\{(c_{i,t})_{i\in I},(x_{i,t})_{i\in I}\},\lambda}{Max} \quad E_0 \sum_{t=0}^{\infty} \beta^t U(c_{l,t}, x_{l,t}),$$

subject to

$$\frac{U_{c,l,t}}{U_{c,h,t}} = \lambda,$$

$$\frac{U_{x,l,t}\phi_h}{U_{x,h,t}\phi_l} = \lambda,$$

$$b_{h,-1} - b_{-1} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_{c,h,t}}{U_{c,h,0}} (\Phi_{h,t} - \Phi_t),$$

[2] As we will do in the numerical exercices in the next section.

and

$$\gamma c_{h,t} + (1-\gamma)c_{l,t} = (\gamma(1-x_{h,t}) + (1-\gamma)(1-x_{l,t})\phi_l)\theta_t,$$

where θ_0, $b_{h,-1}$ and b_{-1} are given.

At this point, we would like to remark that we are not aware of any paper that has characterized and solved this problem in the way we have done here. Garcia-Milà et. al. (2001) have a similar Ramsey problem but they do not solve for the optimal Ramsey allocation. Instead, they calibrate λ to some data features. In what follows, first we are going to solve for the optimal Ramsey allocation. Then, we will perform some numerical exercises to understand which are the main features of the optimal Ramsey allocation.

To do that we assume some functional form for preferences

$$U(c_{i,t}, x_{i,t}) = \frac{c_{i,t}^{1-\sigma}}{1-\sigma} + \frac{x_{i,t}^{1-\sigma}}{1-\sigma},$$

where $\sigma > 0$. If this is the case, we can use the resource constraint (8), and the restrictions (12), (13) to write $c_{l,t}$, $x_{l,t}$ and $x_{h,t}$ as a function of $c_{h,t}$, θ_t and λ, in the following way

$$c_{l,t} = c_l\left(c_{h,t}, \lambda\right) = c_{h,t}\lambda^{-\frac{1}{\sigma}},$$

$$x_{l,t} = x_l\left(c_{h,t}, \lambda, \theta_t\right) = \frac{(\gamma\phi_h + (1-\gamma)\phi_l) - \frac{\left(\gamma + (1-\gamma)\lambda^{-\frac{1}{\sigma}}\right)}{\theta_t} c_{h,t}}{\left(\gamma\phi_h\left(\frac{\phi_h}{\lambda\phi_l}\right)^{-\frac{1}{\sigma}} + (1-\gamma)\phi_l\right)},$$

and

$$x_{h,t} = x_h\left(c_{h,t}, \lambda, \theta_t\right) = \left(\frac{\phi_h}{\lambda\phi_l}\right)^{-\frac{1}{\sigma}} \frac{(\gamma\phi_h + (1-\gamma)\phi_l) - \frac{\left(\gamma + (1-\gamma)\lambda^{-\frac{1}{\sigma}}\right)}{\theta_t} c_{h,t}}{\left(\gamma\phi_h\left(\frac{\phi_h}{\lambda\phi_l}\right)^{-\frac{1}{\sigma}} + (1-\gamma)\phi_l\right)},$$

at the same time, and since taxes, τ_t, are a function of λ and $c_{h,t}$ we can use (10) for $i = h$ to write

$$\tau_t = \tau\left(c_{h,t}, \lambda, \theta_t\right) = 1 - \frac{x_{h,t}\left(c_{h,t}, \lambda, \theta_t\right)^{-\sigma}}{c_{h,t}^{-\sigma}\theta_t\phi_h},$$

so, we can write

$$\tau_t = \tau\left(c_{h,t}, \lambda, \theta_t\right) = 1 - \left(\frac{\phi_h}{\lambda\phi_l}\right) \left[\frac{\frac{(\gamma\phi_h + (1-\gamma)\phi_l)}{\theta_t^{-\frac{1}{\sigma}}\phi_h^{-\frac{1}{\sigma}} c_{h,t}} - \left(\gamma + (1-\gamma)\lambda^{-\frac{1}{\sigma}}\right)\theta_t^{\frac{1}{\sigma}-1}\phi_h^{\frac{1}{\sigma}}}{\gamma\phi_h\left(\frac{\phi_h}{\lambda\phi_l}\right)^{-\frac{1}{\sigma}} + (1-\gamma)\phi_l}\right]^{-\sigma}. \tag{23}$$

This is going to be the most important object of study in this work. Now onwards, this function will be referred to as the policy function.

At this point, it is important to note that the assumption of separability between consumption and leisure decisions allows us to relate λ to the equilibrium income distribution. From (12) we have

$$\lambda = \frac{c_{h,t}^{\sigma}}{c_{l,t}^{\sigma}}.$$

Thus, if $\lambda > 1$, higher λ implies more income inequality. From this point, λ will be referred to as the income distribution parameter.

We can simplify the Ramsey problem as

$$\underset{\{c_{h,t}\},\lambda}{Max} \quad E_0 \sum_{t=0}^{\infty} \beta^t U(c_{l,t}(c_{h,t},\lambda), x_{l,t}(c_{h,t},\lambda,\theta_t)),$$

subject to

$$b_{h,-1} - b_{-1} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_{c,h,t}}{U_{c,h,0}} (\Phi_{h,t}(c_{h,t},\lambda,\theta_t) - \Phi_t(c_{h,t},\lambda,\theta_t)), \tag{24}$$

where θ_0, $b_{h,-1}$ and b_{-1} are given.

If an optimal policy exists and it is interior, the optimal allocations must satisfy the government's first order conditions with respect to $c_{h,t}$ and λ and the restriction (24).

Let η be the langrangian multiplier of (24). Then, the first order conditions of the Ramsey Problem are with respect to $c_{h,t}$ and λ are

$$c_{l,t}^{-\sigma} \frac{\partial c_{l,t}}{\partial c_{h,t}} + x_{l,t}^{-\sigma} \frac{\partial x_{l,t}}{\partial c_{h,t}} + \eta \frac{U_{c,h,t}}{U_{c,h,0}} \left[\sigma \frac{(\Phi_t - \Phi_{h,t})}{c_{h,t}} + (\frac{\partial \Phi_{h,t}}{\partial c_{h,t}} - \frac{\partial \Phi_t}{\partial c_{h,t}}) \right] = 0, \tag{25}$$

and

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\left(c_{l,t}^{-\sigma} \frac{\partial c_{l,t}}{\partial \lambda} + x_{l,t}^{-\sigma} \frac{\partial x_{l,t}}{\partial \lambda} \right) + \eta \frac{c_{h,t}^{-\sigma}}{c_{h,0}^{-\sigma}} (\frac{\partial \Phi_t}{\partial \lambda} - \frac{\partial \Phi_{h,t}}{\partial \lambda}) \right] = 0. \tag{26}$$

Thus, given the optimal λ and η, $c_{h,t}$ only depends on the contemporaneous shock θ_t and it has the same correlation properties as the former.

Given (24), (25) and (26) the solution to the Ramsey problem can be written as

$$\eta = \eta(\phi_l, \theta_0),$$

$$\lambda = \lambda(\phi_l, \theta_0),$$

and

$$c_{h,t} = c_h(\phi_l, \theta_t, \theta_0).$$

2.2. The Asymmetric Shock Model

In the **Symmetric Shock Model** the cycle does not affect the skill ratio. As we will see, this implies that labor income inequality is constant over the cycle. Storesletten, Telmer and Yaron (2002) provide empirical evidence suggesting that labor income inequality is countercyclical. In this section, we present a new version of the model where the aggregate productivity shock, θ_t, only affects type l consumer's skill level and labor income in such a way that labor income inequality will become countercyclical, increasing during contractions and decreasing during expansions. This is the reason why we call this set up the **Asymmetric shock model**. The arising differences are:

1. The production function

$$y_t = \gamma(1 - x_{h,t})\phi_h + (1 - \gamma)(1 - x_{l,t})\phi_l\theta_t. \tag{27}$$

2. Consumers' type i problem

$$\underset{\{c_{i,t},x_{i,t}\}_{t=0}^{\infty}}{Max} \quad E_0 \sum_{t=0}^{\infty} \beta^t U(c_{i,t}, x_{i,t}), \tag{28}$$

subject to

$$c_{i,t} + \int p_t(\theta)b_{i,t}(\theta)d\theta = (1 - \tau_t)\omega_{i,t}(1 - x_{i,t}) + b_{i,t-1}(\theta_t) + T_t, \tag{29}$$

given $b_{i,-1}$ and θ_0.

3. Government's restriction

$$T_t + b_{t-1}(\theta_t) = \tau_t(\gamma(1 - x_{h,t})\omega_{h,t} + (1 - \gamma)(1 - x_{l,t})\omega_{l,t}) + \int p_t(\theta)b_t(\theta)d\theta. \tag{30}$$

4. The bonds market clearing conditions are as in the **symmetric shock model**.

5. The economy resource constraint

$$\gamma c_{h,t} + (1 - \gamma)c_{l,t} = \gamma(1 - x_{h,t})\phi_h + (1 - \gamma)(1 - x_{l,t})\phi_l\theta_t. \tag{31}$$

In this case a competitive equilibrium is defined as:

Definition 2. *Given* $\{\theta_0, b_{l,-1}, b_{h,-1}, b_{-1}\}$ *such that* (6) *holds, a competitive equilibrium is a process for allocations* $\{(c_{i,t})_{i\in I}, (x_{i,t})_{i\in I}, (b_{i,t}(\theta))_{i\in I}, b_t(\theta)\}$*, taxes and transfers* $\{\tau_t, T_t\}$ *and prices* $\{p_t(\theta), (\omega_{i,t})_{i\in I}\}$ *such that:*

1. *For each* $i \in I$, $\{c_{i,t}, x_{i,t}, b_{i,t}(\theta)\}$ *maximizes household's utility function* (28) *subject to the budget constraint* (29) *given* $\{\tau_t, T_t\}$, $\{p_t(\theta), \omega_{i,t}\}$, $b_{i,-1}$ and θ_0.

2. *The equilibrium wages are as follows*

$$\omega_{l,t} = \phi_l \theta_t,$$

and

$$\omega_{h,t} = \phi_h.$$

3. *The government budget constraint* (30)*, the bonds market clearing condition* (5) *and the economy resource constraint* (31) *hold.*

As before, we have to characterize the equilibrium. This is done in the following proposition:

Proposition 2 (Competitive Equilibrium Charaterization). *Given* $\{\theta_0, b_{l,-1}, b_{h,-1}, b_{-1}\}$ *such that* (6) *holds, if the equilibrium is interior and unique, then the equilibrium process for* $\{(c_{i,t})_{i\in I}, (x_{i,t})_{i\in I}, (b_{i,t}(\theta))_{i\in I}, b_t(\theta)\}$, $\{\tau_t, T_t\}$ and $\{p_t(\theta), (\omega_{i,t})_{i\in I}\}$ *is uniquely determined by the following conditions:*

- $\exists\ \lambda$ *such that*

$$\frac{U_{c,l,t}}{U_{c,h,t}} = \lambda,$$

  and

$$\frac{U_{x,l,t}\phi_h}{U_{x,h,t}\phi_l} = \lambda\theta_t,$$

  holds.

- *The next restriction is satisfied*

$$b_{h,-1} - b_{-1} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_{c,h,t}}{U_{c,h,0}} (\Phi_{h,t} - \Phi_t),$$

  where

$$\Phi_{i,t} = c_{i,t} - (1 - x_{i,t}) \frac{U_{x,i,t}}{U_{c,i,t}} \qquad \forall i \in I,$$

  and

$$\Phi_t = (1 - \frac{U_{x,h,t}}{U_{c,h,t}\phi_h})(\gamma(1 - x_{h,t})\phi_h + (1-\gamma)(1 - x_{l,t})\phi_l\theta_t).$$

- *The economy resource constraint* (31) *holds.*

Three are the differences with the **Symmetric Shock Model**. First, the equilibrium wages. Second, the production function (27) and the economy resource constraint (31). An third, the equilibrium characterization: The ratio of marginal utility of leisure is not constant anymore, $U_{x,l,t}\phi_h/\left(U_{x,h,t}\phi_l\right) = \lambda\theta_t$, and therefore, while consumption ratio between the two types of households is constant, worked hours do not need to be. These three differences reflect the fact that the aggregate productivity shocks only affect household's l wage.

But the most important implication of the asymmetric effects of the aggregate shock is reflected in figure 1. Figure 1 plots the equilibrium labor income ratio, $\omega_{l,t}\left(1 - x_{l,t}\right)/\left(\omega_{h,t}\left(1 - x_{h,t}\right)\right)$, for the **Symmetric** and **Asymmetric Models** using the following calibration $(\gamma, \beta, \sigma, \sigma_\varepsilon, \rho, \phi_h, \phi_l) = (0.35, 0.95, 2, 0.05, 0.9, 1, 0.825)$. Therefore, in the **Symmetric Model** the labor income inequality is constant over the cycle, while in the **Asymmetric Model** labor income inequality is countercyclical, increasing in recessions and decreasing in expansions. This result is robust to other parameter choices.

In this **Asymmetric** case we can rewrite (23) as

$$\tau_t = \tau\left(c_{h,t}, \lambda, \theta_t\right) = 1 - \left(\frac{\phi_h}{\lambda\phi_l\theta_t}\right)\left(\frac{\frac{(\gamma\phi_h + (1-\gamma)\phi_l\theta_t)}{\theta_t^{-\frac{1}{\sigma}}\phi_h^{-\frac{1}{\sigma}}c_{h,t}} - \left(\gamma + (1-\gamma)\lambda^{-\frac{1}{\sigma}}\right)\theta_t^{\frac{1}{\sigma}}\phi_h^{\frac{1}{\sigma}}}{\gamma\phi_h\left(\frac{\phi_h}{\lambda\phi_l\theta_t}\right)^{-\frac{1}{\sigma}} + (1-\gamma)\phi_l\theta_t}\right)^{-\sigma}. \quad (32)$$

From this point, let us use "s" as superscript for the symmetric model policy function (23), and "as" for the asymmetric one, (32).

3. Results

Since closed form solutions are not available, we solve the models using numerical simulations.[3] The parameter values choice we consider is very similar to that used in the business cycle literature $(\gamma, \beta, \sigma, \sigma_\varepsilon, \rho, \phi_h) = (0.35, 0.95, 2, 0.05, 0.9, 1)$.[4]

Most models of optimal income redistribution find that the higher the labor income inequality (represented in this model by the skill ratio) the higher the labor income taxes. This model delivers the same result. Table 1 represents the average labor income tax of three different versions of the **Symmetric Shock Model** (identical except by ϕ_l). Using (23), this analysis can be formally written as the determination of

$$E(\tau^s(., \theta_t, 1)),$$

[3]See Appendix for details

[4]Our aim is to study optimal fiscal policy and so much to match the data. This is why we do not calibrate the model to get close to the data. We would like to remark that our qualitative results are robust to different calibrations.

ϕ_l	$E(\tau^s(\phi_l, \theta_t, \theta_0))$	$\lambda^s(\phi_l, \theta_0)$
0.7	0.30875	1.5478
0.825	0.18159	1.2949
0.95	0.05419	1.0773

Table 1: Average Tax Rate, as a Function of Consumer "l"'s Marginal Productivity

i.e. the unconditional mean of the tax level as a function of ϕ_l. As it is shown in the table 1, the higher ϕ_l, the lower the tax. The intuition for this result is as follows: Since ϕ_h is fixed, the lower ϕ_l the lower the tax burden on household type l for any given income tax rate, so the higher the optimal labor income tax rate. Table 1 also reports $\lambda^s(., \theta_0)$ reflecting the fact that although the lower ϕ_l the higher the average labor taxes and, therefore, the higher the income redistributed from households type h to households type l, it is still the case that type h households have higher consumption for all values of ϕ_l, being the difference between type h and l households consumption monotonically decreasing on ϕ_l.

A second standard result in the literature is that optimal labor income tax fluctuate very little (Chari, Christiano, Kehoe (1994) and Gorostiaga (2002)). The model here presented challenges this result. Figure 2 plots the following two functions

$$\tau^s(\phi_l, ., 1), \tag{33}$$

and

$$\tau^{as}(\phi_l, ., 1), \tag{34}$$

i.e., the optimal labor income tax for the **Symmetric** and **Asymmetric Model**, when $\phi_l = 0.825$. Therefore, if we model labor income inequality to be countercyclical, as Storesletten, Telmer and Yaron (2002) empirical evidence suggests, labor income tax also becomes highly countercyclical. This result is very important because it disputes standard labor income tax smoothing results. This result holds for difference choices of ϕ_l.

The intuition for the differences between (33) and (34) is as follows: Type l consumer sets the fiscal policy. She increases taxes until her marginal payments equal her marginal revenues. In the **Symmetric** case type h and l marginal payments are affected in the same way, so taxes do not move. In the **Asymmetric Model**, a positive shocks increases type l skill level, so taxes decrease; a negative shocks decreases type l skill level, so taxes increase.

As Hall (1988) has shown, the intertemporal implications of permanent and non-permanent labor income shocks can be very different. Our results show that this is not the case of optimal redistribution. First, we compare the following two functions

$$\tau^s_p(\phi_l, ., .),$$

and

$$\tau^s(\phi_l, ., 1),$$

where p stand for permanent and $\tau_p^s(\phi_l, ., .)$ is the optimal income tax for an **Symmetric** economy where $\theta_t = \theta_0\ \forall t$. Figure 3 plots this two functions when $\phi_l = 0.825$. As before, this result holds for different choices of ϕ_l.

Second, figure 4 reports the results of the same exercise for the **Asymmetric** model, where we compare the following functions

$$\tau_p^{as}(\phi_l, ., .),$$

and

$$\tau^{as}(\phi_l, ., 1),$$

when $\phi_l = 0.825$. As you can observe in both model the optimal tax is almost the same, regardless of the persistence of the aggregate shock. This result also holds for different choices of ϕ_l.

Finally, we consider a set of **Asymmetric models** that, starting with different skill ratio levels, converge to the same ratio and ask whether the initial skill ratio affect the long run tax rate. Hence, we consider four versions of the **Asymmetric model** with index $j \in \{1, 2, 3, 4\}$ where

$$\tau^{as}(\phi_l, ., \theta_0^j)$$

is the policy function associated with economy j and let $\theta_0^1 = 0.5$, $\theta_0^2 = 0.7$, $\theta_0^3 = 0.8$ and $\theta_0^4 = 0.9$. Note that although $\theta_0^i \neq \theta_0^j$ if $i \neq j$, $\lim_{t\to\infty} E_{t-1}\theta_t^i = \lim_{t\to\infty} E_{t-1}\theta_t^j\ \forall i, j$, thus asymptotic skill ratio is equal across the four models.

Figure 5 reports the four policy functions when $\phi_l = 0.825$. As we can see, the lower θ_0, the higher the taxes. Consequently, even assuming labor income inequality convergence, there is not fiscal policy convergence. This is a very important result because shows how initial income distribution affects long run optimal redistribution policy. An interesting extension would be to test if we observe this pattern in the data. We also compute $\lambda^{as}(\phi_l, .)$ for the four economies. Table 2 reports the results. The higher the initial labor income inequality (the lower the θ_0^j), the higher the consumption inequality (the higher λ).

An intertemporal substitution of consumption argument can explain why the lower the initial inequality level, the higher the long run income taxes. Consumer l wants to smooth consumption, so she increases consumption today via long run taxes. The lower the initial skill level the higher taxes she needs tomorrow. In addition, labor supply's elasticity prevents an excessive increase in tomorrow's taxes, so, as table 2 shows, the initial productivity gap across economies cannot be totally offset. As all the other reported results, this one holds for

θ_0^j	$\lambda^{as}(\phi_l, .)$
0.5	2.15
0.7	1.8
0.8	1.6
0.9	1.2

Table 2: Income Distribution Parameter as a Function of Initial Inequality Level, in the Asymmetric Shocks Model

different choices of ϕ_l.

4. Conclusion

This paper uses a heterogeneous agents version of the neoclassical growth model to study the optimal redistribution policy over the business cycle. We conclude that if we follow the empirical evidence in Storesletten, Telmer, and Yaron (2002) and assume labor income inequality to be countercyclical the standard Chari, Christiano, Kehoe (1994) result- i.e. taxes on labor are roughly constant over the business cycle- does not hold and fiscal policy should be countercyclical

As an intermediate step we also characterize the optimal Ramsey allocation in this heterogeneous agent framework. This is an interesting technical contribution that will allow us to study optimal taxation in a wide range of models.

The analysis is done in a complete markets framework, therefore its extension to an incomplete market environment would be an interesting and valuable line of research.

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5. Appendix

5.1. Numerical Algorithm

We am going to describe the method used for the **Symmetric Model**.

We need to solve (24), (25) and (26).

Step 1 Set θ_0.

Step 2 Guess λ and η.

Step 3 Generate one realization, 5000 periods long, of the Markov Chain. Let

$$A = \begin{bmatrix} \theta_0 & \theta_1 & \theta_2 & ... & \theta_{4999} \end{bmatrix}.$$

Step 4 Generate 100 realization, 21 periods long, of the Markov Chain. Let

$$B = \begin{bmatrix} \theta_0 & \widehat{\theta}_1^1 & \widehat{\theta}_2^1 & ... & \widehat{\theta}_{20}^1 \\ \theta_0 & \widehat{\theta}_1^2 & \widehat{\theta}_2^2 & ... & \widehat{\theta}_{20}^2 \\ ... & ... & ... & ... & ... \\ \theta_0 & \widehat{\theta}_1^{100} & \widehat{\theta}_2^{100} & ... & \widehat{\theta}_{20}^{100} \end{bmatrix}.$$

Step 5 Solve, using (25), $c_h(\theta_i; \lambda, \eta)$ for each one of $\theta_i \in A$.

Step 6 Let

$$y_j = \sum_{t=j}^{5000} \beta^j [c_h(\theta_j; \lambda, \eta)^{-\sigma} \frac{\partial c_l(\theta_j; \lambda, \eta)}{\partial \lambda} + x_l(\theta_j; \lambda, \eta)^{-\sigma} \frac{\partial x_l(\theta_j; \lambda, \eta)}{\partial \lambda} +$$

$$+\eta \frac{c_h(\theta_j; \lambda, \eta)^{-\sigma}}{c_h(\theta_0; \lambda, \eta)^{-\sigma}} (\frac{\partial \Phi(\theta_j; \lambda, \eta)}{\partial \lambda} - \frac{\partial \Phi_h(\theta_j; \lambda, \eta)}{\partial \lambda})].$$

Let $Y = \begin{bmatrix} y_1 & y_2 & ... & y_{2500} \end{bmatrix}$. Let $X_1 = \begin{bmatrix} \theta_0 & \theta_2 & ... & \theta_{2499} \end{bmatrix}$. Let $X_2 = \begin{bmatrix} \theta_0^2 & \theta_1^2 & ... & \theta_{2499}^2 \end{bmatrix}$. Using the standard OLS method, estimate the parameters of

$$Y = \mu + \beta_1 X_1 + \beta_2 X_2 + \epsilon.$$

Note that, given these estimations, we can write

$$E_t(y_{t+1}/\theta_t) \simeq \widehat{\mu} + \widehat{\beta}_1 \theta_t + \widehat{\beta}_2 \theta_t^2.$$

Step 7 Repeat the last step for

$$\widetilde{y}_j = \sum_{t=j}^{5000} \beta^j \frac{c_h(\theta_j; \lambda, \eta)^{-\sigma}}{c_h(\theta_0; \lambda, \eta)^{-\sigma}} (\Phi(\theta_j; \lambda, \eta) - \Phi_h(\theta_j; \lambda, \eta)).$$

Let

$$E_t(\widetilde{y}_{t+1}/\theta_t) \simeq \widehat{\pi} + \widehat{\zeta}_1 \theta_t + \widehat{\zeta}_2 \theta_t^2.$$

Step 8 Solve, using (25), $c_h(\widehat{\theta}_i^j, \lambda, \eta)$ for each one of $\widehat{\theta}_i^j \in B$.

Step 9 Check if

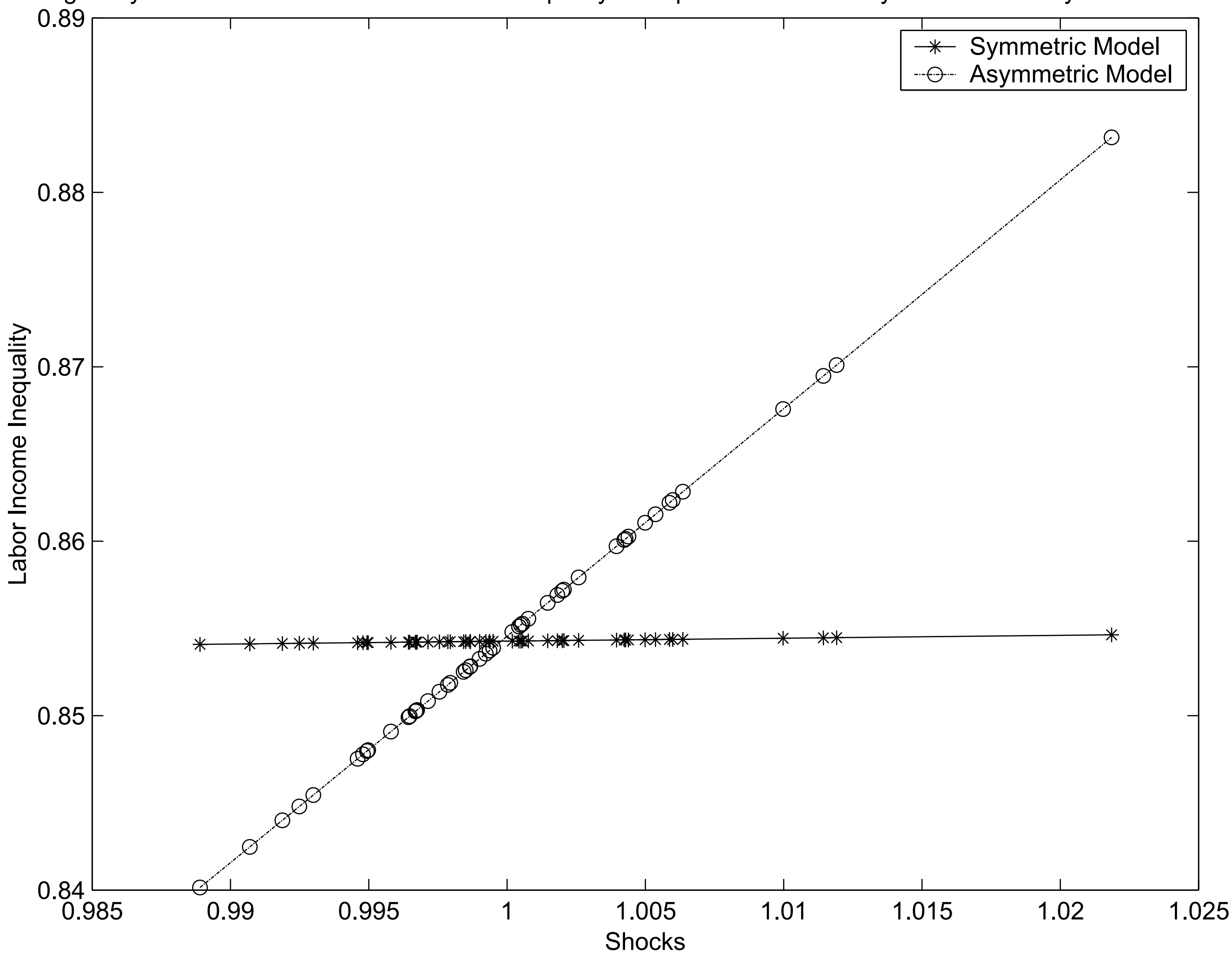
$$\frac{1}{100} \sum_{j=1}^{100} \left(\begin{array}{c} \sum\limits_{i=0}^{19} \beta^i \left(c_h(\widehat{\theta}_i^j; \lambda, \eta)^{-\sigma} \frac{\partial c_l(\widehat{\theta}_i^j;\lambda,\eta)}{\partial \lambda} + x_l(\widehat{\theta}_i^j; \lambda, \eta)^{-\sigma} \frac{\partial x_l(\widehat{\theta}_i^j;\lambda,\eta)}{\partial \lambda} \right) + \\ +\eta \frac{c_h(\widehat{\theta}_i^j;\lambda,\eta)^{-\sigma}}{c_h(\theta_0;\lambda,\eta)^{-\sigma}} \left(\frac{\partial \Phi(\widehat{\theta}_i^j;\lambda,\eta)}{\partial \lambda} - \frac{\partial \Phi_h(\widehat{\theta}_i^j;\lambda,\eta)}{\partial \lambda} \right) + \beta^{20} \left(\widehat{\mu} + \widehat{\beta}_1 \widehat{\theta}_i^{20} + \widehat{\beta}_2 (\widehat{\theta}_i^{20})^2 \right) \end{array} \right) = 0,$$

and

$$\frac{1}{100} \sum_{i=1}^{100} \left(\sum_{j=0}^{19} \beta^i \frac{c_h(\widehat{\theta}_i^j; \lambda, \eta)^{-\sigma}}{c_h(\theta_0; \lambda, \eta)^{-\sigma}} (\Phi(\widehat{\theta}_i^j; \lambda, \eta) - \Phi_h(\widehat{\theta}_i^j; \lambda, \eta)) + \beta \left({}^{20}\widehat{\pi} + \widehat{\zeta}_1 \widehat{\theta}_i^{20} + \widehat{\zeta}_2 (\widehat{\theta}_i^{20})^2 \right) \right) = 0,$$

hold. If it does not, choose new λ and η, and go to step 2 Note that these two equations are approximations to (24), and (26).

Fig 1: Cyclical behaviour of Labor Income Inequality: Comparison between Symmetric and Asymmetric models



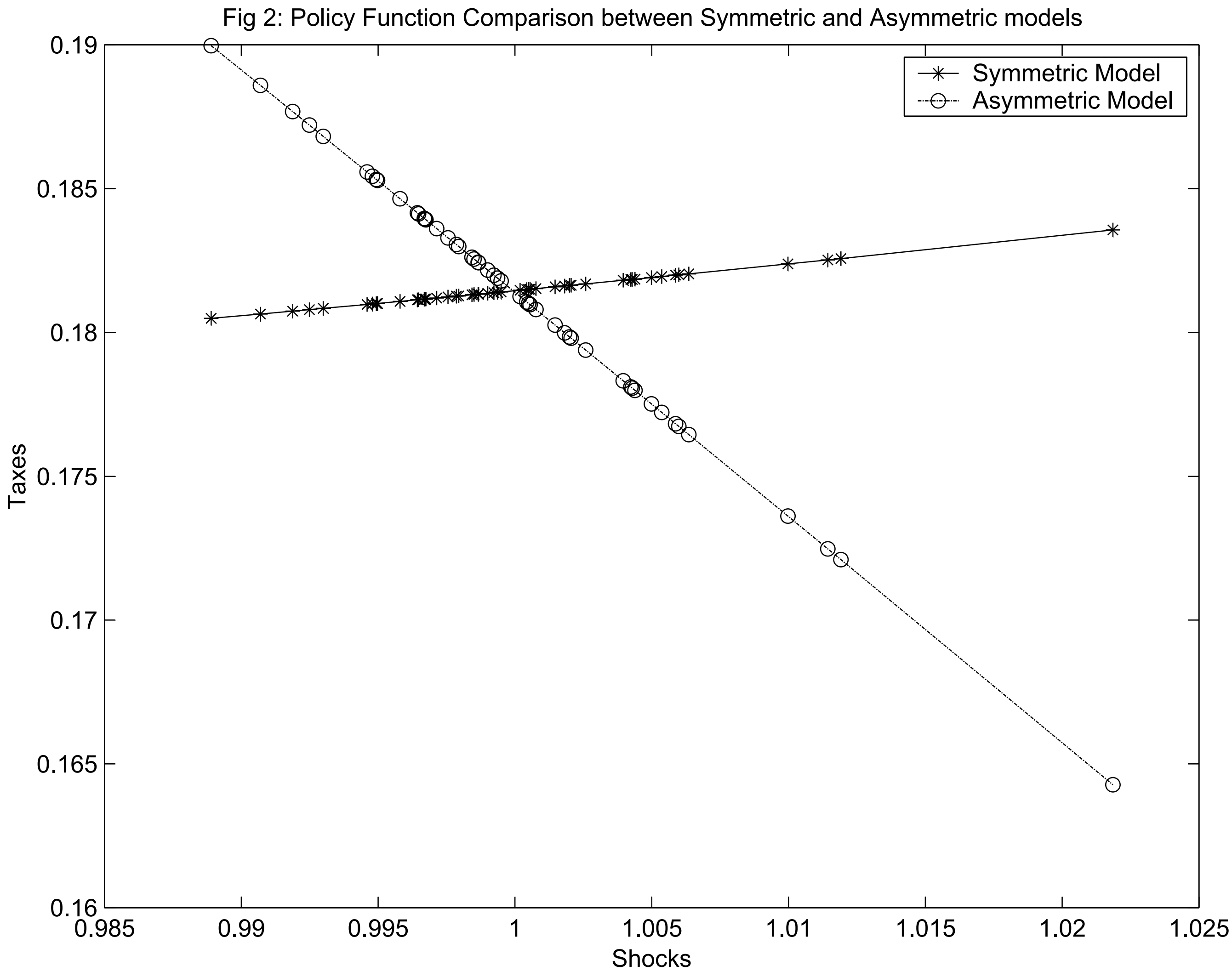


Fig 2: Policy Function Comparison between Symmetric and Asymmetric models
Symmetric Model
Asymmetric Model
Taxes
Shocks
0.19
0.185
0.18
0.175
0.17
0.165
0.16
0.985
0.99
0.995
1
1.005
1.01
1.015
1.02
1.025

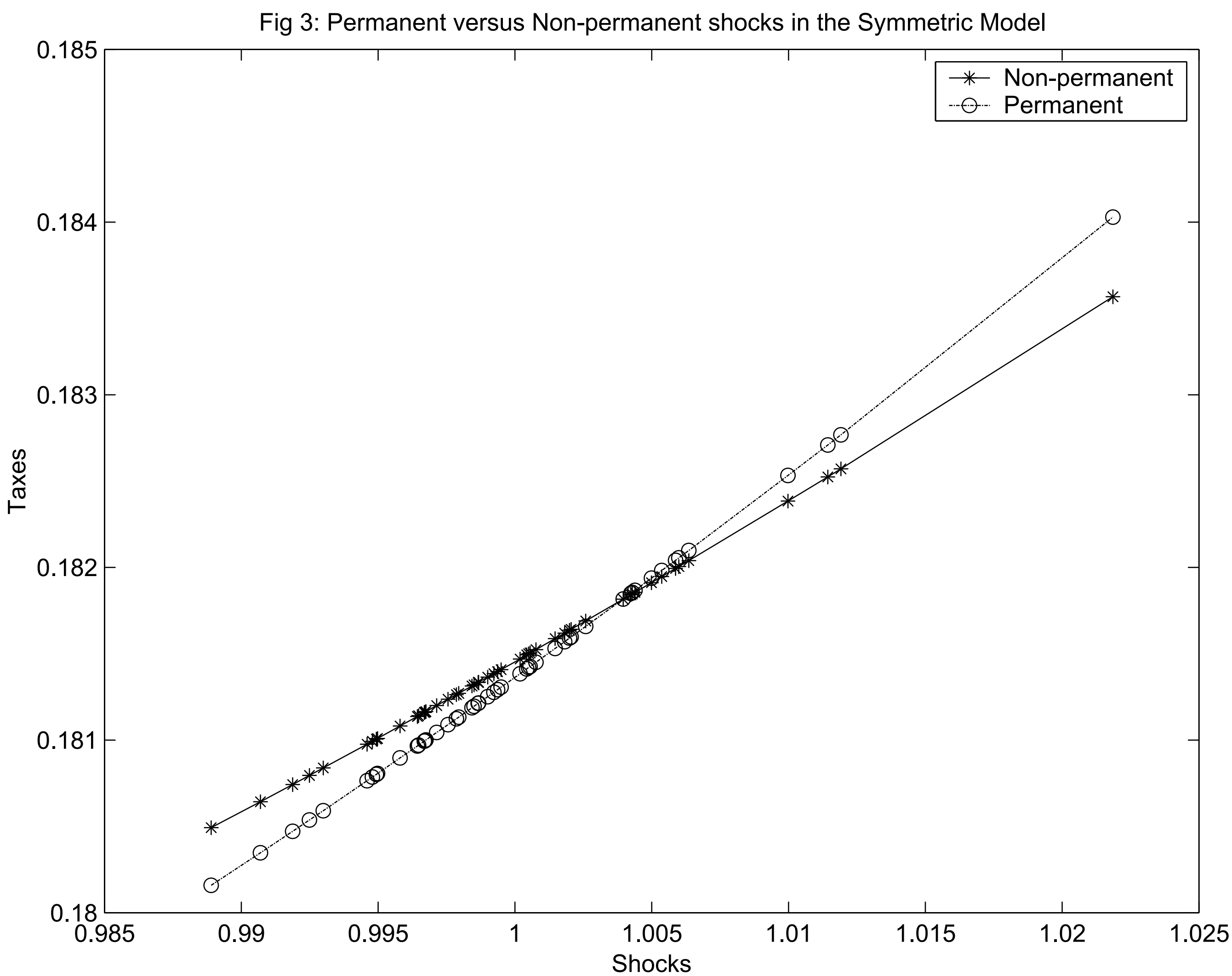


Fig 3: Permanent versus Non-permanent shocks in the Symmetric Model
Non-permanent
Permanent
Taxes
Shocks
0.185
0.184
0.183
0.182
0.181
0.18
0.985
0.99
0.995
1
1.005
1.01
1.015
1.02
1.025

Fig 4: Permanent versus Non-permanent shocks in the Asymmetric Model

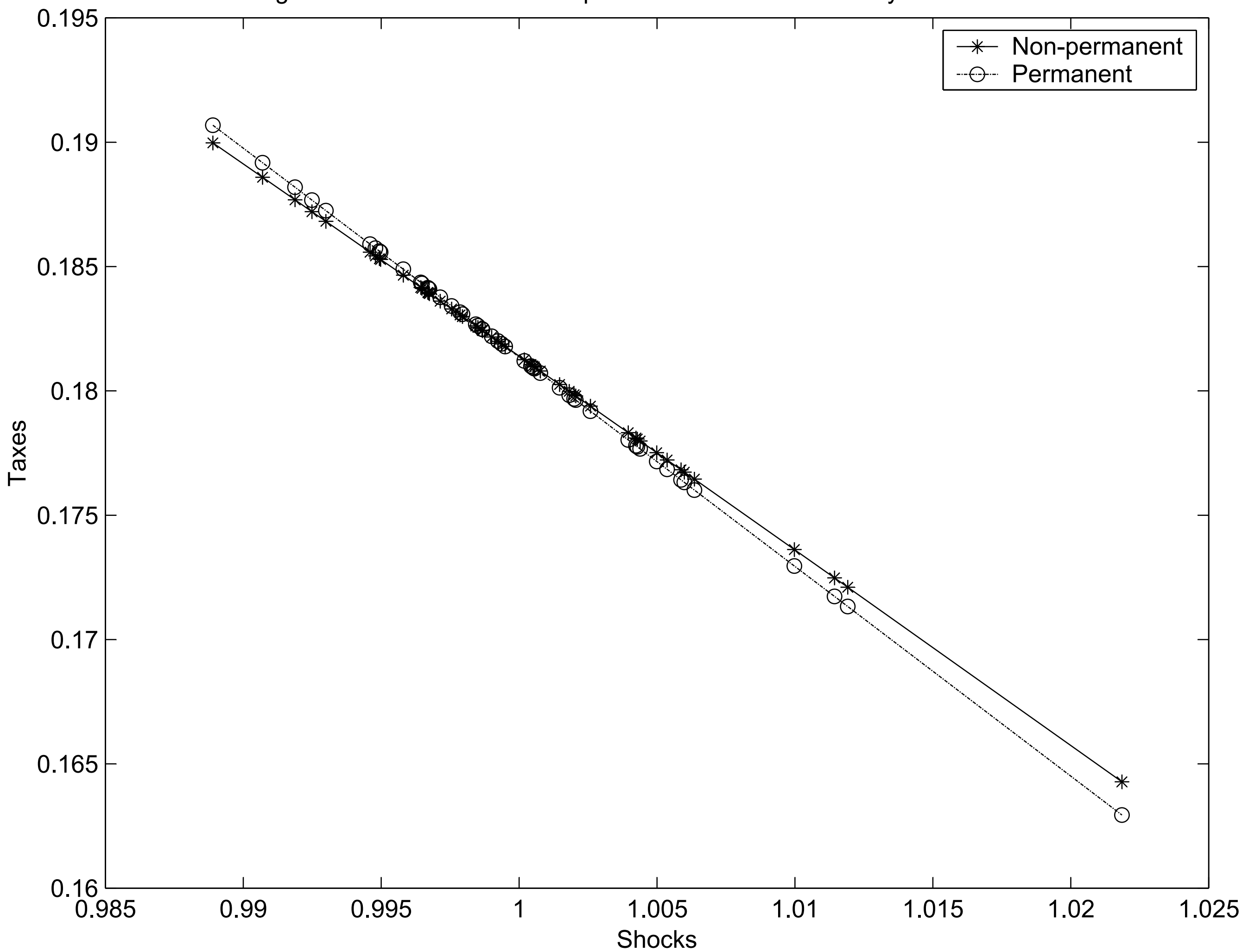


Fig 5: Policy functions for different initial values of the shock in asymmetric model