Are There Optimal Multiple Reserve Requirements?

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Abstract: A number of developing countries have adopted deficit-finance regimes involving multiple reserve requirements. One question the previous literature on this phenomenon has not addressed is whether multiple-reserves regimes can improve on regimes involving single-currency-reserve requirements if the policy settings of the latter regimes are assumed to be chosen optimally. We find that a "conventional" multiple-reserves regime—a regime with positive nominal rates on reservable bonds—cannot Pareto-improve an *optimal* single-currency-regime but can, in some cases, increase social welfare over such a regime.

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1 Introduction

A multiple-reserve-requirements regime is a monetary regime in which the government imposes two types of reserve requirements on the banking system: a currency reserve requirement, which can be satisfied by holdings of government currency, and a bond reserve requirement, which can be satisfied by holdings of government bonds that return below-market rates of interest. Multiple-reserves regimes have been adopted by a number of developing countries at various times in recent years; examples include Chile, Korea, Mexico, and Pakistan. In each case, the country had a large public-sector deficit and was attempting to use seigniorage to finance a substantial portion of that deficit.

One important regularity of observed multiple-reserves regimes is that the real rate of return on reservable government bonds has always been higher than the real rate of return on government currency — that is, the bonds have always yielded positive nominal interest. We will refer to regimes like this as "conventional." While the fact that the nominal interest rates on *private* bonds are always positive makes conventionality seem natural, nothing about the structure of multiple-reserves regimes appears to require positive government bond rates. In a typical regime of this type, the government designates a particular class of bonds as reservable and forces the banks to hold these bonds and no others. Since the government is free, if it wishes, to offer different and higher-yielding bonds to nonbank lenders, it should also be free to impose any reservable bond rate it chooses.[1]

The principal purpose of this paper is to determine whether an optimizing government

[1] In some multiple-reserves regimes, the government gives banks the option of holding either currency or reservable government bonds to satisfy the second reserve requirement. In this case, it is clear that the banks will not purchase reservable bonds unless their nominal interest rate is non-negative. The question then becomes why the government chooses to offer banks this option.

can use multiple reserve requirements to achieve welfare improvements, and whether the type of multiple-reserves regime that such a government selects might tend to be conventional in nature. Thus, this paper extends the literature on optimal single reserve requirements — a literature that includes contributions by Freeman (1987), Brock (1989), Bencivenga and Smith (1992), Mourmouras and Russell (1992), Cothren and Waud (1994), and Espinosa and Yip (1996) — to the case of multiple reserve requirements.

The first theoretical analysis of multiple-reserves regimes was presented by Espinosa (1995). Espinosa's principal goal was to construct a formal model that could be used to evaluate a number of informal claims about the goals and merits of conventional multiple reserve requirements. Although his discussion emphasized the effect of multiple reserve requirements on the equilibrium rate of inflation, he also conducted a welfare analysis. He found that modifying a single-currency-reserve regime by adding a bond reserve requirement can improve economic efficiency, and may increase social welfare even when it does not improve efficiency.

Although Espinosa's analysis involves conducting welfare comparisons across policy regimes, these comparisons are not based on the solutions to optimal-policy-choice problems. In particular, he does not assume that a government contemplating a shift from a single-reserve to a multiple-reserves regime has chosen the policy settings of the initial regime optimally. This feature of his analysis raises the possibility that the government could achieve the efficiency and/or social-welfare improvements he describes by readjusting these policy settings rather than shifting to a different and more complex regime.[2]

[2] For our purposes, a policy is efficient, relative to another policy, if the consumption allocation it supports Pareto dominates the allocation supported by the alternative policy. A policy improves social welfare, relative to another policy, if the consumption allocation it supports has higher social utility, as measured by a social-utility function, than the alternative policy. Our use of terms such as "optimal" (which can refer to efficiency or social welfare) or "social-welfare-maximizing" should be understood as restricted to the context of a particular class of policies. None of the policies we study in this paper are first-best optimal.

Espinosa follows most of the previous literature on single currency reserve requirements by implicitly assuming that there is a unique required reserve ratio that will allow a given deficit to be financed at a particular rate of inflation.[3] It turns out that this may not always be the case. As Fama (1980) has pointed out, in seigniorage economies with a reserve requirement the inflation tax and the reserve requirement combine to impose an indirect tax on bank deposits, with the rate of inflation and the level of the reserve ratio interacting to determine the implicit deposit tax rate. Increases in this tax rate reduce the rate of return received by depositors, which may also reduce the demand for deposits. By analogy with other "Laffer curve" situations in which the level of the tax rate affects the size of the tax base, there are likely to be at least two tax rates that will support a given level of (indirect) deposit-tax revenues. It follows that if the inflation rate is held fixed — which will fix the volume of seigniorage revenues from other sources — then there should be at least two reserve ratios that will support a given level of total revenues. And since the allocation supported by the lower reserve ratio involves a lower implicit tax rate, it should Pareto dominate the allocation supported by the higher reserve ratio.[4]

The possibility of Pareto-inferior single-currency-reserve equilibria with relatively high reserve ratios suggests that imposing conventional multiple reserve requirements may produce efficiency improvements only if the initial single-reserve equilibria are "bad" high-reserve-ratio equilibria, and that conventional multiple-reserves regimes cannot in fact improve on the corresponding "good" low-reserve-ratio equilibria. We show in our Proposition 1 that this is precisely the case — that a conventional multiple-reserves equilibrium can Pareto-dominate a single-currency-reserve equilibrium only if the latter equilibrium is also Pareto-dominated by a single-currency-reserve equilibrium with a different reserve ratio. We also show, by example, that while it *is* possible for a multiple-reserves regime to Pareto-dominate

3 Notable exceptions are Bencivenga and Smith (1992) and Espinosa and Yip (1996).

4 In "pure seigniorage" economies — economies in which the government earns revenue by imposing an inflation tax on public holdings of currency, but does not impose a reserve requirement — there are typically at least two inflation tax rates that will produce a given level of seigniorage revenues. The allocation associated with the high inflation tax rate, moreover, is typically Pareto-dominated by the allocation associated with the low inflation tax rate. See Sargent (1987), chapter 7.

a "good" single-currency-reserve regime, the only type of multiple-reserves regime that can accomplish this feat is an "unconventional" one in which reservable bonds yield negative nominal interest.

The second type of welfare improvement attributed to multiple reserve requirements involves a comparison of multiple-reserves regimes to regimes that combine a single currency reserve requirement with a direct tax on deposits. Espinosa shows that replacing the deposit tax with a bond reserve requirement can increase the welfare of a potentially important group of agents — the "initial old" agents who are holding nominal assets (currency and reservable bonds) at the time of the regime change.[5] The intuition behind this strategy is based on Freeman's (1987) refinement of Fama's insight about the link between reserve requirements and deposit taxation. Freeman points out that during a hyperinflation, when the gross real rate of return on government currency is zero, imposing a proportional reserve requirement raises exactly the same amount of revenue as levying an equal proportional deposit tax, because the government is effectively confiscating the reserves. It follows that replacing a deposit tax with an equal reserve requirement involving positive-gross-real-rate government liabilities — in this case, government bonds — reduces government revenues. To avoid a revenue deficiency, the government must increase its seigniorage tax base by selecting a bond reserve ratio higher than initial deposit tax rate. This increases the demand for reservable bonds, which benefits the current holders of nominal assets.

In this paper we explore yet another implication of Fama's insight, which is that the government may be able to achieve the same policy goals by increasing the currency reserve ratio. An increase in this reserve ratio will also increase nominal-asset demand, and thus will also benefit the initial old agents. A potential problem is that unless the bond and currency return rates happen to be equal, the reserve ratio increase will change the indirect tax rate on bank deposits. The government, however, should be able to exploit the equivalence between

[5] These gains do not involve Pareto improvements, since the welfare of another group of agents declines. However, a government whose social-welfare function gave sufficiently heavy weight to the initial old agents might find the terms of this tradeoff attractive.

direct and indirect deposit taxation by adjusting the direct deposit tax rate in a way that leaves the total tax rate (direct plus indirect) unchanged.

We show in our Proposition 3 that for a wide range of multiple-reserves regimes, there is in fact a single-currency-reserve/deposit-tax regime with a higher currency reserve ratio that will support the same allocation as the multiple reserve requirement. However, Proposition 3 also establishes that some multiple-reserves allocations cannot be supported as single-currency-reserve/deposit-tax equilibria; interestingly enough, these "unsupportable" allocations are the allocations supported by *conventional* multiple reserve requirements. We go on to show, by example, that allocations of this type can be social-welfare maximizing. Thus, our analysis demonstrates that under certain conditions, at least one of the potentially appealing attributes of multiple reserve requirements — their ability to produce increases in social welfare — is robust to the assumption that the government's initial policy settings are selected in an optimal manner.

In the next section of this paper, we begin our formal analysis by presenting an abbreviated description of a basic reserve-requirements model. In Section 2, we investigate the ability of multiple-reserves regimes to produce efficiency improvements over regimes that impose a single currency reserve requirement. In Section 3 we turn to the question of whether multiple-reserves regimes have the potential to increase social welfare. In Part 1 of this section, we show that when the government does not care about the welfare of the initial-old agents, it is optimal for it to abandon reserve requirements entirely in favor of direct taxation of deposits.[6] In Part 2, we consider the more interesting case in which the government cares about all current and future agents, and examine the ability of multiple-reserves regimes to produce social-welfare improvements over single-currency-reserve regimes that also include a direct deposit tax. Section 4 offers some concluding remarks. The proofs of the paper's

6 While this result (our Proposition 1) is not of great significance, it provides a particularly stark illustration the dangers of comparing two regimes without allowing the government to choose the policy settings in the first regime optimally.

propositions are presented in the appendix, along with some illustrative examples.

2 The model

We analyze a two-period overlapping generations model with limited intragenerational heterogeneity and a number of legal/technological constraints on intertemporal trades. The model is essentially similar to the one described by Espinosa (1995). Economic activity occurs at discrete dates $t = 1, 2, \ldots$. At each date t a generation of agents is born; these "members of generation t" live during dates t and $t+1$. Each generation of agents consists of N_p or savers" and N_r "rich savers." Rich savers differ from poor savers only in the magnitude and time-distribution of their endowments of the single consumption good. The endowment patterns of rich and poor savers are invariant to the dates at which these agents are born. At each date an arbitrary number of competitive banks are operating in the economy. These banks may hold one or more of the following types of assets:

- private one-period bonds, which are available on the international credit market at an exogenously-determined gross real interest rate $R > 1$.[7]
- government currency, which yields a gross real return rate $R_m(t) \geq 0$ that is determined by the government through its ability to control the growth rate of the stock of currency.
- government one-period bonds, which yield a gross real return rate $R_b(t) \geq 0$ that is specified by the government.[8]

The liabilities of the banks consist of deposits that are offered to the public at a competitively-determined gross real interest rate $R_d(t)$. The banks are assumed to have zero operating costs and to maximize their date-t profits, which must be zero in equilibrium.

The government is assumed to have imposed a legal minimum denomination on the real market value of a bank deposit. The individual endowments of the poor savers are assumed

[7] Note that $R > 1$ implies that the net rate of return in the international credit market exceeds the net rate of growth of the economy, which under these assumptions is zero.

[8] Since the government sets the nominal interest rate on bonds and has perfect foresight regarding the currency inflation rate (see below), it effectively sets the real interest rate on bonds.

to be too small to permit them to purchase bank deposits; it is further assumed to be illegal and/or infeasible for them to pool their funds to purchase deposits, or to finance deposit purchases with unsecured credit.[9] Rich savers' endowments are assumed to be large enough that this minimum denomination is irrevelant to them. Private and government bonds are assumed to have larger minimum denominations that make them inaccessible to any agents except banks. Thus the only asset available to poor savers is government currency, while rich savers may purchase government currency and/or bank deposits.

The aggregate real savings functions of the poor and rich savers are denoted $m(R_m(t))$ and $d(R_k(t))$, respectively, where $R_k(t) \equiv \max\{R_m(t), R_d(t)\}$. These functions are assumed to be positive, continuous, and strictly increasing over relevant ranges of $R_m(t)$ and $R_k(t)$.

The government is assumed to finance a fixed real deficit of G per period by issuing bonds and/or currency. The aggregate nominal stock of currency in circulation at date t is denoted $M(t)$; the date t price of a unit of the consumption good in terms of government currency (the date t price level) is denoted $p(t)$. Thus $R_m(t) \equiv p(t)/p(t+1)$. Government bonds are payable in government currency: a bond is a title to a quantity of currency next period equal to its nominal price plus its net nominal interest (which may be negative). The aggregate nominal price of the government bonds issued at date t is denoted $B(t)$ and the gross nominal interest rate on these bonds is denoted $R_{nom}(t)$; note that $R_b(t) = R_{nom}(t)R_m(t)$. Government seigniorage revenues at dates $t \geq 2$ are given by

$$[M(t) - M(t\text{-}1)]/p(t) + [B(t) - R_{nom}(t\text{-}1)B(t\text{-}1)]/p(t).$$

The welfare of the poor and rich members of any generation t is assumed to be strictly increasing in $R_m(t)$ and $R_k(t)$, respectively. It is assumed that at date 1 there are an arbitrary number of "initial old" agents (the members of "generation 0") who live for one period and are endowed, in aggregate, with a stock of government currency M_0 and a stock of government bonds B_0. The welfare of these agents is assumed to be strictly increasing in

[9] For earlier examples of the use of minimum denomination restrictions of this type to generate demand for government currency, see Sargent and Wallace (1982) and Bryant and Wallace (1984).

$1/p(1)$, the inverse of the initial price level, which determines the purchasing power of the nominal assets these agents are endowed with.[10]

The government is assumed to impose bond and/or currency reserve requirements on the banks. The fractions of a banks' assets that it is required to hold in the form of currency and government bonds are denoted θ_m and θ_b, respectively. We assume $\theta_m, \theta_b \in [0,1]$ and $\theta \equiv \theta_m + \theta_b \in [0,1]$. Each reserve ratio is the minimum ratio of the market value of a bank's holdings of one of the reservable liabilities (currency or bonds) to the market value of its entire portfolio of liabilities.

We will confine ourselves to the study of *binding stationary equilibria*, which are equilibria in which [1] the rate of return on private credit exceeds the rates of return on government currency and bonds, so that the banks will hold government liabilities only to meet the reserve requirements, and [2] the values of all real variables are constant, while the values of all nominal variables grow at fixed rates (which may be zero).[11] Given R, G, and $M_0 + B_0$, a binding stationary equilibrium can be characterized as values of R_m, R_b, θ_m, θ_b, R_d, and p_1 that satisfy

$$R_d = (1 - \theta_m - \theta_b)R + \theta_b R_b + \theta_m R_m \tag{1}$$

$$G = (1 - R_m)[m(R_m) + \theta_m d(R_d)] + (1 - R_b)\theta_b d(R_d) \tag{2}$$

and

$$[M_0 + B_0]/p_1 = m(R_m) + (\theta_m + \theta_b)\, d(R_d) - G\,. \tag{3}$$

[10]Our treatment of government bonds differs slightly from that of Espinosa (1995). Espinosa defined $B(t)$ as the nominal *face value* of the bonds outstanding at date t and $P_b(t)$ as the nominal price of those bonds, which is the inverse of the gross nominal interest rate. He further assumed that the initial old were endowed with an initial stock of government currency $M(0)$ and an initial stock of government bonds with fixed face value $B(0)$. It follows that the real value of the total asset endowment of the initial old is $[M(0)+P_b(1)B(0)]/p(1)$. Espinosa, however, identifies this value as $[M(0)+B(0)]/p(1)$. This identification implies that the welfare of the initial old depends entirely on the value of $p(1)$ — a property that figures importantly in many of his results. Our revised formulation, in which B_0 is the fixed nominal *market value* of the bond endowment, delivers this property.

[11]The model is easily generalized to cover situations in which the values of real variables grow at fixed, exogenously-determined rates.

The first equation expresses the relationship between the interest rate on bank deposits, the two reserve ratios, and the rates of return on the three nonbank assets that is implied by the requirement that banks earn zero profits. The second and third equations ensure that the government meets its budget constraint at dates $t \geq 2$ and $t=1$, respectively. We also require $\theta_m, \theta_b \in (0,1)$ and $\theta \equiv \theta_m + \theta_b \in (0,1)$. It follows that $R_m < R_d < R$ in any binding stationary equilibrium; the first inequality implies that rich savers' asset portfolios will be composed entirely of bank deposits.

In what follows, it is useful to define

$$A \equiv m(R_m) + \theta\, d(R_d), \tag{4}$$

which represents aggregate real balances of government liabilities, and to note that equations 1-3 imply

$$[M_0 + B_0]/p_1 = A - G = R_m[m(R_m) + \theta_m d(R_d)] + R_b \theta_b d(R_d). \tag{5}$$

In a binding stationary equilibrium we have $p(t)/p(t+1) = R_m$, $R_{nom}(t) = R_{nom} \equiv R_b/R_m$, $M(t)/p(t) = m(R_m) + \theta_m d(R_d)$ and $B(t)/p(t) = \theta_m d(R_d)$ for all $t \geq 1$ [with $p(1) \equiv p_1$]. These equations imply $M(t+1)/M(t) = B(t+1)/B(t) = 1/R_m$ for all $t \geq 2$.

We define a "reserve-requirements policy setting" as a vector $(\overline{\theta}_m, \overline{\theta}_b, \overline{R}_m, \overline{R}_b)$, and an associated "private and public allocation" as values $(\overline{R}_m, \overline{R}_d, \overline{p}_1, \overline{G})$ that this vector of policy settings supports as a binding stationary equilibrium.

3 Can multiple reserve requirements improve efficiency?

The central result reported by Espinosa (1995) is that under certain conditions, the allocation achieved by a single currency reserve requirement can be Pareto-dominated by a multiple reserve requirement with $R_b > R_m$ and the same aggregate reserve ratio. As we noted in the introduction, this result seems to suggest that there is an efficiency rationale for the imposition of conventional multiple-reserves regimes.

As we have also noted, much of literature on the theory of reserve requirements has ignored the possibility of multiple equilibria under single-reserve regimes: given a fixed government deficit and a fixed rate of return on currency, there may be more than one equilibrium reserve ratio.[12] This possibility, it turns out, is critical for understanding the limits of Espinosa's efficiency results. We show in our Proposition 1 that if an allocation supported by a single-reserve regime can be Pareto-dominated by a multiple-reserves regime involving positive nominal bond rates, then there is an alternative single-reserve regime that also Pareto-dominates the original single-reserve regime. Thus, conventional multiple-reserves regimes cannot improve allocational efficiency over single-currency-reserve regimes that are constructed optimally.

Proposition 1 *Suppose the private and public allocation supported by a single reserve requirement is Pareto-dominated by the allocation supported by a multiple reserve requirement with $\widehat{R}_b > \widehat{R}_m$, or $\widehat{R}_b < \widehat{R}_m = \overline{R}_m$. Then there is an alternative single reserve requirement which supports an allocation that Pareto-dominates the allocation supported by the original single reserve requirement.*

In Example 1 of this paper (see the appendix), we apply Proposition 1 to Espinosa's (1995) example of a situation in which a conventional multiple-reserves regime Pareto-dominates a single-currency-reserve regime. We show that the single-reserve regime from this example is also Pareto-dominated by another single-reserve regime with a lower reserve ratio, and that the conventional multiple-reserves regime from the example does not Pareto-dominate the latter regime.

The intuition behind Proposition 1 involves the "reserve-ratio Laffer curve" — the relationship between the value of the required reserve ratio in a single-reserve-requirement regime and the volume of seigniorage revenues produced by that regime, given a fixed value of R_m. In examples of the type presented in Example 1, a reduction in the reserve ratio

[12] Freeman (1987), for example, shows that the optimal reserve requirement involves a gross currency return rate of zero, but does not point out that there are typically at least two reserve ratios that will support an equilibrium at this currency return rate. The equilibrium of this type that involves the lowest reserve ratio Pareto-dominates the others, and the other equilibria may not Pareto-dominate equilibria equilibria involving positive currency return rates.

increases the deposit base so rapidly (by increasing R_d) that demand for reserves actually increases, which means that seigniorage revenues also increase. Thus, in these examples the initial equilibrium is on the "wrong side" of the revenues curve. The increased reserve demand generates higher seigniorage revenues and produces a lower initial price level. Continued reductions in the reserve ratio, however, must eventually reduce reserve demand to the point where seigniorage revenues fall short of their original level: revenues from "reserves seigniorage" are zero when the reserve ratio is cut to zero, so the reserve-ratio Laffer curve must also have a "right side." Consequently there must be an alternative equilibrium in which both total seigniorage revenues and the initial price level match their original levels, but the deposit interest rate is higher — and also, as the proof of Proposition 1 indicates, an alternative equilibrium in which the deposit interest rate is unchanged, but the initial price level is lower and the volume of seigniorage revenues is higher.

A plot of the reserve-ratio Laffer curve associated with Example 1 is displayed in Figure 1. The upper curve displays total seigniorage earnings, and the lower curve displays seigniorage earnings from bank reserves. The upper horizontal line represents the total government deficit, and the lower line represents the total deficit less seigniorage revenues from public currency holdings (which are constant). The vertical lines represent the two equilibrium reserve ratios.

Proposition 1 does not apply to Pareto-dominant multiple-reserves regimes in which $\widehat{R}_m > \overline{R}_m$ and $\widehat{R}_b < \widehat{R}_m$ — regimes in which the rate of return on currency is higher than in the dominated single-reserves regime, but the nominal interest rate on bond reserves is negative. The possibility that regimes of this sort can produce genuine efficiency improvements is associated with the existence of another, more conventional, Laffer curve — the curve that displays the relationship between the amount of seigniorage earned from poor savers and the real rate of return on currency. Consider a single-reserves equilibrium in which the initial value of R_m lies on the "wrong side" of this curve, so that a small increase in R_m increases the amount of seigniorage earned from poor savers. The government may be unable to exploit this situation by increasing R_m because this would cause seigniorage earnings from rich savers to decline more rapidly than earnings from poor savers would increase, producing a decline in total earnings. [While the government can increase its seigniorage earnings from rich savers by increasing θ, if it is seeking a Pareto-improvement these increases must not be rapid enough to cause R_d to fall.] If the government has the option of switching to a multiple-reserves regime, however, it can increase R_m while at the same time reducing R_b. This allows it to cut the inflation tax rate on poor savers — and increase its seigniorage earnings therefrom — while preventing the tax rate on rich savers from falling far enough to produce a net loss of revenue.[13] A situation of this sort is described in our Example 2 (see the appendix).

The finding that the only multiple-reserves regimes that can improve efficiency relative to single-currency-reserve regimes are those in which the nominal yields on reservable bonds are negative is somewhat puzzling, since in practice we do not seem to observe regimes of this type. The solution to this puzzle may involve Proposition 3 below, which establishes that any allocation supported by a multiple-reserves regime involving negative nominal bond rates can also be supported by a regime with a single currency reserve requirement and a proportional deposit tax. It seems conceivable that governments that find themselves in a position of the sort described in Example 2 resort to deposit taxation rather than imposing a second reserve requirement.[14]

[13]Notice the price discrimination aspect to this argument. Bryant and Wallace (1984) use a price discrimination argument of a somewhat different sort to provide an explanation for the minimum denominations on government bonds.

[14]In this model, a tax on on deposit returns amounts to a tax on personal interest income, since deposits are the only assets available to savers.

4 Can multiple reserve requirements increase social welfare?

4.1 Optimal reserve requirements when the government does not care about the initial old

A basic question about multiple reserve requirements is why a government might choose them in preference to less complicated strategies for augmenting the revenues from seigniorage. As we noted in the introduction, one strategy of this type that has received attention in recent years is direct proportional taxation of the returns on bank deposits. While deposit taxation may seem conceptually different from seigniorage, Fama (1980) has argued that the two public-finance strategies are equivalent. An example of this sort of equivalence is presented by Freeman (1987), who studies the optimal level of a single currency reserve requirement in a model where the agents are intragenerationally homogeneous.[15] Freeman assumes that the government is unconcerned or the welfare of the initial old agents, and acts to maximize the steady-state utility the members of generations $t \geq 1$. He shows that it is optimal for a government with this objective to choose the smallest reserve ratio consistent with financing its deficit — a ratio at which the *gross* real rate of return on currency is zero. This policy, he notes, is equivalent to replacing the currency reserve requirement with a proportional tax on deposit returns levied at a rate equal to the required reserve ratio.

Espinosa (1995) also investigates the properties of multiple-reserves regimes in the case where the government does not care about the fate of the initial old. He shows that, in this case, a multiple-reserves regime in which the government sets the gross real rate of return on reservable government bonds at zero always Pareto-dominates a scheme in which the reservable-bond return rate is positive. Thus, the government can always improve efficiency by replacing a bond reserve requirement with a direct deposit tax; stated differently, it can

[15] In Freeman's model, every agent is a "rich saver."

always improve efficiency by replacing a multiple-reserves regime of the conventional type with a regime that is unconventional (because $R_b < R_m$) to the point of being "degenerate" (because $R_b = 0$). In Proposition 2 of this paper, we show that if a government which has imposed a multiple-reserves regime does not care about about the initial old, then it should not be content with reducing its bond reserve requirement to a degenerate state (or equivalently, replacing it with a deposit tax): it can improve efficiency even further by also eliminating its currency reserve requirement. Thus, the optimal financing regime is actually a degenerate *single-reserve* regime.

Proposition 2 *Suppose the government is not concerned about the welfare of the initial old. Then it is optimal for it to set* $\theta_m = 0$.

Both Espinosa's result and our extension thereof are applications of an argument employed by Freeman (1987). In Freeman's model, setting the gross rate of return on currency at zero is optimal because it the strategy for raising the necessary seigniorage revenue that minimizes public holdings of currency. Currency holdings are inefficient because their "before-tax" rate of return, which is equal to the population growth rate, is lower than the rate of return available via physical investment. In Espinosa's model, bond and currency holdings are both inefficient because their common pretax return rate is lower than the rate of return available in the international credit market. However, the government's concern about the welfare of poor savers constrains it to keep the gross currency rate of return R_m positive. Consequently it is optimal for the government to impose reserve requirements only on government bonds, since it can reduce the rate of return on these bonds without affecting the poor savers.

In the optimal equilibrium implicitly described in our Proposition 1, the government levies an unaugmented (reserve-requirement-free) inflation tax on the currency holdings of poor savers, and a proportional tax on the deposits of rich savers. This result is similar to a result obtained by Mourmouras and Russell (1992), who show, in a model with homogeneous agents but risky returns on privately-issued assets, that when the government seeks to maximize steady-state utility the optimal allocation can always be supported by a combination of an unaugmented inflation tax and a proportional tax on deposits.

4.2 Social-welfare-improving multiple reserve requirements when the government cares about the initial old

Espinosa's finding that a government unconcerned about the welfare of the initial old should choose a degenerate multiple-reserves regime confronted him with the challenge of identifying circumstances under which a nondegenerate regime might be optimal. He responded by establishing that if a government with a degenerate multiple-reserves regime cares about the initial old, then it may be able to improve social welfare by increasing the real rate of return on reservable bonds. This result stems from the fact that a gross real bond rate of zero produces the maximum possible bond seigniorage tax rate and consequently requires, *ceteris paribus*, the minimum possible bond seigniorage tax base. Thus, when the gross real bond rate is zero the government imposes the minimum possible bond reserve requirement, which produces the minimum possible demand for reservable bonds. It follows that if multiple-reserves regimes involving $R_b = 0$ are compared to regimes with the same currency reserve requirement that involve $R_b > 0$, the latter regimes produce higher welfare for the initial-old agents, who are assumed to be endowed with stocks of government currency and reservable bonds. Although these regimes produce lower levels of welfare for rich savers, they maintain the same level of total seigniorage revenues, and may be optimal for some government social-welfare functions.

Espinosa does not actually demonstrate that $R_b > 0$ is ever *necessary* for optimality — that is, that there are any potentially social-welfare-maximizing allocations that cannot be achieved when the gross real bond rate is set at zero. This question arises because, as we show in our Proposition 3, many combinations of deposit interest rates and initial price levels that can be achieved by setting $R_b > 0$ can also be achieved by leaving R_b at zero but increasing the currency reserve ratio. These policies meet the government budget constraint by substituting currency seigniorage revenue for bond seigniorage revenue, and improve the welfare of the initial old by increasing the real value of their currency holdings rather their bond holdings. If this sort of substitution was always possible, the option to impose a genuine multiple-reserves regime (that is, one whose bond reserve requirement was not equivalent to a deposit tax) would never be useful to the government. However, Proposition 2 also establishes that it is not always possible. In particular, allocations supported by conventional multiple-reserve-requirements regimes — regimes in which government bonds yield positive nominal interest — cannot be supported by regimes in which the government combines a single currency reserve requirement with a proportional deposit tax.

Proposition 3 *A public and private allocation $(\overline{G}, \overline{R}_m, \overline{R}_d, 1/\overline{p}_1)$ supported by a multiple-reserve-requirement policy setting with $\overline{R}_b > 0$ can be supported by a setting with $\widehat{R}_b = 0$ if and only if $\overline{R}_b \leq \overline{R}_m$.*

In the corollary to this proposition, we show that there exist model specifications and social-welfare functions under which setting $R_b > R_m$ is optimal. We prove the corollary by example: see Example 3 in the appendix.

Corollary 1 *The multiple-reserve-requirements policy setting that maximizes social welfare may involve $R_b > R_m$.*

In this model, a conventional multiple-reserves regime is equivalent to a single-reserve regime in which the government pays interest, at a below-market rate, on a portion of bank reserves. We are not aware of any example of a country that has imposed a regime of the latter type. One explanation for this may be that in developing countries, the common people often tend to view the banking system with a good deal of suspicion.[16] Politically, imposing a multiple reserve requirement may be easier for the government than paying interest on bank reserves. The government may be able to portray imposing a second reserve requirement as a policy that penalizes the banks, while paying interest on reserves may appear too much like a concession to the banking system.

[16]To the extent that real-world developing economies resemble our model economy, this suspicion is amply justified: bank deposits are accessible only to relatively wealthy agents, and these agents use the banks to obtain rates of return on their savings that are considerably higher than those accessible to the common people.

5 Conclusion

In this paper, we have extended the literature on the nature and optimality of reserve requirements [particularly Fama (1980) and Freeman (1987)] to the case of deficit-finance regimes involving multiple reserve requirements. In particular, we have examined the question of whether an optimizing government might prefer multiple reserve requirements to less complex seigniorage-based financing regimes. Our point of departure has been the analysis conducted by Espinosa (1995), who showed that adopting a multiple-reserves regime could increase efficiency and/or social welfare relative to an initial regime involving a single currency reserve requirement. We have investigated whether these results generalize to a situation in which the government has chosen the policy settings of the initial single-reserve regime optimally. Our results suggest that multiple-reserves regimes cannot increase economic efficiency relative to *optimal* single-reserve regimes, but may be able to produce social-welfare improvements.

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Appendix A

Proof of Proposition 1: The deposit-interest-rate equation $R_d = \theta_m R_m + \theta_b R_b + (1-\theta)R$ implies that, in any binding stationary reserve-requirement equilibrium, the government budget constraint can be rewritten

$$G = (1 - R_m)m(R_m) + (1 - R)\theta d(R_d) + (R - R_d)d(R_d)$$

where $\theta \equiv \theta_m + \theta_b$.

In the original single-reserve-requirement equilibrium we have

$$\overline{R}_d = \overline{\theta}_m \overline{R}_m + (1 - \overline{\theta}_m)R$$

$$[M(0) + B(0)]/\overline{p}_1 = \overline{A} - \overline{G}$$

where $\overline{A} = m(\overline{R}_m) + \overline{\theta}_m d(\overline{R}_d)$ and

$$\overline{G} = (1 - \overline{R}_m)m(\overline{R}_m) + (1 - R)\overline{\theta}_m d(\overline{R}_d) + (R - \overline{R}_d)d(\overline{R}_d).$$

A Pareto-dominant multiple-reserve-requirement equilibrium would be

$$\widehat{R}_d = \widehat{\theta}_m \widehat{R}_m + \widehat{\theta}_b \widehat{R}_b + (1 - \widehat{\theta}_m - \widehat{\theta}_b)R$$

$$[M(0) + B(0)]/\widehat{p}_1 = \widehat{A} - \widehat{G}$$

where $\widehat{A} = m(\widehat{R}_m) + \widehat{\theta} d(\widehat{R}_d)$, and

$$\widehat{G} = (1 - \widehat{R}_m)m(\widehat{R}_m) + (1 - R)\widehat{\theta} d(\widehat{R}_d) + (R - \widehat{R}_d)d(\widehat{R}_d)$$

where $\widehat{\theta} \equiv \widehat{\theta}_m + \widehat{\theta}_b$, with $\widehat{R}_m > \overline{R}_m$, $\widehat{R}_d > \overline{R}_d$, $1/\widehat{p}_1 \geq 1/\overline{p}_1$, and $\widehat{G} \geq \overline{G}$. Suppose we choose an alternative single reserve requirement $\widetilde{\theta}_m$ such that

$$\widetilde{R}_d \equiv \widetilde{\theta}_m \widehat{R}_m + (1 - \widetilde{\theta}_m)R = \widehat{R}_d.$$

The multiple-reserves deposit-rate equation implies that this choice requires $\widetilde{\theta}_m \widehat{R}_m + (1 - \widetilde{\theta}_m)R = \widehat{\theta}_m \widehat{R}_m + \widehat{\theta}_b \widehat{R}_b + (1 - \widehat{\theta}_m - \widehat{\theta}_b)R \Leftrightarrow \widetilde{\theta}_m(\widehat{R}_m - R) = (\widehat{\theta}_m + \widehat{\theta}_b)(\widehat{R}_m - R) + \widehat{\theta}_b(\widehat{R}_b - \widehat{R}_m)$.

Case 1: $\widehat{R}_b > \widehat{R}_m$. In this case $\widetilde{\theta}_m(\overline{R}_m - R) = (\widehat{\theta}_m + \widehat{\theta}_b)(\widehat{R}_m - R) + \widehat{\theta}_b(\widehat{R}_b - \widehat{R}_m) \Leftrightarrow (\widetilde{\theta}_m - \widehat{\theta})(\widehat{R}_m - R) > 0 \Leftrightarrow \widetilde{\theta}_m < \widehat{\theta}$. Since $\widetilde{R}_d = \widehat{R}_d$, the equilibrium government budget equations now imply $\widetilde{G} > \widehat{G}$; since $\widehat{G} \geq \overline{G}$ by assumption, they also imply $\widetilde{G} > \overline{G}$. In single reserve requirement equilibria, moreover, we can write $G = (1-R_m)A \Leftrightarrow A = G/(1-R_m) \Leftrightarrow$ [using the money-demand equation] $[M(0)+B(0)]/p_1 = R_m G/(1-R_m)$. Thus $\widetilde{G} > \overline{G}$ and $\widetilde{R}_m > \overline{R}_m$ implies $1/\widetilde{p}_1 > 1/\overline{p}_1$.

Case 2: $\widehat{R}_b < \widehat{R}_m = \overline{R}_m$. In this case $\widetilde{\theta}_m(\overline{R}_m - R) = (\widehat{\theta}_m + \widehat{\theta}_b)(\widehat{R}_m - R) + \widehat{\theta}_b(\widehat{R}_b - \overline{R}_m) \Leftrightarrow \widetilde{\theta}_m > \widehat{\theta}$. Since $\widetilde{R}_d = \widehat{R}_d$, the money demand and government budget equations imply $[M(0)+B(0)]/\widetilde{p}_1 = \widetilde{A} - \widetilde{G} = \overline{R}_m[m(\overline{R}_m) + \widetilde{\theta}_m d(\widehat{R}_d)]$, and $[M(0)+B(0)]/\widehat{p}_1 = \widehat{A} - \widehat{G} = \overline{R}_m[m(\overline{R}_m) + \widehat{\theta}_m d(\widehat{R}_d)] + \widehat{R}_b\widehat{\theta}_b d(\widehat{R}_d)$. Thus $[M(0)+B(0)]/\widetilde{p}_1 - [M(0)+B(0)]/\widehat{p}_1 = [\overline{R}_m\widetilde{\theta}_m - (\overline{R}_m\widehat{\theta}_m + \widehat{R}_b\widehat{\theta}_b)]d(\widehat{R}_d)$, so $\overline{R}_m > \widehat{R}_b$ and $\widetilde{\theta}_m > \widehat{\theta}$ imply $1/\widetilde{p}_1 > 1/\widehat{p}_1$. And since $1/\widehat{p}_1 \geq 1/\overline{p}_1$ by assumption, we have $1/\widetilde{p}_1 > 1/\overline{p}_1$. As we have just seen, for a single reserve requirement $[M(0)+B(0)]/p_1 = R_m G/(1-R_m)$. Thus $1/\widetilde{p}_1 > 1/\overline{p}_1$ and $\widetilde{R}_m = \overline{R}_m \Rightarrow \widetilde{G} > \overline{G}$. □

Example 1 *Let* $M(0)+B(0) = 1$, $R = 1.2$, $m(R_m) = 10 - 4/R_m$, $d(R_d) = 48 - 43/R_d$ *and* $G = 1.9233$. *This economy has a single-reserve-requirement equilibrium that was described by Espinosa (1995); it involves setting* R_m *at* 0.65 *and* θ_m *at* 0.4, *and produces* $R_d = 0.98$ *and* $1/p_1 \doteq 3.5719$. *However, the allocation supported by this equilibrium is Pareto-dominated by the equilibrium allocation associated with a single-reserve policy setting that keeps* R_m *at* 0.65 *but reduces* θ_m *to* 0.18739. *This equilibrium keeps* $1/p_1$ *unchanged but allows* R_d *to increase to* 1.0969.

In his example, Espinosa (1995) showed that the allocation produced by a multiple-reserves regime with $R_b = 1.1R_m$, $\theta_m = 0.3$ *and* $\theta_b = 0.1$ *also Pareto-dominates his single-reserve allocation. However, Espinosa's multiple-reserve-requirement allocation does not Pareto-dominate the allocation supported by the alternative single-reserve equilibrium just described. The equilibrium values of* R_m *and* $1/p_1$ *are* 0.67475 *and* 4.0907, *respectively, but the equilibrium value of* R_d *is only* 0.99665.

Example 2 *Let* $R = 1.2$, $m(R_m) = 10 - 4/R_m$, *and* $d(R_d) = 16\,(48 - 43/R_d)$. *If* $\overline{G} \doteq 12.8943$ *there is a single-reserve equilibrium with* $\theta = 0.2$, $\overline{R}_m = 0.5$, *and* $\overline{R}_d = 1.06$. *This value of* R_m *is on the "wrong side" of the reserve-unaugmented Laffer curve:* $(1 - R_m)m(R_m) > (1 - \overline{R}_m)m(\overline{R}_m)$ *for all* $R_m \in (\overline{R}_m, 0.8)$. *It is readily seen, however, that increases in* R_m *reduce total seigniorage revenue if* θ *is held constant, and even if* θ *is increased at any rate that preserves* $R_d \geq \overline{R}_d$. *[Since the equilibrium is on the "right side" of the reserve-ratio Laffer curve, reducing* θ *as* R_m *increased would produce even greater revenue* *losses.] In a multiple-reserves regime, however, it is possible to reduce R_b as R_m increases, holding the aggregate reserve ratio fixed, in a way that holds $R_d = \overline{R}_d$, keeps $G \geq \overline{G}$, and allows $1/p_1$ to increase with R_m. An example of such a policy setting is $\widehat{R}_m = 0.8$, $\widehat{R}_b = 0.4$, $\widehat{\theta}_m = 0.05$, and $\widehat{\theta}_b = 0.15$. This setting produces $\widehat{R}_d = \overline{R}_d$ and $\widehat{G} = \overline{G}$ but $1/\widehat{p}_1 > 1/\overline{p}_1$ and (obviously) $\widehat{R}_m > \overline{R}_m$ — an allocation that Pareto-dominates the single-reserve allocation.*

Proof of Proposition 2: As Espinosa (1995) showed that $R_b = 0$ was optimal, we will confine our analysis to policy settings with this feature. Equilibria associated with these settings satisfy

$$G = (1 - R_m)[m(R_m) + \theta_m d(R_d)] + \theta_b d(R_d),$$

and

$$R_d = \theta_m R_m + (1 - \theta)R,$$

where $\theta \equiv \theta_m + \theta_b$.

Suppose we have a binding stationary equilibrium in which an allocation $(\overline{G}, \overline{R}_m, \overline{R}_d)$ with the aforementioned characteristics is supported by a policy setting involving $\overline{\theta}_m > 0$. We want to show that there exists a policy setting involving $\theta_m = 0$ that will suport an allocation $(\widehat{G}, \widehat{R}_m, \widehat{R}_d)$, with $\widehat{G} = \overline{G}$, $\widehat{R}_m = \overline{R}_m$, and $\widehat{R}_d > \overline{R}_d$, as a binding stationary equilibrium. Any equilibrium of this type must satisfy

$$\widehat{G} = (1 - \overline{R}_m)m(\overline{R}_m) + \widehat{\theta}_b d(\widehat{R}_d)$$

where $\widehat{R}_d = (1 - \widehat{\theta}_b)R$. And since $\widehat{G} = \overline{G}$, we must have

$$\widehat{\theta}_b d(\widehat{R}_d) = [(1 - \overline{R}_m)\overline{\theta}_m + \overline{\theta}_b]\, d(\overline{R}_d).$$

Define the function $\psi(\theta)$ by $\psi(\theta) \equiv \theta d(R_d(\theta))$, where $R_d(\theta) = (1 - \theta)R$. Suppose we choose $\widetilde{\theta}$ such that $R_d(\widetilde{\theta}) = \overline{R}_d$. Since $\overline{R}_d = \overline{\theta}_m \overline{R}_m + (1 - \overline{\theta})R$, we have $\widetilde{\theta} = \overline{\theta} - \overline{\theta}_m(\overline{R}_m/R) \in (0, \overline{\theta})$. And since $R > 1 \Rightarrow \overline{\theta} - \overline{\theta}_m(\overline{R}_m/R) > (1 - \overline{R}_m)\overline{\theta}_m + \overline{\theta}_b$, we have $\psi(\widetilde{\theta}) > [(1 - \overline{R}_m)\overline{\theta}_m + \overline{\theta}_b]\, d(\overline{R}_d)$.

Since ψ is a continuous function satisfying $\psi(0) = 0$, we know there exists $\widehat{\theta}_b \in (0, \widetilde{\theta})$ such that $\psi(\widehat{\theta}_b) = \widehat{\theta}_b d(\widehat{R}_d) = [(1 - \overline{R}_m)\overline{\theta}_m + \overline{\theta}_b]\, d(\overline{R}_d)$. And since $R_d(\theta)$ is decreasing in θ, $\widehat{\theta}_b < \widetilde{\theta}$ $\Rightarrow \widehat{R}_d > \overline{R}_d$. □

Proof of Proposition 3: $1/\widehat{p}_1 = 1/\overline{p}_1$ requires $\overline{A} - \overline{G} = \widehat{A} - \widehat{G}$, and $\overline{G} = \widehat{G}$ then implies $\overline{A} = \widehat{A}$. This equality, together with $\widehat{R}_d = \overline{R}_d$ and $\widehat{R}_m = \overline{R}_m$, implies $\widehat{\theta} = \overline{\theta}$. Since $\widehat{R}_b = 0$, $\widehat{R}_d = \overline{R}_d$ and $\widehat{\theta} = \overline{\theta}$ imply $\widehat{\theta}_m \overline{R}_m = \overline{\theta}_m \overline{R}_m + \overline{\theta}_b \overline{R}_b$. Suppose we choose $\widehat{\theta}_m$ and $\widehat{\theta}_b$ appropriately: that is,

$$\widehat{\theta}_m = \overline{\theta}_m + \overline{\theta}_b \frac{\overline{R}_b}{\overline{R}_m}$$

and $\widehat{\theta}_b = \overline{\theta} - \widehat{\theta}_m$. This policy is feasible if and only if $\widehat{\theta}_b \geq 0 \Leftrightarrow \widehat{\theta}_m \leq \overline{\theta}$, which is to say iff $\overline{\theta}_m + \overline{\theta}_b \frac{\overline{R}_b}{\overline{R}_m} \leq \overline{\theta}_m + \overline{\theta}_b$. This will be the case iff $\overline{R}_b \leq \overline{R}_m$. □

Proof of Corollary 1:

Example 3 *Consider the economy described in Example 1. Let the social-welfare function be* $W(R_d, 1/p_1, R_m) = \log R_d + \log 1/p_1 + 0.01 \log R_m$. *The social-welfare-maximizing single-reserve equilibrium is* $\overline{\theta}_m \doteq 0.30656$, $\overline{R}_d \doteq 1.0466$, $1/\overline{p}_1 \doteq 4.4785$ *and* $W(\overline{R}_d, 1/\overline{p}_1, \overline{R}_m) \doteq 1.5412$. *The social-welfare-maximizing multiple-reserve equilibrium can be implemented by a range of different policy settings (see below), each of which involves* $R_b > R_m$. *One such setting is* $\widehat{\theta}_m = 0.2$ *and* $\widehat{\theta}_b \doteq 0.11547$, *which produces* $\widehat{R}_b \doteq 0.74577$. *Each optimal policy setting produces* $\widehat{R}_m \doteq 0.69327$, $\widehat{R}_d \doteq 1.0462$, $1/\widehat{p}_1 \doteq 4.4833$, *and* $W(\widehat{R}_d, 1/\widehat{p}_1, \widehat{R}_m) \doteq 1.5419$. □

Figure 1

Reserve-Ratio Laffer Curve