The Cost of Doing Business Abroad and International Capital Market Equilibrium

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Abstract: The implications of the costs of doing business in foreign countries for the resulting capital market equilibrium are studied. When transferring capital goods across national boundaries, the costs incurred are quasi-fixed in a one-good, two-country, intertemporal model with complete financial markets. In our model of the international capital market, deviations from purchasing power parity are endogenously generated. The relative price of physical resources located in one country compared to resources located in another is called the "real exchange rate." The outcome of the model-based analysis is an endogenous generation of a mean-reverting real exchange rate in a continuous-time, general equilibrium model of the international capital market. In dynamic equilibrium, the transfer of capital goods between the two countries is found to be infrequent and lumpy in nature as is observed in foreign direct investment.

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The Cost of Doing Business Abroad and International Capital Market Equilibrium

1.0 Introduction

This paper examines the implications of the costs of doing business in foreign countries for the resulting capital market equilibrium. When transferring capital goods across national boundaries is costly we indicate that the costs incurred are quasi-fixed in a one-good, two-country, intertemporal model with complete financial markets. In our model of the international capital market, deviations from purchasing power parity (PPP) are endogenously generated. The relative price of physical resources located in one country compared to resources located in another is called the "real exchange rate." The outcome of the model-based analysis is an endogenous generation of a mean-reverting, real exchange rate in a continuous-time, general equilibrium model of the international capital market. In dynamic equilibrium, the transfer of capital goods between the two countries is found to be infrequent and lumpy in nature as is observed in foreign direct investment.

Recently, Evans and Lothian (1993) have developed an empirical model to uncover the sources of fluctuations in the real dollar exchange rates of major industrial countries. They find that the real exchange rates did not evolve simply in response to permanent shocks. Transitory shocks played a relatively small but statistically significant role. Earlier, Huizinga (1987) documented mean-reversion of commodity prices to their long-run equilibrium value. This implies negative serial correlation in exchange rates in the longer run in contrast to the serially correlated changes implied by a random walk. Our theoretical model for the real exchange rate is consistent with these empirical findings. As the data indicate, the real exchange rate in our model is influenced by the presence of both temporary and permanent components. Earlier empirical results in Officer (1976), Lee (1976), King (1977), and Cornell (1979) are also reinforced by the analysis in this paper.

The real exchange rate in our model moves within a band between upper and lower barriers, as in a target zone, or like EMS exchange rates. The movement of capital goods occurs when the real exchange rate is reflected back within the band by the upper or lower barrier. With quasi-fixed costs for capital stock transfer, there is mean-reversion in the real exchange rate. Recent empirical studies indicating a significant mean-reversion component in the long-run behavior of real exchange rates by Koedijk and Schotman (1990) as well as Glen (1992), mean-reversion in real exchange rates using modified Dickey-Fuller tests by Cheung and Lai (1994), and mean reversion in EMS exchange rates by Svensson (1991) and Mizrach (1993) validate our theoretical results.

Increasingly, firms are engaged in manufacturing activities located in different countries. This network of activities operating for the multinational corporation derives value because of the ease of transfer of capital goods for shifting production between one country and another. The cost of transferring capital goods for foreign direct investment is but one component of the overall cost of doing business abroad. The entrenchment of incumbents in the foreign market and other barriers to foreign direct investment result in a cost structure different from the one for merely reallocating capital in a domestic situation. As Kogut and Kulatilaka (1994) point out, it is costly to switch production from one country to another due to costs associated with shutdowns and startups, labor contracting and managerial time commitments. Entrenchment of incumbents in foreign markets raises barriers to foreign direct investment because an incumbent has lower distribution costs than an entrant as suggested by Farrell and Shapiro's (1988) switching cost model.

We take one step further in this paper by observing that while technology spillover is quite common between such multinational firms, the competitive advantage of incumbents in foreign markets due to lower distribution costs does not spillover to other firms. As Stein (1994) observes in a model of repeated innovation, there are no spillovers with distribution costs and incumbent firms' existing customer bases give them a competitive advantage over would-be entrants. In the case of transfer of capital goods, extending this idea to the international context, we find that to generate any returns on the capital, multinationals entering these foreign markets would need to incur costs proportional to the amount of such entrenched capital stock in the foreign market. The international capital market equilibrium we analyze does not explicitly include the modeling of a distribution sector but the costs of operating abroad emerge from such barriers to foreign direct investment.

The international capital market has been modeled in recent times using both partial equilibrium and general equilibrium approaches. International capital asset pricing models obtained by putting together the first-order conditions of portfolio choice for various investors of the world or using the small country assumption are good examples of the partial equilibrium approach. The general equilibrium approach requires a full specification of the exogenous variables and their stochastic processes where all endogenous variables can be analyzed. Earlier general equilibrium models of the international capital market like Stockman and Tesar (1990), Backus and Smith (1993) and Backus, Kehoe and Kydland (1994) involved the use of nontraded goods to explain terms of trade and investors' portfolio choice. As a class, general equilibrium models do improve on partial equilibrium approaches by offering more testable results for empirical research.

However, the general equilibrium models mentioned above have proved inadequate in some ways. These models could generate movements in the terms of trade but could not fully explain the high volatility of the terms of trade. Secondly, they could not explain the small unconditional correlation in consumption across countries as well as its relatively small size compared to the correlation in output.

Thus, more recently, these models have given way to an approach which introduces frictions in the specification of production sets. When physical movement of goods in international markets takes place these models include a transfer cost. This cost provides the needed degree of freedom for variation in the terms of trade. For convenience, these models have usually assumed such transfer costs to be proportional to the amount of physical resources being transferred (see, for example, Dumas 1992, Baxter and Crucini 1991, and Uppal 1993). As we discussed above, however, the overall transfer cost can no longer be captured by the simplistic assumption of proportional costs[1] when applied to the movement of capital goods across countries for direct foreign investment. Besides, such proportional costs lead to infrequent movement of physical resources in infinitesimal amounts on each such occasion. This seems contrary to the large, lumpy transfer of capital goods observed in international trade. Our results are consistent with this observation.

The assumption that transfer costs are proportional to the capital stock in the foreign market emerges from the distribution cost advantage of incumbents with existing customer bases. Such customer bases lead to two important consequences: (a) they could reduce the long-run average level of innovation represented by new capital inflows, and (b) they lead to endogenous bunching, or waves, in innovative activity. Our findings (in the international extension) are consistent with such conclusions drawn very early by Schumpeter (1936) and recently researched by Stein (1994): first, we find that finite, lumpy capital transfers indeed take place infrequently and that the higher the transfer cost, lower are such capital transfers; second, we find the wave-like aspect of innovation manifested in the foreign direct investment context as the positive autocorrelatedness of capital stock transfers in the shorter run, which implies that if

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[1]Hollifield and Uppal (1995) derive the reduced-form relation between the forward premium and the change in the spot exchange rate in the presence of deviations from purchasing power parity in the proportional cost-based model by Dumas (1992) and find that when using an instrument set containing both nominal and real returns, the model is rejected by the monthly data.

there is an innovation today, the odds of another innovation tomorrow may be substantially higher.

The incentive for capital goods flow between countries is provided by the opportunity to invest in either location and the need to hedge against consumption shortfalls in one's own country. Though the one good in the two countries in our model is identical, random uncorrelated output shocks in the two countries provide the motivation for transfer of this capital good in spite of its being costly. Firms will be willing to invest capital abroad only if such costs can be traded off against commensurate benefits from going multinational via direct foreign investment. Indeed, the internalization theory posits that direct foreign investment occurs when a firm can increase its value by internalizing markets for certain of its intangible assets. These intangible assets such as marketing abilities, managerial skills, or consumer goodwill have characteristics of public goods in that their value is enhanced in direct proportion to the scale of the firm's markets. Morck and Yeung (1991) find that investors value multinationality for such intangible assets which justify direct foreign investment. In our simple two-country setup, we posit the portfolio diversification motive[2] rather than internalization gains as the benefit. In equilibrium, firms will be willing to transfer capital abroad when the incremental costs equal incremental benefits at the margin.

The rest of the paper is organized as follows. In Section 2.0 the model is set up and an approach for solving it is outlined. Section 3.0 presents the solution which includes the process for the movement of goods from one country to another and the process for deviations from the Law of One Price (LOP). Section 4.0 includes an interpretation of the results for real exchange rates. Section 5.0 concludes the paper and suggests ideas for future research.

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[2]See Adler and Dumas (1983).

2.0 The Model

We work with a simple world economy made up of two countries, home and foreign, each having a single, representative investor. Each representative is restricted to consuming only the domestically available physical good. The home and foreign representative investors are otherwise identical: they begin life with equal endowments, have isoelastic utility functions with the same degree of risk-aversion, and have the same rate of impatience.

The model is based on the costly transfer of storable goods from one country to another. The storable good is identified by its location, but is perfectly substitutable across the two countries for both consumption and production. The alternative to such transfer is consumption in the country of origin or reinvestment in a single, risky constant-returns-to-scale production process. The production processes are identical across the two countries in their instantaneous expected rates of return and instantaneous standard deviations of the rate of return. However, the output shocks[3] in the two countries are uncorrelated. Consumption of the good is location-specific but it is possible for the consumer-investor to own goods in both countries. The good is identified by its location.

The initial endowments of the consumer-investors imply symmetric treatment: in order to diversify their portfolio (i.e. provide a hedge against the risk of being subjected to adverse output shocks), the consumer-investors would move to equate the stocks of goods in the two countries. However, since transferring the good from one country to another (which is tantamount to building and dismantling capital) is costly and, in particular, involves "quasi-fixed" costs, the consumer-investors will "wait and see" before moving the good across national boundaries. The fixed cost of investing (or shipping capital goods) abroad would encourage lumpy investment since the cost of investment is independent of the size of investment. However, such investment abroad

[3]These could be, for example, productivity shocks as in real business cycle models.

will not take place unless the returns (with commensurate diversification of risk) from such investment exceed the fixed cost incurred. In the interim, when there is no foreign investment, there will be deviations from parity in capital stocks across the two countries. In such cases, the representative consumer-investors in the two countries would optimally rebalance their portfolios by consuming at different rates. This implies that consumers in the land of abundance, for example, will consume more rather than investing the good abroad as an immediate response. In this "region of tolerance" they prefer to increase their consumption rates rather than transferring the good to the foreign country. At such times, the relative price of the domestic consumption good with respect to the foreign consumption good will differ from one: usual consumption sharing rules which apply in integrated goods markets will not apply. Individual consumption will be a function not just of aggregate consumption but also of the allocation of physical assets between the two countries.

Following in the tradition of Lucas and Prescott (1971) and Constantinides (1982), the international capital market equilibrium in this paper can be arrived at via a short-cut where a Pareto-optimal consumption allocation that can be achieved by the consumer-investors is studied. The transfer of the good between the two countries being costly causes the Pareto optimum to be constrained. Under these conditions the equilibrium in the goods-market and capital-market can be replaced by an appropriate central planning problem.

2.1 The optimization problem

Let us assume that all consumers start their lives with endowments of goods in amounts such that the appropriate central welfare function devotes equal weights to the utility levels of the households of the two countries. These weights are constant over time and across states of nature only in a Pareto-optimal market. However, equal weights do not imply identical endowments. Since an individual consumer is endowed with goods in both countries, the advantage of location is set off against the disadvantage of smaller quantities.

We denote by (1-γ) the degree of risk-aversion common to all investors, and by c_t and c_t^*, the rates of consumption of the good located in the home location and foreign location at time t respectively. In this problem, K_t and K_t^* are the two state variables (stocks located at home and abroad), ρ is the discount rate of utilities common to all investors, and α and σ are the instantaneous expected value and instantaneous standard deviation of the rates of return in the constant-returns-to-scale production processes; dz and dz^* are the increments of standard Wiener processes, uncorrelated across time, and V(K, K*) is the maximum expected discounted utility constrained by the capital generation processes in the two countries. The consumer-investors do not have to make portfolio decisions for capital.

The central planner's optimization problem then is:

$$V(K,K^*) = \max_{\substack{c,c^* \\ T(\lambda)}} E_t \int_t^{\infty} e^{-\rho(u-t)} \left[\frac{1}{\gamma} c_u^{\gamma} + \frac{1}{\gamma} (c_u^*)^{\gamma} \right] du; \gamma < 1; \tag{1}$$

subject to:

$$dK_t = (\alpha K_t - c_t)dt + \sigma K_t dz_t \quad \text{and} \tag{2}$$

$$dK_t^* = (\alpha K_t^* - c_t^*)dt + \sigma K_t^* dz_t^* \tag{3}$$

where the maximization pertains to the region of tolerance mentioned earlier, and denoted by **T**(λ). Given the presence of fixed shipping costs, such a region **T**(λ) will exist wherein it is optimal for investors not to invest any additional capital abroad. As in Figure 1, in the (K, K*) plane, the region **T**(λ) is bounded by two radial lines, with slopes λ and 1/λ respectively, passing through the origin[4] and symmetrically placed around the 45^{o} line in the first quadrant.[5]

The function *V (K, K*)* exists provided that [6]:

$$r > g\left[a - \frac{1}{2}(1-g)s^{2}\right]. \tag{4}$$

We use X_t and X_t^* as the amounts of the good being shipped or moved from the home location to the foreign location and vice versa. As explained earlier, the fixed cost incurred in shipping the good abroad is proportional to the capital stock of the country to which the good is being shipped. This is denoted by *a*. Take the case of the capital stock *K* located in the home country. The reductions in the capital stock K occur due to consumption in the home country, *c*, and shipments from the home to the foreign country,*X*. Additions to the stock take place due to output $K(a\,dt + s\,dz)$ where dz is white noise and the receipts of goods in amount X^* received from abroad. Thus,

If $X \geq 0$ (when transferring capital to the foreign country):

$$K_t = K_{t-} - X_t \quad \text{and} \tag{5}$$

$$K_t^* = K_{t-}^* + X_t - aK_{t-}^*. \tag{6}$$

If $X^* \geq 0$ (when transferring capital to the domestic country):

$$K_t^* = K_{t-}^* - X_t^* \quad \text{and} \tag{7}$$

$$K_t = K_{t-} + X_t^* - aK_{t-} \tag{8}$$

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[4]This region of tolerance is estimated in Dumas (1992) for the proportional costs case. The formal proof to show that such a region is indeed bounded by two radial lines as mentioned above, see Theorem 4.2 by Davis and Norman (1990).

[5]As we will see in Sec. 2.2, this results from first-order, optimality conditions necessary to solve this problem.

[6]See Merton (1971) or Dumas (1992).

Two properties of the value function *V(K, K*)*, namely, homogeneity of degree γ in *K* and *K** and symmetry with respect to the state variables, are used in solving this problem. The technique used is the same as in Grossman and Laroque (1990), namely, a variant of the dynamic programming approach developed by Krylov (1980): i.e., express the problem given by Equations (1) through (3) as an optimal stopping problem:

$$V(K,K^*) = \max_{\substack{c,c^* \\ x,x^* \geq 0}} E_t \int_t^{\tau} e^{-\rho(u-t)} \left[\frac{1}{\gamma} c_u^{\gamma} + \frac{1}{\gamma} (c_u^*)^{\gamma} \right] du + E_t e^{-\rho(\tau - t)} V(y, y^*) \qquad (9)$$

subject to:

$$dK_t = (\alpha K_t - c_t)dt + \sigma K_t dz_t \quad \text{and}$$

$$dK_t^* = (\alpha K_t^* - c_t^*)dt + \sigma K_t^* dz_t^* .$$

Here

$$y = K_{\tau-} - X_{\tau} + X_{\tau}^* - aK_{\tau-}\varepsilon(X_{\tau}^*), \qquad (10)$$

and $$y^* = K_{\tau-}^* + X_{\tau} - X_{\tau-}^* - aK_{\tau-}^*\varepsilon(X_{\tau}), \qquad (11)$$

and ε is an indicator function:

$$\varepsilon(X) = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \quad \text{if X>0 or X = 0, respectively;} \qquad (12)$$

$$\varepsilon(X^*) = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \quad \text{if X*>0 or X* = 0, respectively;} \qquad (13)$$

where τ is the first time when X or $X^* > 0$.

This optimal stopping problem is different from the Dumas (1992) problem because proportional costs of shipping are analyzed. Here, costs of shipping are fixed. Expressed as a linear stochastic control problem, the shipments can be either an instantaneous control (as in the proportional shipping costs case) or an impulse control (as in the fixed shipping costs case).[7] Both would occur instantaneously. However, instantaneous control processes would be continuous and singular (Karatzas 1983) while impulse control processes would undergo finite jumps. The optimal stopping problem presented here is analogous to such an impulse control problem.

2.2 The optimality conditions

Taking advantage of the homogeneity and symmetry with respect to K and K^* leads us to the conclusion that the first-order conditions will be satisfied along (at least) one pair of rays in the (K, K^*) plane (a formal proof is in JeanBlanc-Picque 1993):

$$K_{t-} = \lambda K^*_{t-} \tag{14}$$

$$K^*_{t-} = \lambda K_{t-} \tag{15}$$

where λ and 1/λ are the slopes of the radial lines, respectively (see Figure 1). Within the cone, *no shipping* or no investment of capital abroad is the optimal decision. When $K \geq \lambda K^*$ or $K^* \geq \lambda K$, the optimal decision is to ship an amount so as to get back to the center of the cone.[8] Flood and Garber (1992) use impulse control applied to fundamentals (money supplies, etc.) where the applicable boundary condition is simply that the level of the exchange rate does not jump when the control is applied. In our case, optimal impulse control requires the imposition of a necessary condition of optimality that the value function of the dynamic program remains continuous at the time of regulation, as in the case of optimal stopping. This is in contrast to the case of proportional shipping where one would only ship infinitesimal amounts just enough to stay within the cone. In solving the problem, symmetry around the 45° line is used. The analysis is undertaken in the region delimited by any one of the two rays and the 45° line. The situation is analogous to a regulated Brownian motion[9] between two

[7]See Harrison and Taksar (1983) and Harrison, Sellke and Taylor (1983) for details.
[8]By formally solving the partial differential equation subject to boundary conditions (value-matching and smooth-pasting conditions) we prove this assertion later in this section.
[9]For an exhaustive treatment of regulated Brownian motion see Harrison (1985), Dixit (1991), and Dumas (1991).

barriers: the 45° line denoted as barrier 0 and the delimiting ray as barrier *q*. The intuition for the boundary conditions is that the value-matching conditions indicate that the investor is indifferent between capital transfer and non-transfer at the margin; the smooth pasting condition ensures continuity of the value function $V(K, K^*)$ of the firm (or investor) where first order conditions are employed.

Value-matching conditions: These ensure continuity between the points from and to which the jump occurs. Flood and Garber (1992) regulate the exchange rate process within a target zone by means of impulse control applied to the fundamentals. However, the applicable boundary conditions are only value-matching conditions requiring that the exchange rate does not jump when the control is applied. In our case, optimal impulse control also requires smooth-pasting conditions as necessary conditions for optimality.

$$V(K_A, K_A^*) = V(K_B, K_B^*) \tag{16}$$

where point A has coordinates (K_A, K_A^*) and point B has coordinates (K_B, K_B^*) in the *(K, K*)* plane (see Figure 1). Thus,

$$V(K_{t-} - X_t, K_{t-}^* + X_t - aK_{t-}^*) = V(K_{t-}, K_{t-}^*). \tag{17}$$

This condition provides the time τ at which to jump,

$$X_t = \frac{K_{t-} - K_{t-}^* + aK_{t-}^*}{2}. \tag{18}$$

Given this condition,

$$V\left[\frac{K_{t-} + K_{t-}^* - aK_{t-}^*}{2}, \frac{K_{t-} + K_{t-}^* - aK_{t-}^*}{2}\right] = V(K_{t-}, K_{t-}^*). \tag{19}$$

At the point A, $K_{t-} = \lambda K_{t-}^*$. It follows that, by symmetry, the target point of the jump is on the 45° line, where,

$$K_t^* = K_t \tag{20}$$

$$= \frac{K_{t-} + K_{t-}^*}{2}. \tag{21}$$

The first-order conditions which serve to determine the unknown λ are the smooth-pasting conditions.

Smooth-pasting conditions: These are first order conditions ensuring that the function is smooth between the points from and to which the jump occurs:

$$\frac{V_K(K_t, K_t^*)}{V_{K^*}(K_t, K_t^*)} = \frac{V_K(K_{t-}, K_{t-}^*)}{V_{K^*}(K_{t-}, K_{t-}^*)}. \tag{22}$$

Since LHS = 1 (by symmetry),

$$V_K(K_{t-}, K_{t-}^*) = V_{K^*}(K_{t-}, K_{t-}^*). \tag{23}$$

This condition gives the answer to the question of the initial and final points of the jump.[10] These should be such that the smooth-pasting conditions on $V(K, K^*)$ should be satisfied between both points since, otherwise, there will be an arbitrage opportunity. Inside the zone of no intervention, the process K (or K^*) moves of its own accord (following equations (2) and (3)). The expected change in V is brought about by the flow payoff $\left[\frac{1}{\gamma} c_u^{\gamma} + \frac{1}{\gamma}(c_u^*)^{\gamma}\right]$ and the effect of discounting at the rate ρ. The Hamilton-Jacobi equation characterizing the function V can be written as follows, for values of K and K^* in the interior of the cone:

$$0 = \left[\frac{1}{\gamma} - 1\right] V_K^{\frac{\gamma}{\gamma-1}} + \left[\frac{1}{\gamma} - 1\right] V_{K^*}^{\frac{\gamma}{\gamma-1}} - \rho V + V_K \alpha K + V_{K^*} \alpha K^* + \frac{1}{2} V_{KK} \sigma^2 K^2 + \frac{1}{2} V_{K^*K^*} \sigma^2 K^{*2} \tag{24}$$

for $\frac{1}{\lambda} \le \frac{K}{K^*} \le \lambda$ subject to value matching conditions and smooth pasting conditions.

3.0 The Solution: Price and LOP deviation processes

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[10]In a set of different economic applications, Brennan and Schwartz (1985) and Grossman and Laroque (1990) use such smooth-pasting conditions for optimal impulse control.

The homogeneity property implies that the problem can be recast on the basis of a new variable representing the build-up of the difference between stocks of the good in the two countries given by $\omega = \ln K - \ln K^*$ and a new function

$$I(\omega) = -\gamma \ln K^* + \ln\left[V(K, K^*)\right]. \tag{25}$$

The transformed ordinary differential equation in I(ω) can be written as:

$$\begin{aligned} &\sigma^2 I''(\omega) - \gamma\sigma^2 I'(\omega) + \sigma^2 [I'(\omega)]^2 \\ &+(\frac{1}{\gamma} - 1)e^{\frac{I(\omega)}{\gamma-1}}[(\gamma - I'(\omega))^{\frac{\gamma}{\gamma-1}} + e^{-\frac{\gamma}{\gamma-1}\omega}(I'(\omega))^{\frac{\gamma}{\gamma-1}}] \quad = \quad 0, \text{ for } \omega > 0. \\ &-\rho + \gamma\alpha + \frac{1}{2}\gamma(\gamma - 1)\sigma^2 \end{aligned} \tag{26}$$

The $I(\omega)$ function reduces the two variable function, $V(K, K^*)$, to one with a single variable. Everything one may want to know about the equilibrium behavior of the economy, can be derived from the knowledge of the function $I(\omega)$.[11] In particular, one can study

1. the dynamics of the physical stocks of capital K and K^* and
2. the dynamics of prices and LOP deviations.

These are dealt with one at a time.

3.1 The dynamics of the physical stocks of capital, K and K^*

From the previous analysis, the need to concentrate on two determining characteristics for the dynamics of the physical stock of capital, the size of the cone of no shipping and the behavior of quantities inside the cone, is evident. Even though no shipping takes place inside the cone, endogenous consumption rates do tend to rebalance the stocks of goods. The size of the cone of no shipping can be obtained for

[11]Note that this equilibrium behavior is within the region of tolerance, **T**(λ).

varying degrees of risk aversion (see Figure 2) and varying levels of uncertainty. This is achieved by the use of a numerical technique: starting with a trial value for λ and using the boundary conditions at the extreme point $\omega = 1n\ \lambda$, get the values of $I'(\omega)$ and $I''(\omega)$. With these as the initial conditions, the partial differential equation (1.24) can be continued[12] until the central point. To do so, the numerical approximation procedure used is the Runge Kutta method of order four.[13] If the chosen trial value for λ satisfies the condition $I'(0) = 0$ at $\omega = 0$ (by symmetry), it is appropriate and the solution reached. Otherwise a new trial value is chosen and the process is repeated.

The behavior of ω (the allocation of the goods between the two countries) inside the cone is the result of production shocks and differential consumption rates, and can be obtained easily from the knowledge of the value function (see Appendix A). It is obvious from equation (A.1) in Appendix A that the behavior of ω is captured entirely by its drift since the diffusion coefficient is constant. The drift gives the conditional expected change in ω. The drift as a function of ω is plotted in Figure 3. Two noteworthy features of the drift are its nonlinearity and tendency for polar attraction. This tendency pulls the ω process towards the edges of the cone. In the figure, this "centrifugal tendency" indicates that the process flies off the center of the cone.[14] The process is non-linear and autoregressive of order 1.

3.2 The dynamics of prices and LOP deviations

The main purpose of determining the value function *V(K, K*)* was to infer the prices which would prevail in a decentralized market economy by looking at the first derivatives of the value function *V(K, K*)*. Now, one can infer prices which would

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[12]This is done by transforming the partial differential equation (1.24) into an ordinary differential equation in $I(\omega)$.

[13]See Abramowitz and Stegun (1972).

[14]The tendency for polar attraction or 'centrifugal tendency' in the ω process is because consumption rates rebalance the stocks of capital, *but less than proportionally*.

prevail in a decentralized market economy. Defining the price p as the price of a unit of K relative to a unit of K^*,

$$p = \frac{V_K(K, K^*)}{V_{K^*}(K, K^*)} \tag{27}$$

since $V(K, K^*)$ is homogenous of degree γ, the price p is a function of ω only. This function is

$$p(\omega) = \frac{I'(\omega)e^{-\omega}}{-I'(\omega)+\gamma}. \tag{28}$$

The law of one price (LOP) prevails when $p = 1$. Since interchanging the two goods changes the price p into $1/p$, which is a non-linear transformation, the definition of p is asymmetric in nature. The symmetry of the problem will be preserved if instead of the process p, one studies the behavior of the relative deviation from the LOP. This is defined as the natural logarithm of p: 1n p. Since the $I(\omega)$ function is known, the knowledge of the process for ω helps in obtaining the process for 1n p. This function is displayed in Figure 4. In equilibrium when there is a perfect balance in the allocation of goods, 1n $p = 0$ and the LOP prevails.

4.0 Interpretation of Results

The main results of the paper describing the price process are presented for the following values of parameters: γ = -1, σ = 0.5, ρ = 0.15, α = 0.11 and the fixed cost a = 0.45%. This price process is also the law of one price (LOP) deviations or real exchange rate process which is known to exhibit mean reversion and heteroskedasticity. It is characterized by its drift and diffusion. The dynamics of the physical stocks of capital, K and K^*, given by ω underlies the evolution of the LOP deviations. Since the function

1n $p(\omega)$ which links 1np to ω is a modified odd function (see Figure 4) it is not strictly monotone. Two properties of such a modified odd function are relevant here:[15]

(i) $\ln p(-\omega) = -\ln p(\omega)$ and

(ii) $\ln p(\omega_1) = \ln p(\omega_2)$; $\ln p(\omega_3) = \ln p(\omega_4)$

1n $p(\omega)$ is a continuous, differentiable function which is periodic like the trigonometric sine function. The implication of (ii) is self-evident: 1n $p(\omega)$ values are generated by either of two values for ω except at the twin peaks, where a unique ω generates the value of 1n $p(\omega)$. Positive (negative) deviations from the Law of One Price are endogenously generated when the capital stock in the domestic country is less (more) than the capital stock in the foreign country: K < K* (K > K*).

The 1n $p(\omega)$ function is an outcome of two given factors: (a) the underlying "change in ω process" is a Brownian motion with drift that exhibits a tendency of polar attraction, and (b) the boundary conditions are imposed at the edges of the cone in keeping with the fixed cost of shipment. The economic interpretation is simple: the ω process provides the dynamics of the physical stocks of capital and resulting consumption changes. The "change in ω" process has a drift because there is net transfer of capital between the two countries. There are consumption claims on capital in both countries. When capital is shipped abroad for investment quasi-fixed costs are incurred, as mentioned earlier. The 1n $p(\omega)$ function is generated only when there is movement of capital. It is an *odd* function of ω because of the fixed cost of capital transfer. As ω moves away from the center of the cone, the LOP deviation increases. However, as ω approaches the edges of the cone (see Figure 1), the LOP deviation or real exchange rate goes through a maximum and starts decreasing because capital transfer is imminent, with the incurrence of just a fixed cost. At the margin (which is

[15]See Kreyszig (1971) for definitions and properties of odd and even functions.

signified by the edge of the cone), the price is at parity as shipment occurs and capital is transferred.

The real exchange rate process generated belongs to a class of dynamic models in which both the conditional mean and the conditional variance are endogenous stepwise functions. In this sense, the process is the continuous-time analog of qualitative threshold ARCH models. The conditional mean and the conditional variance are piecewise linear functions, as a first approximation. However, in this model, there is nonlinearity in both the conditional mean and the conditional variance. Since the 'change in 1n $p(w)$' process is described by such a conditional mean and conditional variance it can be classified as a nonlinear threshold AR(1) model with conditional heteroskedasticity.

The process 1n $p(w)$ is not a martingale: E [d ln p(ω)] is not equal to zero. It switches between regimes 1 and 2. The nature of the process is dependent on the probabilities attached to the occurrence of the two regimes. These probabilities are determined by the parameters of the underlying physical capital transfer process, namely, (1-γ) the risk aversion, σ, the standard deviation and a the fixed cost of capital transfer. These parameters determine the nature of the 1n $p(w)$ process. For example, at very high levels of the fixed cost, there is no net transfer of capital. At very low values of fixed cost, the cone of no shipping or "region of tolerance" collapses to a straight line where, in the limit, consumption changes are instantaneous and the capital stocks are in equilibrium at each instant (as in Merton 1971).

A secondary set of results concerns the variation in the size of the cone of no shipping with variation in the level of risk aversion and the magnitude of the fixed costs. The dependence of this variation is studied as a function of each one of these parametric values (see Figure 2). Given a certain level of fixed costs, the size of the cone of no shipping reduces with increasing levels of risk-aversion. This indicates a higher preference of the consumer-investors for early and fixed-cost shipment in order to diversify away risk. On the other hand, given a certain level of risk aversion, the size of the cone of no shipping increases with increasing levels of fixed transaction costs. Since the fixed cost of shipping is higher than before, the consumer-investor would now prefer rebalancing through consumption changes rather than shipping the good to the other country.

The processes for movement of physical capital and LOP deviations behave in this fashion due to the underlying consumption and shipping patterns of the consumer-investors in the two countries. This economic rationale is the main thrust of these numerical results. For example, the centrifugal tendency of the ω process follows from a basic principle: if there were no net capital investment between the two countries, given isoelastic utility functions, consumption in each country would be strictly proportional to the local stock of capital. When investment of capital goods as observed in this model is allowed, both domestic and foreign consumers have a claim on any increased stock of capital. Foreigners consume more out of their own capital immediately but collect their claim on foreign capital in the next shipment. In the meantime, the local accumulation of goods, as well as the increased foreign consumption, marginally contribute to making the next shipment more imminent. The ω process is stationary because the regular boundaries keep the centrifugal tendency in check. The physical process ω underlies the process for the LOP deviation. At any point in the open region within the cone, the conditional probability of an outward movement is greater than that of an inward movement, like the physical process ω.

5.0 Conclusions and Future Research

The motivation for this paper was provided by the fact that entry costs for firms developing new markets abroad are, in part, fixed costs. Unlike earlier models by Dixit (1989a, 1989b), which were driven by an exchange rate process, the model here is more fundamentally driven by random output shocks in two countries and we obtain the real exchange rate process endogenously. However, as discussed in the previous section, there are some significant differences in the behavior of the price and LOP deviation process. These differences are a consequence of the boundary behavior which depends on whether the cost of capital investment is fixed or variable.

When such capital transfer costs are a concave function (e.g., when there is a fixed and a variable cost component), two barriers are needed--a barrier triggering action and a barrier one moves to--similar to an [s, S] policy. At the outer boundary or barrier, the regulator would be of finite size (as in this paper) and at the inner boundary or barrier the regulator would be an infinitesimal one like the proportional or variable costs case. Starting out with fixed transaction costs is the first step in implementing such a complete model for entry costs. It paves the way for future research on entry costs which are made up of both fixed and variable cost components. In order to build a reputation, firms may spend in the hope that any doubts about their long-term objectives for market development are laid to rest.

5.1 Empirical implications

Models of entry costs, such as this one or Dumas (1992), with fixed or proportional cost structures respectively, find that when capital transfer is costly, relative deviations from LOP will exist and, more importantly, that such deviations are characterized by both mean reversion and conditional heteroskedasticity. Are purchasing power parity (PPP) deviations the result of fixed and variable entry costs or the consequence of the presence of either one of them? This issue needs to be addressed in future research. Simulated data from such models of deviations from PPP and mean-reversion, when put to the test with real exchange rate data, will give us the answer to this question.

Secondly, the twin issues of non-linearity and heteroskedasticity need some empirical investigation. Tests for non-linearity proposed by Keenan (1983), Tsay

(1986) and Hsieh (1989) are examples. Hsieh (1989) has found that a GARCH (1,1) model fits the daily nominal exchange rate data fairly well. Establishing a relationship between such discrete time ARCH/GARCH models and the continuous-time model proposed in this paper is another important avenue for future theoretical and empirical research.

Appendix A

'Allocation of goods' process

The process for allocation of goods is the behavior of ω within the cone. Based on Equations (1.2) and (1.3),

$$d\omega = \left[\frac{c}{K} + \frac{c^*}{K^*}\right]dt + \sigma\sqrt{2}\,d\bar{z} \tag{A.1}$$

where $d\bar{z} = (dz - dz^*)/\sqrt{2}$ is a standardized white noise. Based on Equation (25)

$$V_K = V\frac{I'}{K};$$
$$V_{K^*} = V\frac{-I' + \gamma}{K^*} \qquad \text{(A.2), (A.3)}$$

Therefore,

$$\frac{c}{K} = \left[\frac{VI'}{K^\gamma}\right]^{\frac{1}{1-\gamma}} \qquad \text{and} \tag{A.4}$$

$$\frac{c^*}{K^*} = \left[V\frac{-I' + \gamma}{(K^*)^\gamma}\right]^{\frac{1}{1-\gamma}} \tag{A.5}$$

Substitute Equations (A.4) and (A.5) into (A.1). The equation containing the $I(\omega)$ function fully determines the stochastic differential equation for ω inside the cone. Note that this has the effect of rendering the diffusion coefficient in Equation (A.1) constant.

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