

# Macroeconomic Fluctuations in Europe: Demand or Supply, Permanent or Temporary?

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**Abstract:** We use generalized method of moments to estimate a rational expectations aggregate demand–aggregate supply macroeconomic model for five European economies. Our aim is to examine whether supply or demand shocks have predominated in the major European economies during the post-war era and whether shocks of either type have been primarily temporary or permanent in nature. The estimation procedure is an alternative to estimating and interpreting vector autoregressions under restrictions either of the Bernanke-Sims variety or the Blanchard-Quah variety or to performing calibration exercises.

We find that all four types of shocks (permanent supply, permanent demand, temporary supply, and temporary demand) are needed to account for the data on output and inflation. Permanent or temporary demand shocks have been the dominant source of variance in output growth in four of the five countries, but there is no consistent pattern for inflation.

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## 1. Introduction

In recent years, a number of authors have used vector autoregressions (VAR's) to investigate whether macroeconomic fluctuations are primarily caused by nominal or real shocks. In this paper, we investigate the sources of macroeconomic fluctuations in the major European economies by estimating an aggregate demand/aggregate supply model with rational expectations. Our model allows macroeconomic fluctuations to arise from either supply or demand shocks. We also allow the demand and supply shocks to have permanent and temporary components that are not separately identifiable. A distinctive feature of the analysis therefore is that the number of driving shocks exceeds the number of endogenous variables. Nevertheless, we are able to estimate the structural parameters, including the variances of the underlying shocks, using generalized method of moments.

The classic paper by Sims (1980) found that nominal shocks were a major source of U.S. fluctuations. Sims argued that the exclusion restrictions commonly used to identify parameters in traditional structural models were not reasonable under rational expectations. When expectations are rational, all relevant predictive variables belong in any equation where expectations appear. While a VAR treats all observable variables as endogenous, the parameter estimates are very difficult to interpret. As a substitute for exclusion restrictions, Sims assumed that his data could be ordered in a Wold causal chain. Since then, various other methods of identifying VAR's have been proposed.

Blanchard and Watson (1986) identify a VAR by restricting the contemporaneous correlations of the one-step-ahead forecast errors. They conclude that U.S. fluctuations are due to fiscal, monetary, demand, and supply shocks, in roughly equal proportions.

Several other authors have used long-run restrictions to identify VAR's. After assuming that demand shocks have zero long-run impact on output, Blanchard and Quah (1989) find that demand shocks are the primary source of U.S. fluctuations. By contrast, Shapiro and Watson (1988) find evidence that exogenous labor supply shocks drive U.S. fluctuations. King, Plosser, Stock, and Watson (1991), who use a combination of long and short-run restrictions to identify their VAR's, report that nominal shocks have little importance and find evidence of at least two separate real shocks.

Gali (1992) examines a structural VAR of the IS-LM variety for the US economy. He assumes there are four shocks: supply, money demand, money supply, and an IS shock (that is, three types of "demand" shocks, and one supply/productivity shock). He identifies parameters through a combination of long-run and short-run restrictions. He finds both types of shocks important, but supply shocks are dominant: 70 percent of output variability at business cycle frequencies is accounted for by supply shocks.

Long-run restrictions on VAR's of the Blanchard-Quah variety have also been used by Ahmed and Park (1994), Bergman (1996), Karras (1994), Bayoumi and Eichengreen (1992) and Whitt (1995) to examine evidence on the sources of macroeconomic shocks in other economies. Ahmed and Park focus on seven OECD countries, including five in Europe. They estimate VAR's with four endogenous variables: home country real output, the price level, the balance of trade, and rest-of-world output, proxied by U.S. output. They report strong support for one of the propositions of real-business-cycle theory, namely that supply-side changes explain the bulk of the movements in aggregate output. Bergman studies five countries, including Germany and the United Kingdom, using a bivariate VAR model for output and inflation. Using variance decompositions, he argues that at a typical business cycle frequency (the five-year horizon), supply shocks are the main source of output variance for all his countries.<sup>1</sup>

By contrast, the other three papers find results less favorable to real-business-cycle theory. Karras (1994) estimates VAR's for three European countries, two of which (France and the U.K.) were analyzed by Ahmed and Park. He uses five variables: home country output, the price level, employment, the real interest rate, and the world price of oil. He concludes that real business cycle models are inadequate, because aggregate demand was responsible for over half of the variability of output at a four-quarter horizon in France and Germany, and about 40 percent in the U.K. Bayoumi and Eichengreen (1992) and Whitt (1995) estimate VAR's with two variables, output and prices. Like Karras, they find that aggregate demand shocks account for a substantial portion of output fluctuations in major European countries.

In this paper, we also estimate a small structural model of output fluctuations for several European countries. We follow Hartley and Walsh (1992), however, and use a method of moments procedure to identify the parameters rather than long-run restrictions of the Blanchard-Quah variety. Our results thus are immune from the Lippi and Reichlin (1993) and Faust and Leeper (1994) criticisms of the Blanchard-Quah approach. In addition, structural modeling of the type proposed by Hartley and Walsh (1992), and pursued in this paper, has the advantage of giving the estimated parameters a clear economic interpretation, something often lacking in VAR analyses.

Hartley and Walsh assumed a structure of supply and demand curves where each curve could be affected by more than one shock. When the number of unobserved exogenous shocks exceeds the number of observed endogenous variables, the econometrician cannot recover a time series for the shocks from the data. However, the endogenous variables can be expressed as a vector autoregressive moving average pro-

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<sup>1</sup> It is debatable whether Bergman's results for Germany are entirely supportive of real-business-cycle theory. At a three-year (12-quarter) horizon, only 35 percent of output variance in Germany is attributable to supply shocks.

cess of the shocks. This VARMA representation yields expressions for the contemporaneous and lagged variance and covariances of the endogenous variables as a function of the various supply and demand elasticities and the variances (and possibly covariances) of the underlying shocks.

An initial estimation chooses parameter values to minimize the sum of squared differences between the theoretical second moments and the corresponding sample second moments obtained from the data. A second estimation minimizes a weighted sum of squared deviations with weights chosen “optimally” to yield a test of the parameter restrictions.

The method of moments estimation used by Hartley and Walsh is closely related to the “calibration method” used to evaluate real business cycle models. Whereas the parameters are usually at best just-identified in the typical calibration exercise, however, the number of moments fit in the method of moments estimation can exceed the number of parameters. The over-identifying restrictions can then be tested. The method of moments procedure also allows us to estimate standard errors for the parameter values and this provides further information on the fit between the model and the data.<sup>2</sup>

Hartley and Walsh (1992) were particularly interested in investigating a possible role for inside money in initiating or propagating business cycles. They developed a “non-standard” model that reflected various institutional features of the U.S. money market, and they applied it solely to U.S. data. By contrast, our focus is the relative importance of supply versus demand shocks, and, within each of these categories, permanent versus temporary components. We also want to facilitate cross-country comparisons. We therefore use a simpler and more common structure than Hartley and Walsh (1992), with only demand and supply shocks.

Identification of the different shocks is fundamentally based on the idea that demand shocks tend to push prices and output in the same direction, while supply shocks push them in opposite directions. Since we allow for expectations and lags, and permanent and temporary components of each type of shock, however, our model can account for more complicated patterns of correlations between prices and output.

We find that all four types of shocks (permanent supply, permanent demand, temporary supply, and temporary demand) are needed to account for the data. Permanent demand shocks have been the dominant source of variance in output growth in Germany, the UK and the Netherlands. Temporary demand shocks

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<sup>2</sup> Another difference is that, in the typical calibration exercise, the model is kept simple enough that each moment is primarily dependent on the values of a small number of key parameters. In the models examined in this paper, and in Hartley and Walsh (1992), however, the second moments are complicated functions of the parameters. It is then no longer obvious how parameter values should be set so as to optimally match the theoretical second moments to their corresponding sample values. The main source of this non-linearity is the assumption that expectations are formed rationally.

have been about twice as important a source of variance in output growth in Italy, while all four shocks have been of roughly equal importance in France. However, permanent supply shocks have been the main source of longer run positive autocorrelation in output growth in all countries. In all countries demand shocks contribute to longer run negative autocorrelations in output growth.

Inflation variances and autocovariances have been dominated by permanent supply shocks in France. In the remaining countries, permanent supply and demand shocks have been of roughly similar importance.

We find net positive covariances between output growth and *future* inflation in the data from Germany, France, and Italy. These results stand in contrast to the findings of Cooley and Ohanian (1991, Table 1) and Kydland and Prescott (1990, Table 4) for the United States that these covariances are negative at nearly all leads and lags. Our parameter estimates imply that in all economies, permanent supply shocks have contributed to a negative covariance between output growth and current, lagged, and future inflation. By contrast, permanent demand shocks have contributed to a negative covariance between output growth and *lagged* inflation, but a positive covariance between output growth and *current and future* inflation. Temporary shocks have also influenced the contemporaneous correlations between output growth and inflation in all countries.

## **2. Integration and co-integration tests**

There are few a priori theoretical restrictions on the possible number, or stationarity properties, of the shocks affecting the macroeconomy. Before developing and estimating the model, therefore, the data need to be examined for stationarity and possible co-integration features. The assumed stochastic structure of the theoretical model then needs to be consistent with the stationarity properties of the data.

Quarterly data on industrial production and producer prices, both seasonally-adjusted, were obtained from the IFS or the BIS for the five largest West European economies – Germany, France, UK, Netherlands and Italy. The data are described in more detail in Appendix 2. The well-known augmented Dickey-Fuller test was applied to the quarterly series, logged, in order to assess the number of unit roots (permanent shocks) in the data. The results are presented in Table 1.

The pattern for Germany is clear: we fail to reject a single unit root in each series, we do reject two unit roots in each, and we fail to reject an absence of co-integration. For the other 20 tests, 17 conform to the German pattern.

The first exception is France (column 1). The test indicates a weak rejection of a single unit root in output (the 1 percent critical value is about -3.97), suggesting that the output level might be stationary.

The other two exceptions are both in column 4 for the UK and the Netherlands. In these two instances, we fail to reject the null hypothesis of two unit roots in the price series at the five (or even the 10) percent levels. Graphs of the data indicated that for a lengthy period in the middle of the sample (roughly 1973 to 1981), the mean rate of growth of prices was substantially higher than at other times. We considered using dummy variables to create adjusted series, but chose not to do so for two reasons: first, such dummy variables might well remove from the data major supply or demand shocks, and second, we thought it desirable to maintain cross-country consistency by using the same pre-filter for all countries.

**TABLE 1. Dickey-Fuller tests of stationarity and co-integration<sup>a</sup>**

	Industrial production		Producer prices		Co-integration
	1 unit root <sup>b</sup>	2 unit roots	1 unit root	2 unit roots	
Germany	-2.42 (-3.45)	-8.33	-1.31	-3.97	-1.87 (-3.86)
France	-3.54 (-3.46)	-8.14	-1.08	-4.72	-3.31 (-3.87)
UK	-2.89 (-3.44)	-10.76	-1.68	-2.32	-2.78 (-3.85)
Netherlands	-1.92 (-3.44)	-4.78	-1.22	-3.03	-1.60 (-3.85)
Italy	-1.95 (-3.45)	-11.90	-1.56	-4.90	-0.78 (-3.86)

a. Tests were done including zero to eight lags of the dependent variables in order to deal with the possibility of serially-correlated residuals. The test statistics in the table use the specification that was the “best” according to the Schwarz criterion, subject to having residuals with a Ljung-Box Q statistic that failed to reject the null hypothesis of no serial correlation at a 10 percent or higher significance level.

b. The 5 percent critical value for each country's unit root tests are given in parentheses below the test statistics in column 1; the same critical value also applies to columns 2 to 4. The 5 percent critical values for the Engle-Granger co-integration tests are in parentheses below the test statistics in column 5. All critical values were obtained from Mackinnon (1991).

Based on these results, we constructed an aggregate demand/aggregate supply model with two independent permanent shocks.<sup>3</sup> We shall assume one of these shocks is a supply shock and one is demand shock. We make no assumptions, however, about whether the demand shock is real or nominal.

### 3. Aggregate Supply

We assume there is a supply shock  $s_t$  at  $t$  that is a *combined* temporary and permanent shock. Temporary supply shocks could represent the effect of strikes, severe weather or other temporary influences on aggregate production. Permanent supply shocks represent long-lasting shifts in aggregate supply associ-

<sup>3</sup> While aggregate demand/aggregate supply models that use the IS-LM framework have been criticized in recent years, McCallum (1989, pp. 102-107) argues that if the supply function has classical properties, as is the case in the model in this paper, then the resulting model is for many purposes rather similar to models derived from explicit maximization of agents' choice problems.

ated, for example, with changes in technology and factor supplies.

While  $s_t$  is known at  $t$ , neither agents in the economy nor the observing econometrician know for sure what part of  $s_t$  will be permanent.<sup>4</sup> We initially assume that agents learn the temporary versus permanent composition of supply shocks after one period. Later in the paper, we consider a model where agents do not know the composition of the supply and demand shocks for two periods.

We also assume supply increases when current prices rise above the rationally expected prices based on the previous period's information. Lucas (1973) provides a justification for such an effect when suppliers are confused about whether shocks are primarily local (and real) or aggregate (and nominal). Our model does not distinguish between local and aggregate shocks, while agents always know the current demand and supply shocks. They are confused only about the permanence of those shocks. Nevertheless, we can obtain an analog of the Lucas supply curve if we assume suppliers base their expectations on last period's information. Alternatively, Fischer (1977) generates such a supply curve in a model where suppliers pre-commit to contracts one period in advance.

Finally, we allow supply to be autocorrelated. This could result, for example, from investments that transmit current deviations of supply into future periods. Thus, the aggregate supply curve can be written (where all variables are in logarithms):

$$y_t = \rho y_{t-1} + \gamma(p_t - E_{t-1}p_t) + s_t \quad (1)$$

with

$$s_t = \xi_t + \varepsilon_t = \xi_{t-1} + \zeta_t + \varepsilon_t \quad (2)$$

The shocks  $\zeta$  (the innovation to the permanent component of the overall supply shock) and  $\varepsilon$  (the temporary component) are assumed to be uncorrelated at all leads and lags and each of them is assumed to be independently identically distributed. Because we use GMM for estimation, we do not need to specify a distribution for the shocks  $\zeta$  and  $\varepsilon$ . We merely need to assume that both shocks have finite second moments. The same is true of the components of the demand shocks that are specified below.

We shall assume that the number of integrated random variables among the driving shocks matches the number of non-stationary driving shocks indicated by the unit-root and co-integration analysis. The structural model then must be constructed so that it would yield stationary endogenous variables if the driving shocks had also been stationary. In particular, the autocorrelation parameter,  $\rho$  needs to lie in the interval

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<sup>4</sup> Brunner, Cukierman and Meltzer (1980) developed a similar theoretical model in which macroeconomic fluctuations arise because agents cannot distinguish permanent from temporary shocks.

(-1,1). We expect the elasticity coefficient  $\gamma$  to be positive.

#### 4. Aggregate Demand

We allow the aggregate demand for output to respond negatively to the real interest rate. Following Sieper (1989), we assume that, in contrast to factor markets where expectations are based on information available at  $t-1$ , expectations in capital markets are based on information available at  $t$ . There is also a real demand shock  $\chi_t$  that represents shifts in the IS curve. Examples of such shifts are changes in demographics, fiscal policy or export demand. As with aggregate supply, we allow aggregate demand to be autocorrelated. Thus, the aggregate demand curve can be written (with variables other than the interest rate in logarithms):

$$y_t = \eta y_{t-1} - \alpha(i_t - E_t p_{t+1} + p_t) + \chi_t \quad (3)$$

We expect  $\alpha$  to be positive and again require the autocorrelation parameter,  $\eta$  to lie in the interval (-1,1).

##### *Money Market*

We also postulate a conventional aggregate demand for money balances:

$$m_t - p_t = \beta y_t - \delta^{-1} i_t + \omega_t \quad (4)$$

where  $\omega$  is a shock to money demand and  $\delta^{-1}$  is the interest semi-elasticity of the demand for money.

##### *Reduced form aggregate demand curve*

We assume equilibrium  $p_t$  and  $i_t$  equate aggregate supply and aggregate demand for goods and money.

From the money market equilibrium condition we can conclude that

$$i_t = \beta \delta y_t - \delta(m_t - p_t - \omega_t). \quad (5)$$

Substitute (5) into the aggregate demand curve (3) to deduce that it can be written:

$$y_t = \eta y_{t-1} - \alpha \left[ \beta \delta y_t - \delta \left( m_t - \omega_t + \frac{\chi_t}{\alpha \delta} - p_t \right) - (E_t p_{t+1} - p_t) \right] \quad (6)$$

Equation (6) can be re-arranged to yield

$$y_t = \frac{\eta}{1 + \alpha \beta \delta} y_{t-1} + \frac{\alpha \delta}{1 + \alpha \beta \delta} \left( m_t - \omega_t + \frac{\chi_t}{\alpha \delta} - p_t \right) + \frac{\alpha}{1 + \alpha \beta \delta} (E_t p_{t+1} - p_t) \quad (7)$$

As shown in the middle term in (7), shocks to aggregate demand can arise in many ways: besides the real demand (IS) shocks represented by  $\chi_t$ , monetary policy can generate nominal shocks by changing the true money supply  $m$ , and changes in financial intermediation technology among other factors can produce



real shocks to money demand  $\omega$ .

We assume neither the public nor the econometrician observe  $m$ ,  $\omega$  or  $\chi$ . Nevertheless, using  $y$ ,  $p$ ,  $E_t p_{t+1}$  and (7) the public can infer the value of the amalgamated demand shock  $d$  (defined as  $m_t - \omega_t + \chi_t / \alpha \delta$ ). We can write the aggregate demand curve in terms of prices and the demand shock  $d$  alone in the form

$$y_t = \psi y_{t-1} + \Gamma(d_t - p_t) + \Phi(E_t p_{t+1} - p_t) \quad (8)$$

We conduct the subsequent analysis using (8) for the aggregate demand curve. Analogously to the supply shock  $s_t$  we assume that the demand shock  $d_t$  is a *combined* temporary and permanent shock.

$$d_t = \mu_t + \tau_t = \mu_{t-1} + \nu_t + \tau_t \quad (9)$$

The shocks  $\nu$  and  $\tau$  are assumed to be independently identically distributed and uncorrelated at all leads and lags with each other and with the supply shocks.

We again assume that neither the econometrician nor the agents in the economy know how much of a current demand shock is temporary and how much is permanent. Specifically, while  $d_t$  is known, the components  $\mu_{t-1}$ ,  $\nu_t$  and  $\tau_t$  are not. We again assume, however, that agents learn the temporary versus permanent (but not the real versus nominal) composition of  $d_t$  after one (or, later in the paper, two) period(s).

## 5. Equilibrium

Using the lag operator  $L$ , the aggregate supply curve (1) can be written:

$$(1 - \rho L)y_t = \gamma(p_t - E_{t-1}p_t) + s_t \quad (10)$$

while the aggregate demand curve (8) can be written

$$(1 - \psi L)y_t = \Phi E_t p_{t+1} - (\Phi + \Gamma)p_t + \Gamma d_t. \quad (11)$$

Multiplying (10) by  $(1 - \psi L)$  and (11) by  $(1 - \rho L)$  we deduce that product market equilibrium requires

$$\begin{aligned} (1 - \psi L)[\gamma(p_t - E_{t-1}p_t) + s_t] &= (1 - \rho L)[\Phi E_t p_{t+1} - (\Phi + \Gamma)p_t + \Gamma d_t] \\ &= \Phi E_t p_{t+1} - \Phi \rho E_{t-1} p_t - (\Phi + \Gamma)(1 - \rho L)p_t + \Gamma(1 - \rho L)d_t \end{aligned} \quad (12)$$

Since the composite shocks  $s_t$  and  $d_t$  are non-stationary,  $p_t$  is also non-stationary. To solve for the equilibrium price and output, we need to manipulate equation (12) to ensure we are working in spaces of stationary processes. By adding and subtracting  $\Phi \rho p_t$ , equation (12) can be re-arranged to obtain

$$\Phi E_t p_{t+1} - (\Phi + \Gamma)(1 - \rho L)p_t - \Phi \rho p_t = (1 - \psi L)s_t - \Gamma(1 - \rho L)d_t + (\gamma - \Phi \rho - \psi \gamma L)(p_t - E_{t-1}p_t). \quad (13)$$

Now observe that  $p_t - E_{t-1}p_t = P_t - E_{t-1}P_t$  is stationary<sup>5</sup> while

$$(1 - L)E_t p_{t+1} = E_t p_{t+1} - E_{t-1}p_t = E_t p_{t+1} - p_t + p_t - E_{t-1}p_t = E_t[(1 - L)p_{t+1}] + (p_t - E_{t-1}p_t). \quad (14)$$

Thus, differencing (13), we obtain a stochastic difference equation for  $P_t = (1-L)p_t$ :

$$\Phi E_t P_{t+1} - (\Phi + \Gamma)(1 - \rho L)P_t - \Phi \rho P_t = (1 - \psi L)(\zeta_t + \varepsilon_t - \varepsilon_{t-1}) - \Gamma(1 - \rho L)(v_t + \tau_t - \tau_{t-1}) + [(\gamma - \Phi \rho - \Phi) - (\psi \gamma + \gamma - \Phi \rho)L + \psi \gamma L^2](P_t - E_{t-1}P_t) \quad (15)$$

## 6. Information processing

Individuals know the functional forms of the aggregate demand and supply curves. They also know  $p_t$  and  $y_t$ , and therefore the values of  $s_t$  and  $d_t$ , at time  $t$ . We assume to begin with, however, they do not know the decomposition of  $s_t$  or  $d_t$  into their components  $\zeta_t$ ,  $\varepsilon_t$ ,  $\mu_t$  or  $\tau_t$  until period  $t+1$ . From these assumptions about information, and the form of (15), we deduce that  $P_t$  will be a linear function of current and lagged  $\zeta$ ,  $\varepsilon$ ,  $v$  and  $\tau$ . Since individuals know, at  $t-1$ , all shocks dated  $t-2$  or earlier,  $(P_t - E_{t-1}P_t)$  will be a linear sum:

$$P_t - E_{t-1}P_t = \pi_{10}\zeta_t + \pi_{20}\varepsilon_t + \pi_{30}v_t + \pi_{40}\tau_t + \pi_{11}(\zeta_{t-1} - E_{t-1}\zeta_{t-1}) + \pi_{21}(\varepsilon_{t-1} - E_{t-1}\varepsilon_{t-1}) + \pi_{31}(v_{t-1} - E_{t-1}v_{t-1}) + \pi_{41}(\tau_{t-1} - E_{t-1}\tau_{t-1}) \quad (16)$$

Since individuals know  $\xi_{t-2}$ ,  $\mu_{t-2}$ ,  $\varepsilon_{t-2}$ ,  $\tau_{t-2}$ ,  $\Delta s_{t-1} = \zeta_{t-1} + \varepsilon_{t-1} - \varepsilon_{t-2}$  and  $\Delta d_{t-1} = v_{t-1} + \tau_{t-1} - \tau_{t-2}$  at  $t-1$ , they will also observe  $\zeta_{t-1} + \varepsilon_{t-1}$  and  $v_{t-1} + \tau_{t-1}$ . Projecting onto these variables they would obtain:

$$E_{t-1}\zeta_{t-1} = a_1(\zeta_{t-1} + \varepsilon_{t-1}) \quad (17)$$

$$E_{t-1}\varepsilon_{t-2} = a_2(\zeta_{t-1} + \varepsilon_{t-1}) \quad (18)$$

$$E_{t-1}v_{t-1} = b_1(v_{t-1} + \tau_{t-1}) \quad (19)$$

$$E_{t-1}\tau_{t-1} = b_2(v_{t-1} + \tau_{t-1}) \quad (20)$$

where

$$a_1 = \frac{\sigma_\zeta^2}{\sigma_\zeta^2 + \sigma_\varepsilon^2}, a_2 = \frac{\sigma_\varepsilon^2}{\sigma_\zeta^2 + \sigma_\varepsilon^2}, b_1 = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\tau^2} \text{ and } b_2 = \frac{\sigma_\tau^2}{\sigma_v^2 + \sigma_\tau^2}. \quad (21)$$

Observe that  $a_2 = 1 - a_1$  and  $b_2 = 1 - b_1$ .

## 7. ARIMA representations for $p_t$ and $y_t$

We define the inverse of the lag operator by

$$L^{-1}x_{t-i} = \begin{cases} x_{t-i+1} & i > 0 \\ E_t x_{t-i+1} & i \leq 0 \end{cases} \quad (22)$$

where  $x_t$  is known at time  $t$ . Then the equilibrium solution for  $P_t$  can be written in terms of current and

<sup>5</sup> Thus, while  $p_t$  and  $E_{t-1}p_t$  are both non-stationary, they are co-integrated.

lagged shocks using the operators  $L$  and  $L^{-1}$ :

**Lemma 1:** The equilibrium inflation rate  $P_t$  satisfies the stochastic difference equation:

$$(\Phi + \Gamma) \left( 1 - \frac{\Phi}{\Phi + \Gamma} L^{-1} \right) (1 - \rho L) P_t = \Gamma (1 - \rho L) (v_t + \tau_t - \tau_{t-1}) - (1 - \psi L) (\zeta_t + \varepsilon_t - \varepsilon_{t-1}) - \left[ (\gamma - \Phi \rho - \Phi) - (\psi \gamma + \gamma - \Phi \rho) L + \psi \gamma L^2 \right] \left[ \sum_{i=0}^1 (\kappa_{1i} \zeta_{t-i} + \kappa_{2i} \varepsilon_{t-i} + \kappa_{3i} v_{t-i} + \kappa_{4i} \tau_{t-i}) \right] \quad (23)$$

for constant coefficients  $\kappa_{i0} = \pi_{i0}$ ,  $i = 1, \dots, 4$ , and

$$\kappa_{11} = (\pi_{11} - \pi_{21}) a_2, \quad \kappa_{21} = -(\pi_{11} - \pi_{21}) a_1, \quad \kappa_{31} = (\pi_{31} - \pi_{41}) b_2 \quad \text{and} \quad \kappa_{41} = -(\pi_{31} - \pi_{41}) b_1. \quad (24)$$

**Proof.** The left side of (15) can be written

$$\begin{aligned} \Phi E_t P_{t+1} - (\Phi + \Gamma) (1 - \rho L) P_t - \Phi \rho P_t &= -(\Phi + \Gamma) \left[ -\frac{\Phi}{\Phi + \Gamma} E_t P_{t+1} + \left( 1 + \frac{\Phi}{\Phi + \Gamma} \rho \right) P_t - \rho P_{t-1} \right] \\ &= -(\Phi + \Gamma) \left[ \left( 1 - \frac{\Phi}{\Phi + \Gamma} L^{-1} \right) P_t - \left( 1 - \frac{\Phi}{\Phi + \Gamma} L^{-1} \right) \rho P_{t-1} \right] \end{aligned} \quad (25)$$

Also, substitute (17)–(20) into the right side of (16) and then substitute the result into (15).

Now define  $F = \Phi / (\Phi + \Gamma)$  and observe that for  $\Phi$  and  $\Gamma$  positive,  $F < 1$ . Also, all the shocks on the right side of (23) are stationary. The polynomial in  $L^{-1}$  on the left side of (23) can therefore be expanded as a geometric series on the right side of (23). Then by using (22), and the fact that the shocks on the right side of (23) are independently distributed we can show:

**Theorem 1:** When the composition of shocks is unknown for one period, equilibrium inflation satisfies

$$(1 - \rho L) P_t = \sum_{i=0}^3 [\pi_{1i} \zeta_{t-i} + \pi_{2i} \varepsilon_{t-i} + \pi_{3i} v_{t-i} + \pi_{4i} \tau_{t-i}] \quad (26)$$

where the coefficients  $\pi_{ij}$ ,  $i = 1, \dots, 4$ ,  $j = 0, \dots, 3$  satisfy equations (65)–(78) in appendix 1.

**Proof.** The proof is given in appendix 1.

**Comment:** Note that the solution (26) is consistent with the unanticipated inflation rate given in (16).

Use  $\Pi_1$  for the  $4 \times 4$  matrix of MA coefficients with  $\Pi_{1j}$  the  $j$ th column of  $\Pi_1$ , so the 4 polynomials multiplying  $z_t = [\zeta_t \ \varepsilon_t \ v_t \ \tau_t]'$  are the rows of

$$\Pi_1(L) = \sum_{j=1}^4 \Pi_{1j}' L^{j-1}$$

Then we can write the ARMA(1,3) representation for  $P_t$  as:

$$(1 - \rho L)P_t = \Pi_1(L)z_t. \quad (27)$$

From the supply curve (1), (16) and (17)–(20) we obtain an expression for equilibrium output:

**Theorem 2:** When the composition of shocks is unknown for one period, equilibrium output  $y_t$  satisfies:

$$(1 - \rho L)y_t = s_t + \gamma \sum_{i=0}^1 (\kappa_{1i}\zeta_{t-i} + \kappa_{2i}\varepsilon_{t-i} + \kappa_{3i}\nu_{t-i} + \kappa_{4i}\tau_{t-i}) \quad (28)$$

where  $\kappa_{i0} = \pi_{i0}$ ,  $i = 1, \dots, 4$  while  $\kappa_{i1}$ ,  $i = 1, \dots, 4$ , satisfy (24).

**Proof.** Substitute (17)–(20) and the right hand side of (16) into the aggregate supply curve (1).

**Corollary:** The first difference of the equilibrium output  $Y_t = \Delta y_t$  follows an ARMA(1,2) process.<sup>6</sup>

**Proof.** Multiply (28) through by  $(1-L)$ .

If we define a  $4 \times 3$  matrix  $\Pi_2$  of MA coefficients, we can write the ARMA(1,2) representation for  $Y_t$ :

$$(1 - \rho L)Y_t = \Pi_2(L)z_t. \quad (29)$$

Appendix 1 then shows how the ARMA representations (27) for  $P_t = \Delta p_t$  and (29)  $Y_t = \Delta y_t$  can be used to derive theoretical expressions for the variances and autocovariances of  $P_t$  or  $Y_t$  and the cross covariances between current and lagged values of  $P_t$  and  $Y_t$ .

## 8. Estimating the parameters using GMM

We examined lags up to six quarters for the autocovariances and cross covariances. We expected that this would cover a substantial part of typical cyclical fluctuations while leaving us a reasonable sample size (from the original roughly 100 to 130 quarters). We thus obtained theoretical expressions for 2 variances and 25 covariances of rates of change of equilibrium output and price. There are 9 parameters in these expressions. We can write the vector of parameters to be estimated as<sup>7</sup>

$$b = [\rho, \psi, \gamma, \Gamma, \Phi, \sigma_\zeta, \sigma_\varepsilon, \sigma_\nu, \sigma_\tau] \quad (30)$$

and we can denote the  $27 \times 1$  vector of theoretical second moments by  $\theta(b)$ .

From the data, we have  $N$  observations on trend-corrected and seasonally adjusted quarterly rates of

<sup>6</sup> Since output growth follows a stationary ARMA process, demand or supply shocks cannot permanently affect output *growth* – the coefficients on the permanent shocks must eventually decline to zero. The long run effect of a demand or supply shock on the *level* of output, however, is given by the *sum* of the coefficients in the output growth ARMA, which can be non-zero. Buiter (1995, note 13) has argued that the restriction, used by Blanchard and Quah (1989) and others, that demand shocks have no long-run real effects, makes sense for nominal, but not real, demand shocks.

<sup>7</sup> We estimated standard deviations instead of variances to impose the restriction that the variances are non-negative.

change in industrial production and producer prices. Using this data, we calculate  $27 \times N$  cross products corresponding to our 27 theoretical second moments, with one set of cross products for each period  $n$ . Following the notation of Hansen (1982), we write  $f(\Delta x_n, b)$  for the  $27 \times 1$  vector of differences between the sample cross products in period  $n$  and the corresponding theoretical second moments in  $\theta(b)$ . Under the null hypothesis,  $E[f(\Delta x_n, b)] = 0$ . We form

$$g_N(b) = \frac{1}{N} \sum_{n=1}^N f(\Delta x_n, b), \quad (31)$$

which, in our case, equals the vector of differences between the empirical second moments and the corresponding theoretical second moments.

Initial estimates  $\hat{b}$  of  $b$  are obtained by minimizing the sum of squared errors  $g_N(b)'g_N(b)$ .<sup>8</sup> Following Hansen (1982), Cumby, Huizinga and Obstfeld (1983) and White and Domowitz (1984) we conclude that  $\sqrt{N}(\hat{b} - b)$  will converge in distribution to a random vector with mean zero and covariance matrix

$$(d'd)^{-1}d'Sd(d'd)^{-1} \quad (32)$$

where

$$d = E\left[\frac{\partial}{\partial b}f(b)\right] \quad (33)$$

and the matrix  $S$  is defined by

$$S = \sum_{j=-\infty}^{\infty} E[f(\Delta x_0, b)f(\Delta x_{-j}, b)']. \quad (34)$$

An estimate of  $d$  can be obtained using the least square parameter estimates  $\hat{b}$ :

$$\hat{d} = \left[\frac{\partial}{\partial b}g_N(\hat{b})\right] = -\left[\frac{\partial}{\partial b}\theta(\hat{b})\right] \quad (35)$$

Following Newey and West (1987) we estimate  $S$  by<sup>9</sup>

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<sup>8</sup>. In practice, the numerical minimization algorithm worked better when we normalized by re-scaling parameter values and dividing  $g_N(b)'g_N(b)$  by the sum of squared values of the sample moments. We used a combination of a derivative-based quasi-Newton method and the Nelder-Mead simplex algorithm to minimize the highly non-linear objective function. The simplex algorithm proved more effective at finding the general region of parameter space where a minimum lies, while the derivative-based algorithm was more effective at actually attaining the local minimum to be found in that region. To ensure we obtained a global minimum of the objective function, we tried many different starting values for the parameters.

<sup>9</sup>. In our empirical analysis, we used  $J = 12$ .

$$\hat{S}_J = \hat{\Omega}_0 + \sum_{j=1}^J w(j, J) [\hat{\Omega}_j + \hat{\Omega}_j'] \quad (36)$$

where  $w(j, J) = 1 - [j/(J+1)]$  is a linearly declining weighting function and

$$\hat{\Omega}_j = \frac{1}{N} \sum_{n=j+1}^N f(\Delta x_n, b) f(\Delta x_{n-j}, b)'. \quad (37)$$

Hansen (1982) shows that the optimal GMM estimator (in the sense that the asymptotic covariance matrix of  $b$  is as small as possible) is obtained by minimizing a weighted sum of squares<sup>10</sup>  $g_N(b)' W g_N(b)$ , for a symmetric weighting matrix  $W$  which is a consistent estimator of  $S^{-1}$ . If we let  $\tilde{b}$  be the parameter vector which minimizes this weighted sum of squares then  $\sqrt{N}(\tilde{b} - b)$  will converge in distribution to a random vector with mean zero and covariance matrix  $(d'Sd)^{-1}$ , which can be estimated by

$$(\hat{d}' \hat{S}_J \hat{d})^{-1} \quad (38)$$

Following the suggestion in Hansen (1982), we test the over-identifying restrictions by evaluating

$$N g_N(\tilde{b})' (\hat{S}_N)^{-1} g_N(\tilde{b}), \quad (39)$$

which converges in distribution to a chi-square random variable with  $r - q$  degrees of freedom where  $r$  is the number of moment conditions (27 in our case) and  $q$  the number of parameters (9 in our case).

By analogy with variance decompositions in VAR's, we shall use the final parameter estimates to decompose the variances and covariances into the components due to each of the underlying shocks. This will provide our measure of the relative importance of supply and demand, and temporary and permanent shocks in driving output and prices over the sample period.

## 9. Results for the first model

The 27 moments used to estimate the model were the variance of output growth, the contemporaneous covariance between output growth and producer-price inflation, the variance of inflation, each variable's autocovariances up to six quarters, the contemporaneous cross-covariance, and other cross-covariances going forward and back up to six quarters. In all countries the sample variance of output growth is greater than the variance of inflation, but the disparity varies considerably across countries. The ratio of the variance of output growth to the variance of inflation ranges from 4.2 for Germany to 1.4 for Italy.

The pattern of sample cross-covariances warrants discussion. Kydland and Prescott (1990) and Cooley

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<sup>10</sup> In effect, the weighting matrix emphasizes those moments that can be estimated more precisely from the data.

and Ohanian (1991) report negative cross-covariances between filtered prices and output for the United States at nearly all leads and lags. This led Kydland and Prescott to call the notion of a positive relationship between prices and output a monetary myth.

For our countries, we find somewhat different patterns. The contemporaneous cross-covariance is sizeable and negative for the United Kingdom, but small and positive for France and Italy, and small and negative for Germany and Netherlands. The cross-covariances between output growth and positive lags of inflation are consistently negative, thereby conforming to the U.S. pattern: the negative sign means that when inflation rises, output tends to fall several quarters later. However, the cross-covariances in the other direction, between output growth and future (negative lags of) inflation, are quite variable: mostly negative for the United Kingdom and the Netherlands, but mostly positive for the other three countries.<sup>11</sup>

**TABLE 2. Least squares parameter estimates**

Parameter	Germany	Germany <sup>a</sup>	France	U. K.	Netherlands	Italy
$\tanh^{-1}(\rho)$	1.1893	1.1799	1.2438	1.5784	1.7080	1.2144
$\tanh^{-1}(\psi)$	0.4678	0.2483	0.4219	-0.0136	0.3251	0.0198
$\gamma$	4.2610	3.5385	2.1389	9.2372	3.7430	2.1372
$\Gamma$	0.8208	1.1034	0.2943	0.4533	0.4877	0.6219
$\tanh^{-1}(F)$	0.2464	0.4600	1.4696	-2.9552	1.6942	-3.0595
$\sigma_{\zeta}$	0.004636	0.004694	0.002508	0.001812	0.001397	0.003311
$\sigma_{\varepsilon}$	0.005384	0.015243	0.016526	0.060779	0.035365	0.029791
$\sigma_{\nu}$	0.014694	0.014690	0.000010	0.043592	0.009277	0.056177
$\sigma_{\tau}$	0.012309	0.0000003	0.002442	0.000323	0.011721	0.000050
LS objective	0.09150	0.09181	0.13900	0.02689	0.05300	0.08444

a. We found two sets of estimates for Germany with similar least squares values. While the first set had a lower minimized least squares objective, the second set a lower weighted least squares objective and lower estimated standard errors.

The least squares estimates of the parameters, and the corresponding minimized value for the (normalized) sum of squares objective function, are presented in Table 2. We defined the parameters so that all except the autocorrelation coefficients ( $\rho$  and  $\psi$ ) should be positive. If  $\rho$  represents lags in the capital accumulation process, however, we would expect it also to be positive. We do require both  $\rho$  and  $\psi$  to be less than 1 in absolute value. As with ARIMA models, the same autocorrelation structure can be explained either by stationary or non-stationary, and invertible or non-invertible processes. We have elim-

<sup>11</sup>. Several factors may account for the differences between our results and those of Cooley and Ohanian and Kydland and Prescott: the countries, the measures of output and inflation, and the way the data were filtered all differ. Cooley and Ohanian use real GNP and implicit price deflators, while Kydland and Prescott use real GNP and two price measures, the implicit price deflator and consumer price. We use industrial production and producer prices. As for filters, Kydland and Prescott use only the Hodrick-Prescott filter, while Cooley and Ohanian use three filters: linear detrending, differencing, and the Hodrick-Prescott filter. We use differencing but in addition we remove a linear trend and seasonal effects.

inated this identification problem by ensuring the numerical algorithm concentrates on stationary and invertible representations of the data. Similarly, the coefficient  $F$  on the forward operator is required to be less than 1 in absolute value.<sup>12</sup>

We normalized the sum of squared differences between the sample and theoretical second moments by dividing by the sum of the squared second moments. The least squares objective function can thus be thought of as a type of  $R^2$  measure. It tells us the proportion of the “variation” in the second moments that the theoretical model explains. Except for France, the estimated model accounts for over 90 percent of the variability of the 27 moments.

The minimized value for the least squares objective function was lowest for the UK, highest for France. One might conclude that the model performs best for the UK and worst for France, with the other countries in between. Such a conclusion would be unwarranted, however, since the least squares objective function is not the best measure of the fit between the theoretical model and the data. The minimized least squares objective function, in common with “calibration” exercises, places greater weight on explaining the larger moments (in absolute value). By contrast, the GMM, or weighted least squares procedure, places greater weight on explaining the moments that can be estimated more precisely from the data in the sense that they have a lower sample variance.

The weighted least squares estimates, together with their standard errors estimated according to (38), are presented in Table 3. In all countries, the minimized weighted least squares objective function (39) was well below conventional significance levels for a chi-squared random variable with 18 degrees of freedom. However, the distribution of this statistic in samples as small as ours is unlikely to be chi-squared with the hypothesized degrees of freedom.<sup>13</sup>

While the overall fit appears good, some of the estimated parameter values do not accord with our prior expectations, notably the negative estimates of  $F$  (and hence  $\Phi$ ) for Italy and the U.K. Also, some of the estimated standard errors are large.

While unexpected parameter estimates, or large standard errors, may indicate an inadequate theoretical framework, other factors might also be relevant. While many lag structures could be consistent with the basic theoretical framework, we did not adjust the lags in the model to better fit the data.<sup>14</sup> Also, the lack of a specified distribution for the shocks may have reduced our ability to obtain tightly estimated standard

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<sup>12</sup> We estimated the inverse hyperbolic tangents of  $F = \Phi/(\Phi+\Gamma)$ ,  $\rho$  and  $\psi$  to impose the conditions  $|F|<1$ ,  $|\rho|<1$  and  $|\psi|<1$ .

<sup>13</sup> Burnside and Eichenbaum (1994) examine the small sample properties of GMM estimators.

<sup>14</sup> In many studies, lag lengths are chosen ex-post using the Akaike or a similar goodness-of-fit criterion.



errors. Finally, many of the parameters are unlikely to have been constant over the sample period. We are not estimating “deep structural parameters” (arising from a specification of relatively stable taste and technology functions), and policies and other sources of shocks are likely to have varied over time.

**TABLE 3. Weighted least squares parameter estimates<sup>a</sup>**

Parameter	Germany	Germany <sup>b</sup>	France	U. K.	Netherlands	Italy
$\tanh^{-1}(\rho)$	1.1971 (0.0777)	1.1866 (0.0802)	1.2842 (0.0974)	1.5938 (0.1339)	1.6603 (0.2964)	1.2901 (0.1055)
$\tanh^{-1}(\psi)$	0.4218 (4.7715)	0.1881 (0.1279)	0.4084 (1.3584)	-0.0105 (0.0919)	0.2798 (0.2348)	0.0203 (0.1370)
$\gamma$	4.4581 (18.085)	3.6006 (0.4013)	2.1817 (0.9649)	10.3710 (2.6864)	3.5410 (0.7647)	2.2602 (0.2764)
$\Gamma$	0.8722 (5.8709)	1.1765 (0.1940)	0.3304 (0.6180)	0.4676 (0.0924)	0.5099 (0.3868)	0.5810 (0.3077)
$\tanh^{-1}(F)$	-0.0379 (16.404)	0.2049 (0.6907)	1.3701 (0.5966)	-2.8535 (1276.6)	1.6610 (0.6960)	-2.2777 (338.12)
$\sigma_{\zeta}$	0.004445 (0.00042)	0.004467 (0.00043)	0.002437 (0.00028)	0.001746 (0.00067)	0.001369 (0.00075)	0.003001 (0.00054)
$\sigma_{\varepsilon}$	0.004968 (76.667)	0.015464 (0.001709)	0.015551 (0.00518)	0.062807 (0.01622)	0.033528 (0.00557)	0.02950 (0.00282)
$\sigma_{\nu}$	0.015221 (0.00267)	0.015676 (0.002957)	0.010426 (0.01003)	0.039376 (0.01661)	0.010572 (0.01860)	0.053405 (0.00815)
$\sigma_{\tau}$	0.011318 (0.20370)	1.4e-08 (6.6480)	0.004775 (0.35501)	0.000002 (1.33870)	0.000059 (0.74565)	0.000014 (1.7011)
$\chi^2$ (18 d.f.) (P-value)	9.447 (0.949)	9.372 (0.951)	6.946 (0.991)	8.120 (0.977)	9.799 (0.938)	8.709 (0.966)
Implied parameter values <sup>c</sup>						
$\rho$	0.8328	0.8301	0.8576	0.9207	0.9303	0.8591
$\psi$	0.3985	0.1859	0.3871	-0.0105	0.2727	0.0203
$F$	-0.0379	0.2021	0.8787	-0.9934	0.9304	-0.9792
$\Phi$	-0.0319	0.298	2.394	-0.233	6.812	-0.2874
$\delta^{-1}$	-0.037	0.253	7.246	-0.498	13.359	-0.495
$\beta + \delta^{-1}\alpha^{-1}$	1.147	0.850	3.027	2.139	1.961	1.721

a. Standard errors are in parentheses below each parameter estimate.

b. We found two sets of estimates for Germany with similar minimized least squares and weighted least squares values.

c. The income elasticity of money demand  $\beta$  and the real interest elasticity of demand  $\alpha$  cannot be recovered.

## 10. An alternative information assumption

A change in the amount of information available to individuals substantially alters equilibrium prices and output. To illustrate this, we now assume that agents do not know the decomposition of  $s_t$  or  $d_t$  into their components  $\xi_t$ ,  $\varepsilon_t$ ,  $\mu_t$  or  $\tau_t$  until period  $t+2$ . In appendix 1, we derive the following analogs to the above results for equilibrium prices and output.

**Theorem 3:** When the composition of shocks is unknown for two periods, equilibrium inflation satisfies

$$(1-\rho L)P_t = \sum_{i=0}^4 [\pi_{1i}\zeta_{t-i} + \pi_{2i}\varepsilon_{t-i} + \pi_{3i}\nu_{t-i} + \pi_{4i}\tau_{t-i}] \quad (40)$$

where the 18 distinct coefficients  $\pi_{ij}$ ,  $i = 1, \dots, 4, j = 0, \dots, 4$  (with  $\pi_{10} = \pi_{20}$  and  $\pi_{30} = \pi_{40}$ ) are given by the 18 equations (106)–(123) in appendix 1.

**Proof.** The proof is given in appendix 1.

Thus, under the modified information assumptions,  $P_t$  follows an ARMA(1,4) process. From the supply curve (1) and (85) we obtain an expression for equilibrium output:

**Theorem 4:** When the composition of shocks is unknown for two periods, equilibrium output  $y_t$  satisfies:

$$(1-\rho L)y_t = s_t + \gamma \sum_{i=0}^2 (\kappa_{1i}\zeta_{t-i} + \kappa_{2i}\varepsilon_{t-i} + \kappa_{3i}\nu_{t-i} + \kappa_{4i}\tau_{t-i}) \quad (41)$$

where  $\kappa_{i0} = \pi_{i0}$ ,  $i = 1, \dots, 4$  while  $\kappa_{ij}$ ,  $i = 1, \dots, 4, j = 1, 2$  satisfy (96) and (97).

Multiplying (41) through by  $(1-L)$ , we conclude that, under the modified information assumptions, the first difference of equilibrium output  $Y_t = \Delta y_t$  follows an ARMA(1,3) process.

The additional MA terms in the ARMA processes for  $P_t$  and  $Y_t$  lead to straightforward modifications for the expressions in appendix 1 for variances and covariances. These have been omitted for brevity.

## 11. Results for the alternative model

**TABLE 4. Least squares parameter estimates for the alternative model**

Parameter	Germany	France	U. K.	Netherlands	Italy
$\tanh^{-1}(\rho)$	1.1955	1.2388	1.5784	1.5621	1.2463
$\tanh^{-1}(\psi)$	0.5572	0.6068	-0.0138	0.1615	0.4961
$\gamma$	4.3078	3.0906	9.1982	3.8273	2.9563
$\Gamma$	0.7203	0.2368	0.4534	0.7632	0.2877
$\tanh^{-1}(F)$	0.0796	1.3356	-2.2358	1.2361	-0.1399
$\sigma_\zeta$	0.004598	0.002595	0.001812	0.001851	0.003008
$\sigma_\varepsilon$	0.007858	0.006755	0.061102	0.035266	0.006369
$\sigma_\nu$	0.014554	0.006578	0.043581	0.016609	0.054067
$\sigma_\tau$	0.016308	0.058139	0.000048	0.000002	0.068094
LS objective	0.09126	0.13736	0.02689	0.05283	0.08388

The least squares parameter estimates for the alternative model are presented in Table 4. In all countries the minimized least squares objective function is lower in Table 4 than in Table 2, although the differ-

ences are slight. This could reflect the fact that the alternative model has additional MA terms for equilibrium inflation and output growth, although these additional terms are constrained to be functions of the same number of underlying parameters.

The weighted least squares estimates are in Table 5. These are below the corresponding minimized values of the weighted least squares objective function in Table 3 only for France and the UK. Nevertheless, the differences are again small.

**TABLE 5. Weighted least squares parameter estimates<sup>a</sup> for the alternative model**

Parameter	Germany	France	U. K.	Netherlands	Italy
$\tanh^{-1}(\rho)$	1.2022 (0.0787)	1.2778 (0.0975)	1.5935 (0.1235)	1.5071 (0.2154)	1.3298 (0.1096)
$\tanh^{-1}(\psi)$	0.5247 (0.9572)	0.6284 (0.0596)	-0.0268 (0.0639)	0.1489 (0.1391)	0.5202 (0.1908)
$\gamma$	4.4632 (3.9632)	2.7966 (0.7171)	10.140 (2.4995)	3.6867 (0.7617)	3.2231 (0.4769)
$\Gamma$	0.7525 (0.8999)	0.2501 (0.0294)	0.4651 (0.0805)	0.7935 (0.3586)	0.2619 (0.1468)
$\tanh^{-1}(F)$	-0.3672 (8.3042)	1.2128 (0.2629)	0.1955 (1.9226)	1.1184 (0.6628)	0.1501 (2.1774)
$\sigma_{\zeta}$	0.004418 (0.00043)	0.002515 (0.00029)	0.001751 (0.00038)	0.001872 (0.00076)	0.002272 (0.00055)
$\sigma_{\varepsilon}$	0.007292 (1.3160)	0.002264 (0.00597)	0.062086 (0.01522)	0.033705 (0.00540)	0.006335 (1.4533)
$\sigma_{\nu}$	0.015046 (0.00319)	0.011677 (0.00842)	0.037835 (0.00718)	0.017061 (0.00427)	0.050400 (0.00911)
$\sigma_{\tau}$	0.015608 (0.04302)	0.047338 (0.01529)	1.5e-06 (3.7133)	1.9e-09 (214.89)	0.068144 (0.04257)
$\chi^2$ (18 d.f.)	9.450	6.939	8.031	9.907	8.723
(P-value)	(0.948)	(0.991)	(0.978)	(0.935)	(0.966)
Implied parameter values <sup>b</sup>					
$\rho$	0.8343	0.8559	0.9207	0.9064	0.8692
$\psi$	0.4813	0.5570	-0.0268	0.1478	0.4779
$F$	-0.3516	0.8375	0.1930	0.807	0.1490
$\Phi$	-0.1957	1.2890	0.1113	3.318	0.0459
$\delta^{-1}$	-0.2601	5.1540	0.2392	4.181	0.1751
$\beta + \delta^{-1}\alpha^{-1}$	1.329	3.999	2.150	1.260	3.818

a. Standard errors are in parentheses below each parameter estimate.

b. The income elasticity of money demand  $\beta$  and the real interest elasticity of demand  $\alpha$  cannot be recovered.

It is comforting that in many cases, the estimated parameters in Table 5 are quite similar to the ones in Table 3, suggesting that the specification of the length of the information lag does not make a huge difference. The most notable changes are in Italy and the UK, where the alternative specification leads to the

expected positive values for  $F$  and  $\Phi$ . By contrast, for Germany the alternative specification leads to counter-intuitive negative values for  $F$  and  $\Phi$ , as in the first column of Table 3. As for France and the Netherlands, the results in Table 5 are preferable to those in Table 3 because various parameters are more tightly estimated in Table 5. Accordingly, in the subsequent discussion, we shall take the second model from Table 3 for Germany but the models from Table 5 for the remaining countries.

## 12. Discussion of the preferred models for each country

For convenience, the parameter estimates in the preferred models for each country are repeated in Table 6. Figures 1–5 graph, for the preferred models for each country, the fit between the sample and the estimated moments, the decomposition of each of the moments into the components arising from each type of shock, and the implied impulse response functions from each type of shock.

The fit between the sample moments and the weighted least squares theoretical moments is presented in the upper left chart of Figures 1–5. The graphs indicate a reasonably close fit between the theoretical and sample moments for all countries. The most difficult problem seemed to be matching the autocovariances of output growth. In all countries, some autocovariances of output growth were positive, while at other lags they were negative. The estimated models generally could not match such patterns well.

**TABLE 6. Parameter estimates for the preferred model for each country**

Parameter	Germany	France	U. K.	Netherlands	Italy
$\rho$	0.8301	0.8559	0.9207	0.9064	0.8692
$\psi$	0.1859	0.5570	-0.0268	0.1478	0.4779
$\gamma$	3.6006	2.7966	10.140	3.6867	3.2231
$\Gamma$	1.1765	0.2501	0.4651	0.7935	0.2619
$\Phi$	0.298	1.2890	0.1113	3.318	0.0459
$F$	0.2021	0.8375	0.1930	0.8070	0.1490
$\sigma_{\zeta}$	0.004467	0.002515	0.001751	0.001872	0.002272
$\sigma_{\varepsilon}$	0.015464	0.002264	0.062086	0.033705	0.006335
$\sigma_{\nu}$	0.015676	0.011677	0.037835	0.017061	0.050400
$\sigma_{\tau}$	1.4e-08	0.047338	1.5e-06	1.9e-09	0.068144
$\delta^{-1}$	0.253	5.1540	0.2392	4.181	0.1751
$\beta + \delta^{-1}\alpha^{-1}$	0.850	3.999	2.150	1.260	3.818

The most consistent and tightly-estimated parameter is  $\rho$ , the autocorrelation coefficient in the aggregate supply curve. Apparently the data are indicating that supply disturbances, whether temporary or permanent, exhibit strong persistence. By contrast, the autocorrelation coefficient in aggregate demand ( $\psi$ ) implies moderate persistence in France and Italy, weak persistence in Germany and the Netherlands and no persistence at all in the UK.

The estimated elasticities of supply with respect to unexpected inflation,  $\gamma$ , are all of the hypothesized positive sign. The inverse of  $\gamma$  can be interpreted as the slope coefficient in an expectations-augmented Phillips curve. The estimates appear reasonable for all countries except the U.K.

The effect of expected future shocks on current output and prices is determined by  $F = \Phi/(\Phi+\Gamma)$ . While the estimates of  $F$  appear reasonable for all countries in so far as they are all between zero and one, the combined estimates of  $F$  and  $\Gamma$  imply unreasonably low interest semi-elasticities of money demand ( $\delta^{-1}$ ) for Germany, the UK and Italy. On the other hand, the estimates of  $\Gamma$  for all countries except Germany can accommodate an income elasticity of money demand ( $\beta$ ) of around unity.

The estimated standard deviations of the shocks suggest that temporary demand shocks have been relevant only in France and Italy, where they have been the largest type of shock. Permanent demand shocks appear to have been relatively large in all countries, but particularly so in Italy and the U.K. On the other hand, temporary supply shocks have tended to exceed permanent supply shocks in all countries except France, where the estimated standard errors are similar. The temporary supply shocks are estimated to be substantially larger than the permanent ones in Germany, the UK and the Netherlands. The estimated standard error of permanent supply shocks is more similar across the economies than is the case for any of the other shocks. The similarity in size of permanent supply shocks across economies might suggest that the economies have faced common technology, oil price or other permanent supply shocks.

The relative contributions of the different shocks to variances, autocovariances and cross-covariances in output growth and inflation depend not only on the estimated standard errors of the shocks but also on the autoregressive and moving average coefficients. In the VAR literature, the traditional way to present the information contained in the estimated coefficients is to graph the impulse response functions. Using the parameter estimates in Table 6 we can calculate the effects on  $Y$  and  $P$  of a unit shock to  $\zeta$ ,  $\varepsilon$ ,  $\nu$  or  $\tau$ . The resulting impulse response functions for a period of 12 quarters (3 years) are graphed for each country in the final two panels of Figures 1 through 5.

In all countries, permanent supply shocks have the longest lasting effects on output growth, with the peak positive effects occurring after a two or three quarter lag. The effects of the remaining shocks on output growth are negligible beyond two or three quarters after the period of the shock. Permanent supply shocks also have the longest lasting effects on inflation, although permanent demand shocks also have a cumulative positive impact on inflation in all countries.

The *cumulative* effects of shocks on output growth and inflation can also be interpreted as long run effects on the output and price *levels*. From the sums of the impulse responses in Figures 1 through 5, and using

the fact that subsequent coefficients will decline exponentially from the final coefficients at lag 12, we can conclude that only permanent supply shocks will have a long run positive impact on output. The long run effects of the remaining shocks are all effectively zero. Furthermore, the long run effects of permanent demand shocks on the price level are very close to unity in all countries. This suggests that the permanent demand shocks we are estimating are predominantly nominal in character. Permanent supply shocks have a substantial negative impact on the long run price level in all countries.

The middle panels of Figures 1 through 5 graph the contribution of each shock to the variances and autocovariances of  $Y$  and  $P$  and the contemporaneous and lagged covariances between  $Y$  and  $P$ . These are not variance decompositions as usually derived and discussed in the VAR literature. Instead of presenting the proportion of forecast error variances resulting from each shock, the figures simply decompose the different variances and covariances into the components coming from each type of shock. In the figures, vertical bars give the contribution by the four types of shock to each variance or covariance. In all cases the bars are ordered, from left to right, as follows: permanent supply shocks ( $\zeta$ ), temporary supply shocks ( $\epsilon$ ), permanent demand shocks ( $\nu$ ), and temporary demand shocks ( $\tau$ ). In some cases (usually involving the temporary shocks) a bar is so small relative to the others that it is not visible on the chart.

In four out of five countries, demand shocks (either permanent or temporary) are the predominant source of variance in output growth. In Germany, the UK and the Netherlands, permanent demand shocks are the largest contributor to variance in output growth. While these shocks are also an important source of output growth variance in Italy, temporary demand shocks are about twice as important. In France, all four shocks contribute a roughly similar amount to the variance of output growth.

The variance of inflation shows no consistent pattern. In Germany and especially Italy, the variance of inflation is predominantly attributable to permanent demand shocks, while in France and the UK, permanent supply shocks are more important. In the Netherlands, temporary supply shocks are the largest source of variance in inflation.

As for the patterns of autocorrelation in output growth and inflation, inspection of the upper-left charts in Figures 1-5 shows that in most cases, inflation is more serially-correlated than output growth. The charts of decompositions show that our model accounts for these patterns by having both permanent supply and demand shocks contributing to positive autocovariances of inflation, whereas in the case of output growth, the two permanent shocks have tended to offset one another, at least at the longer lags. In particular, at the longer lags (and sometimes at all non-zero lags) the permanent supply shocks contribute toward positive autocorrelation of output growth, but permanent demand shocks contribute toward nega-

tive autocorrelation.

As discussed earlier, the cross-covariances between output growth and current or future inflation are small and variable (sometimes positive, sometimes negative). By contrast, the cross-covariances between output growth and past inflation are consistently negative and relatively large. Our estimated model largely attributes this difference to a switch in the sign of the effect of permanent demand shocks.<sup>15</sup> In all countries, permanent supply shocks are an important contributor to negative covariance between  $Y$  and  $P$  at all leads and lags. Permanent demand shocks tend to reinforce the effect of permanent supply shocks in the case of covariances between  $Y$  and lagged values of  $P$ , but they tend to have an offsetting positive effect on covariances between  $Y$  and current or future values of  $P$ .

We do not present time series of the driving shocks for each country since the sample values of these shocks are not identified. We have four driving shocks, but only two endogenous variables (output and prices). As we remarked in the introduction, an advantage of the method of moments procedure used in this paper is that the number of driving shocks can exceed the number of endogenous variables.

### **13. Concluding remarks**

This paper uses a method of moments procedure to estimate an aggregate demand/aggregate supply model with rational expectations for various European economies. The results indicate that permanent demand shocks are the predominant source of variance in output growth in most of these economies (France is the exception) though permanent supply shocks have important effects on covariance patterns, while the temporary shocks are also significant in France and Italy. Permanent supply shocks are also very significant determinants of the variance and autocorrelation in inflation.

### **14. Appendix 1**

#### *Proof of Theorem 1*

For convenience, we rewrite the equation (23) that is to be solved for the equilibrium inflation rate  $P_t$  as:

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<sup>15</sup> Temporary supply shocks contribute to the negative contemporaneous covariance between output growth and inflation in all countries, but affect other lead or lag covariances only in France, and even then only slightly. In Italy, temporary demand shocks are an important contributor to covariance between output growth and inflation at a number of leads and lags.

$$(\Phi + \Gamma)(1 - FL^{-1})(1 - \rho L)P_t = \Gamma(1 - \rho L)(v_t + \tau_t - \tau_{t-1}) - (1 - \psi L)(\zeta_t + \varepsilon_t - \varepsilon_{t-1}) - \quad (42)$$

$$[\theta_0 - \theta_1 L + \theta_2 L^2] \left[ \sum_{i=0}^1 (\kappa_{1i} \zeta_{t-i} + \kappa_{2i} \varepsilon_{t-i} + \kappa_{3i} v_{t-i} + \kappa_{4i} \tau_{t-i}) \right]$$

We have, for  $i \geq 0$ ,

$$\frac{1}{1 - FL^{-1}} \zeta_{t-i} = \zeta_{t-i} + F \zeta_{t-i+1} + \dots + F^i a_1 (\zeta_t + \varepsilon_t) \quad (43)$$

$$\frac{1}{1 - FL^{-1}} \varepsilon_{t-i} = \varepsilon_{t-i} + F \varepsilon_{t-i+1} + \dots + F^i a_2 (\zeta_t + \varepsilon_t) \quad (44)$$

$$\frac{1}{1 - FL^{-1}} v_{t-i} = v_{t-i} + F v_{t-i+1} + \dots + F^i b_1 (v_t + \tau) \quad (45)$$

$$\frac{1}{1 - FL^{-1}} \tau_{t-i} = \tau_{t-i} + F \tau_{t-i+1} + \dots + F^i b_2 (v_t + \tau) \quad (46)$$

Since the stochastic processes are now stationary, we can use (43)–(46) to invert  $(1 - FL^{-1})$  on the right side of (42). We find that equilibrium  $P_t$  will indeed be given by (26) so long as the  $\pi_{ij}$  coefficients satisfy:

$$(\Phi + \Gamma)\pi_{10} = -(1 - \psi F)(1 - a_2 F) - (\theta_0 - \theta_1 F + \theta_2 F^2)[(\kappa_{10} a_1 + \kappa_{20} a_2) + (\kappa_{11} a_1 + \kappa_{21} a_2)F] \quad (47)$$

$$(\Phi + \Gamma)\pi_{11} = \psi + (\theta_1 - \theta_2 F)\kappa_{10} - (\theta_0 - \theta_1 F + \theta_2 F^2)\kappa_{11} \quad (48)$$

$$(\Phi + \Gamma)\pi_{12} = \theta_1 \kappa_{11} - \theta_2 (\kappa_{10} + \kappa_{11} F) \quad (49)$$

$$(\Phi + \Gamma)\pi_{13} = -\theta_2 \kappa_{11} \quad (50)$$

$$(\Phi + \Gamma)\pi_{20} = -(1 - \psi F)(1 - a_2 F) - (\theta_0 - \theta_1 F + \theta_2 F^2)[(\kappa_{10} a_1 + \kappa_{20} a_2) + (\kappa_{11} a_1 + \kappa_{21} a_2)F] \quad (51)$$

$$(\Phi + \Gamma)\pi_{21} = 1 + \psi(1 - F) + (\theta_1 - \theta_2 F)\kappa_{20} - (\theta_0 - \theta_1 F + \theta_2 F^2)\kappa_{21} \quad (52)$$

$$(\Phi + \Gamma)\pi_{22} = -\psi + \theta_1 \kappa_{21} - \theta_2 (\kappa_{20} + \kappa_{21} F) \quad (53)$$

$$(\Phi + \Gamma)\pi_{23} = -\theta_2 \kappa_{21} \quad (54)$$

$$(\Phi + \Gamma)\pi_{30} = \Gamma(1 - \rho F)(1 - b_2 F) - (\theta_0 - \theta_1 F + \theta_2 F^2)[(\kappa_{30} b_1 + \kappa_{40} b_2) + (\kappa_{31} b_1 + \kappa_{41} b_2)F] \quad (55)$$

$$(\Phi + \Gamma)\pi_{31} = -\rho \Gamma + (\theta_1 - \theta_2 F)\kappa_{30} - (\theta_0 - \theta_1 F + \theta_2 F^2)\kappa_{31} \quad (56)$$

$$(\Phi + \Gamma)\pi_{32} = \theta_1 \kappa_{31} - \theta_2 (\kappa_{30} + \kappa_{31} F) \quad (57)$$

$$(\Phi + \Gamma)\pi_{33} = -\theta_2 \kappa_{31} \quad (58)$$

$$(\Phi + \Gamma)\pi_{40} = \Gamma(1 - \rho F)(1 - b_2 F) - (\theta_0 - \theta_1 F + \theta_2 F^2)[(\kappa_{30} b_1 + \kappa_{40} b_2) + (\kappa_{31} b_1 + \kappa_{41} b_2)F] \quad (59)$$

$$(\Phi + \Gamma)\pi_{41} = -\Gamma - \rho \Gamma(1 - F) + (\theta_1 - \theta_2 F)\kappa_{40} - (\theta_0 - \theta_1 F + \theta_2 F^2)\kappa_{41} \quad (60)$$

$$(\Phi + \Gamma)\pi_{42} = \rho \Gamma + \theta_1 \kappa_{41} - \theta_2 (\kappa_{40} + \kappa_{41} F) \quad (61)$$

$$(\Phi + \Gamma)\pi_{43} = -\theta_2 \kappa_{41} \quad (62)$$



Now use  $\kappa_{i0} = \pi_{i0}$ , and expressions (24) for  $\kappa_{i1}$ , to find:

$$\kappa_{11}a_1 + \kappa_{21}a_2 = (\pi_{11} - \pi_{21})a_2a_1 - (\pi_{11} - \pi_{21})a_1a_2 = 0 \quad (63)$$

$$\kappa_{31}b_1 + \kappa_{41}b_2 = (\pi_{31} - \pi_{41})b_2b_1 - (\pi_{31} - \pi_{41})b_1b_2 = 0 \quad (64)$$

so that equations (47)–(62) can be simplified to:

$$(\Phi + \Gamma + \theta_0 - \theta_1 F + \theta_2 F^2)\pi_{10} = (\Phi + \Gamma + \theta_0 - \theta_1 F + \theta_2 F^2)\pi_{20} = -(1 - \psi F)(1 - a_2 F) \quad (65)$$

$$[\Phi + \Gamma + a_2(\theta_0 - \theta_1 F + \theta_2 F^2)]\pi_{11} = \psi + (\theta_1 - \theta_2 F)\pi_{10} + (\theta_0 - \theta_1 F + \theta_2 F^2)\pi_{21}a_2 \quad (66)$$

$$[\Phi + \Gamma + a_1(\theta_0 - \theta_1 F + \theta_2 F^2)]\pi_{21} = \psi + (1 - \psi F) + (\theta_1 - \theta_2 F)\pi_{10} + (\theta_0 - \theta_1 F + \theta_2 F^2)\pi_{11}a_1 \quad (67)$$

$$(\Phi + \Gamma)\pi_{12} = (\theta_1 - \theta_2 F)(\pi_{11} - \pi_{21})a_2 - \theta_2\pi_{10} \quad (68)$$

$$(\Phi + \Gamma)\pi_{22} = -\psi - (\theta_1 - \theta_2 F)(\pi_{11} - \pi_{21})a_1 - \theta_2\pi_{10} \quad (69)$$

$$(\Phi + \Gamma)\pi_{13} = -\theta_2(\pi_{11} - \pi_{21})a_2 \quad (70)$$

$$(\Phi + \Gamma)\pi_{23} = \theta_2(\pi_{11} - \pi_{21})a_1 \quad (71)$$

$$(\Phi + \Gamma + \theta_0 - \theta_1 F + \theta_2 F^2)\pi_{30} = (\Phi + \Gamma + \theta_0 - \theta_1 F + \theta_2 F^2)\pi_{40} = \Gamma(1 - \rho F)(1 - b_2 F) \quad (72)$$

$$[\Phi + \Gamma + b_2(\theta_0 - \theta_1 F + \theta_2 F^2)]\pi_{31} = -\rho\Gamma + (\theta_1 - \theta_2 F)\pi_{30} + (\theta_0 - \theta_1 F + \theta_2 F^2)\pi_{41}b_2 \quad (73)$$

$$[\Phi + \Gamma + b_1(\theta_0 - \theta_1 F + \theta_2 F^2)]\pi_{41} = -\rho\Gamma - \Gamma(1 - \rho F) + (\theta_1 - \theta_2 F)\pi_{30} + (\theta_0 - \theta_1 F + \theta_2 F^2)\pi_{31}b_1 \quad (74)$$

$$(\Phi + \Gamma)\pi_{32} = (\theta_1 - \theta_2 F)(\pi_{31} - \pi_{41})b_2 - \theta_2\pi_{30} \quad (75)$$

$$(\Phi + \Gamma)\pi_{42} = \rho\Gamma - (\theta_1 - \theta_2 F)(\pi_{31} - \pi_{41})b_1 - \theta_2\pi_{30} \quad (76)$$

$$(\Phi + \Gamma)\pi_{33} = -\theta_2(\pi_{31} - \pi_{41})b_2 \quad (77)$$

$$(\Phi + \Gamma)\pi_{43} = \theta_2(\pi_{31} - \pi_{41})b_1 \quad (78)$$

### *Theoretical second moments*

To obtain the autocovariances of  $P_t$  or  $Y_t$ , first solve for the covariances between  $P_t$  or  $Y_t$  and the first three lags of  $z_t' = [\Delta s_t \ \eta_t \ \Delta d_t \ \omega_t]$ . Multiply (27) and (29) by each shock and take expectations to obtain two  $4 \times 3$  matrices  $N_1$  and  $N_2$  of covariances.<sup>16</sup> For example, write the covariances of  $P_t$  with  $z_t, z_{t-1}, z_{t-2}, z_{t-3}$  as

$$\begin{bmatrix} \text{cov}(P_t, z_t) & \text{cov}(P_t, z_{t-1}) & \text{cov}(P_t, z_{t-2}) & \text{cov}(P_t, z_{t-3}) \end{bmatrix} = \begin{bmatrix} \sigma_\xi^2 & 0 & 0 & 0 \\ 0 & \sigma_\varepsilon^2 & 0 & 0 \\ 0 & 0 & \sigma_v^2 & 0 \\ 0 & 0 & 0 & \sigma_w^2 \end{bmatrix} N_1 \quad (79)$$

where the coefficient matrices  $N_i$  are given by (80):

<sup>16</sup> For notational convenience, we extend  $\Pi_2$  with a column of zeros so both  $\Pi_1$  and  $\Pi_2$  are  $4 \times 4$ .

$$N_i = \Pi_i \begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 \\ 0 & 1 & \rho & \rho^2 \\ 0 & 0 & 1 & \rho \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (80)$$

Now multiply (27) and (29) by lagged values of  $P_t$  or  $Y_t$  and take expectations to find:

$$\begin{bmatrix} 1 & -\rho & 0 & 0 & 0 & 0 & 0 \\ -\rho & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\rho & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\rho & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\rho & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\rho & 1 \end{bmatrix} \begin{bmatrix} \gamma_{i0} \\ \gamma_{i1} \\ \gamma_{i2} \\ \gamma_{i3} \\ \gamma_{i4} \\ \gamma_{i5} \\ \gamma_{i6} \end{bmatrix} = \sum_{j=1}^4 M_{ij} \Pi_i^j \sigma_{z_j}^2 \quad (81)$$

where  $\sigma_{z_j}^2$  is the variance of the  $j$ th component of  $z$ ,  $\Pi_i^j$  (of dimension  $4 \times 1$ ) is the transpose of the  $j$ th row of  $\Pi_i$  and, if we use  $N_i^{jk}$  for the  $(j,k)$ -th element of  $N_i$ , the  $7 \times 4$  matrices  $M_{ij}$  are defined by:

$$M_{ij} = \begin{bmatrix} N_i^{j1} & N_i^{j2} & N_i^{j3} & N_i^{j4} \\ 0 & N_i^{j1} & N_i^{j2} & N_i^{j3} \\ 0 & 0 & N_i^{j1} & N_i^{j2} \\ 0 & 0 & 0 & N_i^{j1} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (82)$$

Finally, multiply (27) by (29), (27) by  $Y_{t-1}, \dots, Y_{t-6}$ , and (29) by  $P_{t-1}, \dots, P_{t-6}$  and take expectations to get 13 equations for  $cov(P_t, Y_t)$ ,  $\gamma_k^{YP} = cov(Y_t, P_{t-k})$  and  $\gamma_k^{PY} = cov(P_t, Y_{t-k})$ ,  $k = 1, 2, \dots, 6$ :

$$\begin{bmatrix}
1+\rho^2 & -\rho & -\rho & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\rho & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\rho & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\rho & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\rho & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\rho & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\rho & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\rho & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\rho & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\gamma_0^{PY} \\
\gamma_1^{YP} \\
\gamma_1^{PY} \\
\gamma_2^{YP} \\
\gamma_2^{PY} \\
\gamma_3^{YP} \\
\gamma_3^{PY} \\
\gamma_4^{YP} \\
\gamma_4^{PY} \\
\gamma_5^{YP} \\
\gamma_5^{PY} \\
\gamma_6^{YP} \\
\gamma_6^{PY}
\end{bmatrix}
=
\begin{bmatrix}
H_0 \\
H_1^{YP} \\
H_1^{PY} \\
H_2^{YP} \\
H_2^{PY} \\
H_3^{YP} \\
H_3^{PY} \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\sigma_\zeta^2 \\
\sigma_\varepsilon^2 \\
\sigma_v^2 \\
\sigma_\tau^2
\end{bmatrix}
\quad (83)$$

$$H_0 = \text{diag}[\Pi_1 \cdot \Pi_2'] \quad H_1^{YP} = \text{diag}[N_1(:, 1:3) \cdot \Pi_2(:, 2:4)'] \quad H_1^{PY} = \text{diag}[N_2(:, 1:3) \cdot \Pi_1(:, 2:4)']$$

where

$$\begin{aligned}
H_2^{YP} &= \text{diag}[N_1(:, 1:2) \cdot \Pi_2(:, 3:4)'] & H_2^{PY} &= \text{diag}[N_2(:, 1:2) \cdot \Pi_1(:, 3:4)'] \\
H_3^{YP} &= \text{diag}[N_1(:, 1) \cdot \Pi_2(:, 4)'] & H_3^{PY} &= \text{diag}[N_2(:, 1) \cdot \Pi_1(:, 4)']
\end{aligned}$$

*Proof of Theorem 3*

The equation to be solved for the equilibrium inflation rate  $P_t$  can now be written as:

$$(\Phi + \Gamma)(1 - FL^{-1})(1 - \rho L)P_t = \Gamma(1 - \rho L)(v_t + \tau_t - \tau_{t-1}) - (1 - \psi L)(\zeta_t + \varepsilon_t - \varepsilon_{t-1}) - [\theta_0 - \theta_1 L + \theta_2 L^2][P_t - E_{t-1}P_t] \quad (84)$$

Equilibrium  $P_t$  will again be a linear function of current and lagged  $\zeta$ ,  $\varepsilon$ ,  $v$  and  $\tau$ , but it will now involve an additional lag. In particular, in place of (16),  $(P_t - E_{t-1}P_t)$  will now be a linear sum involving two lags:

$$\begin{aligned}
P_t - E_{t-1}P_t &= \pi_{10}\zeta_t + \pi_{20}\varepsilon_t + \pi_{30}v_t + \pi_{40}\tau_t + \pi_{11}(\zeta_{t-1} - E_{t-1}\zeta_{t-1}) + \pi_{21}(\varepsilon_{t-1} - E_{t-1}\varepsilon_{t-1}) \\
&+ \pi_{31}(v_{t-1} - E_{t-1}v_{t-1}) + \pi_{41}(\tau_{t-1} - E_{t-1}\tau_{t-1}) + \pi_{12}(\zeta_{t-2} - E_{t-1}\zeta_{t-2}) + \pi_{22}(\varepsilon_{t-2} - E_{t-1}\varepsilon_{t-2}) \\
&+ \pi_{32}(v_{t-2} - E_{t-1}v_{t-2}) + \pi_{42}(\tau_{t-2} - E_{t-1}\tau_{t-2}) \equiv \sum_{i=0}^2 (\kappa_{1i}\zeta_{t-i} + \kappa_{2i}\varepsilon_{t-i} + \kappa_{3i}v_{t-i} + \kappa_{4i}\tau_{t-i})
\end{aligned} \quad (85)$$

Since at  $t-1$  individuals now know  $\xi_{t-3}, \mu_{t-3}, \varepsilon_{t-3}, \tau_{t-3}, \Delta s_{t-2} = \zeta_{t-2} + \varepsilon_{t-2} - \varepsilon_{t-3}, \Delta s_{t-1} = \zeta_{t-1} + \varepsilon_{t-1} - \varepsilon_{t-2}, \Delta d_{t-2} = v_{t-2} + \tau_{t-2} - \tau_{t-3}$  and  $\Delta d_{t-1} = v_{t-1} + \tau_{t-1} - \tau_{t-2}$ , they effectively observe  $\zeta_{t-2} + \varepsilon_{t-2}, \zeta_{t-1} + \varepsilon_{t-1} - \varepsilon_{t-2}, v_{t-2} + \tau_{t-2}$  and  $v_{t-1} + \tau_{t-1} - \tau_{t-2}$ . Projecting onto these variables they would obtain:

$$E_{t-1}\zeta_{t-1} = a_{11}(\zeta_{t-1} + \varepsilon_{t-1} - \varepsilon_{t-2}) + a_{12}(\zeta_{t-2} + \varepsilon_{t-2}) \quad (86)$$

$$E_{t-1}\varepsilon_{t-2} = a_{21}(\zeta_{t-1} + \varepsilon_{t-1} - \varepsilon_{t-2}) + a_{22}(\zeta_{t-2} + \varepsilon_{t-2}) \quad (87)$$

$$E_{t-1}\zeta_{t-2} = a_{31}(\zeta_{t-1} + \varepsilon_{t-1} - \varepsilon_{t-2}) + a_{32}(\zeta_{t-2} + \varepsilon_{t-2}) \quad (88)$$

$$E_{t-1}\varepsilon_{t-2} = a_{41}(\zeta_{t-1} + \varepsilon_{t-1} - \varepsilon_{t-2}) + a_{42}(\zeta_{t-2} + \varepsilon_{t-2}) \quad (89)$$

$$E_{t-1}v_{t-1} = b_{11}(v_{t-1} + \tau_{t-1} - \tau_{t-2}) + b_{12}(v_{t-2} + \tau_{t-2}) \quad (90)$$

$$E_{t-1}\tau_{t-1} = b_{21}(v_{t-1} + \tau_{t-1} - \tau_{t-2}) + b_{22}(v_{t-2} + \tau_{t-2}) \quad (91)$$

$$E_{t-1}v_{t-2} = b_{31}(v_{t-1} + \tau_{t-1} - \tau_{t-2}) + b_{32}(v_{t-2} + \tau_{t-2}) \quad (92)$$

$$E_{t-1}\tau_{t-2} = b_{41}(v_{t-1} + \tau_{t-1} - \tau_{t-2}) + b_{42}(v_{t-2} + \tau_{t-2}) \quad (93)$$

where the coefficients  $a_{ij}$  and  $b_{ij}$  satisfy<sup>17</sup>

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{bmatrix} \begin{bmatrix} \sigma_\xi^2 + 2\sigma_\varepsilon^2 & -\sigma_\varepsilon^2 \\ -\sigma_\varepsilon^2 & \sigma_\xi^2 + \sigma_\varepsilon^2 \end{bmatrix} = \begin{bmatrix} \sigma_\xi^2 & 0 \\ \sigma_\varepsilon^2 & 0 \\ 0 & \sigma_\xi^2 \\ -\sigma_\varepsilon^2 & \sigma_\varepsilon^2 \end{bmatrix} \quad (94)$$

$$\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ b_{41} & b_{42} \end{bmatrix} \begin{bmatrix} \sigma_v^2 + 2\sigma_\tau^2 & -\sigma_\tau^2 \\ -\sigma_\tau^2 & \sigma_\xi^2 + \sigma_\tau^2 \end{bmatrix} = \begin{bmatrix} \sigma_v^2 & 0 \\ \sigma_\tau^2 & 0 \\ 0 & \sigma_v^2 \\ -\sigma_\tau^2 & \sigma_\tau^2 \end{bmatrix}. \quad (95)$$

Substituting (86)–(93) into (85) we also deduce that  $\kappa_{ji}$  and  $\pi_{ji}, i = 1, 2$  are related by the equations:

$$\begin{bmatrix} \kappa_{11} \\ \kappa_{21} \\ \kappa_{12} \\ \kappa_{22} \end{bmatrix} = \begin{bmatrix} 1 - a_{11} & -a_{21} & -a_{31} & -a_{41} \\ -a_{11} & 1 - a_{21} & -a_{31} & -a_{41} \\ -a_{12} & -a_{22} & 1 - a_{32} & -a_{42} \\ a_{11} - a_{12} & a_{21} - a_{22} & a_{31} - a_{32} & 1 + a_{41} - a_{42} \end{bmatrix} \begin{bmatrix} \pi_{11} \\ \pi_{21} \\ \pi_{12} \\ \pi_{22} \end{bmatrix} \equiv A \begin{bmatrix} \pi_{11} \\ \pi_{21} \\ \pi_{12} \\ \pi_{22} \end{bmatrix} \quad (96)$$

<sup>17</sup>. Note that equations (94) and (95) imply that  $a_{31} + a_{41} = 0, b_{31} + b_{41} = 0, a_{32} + a_{42} = 1$  and  $b_{32} + b_{42} = 1$ .

$$\begin{bmatrix} \kappa_{31} \\ \kappa_{41} \\ \kappa_{32} \\ \kappa_{42} \end{bmatrix} = \begin{bmatrix} 1-b_{11} & -b_{21} & -b_{31} & -b_{41} \\ -b_{11} & 1-b_{21} & -b_{31} & -b_{41} \\ -b_{12} & -b_{22} & 1-b_{32} & -b_{42} \\ b_{11}-b_{12} & b_{21}-b_{22} & b_{31}-b_{32} & 1+b_{41}-b_{42} \end{bmatrix} \begin{bmatrix} \pi_{31} \\ \pi_{41} \\ \pi_{32} \\ \pi_{42} \end{bmatrix} \equiv B \begin{bmatrix} \pi_{31} \\ \pi_{41} \\ \pi_{32} \\ \pi_{42} \end{bmatrix}. \quad (97)$$

Using the projection equations (86)–(93) updated one period we find, for  $i \geq 2$ ,

$$\frac{1}{1-FL^{-1}}\zeta_{t-i} = \zeta_{t-i} + F\zeta_{t-i+1} + \dots + F^{i-2}\zeta_{t-2} \quad (98)$$

$$+ F^{i-1}[a_{31}(\zeta_t + \varepsilon_t - \varepsilon_{t-1}) + a_{32}(\zeta_{t-1} + \varepsilon_{t-1})] + F^i[a_{11}(\zeta_t + \varepsilon_t - \varepsilon_{t-1}) + a_{12}(\zeta_{t-1} + \varepsilon_{t-1})]$$

$$\frac{1}{1-FL^{-1}}\varepsilon_{t-i} = \varepsilon_{t-i} + F\varepsilon_{t-i+1} + \dots + F^{i-2}\varepsilon_{t-2} \quad (99)$$

$$+ F^{i-1}[a_{41}(\zeta_t + \varepsilon_t - \varepsilon_{t-1}) + a_{42}(\zeta_{t-1} + \varepsilon_{t-1})] + F^i[a_{21}(\zeta_t + \varepsilon_t - \varepsilon_{t-1}) + a_{22}(\zeta_{t-1} + \varepsilon_{t-1})]$$

while

$$\frac{1}{1-FL^{-1}}\zeta_{t-1} = a_{31}(\zeta_t + \varepsilon_t - \varepsilon_{t-1}) + a_{32}(\zeta_{t-1} + \varepsilon_{t-1}) + F[a_{11}(\zeta_t + \varepsilon_t - \varepsilon_{t-1}) + a_{12}(\zeta_{t-1} + \varepsilon_{t-1})] \quad (100)$$

$$\frac{1}{1-FL^{-1}}\varepsilon_{t-1} = a_{41}(\zeta_t + \varepsilon_t - \varepsilon_{t-1}) + a_{42}(\zeta_{t-1} + \varepsilon_{t-1}) + F[a_{21}(\zeta_t + \varepsilon_t - \varepsilon_{t-1}) + a_{22}(\zeta_{t-1} + \varepsilon_{t-1})] \quad (101)$$

$$\frac{1}{1-FL^{-1}}\zeta_t = a_{11}(\zeta_t + \varepsilon_t - \varepsilon_{t-1}) + a_{12}(\zeta_{t-1} + \varepsilon_{t-1}) \quad (102)$$

$$\frac{1}{1-FL^{-1}}\varepsilon_t = a_{21}(\zeta_t + \varepsilon_t - \varepsilon_{t-1}) + a_{22}(\zeta_{t-1} + \varepsilon_{t-1}). \quad (103)$$

Also note that, since the combination of shocks  $\zeta_t + \varepsilon_t - \varepsilon_{t-1}$  is known at  $t$ :

$$\frac{1}{1-FL^{-1}}(\zeta_t + \varepsilon_t - \varepsilon_{t-1}) = \zeta_t + \varepsilon_t - \varepsilon_{t-1} - F[a_{21}(\zeta_t + \varepsilon_t - \varepsilon_{t-1}) + a_{22}(\zeta_{t-1} + \varepsilon_{t-1})] \quad (104)$$

$$\frac{1}{1-FL^{-1}}(\zeta_{t-1} + \varepsilon_{t-1} - \varepsilon_{t-2}) = \zeta_{t-1} + \varepsilon_{t-1} - \varepsilon_{t-2} + F(\zeta_t + \varepsilon_t - \varepsilon_{t-1}) - F^2[a_{21}(\zeta_t + \varepsilon_t - \varepsilon_{t-1}) + a_{22}(\zeta_{t-1} + \varepsilon_{t-1})]. \quad (105)$$

Similar expressions can be derived for the demand shocks.

The operator  $(1-FL^{-1})$  can again be inverted on the right side of (84), allowing us to deduce that equilibrium  $P_t$  will indeed be given by (40) so long as the  $\pi_{ij}$  coefficients satisfy the following system of equations (with matrices  $A$  and  $B$  defined in (96) and (97)).

The five coefficients  $\pi_{10} = \pi_{20}$ , and  $\Pi_s = [\pi_{11} \ \pi_{21} \ \pi_{12} \ \pi_{22}]$  satisfy the five simultaneous equations:

$$\begin{aligned}
& (\theta_0 - \theta_1 F + \theta_2 F^2) \Pi_s' A' \begin{bmatrix} a_{31} + a_{11} F \\ a_{41} + a_{21} F \\ (a_{31} + a_{11} F) F \\ (a_{41} + a_{21} F) F \end{bmatrix} + [\Phi + \Gamma + (\theta_0 - \theta_1 F + \theta_2 F^2)(a_{11} + a_{21})] \pi_{10} \\
& = -(1 - \psi F)(1 - a_{21} F)
\end{aligned} \tag{106}$$

$$\begin{aligned}
& (\theta_0 - \theta_1 F + \theta_2 F^2) \Pi_s' A' \begin{bmatrix} a_{32} + a_{12} F \\ a_{42} + a_{22} F \\ (a_{32} + a_{12} F) F \\ (a_{42} + a_{22} F) F \end{bmatrix} + \{\theta_0(a_{12} + a_{22}) - \\
& (\theta_1 - \theta_2 F)[1 + (a_{12} + a_{22})F]\} \pi_{10} + (\Phi + \Gamma) \pi_{11} = \psi + a_{22} F(1 - \psi F)
\end{aligned} \tag{107}$$

$$\begin{aligned}
& (\theta_0 - \theta_1 F + \theta_2 F^2) \Pi_s' A' \begin{bmatrix} a_{32} - a_{31} + (a_{12} - a_{11}) F \\ a_{42} - a_{41} + (a_{22} - a_{21}) F \\ [a_{32} - a_{31} + (a_{12} - a_{11}) F] F \\ [a_{42} - a_{41} + (a_{22} - a_{21}) F] F \end{bmatrix} + \{\theta_0(a_{12} + a_{22} - a_{11} - a_{21}) - \\
& (\theta_1 - \theta_2 F)[1 + (a_{12} + a_{22} - a_{11} - a_{21})F]\} \pi_{10} + (\Phi + \Gamma) \pi_{21} \\
& = 1 + \psi(1 - F) + (a_{22} - a_{21})F(1 - \psi F)
\end{aligned} \tag{108}$$

$$\Pi_s' A' \begin{bmatrix} -(\theta_1 - \theta_2 F) \\ 0 \\ (\theta_0 - \theta_1 F + \theta_2 F^2) \\ 0 \end{bmatrix} + \theta_2 \pi_{10} + (\Phi + \Gamma) \pi_{12} = 0 \tag{109}$$

$$\Pi_s' A' \begin{bmatrix} 0 \\ -(\theta_1 - \theta_2 F) \\ 0 \\ (\theta_0 - \theta_1 F + \theta_2 F^2) \end{bmatrix} + \theta_2 \pi_{10} + (\Phi + \Gamma) \pi_{22} = -\psi \tag{110}$$

Similarly,  $\pi_{30} = \pi_{40}$ , and  $\Pi_d = [\pi_{31} \pi_{41} \pi_{32} \pi_{42}]$  satisfy the five simultaneous equations:

$$\begin{aligned}
& (\theta_0 - \theta_1 F + \theta_2 F^2) \Pi_d' B' \begin{bmatrix} b_{31} + b_{11} F \\ b_{41} + b_{21} F \\ (b_{31} + b_{11} F) F \\ (b_{41} + b_{21} F) F \end{bmatrix} + [\Phi + \Gamma + (\theta_0 - \theta_1 F + \theta_2 F^2)(b_{11} + b_{21})] \pi_{30} \\
& = \Gamma(1 - \rho F)(1 - b_{21} F)
\end{aligned} \tag{111}$$

$$\begin{aligned}
& (\theta_0 - \theta_1 F + \theta_2 F^2) \Pi_d' B' \begin{bmatrix} b_{32} + b_{12} F \\ b_{42} + b_{22} F \\ (b_{32} + b_{12} F) F \\ (b_{42} + b_{22} F) F \end{bmatrix} + \{\theta_0(b_{12} + b_{22}) - \\
& (\theta_1 - \theta_2 F)[1 + (b_{12} + b_{22})F]\} \pi_{30} + (\Phi + \Gamma) \pi_{31} = -\Gamma \rho - \Gamma b_{22} F(1 - \rho F)
\end{aligned} \tag{112}$$

$$\begin{aligned}
& (\theta_0 - \theta_1 F + \theta_2 F^2) \Pi_d' B' \begin{bmatrix} b_{32} - b_{31} + (b_{12} - b_{11}) F \\ b_{42} - b_{41} + (b_{22} - b_{21}) F \\ [b_{32} - b_{31} + (b_{12} - b_{11}) F] F \\ [b_{42} - b_{41} + (b_{22} - b_{21}) F] F \end{bmatrix} + \{\theta_0(b_{12} + b_{22} - b_{11} - b_{21}) - \\
& (\theta_1 - \theta_2 F)[1 + (b_{12} + b_{22} - b_{11} - b_{21})F]\} \pi_{30} + (\Phi + \Gamma) \pi_{41} \\
& = -\Gamma - \rho \Gamma(1 - F) - \Gamma(b_{22} - b_{21})F(1 - \rho F)
\end{aligned} \tag{113}$$

$$\begin{aligned}
& \Pi_d' B' \begin{bmatrix} -(\theta_1 - \theta_2 F) \\ 0 \\ (\theta_0 - \theta_1 F + \theta_2 F^2) \\ 0 \end{bmatrix} + \theta_2 \pi_{30} + (\Phi + \Gamma) \pi_{32} = 0
\end{aligned} \tag{114}$$

$$\begin{aligned}
& \Pi_d' B' \begin{bmatrix} 0 \\ -(\theta_1 - \theta_2 F) \\ 0 \\ (\theta_0 - \theta_1 F + \theta_2 F^2) \end{bmatrix} + \theta_2 \pi_{10} + (\Phi + \Gamma) \pi_{42} = \rho \Gamma
\end{aligned} \tag{115}$$

The remaining  $\pi$  coefficients satisfy the following equations that are separable from the above systems:

$$\Pi_s'A' \begin{bmatrix} \theta_2 \\ 0 \\ -(\theta_1 - \theta_2 F) \\ 0 \end{bmatrix} + (\Phi + \Gamma)\pi_{13} = 0 \quad (116)$$

$$\Pi_s'A' \begin{bmatrix} 0 \\ \theta_2 \\ 0 \\ -(\theta_1 - \theta_2 F) \end{bmatrix} + (\Phi + \Gamma)\pi_{23} = 0 \quad (117)$$

$$\Pi_s'A' \begin{bmatrix} 0 \\ 0 \\ \theta_2 \\ 0 \end{bmatrix} + (\Phi + \Gamma)\pi_{14} = 0 \quad (118)$$

$$\Pi_s'A' \begin{bmatrix} 0 \\ 0 \\ 0 \\ \theta_2 \end{bmatrix} + (\Phi + \Gamma)\pi_{24} = 0 \quad (119)$$

$$\Pi_d'B' \begin{bmatrix} \theta_2 \\ 0 \\ -(\theta_1 - \theta_2 F) \\ 0 \end{bmatrix} + (\Phi + \Gamma)\pi_{33} = 0 \quad (120)$$

$$\Pi_d'B' \begin{bmatrix} 0 \\ \theta_2 \\ 0 \\ -(\theta_1 - \theta_2 F) \end{bmatrix} + (\Phi + \Gamma)\pi_{43} = 0 \quad (121)$$

$$\Pi_d'B' \begin{bmatrix} 0 \\ 0 \\ \theta_2 \\ 0 \end{bmatrix} + (\Phi + \Gamma)\pi_{34} = 0 \quad (122)$$

$$\Pi_d'B' \begin{bmatrix} 0 \\ 0 \\ 0 \\ \theta_2 \end{bmatrix} + (\Phi + \Gamma)\pi_{44} = 0 \quad (123)$$



## 15. Appendix 2 - data sources

The data for this paper were obtained from the International Monetary Fund's International Financial Statistics (IFS) or from the BIS. In the case of West Germany, industrial production was taken from the BIS, series SBBBDE91, and producer prices were also taken from the BIS, series VBBBDE02; data were available from 1962 through 1994. In the cases of the Netherlands and the United Kingdom, data were taken from IFS (line numbers 66-c and 63), and data were available from 1960 through 1994. For Italy, data were taken from IFS and were available from 1960 through the third quarter of 1993. For France, data on industrial production were taken from IFS, while data on producer prices were taken from the BIS, series VBNBFR02; data were available from 1970 through 1994.

There were several reasons for our choice of quarterly data. First, preliminary analysis of the data for our largest economy, Germany, showed that after conversion to logs, first differencing and removal of a trend, the output series contained considerable month-to-month negative autocorrelation. In our view, this month-to-month negative correlation represents the effects of weather, changes in the number of working days per month as we go from year to year, and perhaps measurement error, and not the business cycle phenomena that are our focus. Second, we expect the lags involved in business cycle fluctuations to last more than a year and perhaps several years. However, the computational burden of fitting long lags is increased when monthly data are used and there are many more autocorrelations and cross correlations to be fit. Third, in the case of France, the only consistent data series on producer prices that covered the period we wished to focus on was not available on a monthly basis. And finally, using quarterly data makes it easier to make comparisons with the results in Cooley and Ohanian (1991) and Kydland and Prescott (1990).

As discussed in the text, unit-root tests indicated that the log of the raw series contained a single unit root; accordingly, all the series were first-differenced. To further ensure stationarity, each differenced series was regressed onto a constant, a linear trend, and seasonal dummies, and the residuals from these regressions were used as our measures of output and prices in the manufacturing sector.

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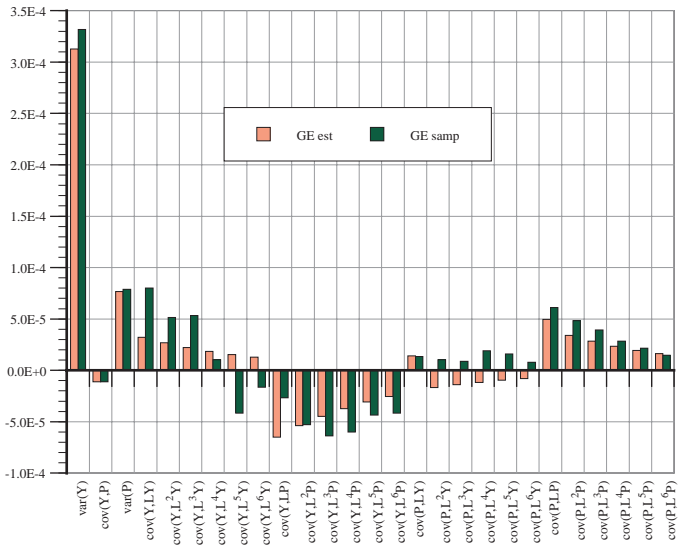
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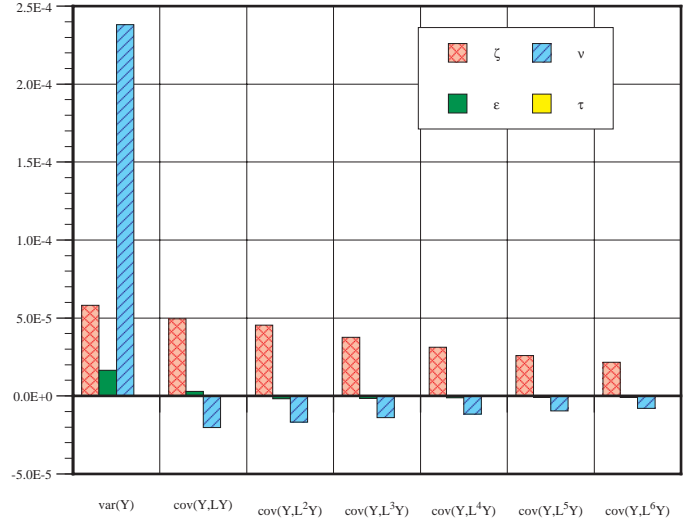
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Figure 1: Germany

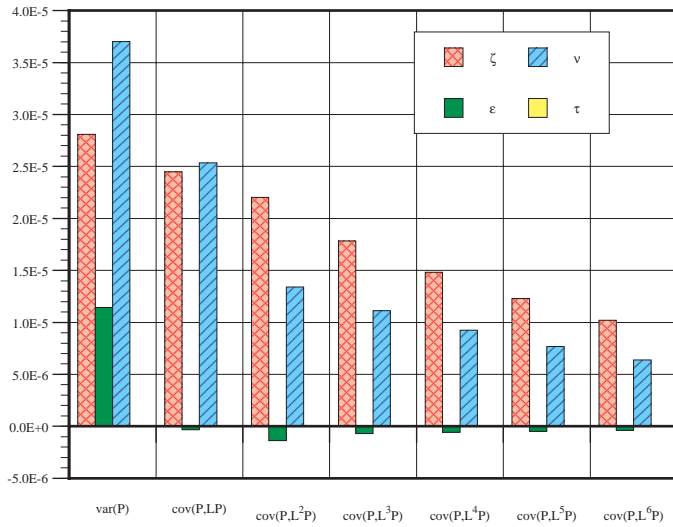
Estimated and Sample Moments



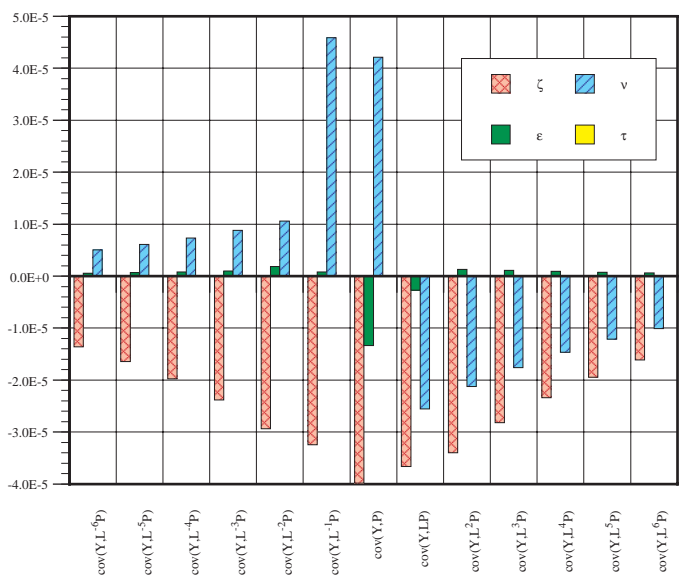
Output Growth Decompositions



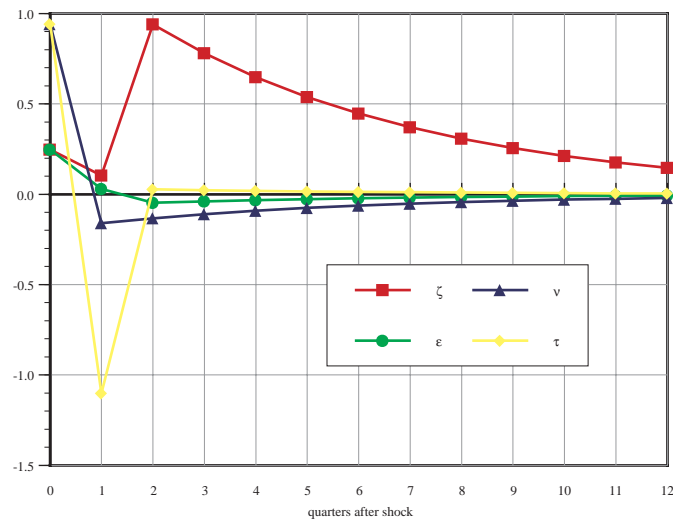
Inflation Decompositions



Covariance Decompositions



Output Growth Impulse Response



Inflation Impulse Response

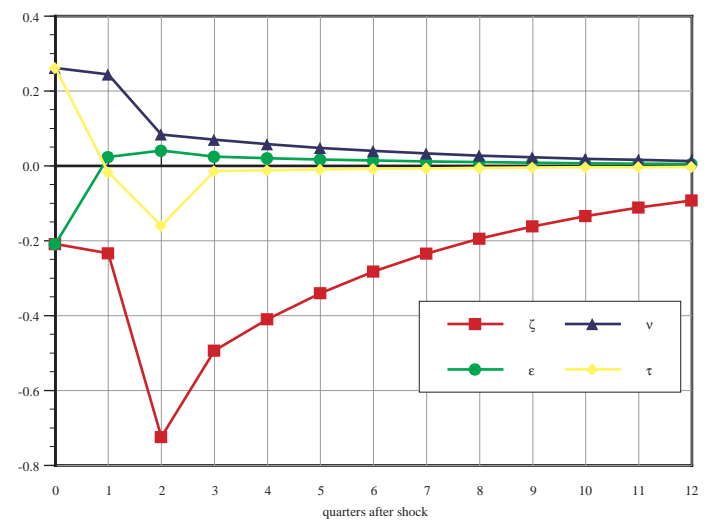
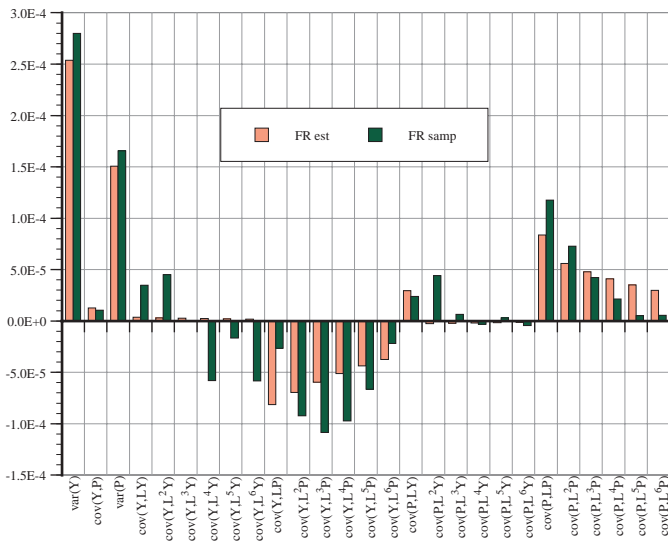
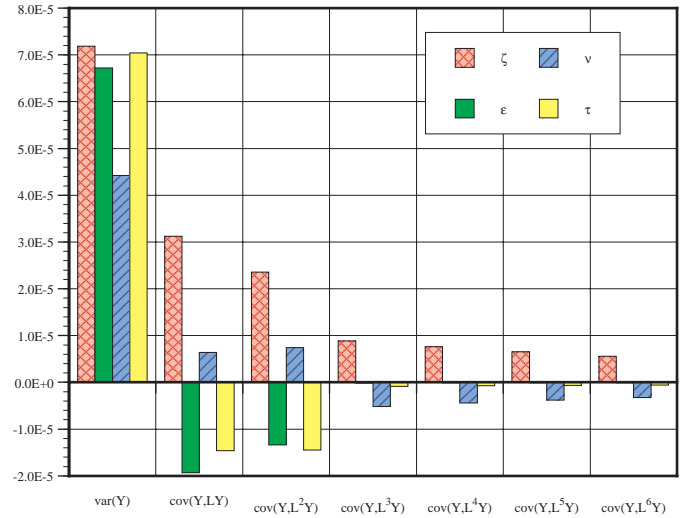


Figure 2: France

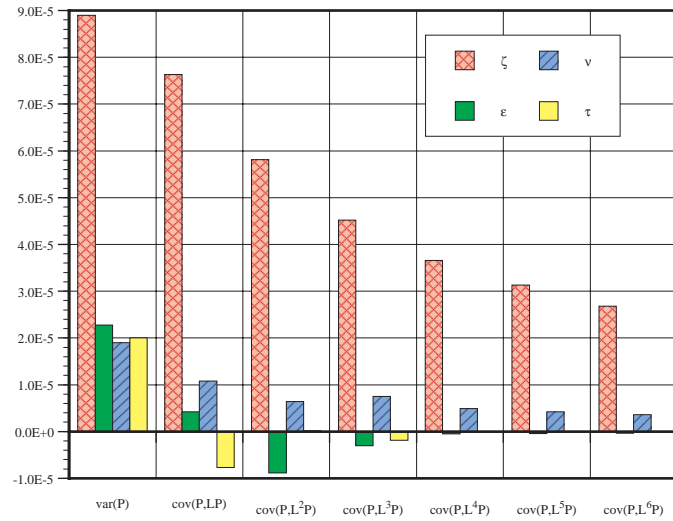
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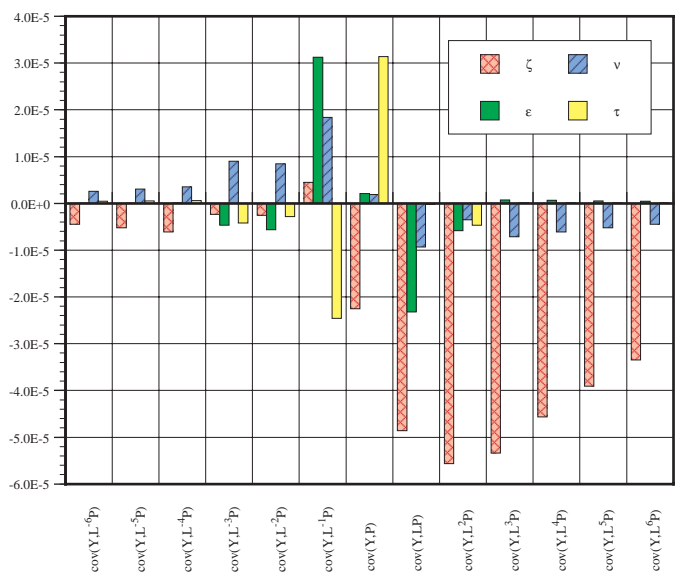
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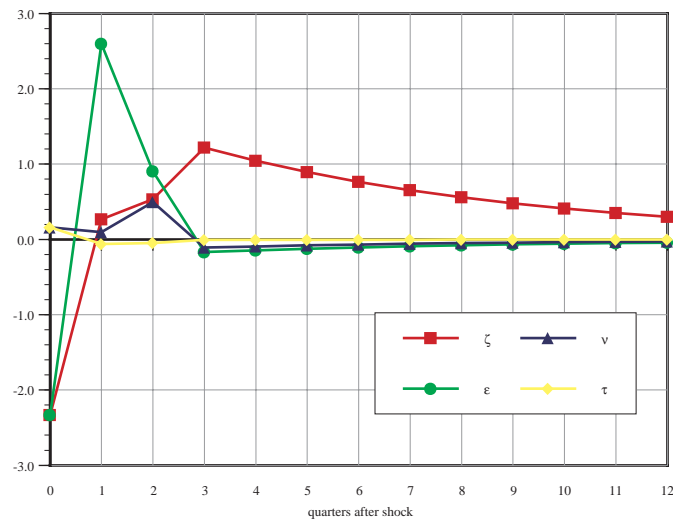
Inflation Decompositions



Covariance Decompositions



Output Growth Impulse Response



Inflation Impulse Response

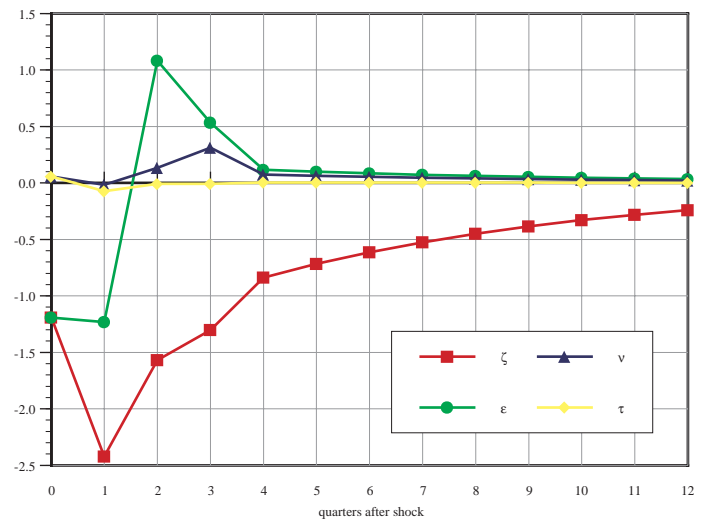
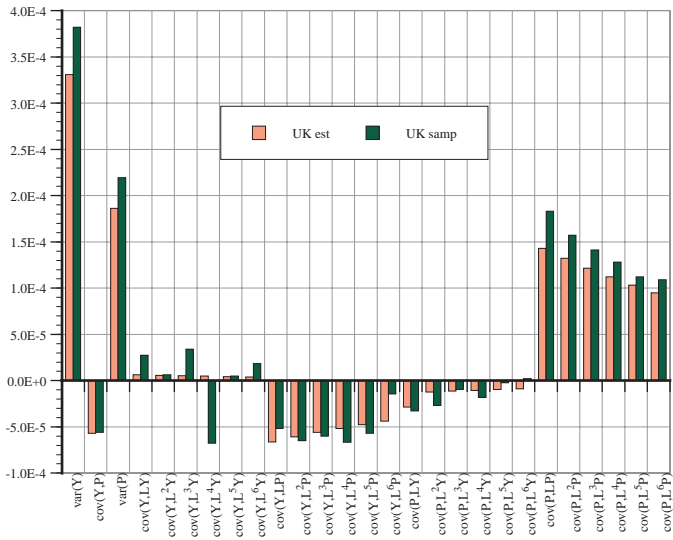
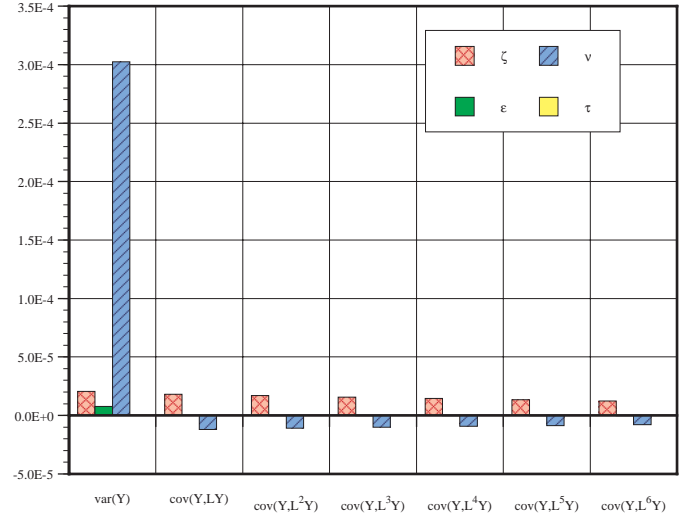


Figure 3: United Kingdom

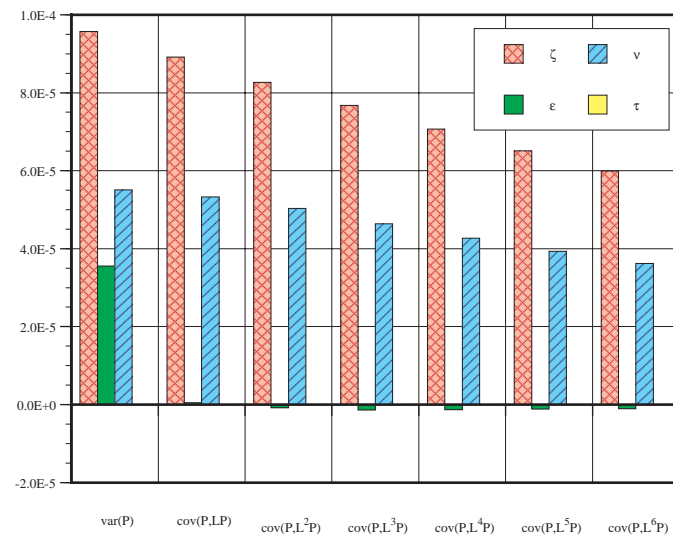
Estimated and Sample Moments



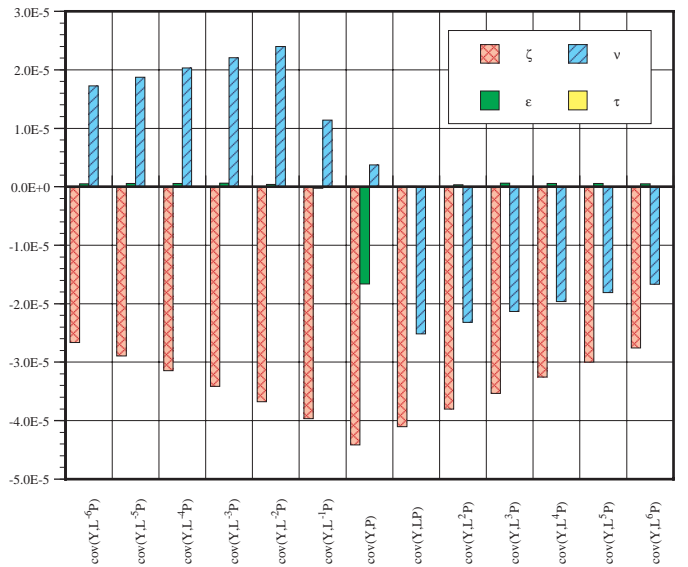
Output Decompositions



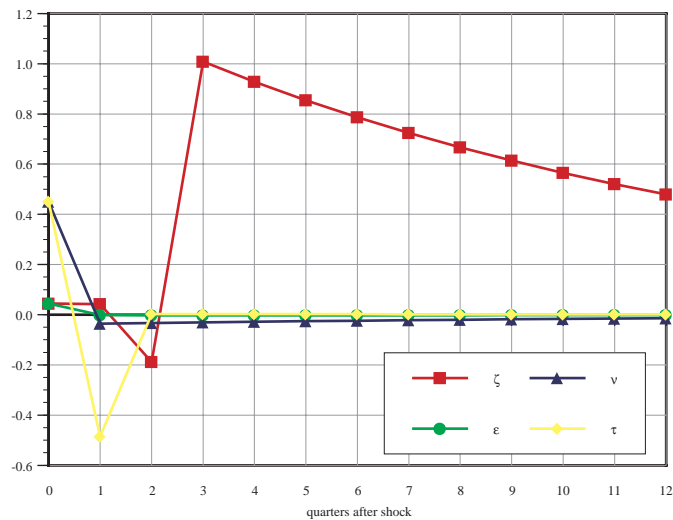
Inflation Decompositions



Covariance Decompositions



Output Impulse Response



Inflation Impulse Response

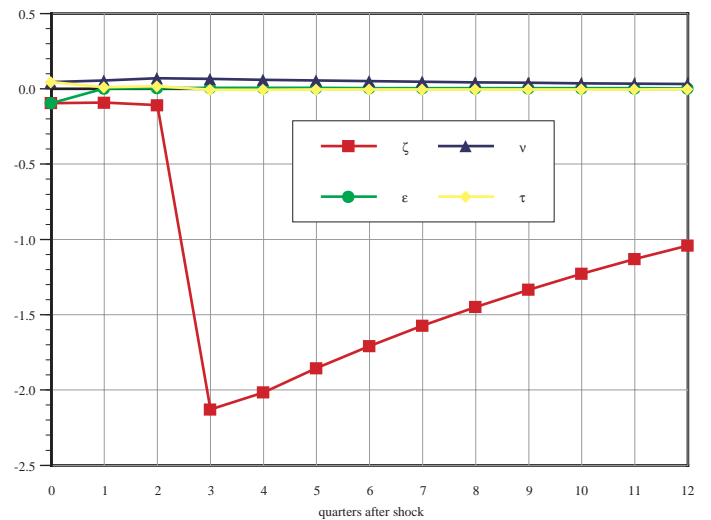
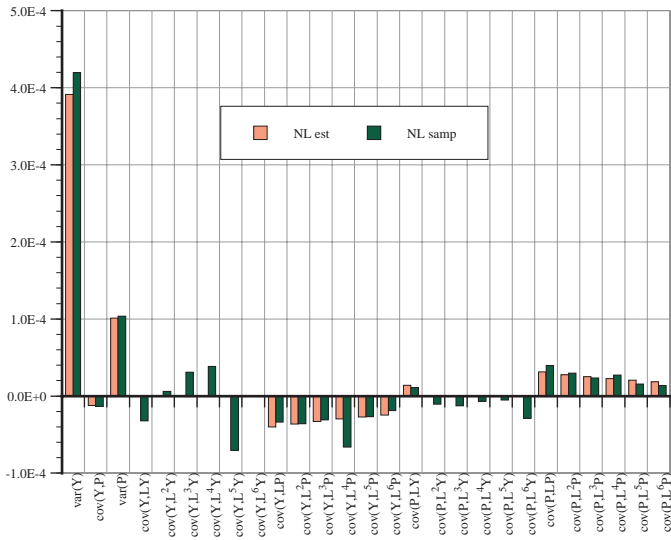
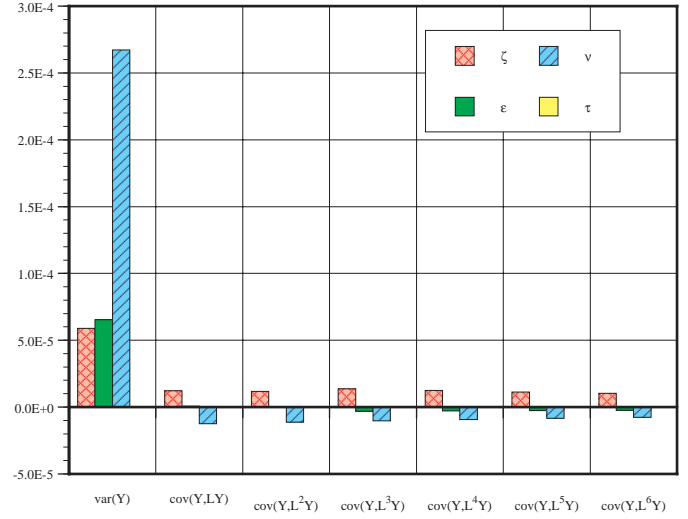


Figure 4: Netherlands

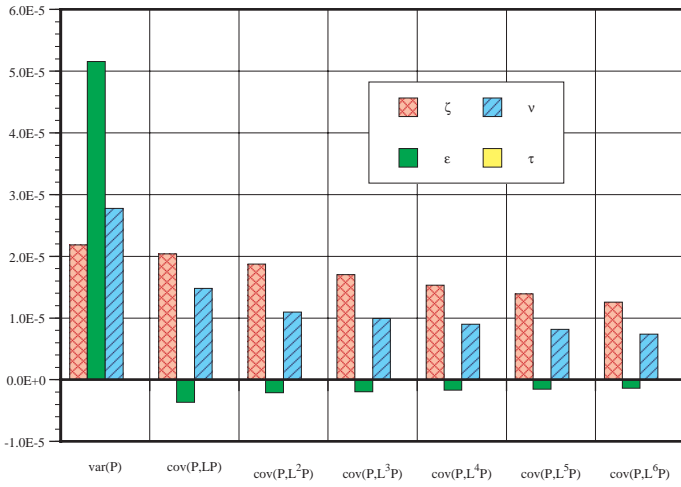
Estimated and Sample Moments



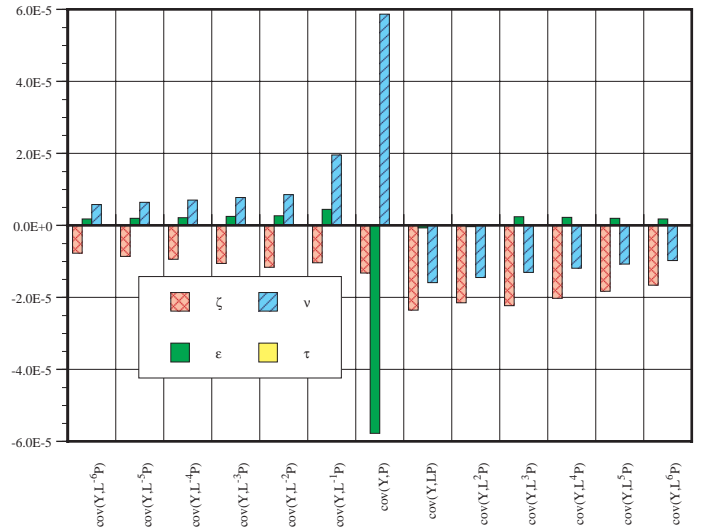
Output Decompositions



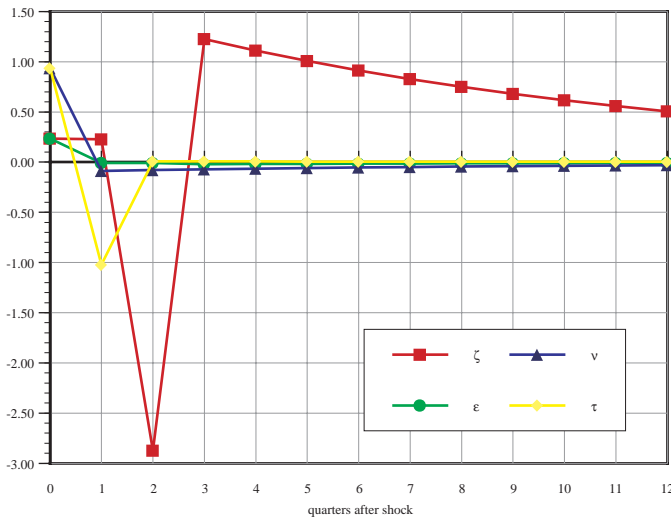
Inflation Decompositions



Covariance Decompositions



Output Impulse Response



Inflation Impulse Response

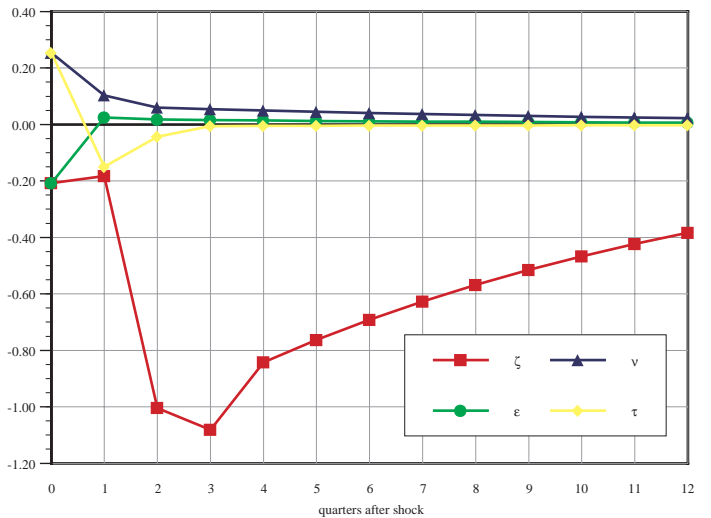
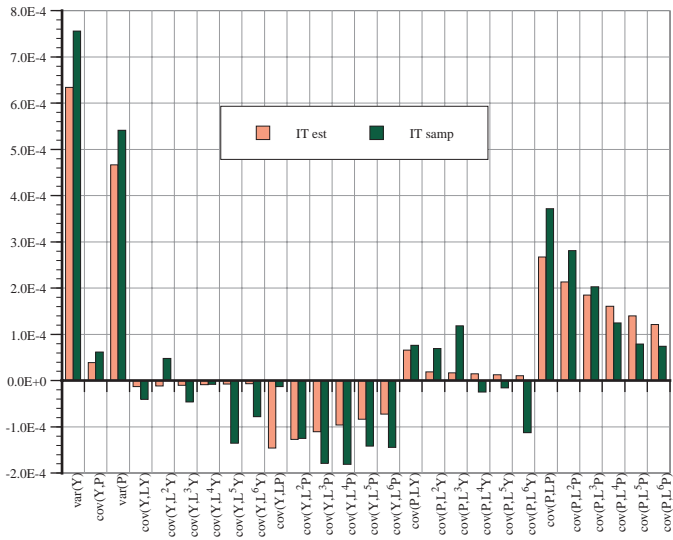
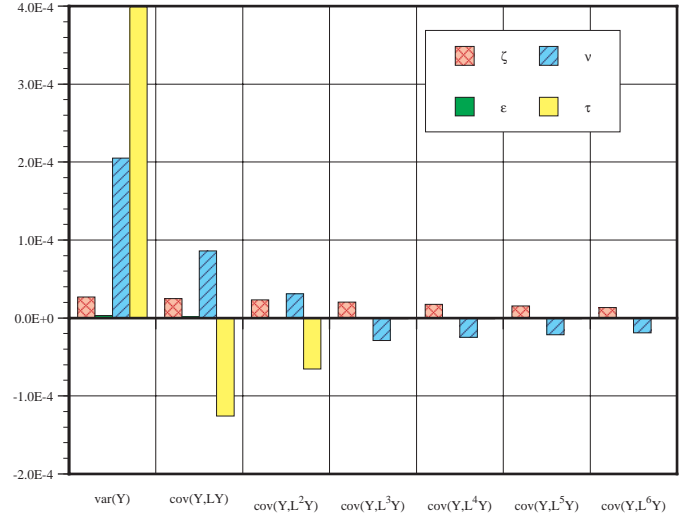


Figure 5: Italy

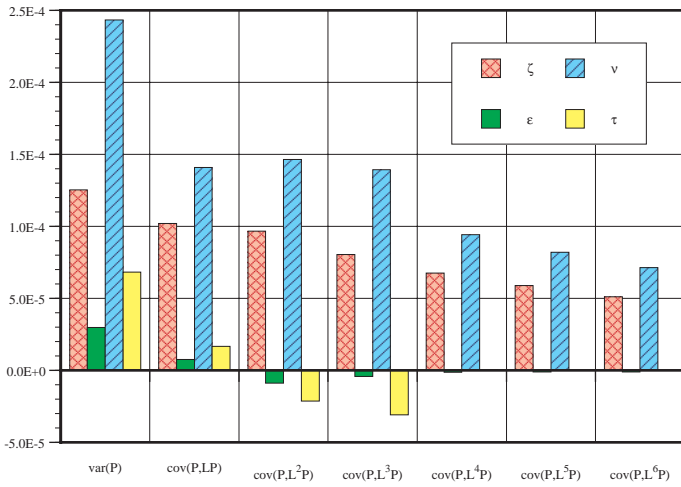
Estimated and Sample Moments



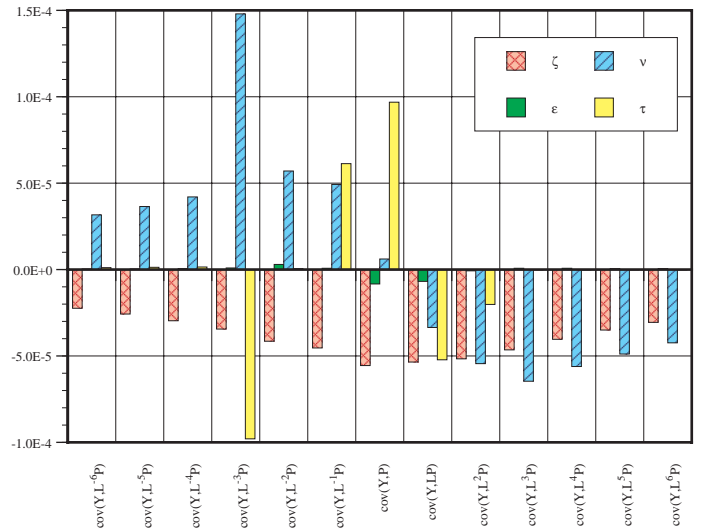
Output Decompositions



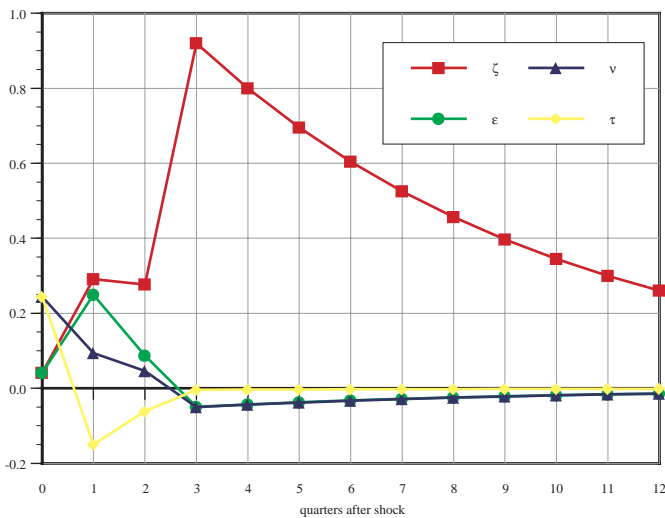
Inflation Decompositions



Covariance Decompositions



Output Impulse Response



Inflation Impulse Response

