Political Party Negotiations, Income Distribution, and Endogenous Growth

Roberto Chang

Federal Reserve Bank of Atlanta
Working Paper 95-3
June 1995

Abstract: This paper examines the determination of the rate of growth in an economy in which two political parties, each representing a different social class, negotiate the magnitude and allocation of taxes. Taxes may increase growth if they finance public services but reduce growth when used to redistribute income between classes. The different social classes have different preferences about growth and redistribution. The resulting conflict is resolved through the tax negotiations between the political parties. I use the model to obtain empirical predictions and policy lessons about the relationship between economic growth and income inequality. The model is consistent with the observation that differences in growth rates across countries are negatively related to income inequality. However, government policy cannot simultaneously increase growth and reduce inequality.

JEL Classification: D7, E6, O1

The views expressed here are those of the author and not necessarily those of the Federal Reserve Bank of Atlanta or the Federal Reserve System. This paper originated in conversations with Jess Benhabib and Rao Aiyagari. Early stages of this research were supported by NSF Grant SES-9122454 and by the C.V. Center for Applied Economics at New York University. The author thanks Charles De Bartolome, Ken Judd, Per Krusell, Boyan Jovanovic, Torsten Persson, B. Ravikumar, Anne Sibert, and seminar participants at the Federal Reserve Bank of Minneapolis, the Federal Reserve Bank of Atlanta, Ohio State University, the University of Maryland, the University of Colorado, and the Econometric Society Winter Meetings for comments and suggestions. Any remaining errors are the author's responsibility.

Please address questions of substance to Roberto Chang, Research Department, Federal Reserve Bank of Atlanta, 104 Marietta Street, N.W., Atlanta, Georgia 30303-2713, 404/521-8057.

POLITICAL PARTY NEGOTIATIONS, INCOME DISTRIBUTION, AND ENDOGENOUS GROWTH

1. Introduction

This paper examines the determination of the rate of growth in an economy in which two political parties, each representing a different social class, negotiate the magnitude and allocation of taxes. Taxes may increase growth if they finance public services but reduce growth when used to redistribute income between classes. The different social classes have different preferences about growth and redistribution. The resulting conflict is resolved through the tax negotiations between the political parties. I use the model to obtain empirical predictions and policy lessons about the relationship between economic growth and income inequality.

A main implication of the analysis is that, in equilibrium, differences in growth rates across countries may be negatively related to measures of income inequality. This is consistent with the recent empirical findings of Persson and Tabellini (1994) and Alesina and Rodrik (1994).[1] However, in the model studied below government policy cannot simultaneously reduce inequality and increase growth. This surprising result is possible because growth and income inequality are endogenously determined by the outcome of tax negotiations, which in turn does not depend on the initial allocation of assets. That the empirical relation between income inequality and growth may be a reduced form providing little information about the effects of government policies on growth and inequality is, of course, reminiscent of Lucas's (1976)

---

[1]As discussed below, these papers show that the inverse relation between income inequality and economic growth may be the outcome of the interaction between economics and politics. Alternative explanations can be constructed by assuming capital market imperfections. See, for example, Greenwood and Jovanovic (1990) and Galor and Zeira (1993). For a summary of this line of research, see Aghion and Bolton (1992).

celebrated critique of econometric policy evaluation.

In addition to the above policy message, this paper presents a bargaining approach that may be of interest for students of the positive theory of economic policy. I assume that the political parties negotiate taxes by playing a bargaining game whose structure is similar to that of Rubinstein (1982). A novel aspect of my model is that tax negotiations and private investment may occur simultaneously. This results in a complex but realistic interplay between the behavior of the private sector and the tax negotiations: private investment is affected by the taxes expected to emerge from the negotiations, which in turn depend on the expected behavior of investment.[2] I show how to identify the bargaining outcomes using the concept of sustainable bargaining equilibrium (SBE).[3]

The bargaining approach of this paper contrasts with recent voting models of the determination of public policy. In particular, Persson and Tabellini (1994) and Alesina and Rodrik (1994) attempted to explain the negative correlation between income inequality and growth as the politico-economic equilibrium of an economy in which people vote for taxes.[4] My analysis complements theirs in several dimensions. One is realism: it is often the case that taxes are not the direct result of a popular vote but of a negotiation between representatives of different groups.[5] The second dimension is the

---

[2] A recent example of this kind of interaction is the behavior of business investment in response to the Clinton deficit reduction proposals in 1993. Analysts observed that many firms postponed their investment plans waiting for new business taxes to be agreed upon. At the same time, sluggish investment was believed to be slowing economic recovery, which affected the deficit reduction negotiations.

[3] The SBE concept is the natural extension, to bargaining problems, of the concept of "sustainable plans" developed by Barro and Gordon (1983), Chari and Kehoe (1990), and Stokey (1991). Chang (1995a, 1995b) used the SBE concept to study negotiations about a monetary union and sovereign debt, respectively.

[4] See also Persson and Tabellini (1992) and Perotti (1992).

[5] It may be argued that, although people do not vote for taxes, they vote for

scope of the theory. Although for concreteness I will talk about negotiations between political parties, it should become clear that the model in this paper is applicable to any economy in which government decisions emerge from the consensus between two players that represent different constituencies. Thus the model yields lessons for some kinds of dictatorships, as well as bipartisan democracies. Finally, while the voting models in Alesina and Rodrik (1994) and Persson and Tabellini (1994) imply that "inequality hurts growth," my model lends no support to that conclusion.

A central assumption of this paper is that the political parties have some power to appropriate resources for their respective constituencies. In adopting this view I follow Lancaster (1973), Benhabib and Rustichini (1991) and Tornell and Velasco (1992). But while they model each social class as a single, strategic player, I assume that the private sector is atomistic and behaves competitively. In addition, the mechanisms by which a social class can appropriate resources from the other are different. In my model, each group's appropriating power emanates from the assumptions that a tax agreement requires approval by both parties and that delaying an agreement implies an inefficient status quo situation. In theirs, one of the social groups has the right to directly expropriate resources from others, a power that is limited only to the extent that other groups can choose outside options.

The paper proceeds as follows. Section 2 presents the world in which tax negotiations take place. Section 3 describes the negotiation process and defines the SBE concept. Section 4 provides sufficient conditions for the existence of a stationary SBE and characterizes it. Section 5 examines the implications of the model for empirical issues and policy analysis. Section 6

---

the representatives who decide on taxes. Often, however, the elected representatives must respond to different constituencies and do not share a common view about taxes. This conflict is resolved through bargaining.

concludes. Some technical proofs are delayed to an Appendix.

2. The Model

This section describes the economy under consideration. In the model below, based on Barro (1990), the government provides public services which affect production and the rate of growth of the economy. Public services must be financed with income taxes, which deter investment. My point of departure from Barro's model will be to assume that some or all of the tax revenues can be transferred to a class of agents (called "workers" below), who use these transfers to increase their own consumption. It turns out that workers and capitalists have partly conflicting interests about taxes and the allocation of tax revenues. How such conflict is resolved depends on the political structure and is the subject of later sections.

Although it is possible to state the main points of this paper in a variety of frameworks, Barro's model is convenient for at least two reasons. First, Barro's model includes a direct and intuitive relationship between government policy, including redistributive policy, and economic growth. Second, since Alesina and Rodrik (1994) also used Barro's model, readers may easily compare my analysis against theirs.

Time is discrete and indexed by $t = 0,1,2,\ldots$ I shall consider a closed economy populated by two types of agents: "capitalists" and "workers." Each class has a large number of identical agents. The number of workers as a fraction of the population will be denoted by λ.

The representative capitalist owns, at the start of each period t, an amount k_t of a durable good called "capital." In period t she can produce more capital according to the Cobb Douglas production function:

$$y_t = Ag_t^{\alpha}k_t^{(1-\alpha)}, \tag{2.1}$$

where g_t denotes (per capitalist) government provision of public services at t and $A > 0$, $\alpha \in (0,1)$ are technological parameters. The production function (2.1) incorporates the fact that some government expenditures are important for production. One may think of g_t as infrastructure, police, or fire prevention provided by the government. Under some interpretations, one may want to assume that α is fairly small; I will indeed assume a small α in my numerical exercises later.

I assume that the typical capitalist takes as given the (possibly random) path of the ratio of public services to output, $\theta_t = g_t/y_t$. This formulation is plausible for some public services that are enjoyed by different users in proportion to their respective activities. Alternatively, I could have assumed that the capitalist takes the level of public services, g_t, as given, but such assumption introduces some complications that are peripheral to my discussion.[6]

In period t, the typical capitalist must pay a proportional tax τ_t on her current income and decide how much capital to consume and to leave for the next period. There is no depreciation, and therefore the evolution of capital is given by:

$$k_{t+1} = k_t + (1-\tau_t)\, y_t - c_t = R_t k_t - c_t, \tag{2.2}$$

where c_t denotes the capitalist's consumption and, as implied by (2.1) and the definition of θ_t :

[6]Assuming that the capitalist takes the level of public services as given introduces an externality effect. See Barro (1990).

$$R_t = 1 + (1-\tau_t)\, A^{1/(1-\alpha)}\theta_t^{\alpha/(1-\alpha)}. \qquad (2.3)$$

R_t is the (gross) after tax rate of return on investment. As shown by (2.3), the rate of return in period t is determined by the parameters θ_t and τ_t. A stochastic process $\{(\theta_t,\tau_t)\}_{t=0}^{\infty}$ will be called a <u>fiscal</u> <u>policy</u>.[7]

I assume that the capitalist's preferences are described by:

$$E\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right] = E\left[\sum_{t=0}^{\infty}\beta^t \frac{c_t^{1-\sigma}}{1-\sigma}\right], \qquad (2.4)$$

where $0 < \beta < 1$ and $\sigma > 0$, $\sigma \neq 1$ and E(.) denotes expectation. The representative capitalist's problem is to maximize (2.4) subject to (2.2) and (2.3), given a fiscal policy and the initial quantity of capital.

The rest of the economy is specified to make the analysis as simple as possible. In each period, the government transfers the difference between tax revenues and government expenditures to the workers.[8] Workers do not have another source of income[9] and cannot borrow or lend; both assumptions can be relaxed at the expense of heavier calculations. Hence each worker's transfer and consumption is given by:

$$\begin{aligned} c_t^w &= (1-\lambda)(\tau_t y_t - g_t)/\lambda \\ &= [1 + A^{1/(1-\alpha)}(\theta_t^{\alpha/(1-\alpha)} - \theta_t^{1/(1-\alpha)}) - R_t]\,(1-\lambda)k_t/\lambda, \end{aligned} \qquad (2.5)$$

where the last equality is easily deduced from (2.1)-(2.3).

---

[7]Hereafter I will assume that $\tau \in [0,1]$ and $0 \le \theta \le \tau$.

[8]Hence the budget is balanced in each period.

[9]Hence workers do not "work." This is mainly for computational reasons. I could include labor in the production function, as in Alesina and Rodrik (1994), but the algebra would become much more tedious.

Given a fiscal policy, workers' lifetime consumption is defined by (2.3), (2.5), and the evolution of k_t determined by the behavior of capitalists. I will assume, for simplicity, that workers' preferences are linear and given by $E \sum_{t=0}^{\infty} \delta^t c_t^w$.[10] Note that, for this sum to converge, δ has to be small enough relative to the rate of growth of consumption. Such restriction will always be satisfied below.

The evolution of this economy will, from the preceding description, depend on fiscal policy. But nothing so far reveals what fiscal policy will prevail. What is clear is that workers and capitalists have partly conflicting interests about fiscal policies. Capitalists benefit from government services but are hurt by taxes. It is intuitively obvious that their most preferred fiscal policy involves some positive level of government services and taxes but zero transfers to workers. Workers benefit from transfers and therefore would like taxes to be strictly larger than the amount needed to finance government services but small enough not to cause too large a fall in production and investment.

Later I will assume that this conflict is resolved by a negotiation between two political parties, each representing a social class. The political parties will bargain over tax agreements. If implemented, an agreement specifies a fiscal policy for the rest of time. For simplicity, I will restrict attention to constant agreements, that is, agreements that specify a constant θ and a constant τ. In the rest of this section I will state and discuss some facts needed later.

Suppose that a constant fiscal policy (θ,τ) is implemented without delay, starting in period zero. Associated with this policy there is a perfect

---

[10] One reason for assuming linear utility is that, below, I want to assume that c_t^w may be zero. This would be less easy to handle if workers had a CRRA utility function with intertemporal elasticity greater than one.

foresight equilibrium of this economy, whose main features are described by:[11]

Fact One: Let $\theta_t = \theta$ and $\tau_t = \tau$ be such that $\beta R^{1-\sigma} < 1$ and $\delta(\beta R)^{1/\sigma} < 1$, where $R = 1+(1-\tau)A^{1/(1-\alpha)}\theta^{\alpha/(1-\alpha)}$ is the equilibrium rate of interest. Also, let $k_0 = k$. Then the discounted utility of the representative capitalist is given by:

$$v(k,\theta,\tau) = \Psi(R)\,\frac{k^{1-\sigma}}{1-\sigma}\,, \tag{2.6a}$$

where:

$$\Psi(x) = x^{1-\sigma}\,\{\,1 - (\beta x^{1-\sigma})^{1/\sigma}\,\}^{-\sigma}\,. \tag{2.6b}$$

The utility of workers is:

$$w(k,\theta,\tau) = \frac{(1 + A^{1/(1-\alpha)}(\theta^{\alpha/(1-\alpha)} - \theta^{1/(1-\alpha)}) - R)}{1 - \delta\ (\beta R)^{1/\sigma}}\,[\ (1-\lambda)k/\lambda]\,. \tag{2.7}$$

Moreover, the rate of growth of the economy is given by $k_{t+1}/k_t = (\beta R)^{1/\sigma}$. ∎

Some remarks are in order. First, Fact One will be useful in describing the discounted payoffs to workers and capitalists of an immediate tax agreement (θ,τ) when the stock of capital is k. In particular, because this economy is recursive, $v(k,\theta,\tau)$ and $w(k,\theta,\tau)$ are the payoffs to workers and capitalists, from any period t on and discounted to period t, of an agreement to set $\theta_s = \theta$ and $\tau_s = \tau$ for $s \geq t$, if the stock of capital at t is $k_t = k$.

Second, this economy displays unbounded growth provided θ and τ are such

[11] The proof of Fact One follows easily from Barro (1990) and is left to the reader.

that $\beta R > 1$. Thus the economy may exhibit long-run growth, but whether this is actually the case depends on fiscal policy.

Third, the existence of a perfect foresight equilibrium requires v and w to be finite. This is the reason for requiring $\beta R^{1-\sigma} < 1$ and $\delta(\beta R)^{1/\sigma} < 1$ in Fact One. Note that the first requirement is satisfied if $R > 1$ and $\sigma > 1$ and the second is satisfied if $\delta(\beta R^{*})^{1/\sigma} < 1$, where R^{*} is defined below.

Fourth, what would the median voter theorem tell us about this economy? Suppose that at the beginning of time there was a vote to pick a constant fiscal policy. Then, clearly, the resulting policy would maximize the utility of the representative capitalist if there are more capitalists than workers, that is, if $\lambda < 0.5$. As in Barro (1990), it is easy to show that the capitalist's most preferred policy is given by $\theta = \tau = \alpha$. The condition $\theta = \alpha$ is a condition of productive efficiency,[12] while $\tau = \alpha$ says that taxes are just enough to finance government expenditure and that no resources are transferred to workers. It is also easily checked that such policy maximizes the rate of growth in this economy and that the associated interest rate is given by $R^{*} \equiv 1 + A^{1/(1-\alpha)}(\alpha^{\alpha/(1-\alpha)} - \alpha^{1/(1-\alpha)})$.

On the other hand, if $\lambda > 0.5$, the winning policy would maximize the utility of workers, $w(k,\theta,\tau)$. Using Fact One, it can be shown that the workers' most preferred policy requires the productive efficiency condition $\theta = \alpha$. Then, by (2.7), the workers' most preferred policy requires picking τ such that the interest rate is $R_{*} = 1 + (1-\tau)\, A^{1/(1-\alpha)}\alpha^{\alpha/(1-\alpha)}$, where R_{*} is the solution of:[13]

[12] Efficiency requires that the government share, θ, be equal to the share that would obtain if government services were a competitively provided input (Barro 1990, page S109).

[13] If (2.8) has many solutions, we take the largest one.

$$\mathrm{Max}_{R\in[1,R^*]} \quad \frac{R^* - R}{1-\delta(\beta R)^{1/\sigma}} \quad . \tag{2.8}$$

Since (2.8) implies that $R_* < R^*$, it follows that the policy most preferred by workers would not maximize growth. In fact, $R_* = 1$ for the parameterizations studied later, which implies that workers would be willing to sacrifice economic growth for bigger transfers.

Finally, consider agreements (θ,τ) that satisfy the productive efficiency condition $\theta = \alpha$ and the condition $R = 1+ (1-\tau)\ A^{1/(1-\alpha)}\alpha^{\alpha/(1-\alpha)} \in [R_*\ ,\ R^*]$. It can be shown (and it should be evident from the preceding paragraphs) that any agreement satisfying both conditions is <u>constrained</u>[14] <u>efficient</u> in the sense that there cannot be another (constant) agreement that makes both workers and capitalists better off.

Fact One summarizes the essential facts of the economy under study, assuming that a tax agreement (θ,τ) is implemented at the beginning of time. What happens, however, if there is no immediate agreement? Below, I shall assume that parties will keep negotiating taxes until they reach an agreement, or forever. Also, for all agents to have an incentive to reach agreement quickly, I will assume that absence of agreement implies a loss of potential output: as long as there is no agreement about taxes, τ and θ are both zero. Therefore, in the absence of a tax agreement, workers do not consume and the economy does not grow (because production possibilities are given by $k_{t+1} = k_t - c_t$, where c_t denotes capitalists' consumption).[15]

There is one final aspect to take into account. In the absence of a tax

---

[14]"Constrained" because lump sum taxes are not allowed.

[15]More generally, one may assume that there are $\hat{\tau}$ and $\hat{\theta}$, given by history, that prevail until an agreement is reached. For all parties to have an incentive to agree quickly, τ and θ must be inefficient. Assuming that they are zero simplifies calculations and notation considerably.

agreement, capitalists must decide how much to consume and invest based, presumably, on their expectations about current and future rates of return. But the future rate of return, and therefore private investment, may depend on when a tax agreement will be reached and what the agreement will be. For instance, suppose that there is no agreement at $t = 0$ but that it is common knowledge that agreement will be reached in period $t = 1$. Then, the consumption-saving plans of capitalists in period zero will depend on their expectations about the fiscal policy (θ', τ') that will be agreed upon for $t = 1$ on. My model will have to take into account the effect of the tax negotiations on investment behavior.

Summarizing, in this economy different tax agreements imply different rates of growth and different degrees of income redistribution. Capitalists would like to choose taxes so as to maximize growth and minimize redistribution. Workers would prefer slower growth but some redistribution. Both sides may benefit from a tax agreement because there is no production in its absence.

The fiscal policy and, therefore, the growth rate and the degree of income inequality that would be observed in this economy depend on political institutions. We have already indicated what fiscal policy would be chosen by a popular vote; likewise, one could examine what policy would be chosen by a dictator. Our objective in the next sections will be to analyze the determination of fiscal policy and economic equilibrium in a bipartisan government.

3. The Bargaining Problem and a Definition of Equilibrium

In the remaining sections I will assume that the government is controlled by two parties called L (representing workers) and C (representing capitalists). These parties will be assumed to exchange offers and counter-offers over time about fiscal policy. Implementation of a particular proposal requires the consent of both parties. The main objective of this section is to describe the bargaining mechanism and to discuss how to characterize its outcomes.

The bargaining procedure is as follows. Let party L represent workers and party C capitalists. At $t = 0$, with k_0 given, one of the parties is chosen randomly (in a way to be specified below) to make an offer to the other. After the offering party has chosen an offer, say $a_0 = (\theta_0, \tau_0)$, the responding party may then accept the offer (Y) or reject it (N). If a_0 is accepted, bargaining ends, and a (constant) fiscal policy $\theta_t = \theta^0$, $\tau_t = \tau^0$, $t \geq 0$, is immediately implemented.

If the offer is rejected, bargaining continues at $t = 1$. During the remainder of period zero, however, capitalists decide how much to consume (c_0) or to invest (k_1). This decision determines the amount of capital k_1 at $t = 1$. Then, one of the players is chosen randomly to make an offer $a_1 = (\theta^1, \tau^1)$. If the offer is accepted, bargaining ends, and the agreement a_1 is implemented. If the offer is rejected, capitalists decide their consumption (c_1) and investment (k_2). Period two then starts with capital k_2 and one of the parties being chosen randomly to make an offer. This process continues until an offer is accepted, or forever.[16] Figure 1 depicts period t of the model.[17]

To complete the description of the bargaining procedure, I need to describe the process by which a party is chosen to make an offer in each

---

[16]Note that this statement of the model assumes that agreements are final: after an offer is accepted, the corresponding tax agreement is written in stone. However, this aspect of the model is not crucial, as I discuss at the end of Section 4.

[17]Figures and tables can be found at the end of the paper.

period. I will simply assume that, in each period t, the L party is chosen with probability λ to make an offer. One story that makes this assumption plausible is that government negotiations take place in a "Senate." In each round of the negotiations, one of the senators is randomly given the Senate floor to propose a tax package. The number of senators per party reflects the structure of the population.

This bargaining model makes some realistic and concrete assumptions about the environment. Individuals do not vote directly on fiscal policy; rather, their interests are represented in the government by the political parties. Within the government, there is a need for consensus among the two parties in order to change the tax regime. The need for consensus grants workers a degree of power that may not be apparent from the economic aspects of the model: the workers' party can veto tax proposals and cause damage to both sides (because there is no production in the absence of an agreement). Using this veto power, workers may try to appropriate some of the benefits of economic growth. But the workers' power is limited by the fact that their vetoing tax proposals hurts not only capitalists but also themselves.

The model also assumes that the "voice" of each social group in the government becomes stronger the greater the group's size. This is captured by the assumption that λ, the relative number of workers, is also equal to the probability that the workers' party makes offers. Hence the model implies that the composition of the population is an important factor for the determination of public policy (although, in contrast with voting models, it is not the only factor).

Finally, the model is constructed so that tax negotiations occur in "real time." This means, in particular, that market activity does not wait until an agreement has been reached: consumption, production, and investment take place while the parties negotiate.

The bargaining procedure is similar to Binmore's (1987) version of the celebrated Rubinstein (1982) bargaining model. There is an important difference, however. Rubinstein and Binmore analyzed a game between two players. In my model, there are two strategic players (the political parties) and a large number of competitive agents (workers and capitalists). Capitalists, in particular, have to decide how much to save in each period as long as there is no tax agreement, based on their expectations about the future of the tax negotiations, which determine the expected return on capital. Conversely, private investment determines the evolution of capital and, therefore, the bargaining stakes every period. Thus there is an interplay between private investment and tax negotiations that is absent from models of the Rubinstein type. This interplay adds realism to the model but adds complexity also. In particular, how to characterize the outcomes of the model is not obvious.

In Chang (1995a, 1995b) I have argued that an appropriate equilibrium concept to characterize the solution of this kind of problem is that of <u>sustainable bargaining equilibrium</u> (SBE). In fact, this bargaining model is similar to the models in those two papers and therefore I can adapt the tools developed there to solve the present problem. In the remainder of this section I will provide an intuitive description of the SBE concept.[18] Readers familiar with the arguments of Chang (1995a, 1995b) may want to go directly to Section 4.

A <u>history</u> at t, denoted by h^t, is a description of the evolution of the world up to and including period t; in particular, h^t describes which party was chosen to make an offer and what offer was made in each period $s = 0,1,..,t$. An <u>allocation rule</u> F is a description of capitalists' consumption

---

[18] All the concepts below can be formalized in a precise way. See Chang (1995a, 1995b) for details in a similar context.

(c_t) and investment (k_{t+1}) in each period t as a function of history h^t. Intuitively, an allocation rule tells us the behavior of the private sector if no agreement has been reached up to and including period t.

For j = L,C, a <u>strategy for player j</u>, denoted S_j, is a description of what offer to make (if it is j's turn to offer) and which offers to accept (if j has to respond to an offer) in every period after any history.

Given k_0, an allocation rule F and a strategy pair $S = (S_L, S_C)$ determine the expected payoff to each party. A strategy pair S is a <u>Nash equilibrium</u> if S_L maximizes L's expected payoff given S_C, F, and k_0, and vice versa. A Nash equilibrium is <u>subgame perfect</u> if its continuation is a Nash equilibrium after any history.

To complete the definition of SBE, I need to pose natural restrictions on equilibrium allocation rules. Consider any period t after history h^t. The representative capitalist has capital $k_t(h^{t-1})$ and has to decide how much to consume and invest in period t. Suppose that a strategy pair S is given. Then the continuation of S after h^t, call it $S|h^t$, induces a distribution over future bargaining outcomes, that is, which agreement will be reached and when. For the allocation rule F to be consistent with optimizing behavior, the behavior that F prescribes after history h^t, $c_t(h^t)$ and $k_{t+1}(h^t)$, must maximize the expected utility of a capitalist that starts period t with capital $k_t(h^{t-1})$, given the distribution over the bargaining outcomes induced by $S|h^t$. If the allocation rule F satisfies this requirement, I will say that F is <u>competitive</u> given S.

A <u>sustainable bargaining equilibrium</u> is an allocation rule F and a strategy pair S such that F is competitive given S and, given F, S is a subgame perfect Nash equilibrium. By construction, an SBE implies that, given the behavior of the private sector, the two parties' strategies are optimal against each other after any history. Conversely, given the parties' strategies, the behavior of the private sector is consistent with a perfect foresight equilibrium, also after every history. Thus an SBE is a natural concept based on the assumption that each agent behaves optimally and rationally after any contingency.

The rest of the paper characterizes the outcomes of my bargaining model by its SBEs and studies empirical and policy implications.

4. Existence and Characterization of a Sustainable Bargaining Equilibrium

This section provides sufficient conditions for the existence of Pareto optimal (PO) stationary SBEs. The main result is that these SBEs solve a pair of normal equations. Although the normal equations are highly nonlinear, they can be analyzed numerically for different values of the underlying parameters of the economy. Such calculation is performed in Section 5.

The SBEs studied in this section will be stationary in the sense that the players' strategies and the savings rate implied by the allocation rule will be independent of previous history. The SBEs of this section will also be Pareto optimal in implying that agreement is reached without delay after any history.[19]

Focusing on PO stationary SBEs is justified on several grounds. First, they are important per se: one may want to restrict attention to stationary equilibria because they depend on history only through the physically relevant aspects of the environment, in this case the amount of capital. Second, they are relatively easy to compute, which not only allows us to study them but also lends them plausibility as an equilibrium concept. Finally, I conjecture that PO stationary SBEs are the only SBEs in this model, at least for the

---

[19] Note that below I do not restrict strategies to be Pareto optimal. Pareto optimality is postulated to be a property of the outcome of the bargaining.

parameters studied in Section 5. Although I do not have a proof that the normal equations below have a unique solution, in the numerical analysis of Section 5 I was unable to find multiple solutions. This, plus the assumption of complete information, indicates that uniqueness of SBEs is a likely possibility.

To derive candidate strategies and allocation rules, suppose that there is a PO stationary SBE. Stationarity of the SBE strategies implies that L offers $a_0 = (\theta_0, \tau_0)$ whenever L is selected to make an offer and that C offers $a_1 = (\theta_1, \tau_1)$ if C is selected. Pareto optimality implies that agreement is immediate. To proceed further, note that θ_0 and θ_1 must be equal to α, that is, Pareto optimality and stationarity imply that fiscal outcomes must be constrained efficient (in the sense of Section 2).[20] This fact and the fact that agreement is immediate then imply that, after any history, there must be an agreement with an interest rate given by:

$$x = 1 + (1-\tau_0)\, A^{1/(1-\alpha)} \alpha^{\alpha/(1-\alpha)}\ , \tag{4.1}$$

$$y = 1 + (1-\tau_1)\, A^{1/(1-\alpha)} \alpha^{\alpha/(1-\alpha)}\ , \tag{4.2}$$

whenever the L party or the C party, respectively, makes an offer.

What allocation rules can be competitive, given these strategies? The postulated strategies imply that, after any history h^t, an agreement will certainly be reached in period (t+1) and that the agreed upon policy will be a_0 if L is chosen to make an offer and a_1 if C is chosen. Fix any h^t and k_t,

---

[20] Suppose, for instance, that L's offer is $a_0 = (\theta_0, \tau_0)$, where $\theta_0 \neq \alpha$. Then, since a_0 is not constrained efficient, there must be an offer (α, τ') that, if accepted, gives C at least as much utility as a_0 while giving L strictly more utility. But this implies that offering a_0 cannot be optimal for L.

and recall that the probability that the L party makes an offer is equal to λ. The typical capitalist's problem is then to choose c_t and k_{t+1} to maximize her expected utility, that is, $u(c_t) + \beta [\lambda v(k_{t+1}, a_0) + (1-\lambda) v(k_{t+1}, a_1)]$, with v given by (2.6). The solution of this problem is easily shown to be:

$$k_{t+1} = [\zeta(x,y)^{1/\sigma} / (1+ \zeta(x,y)^{1/\sigma})] k_t \tag{4.3a}$$

and $c_t = k_t - k_{t+1}$, where x and y are defined in (4.1) and (4.2),

$$\zeta(x,y) = \beta [\lambda \Psi(x) + (1-\lambda) \Psi(y)] \tag{4.3b}$$

and Ψ is given by (2.6b).

It remains to show that, given the allocation rule (4.3), the postulated strategies are a subgame perfect Nash equilibrium. As in Rubinstein (1982), this will be shown to be the case if in each period the proposer makes an offer that leaves the responder indifferent between accepting or rejecting the offer. This condition means that when it is C's turn to answer to an offer a_0, the value to C of accepting the offer, $v(k_t,a_0)$, must equal C's value of rejecting it, $u(c_t) + \beta [\lambda v(k_{t+1}, a_0) + (1-\lambda) v(k_{t+1}, a_1)]$, with c_t and k_{t+1} determined by the allocation rule (4.3), and v given by Fact One. After some simplification and using the definition of x and y, it turns out that C is indifferent between taking and rejecting a_0 if and only if:

$$\Psi(x) = [1 + (\beta(\lambda \Psi(x) + (1-\lambda) \Psi(y)))^{1/\sigma}]^{\sigma}. \tag{4.4}$$

Similarly, L is indifferent between accepting or rejecting a_1 when it is L's turn to respond if and only if $w(k_t, a_1) = \delta [\lambda w(k_{t+1}, a_0) + (1-\lambda) w(k_{t+1}, a_1)]$ which, using (2.7), (4.1), (4.2) and (4.3), reduces to:

$$\gamma(y) = \delta \; [\; \lambda \; \gamma(x) + (1-\lambda) \; \gamma(y) \;] \quad \zeta(x,y)^{1/\sigma} / \; (1 + \zeta(x,y)^{1/\sigma}) \; , \qquad (4.5a)$$

where ζ is given by (4.3b) and:

$$\gamma(x) = \frac{R^* - x}{1 - \delta(\beta x)^{1/\sigma}} \; . \qquad (4.5b)$$

The two equations (4.4) and (4.5) are crucial to characterize the SBE postulated here. Following Rubinstein (1982), I will call (4.4) and (4.5) the <u>normal equations.</u>

The following theorem shows that if x and y satisfy the normal equations, are constrained efficient outcomes, and are not too large, there is an SBE with the anticipated properties:

<u>Proposition 1:</u> Suppose x and y belong to $[R_*, R^*]$ and satisfy the normal equations. Also, assume that $\beta x^{1-\sigma}$, $\beta y^{1-\sigma}$, $\delta(\beta x)^{1/\sigma}$, and $\delta(\beta y)^{1/\sigma}$ are all less than one. Then the following is a sustainable bargaining equilibrium:

Allocation rule F: defined by (4.3) above.

Strategy for L: Offer $a_0 = (\alpha, \tau_0)$ when selected to make an offer, where τ_0 satisfies (4.1); when selected to answer to an offer, accept any offer a such that $w(k_t,a) \geq \delta \; [\; \lambda \; w(k_{t+1}, \; a_0) + (1-\lambda) \; w(k_{t+1}, \; a_1) \;]$.

Strategy for C: Offer $a_1 = (\alpha, \tau_1)$ when selected to make an offer, where τ_1 satisfies (4.2); when selected to answer to an offer, accept any offer a such that $v(k_t,a) \geq u(c_t) + \beta \; [\; \lambda \; v(k_{t+1}, \; a_0) + (1-\lambda) \; v(k_{t+1}, \; a_1) \;]$.█

The proof is in the Appendix. The usefulness of Proposition 1 is that, under slightly milder restrictions, one can show that a solution to the normal equations exists and identifies an SBE:

<u>Proposition 2:</u> Assume that $R_* = 1$ and that $\sigma > 1$. Then the normal equations (4.4) and (4.5) have a solution (x,y) in $[1, R^*]^2$.▮

The proof, an application of Brouwer's Fixed Point Theorem, is also in the Appendix.

Joining Proposition 1 and Proposition 2 we obtain the following:

<u>Corollary:</u> If $\delta(\beta R^*)^{1/\sigma} < 1$, $R_* = 1$, and $\sigma > 1$, then there is an SBE characterized by the solutions of the normal equations.

In the SBE of the Corollary, bargaining stops in the first period, and the interest rate agreed upon is x if L is chosen to make an offer in the first period and y if C moves first. The outcome is Pareto optimal, as anticipated.

The Corollary suggests how one may study the set of SBEs discussed in this section: if one chooses parameters such that $\sigma > 1$, $R_* = 1$ and $\delta(\beta R^*)^{1/\sigma} < 1$, then finding an SBE corresponding to these parameters reduces to finding a solution of the normal equations. This is the procedure used in the next section.

The Corollary ensures existence but not uniqueness. However, in the numerical exercises of the next section I was unable to find multiple solutions of the normal equations. Note also that one should expect that the normal equations have isolated solutions.

Before leaving this section, note that the SBEs characterized in this section are independent of the initial quantity of capital k_0. This fact is a result of our assumption about functional forms and has many consequences. One of them is that we can allow for renegotiation after an initial agreement without changing the results. Suppose that the original model has a unique SBE, and consider a modified model in which any agreement can be broken in any period. Assume that, if an agreement is broken in period t, new tax negotiations start in period (t+1) and that taxes are set to zero again until there is a new agreement. Then it should be intuitively clear that, in the modified model, the parties agree on the SBE of the original model and there is never an incentive for either party to break this agreement.

5. Empirical Implications and Policy Issues

This section discusses some of the implications of the model for empirical analysis and policy formulation. Subsection 5.1 analyzes the empirical properties of the model; in particular, it is shown that the model is capable of generating a negative observable relation between economic growth and income inequality. Subsection 5.2 asks whether such empirical relation implies that public policy can simultaneously increase growth and reduce inequality. The answer, unexpectedly, turns out to be negative.

5.1 Explaining the Cross-Country Relation between Income Inequality and Growth

I start by analyzing the dependence of the set of SBEs discussed in the previous section to changes in the underlying parameters of the economy. I proceed as follows: I choose an empirically plausible benchmark set of parameters for which the Corollary of the last section applies. Then I find an associated SBE by solving the normal equations and analyze how it is affected by changes in each of the parameters of the model. The objective of this procedure is to examine what one would observe in a cross-section sample of countries which may differ in their fundamental parameters .

In this discussion, I proceed by solving the normal equations numerically. An alternative procedure would have been to find an SBE and then to use the Implicit Function Theorem to study its properties analytically. However, since the normal equations are highly nonlinear, the analytic procedure becomes messy very quickly and yields little insight.

The parameters of the model are the inverse of the elasticity of intertemporal substitution σ, the discount factors of workers and capitalists δ and β, the parameters of the production function A and α, and the relative number of workers λ. Based on studies of consumer behavior, an initial choice of σ equal to 2 seems plausible. As for the parameter α of the production function (2.1), note that in an SBE the efficiency condition $\alpha = \theta = g/y$ holds, which suggests that α can be chosen from National Income Accounts. For the United States, government purchases of goods and services are in the order of 20 percent of GNP. However, one may argue that that figure includes a lot of "unproductive" government spending, while in the model g stands for "productive" public services. Given this measurement problem and the discussion in Section 2, I conservatively choose $\alpha = 0.1$ for the benchmark. I choose $\delta = 0.9$, $\beta = 0.95$, and $A = 0.25$. In choosing these values I assume that the relevant period is a year; accommodating a different period length is easy, by varying δ, β, and A. Finally, the benchmark value of λ is set at 0.5. This is mostly for simplicity: the results are essentially the same for any λ between zero and one.

With these benchmark values, one can solve the normal equations [21] and find that the corresponding solutions of x and y are 1.099 and 1.117. That is,

---

[21]Here I solve the normal equations numerically with the help of MATHEMATICA nonlinear equation procedure. The MATHEMATICA program that performs the calculations is available on request.

these parameters imply that, in equilibrium, the average rate of return on capital is around 11 percent. The implied growth rate of output can be calculated as $(\beta x)^{1/\sigma}$ or $(\beta y)^{1/\sigma}$ depending on who starts the negotiation; for this discussion, I report the expected growth rate, $\lambda(\beta x)^{1/\sigma} + (1-\lambda)(\beta y)^{1/\sigma}$. Expected growth in the benchmark case is 1.026. That is, these parameters predict that GNP would grow at a rate of 2.6 percent per year.[22] Note that tax negotiations may have a sizable effect on the rate of growth of the economy: the benchmark parameters imply that the economy's maximum possible rate of growth is about 4.5 percent, but in equilibrium the growth rate is less than 3 percent.

Each particular SBE determines the flows of income earned by capitalists and workers, and therefore the extent of income inequality. For our discussion, income distribution will be summarized by the expected value of the ratio of workers' income to total income in this economy, denoted by "L share:"

$$\text{L share} = \lambda c^{W}/(1-\lambda)y = (\tau y - g)/y = (R^{*}-R)/(\alpha^{\alpha/(1-\alpha)}) .$$

For the benchmark parameters, L share is equal to 0.055.

In this model, L share is a useful number because it corresponds to different income distribution concepts. L share is a measure of the size distribution of income as well as of the functional distribution: L share gives the percentage of national income that goes to the poorest (λ x 100) percentage of the population. In my computations L share is always less than λ, and hence an increase in L share implies a more egalitarian income

---

[22]For these benchmark parameters, one can calculate that $R = 1.183$ and $R_{*} = 1$; the conditions of the Corollary are then satisfied, and the values of x and y correspond in fact to an SBE.

distribution.

Readers may correctly note that L share is, strictly speaking, a measure of the distribution of income after tax, while some empirical studies (such as Persson and Tabellini 1994) use before-tax measures. How much my arguments may be affected by this discrepancy depends on the empirical interpretation of the model's "transfers." My view is that many governments give transfers that show up as income for some social groups. For example, populist governments in Latin America used to reward their constituents by hiring them for unproductive jobs in the public sector. In cases like these, before-tax income distribution measures would include a transfers component.

One can repeat the calculations described in the preceding paragraphs for each possible constellation of parameters and study how the SBEs depend on different assumptions. Table 1 shows the SBEs associated with different values of the capitalists' discount rate β.

In Table 1, the rows labeled x and y show the solution of the normal equations. The row labelled "Growth" shows the implied expected growth rate of the economy. One can see that as β increases growth increases and the workers' share of income decreases. An increase in β has two effects in this model. First, it increases the amount of investment given interest rates. Thus a larger β implies a larger "growth cake" to be divided between the two social classes. By itself, however, this effect would not imply a smaller L share. The second effect is familiar from the bargaining literature. A larger β implies that the C party becomes more patient in the tax negotiation. The bargaining situation then becomes relatively more favorable to C, as in other bargaining models.[23] Hence the workers' share decreases.

Note, in passing, that for β = 0.9 the SBE growth rate is essentially

[23]For instance, Rubinstein (1982).

zero. However, potential growth is positive (since $(\beta R^{*})^{1/\sigma} > 1$). Hence the model implies that political considerations alone may cause countries to stagnate.

Table 2 shows the results for different values of the workers' discount factor δ. Intuitively, one would expect a larger δ, which makes workers more patient, to favor the workers' party in the tax negotiations. Therefore a larger δ should imply a larger transfer to workers, slower growth, and a greater workers' share. This is in fact the outcome of the model, as Table 2 shows.

Table 3 shows the effects of changing the inverse of the elasticity of intertemporal substitution σ. The effects of a larger σ are similar to those of assuming a smaller β: As σ increases growth slows down, and workers get a larger share of the pie. The intuition is as follows. As σ increases, the elasticity of intertemporal substitution falls and capitalists save less given any interest rate. Therefore, should there be a disagreement at $t = 0$, the stock of capital from which the economy starts growing at $t = 1$, if agreement is reached, becomes smaller. This hurts capitalists more than workers because workers are not as concerned about growth as with redistribution. So the C party has to offer a more generous tax package for the L party to reach agreement quickly.

Table 4 shows the effects of changing λ, the composition of the population. Larger values of λ imply that the relative number of workers increase; as a consequence, L share improves, but growth worsens. This reflects the assumption that the probability that the L party makes offers is increasing in λ. As λ increases, the L party gets a "stronger voice" in the government and uses its increased power to obtain more transfers. But this is costly in terms of growth.

At this point, note that L share and growth move in opposite directions as I vary parameters in Tables 1-4. Thus, if the main sources of variation in the cross-country data were the preference parameters β, δ, and σ, or the population parameter λ, one would observe that "inequality helps growth." This would be, of course, inconsistent with the recent empirical findings of Alesina and Rodrik (1994) and Persson and Tabellini (1994).

Finally, Tables 5 and 6 show the effect of varying the technological parameters α and A. Note that these are the only two parameters that determine R^* in this economy. As noted by the rows labeled "R^*" in the tables, increases in α decrease R^*, and increases in A increase R^*. Since the "growth pie" depends on R^*, one would expect parameter changes that increase R^* to result in faster growth. Tables 5 and 6 show that this is in, in fact, the case.

Tables 5 and 6 also show that technological changes associated with a larger R^* imply a larger income share for workers. Again, the intuition is that workers care less about growth than capitalists, so that if the "growth pie" becomes larger the C party has to offer a more than proportionally generous offer to the L party for the right to pass a tax reform.

In equilibrium, $g/y = \theta = \alpha$. Hence Table 5 implies that observing a negative relation between growth and productive government expenditure (as a share of income) is consistent with the model. This would be the case if one had observations of growth and g/y for countries with different αs. If countries had the same αs but differed in other parameters, Tables 1-4 and 6 imply that one would observe no correlation between growth and government expenditure.[24]

---

[24]See Barro (1990) for a discussion of the relevant empirical evidence. Barro observes that his model would generate a negative relation between growth and g/y if θ is chosen to satisfy productive efficiency. He also discusses problems associated with the need to measure "productive" government services. Perotti (1992) reports finding a negative relation between public investment and the third quintile share in cross section data. Such a relation is consistent with my model if g/y is public investment.

Tables 5 and 6 also show that my model is consistent with the negative empirical correlation between income inequality and economic growth emphasized by Alesina and Rodrik and Persson and Tabellini. Such correlation would be observed in a world in which cross-country variation in technological parameters (the As and the αs) dominated variation in preference parameters (the βs, δs, and σs). The intuition, again, is straightforward. Countries with larger As and/or lower αs have higher potential growth rates. The political system implies that some of this advantage is translated into higher actual growth rates and some of it into more redistribution from capitalists to workers.

In closing this subsection, note that growth and income inequality do not depend on the initial amount of capital per capitalist, k_0. This is surprising: one could have expected increases in either k_0 to make the C party stronger, implying less redistribution and more growth. The reason that such intuition fails is that changes in k_0 have ambiguous effects on relative bargaining power. An increase in k_0 both strengthens and weakens capitalists: it strengthens them because they can consume more in the absence of a tax agreement, but it weakens them because each agreement becomes more valuable and the cost of waiting increases. Given my assumptions about preferences and technology, these opposite forces exactly balance each other.

5.2. Policy Implications

It has been shown that the model can explain a negative cross-country correlation between income inequality and growth. A natural question emerges: can such a correlation be exploited by public policy? More precisely, can government policy simultaneously increase growth and reduce inequality? This subsection discusses these questions.

A first observation is that the empirical relationship between income inequality and growth implied by the model is not a casual relationship. In my model, income distribution and growth are endogenous outcomes of the same political process. Saying that "income inequality hurts growth" does not make more sense than saying that "growth hurts income inequality."

In principle, the preceding observation does not preclude the possibility that redistributing initial wealth, in this model the initial capital stock, may enhance the rate of economic growth. In this model, however, such policies would not affect growth or income distribution in the long run.

To see this, imagine that at the beginning of time, before bargaining starts, an omnipotent dictator were to take some amount of capital $0 < \varepsilon < k_0$ from each capitalist and divide it equally among the workers. Then the bargaining outcome would be the same as if the economy had started with $k_0-\varepsilon$ units of capital per capitalist instead of k_0, and the growth rate would not change. At this growth rate, capitalists are worse off in absolute terms but their relative assessment of accepting the SBE agreement versus waiting is unchanged. This is expressed by the fact that the normal equation for capitalists, (4.4), does not depend on capital in any period. As for workers, since they cannot save, they would consume the transfer $(1-\lambda)\varepsilon/\lambda$ regardless of whether there is an agreement at $t = 0$ or not. The transfer makes workers better off, but their normal equation, (4.5), would also be unaffected. Hence, the SBE taxes, L share, and growth rate do not change with this kind of redistribution.

Although the above argument depends on the assumptions that workers do not save, it would survive if I allowed workers to save or even to lend their capital to the capitalists, as long as workers cannot borrow[25] and if I imposed

---

[25]If workers were allowed to borrow, the arguments in Section 2 would no longer suffice to characterize the competitive equilibria of the model. Although modifying the analysis to cover this case is straightforward, it involves a considerable amount of tedious work that is outside the scope of this paper.

the slightly stronger assumption that $\delta R^* < 1$. In such case, if workers could save or lend to capitalists, the equilibrium interest rate would be not larger than R^*, but $\delta R^* < 1$ implies that workers would consume their resources as fast as possible.

For a more "radical" example, suppose that at the beginning of time the omnipotent dictator could transform some of the workers into new capitalists, dividing at the same time the existing amount of capital among all (old and new) capitalists equally. Such a policy would certainly improve income distribution in the short run and deliver faster long-run growth, but in the long run one would observe a <u>less egalitarian</u> income distribution. This follows because the new economy would be just like the old one, except that $(1-\lambda)$ would be larger and k_0 smaller. The change in k_0 would not affect the equilibrium values of growth or L share, but, as implied by Table 4, the increase in $(1-\lambda)$ would reduce L share and increase growth.

Hence I obtain a version of the "Lucas Critique" for the interpretation of the cross-country evidence on income distribution and growth. The model can yield a negative relation between income inequality and growth. This evidence would be, however, a reduced form with no implications for evaluating the impact of public policy on long-run growth and income distribution.

As with other "policy ineffectiveness" results, the above discussion is naturally subject to some qualifications. For instance, it depends on the fact that the normal equations do not depend on capital, which in turn hinges on the specific functional forms that I have postulated. It may be possible to destroy such homogeneity by assuming different functional forms; the effect on policy remains unknown.

A second qualification is that my analysis has implicitly assumed the local uniqueness of SBEs. The discussion in Section 4 implies that stationary SBEs are, in fact, locally unique, and hence my arguments are complete provided that one is willing to assume stationarity. Nevertheless, the uniqueness of SBEs without the stationarity assumption remains an open question.[26]

Although one should keep the above qualifications in mind, neither of them eliminates the conclusion that a negative correlation between income inequality and growth does not imply that government policy can attain faster growth and more equality. The aspect of my model that is crucial for this argument is that both growth and income distribution are endogenous outcomes of the politico-economic process. Since income inequality and growth are both endogenous, their observed correlation is not informative about the long-run effects of redistributive policies.

6. Final Remarks

This paper has studied a model in which taxes determine economic growth and income distribution and emerge from negotiations between political parties. The model is consistent with the existence of an empirical negative relation between income inequality and growth. Such relation, however, does not imply that government policy can simultanoeusly increase growth and reduce inequality.

In what sense do these results challenge the studies by Alesina and Rodrik (1994) and Persson and Tabellini (1994)? Their models and the one in this paper may all be consistent with the empirical correlation about income distribution and growth, but they have very different policy implications. This implies that efforts should be made to discriminate between the alternative models on the basis of theoretical or empirical criteria other

[26]Although, as indicated in Section 4, I suspect that SBEs are unique in this model.

than the income distribution-growth evidence.

In this regard, note that in interpreting their empirical evidence Alesina and Rodrik (1994) and Persson and Tabellini (1994) implicitly assumed that income distribution is an exogenous variable.[27] In fact, they see income distribution as the main determinant of the political structure. My model assumes, in contrast, that income distribution is an endogenous outcome of the politico-economic process. Income distribution and growth are determined simultaneously.

In addition to its lessons for policy, the bargaining approach advanced in this paper should prove useful in analyzing many other problems in which politics and economics interact. In Chang (1995a,b) I have shown how the analytical tools illustrated here can be fruitful for studying the European monetary union and sovereign debt, respectively; other applications clearly abound.

---

[27]More precisely, Alesina and Rodrik assume that income distribution is a good proxy for the distribution of wealth, which is given exogenously and determines growth. In Persson and Tabellini's model, income is directly related to "basic skills," whose distribution is given and determines growth.

Appendix

Proof of Proposition 1: As discussed in the text, the allocation rule F is competitive by construction. Hence, it is sufficient to verify the optimality of C's and L's strategies after any history.

By the recursive structure of the proposed strategies and allocation rule, it is sufficient to verify that no player can unilaterally gain from a one-shot deviation from the proposed strategies. Thus, consider a period t in which C must respond to an offer a. If C takes the offer, its payoff is $v(k_t, a)$. If C rejects it, the continuation strategies imply that an agreement a_0 (resp. a_1) will be reached with probablity λ (resp. $(1-\lambda)$) at $(t+1)$, giving C an expected payoff of $u(c_t) + \beta [\lambda v(k_{t+1},a_0) + (1-\lambda) v(k_{t+1}, a_1)]$. As discussed above, (4.4) implies that C is indifferent between the two options.

Likewise, consider L's decision when L has to make an offer. Suppose L does not offer a_0. Any offer that gives L strictly more than $w(k_t, a_0)$ will be rejected because a_0 is Pareto optimal. So it must be the case that L wants to delay the agreement. The continuation strategies then imply an agreement a_0 or a_1, with probabilities λ and $(1-\lambda)$, at $(t+1)$; the implied expected payoff for L is then $\delta[\lambda w(k_{t+1}, a_0) + (1-\lambda) w(k_{t+1}, a_1)]$. But it is easily verified that $w(k_t, a_0) > \delta[\lambda w(k_{t+1}, a_0) + (1-\lambda) w(k_{t+1}, a_1)]$. So offering a_0 is in fact optimal for L.

Showing that L's acceptance rule is optimal is easy and left to the reader. Consider C's decision about what offer to make when called to make an offer. Suppose that C does not offer a_1. Any offer that gives C more than $v(k_t, a_1)$ will be rejected because a_1 is Pareto optimal. Therefore, if C does not offer a_1, it must be the case that C plans to delay agreement. The continuation strategies then imply that an agreement a_0 or a_1 will be reached at (t+1) with probabilities λ and $(1-\lambda)$. The payoff to C, discounted to t, of delaying agreement is $u(c_t) + \beta[\ \lambda v(k_{t+1}, a_0) + (1-\lambda)\ v(k_{t+1}, a_1)\ ]$. Since the latter expression is equal to $v(k_t, a_0)$, it suffices to show that $v(k_t, a_1) \geq v(k_t, a_0)$ or, using (2.6), that $\Psi(y) \geq \Psi(x)$. But (4.5a) implies, as the interested reader can easily check, that $\gamma(y) < \gamma(x)$ which, since γ is stricly decreasing, implies that $y > x$. Since Ψ is strictly increasing, $\Psi(y) \geq \Psi(x)$ and C's strategy is optimal.

<u>Proof of Proposition 2</u>: One can show that $\Psi(x)$ is strictly increasing on $[1, R^*]$, with $\Psi(1) = (1-\beta^{1/\sigma})^{-\sigma}$ and $\Psi(R^*) = \{1-(\beta(R^*)^{1-\sigma})^{1/\sigma}\}^{-\sigma}\ (R^*)^{1-\sigma}$. Given any $(x,y) \in [1, R^*]^2$, the RHS of (4.4) is a number in the interval $[(1-\beta^{1/\sigma})^{-\sigma}, \{1-(\beta(R^*)^{1-\sigma})^{1/\sigma}\}^{-\sigma}]$. Call this number z. Since f is continuous, strictly decreasing, and satisfies $\Psi(1) \leq z \leq \Psi(R^*)$, there is a unique number $T_1(x,y)$ in $[1, R^*]$ such that $\Psi(T_1(x,y)) = z$. Thus T_1 maps $[1, R^*]^2$ to $[1,R^*]$. T_1 is clearly continuous.

Since $R_* \leq 1$, $\gamma(y)$ is decreasing on $[1, R^*]$ and continuous. Also, $\gamma(1) = (R^*-1)/(1-\delta\beta^{1/\sigma})$ and $\gamma(R^*) = 0$. Given any (x,y) in $[1, R^*]^2$, the RHS of (4.5) is a number in the interval $[0, \delta\beta^{1/\sigma}\gamma(1)]$; call this number z'. Now $\gamma(1) > \delta\beta^{1/\sigma}\gamma(1) \geq z \geq 0 = \gamma(R^*)$, so that there is a unique number $T_2(x,y)$ in $[1, R^*]$ such that $\gamma(T_2(x,y)) = z'$. Thus T_2 maps $[1, R^*]^2$ to $[1,R^*]$; T_2 is clearly continuous.

Finally, consider the mapping ξ from $[1,R^*]^2$ to itself defined by $\xi(x,y) = (T_1(x,y), T_2(x,y))$. It is easy to check that a fixed point of this mapping is a solution of the normal equations. But all the conditions of Brouwer's Fixed Point Theorem are satisfied, and therefore the Proposition is proved. ∎

References

Aghion, Phillipe, and Patrick Bolton (1992). "Distribution and Growth in Models of Imperfect Capital Markets." *European Economic Review* 36, 603-612.

Alesina, Alberto, and Dani Rodrik (1994). "Distributive Politics and Economic Growth." *Quarterly Journal of Economics* 109, 465-490.

Barro, Robert (1990). "Government Spending in a Simple Model of Endogenous Growth." *Journal of Political Economy* 98, S103-S125.

Barro, Robert, and David Gordon (1983). "Rules, Discretion and Reputation in a Model of Monetary Policy." *Journal of Monetary Economics* 12, 101-21.

Benhabib, Jess, and Aldo Rustichini (1991). "Social Conflict, Growth, and Income Distribution." C.V. Starr Center for Applied Economics RR 91-22, New York University.

Chang, Roberto (1995a). "Bargaining a Monetary Union." *Journal of Economic Theory*, forthcoming.

Chang, Roberto (1995b). "Private Investment and Sovereign Debt Negotiations." *International Economic Review* 36, 387-406.

Chari, V.V., and Patrick Kehoe (1990). "Sustainable Plans." *Journal of Political Economy* 98, 783-802.

Galor, Oded, and Joseph Zeira (1993). "Income Distribution and Macroeconomics." *Review of Economic Studies* 60, 35-52.

Greenwood, Jeremy, and Boyan Jovanovic (1990). "Financial Development, Growth, and the Distribution of Income." *Journal of Political Economy* 98, 1076-1107.

Lancaster, Kelvin (1973). "The Dynamic Inefficiency of Capitalism." *Journal of Political Economy* 81, 1098-1199.

Lucas, Robert (1976). "Econometric Policy Evaluation: A Critique." *Carnegie-Rochester Conference in Public Policy* 1, 19-46.

Perotti, Roberto (1992). "Income Distribution, Politics and Growth." *American Economic Review* 82, 311-316.

Persson, Torsten, and Guido Tabellini (1992). "Growth, Distribution, and Politics." In *Political Economy, Growth, and Business Cycles*, Cukierman, Hercowitz and Leiderman (eds.). Cambridge: MIT Press.

Persson, Torsten, and Guido Tabellini (1994). "Is Inequality Harmful for Growth?" *American Economic Review* 84, 600-621.

Rubinstein, Ariel (1982). "Perfect Equilibrium in a Bargaining Model." *Econometrica* 50, 97-109.

Stokey, Nancy (1991). "Credible Fiscal Policy." *Journal of Economic Dynamics and Control* 15, 627-656.

Tornell, Aaron, and Andres Velasco (1992). "The Tragedy of the Commons and Economic Growth: Why Does Capital Flow from Poor to Rich Countries?" *Journal of Political Economy* 100, 1208-1231.

FIGURE 1

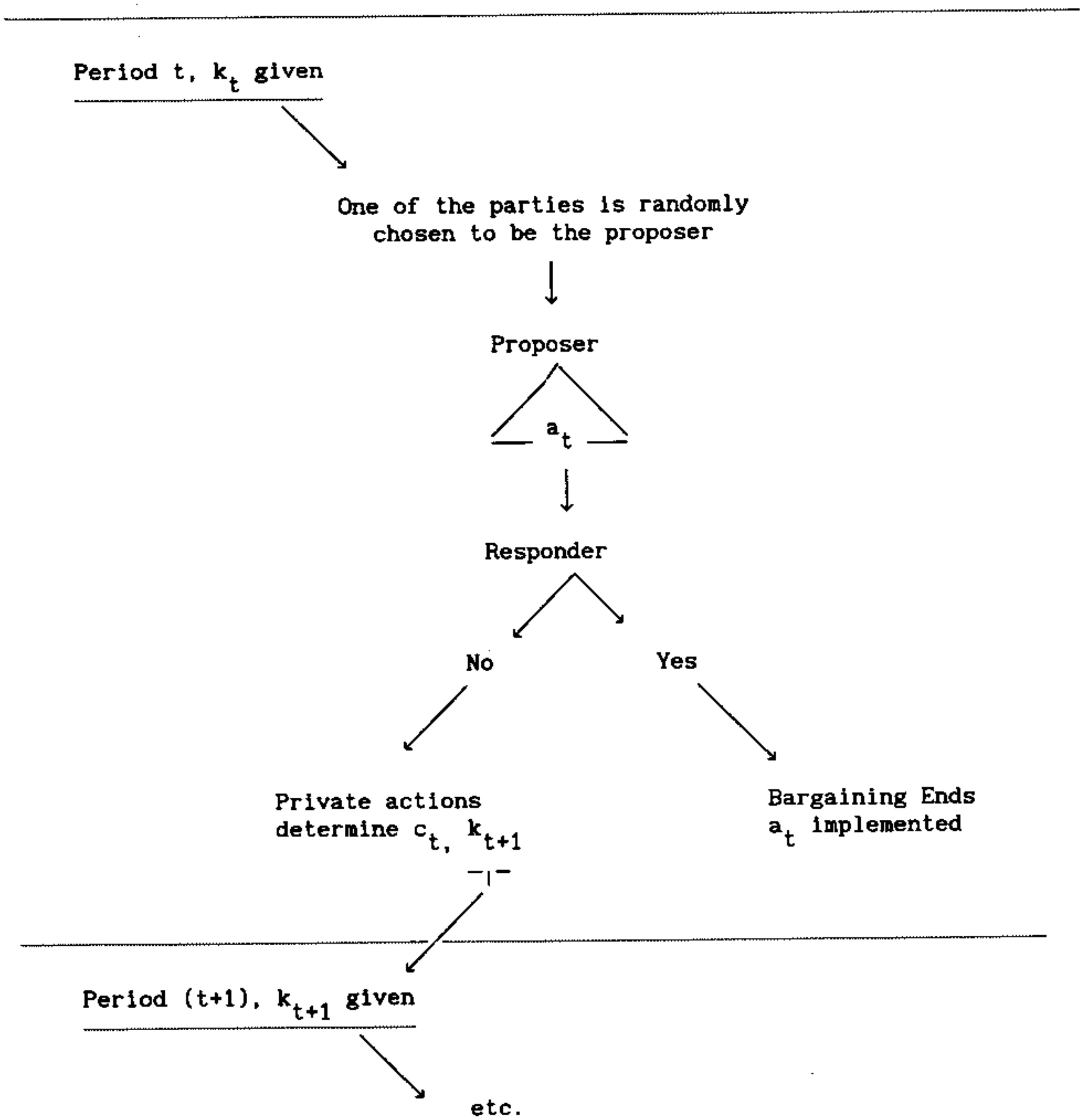
Period t, k_t given
One of the parties is randomly chosen to be the proposer
Proposer
a_t
Responder
No
Yes
Private actions determine c_t, k_{t+1}
Bargaining Ends a_t implemented
Period (t+1), k_{t+1} given
etc.

Table 1
Effect of Capitalists' Discount Rate β

	β			
	0.90	0.93	0.95	0.97
x	1.091	1.096	1.099	1.103
y	1.113	1.115	1.117	1.119
Growth	0.996	1.014	1.026	1.038
L share	0.061	0.057	0.055	0.050
R^*	1.149	1.149	1.149	1.149

<u>Fixed</u> <u>Parameters:</u> $A = 0.25$, $\alpha = 0.1$, $\delta = 0.9$, $\sigma = 2$, $\lambda = 0.5$

Table 2
Effect of Workers Discount Rate δ

	δ			
	0.80	0.84	0.87	0.90
x	1.105	1.102	1.101	1.099
y	1.124	1.121	1.119	1.117
Growth	1.029	1.028	1.027	1.026
L share	0.045	0.049	0.052	0.055
R^*	1.149	1.149	1.149	1.149

<u>Fixed</u> <u>Parameters:</u> $A = 0.25$, $\alpha = 0.1$, $\beta = 0.95$, $\sigma = 2$, $\lambda = 0.5$

Table 3
Effect of Inverse of Elasticity of Substitution σ

	σ			
	2.0	2.5	3.0	3.5
x	1.099	1.096	1.093	1.091
y	1.117	1.114	1.112	1.110
Growth	1.026	1.020	1.016	1.013
L share	0.055	0.058	0.061	0.063
R^*	1.149	1.149	1.149	1.149

Fixed Parameters: $A = 0.25, \alpha = 0.1, \delta = 0.9, \beta = 0.95, \lambda = 0.5$

Table 4
Effect of Population Makeup λ

	λ			
	0.4	0.5	0.6	0.7
x	1.108	1.099	1.089	1.077
y	1.124	1.117	1.108	1.098
Growth	1.030	1.026	1.021	1.014
L share	0.041	0.055	0.068	0.085
R^*	1.183	1.183	1.183	1.183

Fixed Parameters: $\alpha = 0.1, \delta = 0.9, \sigma = 2, \beta = 0.95, A = 0.3$

Table 5
Effects of Marginal Productivity α

	α			
	0.1	0.15	0.2	0.25
x	1.099	1.080	1.065	1.053
y	1.117	1.093	1.074	1.059
Growth	1.026	1.016	1.008	1.001
L share	0.055	0.045	0.037	0.030
R^*	1.149	1.119	1.095	1.074

Fixed Parameters: $A = 0.25$, $\delta = 0.9$, $\beta = 0.95$, $\sigma = 2$, $\lambda = 0.5$

Table 6
Effect of Overall Productivity A

	A			
	0.25	0.30	0.35	0.40
x	1.099	1.120	1.142	1.168
y	1.117	1.145	1.177	1.216
Growth	1.026	1.037	1.050	1.064
L share	0.055	0.065	0.074	0.077
R^*	1.149	1.183	1.217	1.252

Fixed Parameters: $\alpha = 0.1$, $\delta = 0.9$, $\sigma = 2$, $\beta = 0.95$, $\lambda = 0.5$