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Expected Stock Returns and Volatility in a Production Economy: A Theory and Some Evidence

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Abstract: The sign of the relationship between expected stock market returns and volatility appears to vary over time, a result that seems at odds with basic notions of risk and return. In this paper we construct an economy where production involves the use of both labor and capital as inputs. We show that when capital investment is "sticky," the sign of the relation between stock market risk and return varies in accordance with the supply of labor but requires no time variation in preferences. In particular, we show that for asset market equilibria where firms face an elastic supply of labor, the traditional positive risk-return relation obtains. Conversely, a negative relation obtains for asset market equilibria where there is positive probability that labor supply will be highly inelastic. A nice feature of our model is that, unlike earlier work, the sign of the stock market risk-return relation can be associated with observable features of the business cycle. Post–World War II macroeconomic and stock return data are used to test the predictions from the model. Using standard measures of stock market volatility, our results provide support for a stock market risk-return relation that is negative at the peaks of business cycles and positive at the troughs.

JEL classification: E23, E34, G12

Key words: stock markets, risk, return, production and business cycles

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Please address questions regarding content to Padamja Singal, Department of Finance, College of Business Administration, Ohio University, Athens, Ohio 45701, 740/597-2901, 740/593-9539 (fax), singal@ohiou.edu; and Stephen D. Smith, Department of Finance, College of Business Administration, Georgia State University, Atlanta, Georgia 30303, and Research Department, Federal Reserve Bank of Atlanta, 404/651-1236, 404/651-2630 (fax), fncsds@langate.gsu.edu.

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1. Introduction

Standard models for pricing aggregate risk imply, for fixed preferences and a stationary supply of output, that changes in aggregate risk should always be positively associated with changes in expected return. However, the empirical evidence, starting with Black (1976), suggests otherwise. Since then different authors have found conflicting signs for the time series relationship between expected stock returns and return volatility. For example, French, Schwert and Stambaugh (FSS, 1987) find evidence of a positive relation between monthly expected returns on a broad index of stocks and the level of predictable volatility. Conversely, Glosten, Jagannathan and Runkle (GJR, 1993) find support for a negative relation between the first two conditional moments. More recently, Whitelaw (1994) provides evidence that the conditional volatility of stock market returns leads conditional returns. Specifically, he finds that returns are positively correlated with past values of conditional volatility and negatively correlated with future values of conditional volatility. Moreover, his measures of expected return vary countercyclically in the context of the business cycle, a result consistent with the earlier work of Fama and French (1989) and others.

In this regard, it is reasonable to expect business cycle variables such as investment and employment to have an impact on the stock market since stocks are just claims to cash flows from real assets.[1] Indeed, Barro (1990) finds that changes in stock prices have substantial explanatory power for one period ahead gross private domestic investment. These papers, when combined,

[1] The procyclical nature of investment and employment has been extensively documented in the business cycle literature. For instance, Boldrin and Horvath (1995) report correlations of 0.81 between investment and output, and a correlation of 0.88 between labor hours and output.

suggest an interesting approach that one might use to explain how the risk-return relation in securities markets changes over the business cycle.

First, we believe that Whitelaw's (1994) results may be evidence that the relation between the first two conditional moments of stock returns changes signs over specific phases of the business cycle. Second, the results in Barro (1990) are suggestive of the idea that expected stock returns are determined, at least in part, by the factors of real production, including investment, employment, and the underlying risk in the economy. Finally, it is known that there is at least a weak link between conditional volatility in financial markets and measures of macroeconomic risk (Schwert (1989)).[2] The challenge is to explain if and why the sign of the relationship between expected returns and macroeconomic risk switches, depending on the level of investment and employment. The purpose of this paper, therefore, is to construct a theoretical model that explains the time varying relation between expected stock market returns and volatility, link this variation to changes in investment and employment, and provide some empirical validation for the model using standard data and econometric techniques.

Previous theoretical work (Barsky 1989, Abel 1988, and Naik 1994) establishes conditions under which expected stock market returns will be negatively or positively related to risk, depending on investor preferences regarding intertemporal substitution.[3] However, these models do not, for *fixed* preferences, generate *changes* in the sign of the relation between risk and

[2] For purposes of discussion, we distinguish between macroeconomic risk and stock market risk. Macroeconomic risk is referred to as "risk", while stock market risk is referred to as "volatility in the stock market" or "volatility" when there is no ambiguity.

[3] With Expected Utility preferences, the coefficient of relative risk aversion governs both risk aversion and the aversion to intertemporal substitution. In this case, a sufficient condition for a negative relation between risk and stock returns is that the coefficient of relative risk aversion be greater than one (Barsky 1989, Abel 1988). For non EU preferences, a sufficient condition is that the elasticity of intertemporal substitution be less than one (Barsky 1989, Naik 1994.)

expected returns over time.[4] The reason that the risk-return relation can not switch signs in these models is the assumed independence between the mean and variance of the dividend process.

Backus and Gregory (1993) drop this independence assumption and are therefore able to generate the changes in sign. In their framework, the correlation between the mean and volatility of the dividend process is exogenous. Indeed, one of the lessons from their work is that by assuming a richer, albeit exogenous, dividend process one is able to generate a positive, negative or no relationship between the moments of stock returns. Thus, in order to derive testable restrictions on the data, we model an economy where the moments of dividends are endogenous.[5] In particular, we show that the correlation between the mean and the volatility of dividends will depend on the level of investment and employment, as well as the underlying risk in the economy.

Our results linking the moments of the endogenous dividend process to risk draw on the insights developed in the literature regarding investment and uncertainty by Caballero (1991), Pindyck (1993) and others. Specifically, Caballero provides conditions under which an increase in risk may lead to a decrease in investment. We are aware of Pindyck's (1993) critique of Caballero's results that the conclusions are limited to isolated firms, and are not valid for industry wide increases in demand uncertainty.

We overcome this criticism by analyzing a model with production uncertainty whereby the finiteness of factor inputs, labor and capital, creates both constant and decreasing returns to scale. A high level of endogenously determined investment implies decreasing returns, whereas a low level of investment implies constant returns to scale, since the demand for labor is increasing in the level

[4] Intuitively, in these models, an increase in risk affects expected returns in the following manner: Risk averse investors suffer a decline in utility expected from future consumption. If they are (not) very averse to intertemporal substitution, they allocate greater (smaller) resources to future consumption in response. Since the supply of future consumption is imperfectly elastic, an increase (decrease) in demand leads to a decrease (increase) in expected returns.

of real investment. As in Caballero, we obtain a positive as well as a negative investment-uncertainty relation in our framework. Moreover, by introducing a simple financial market, we are able to trace out the implications of this last result in terms of the first two conditional moments of stock returns. In particular, we are able to show that in economies characterized by low levels of investment (representing "poor" economic conditions), the relationship between the moments is positive and vice versa in economies where the level of investment is high.

We are able to derive the above results without assuming that any of the agents in the economy are risk averse, although a subset of agents (investors) are assumed to be averse to intertemporal substitution. When combined with earlier work by, for example, Barsky (1989), who shows that risk aversion alone is not sufficient to generate a determinate sign on the risk-return relationship, our model highlights the importance of aversion to intertemporal substitution for explaining co-movements in the moments of stock returns over time.

The framework used here also allows us to test several predictions relating the co-movements of risk and return to simple proxies for investment, employment and risk. Preliminary results suggest that the data are entirely consistent with the theory presented here. Indeed, we find that time varying relationship between the first two moments of stock returns exhibits the same time series pattern and lead-lag structure documented by Whitelaw (1994).

The outline of the remainder of the paper is as follows. Section 2 contains a description of the model and the development of equilibrium in real and financial markets. In section 3 we provide comparative static results with respect to the underlying parameters in the economy. Section

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[5] We are aware of the argument that any exogenous dividend process can be linked back to an equilibrium in some underlying economy (see, e.g., Duffie (1992)) Our purpose here is to investigate the properties of a dividend process with time varying equilibrium levels of investment and employment.

4 contains the empirical specification, data definitions and a discussion of the results. A summary and suggestions for future research conclude the paper.

2. The Model

We consider a two-date production economy with three types of agents: investors, firms and workers.[6] All agents are assumed to be risk neutral, although investors are assumed to be averse to intertemporal substitution. There are a large but finite number of agents of each type and they behave in a competitive manner. All agents are assumed to have the same information and are homogeneous within their class of agents, so we need only consider the decisions of three representative agents. We discuss each in turn.

Investors:

The representative investor is endowed with *W* units of the single good, which she optimally allocates between current consumption and purchase of shares in the firm at date 0.[7] The ownership of shares entitles the investor to future dividends. We assume that her time additive utility of consumption is given by

$$U(C_0, C_1) = [C_0]^{1-\gamma} + [E(C_1)]^{1-\gamma}, \quad 0 < \gamma < 1 \qquad (1)$$

[6] The degree of heterogeniety in agents is innocuous, of course, if the set of available financial contracts is complete so that each agent's payoffs in equilibrium depend only on aggregate output. For our problem to be of interest, some incompleteness of contracting is required. Considering three types of agents and restricting the ability of agents to write contracts on labor's alternative production technology represents a sufficient level of incompleteness for our purposes. Otherwise, if the contracting space is complete, it is always possible to construct a "representative" agent and prices and production decisions will be set as if there were but a single agent in the economy. Notice that we could, and do in the empirical section, combine the firm and the investor without any qualitative change in our results.

[7] In this model, all financial assets constitute claims to cash flows from real assets. No financial assets are independent of real assets. The investor can store output risklessly, but this will be dominated if gross expected returns from investment are greater than one.

, where C_0 = consumption at date 0 and C_1 = consumption at date 1.[8] The parameter γ takes values on the open interval (0,1). The elasticity of intertemporal substitution is measured by $1/\gamma$. However, unlike earlier work in this area (for example, Barsky (1989) and Naik (1994)), preferences do not determine the sign of the risk-return relation in this paper since the investor is assumed to be risk neutral. Thus, we will show that, in the context of our model, risk aversion is not necessary in order to generate a time varying relationship between the conditional expectation and volatility of stock returns.

We define the investor's demand for expected future dividends as $\overline{D}^D$. Defining the gross required (and in equilibrium, expected) returns on the stock as $\overline{R}$, a price taking investor will offer a price $P = \frac{\overline{D}^D}{\overline{R}}$ for the shares. Normalizing the number of shares to one and recognizing her budget constraint, the relevant optimization problem may be equivalently written as

$$\underset{\overline{D}^D}{\text{Maximize}}\ U(\overline{D}^D) = [W - \frac{\overline{D}^D}{\overline{R}}]^{1-\gamma} + [\overline{D}^D]^{1-\gamma} \qquad (2)$$

Firms:

The representative firm possesses a Cobb-Douglas constant returns to scale technology which uses both labor, L, and physical capital, K, to produce date 1 output $Y = K^{\alpha} L^{1-\alpha}\varepsilon$, where $\varepsilon \geq 0$ is a random shock, with $E[\varepsilon] = 1$, and α lies on the interval (0,1). Investment is made at date 0 and is irreversible. The firm finances this investment by selling all shares in the firm to the investor at a price P. The objective of the firm is to choose a value of K such that it maximizes the net present value of investment, i.e., maximizes (P - K).

[8] In line with earlier work, we assume that investors display aversion to intertemporal substitution but separate this from risk aversion. In particular, we show that risk aversion is not a necessary condition to get a time varying relationship between expected returns and risk. Moreover, adding risk aversion, but at the same time keeping our contracting frictions (see footnote 3) will not change the results.

Denoting a particular realization of dividends supplied by D^S, the firm receives $P = \frac{\overline{D}^S}{\overline{R}}$ for its shares, where $\overline{D}^S$ is the expected future dividend supplied. Since the firm takes $\overline{R}$ as given, its optimization problem can be written as

$$\underset{K}{\text{Maximize}} \ \frac{\overline{D}^S}{\overline{R}} - K \tag{3}$$

At date 1, the firm experiences a shock of ε and since capital investment is irreversible, only labor demand can be adjusted by the firm ex post. At this point the labor market also clears, production takes place and the resulting output is distributed in the form of wages and dividends.[9]

Workers:

The representative worker is endowed with N units of labor along with an alternative production technology which requires only labor input and yields $w > 0$, per unit of labor input. We also assume that the worker has no disutility of effort. Hence, the supply curve of labor is perfectly elastic (at a wage of w) in states where the demand for labor is less than N units and is perfectly inelastic in states where labor demand exceeds N units.[10]

Our assumption regarding the irreversibility of investment, when combined with a finite supply of labor, has real economic consequences in equilibrium. In particular, a higher level of investment ex ante is more likely to result in high demand for labor ex post and in higher wages. It follows from these assumptions that, unlike results in the previous literature, equilibrium expected returns on physical capital are decreasing or constant in the level of investment.

Equilibrium:

[9] Since the firm does not carry any debt, the only claimants to future cash flows are the worker and the investor.

[10] If $w = 0$, then the firm's demand for labor will be unbounded in all states of the world, since labor's marginal product is always positive with a Cobb-Douglas production technology.

It is clear from the foregoing discussion that both the firm and the investor's decisions are, in equilibrium, determined by the required rate of return. In particular, we define an equilibrium as a pair $\left(K^*(\overline{R}^*), \overline{R}^*\right)$ such that the investor's optimal demand for expected future dividends $\overline{D}^{D*}$ and the firm's optimal supply of expected future dividends $\overline{D}^{S*}$ are equal, where the "*" sign denotes equilibrium values of the variables. We first discuss the supply correspondence and then turn our attention to the demand function and the properties of our economy in equilibrium.

Supply of Expected Dividends:

Since investment is irreversible and is made before ε is realized, the firm chooses K^* by recursively taking into consideration the optimal date 1 labor input decision. As always, the date 1 hiring decision is optimal only if the marginal product of labor equals the wage rate. In our set-up the marginal product of labor is given by $MP_L = (1-\alpha) K^{\alpha} L^{-\alpha}\varepsilon$. Since N is finite and there is no disutility of effort, there are two possibilities:

i. $MP_L = (1-\alpha) K^{\alpha} L^{-\alpha}\varepsilon = w$, for $L < N$, or

ii. $MP_L = (1-\alpha) K^{\alpha} L^{-\alpha}\varepsilon \geq w$, for $L = N$.

Clearly, there exists an ε, call it "c", that solves $(1-\alpha) K^{\alpha}N^{-\alpha}\varepsilon = w$, i.e.,

$$c \equiv \frac{wN^{\alpha}}{(1-\alpha)K^{\alpha}} \tag{4}$$

For $\varepsilon < c$, the firm hires $L^* = K\left(\frac{(1-\alpha)\varepsilon}{w}\right)^{\frac{1}{\alpha}}$ units of labor at a wage of w. Since $L^* < N$, we label such labor market conditions as "slack." For $\varepsilon > c$, wages adjust upward to equal the worker's marginal product, and $L^* = N$. We label these labor market conditions "tight." For fixed N, w, and α, it is clear that the probability of realizing a slack or tight labor market is determined by K, e.g., high levels of K are associated with a lower probability of slack labor markets. More generally, the probability that labor market conditions will be either slack or tight is determined endogenously through the choice of *K*.

We assume, for specificity, that the random shock ε is distributed uniformly on the interval [1-θ, 1+θ] where $\theta \in (0,1)$. The density function is given by f (ε) = 1/(2θ).[11] Notice that $c \geq 1+\theta$, $c \leq 1-\theta$, or $c \in (1-\theta, 1+\theta)$. Incorporating the optimal hiring decision for the firm, it follows that $\overline{D}^S$ can be written as

$$\overline{D}^S = \begin{cases} \int_{1-\theta}^{1+\theta} \alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \frac{K\varepsilon^{\frac{1}{\alpha}}}{2\theta} d\varepsilon & c \geq 1+\theta \\ \int_{1-\theta}^{c} \alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \frac{K\varepsilon^{\frac{1}{\alpha}}}{2\theta} d\varepsilon + \int_{c}^{1+\theta} \frac{\alpha K^{\alpha} N^{1-\alpha}\varepsilon}{2\theta} d\varepsilon & 1-\theta < c < 1+\theta \\ \int_{1-\theta}^{1+\theta} \frac{\alpha K^{\alpha} N^{1-\alpha}\varepsilon}{2\theta} d\varepsilon & c \leq 1-\theta \end{cases} \tag{5}$$

Equation (5) formalizes the notion that expected future dividends are a function of initial investment, labor endowment and the probability of being in tight labor markets. It is also easy to see from (5) that $\overline{D}^S$ is strictly increasing and continuous in *K*.

Solving for the optimal level of investment in the standard recursive fashion, and denoting partial differentiation by subscripts, we have, from (3),

$$\overline{D}_K^S = \overline{R} \tag{6}$$

as a necessary condition for equilibrium, where,

[11] While we have worked out some of the results using arbitrary probability measures, the uniform is both tractable and provides a finite bound for output.

$$\overline{D}_K^S = \begin{cases} \int_{1-\theta}^{1+\theta} \alpha \left(\frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} \frac{\varepsilon^{\frac{1}{\alpha}}}{2\theta} d\varepsilon & c \geq 1+\theta \\ \int_{1-\theta}^{c} \alpha \left(\frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} \frac{\varepsilon^{\frac{1}{\alpha}}}{2\theta} d\varepsilon + \int_{c}^{1+\theta} \frac{\alpha^2 K^{\alpha-1} N^{1-\alpha} \varepsilon}{2\theta} d\varepsilon & 1-\theta < c < 1+\theta \\ \int_{1-\theta}^{1+\theta} \frac{\alpha^2 K^{\alpha-1} N^{1-\alpha} \varepsilon}{2\theta} d\varepsilon & c \leq 1-\theta \end{cases} \qquad (6`)$$

It is easy to see, from (6`), that over the range $c \geq 1+\theta$, $\overline{D}_K^S$ is independent of K, while for $c < 1+\theta$, $\overline{D}_K^S$ is declining in K.[12] The optimal level of investment is shown in Figure 1. It is clear that the lower the required return on financial assets, the higher the level of desired investment. Note that the firm optimally invests zero when expected returns on financial assets are greater than marginal expected returns on real assets.[13] The case where $c \geq 1+\theta$ (slack labor markets in all states) corresponds to a situation where the firm faces constant returns to scale with respect to K and is therefore indifferent between various possible levels of investment. Alternatively, for $c < 1+\theta$, there is positive probability of tight labor markets ex post and the firm faces decreasing returns to scale with respect to K. In this case $K^*(\overline{R})$ is a strictly decreasing function of $\overline{R}$.

From (5), for fixed values of the exogenous parameters, $\overline{D}^{S*}(\overline{R})$ is strictly increasing in, and uniquely determined by $K^*(\overline{R})$. Therefore, we are able to prove the following lemma, which will be useful in characterizing the existence and uniqueness of equilibrium.

Lemma 1: $K^*(\overline{R})$ and $\overline{D}^{S*}(\overline{R})$ are both convex valued upper hemicontinuous correspondences in $\overline{R}$.

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[12] We note that $\overline{D}_K^S$ is continuous, but not everywhere differentiable in initial investment. In particular, $\overline{D}_{KK}^S$ is not defined for $c \in \{1-\theta, 1+\theta.\}$. This fact is taken into account when we later do the comparative static exercises.

[13] Despite the fact that the Cobb-Douglas production function meets the Inada conditions, marginal expected returns on real assets are bounded above, owing to the expost adjustability of labor.

Proof: See Appendix A.

Demand for Expected Dividends:

We now consider how the investor determines her demand for expected future dividends. The demand schedule can be specified by using (2) to derive the first order necessary condition for equilibrium. These operations yield

$$\overline{D}^{D*}(\overline{R}) = \frac{W}{\overline{R}^{-1} + \overline{R}^{\frac{-1}{\gamma}}} \tag{7}$$

Given equation (7), it is straightforward to prove the following lemma.

Lemma 2: $\overline{D}^{D*}$ is a continuous and strictly increasing function of $\overline{R}$, for given *W*.

Proof: Differentiate both sides of equation (7) with respect to $\overline{R}$. This yields

$$\overline{D}^{D*}_{\overline{R}} = \frac{-W}{\left(\overline{R}^{-1} + \overline{R}^{\frac{-1}{\gamma}}\right)^2}\left(-\overline{R}^{-2} - \frac{1}{\gamma}\overline{R}^{\frac{-1}{\gamma}-1}\right) = \frac{W}{\left(\overline{R}^{-1} + \overline{R}^{\frac{-1}{\gamma}}\right)^2}\left(\overline{R}^{-2} + \frac{1}{\gamma}\overline{R}^{\frac{-1}{\gamma}-1}\right)$$

which is clearly positive since γ, *W*, and $\overline{R}$ are strictly positive.♦

We note that Lemma 2 follows directly from the assumption that the investor is averse to intertemporal substitution. We are now prepared to prove that an equilibrium exists and that it is unique. These results are summarized in the following theorem and corollary.

Theorem 1: There exists a pair $\left(K^*\left(\overline{R}^*\right), \overline{R}^*\right)$ for which $\overline{D}^{S*}\left(\overline{R}^*\right) = \overline{D}^{D*}\left(\overline{R}^*\right)$.

Proof: See Appendix A.

Corollary 1: The equilibrium in Theorem 1 is unique.

Proof: See Appendix A.

Intuitively, existence follows from the fact that $\overline{R}$ determines both $\overline{D}^{S*}\left(\overline{R}\right)$ and $\overline{D}^{D*}\left(\overline{R}\right)$. For very low values of $\overline{R}$, $\overline{D}^{D*}\left(\overline{R}\right)$ is close to zero. Conversely, for very high values of $\overline{R}$, $\overline{D}^{S*}\left(\overline{R}\right)$ is zero. From the continuity and convex valuedness of $\overline{D}^{S*}(\overline{R})$ and $\overline{D}^{D*}(\overline{R})$, it follows that there will exist an intermediate level of $\overline{R} = \overline{R}^*$, for which $\overline{D}^{S*}(\overline{R}^*)$ and $\overline{D}^{D*}(\overline{R}^*)$ are equal. Uniqueness follows from the fact that $\overline{D}^{D*}(\overline{R})$ is strictly increasing in $\overline{R}$. Figure 2 provides a graphical representation of this discussion.

Note that the equilibrium pair $(K^*(\overline{R}^*), \overline{R}^*)$ can be identified by $K^*(\overline{R}^*)$ alone since the inverse mapping $\overline{R}^*(K^*)$ is a single valued non-increasing continuous function (this can also be seen from Figure 1.) The identification of an equilibrium by K^* alone is useful since it implies, for given *w*, *N*, and α, a unique probability of being in slack labor markets. Moreover, different values of K^* imply different probabilities of being in slack or tight labor markets. We now show that, for different levels of *W*, different levels of K^* are feasible equilibria.

Theorem 2: Different levels of *W* result in different levels of K^*.

Proof of Theorem 2: Pick a *W*. It is clear from (7) that the mapping between $\overline{D}^{D*}(\overline{R})$ and *W* is, for a given $\overline{R}$, one to one. Solve equation (7) to find the associated $\overline{D}^{D*}(\overline{R})$ Also, from the existence and uniqueness of equilibrium, the mapping between $\overline{D}^{D*}(\overline{R})$ and K^* is also one to one. Therefore, there exists a unique K^* associated with a given value of *W*.♦

Theorem 2 shows that any level of investment is a feasible equilibrium, given an appropriate initial endowment, *W*. This result makes intuitive sense because the investor can convert current resources into future consumption only through investment in the firm. Therefore, by varying her endowment, it is possible to obtain any level of equilibrium investment and its associated probability of slack labor markets. This result is important since we argue that the different values of

K^* and c are representative of different phases of the business cycle. Empirical business cycle research suggests that this is indeed the case.[14,15]

3. Comparative Static Results

In this section we examine how a small change in any of the exogenous parameters (N, w, W, θ) may affect our equilibrium level of $\overline{R}^*$.[16] In addition, we also present results about the impact of a change in risk, θ, on stock market volatility, σ^2. We will use these comparative static results later to help specify our testable hypotheses. The two different sets of equilibria that are of interest to us are $c > 1+\theta$, where the equilibrium level of investment is said to be *low*, and $1-\theta < c < 1+\theta$, where the equilibrium level of investment is said to be *high.* The comparative static results pertain to these two cases.

The primary result of interest is the impact of changes in θ on $\overline{R}^*$ and σ^2. Given our distributional assumptions, risk is uniquely defined by θ. We first establish some lemmas that relate $\overline{D}^S$ to θ. Specifically, we show that for *low* levels of investment, $\overline{R}^*$ and σ^2 are increasing in θ. We also show that there exist *high* levels of investment such that $\overline{R}^*$ is decreasing and σ^2 is increasing in θ. Thus, we establish that there exist asset market equilibria where the traditional theory holds, and there exist equilibria where it is "reversed". We first prove the following lemma.

Lemma 3: If $c > 1 + \theta$, $\overline{D}^S_\theta > 0$.

Proof: See Appendix A.

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[14] As discussed in the introduction, investment is very procyclical over the business cycle, as is employment. Clearly, a low *c* would be associated with high investment and employment (as in a boom.) On the other hand, a high *c* would be associated with low investment and employment (as in a recession.)

[15] We are aware of Abel's (1988) argument that, strictly speaking, a two-date model such as this can only be used to compare two different economies and not the same economy over time. However, we still feel that these results are suggestive to the extent that certain values of the observed equilibrium are typically associated with certain phases of the business cycle.

When levels of investment are low, realized dividends, D^S, are convex and increasing in ε for all possible values of ε, on account of the hiring flexibility in labor markets (see Figure 3). Therefore, an increase in risk in the sense of Rothschild and Stiglitz (1971) shifts the weight of the distribution towards the tails. Using Jensen's Inequality and the convexity of dividends in ε, it follows that an increase in risk actually results in higher values of $\overline{D}^S$.[17] Using this result, we can now prove the following theorem.

Theorem 3: If $c > 1 + \theta$, $\frac{d\overline{R}^*}{d\theta} > 0$, and $\frac{d\sigma^2}{d\theta} > 0$.

Proof: See Appendix A.

Intuitively, if equilibrium investment levels are *low*, production is constant returns to scale and the firm earns no economic profits. Therefore, returns to real assets are the same, in equilibrium, as returns on financial assets.[18] Moreover, Lemma 3 shows that an increase in θ results in an increase in $\overline{D}^S$. This increase flows through to holders of financial assets. Also, as one might expect, an increase in θ leads to an increase in σ^2.

We next consider equilibria where the level of investment is *high*, so that the firm faces tight labor markets with positive probability. In this case, even though D^S is piecewise convex in ε (see Figure 4), there exist equilibria such that it is globally concave in the state realization. It follows that a mean preserving spread in risk will not always increase $\overline{D}^S$, i.e., there exist equilibria where $\overline{D}^S$ is decreasing in θ. In this case the result in Lemma 3 is reversed. More formally, we can prove

[16] Comparative statics have been derived for the impact of the exogenous parameters on both equilibrium investment and expected stock returns. However, only the latter are presented here, since those are of primary interest in this paper.

[17] In this case it turns out that investment is also increasing in θ, a result first obtained by Hartman (1972).

Lemma 4: If $(1-\theta)^2 < c^2 < 1-\theta^2$, $\overline{D}_{\theta}^{S} < 0$.

Proof: See Appendix A.

We are now prepared to prove our main result.

Theorem 4: If $(1-\theta)^2 < c^2 < 1-\theta^2$ and $\theta \geq 0.5$, there exist equilibria such that $\frac{d\overline{R}^*}{d\theta} < 0$ and $\frac{d\sigma^2}{d\theta} > 0$.

Proof: See Appendix A.

The intuition behind the proof of Theorem 4 comes from the fact that there exist equilibria such that an increase in θ reduces $\overline{D}^S$. Moreover, Lemma 2 shows that the investor's required rate of return is strictly increasing in $\overline{D}^D$. From the existence of equilibrium, $\overline{D}^*$ will decline, as will $\overline{R}^*$. This generates the first part of Theorem 4. Finally, when $\theta \geq 0.5$, we are able to show that σ^2 is an increasing function of θ, thus establishing the second part of the theorem.[19]

In the context of existing literature, the importance of Theorems 3 and 4 comes from the fact that they provide an equilibrium-based rationale for time variation in the correlation between the mean and volatility of stock returns over different phases of the business cycle.

We now present comparative static results with respect to changes in *N*, *W*, and *w*. First, we consider *low* levels of investment so that $c > 1+\theta$. As one might expect, an increase in *w* reduces $\overline{D}^S$ since an increase in *w* results in a higher wage bill. On the other hand, a small change in *N* does not change $\overline{D}^S$ since the firm already has maximum hiring flexibility. Finally, a change in

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[18] Restoy and Rockinger (1994) formalize a simple model where returns on real and financial assets are equal state by state, if production is constant returns to scale.

[19] $\theta \geq 0.5$ is a sufficient condition to guarantee that stock market volatility and risk will be directly related. While we have not established the "genericity" of the result (i.e. stock market volatility always moves in the same direction as macroeconomic risk,) we do show this result holds for the equilibria under consideration.

initial endowment, *W*, does not impact $\overline{D}^S$ since output is independent of *W*. The foregoing discussion is summarized in Lemma 5.

Lemma 5: If $c > 1+\theta$, $\overline{D}^S_w$ < 0, and $\overline{D}^S_N = \overline{D}^S_W$ = 0.

Proof: See Appendix A.

It is clear from Lemma 5 that an increase in *w* worsens the opportunity set of the firm (i.e., $\overline{D}^S$ declines). We next show that this effect flows through directly to financial assets and results in a decrease in $\overline{R}^*$. Moreover, since *N* influences the equilibrium only through $\overline{D}^S$, it turns out that, not surprisingly, changes in *N* have no effect on $\overline{R}^*$ at these equilibria. Conversely, changes in *W* influence the equilibrium only through $\overline{D}^D$. However, at these equilibria, production is constant returns to scale, so that changes in $\overline{D}^D$ are matched by changes in $\overline{D}^S$ without any change in $\overline{R}^*$. A formal statement of these results is presented in Theorem 5.

Theorem 5: If $c > 1+\theta$, $\frac{d\overline{R}^*}{dw}$ < 0, and $\frac{d\overline{R}^*}{dN} = \frac{d\overline{R}^*}{dW} = 0$.

Proof: See Appendix A.

Finally, we investigate the case where the equilibrium level of investment is *high*, so that 1-θ < *c* < 1+θ. In this case an increase in *w* increases the firm's wage bill in slack labor markets, while an increase in *N* lets the firm increase hiring when labor markets turn out to be tight. It follows that $\overline{D}^S$ is inversely related to *w* and directly related to *N*. For reasons discussed earlier, an increase in *W* leaves $\overline{D}^S$ unchanged. These results are summarized in the next Lemma.

Lemma 6: If 1-θ < *c* < 1+θ, $\overline{D}^S_w \leq 0$, $\overline{D}^S_N$ > 0, and $\overline{D}^S_W$ = 0.

Proof: See Appendix A.

We use this lemma to help prove the following theorem.

Theorem 6: If 1-θ $< c <$ 1+θ, $\frac{d\overline{R}^*}{dN} > 0$ and $\frac{d\overline{R}^*}{dW} < 0$. Also, there exist equilibria such that $\frac{d\overline{R}^*}{dw} > 0$.

Proof: See Appendix A.

Theorem 6 is true because an increase in N increases $\overline{D}^S$. Ceteris paribus, this leads to a higher price for shares, which causes the investor to demand a higher expected return on the shares. An increase in W also affects equilibrium expected returns, since it increases the investor's demand for current and future consumption. Moreover, since the firm faces decreasing returns to scale at *high* levels of investment, it follows that the desired higher supply of future consumption will be forthcoming only if the investor is willing to accept a lower expected return.

It is interesting in this framework that at very *high* levels of investment (i.e. $c \rightarrow 1-\theta$), when labor markets are expected to be tight almost with probability one, an increase in w actually leads to an increase in $\overline{R}^*$. At these equilibria, an increase in w has very little impact on $\overline{D}^S$ since the firm pays higher than w wages in most states, anyway. However, an increase in w increases the probability of being in slack labor markets, which is valuable at the margin and hence increases $\overline{D}^*$ as well as $\overline{R}^*$.

Empirical Specification, Data and Results

Econometric Issues:

In order to specify a general empirical formulation of the model presented in section 2, one would need to specify a system of equations that generates expected stock returns, volatility and the level of investment. Clearly, our focus is the time varying relationship between conditional expected returns and volatility. Thus, we are not particularly interested in looking at determinants of the *level* of investment, but rather incorporating the role of investment in determining the probability of being in slack vs. tight labor markets. From this perspective, the endogeneity of K^* presents the usual econometric problems associated with inference in a single structural equation.

Interestingly, expected returns can be written in an equation as a function of only w, θ, α and c, where c summarizes all of the relevant information contained in K^*, w, and N and represents the probability of being in slack or tight labor markets. Using (6) and (6`), and the definition of c, we get:

$$\overline{R}^* = \begin{cases} \int_{1-\theta}^{1+\theta} \alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \frac{\varepsilon^{\frac{1}{\alpha}}}{2\theta} d\varepsilon & c \geq 1+\theta \\ \alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \left(\int_{1-\theta}^{c} \frac{\varepsilon^{\frac{1}{\alpha}}}{2\theta} d\varepsilon + \int_{c}^{1+\theta} \frac{\alpha c^{\frac{1-\alpha}{\alpha}} \varepsilon}{2\theta} d\varepsilon \right) & 1-\theta < c < 1+\theta \\ \alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \int_{1-\theta}^{1+\theta} \frac{\alpha c^{\frac{1-\alpha}{\alpha}} \varepsilon}{2\theta} d\varepsilon & c \leq 1-\theta \end{cases} \quad (8)$$

The endogeneity problem associated with c is eliminated if we simplify the model by combining the investor and the firm into one agent. This follows from the fact that, in such an economy, equilibrium is determined by K^* alone. However, average returns are still well defined and can be used in the empirical tests of the model. Importantly, such a simplification does not alter earlier predictions regarding the co-movement between the first two moments of stock returns. [20, 21]

A second point involves the fact that our model explains expected returns, not expected excess returns. The extant literature, however, documents a time varying relation between excess returns and volatility. This difference may limit comparability with earlier work. Furthermore, our model does not address issues of information arrival in the stock market. If investors are rational

[20] These proofs are available upon request from the authors.

[21] The only qualitative difference in the results between the two models is that $\frac{d\overline{R}^*}{dw} \leq 0$ everywhere in the simplified version. Other comparative static results are similar across the two versions. The reason we present a more extended

and can anticipate real activity, information about future expected investment and dividends should get incorporated in current returns.[22] Therefore, our empirical tests, which do not adjust for the information arrival process, are actually biased against finding evidence in support of our hypotheses.

An empirical model of volatility must also be specified. There is substantial evidence that conditional volatility is serially correlated. Therefore, we model stock return volatility using a GARCH(1,1) process, although the results appear to be robust across various measures of volatility[23]. It follows from this assumption that we can write realized returns as

$$\tilde{R}_{t+1} = \overline{R}_t + \xi_{t+1} \text{, where} \tag{9}$$

$$\xi_{t+1} = z_{t+1}\sigma_{t+1}, \quad z_t \approx \mathrm{N}(0,1), \quad \sigma^2_{t+1} = b_0 + b_1\xi_t^2 + b_2\sigma_t^2 \tag{10}$$

, where $\tilde{R}_{t+1}$ and $\overline{R}_t$ denote the realized and expected returns from period t to t+1.

Specification of the Model:

While the above simplifications are sufficient to eliminate any simultaneous equation bias and provide a widely used measure of volatility, $\overline{R}_t$ is still a highly nonlinear function and depends on a number of parameters, some of which may be difficult to estimate. The key economic insight from Theorems 3 and 4, however, is that the relationship between expected returns and risk depends on *c*, which represents the phase of the business cycle. In these preliminary tests we provide an empirical specification that lets us test for this nonlinearity in the simplest manner possible. First, we make no attempt to estimate the production parameter, α. We recognize that the omission of α, to the extent that it varies through time, will cause potential biases in the remaining

model in Section 2 is that it facilitates future investigation of other interesting relationships, e. g. the relationship between the level of stock prices and investment over the business cycle.

[22] For example, see Fama (1981.)

parameter estimates. Second, we focus our attention on the simplest non-linear specification that captures the cyclical variation between risk and expected return. In particular, using the symbols $\hat{w}_t$, $\hat{\theta}_t$ and $\hat{c}_t$ to denote empirical proxies for w, θ, and c, we estimate two functional forms for $\overline{R}_t$.[24]

$$\overline{R}_t = a_0 + a_1\hat{\theta}_t + a_2(\hat{\theta}_t * \hat{c}_t) + a_3\hat{w}_t \qquad (11)$$

$$\overline{R}_t = a_0 + a_1\hat{\theta}_t + a_2(\hat{\theta}_t * \log(\hat{c}_t)) + a_3\hat{w}_t \qquad (12)$$

This empirical specification allows us to smooth out the discontinuities associated with our theoretical model without losing the ability to test our main hypotheses.[25] As predicted by the model, a_1 and a_3 should be negative, while a_2 should be positive. As is typical, these hypotheses are specified as alternatives in one-tailed tests against the null hypotheses that a_1, a_2, and $a_3 = 0$.

Note that (11) and (12) differ from each other only in that the former uses $\hat{c}_t$, while the latter uses log ($\hat{c}_t$) in the specification. The latter allows us to incorporate an additional potential source of non-linearity in the dependence of $\overline{R}_t$ on $\hat{\theta}_t$.

Data

The post World War II period is the longest period in U.S. history for which continuous data on real investment and labor force are available. Furthermore, data measurement techniques in this time period are much more reliable than those employed before the war.[26] Since compensation

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[23] We worked with alternative specifications of volatility as can be found in Glosten, Jagannathan and Runkle (1993) , Schwert (1989), or Whitelaw (1994). Our results are robust under these alternate specifications.

[24] Different specifications yielded surprisingly robust results. Results from alternative specifications are available upon request.

[25] According to the theoretical form of (8), a Markov-Switching regime model might appear to be the best way to test our model. However, if one believes that there is some element of aggregation across industries, our approach would be valid for aggregate stock returns. In future work, we intend to check this assumption by comparing the two approaches.

[26] See, for example, Romer (1986) for a discussion of this issue in the context of employment data.

and investment data are reported only on a quarterly basis and are needed to calculate $\hat{c}_t$, we use quarterly macroeconomic data for the years 1948 to 1995.

The empirical proxy we employ for the workers' reservation wage, *w*, at time t, denoted $\hat{w}_t$, is the real compensation per employee between dates t-1 and t. To the extent that wages are sticky downwards, last period's real compensation per employee ought to reflect the average worker's reservation wage for the current time period. Real compensation per employee is the total compensation of employees divided by the number of people employed, deflated by the Consumers Price Index. The time series for $\hat{w}_t$ is plotted in Figure 5.

From (4), *c* is given by $\frac{wN^{\alpha}}{(1-\alpha)(K^{*})^{\alpha}}$. The empirical proxy we use at date t, $\hat{c}_t$, closely adheres to this functional form except for the omission of α. In particular, $\hat{c}_t$ is constructed as the ratio of last period's unemployment ($UNEMP_{t-1,t}$) to last period's real gross private domestic investment ($RGPDI_{t-1,t}$), multiplied by the real compensation per employee ($\hat{w}_t$.)[27] Last period's unemployment reflects the current period's availability of labor and last period's investment may reflect the level of investment that is likely to be made this period.[28] Given that $\hat{c}_t$ is a continuous variable, it may be a good measure of the probability of being in slack or tight labor markets. In particular, a high level of $\hat{c}_t$ should be associated with a high probability of a slack labor market, and vice versa. Figures 6, 7, and 8 plot the time series of real gross private domestic investment, unemployment and $\hat{c}_t$, respectively. Summary statistics for these variables are presented in Table 1.

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[27] We would like to thank the Federal Reserve Bank of St. Louis for making available the relevant macroeconomic data on their website. Definitions of the macroeconomic aggregates are contained in appendix B.

[28] We recognize that there may be better ways to form expectations of investment, using either the information contained in past stock market returns or other macroeconomic variables. However, such extensions should only improve our results.

Two facts stand out from figures 5 to 8. First, as expected, all four series exhibit some degree of a time trend, although the time series of $\hat{c}_t$ displays a far lower trend than the other three series. Indeed, the relatively flat trend and the presence of cyclicality in $\hat{c}_t$ give us reason to believe that it may act as a good proxy for phases of the business cycle.[29]

We follow Schwert (1989) and our proxy for risk at time t, denoted $\hat{\theta}_t$, is constructed as the volatility of growth rate of the industrial production index (GIP), for twelve quarters prior to the current period.[30] Formally,

$$\hat{\theta}_t = \sqrt{\frac{\sum_{i=t}^{t-11} (\mathrm{GIP}_{i\text{-}1,i} - \overline{\mathrm{GIP}})^2}{11}}$$

A continuous series of daily stock returns on an index is obtained by splicing a return series obtained from Schwert (for the period, January 1948 to July 1962) with daily returns on the CRSP value weighted index (July 1962 to December 1995).[31] Nominal quarterly returns are computed by taking daily returns and compounding them over quarterly intervals. The inflation rate is subtracted from these measures to yield estimates of real stock returns. Finally, as noted earlier, our proxy for conditional stock return volatility comes from fitting a GARCH(1,1) process for the

error variance.

Results

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[29] Augmented Dickey Fuller tests that include a constant, reject the null hypothesis of a unit root for both $\hat{c}_t$ and $\hat{w}_t$.

[30] Schwert actually uses the absolute deviation from the mean but the results are qualitatively similar to those reported here.

[31] Schwert (1990) contains a detailed description of the construction of the return series.

In Table 2, we present maximum likelihood estimates of (11) and (12) with t-statistics computed using robust errors as per Bollerslev and Wooldridge (1992). As can be seen from these results, the sign of the relation between stock returns and risk, $\hat{\theta}_t$, depends on the phase of business cycle ($\hat{c}_t$.) For high values of $\hat{c}_t$, we see that the relation is positive, while for low values of $\hat{c}_t$, the relation is negative.

These regression results are clearly consistent with our hypotheses. What is less clear is whether this behavior of expected returns would manifest itself in the sort of patterns documented by Whitelaw (1994). Figures 9 and 10 present the correlations between conditional returns and different leads (lags) of conditional variance, for the specifications of $\overline{R}_t$ given by (11) and (12) respectively. The number on the horizontal axis denotes the number of leads of conditional volatility that are correlated with conditional return. For example, -6 denotes the correlation between $\overline{R}_t$ and σ^2_{t-6}. The patterns in Figures 9 and 10 obviously display the same lead-lag relation between conditional returns and volatility as those found in figures 3 and 5 in Whitelaw. In particular, conditional returns are positively related to past values of conditional volatility and negatively related to future values of conditional volatility.

Finally, we provide some direct evidence that the sign of the relation between conditional returns and volatility depends on the phase of the business cycle. Using conditional returns, $\overline{R}_t$, estimated from (11) and the associated conditional volatility from (10), we run the following least-squares regressions.[32]

[32] The use of conditional volatility estimates as a regressor poses the usual problems associated with generated regressors. However, as shown in Pagan (1984), the asymptotic t-statistics are valid in our approach.

$$\overline{R}_t = a_0 + a_1\sigma_t^2 + \nu_t \tag{13}$$

$$\overline{R}_t = a_0 + a_1\sigma_t^2 + a_2(\sigma_t^2 * \hat{c}_t) + \eta_t \tag{14}$$

The results for equations (13) and (14) are contained in Table 3, Panel A. The evidence provides strong support for the hypothesized relation between conditional returns and volatility that depends on the phase of the business cycle. However, the low value of the Durbin-Watson statistic is a cause for concern. To correct for autocorrelation, we estimate equation (15), which is equation (14) in first differences.

$$\overline{R}_t - \overline{R}_{t-1} = a_1(\sigma_t^2 - \sigma_{t-1}^2) + a_2(\sigma_t^2\hat{c}_t - \sigma_{t-1}^2\hat{c}_{t-1}) + \chi_t \tag{15}$$

The results for (15) are also reported in Table 3, panel A. These results provide further evidence in support of our hypotheses.

In Panel B, we present results for equations (13), (14) and (15) using log($\hat{c}_t$) instead of $\hat{c}_t$. The results are quite similar to those obtained with $\hat{c}_t$. We also tried using the sum of the squared daily returns over the quarter as a proxy for conditional volatility. This measure is identical to the one in Whitelaw (1994). Tests employing this proxy yielded results that were qualitatively similar to those reported in the text (see footnote 23).

5. Summary and Conclusions

In this paper, we provide an equilibrium model where the sign of the co-movement between the first two conditional moments of stock returns depends on the level of investment and employment. Specifically, we prove that when the level of investment is high and unemployment is expected to be low, there exist asset market equilibria such that the relation may be negative. Conversely, when the level of investment is low and unemployment is expected to be high, there exist asset market equilibria such that the relation is positive. Thus, we are able to rationalize, in the context of an equilibrium model, the fact that the contemporaneous correlation between stock market volatility and expected stock returns can, for *fixed* preferences, vary over the business cycle. Moreover, unlike previous authors, we show that risk aversion on the part of investors is not necessary to generate a negative risk-return relation.

We also find strong empirical support for the model's predictions. In fact, the empirical specification of our model generates a lead-lag relationship between conditional stock returns and volatility that is strikingly similar to that documented in the extant literature. Thus, we have provided one explanation for why risk and return do not always move in the same direction over time.

In this paper we have intentionally avoided looking at the determinants of the level of investment and stock prices from an empirical perspective. However, the estimation of a complete system of equations, as developed here, may help us to better understand the work that documents the fact that current stock prices provide information concerning future levels of investment and output. Moreover, the framework used here may also be used to study questions concerning the business cycle related variation in Q ratios.

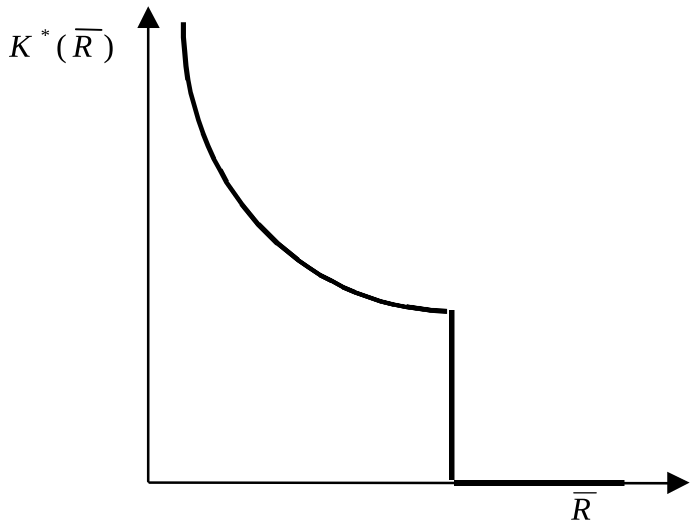

Figure 1: shows that optimal investment by the firm, $K^*(\overline{R})$, is a convex valued upper hemi continuous correspondence in $\overline{R}$.

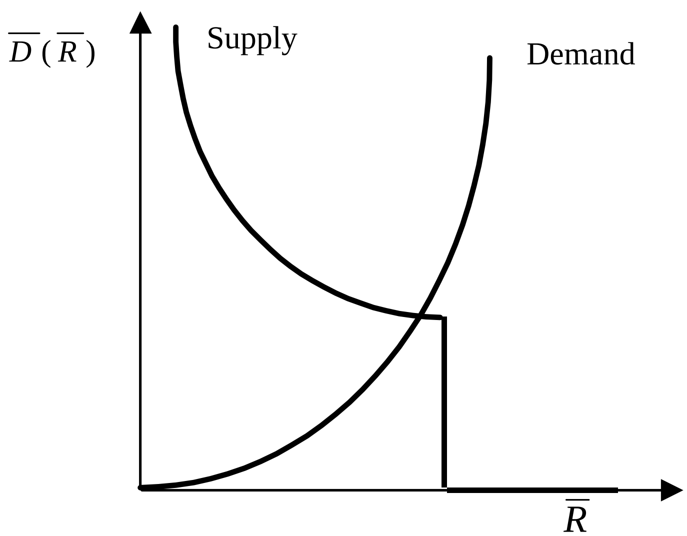

Figure 2: shows the Demand and Supply of $\overline{D}$, as a function of $\overline{R}$. The existence and uniqueness of equilibrium can be inferred from the picture.

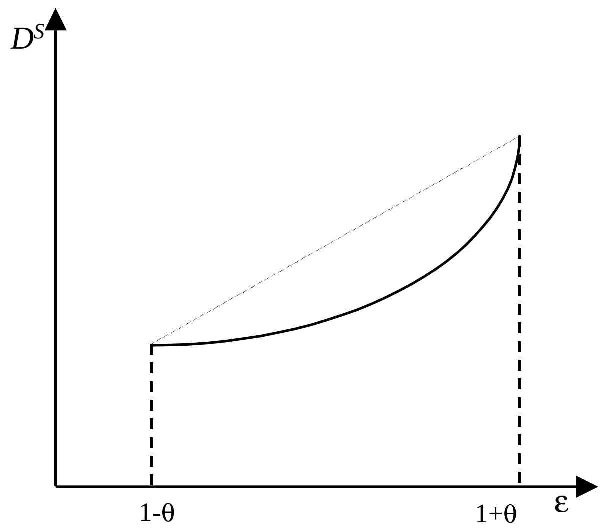

Figure 3: Dividends (D^S) realized in different states (ε). When the equilibrium level of investment is low, dividends are convex in the state realization.

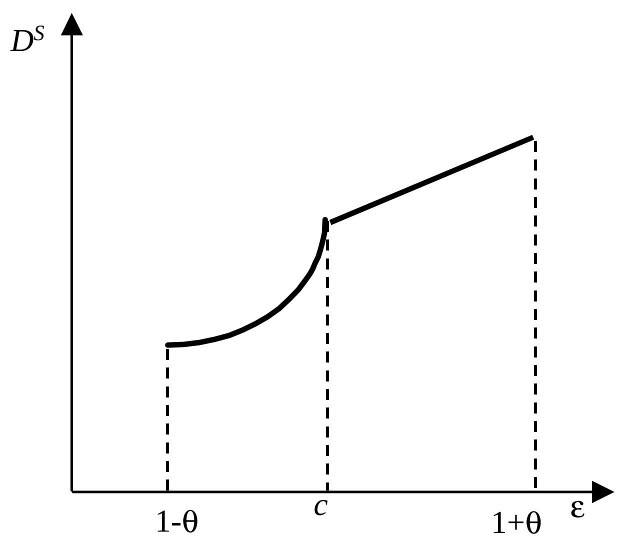

Figure 4: Dividends (D^S) realized in different states (ε). When the equilibrium level of investment is *high*, dividends are convex in the state realization for states poorer than *c*. When the realized states are better than *c*, dividends are linear in the state realization.

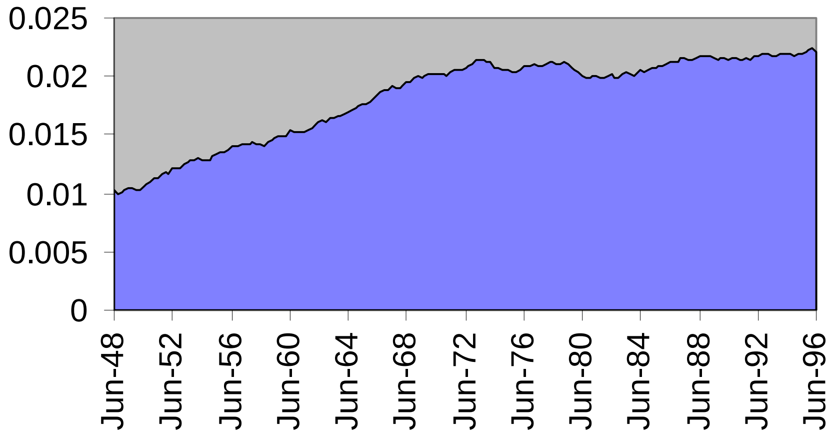

Figure 5: $\hat{w}_t$, real compensation per employee, for the years 1948 and 1995, measured in millions of dollars.

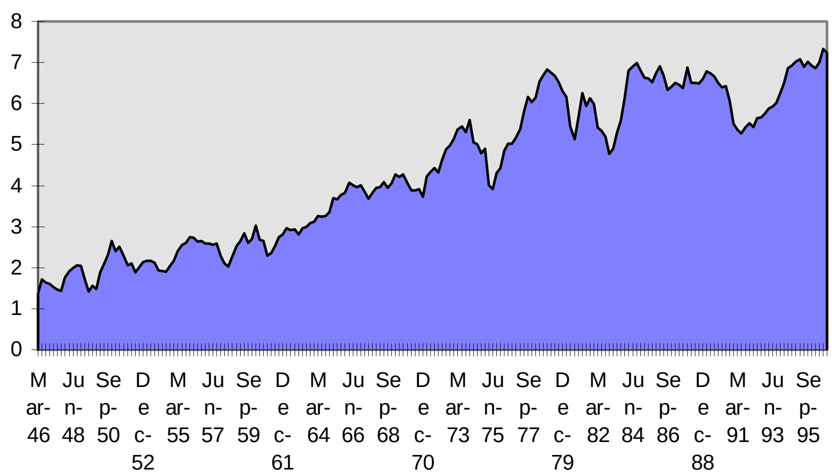

Figure 6: Real Gross Private Domestic Investment (RGPDI) for the years 1946 to1995, measured in hundreds of billions of dollars.

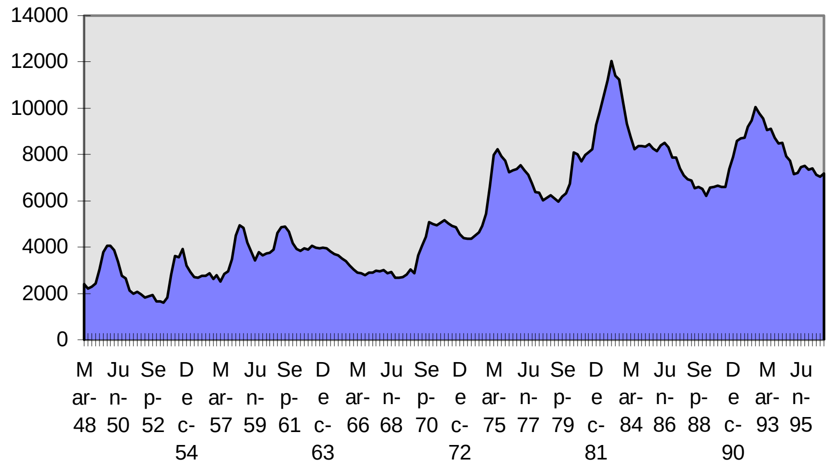

Figure 7: Unemployment (UNEMP) for the years 1948 to 1995. Unemployment is measured in thousands.

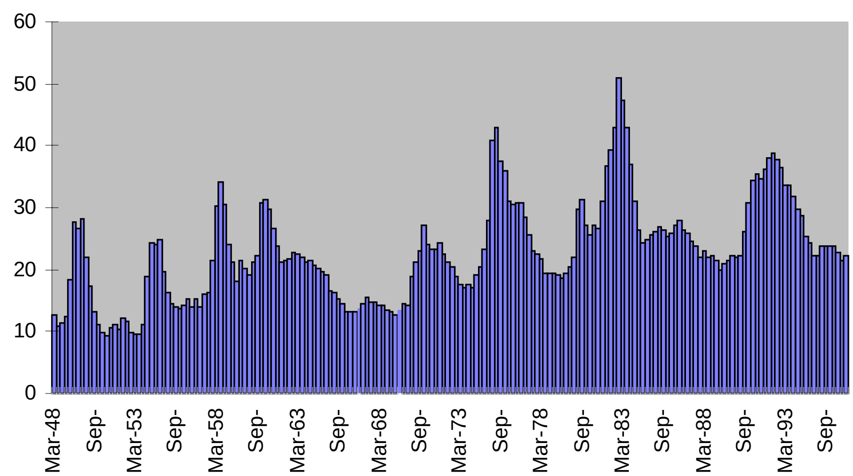

Figure 8: $\hat{c}_t$, is the ratio of $UNEMP_{t-1,t}$ to $RGPDI_{t-1,t}$, multiplied by $\hat{w}_t$, for the years 1948 to 1995.

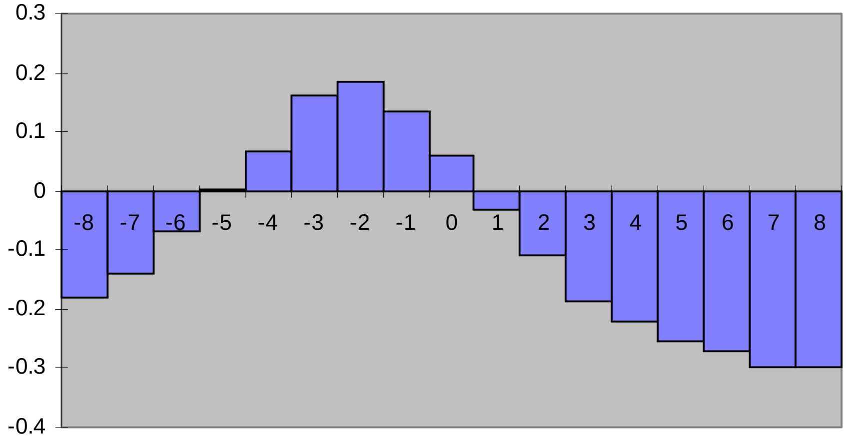

Figure 9: Correlations between the conditional expected returns and leads of conditional expected volatility for an aggregate stock index. For example, -6 denotes the correlation between $\overline{R}_t$ and σ^2_{t-6}. The conditional returns and variance were obtained from maximum likelihood estimates of 11, including a GARCH(1,1) process for the variance of the error terms.

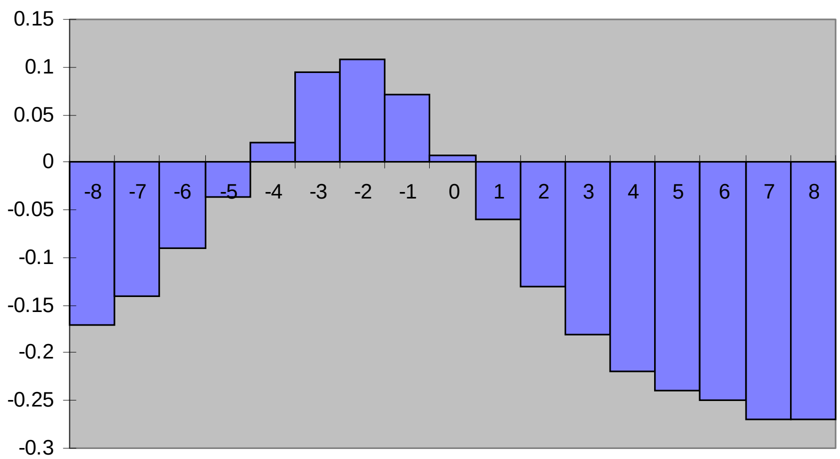

Figure 10: Correlations between the conditional expected returns and leads of conditional expected volatility for an aggregate stock index. For example, -6 denotes the correlation between $\overline{R}_t$ and σ^2_{t-6}. The conditional returns and variance were obtained from maximum likelihood estimates of 12, including a GARCH(1,1) process for the variance of the error terms.

APPENDIX A

Proof of Lemma 1: The firm's choice of optimal investment $K^*(\overline{R})$is defined implicitly in its first order condition (6 and 6\`). Using (6 and 6\`) and recognizing that *c* is also a function of *K*, we can show that $K^*(\overline{R})$ is a convex valued upper hemi continuous (uhc) correspondence in $\overline{R}$.

First, it is clear that the first order condition cannot be met for any $\overline{R} > \int_{1-\theta}^{1+\theta} \alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \frac{\varepsilon^{\frac{1}{\alpha}}}{2\theta} d\varepsilon$ since

$\overline{D}_K^S$ is bounded above by $\int_{1-\theta}^{1+\theta} \alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \frac{\varepsilon^{\frac{1}{\alpha}}}{2\theta} d\varepsilon$. In this case, the firm behaves optimally by not investing (i.e., $K^*(\overline{R})$= 0).

Second, for $\overline{R} = \int_{1-\theta}^{1+\theta} \alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \frac{\varepsilon^{\frac{1}{\alpha}}}{2\theta} d\varepsilon$, $K^*(\overline{R}) \in \left[0, \left(\frac{wN^{\alpha}}{(1-\alpha)(1+\theta)}\right)^{\frac{1}{\alpha}}\right]$. Finally, when

$\overline{R} \in \left(0, \int_{1-\theta}^{1+\theta} \alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \frac{\varepsilon^{\frac{1}{\alpha}}}{2\theta}\right)$, the mapping between $\overline{R}$ and K^* is one-to-one and continuous.

Moreover $K^*(\overline{R}) \in \left[\left(\frac{wN^{\alpha}}{(1-\alpha)(1+\theta)}\right)^{\frac{1}{\alpha}}, \infty\right]$ and is strictly decreasing in $\overline{R}$.

From the above, it is clear that $K^*(\overline{R})$ always maps into convex sets and, hence, is convex valued.

Moreover, for $\overline{R} \neq \int_{1-\theta}^{1+\theta} \alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \frac{\varepsilon^{\frac{1}{\alpha}}}{2\theta} d\varepsilon$, $K^*(\overline{R})$ is a continuous function in $\overline{R}$.

To establish the upper hemicontinuity of $K^*(\overline{R})$, we need to show that for any sequence

$\overline{R}_n \to \overline{R} = \int_{1-\theta}^{1+\theta} \alpha \left(\frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} \frac{\varepsilon^{\frac{1}{\alpha}}}{2\theta} d\varepsilon$, there exists a subsequence and a $K^*(\overline{R})$ such that $K^*(\overline{R}_n) \to K^*(\overline{R})$.

In order to do this, we show that for sequences $\overline{R}_n$ approaching $\overline{R}$ from above or below, $K^*(\overline{R}_n) \to$ $K^*(\overline{R})$.

First, construct a sequence $\overline{R}_n \to \overline{R} = \left(\int_{1-\theta}^{1+\theta} \alpha \left(\frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} \frac{\varepsilon^{\frac{1}{\alpha}}}{2\theta} d\varepsilon \right)^+$. Since $K^*(\overline{R}_n) = 0 \; \forall \; \overline{R}_n$ and

$K^*(\overline{R}) \in \left[0, \left(\frac{wN^{\alpha}}{(1-\alpha)(1+\theta)} \right)^{\frac{1}{\alpha}} \right]$ then, $K^*(\overline{R}_n) \to K^*(\overline{R}) = 0$.

Next, construct a sequence $\overline{R}_n \to \overline{R} = \left(\int_{1-\theta}^{1+\theta} \alpha \left(\frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} \frac{\varepsilon^{\frac{1}{\alpha}}}{2\theta} d\varepsilon \right)^-$. In this case, $K^*(\overline{R}_n) \to$

$\left(\left(\frac{wN^{\alpha}}{(1-\alpha)(1+\theta)} \right)^{\frac{1}{\alpha}} \right)^+$ and this limit is contained in the set $\left[0, \left(\frac{wN^{\alpha}}{(1-\alpha)(1+\theta)} \right)^{\frac{1}{\alpha}} \right]$.

Last, since $\overline{D}^{S*}(\overline{R})$ is a monotonically increasing function of $K^*(\overline{R})$, it will also be a convex valued uhc correspondence in $\overline{R}$. ♦

Proof of Theorem 1: Define excess demand as

$$\overline{D}^{D*}(\overline{R}) - \overline{D}^{S*}(\overline{R}) = \frac{W}{\overline{R}^{-1} + \overline{R}^{\frac{-1}{\gamma}}} - \overline{D}^{S*}\left(K^*(\overline{R})\right), \qquad \forall \, \overline{R} \in (0, \infty)$$

From Lemmas 1 and 2, we know that excess demand is a convex valued uhc correspondence in $\overline{R}$.

For $\overline{R} > \int_{1-\theta}^{1+\theta} \alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \frac{\varepsilon^{\frac{1}{\alpha}}}{2\theta} d\varepsilon$, $K^{*}(\overline{R}) = 0$, and excess demand equals

$$\overline{D}^{D*}(\overline{R}) - \overline{D}^{S*}(\overline{R}) = \frac{W}{\overline{R}^{-1} + \overline{R}^{\frac{-1}{\gamma}}} - 0 > 0.$$

Moreover, as $\overline{R} \to 0$, $K^{*}(\overline{R}) \to \infty$, and $\overline{D}^{D*} \to 0$. Therefore, excess demand is given by

$\overline{D}^{D*}(\overline{R}) - \overline{D}^{S*}(\overline{R}) = 0 - \infty = -\infty < 0.$

Clearly, there exist $\left(K^{*}\left(\overline{R}^{*}\right), \overline{R}^{*}\right)$ for which excess demand takes on both positive and negative values. Therefore, there must exist $\left(K^{*}\left(\overline{R}^{*}\right), \overline{R}^{*}\right)$ such that excess demand is zero. ♦

Proof of Corollary 1: By contradiction.

Suppose there exist two distinct equilibria, $\left(K^{1*}\left(\overline{R}^{1*}\right), \overline{R}^{1*}\right)$ and $\left(K^{2*}\left(\overline{R}^{2*}\right), \overline{R}^{2*}\right)$, such that excess demand at both equilibria equals zero. Note that $\left(K^{1*}\left(\overline{R}^{1*}\right), \overline{R}^{1*}\right)$ and $\left(K^{2*}\left(\overline{R}^{2*}\right), \overline{R}^{2*}\right)$ are distinct only if $K^{1*}\left(\overline{R}^{1*}\right) \neq K^{2*}\left(\overline{R}^{2*}\right)$ since $\overline{R}^{*}$ is a single valued and non-increasing function of K^{*}.

Therefore, assume without loss of generality that $K^{1*}\left(\overline{R}^{1*}\right) < K^{2*}\left(\overline{R}^{2*}\right)$ so that $\overline{R}^{1*} \geq \overline{R}^{2*}$.

But this implies that excess demand in the first equilibrium exceeds that in the second, a contradiction. ♦

Proof of Lemma 3: We use risk in the sense of a mean preserving spread (Rothschild and Stiglitz, 1971). In our model, an increase in risk is represented by an increase in θ.

Note that D^{S} is a real-valued, increasing and strictly convex function of ε. Therefore, by Jensen's inequality, $\overline{D}_{\theta}^{S} > 0$. ♦

Proof of Theorem 3:

From equations (6) and (7) and Theorem 1 and Corollary 1 in the text, we know that an equilibrium must satisfy

$$G^1 \equiv \frac{W}{\overline{D}^D} - \frac{1}{\overline{R}} - \left(\overline{R}\right)^{-\frac{1}{\gamma}} = 0 \tag{A1}$$

$$G^2 \equiv \overline{D}_K^S - \overline{R} = 0 \tag{A2}$$

$$\overline{D}^S = \overline{D}^D \tag{A3}$$

Using the implicit function theorem, the impact of a change in an arbitrary exogenous parameter, *x*, on $K^*\left(\overline{R}^*\right)$ and $\overline{R}^*$ can be determined by totally differentiating both G^1 and G^2, and solving for $\frac{dK^*}{dx}$ and $\frac{d\overline{R}^*}{dx}$. This operation yields

$$\begin{aligned} \frac{dK^*}{dx} &= \frac{-G_{\overline{R}}^2 G_x^1 + G_{\overline{R}}^1 G_x^2}{G_{\overline{R}}^2 G_K^1 - G_{\overline{R}}^1 G_K^2} \\ \frac{d\overline{R}^*}{dx} &= \frac{G_K^2 G_x^1 - G_K^1 G_x^2}{G_{\overline{R}}^2 G_K^1 - G_{\overline{R}}^1 G_K^2} \end{aligned} \tag{A4}$$

Choose $x = \theta$. Using (A1), (A2) and (A3), $G_K^2 = \overline{D}_{KK}^S$, $G_K^1 = \frac{-W}{\left(\overline{D}^S\right)^2}\overline{D}_K^S$, $G_\theta^2 = \overline{D}_{K\theta}^S$,

$G_\theta^1 = \frac{-W}{\left(\overline{D}^S\right)^2}\overline{D}_\theta^S$, $G_{\overline{R}}^1 = \frac{1}{\overline{R}^2} + \frac{\overline{R}^{-\frac{1}{\gamma}-1}}{\gamma}$ and $G_{\overline{R}}^2 = -1$. If equilibrium investment is *low*, $c > 1+\theta$. In this case, $\overline{D}_{KK}^S = 0$ and $\overline{D}_K^S = \frac{\overline{D}^S}{K}$. Moreover, $\overline{D}_{K\theta}^S = \frac{\overline{D}_\theta^S}{K}$. Using these facts and (A4) it follows that $\operatorname{sign}\frac{d\overline{R}^*}{d\theta} = \operatorname{sign}\left(-\left[-\frac{W}{\left(\overline{D}^S\right)^2}\overline{D}_K^S\right]\overline{D}_{K\theta}^S\right) = \operatorname{sign}\left(\frac{W}{\overline{D}^S}\frac{1}{K^2}\overline{D}_\theta^S\right) = \operatorname{sign}\overline{D}_\theta^S > 0.$

To prove the second part of the theorem, define the variance of stock returns as σ^2. From the first order condition for the firm $\sigma^2 = \text{variance}\left(\frac{D}{P}\right)$, which is independent of *K* since *P* =*K*. It follows that $\frac{d\sigma^2}{d\theta} = \sigma_\theta^2$. After some algebra, it can be shown that $\frac{d\sigma^2}{d\theta} = \sigma_\theta^2 > 0$.♦

Proof of Lemma 4: Using Liebnitz' rule, we have that

$$\overline{D}_\theta^S = -\frac{\overline{D}^S}{\theta} + \frac{D^S\{\varepsilon = 1+\theta\}}{2\theta} + \frac{D^S\{\varepsilon = 1-\theta\}}{2\theta},$$

$$= \frac{1}{2\theta}\left[-\frac{1}{\theta}\left(\int_{1-\theta}^{c} D^S d\varepsilon + \int_{c}^{1+\theta} D^S d\varepsilon\right) + \left(D^S\{\varepsilon = 1-\theta\} + D^S\{\varepsilon = 1+\theta\}\right)\right]$$

where $D^S\ \{\varepsilon = a\}$ denotes dividends generated in state *a*.

Note that $-\frac{1}{\theta}\left[\int_{1-\theta}^{c} D^S d\varepsilon\right] + \left[D^S\{\varepsilon = 1-\theta\}\right] < 0$ since $\theta \in (0,1)$ and $D^S\ \{\varepsilon = 1\text{-}\theta\} < D^S\ \{\varepsilon < c\}$.

Moreover, $-\frac{1}{\theta}\left[\int_{c}^{1+\theta} D^S d\varepsilon\right] + \left[D^S\{\varepsilon = 1+\theta\}\right] = \alpha K^\alpha N^{1-\alpha}\left[-\int_{c}^{1+\theta} \frac{\varepsilon}{\theta} d\varepsilon + (1+\theta)\right] = \alpha K^\alpha N^{1-\alpha}\left[\theta^2 - 1 + c^2\right]$.

Therefore, for $(1-\theta)^2 < c^2 < 1-\theta^2$, $-\frac{1}{\theta}\left[\int_{c}^{1+\theta} D^S d\varepsilon\right] + \left[D^S\{\varepsilon = 1+\theta\}\right] < 0$ and $\overline{D}_\theta^S < 0$.♦

Proof of Theorem 4: From (A4),

$$\frac{d\overline{R}^*}{d\theta} = \left[\overline{D}_{KK}^S \overline{D}_\theta^S - \overline{D}_K^S \overline{D}_{K\theta}^S\right]\left[-\frac{W}{\left(\overline{D}^S\right)^2}\right] \tag{A5}$$

and $$\frac{dK^*}{d\theta} = -\frac{W}{(\overline{D}^S)^2}\overline{D}_\theta^S + \left[\frac{1}{\overline{R}^2} + \frac{\overline{R}^{-\frac{1}{\gamma}-1}}{\gamma}\right]\overline{D}_{K\theta}^S \tag{A6}$$

From Lemma 4, we know that $\overline{D}_{\theta}^{S} < 0$ over this range. Moreover, at $c^2 = 1-\theta^2$, it is easy to show that $\overline{D}_{K\theta}^{S} = \frac{\overline{D}_{\theta}^{S}}{K} < 0$. Therefore, $\frac{d\overline{R}^*}{d\theta} < 0$ at $c^2 = 1-\theta^2$.

It can also be shown that as $c^2 \to \left((1-\theta)^2\right)^+$, $\overline{D}_{K\theta}^{S} > 0$.

Note that $\overline{D}_{KK}^{S} < 0$, $\overline{D}_{K}^{S} > 0$ and $\left[-\frac{W}{\left(\overline{D}^{S}\right)^2}\right] < 0$. This implies that as $c^2 \to \left((1-\theta)^2\right)^+$, $\frac{dK^*}{d\theta} > 0$.

We now show that there exist equilibria over the range $(1-\theta)^2 < c^2 < 1-\theta^2$ such that

$\frac{d\overline{R}^*}{d\theta} < 0$ and $\frac{dK^*}{d\theta} > 0$. The argument is as follows. First, suppose $\exists\, c = c'$ such that $\frac{d\overline{R}^*}{d\theta} = 0$. Since

$\frac{d\overline{R}^*}{d\theta} = 0$, and $\overline{D}_{\theta}^{S} < 0$, we can infer from (A4) that, at c', $\overline{D}_{K\theta}^{S} > 0$. From (A5), at c', $\frac{dK^*}{d\theta} > 0$.

By continuity, there exists a $c \to c'$, $\frac{d\overline{R}^*}{d\theta} < 0$ while $\frac{dK^*}{d\theta} > 0$.

Alternatively, suppose that $\frac{d\overline{R}^*}{d\theta} < 0 \quad \forall\, c^2$ such that $(1-\theta)^2 < c^2 < 1-\theta^2$. In this case, there exists a

$c^2 \to \left((1-\theta)^2\right)^+$, $\frac{dK^*}{d\theta} > 0$ while $\frac{d\overline{R}^*}{d\theta} < 0$.

To prove the second part of the theorem, take the total differential of σ^2 with respect to θ. This

yields $\frac{d\sigma^2}{d\theta} = \frac{1}{P^2}\frac{\partial \mathrm{Var}(D^S)}{\partial\theta} + \frac{1}{P^2}\frac{\partial \mathrm{Var}(D^S)}{\partial K}\frac{dK^*}{d\theta} - \frac{\mathrm{Var}(D^S)}{P^3}\frac{dP}{d\overline{R}}\frac{d\overline{R}^*}{d\theta}$.

It is straightforward to see that $\frac{\partial \mathrm{Var}(D^S)}{\partial K} > 0$ and $\frac{dP}{d\overline{R}} > 0$ over this range of equilibria.

With some algebra, it can also be shown that $\frac{\partial \text{Var}(D^S)}{\partial \theta} > 0 \;\forall \theta > 0.5$. It follows that

$\frac{d\sigma^2}{d\theta} > 0$, and $\frac{d\overline{R}^*}{d\theta} < 0$ at an equilibrium where $\theta > 0.5$ and $\frac{dK^*}{d\theta} > 0$. ♦

Proof of Lemma 5: When $c > 1+\theta$, $\overline{D}^S$, from 5\`, is given by $\overline{D}^S = \int_{1-\theta}^{1+\theta} \alpha \left(\frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} K \frac{\varepsilon^{\frac{1}{\alpha}}}{2\theta} d\varepsilon$.

Differentiating first with respect to w, we have that $\overline{D}_w^S = -\left(\frac{1-\alpha}{w} \right)^{\frac{1}{\alpha}} \int_{1-\theta}^{1+\theta} \frac{K\varepsilon^{\frac{1}{\alpha}} d\varepsilon}{2\theta} < 0$.

Second, by inspection, we have that in this case $\overline{D}_W^S = \overline{D}_N^S = 0$. ♦

Proof of Theorem 5: Recall that, when $c > 1+\theta$, $\overline{R} = \overline{D}_K^S = \frac{\overline{D}^S}{P} = \frac{\overline{D}^S}{K}$, so, $\overline{D}_{K\theta}^S = \frac{\overline{D}_\theta^S}{K}$, and

$\overline{D}_{KK}^S = 0$. It follows, using (A4) and Lemma 5, that

$$\text{sign } \frac{d\overline{R}^*}{dw} = \text{sign}\left(-\left[-\frac{W}{\left(\overline{D}^S\right)^2} \overline{D}_K^S \right] \overline{D}_{Kw}^S \right) = \text{sign}\left(\frac{W}{\overline{D}^S} \frac{\overline{D}_w^S}{K^2} \right) = \text{sign } \overline{D}_w^S < 0.$$

Furthermore, $\text{sign } \frac{d\overline{R}^*}{dW} = \text{sign}\left(-\left[-\frac{W}{\left(\overline{D}^s\right)^2} \overline{D}_K^S \right] \overline{D}_{KW}^S \right) = 0$.

Finally, $\text{sign}\frac{d\overline{R}^*}{dN} = \text{sign}\left(-\left[-\frac{W}{\left(\overline{D}^S\right)^2} \overline{D}_K^S \right] \overline{D}_{KN}^S \right) = \text{sign } \left(\frac{W}{\overline{D}^S} \frac{\overline{D}_N^S}{K^2} \right) = 0.$ ♦

Proof of Lemma 6: Note that for $1-\theta < c < 1+\theta$, $\overline{D}^S = \int_{1-\theta}^{c} \alpha \left(\frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} \frac{K\varepsilon^{\frac{1}{\alpha}} d\varepsilon}{2\theta} + \int_{c}^{1+\theta} \frac{\alpha K^\alpha N^{1-\alpha} \varepsilon \, d\varepsilon}{2\theta}$.

In this case, it is easy to see that $\overline{D}_N^S = \int_{c}^{1+\theta} \frac{\alpha(1-\alpha)K^\alpha N^{-\alpha} \varepsilon \, d\varepsilon}{2\theta} > 0$.

Similarly, $\overline{D}_w^S = -\left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}} \int_{1-\theta}^{c} K\varepsilon^{\frac{1}{\alpha}} \frac{d\varepsilon}{2\theta} < 0$. Finally, by inspection $\overline{D}_W^S = 0$. ♦

Proof of Theorem 6: When 1-θ < c < 1+θ, $\overline{D}_{KK}^S < 0$.

Therefore, sign $\frac{d\overline{R}^*}{dN}$ = sign $\left[\overline{D}_{KK}^S \overline{D}_N^S - \overline{D}_K^S \overline{D}_{NK}^S\right]\left[-\frac{W}{\left(\overline{D}^S\right)^2}\right]$

It is straightforward to show that $\overline{D}_{KN}^S = \int_c^{1+\theta} \frac{\alpha^2(1-\alpha)K^{\alpha-1}N^{-\alpha}\varepsilon\, d\varepsilon}{2\theta} + \frac{\alpha^2 w^2 N^\alpha}{2\theta(1-\alpha)K^{1+\alpha}} > 0$. Moreover,

from our previous analysis, we know that $\overline{D}_N^S$ and $\overline{D}_K^S$ are both positive. This implies that

sign $\frac{d\overline{R}^*}{dN} > 0$.

Second, since $\overline{D}_{KK}^S < 0$ and $\overline{D}_{KW}^S = 0$, it follows from (A4) that $\frac{d\overline{R}^*}{dW} < 0$.

Finally, as $c \to 1-\theta$, $\overline{D}_w^S \to 0$, and $\overline{D}_{Kw}^S = -\left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}} \left(\int_{1-\theta}^{c} \frac{\varepsilon^{\frac{1}{\alpha}}\, d\varepsilon}{2\theta} - \frac{\alpha c^{\frac{1+\alpha}{\alpha}}}{2\theta} \right) > 0$.

Therefore, sign $\frac{d\overline{R}^*}{dw}$ = sign $\left[\overline{D}_{KK}^S \overline{D}_w^S - \overline{D}_K^S \overline{D}_{Kw}^S\right]\left[-\frac{W}{\left(\overline{D}^S\right)^2}\right] > 0$. ♦

APPENDIX B

Data Definitions*

Compensation of Employees is the income accruing to employees as remuneration for their work. It is the sum of Wages and Salaries and Supplements to Wages and Salaries.

Wages and Salaries consists of the monetary remuneration of employees, including the compensation of corporate officers; commissions, tips and bonuses; and receipts in kind that represent income to the recipients.

Supplements to Wages and Salaries consists of employer contributions to social insurance and of other labor income.

The Civilian Labor Force: consists of all civilians 16 to 65 years of age not confined to an institution. The term civilian is used to exclude members of the armed forces who make up a small (under 2 percent), but significant part of the labor force. Effective January 1994, the labor force does not include discouraged workers. Discouraged workers are officially defined as "persons who want a job, are available to take a job, and who had looked for work within the past year but not within the prior 4 weeks because they believed their search would be futile (Employment and Earnings, 1994.)

Employment: A person is defined as being employed if he or she worked at least one hour per week for pay or profit during the survey period. A person is also considered as employed if he or she worked at least 15 hours during the survey week for no pay in a family-owned business.

Unemployment: A person is defined as being unemployed if he or she is a part of the civilian labor force, but does not qualify to be considered employed.

Gross Private Domestic Investment is fixed capital goods purchased by private business and nonprofit institutions, and the value of the change in the physical volume of inventories held by private business. The former includes all private purchases of dwellings, whether purchased for tenant or owner occupancy. Net purchases of used goods are also included.

* The above definitions are quoted from either the National Income and Product Accounts of the United States - 1929 to1974 (published by the National Income Division of the Bureau of Economic Analysis, U.S. Department of Commerce), or from A Guide to Everyday Economic Statistics by Gary E. Clayton and Martin Gerhard Giesbrecht (McGraw-Hill, Inc.,1995.)

Table 1

Summary statistics

$\hat{\theta}_t$: standard deviation in the growth rates of industrial production for the past 12 quarters.

$UNEMP_{t-1,t}$: number of people unemployed in the United States (000s) between dates t-1 and t.

$RGPDI_{t-1,t}$: real gross private domestic investment (in billions of dollars) between dates t-1 and t.

$\hat{w}_t$: real compensation per employee, between dates t-1 and t.

$\hat{c}_t$: ratio of $UNEMP_{t-1,t}$ to $RGPDI_{t-1,t}$, multiplied by $\hat{w}_t$.

$\tilde{R}_{t+1}$: quarterly stock returns between dates t and t+1.

Variable Name	Mean	Standard Deviation	Minimum	Maximum
$\hat{\theta}_t$	0.02218	0.01234	0.00570	0.07457
$UNEMP_{t-1,t}$	5487.62	2489.38	1607.00	12051.00
$RGPDI_{t-1,t}$	4.449	2.249	1.371	23.70
$\hat{w}_t$	0.0183	0.0036	0.00988	0.0224
$\hat{c}_t$	22.802	7.964	9.302	50.94
Log ($\hat{c}_t$)	3.065	0.355	2.23	3.93
Unemployment (percent)	5.747	1.591	2.548	10.849
$\tilde{R}_{t+1}$	0.0213	0.0803	-0.275	0.2238

Table 2

Maximum Likelihood Estimates of the time varying relation between stock returns and risk, with a GARCH(1,1) process to adjust for heteroskedasticity.

$\tilde{R}_{t+1}$: quarterly stock returns, between dates t and t+1.

$\hat{\theta}_t$: standard deviation in the growth rates of industrial production for the last 12 quarters.

$\hat{w}_t$: real compensation per employee, between dates t-1 and t.

$\hat{c}_t$: ratio of $UNEMP_{t-1,t}$ to $RGPDI_{t-1,t}$, multiplied by $\hat{w}_t$.

$$\tilde{R}_{t+1} = a_0 + a_1\hat{\theta}_t + a_2(\hat{\theta}_t * \hat{c}_t) + a_3\hat{w}_t + \xi_{t+1} \quad (11)$$

$$\tilde{R}_{t+1} = a_0 + a_1\hat{\theta}_t + a_2(\hat{\theta}_t * \log(\hat{c}_t)) + a_3\hat{w}_t + \xi_{t+1} \quad (12)$$

$$\xi_{t+1} = z_{t+1}\sigma_{t+1}, \quad z_t \approx \mathrm{N}(0,1), \quad \sigma^2_{t+1} = b_0 + b_1\xi^2_t + b_2\sigma^2_t$$

	a_0	a_1	a_2	a_3	b_0	b_1	b_2	Log-likelihood Function
(11)	0.17 (4.62)*	-3.108 (-3.55)*	0.103 (4.50)*	-7.55 (-4.52)*	0.001 (3.17)*	0.15 (1.60)	0.575 (5.39)*	395.824
(12)	0.16 (4.24)*	-6.51 (-3.30)*	1.92 (3.41)*	-7.06 (-3.99)*	0.001 (2.85)*	0.15 (1.48)	0.559 (4.97)*	394.708

* indicates significance at the 5 percent level.

** t-statistics are reported in parentheses, computed as in Bollerslev and Wooldridge (1992.)

Table 3

Evidence on the time varying relation between the first two conditional moments of stock returns.

$\overline{R}_t$: predicted quarterly stock returns between dates t and t+1, estimated from (11) and (12).

$\hat{c}_t$: ratio of $UNEMP_{t-1,t}$ to $RGPDI_{t-1,t}$, multiplied by $\hat{w}_t$.

Panel A: σ_t^2 is estimated from (11) and a GARCH(1,1) process on the errors.

$$\overline{R}_t = a_0 + a_1\sigma_t^2 + \nu_t$$

$$\overline{R}_t = a_0 + a_1\sigma_t^2 + a_2(\sigma_t^2 * \hat{c}_t) + \eta_t$$

$$\overline{R}_t - \overline{R}_{t-1} = a_1(\sigma_t^2 - \sigma_{t-1}^2) + a_2(\sigma_t^2 * \hat{c}_t - \sigma_{t-1}^2 * \hat{c}_{t-1}) + \chi_t$$

	a_0	a_1	a_2	R^2	F statistic (p-value)	DW
(13)	0.02 (4.66)*	0.259 (0.37)		0.00	0.1420 (0.71)	0.262
(14)	0.031 (7.70)*	-6.49 (-5.94)*	0.202 (7.42)*	0.227	27.66 (0.00)*	0.23
(15)		-6.69 (-9.70)*	0.27 (11.42)*	0.41		1.92

* indicates significance at the 5 percent level.

Panel B: σ_t^2 is estimated from (12) and a GARCH(1,1) process on the errors.

$$\overline{R}_t = a_0 + a_1\sigma_t^2 + \nu_t$$

$$\overline{R}_t = a_0 + a_1\sigma_t^2 + a_2(\sigma_t^2 * \log(\hat{c}_t)) + \eta_t$$

$$\overline{R}_t - \overline{R}_{t-1} = a_1(\sigma_t^2 - \sigma_{t-1}^2) + a_2(\sigma_t^2 * \log(\hat{c}_t) - \sigma_{t-1}^2 * \log(\hat{c}_{t-1})) + \chi_t$$

	a_0	a_1	a_2	R^2	F statistic (p-value)	DW
(13)	0.022 (5.66)*	-0.183 (-0.28)		0.00	0.081 (0.77)	0.23
(14)	0.031 (7.52)*	-4.68 (-4.45)*	0.135 (5.21)*	0.12	13.60 (0.00)*	0.22
(15)		-4.69 (-7.24)*	0.19 (8.43)*	0.27		1.84

* indicates significance at the 5 percent level.

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