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Abstract: What are the benefits provided by a payment system? What are the tradeoffs in public versus private payment systems and in restricted versus open payments arrangements? Modern payment systems encompass a variety of institutional designs with varying degrees of counterparty protection. We develop a framework which allows for an examination and comparison of payment systems and specification of conditions leading to their adoption. We relate these conditions to the design of present large-value payment systems (Fedwire, CHIPS, Target, etc.).

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Key words: payment systems, limited enforcement, settlement risk

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1 Introduction

When trade is decentralized, and agents are not completely reliable, there can be temporary shortages of collateral so that spot transactions— transactions of goods against collateral—are not always possible. In such a case, payments systems become relevant, as the exchange of goods becomes temporally separated from the exchange of collateral. Payments systems become a means of economizing on the amount of collateral needed to enforce the pledges made. The world has seen a variety of payments mechanisms, under both public and private arrangements, centralized and decentralized. What are the optimal arrangements for such payments systems? This paper investigates an environment in which we can begin to address this question. We start by analyzing the development of a private arrangement for effecting payments. We consider the questions of membership in the organization, as well as requirements for monitoring and for the posting of collateral. We then consider a central bank-sponsored payments arrangement, where the central bank can exploit its taxation powers to back up its promises. We examine the advantages and disadvantages of including the central bank in the payments mechanism, and how its presence causes a system to change.

Our analysis is most relevant for the design of "large-value" or "wholesale" payments systems. Traditionally, these systems are used to settle obligations between banks, as may arise from large-value commercial transactions, the operation of "small-value" or "retail" payment systems (e.g., checks and credit cards), or from the need to settle financial market transactions. Fedwire (operated by the Federal Reserve)[1] and CHIPS (operated by the New York Clearing House) are the two preeminent large-value payment systems in the U.S.[2] Large-value payment systems typically have a hierarchical structure. At the core of these systems is either a single institution (a central bank in the case of Fedwire and similar systems) or a relatively small group of institutions (as in the case of CHIPS) with special settlement privileges. A second level of the hierarchy has institutions which may access the system, but with restrictions on access such as position limits or additional collateral requirements. At the bottom of the hierarchy are institutions who are not members of the system or are customers of member institutions, both of whom clear and settle payments through member institutions. A principal focus of our analysis will be to understand the purpose of this hierarchical structure.

In ordinary circumstances the operation of public and private payment systems is, at a practical level, much the same. There are, however, some notable differences in the legal and institutional underpinnings of public versus private arrangements. These distinctions, largely inconsequential during normal times,

[1] We use the term "Fedwire" to refer to all Federal Reserve large-value payment services, including both Fedwire and net settlement.

[2] See, for example, U.S. General Accounting Office (2002) for a description of Fedwire and CHIPS. The value of payments passing through these systems is considerable: average daily payments for 2001 were \$1.2 trillion for CHIPS and \$1.7 trillion for Fedwire. Other notable large-value systems include Target (Euro area, 2001 average daily payments of about \$1.3 trillion) and BOJ-Net (Japan, 2001 average daily payments of about \$520 billion).

can give rise to critical distinctions in the functionality of the two types of system during times of duress. The first such distinction is in the area of "finality." A funds transfer over a public system typically represents an unconditional transfer of a claim on a central bank. As such, it unconditionally discharges an obligation, whereas a transfer of funds over a private system may not. For example, in the U.S. a funds transfer over the Fedwire system automatically and immediately becomes a liability of the Federal Reserve, and is virtually always final. Payments made over private systems may not carry the same degree of finality.[3]

A second distinction between public and private systems can arise in the area of credit policy. Intraday credit is an essential component of many large-value payment systems. The demand for such credit largely arises from payment system participants' inability to coordinate incoming and outgoing payments.[4] Both private and public payment systems may grant intraday credit, and in both types of systems, credit risk is commonly controlled through such devices as position limits, monitoring (e.g. bank supervision), and collateral requirements. Membership in a public payment system, however, necessarily carries with it a form of "credit insurance" that has no analog in a private system: that is, while a central bank may limit the availability of intraday credit during normal times, it cannot credibly commit to withhold credit during times of duress. In a crisis, a central bank will always be tempted to enable the settlement of ex post welfare-improving trades.[5] This can lead to the central bank granting credit in circumstances where, during normal times, the granting of such credit would lead to unacceptable level of credit exposure.

In our analysis of private payments arrangements, we consider the effects of three devices in the enforcement of settlement obligations: netting, monitoring, and collateral. For cost of monitoring sufficiently low, we show that these devices should be applied in roughly in that order. Netting alone will be adequate if all counterparties are known to be sufficiently reliable. If some of the counterparties involved may be too undependable, monitoring of these counterparties–enforced by a requirement that they settle through a more re-

[3] In the U.S., the finality of funds transfers over private large-value systems is governed by Article 4a of the Uniform Commercial Code (UCC). UCC 4a says, in effect, that a payment becomes final as soon as the recipient's bank (or the recipient, if the recipient is a bank) accepts a payment instruction from the large-value payment system. In practice, such payments are virtually always accepted. Nonetheless, the recipient (or recipient's bank) retains the option to refuse such payments.

The finality rules contained in UCC 4a are of course, somewhat specific to the U.S. legal system. What is relevant for our analysis is not the finality rules themselves, but the existence of the underlying (primarily credit) risks that these rules seek to allocate.

[4] For example, McAndrews and Rajan (2000) and Coleman (2002) show that payments over Fedwire tend to peak late in the day; a lack of coordination between incoming and outgoing payments is one reason commonly given for this pattern. A number of theoretical models beginning with Freeman (1996) address this lack of coordination and available remedies. See Zhou (2000) for a survey of this literature.

[5] The theory behind such temptation is laid out by Rochet and Tirole (1996). See also McAndrews and Potter (2002) for a description of the Fed's liquidity provision in the wake of the 9/11 shock.

liable agent–may be necessary. For still less reliable counterparties, posting of collateral will be necessary to ensure settlement.

In most situations these three devices, when combined with sufficient availability of collateral, will enable agents to organize trade efficiently. But as emphasized by Kocherlakota (2001), the efficacy of these devices is ultimately tied to the value of the collateral good. If there is a downward shock to collateral value, then trade will break down even under net settlement, because net settlement provides no inducement to deliver goods when there is no incentive to pay for goods received. Settlement on the books of a central bank, by contrast, always provides an incentive to deliver, because the value offered in exchange does not derive from the value of collateral, but instead the taxation powers of the government. As long as the central bank makes credit freely available (and ex post, the central bank will have an incentive to grant such credit), confidence in the value of central bank liabilities will be sufficient to sustain trade. And to the extent that obligations incurred by payment system participants are offsetting, the central bank in equilibrium bears no loss.

The liquidity provided by public payment systems has a downside, however. Although confidence in the liabilities of the central bank can sustain trade during crises, that same confidence can undermine the incentives of payment system participants for mutual monitoring. This is of concern if one believes the public sector is worse at monitoring, or is less inclined to act upon the basis of information received. As a result, this disadvantage must be given due weight in the consideration of the relative merits of public versus private systems.

2 Literature survey

Central to the analysis below is the notion of delegated monitoring (Diamond 1984), specifically the delegation of monitoring within a payment system. The study of strategic interactions that may arise between monitoring incentives on the one hand, and the settlement of outstanding obligations on the other, was introduced by Rochet and Tirole (1996) [RT], and has been extended by Fujiki, Green, and Yamazaki (1999) [FGY]. RT show how a central bank's "too-big-to-fail" (TBTF) policy may dilute banks' incentives to monitor their exposures with other banks, including those that may arise in a payments context. FGY, by contrast, are concerned with providing a more fundamental justification for public sector involvement in the payment system. To this end they show that such involvement can be seen as a feature of (generalized) core allocations in an economy that incorporates private information and other trading fractions. Our approach is generally closer to that of RT in the sense that we will take certain aspects of public sector involvement in large-value payment systems (including TBTF) as parametric; we then consider potential interactions of such involvement with monitoring incentives. Details of the model environment are closer to FGY, however, in the sense that monitored information may be conveyed by explicit reports, and does not have to be inferred from observed behavior.

Another crucial element of our analysis is the notion that limited enforcement frictions can sometimes be overcome by substituting public obligations for private ones. This idea is by now a very familiar one, following such papers as Woodford (1990), Holmström and Tirole (1998), Kocherlakota (2001), and Köppl and MacGee (2001). What is different below is that we explore how such substitution may relate to the efficacy of other devices for overcoming limited enforcement, as are commonly employed in large-value payment systems.

By virtue of its subject matter, our paper is also connected to many other papers in the burgeoning literature on the design of payments systems. Two of the most relevant papers are Freixas, Parigi, and Rochet (2000) [FPR] and Holthausen and Rønde (2002) [HR]. FPR analyze the interplay between patterns of settlement obligations, their associated potential for creating "systemic risk" scenarios, and the efficacy of various policy interventions designed to prevent the spread of systemic risk. Our setup is similar in the sense that certain alignments of preference shocks (and their resulting settlement obligations) can give rise to scenarios with an elevated potential for settlement failures. The focus of our analysis is somewhat different, however, as we are less concerned with the desirability of public sector bailouts per se, but instead on how the potential for bailouts may interact with other means for lessening settlement risk, particularly restrictions on full-fledged membership in payment systems.

HR also look at issues of membership in large-value payment systems that utilize net settlement. They show that under limited liability, there exists an incentive for overly broad membership in these systems, since member institutions may not internalize the costs of potential settlement failures. Nonetheless decisions on membership in these systems may best be left up to the private sector, if the private sector enjoys a significant informational advantage over regulators. Our analysis provides a somewhat complementary result (Corollary 16 below), i.e., if the private sector has an informational advantage over regulators, this advantage may best be exploited by restricting access to public settlement systems.

3 The model

There are three time periods 0, 1, and 2; agents establish a payments system in period 0, trade in period 1, and settle and consume in period 2. There are a large and equal number of agents of three types: A, B, and C (Below we will sometimes use A to mean "an agent of type A," etc.). We will regard these agents as centralized during periods 0 and 2, but separated on different "islands" during the trade period. Conceptually, it is most natural to imagine that each trader is a two-agent household, and that at the trading round the two agents separate, one meeting with an agent of each of the other two potential counterparties. Each agent is also a potential member of various types of payment arrangements, which will be described in more detail below.

There are four goods: an indivisible endowment good unique to each type (goods A, B, and C) and a collateralizable, numeraire good. Each agent is endowed with one unit of his type's good (collectively known as "eponymous" goods) and L units of the numeraire good.

The numeraire will always be a desirable consumption good for all agents. The endowment of the numeraire good can only be shipped to another agent in period 2. Each of the eponymous goods is also a potential consumption good. Agents' preferences over these goods are determined by preference shocks whose structure we describe below. The eponymous goods can only be shipped to another agent in period 1. Goods differ in terms of their "attachability." Specifically, only the numeraire good may be attached by another agent, and then only if it has been placed in a special collateral facility. Placement of collateral within the facility incurs the cost of a fraction λ of the good stored as collateral.[6]

The economy is subject to shocks (D, E, F), where D, E, and F are independent. The triple $D = \left(D^A, D^B, D^C\right)$ determines the costs of default for agents of types A, B, and C, respectively, where this cost is measured in equivalent units of the numeraire. The individual components $D^i \in \{D_L, D_H\}$ where $D_H \gg D_L = 0$. An agent whose cost is D_H (D_L) is said to be "reliable" ("unreliable"). The probability that an agent has a high default cost is known as his "reliability."

The shock $E \in \{0, 1\}$ determines the "orientation" of agents' preferences. When $E = 0$, the orientation is "counterclockwise": a type A will want to consume type B's endowment good, type C will want to consume type A's endowment good, and type B will sometimes want to consume type C's endowment good. When $E = 1$ (as occurs with probability e, $0 < e < 1$), the orientation is "clockwise": type B wants to consume type A's endowment good, type C wants to consume type B's endowment good, and type A sometimes wants to consume type C's endowment good.

The shock $F \in \{0, 1\}$ determines, for a given orientation, whether another type agent (B under counterclockwise orientation, A under clockwise orientation) wants to consume C's endowment good (in which case $F = 1$, and there is "balanced demand") or not ($F = 0$, and there is "unbalanced demand"). The probability of balanced demand is given by f. Absent monitoring, the realization of the preference shock F is private to to the type C agents. See Figure 1 for an illustration of agents' preferences under various realizations of shocks E and F.[7]

The probability distribution of the shocks D depends on the "state of the economy," which is publicly revealed in period 0. If there is a panic (as occurs with probability $1 - n$), then all agents are known to be unreliable. During normal times (which occur with probability n) types A and B have default costs equal to D_H with probability one—i.e., A and B are known to be reliable.

[6] Think of this as a simple production process for generating collateral. Many papers have dealt with the benefits of collateral in allowing trade to occur between unreliable parties. This paper will focus on collateral as the expensive alternative to other payments arrangements.

[7] Clearly it is possible to construct models with more types of agents and more complex patterns of payments; see, e.g., Freixas, Parigi, and Rochet (2000). What is important for our setup is that some risky agents sometimes desire to make payments that are not always offset by incoming payments by other agents.

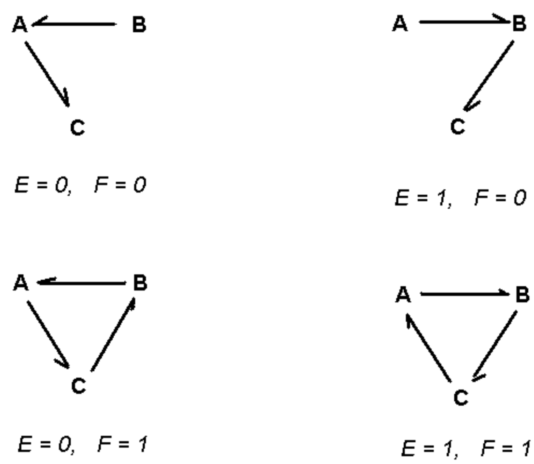

Figure 1: The flow of eponymous goods

Type C is reliable with probability r^C. Type C's learn whether or not they are reliable in period 0. By incurring a disutility equal to $M > 0$, agents A or B can monitor and thereby learn whether C is reliable, with certainty, at the same time that C learns this information.

The information obtained by monitoring cannot be verified by an outside party, hence the costs of monitoring cannot be shared. Information obtained by a private monitor thus remains private until it is willingly revealed by the monitor. A penalty equal to a disutility of $X \gg 0$ may be applied by an enforcement authority (a social planner or a clearing organization) to monitors whose reports turn out to be false, i.e., in cases where a monitor reports a type C to be reliable who subsequently defaults.[8]

Events proceed as follows (see Table 1 below for a summary). In period 0, if a panic has not occurred, then agents of type A or B, or both, can decide to monitor agents of type C. Agents then have the option of placing collateral in the collateral facility. Next, at the beginning of period 1, preference shock E is learned and each agent has a representative travel to the location where his potential supplier produces. Once there, purchaser and supplier learn whether their trade will be desired (in other words, C and the demander of C's good learn F), immediately followed by trading in commodities A, B, and C in return for promises of period 2 transfers of the numeraire good. Given the separation

[8] We assume that the application of the penalty for false reports is deterministic. Introducing stochastic penalties could lead to welfare gains but would not substantively change the results derived below.

during the trading period, the only feasible trades are trades of current delivery of an eponymous good in return for future delivery of the numeraire good. Contingent trades (for example, of the form, "I will deliver good A to you provided someone else delivers good B to me") are not permitted, i.e., the representative must make an arrangement independent of the arrangement made by his partner. For the moment we are ruling out pure "spot" (aka "delivery versus payment" or DVP) transactions of goods for immediate payment. Trades are private information between the two agents involved while the trades are occuring, but become known to other agents as soon as all trading is finished. In period 2, agents may then make their pledged numeraire transfers, or default. If default occurs, the defaulting agent's creditor may attach the agent's collateral up to the amount of the pledge. Finally, consumption occurs.

Table 1: Sequence of events in the model

Period 0a.	State of the economy is learned (panic or no panic)
0b.	Agents agree to be monitored (or not)
0c.	Reliability revealed to agents and monitor
0d.	Announcements of monitors & collateral choice
Period 1a.	Orientation of trade E is learned
1b.	Agents travel to other islands
1c.	Preference shocks F revealed (balance of demand)
1d.	Trade in eponymous goods
1e.	Trades revealed
Period 2a.	Transfer of numeraire good (or default)
2b.	Attachment of collateral
2c.	Consumption

Formally, preferences of a type-i agent ($i = A, B, C$) are given by the expectation of i's period 2 utility, i.e.,

$$u^i = v^i \left(\sum_{j=A,B,C,\ell} s^i_j c^i_j - \delta^i D^i \right) - \mu^i \left(M + \xi^i X \right) \tag{1}$$

where c^i_j is agent i's consumption of good j, δ^i is an indicator for agent i's default decision, μ^i is an indicator for agent i's decision ($i = A, B$) to monitor, and ξ^i is an indicator of whether the monitor is discovered to have filed a false report. The consumption weights s^i_j are (E, F) - measurable and indicate "single coincidences" of wants; in particular

$$s^i_j = 1 \text{ if } j = \ell \tag{2}$$

$$s^i_j = \alpha \in (0,1) \text{ if } i = j \tag{3}$$

In addition,

$$s^A_B = 1 - E \tag{4}$$

$$s^A_C = EF \tag{5}$$

$$s_A^B = E \tag{6}$$
$$s_C^B = (1 - E)\,F \tag{7}$$

$$s_A^C = 1 - E \tag{8}$$
$$s_B^C = E \tag{9}$$

The utility of consumption v^i is given by

$$v^i(c) = w(c) \tag{10}$$

for $i = A, B$, where $w'(c)$ is a step function:

$$\begin{aligned} w'(c) &= \overline{w} \text{ if } c < \alpha + L \\ &= \underline{w} \text{ if } c > \alpha + L \end{aligned} \tag{11}$$

and $0 < \underline{w} < 1 < \overline{w}$. For agent C the utility of consumption is given by

$$v^C(c) = c \tag{12}$$

4 The planner's problem

An allocation in this economy is a vector (μ, ℓ, c, δ) that determines respectively agents' monitoring, collateral postings, consumption and default decisions. An optimal allocation maximizes the expected utility of type C agents

$$Eu^C \tag{13}$$

subject to the (per capita) feasibility constraints

$$\sum_i c_j^i \leq 1 \text{ for } j = A, B, C \tag{14}$$
$$\sum_i c_\ell^i \leq 3L - \lambda \sum_i \ell^i \tag{15}$$

and the individual rationality constraints for A and B

$$Eu^A \geq w(\alpha + L) \tag{16}$$
$$Eu^B \geq w(\alpha + L) \tag{17}$$

We will study non-autarkic allocations in which all opportunities for trade are exploited and default does not occur. For such allocations consumption of the eponymous goods will be given by

$$c_{s(A)}^A = 1 - E(1 - F) \tag{18}$$
$$c_{s(B)}^B = E + (1 - E)F \tag{19}$$
$$c_{s(C)}^C = 1 \tag{20}$$
$$c_C^C = 1 - F \tag{21}$$

where

$$s(i) = i + 2E - 1 \pmod 2 \tag{22}$$

i.e., $s(i)$ denotes i's "supplier" under a given orientation of trade.[9]

Limited enforcement (non-attachability of goods not posted) imposes additional constraints on non-autarkic, no-default allocations. When demand is balanced ($F = 1$) then absence of default in a given state (D, E, F) requires that

$$v^i\left(c^i_{s(i)} + c^i_\ell\right) \geq v^i\left(c^i_{s(i)} + \alpha + L - \ell^i - D^i\right) \text{ for } i = A, B, C \tag{23}$$

When demand is balanced, the limited enforcement constraints (23) guarantee that agent i will have an incentive to trade his eponymous good to the next person in the "credit chain" (i.e., $s^{-1}(i) = i - 2E + 1$), rather than to simply accept $s(i)$'s eponymous good and hang on to his own. In other words constraints (23) require that

$$\begin{aligned} & v^i(\text{consuming } s(i)\text{'s eponymous good} + \text{consuming numeraire good}) \\ \geq\ & v^i(\text{cons. } s(i)\text{'s eponymous good} + \text{cons. own eponymous good} \\ & +\text{numeraire endowment} - \text{loss of collateral} - \text{default cost}) \end{aligned} \tag{24}$$

For a reliable agent ($D^i = D_H$) the limited enforcement constraint will never bind, but it may bind for an unreliable agent ($D^i = 0$). In the latter case the constraints reduce to

$$c^i_\ell \geq \alpha + L - \ell^i \tag{25}$$

When demand is not balanced ($F = 0$) then the limited enforcement constraint above must be modified for agent C. In particular, it is then written as

$$c^C_{s(C)} + c^C_\ell + \alpha \geq c^C_{s(C)} + \alpha + L - \ell^C \tag{26}$$

which reduces to

$$c^C_\ell \geq L - \ell^C \tag{27}$$

This constraint says that C's consumption of the numeraire good cannot fall below what he could get by walking away from his collateral.

For monitoring to occur, additional restrictions must be placed on the planner's problem. Without loss of generality suppose that an agent A agrees to monitor an agent C. Let $\underline{D}^C$ indicate A's report on C's reliability. By a slight abuse of notation, let $c^A\left(\underline{D}^C\right)$ represent the appropriately weighted sum of A's consumption, conditional on his report. Then, truth-telling conditions on the monitor A can be shown to reduce to[10]

$$r^C w\left(c^A\left(\underline{D}^C = D_H\right)\right) - M \geq r^C w\left(c^A\left(\underline{D}^C = D_L\right)\right) \tag{28}$$

[9] To keep notation compact, the expression for $s(i)$ makes use of the obvious mapping from $\{A, B, C\}$ to the integers (mod 2).

[10] In general we should see expected utilities in the truth-telling conditions (28) and (29), i.e., c^A should be stochastic. However, in the solutions studied below, A and B will be completely insured by C against all risks (other than that of monitoring a type C and finding him to be unreliable). Hence we can state the truth-telling conditions in the somewhat simplified form given above.

$$(1-r^C)w\left(c^A\left(\underline{D}^C=D_L\right)\right)-M \geq (1-r^C)w\left(c^A\left(\underline{D}^C=D_H\right)\right)-X \qquad (29)$$

Condition (28) guarantees that the monitor will report a reliable party as reliable, whereas (29) guarantees that an unreliable party will also be reported as such. For a sufficiently stringent penalty X on false reports, it can be shown that (28) will bind while (29) will never bind. For monitoring to occur, it must also be individually rational for A to undertake the monitoring, i.e., the expected utility from monitoring must be at least as great as that of autarky

$$r^C w\left(c^A\left(\underline{D}^C=D_H\right)\right)+(1-r^C)w\left(c^A\left(\underline{D}^C=D_L\right)\right)-M \geq w(\alpha+L) \qquad (30)$$

Substituting (28) at equality into (30) at equality we obtain

$$w\left(c^A\left(\underline{D}^C=D_L\right)\right)=w(\alpha+L) \qquad (31)$$

In other words, a monitor who reports a type C to be unreliable simply receives his autarky level of consumption. The consumption of a monitor who reports a type C to be reliable may then be from calculated from (28) as

$$w\left(c^A\left(\underline{D}^C=D_H\right)\right)=w(\alpha+L)+M/r^C \qquad (32)$$

4.1 Full-enforcement benchmark

We begin by considering a benchmark allocation, the solution to the planner's problem under full enforcement (i.e., what would be obtainable if all goods were attachable at zero cost, or if all agents were always reliable). In this case, if demand is balanced, then all agents both deliver a good and receive a good. If demand is not balanced, then C and $s(C)$ receive goods, while $s(C)$ and $s^{-1}(C)$ deliver them. Any numeraire good beyond what is required to fulfill the individual rationality constraints (16) and (17) is transferred to C. Straightforward algebra then yields the following as the optimal allocation under full enforcement (in addition to conditions (18)-(21))

$$\begin{aligned}
\ell^{A,B,C} &= 0 && (33)\\
c_\ell^A &= c_\ell^{A*}=L-((1-E)+EF)+\alpha && (34)\\
c_\ell^B &= c_\ell^{B*}=L-(E+(1-E)F)+\alpha && (35)\\
c_\ell^C &= c_\ell^{C*}=L+(1+F)(1-\alpha)-(1-F)\alpha && (36)
\end{aligned}$$

4.2 Optimal allocations under limited enforcement

We begin our analysis of allocations under limited enforcement by considering a special case. Suppose that panics do not occur and C is always unreliable. In this case, there is no return to monitoring, and the planner's problem reduces to the constrained-optimal choice of consumption and collateral.

We first consider the subcase where demand is always balanced. Assuming that all trades take place, let T be the maximum total amount of numeraire that A and B are willing to transfer to C under balanced demand, i.e., $T=2(1-\alpha)$.

Then the limited enforcement constraint (25) implies that C will only need to post collateral of

$$\ell^{Cb} = \max\left\{0, \frac{\alpha - T}{1-\lambda}\right\} = \max\left\{0, \frac{3\alpha - 2}{1-\lambda}\right\} \tag{37}$$

to ensure that he will deliver his eponymous good.

Next we consider the subcase where demand is never balanced ($f = 0$). Then C's limited enforcement constraint becomes, from (27)

$$\begin{aligned} c_\ell^C &\geq L - \ell^C \Longleftrightarrow \\ L - \lambda \ell^C - Q &\geq L - \ell^C \Longleftrightarrow \\ \ell^C &\geq \frac{Q}{1-\lambda} \end{aligned} \tag{38}$$

where Q indicates a net transfer of the numeraire good from C to other agents. In this case an "open credit chain" runs from C to C's supplier, i.e., $s(C)$, to $s(C)$'s supplier, $s^{-1}(C)$. Individual rationality constraints (16) and (17) require that C's supplier receive at least α of the numeraire good as compensation. Not all of this need come from C. However the surplus available for redistribution is only half of what is available in the case of balanced trade, i.e., in this case we set $T = 1 - \alpha$, yielding

$$\ell^{Cu} = \max\left\{0, \frac{2\alpha - 1}{1-\lambda}\right\} \geq \ell^{Cb} \tag{39}$$

More generally, when there is a positive probability of both balanced and unbalanced demand, the first-best allocation is described in the following proposition.

Proposition 1 *Suppose that demand is not always balanced ($f < 1$), that C is known to be unreliable ($r^C = 0$), and that panics are not possible ($n = 1$). Then for $\lambda > 0$ sufficiently small and $\overline{w}$ sufficiently large, the solution to the planner's problem is given by (18)-(21) and*

$$\ell^{A,B} = 0 \tag{40}$$

$$\ell^C = \ell^{Cu} = \max\left\{0, \frac{\alpha - T}{1-\lambda}\right\} \tag{41}$$

$$c_\ell^A = c_\ell^{A*} \tag{42}$$

$$c_\ell^B = c_\ell^{B*} \tag{43}$$

$$c_\ell^C = c_\ell^{C*} - \lambda \ell^{Cu} \tag{44}$$

$$T = 1 - \alpha \tag{45}$$

Proof. (Sketch) This is simply the solution to the planner's problem when demand is not balanced; since it incorporates a collateral requirement that is more stringent than for balanced demand, it insures that type C agents will always honor their obligations. If C posts any less collateral, C will default with positive probability, with an arbitrarily large social cost as $\overline{w}$ grows without bound. If λ=0, then by posting collateral C can costlessly insure A and B against default; by standard arguments such insurance is always socially preferred. By continuity, such insurance is also preferred so long as it is sufficiently cheap (i.e., λ is sufficiently small). ■

We now consider the case where $0 < r^C < 1$ so that C is potentially reliable, but not always so. Then C's performance can always be insured by requiring collateral as described above, but this requirement carries with it an expected social cost of $r^C \lambda \ell^{Cu}$, relative to full information.

An alternative to collateral is for one of C's potential counterparties (either A or B but not both) to monitor. In general monitoring will be socially preferable to collateral (i.e., provide cheaper performance on the part of C) as long as the cost of monitoring M is outweighed by the benefits of posting less collateral.

Relative to a world without monitoring, there are two potential complications. The first is the non-verifiability of monitored information. In other words the designated monitor must have an incentive to truthfully reveal the outcome of the monitoring process to the social planner. From condition (32) above, this requires that the type C's must pay a "fee" of h units of the numeraire when they are discovered to be reliable, where h is implicitly defined as

$$w\,(\alpha + L + h) = w(\alpha + L) + M/r^C \qquad (46)$$

The second complication is that the cost of monitoring M must somehow be offset (in expectation), so that monitoring is individually rational. From (31), however, it follows that A or B is willing to bear this cost for the prospect of receiving the fee h when the monitored party C turns out to be reliable.

The solution to the planner's problem in the case of monitoring thus has a particularly simple form. If the type C's are found to be reliable, the solution is the same as under full enforcement, except that each type C must pay the fee h to his monitor. If the type C's are found to be unreliable, the solution is the same as in Proposition 1. Formally we have:

Proposition 2 *Suppose that demand is not always balanced ($f < 1$), that C is sometimes but not always reliable ($0 < r^C < 1$), and that panics are not possible ($n = 1$). Then for $\lambda, M > 0$ sufficiently small and $\overline{w}$ sufficiently large,* *the solution to the planner's problem is given by (18)-(21) and*

$$\ell^{A,B} = 0 \tag{47}$$

$$\ell^{C} = \ell^{Cm} = \chi\left\{D^C = D_L\right\} \max\left\{0, \frac{\alpha - T}{1-\lambda}\right\} \tag{48}$$

$$c_\ell^A = c_\ell^{A*} + \mu^A \chi\left\{D^C = D_H\right\} h \tag{49}$$

$$c_\ell^B = c_\ell^{B*} + \mu^B \chi\left\{D^C = D_H\right\} h \tag{50}$$

$$c_\ell^C = c_\ell^{C*} - \lambda \ell^{Cm} - \chi\left\{D^C = D_H\right\} h \tag{51}$$

$$\mu^A \mu^B = 0 \tag{52}$$

$$\mu^A + \mu^B = 1 \tag{53}$$

$$T = 1 - \alpha \tag{54}$$

We now consider optimal allocations in panics. Recall that all agents are known to be unreliable when a panic occurs. Accordingly trade can only proceed if collateral is posted. In the case of balanced demand, agents' incentive constraints are given by (25). These cannot hold simultaneously in the absence of collateral. To see this, consider a candidate allocation in which no collateral is posted and none of the numeraire good changes hands, i.e., $\ell^i = 0$ and $c^i = L$ for all i. Clearly this allocation violates the incentive constraint (25) for all agents. And, unless collateral is posted, there is no way the planner can redistribute the numeraire good across agents so that all incentive constraints can simultaneously hold.

What is the minimum amount of collateral needed? We can sum the balanced-demand incentive constraints (25) to obtain

$$\sum_i c_\ell^i \geq 3\left(L + \alpha\right) - \sum_i \ell^i \tag{55}$$

Substituting (55) into the resource constraint for the numeraire good (15) yields an expression for the total (per capita) amount of collateral that will have to be posted

$$\sum_i \ell^i \geq \frac{3\alpha}{1-\lambda} \tag{56}$$

Invoking symmetry, we obtain

$$\ell^i \geq \frac{\alpha}{1-\lambda} \tag{57}$$

Mimicking our approach for the normal state, we now calculate the minimum level of collateral necessary for C to completely insure A and B against default, without violating individual rationality for A and B. When demand is balanced, this is given by (57) at equality. When demand is not balanced, then $s^{-1}(C)$ requires compensation equal to the reservation value of $s^{-1}(C)$'s eponymous good, plus the cost of posting sufficient collateral to satisfy (57), i.e., a transfer

R of numeraire good given by

$$R = \alpha + \lambda \left(\frac{\alpha}{1-\lambda} \right) = \left(\frac{1+\lambda}{1-\lambda} \right) \alpha \tag{58}$$

Not all of this need come from C; in fact $s(C)$ can contribute up to

$$S = \frac{1-\alpha-\lambda}{1-\lambda} \tag{59}$$

without violating his incentive or individual rationality constraints (as long as $\alpha > 1/2$). Accordingly, C's collateral posting for the panic state must satisfy

$$\ell^C \geq R - \left(\frac{1-\alpha-\lambda}{1-\lambda} \right) \tag{60}$$

but (60) is redundant given (57) for λ close to zero. We can now state the solution to the planner's problem in a more general case:

Proposition 3 *Suppose that demand is not always balanced ($f < 1$), that C is sometimes but not always reliable ($0 < r^C < 1$), that panics are possible ($n < 1$), and that $\alpha > 1/2$. Then for $\lambda, M > 0$ sufficiently small and $\overline{w}$ sufficiently large, when a panic does not occur, the solution to the planner's problem is given in Proposition 2; when a panic occurs, the solution is given by (18)-(21) and*

$$\begin{aligned}
\ell^{A,B,C} &= \ell^p = \frac{\alpha}{1-\lambda} && (61)\\
c_\ell^A &= L - ((1-E) + EF)\, S + E(1-F)R - \lambda \ell^p && (62)\\
c_\ell^B &= L - (E + (1-E)F)\, S + (1-E)(1-F)R - \lambda \ell^p && (63)\\
c_\ell^C &= L - \lambda \ell^p + (1+F)\, S - (1-F)R && (64)
\end{aligned}$$

where R and S are given by (58) and (59) respectively.

Proof. (Sketch) The proposed panic-state solution maximizes C's utility subject to the constraint that no agent ever defaults. For $\overline{w}$ large, this outcome is socially desirable if $\lambda = 0$, and by continuity, for λ close to zero. ■

5 Implementation with private payment systems

In this section we consider whether certain arrangements (games) can implement the optimal allocations described above. We consider three sorts of arrangements which differ according to the structure of their "payment systems." A *payment system* is taken as a set of rules that determines how obligations arising during the trading stage may be extinguished. In the initial period, the agents jointly decide whether to join the payment system; we will not model this participation game in detail.

In the first two arrangements, there is no possibility of monitoring: an agent i's strategy is given by the vector $\sigma^i = (\ell^i, b^i, t^i, d^i)$, where ℓ^i denotes i's collateral posting, b^i denotes i's decision to enter in a period 1 trade to buy an eponymous good of type $s(i)$ in return for a pledged delivery of a certain amount of the numeraire good, t^i denotes i's period 1 decision to trade his eponymous good to an $s^{-1}(i)$ type agent for a similar pledge, and d^i indicates an agent's period 2 decision to either settle or default on the obligations that arise during the trading stage. We restrict our analysis to symmetric equilibria in pure strategies, in which both trade and settlement occur, and in which the prices of the eponymous goods are symmetric across markets. The price of each eponymous good i, in terms of the numeraire good, must clear the market for that good in the standard Walrasian sense, and is denoted P^i.

5.1 Arrangement 1: trade with a collateral facility

In the first of the three arrangements, there is no "payment system" in the traditional sense, but the collateral facility is available to facilitate settlement. That is, each trade is settled independent of all other trades, by the transfer of an appropriate amount of the numeraire good. Agents can post collateral as a way of committing to settle their obligations.[11] If a default occurs, the defaulter's collateral is seized by "the legal system" and transferred to the creditor of the defaulter.

A potentially unreliable agent cannot be trusted to settle a trading obligation, without having posted collateral. Suppose demand is balanced, and that an agent C has delivered his eponymous good to a type A or B in return for a promised payment of $P^C \in [\alpha, 1]$, and has likewise taken delivery of an eponymous good from another agent, for a promised payment $P^{s(C)}$. Then in order for C to always settle in period 2, the following condition must be satisfied

$$1 + P^C - P^{s(C)} + L - \lambda \ell^C \geq 1 + P^C + L - \ell^C \tag{65}$$

which in words says that for the type C agent,

$$\begin{aligned} &(\text{consumption of supplier's good}) + (\text{payment from demander}) \\ &-(\text{payment to supplier}) + (\text{endowment of numeraire good}) \\ &-(\text{loss from posting collateral}) \\ &\geq (\text{consumption of supplier's good}) + (\text{payment from demander}) \\ &+(\text{numeraire endowment}) - (\text{collateral lost in default}) \end{aligned} \tag{66}$$

When prices of eponymous goods are symmetric so that $P^C = P^{s(C)} = P$, this is equivalent to

$$\ell^C \geq \frac{P}{1-\lambda} \tag{67}$$

[11] We are assuming that the collateral facility can keep track of trading postions so that no one can trade for more than one unit of an eponymous good.

When demand is not balanced ($F = 0$), the same condition can be shown to apply. Sufficient collateral must be posted so that condition (67) will hold for all market-clearing prices $P \in [\alpha, 1]$, i.e., C must post sufficient collateral so that

$$\ell^C \geq \ell^{C1} = \frac{1}{1-\lambda} \tag{68}$$

The potentially unreliable agent C must also have an incentive to deliver his eponymous good when it is demanded by another agent (i.e., when demand is balanced). This requires that

$$1 + P^C - P^{s(C)} + L - \lambda \ell^C \geq 1 + \alpha + L - \ell^C \tag{69}$$

which under symmetric prices is equivalent to

$$\ell^C \geq \ell^{C2} = \frac{\alpha}{1-\lambda} \tag{70}$$

Condition (70) is implied by the previous condition (68). Equilibrium allocations for arrangement 1 can now be described:

Proposition 4 *Suppose that C is not always reliable ($r^C < 1$), and that panics are not possible ($n = 1$). Then under arrangement 1, for $\lambda > 0$ sufficiently small and $\overline{w}$ sufficiently large, equilibrium allocations are given by (18)-(21) and*

$$\begin{aligned} \ell^{A,B} &= 0 && (71)\\ \ell^C &= \ell^{C1} = \frac{1}{1-\lambda} && (72)\\ c_\ell^A &= c_\ell^{A**} = L + E(1-F)P && (73)\\ c_\ell^B &= c_\ell^{B**} = L + (1-E)(1-F)P && (74)\\ c_\ell^C &= c_\ell^{C**} = L - \lambda \ell^{C1} - (1-F)P && (75)\\ P &\in [\alpha, 1] && (76) \end{aligned}$$

5.2 Arrangement 2: multilateral net settlement

The next arrangement we consider is "multilateral net settlement" among all agents. After trading, settlement occurs in two stages. In the first stage (at the end of period 1), agents' net positions are calculated vis-a-vis all other agents. Each agents' original obligation is then replaced by his net obligation. In the second stage (in period 2), agents having positive net obligations may discharge these by transfer of their numeraire good to the payment system; these transfers are then distributed to net creditors.[12]

The crucial difference between this arrangement and the arrangement 1 is that under balanced demand, settlement is automatic in symmetric equilibrium,

[12] What we have in mind is a "strong" form of net settlement where substitution of the net obligation for the original (gross) obligation is legally binding. In other words, a default at the second stage does not affect the recasting of obligations that took place at the first stage. In securities industry parlance this is sometimes known as "netting by novation."

so that the "settlement" constraint (68) is no longer applicable for that case. That is, under net settlement rules, an agent who is in a zero net position after period 1 has no opportunity for default. When demand is not balanced, however, then (68) continues to apply–an agent in a net debit position after trading must still have an incentive to settle. In addition, the "delivery" incentive constraint (70) still applies–agents must have an incentive to trade, no matter what the rules are concerning settlement. In short, under multilateral net settlement the following results are immediate:

Proposition 5 *Suppose that demand is not always balanced ($f < 1$), that C is not always reliable ($r^C < 1$), and that panics are not possible ($n = 1$). Then for $\lambda > 0$ sufficiently small and $\overline{w}$ sufficiently large, equilibrium allocations of arrangement 2 are the same as those under arrangement 1. If however, demand is always balanced ($f = 1$), the unique equilibrium allocation of arrangement 2 is given by (18)-(21) and*

$$c^A_{s(A)} = c^B_{s(B)} = c^C_{s(C)} = 1 \tag{77}$$

$$\ell^{A,B} = 0 \tag{78}$$

$$\ell^C = \ell^{C2} = \frac{\alpha}{1-\lambda} \tag{79}$$

$$c^A_\ell = c^B_\ell = L \tag{80}$$

$$c^C_\ell = L - \lambda \ell^{C2} \tag{81}$$

Corollary 6 *When demand is always balanced ($f = 1$), the equilibrium allocation under multilateral net settlement dominates the equilibrium allocation in the absence of a payment system.*

Corollary 7 *The first-best allocation of Proposition 1 is not attainable as an equilibrium of arrangement 1 (trading without a payment system) or of arrangement 2 (multilateral net settlement).*

Corollary 8 *The equilibrium allocations of both arrangement 1 (no payment system) and arrangement 2 (multilateral net settlement) converge (almost surely) to the first-best allocation of Proposition 1 as $\alpha \uparrow 1$.*

Uncoordinated (symmetric) outcomes are inefficient under either arrangements 1 or 2, essentially because these trading arrangements provide no opportunity for agents to commit to trade their eponymous goods at their reservation value; nor do A and B have an incentive to "donate" surplus to the unreliable type C's so as to lower C's collateral requirement. However net settlement is always preferred for the special case where demand is always balanced. Also both equilibrium allocations approach efficiency as the reservation value of the eponymous goods more closely matches their market value.

5.3 Arrangement 3: hierarchical net settlement

If C is potentially reliable ($0 < r^C < 1$), there can be payoffs to monitoring (less resources tied up in collateral) when monitoring is sufficiently inexpensive.

The value of any information obtained via monitoring is limited by its non-verifiability, however. A type-C agent may agree to be monitored by a type-A or type-B agent, but when the orientation of demand is uncertain ($0 < e < 1$), such information may be useless ex post. Monitoring by both types of potential counterparties is clearly duplicative and cannot be efficient. We therefore consider the third type of arrangement, hierarchical net settlement.

Under this arrangement, trading and settlement are identical to arrangement 2, except that type C's are excluded from directly settling with other agents. Accordingly, a type A or B must serve as the "settlement agent" for each agent of type C. Becoming a settlement agent means that the settlement agent must monitor C and make a report to other payment system participants on C's reliability. The monitor must also honor C's obligations as if they were his own. Settlement consists of an instantaneous sequence of net settlements: first, between C and C's settlement agent, then among A and B. The burden of any default by C is thus borne by C's monitor; in addition to the loss of numeraire good, the payment system may impose a nonpecuniary penalty X on monitors when their "customers" default.

C may either post collateral or agree to be monitored first. If C is monitored and found to be unreliable, ensuring settlement in all cases requires that C post sufficient collateral such that (68) holds. If C is found to be reliable, C's monitor may charge C a fee h' to offset the costs of monitoring (as was the case with planner's problem analyzed above). Truth-telling conditions analogous to (28) and (29), and an individual rationality condition analogous to (30) allow us to derive an implicit definition of h' (analogous to (46) in the planner's problem)

$$E_{E,F}w\left(c_*^A + h'\right) = E_{E,F}w\left(c_*^A\right) + M/r^C \tag{82}$$

where $E_{E,F}$ denotes expectation with respect to the random variables E and F, and c_*^A denotes the sum of the monitor's consumption in an allocation without monitoring, such as described in Proposition 4.[13] We can now state:

Proposition 9 *Suppose that demand is not always balanced ($f < 1$), that C is sometimes but not always reliable ($0 < r^C < 1$), and that panics are not possible ($n = 1$). Then for $\lambda, M > 0$ sufficiently small and $\overline{w}$ sufficiently large,*

[13] As in the planner's problem, (82) can be stated in somewhat simplified form because we are analyzing equilibria in which the monitor's equilibrium consumption does not fall below his autarky level.

equilibrium allocations of arrangement 3 are given by (18)-(21) and

$$\begin{aligned}
\ell^{A,B} &= 0 & (83)\\
\ell^{C} &= \ell^{C1} = \left(\frac{1}{1-\lambda}\right) & (84)\\
c_\ell^{A} &= c_\ell^{A**} + \mu^{A}\chi\{D^{C} = D_H\}h' & (85)\\
c_\ell^{B} &= c_\ell^{B**} + \mu^{B}\chi\{D^{C} = D_H\}h' & (86)\\
c_\ell^{C} &= c_\ell^{C**} + \chi\{D^{C} = D_H\}\left(\lambda\ell^{C1} - h'\right) & (87)\\
\mu^{A}\mu^{B} &= 0 & (88)\\
\mu^{A} + \mu^{B} &= 1 & (89)
\end{aligned}$$

Corollary 10 *The first-best allocation of Proposition 2 is not attainable as an equilibrium of arrangement 3 (hierarchical net settlement), but equilibrium allocations of arrangement 3 approach the first-best allocation (almost surely) as* $\alpha \uparrow 1$.

Corollary 11 *Under the hypotheses of the proposition, arrangement 3 (hierarchical net settlement) dominates both arrangement 1 (no payment system) and arrangement 2 (multilateral net settlement).*

Proof. (Sketch) The proposition and corollary 11 follow immediately from the discussion above. Corollary 10 follows from Proposition 3 and the observation that $\ell^{C3} > \ell^{Cm}$ when C is found to be unreliable. ■

Finally we consider the impact of panics on equilibrium allocations. In a panic all parties are unreliable and must post collateral satisfying at least the delivery incentive condition (70). In the absence of a payment system, additional collateral would be required at the settlement stage, i.e., each agent would post collateral sufficient to satisfy (68). Under net settlement (arrangements 2 or 3), this more stringent collateral position would only apply to agents who would normally be expected to end period 1 in a net debit position, i.e. agent C. Accordingly, in a panic state, equilibrium allocations of all three arrangements would be given as in the following:

Proposition 12 *Suppose that panics are possible* ($n < 1$)*. Then when a panic occurs, equilibrium allocations under arrangement 1 is given by (18)-(21) and*

$$\begin{aligned}
\ell^{A,B,C} &= \ell^{C1} = \frac{1}{1-\lambda} & (90)\\
c_\ell^{A} &= c_\ell^{A**} - \lambda\ell^{A} & (91)\\
c_\ell^{B} &= c_\ell^{B**} - \lambda\ell^{B} & (92)\\
c_\ell^{C} &= L - \lambda\ell^{C1} - (1-F)\,P & (93)\\
P &\in [\alpha, 1] & (94)
\end{aligned}$$

Under arrangements 2 or 3, identical allocations obtain except that

$$\ell^{A,B} = \frac{\alpha}{1-\lambda} \qquad (95)$$

when demand is not always balanced ($f < 1$); when demand is always balanced ($f = 1$), collateral requirements for all types are given by

$$\ell^{A,B,C} = \frac{\alpha}{1-\lambda} \tag{96}$$

Corollary 13 *The first-best allocation of Proposition 3 is not attainable as an equilibrium of any of the arrangements; however equilibrium allocations of arrangement 3 approach the first-best (almost surely) as $\alpha \uparrow 1$.*

Corollary 14 *When a panic occurs, any equilibrium allocation of arrangement 1 is dominated by some equilibrium allocation of arrangements 2 or 3.*

In summary, relative to a world without a payment system, multilateral and hierarchical net settlement may be seen as successively better approximations to first-best outcomes. Netting by itself can economize on collateral as long as demand is balanced (Corollary 6). Imposing a hierarchy on net settlement generates further gains, by restricting the use of collateral to bad credits (Corollary 11). However netting is less effective in a panic, when collateral is necessary to generate incentives to trade (Proposition 12).

6 Settling on the books of a central bank

We now analyze a trading game where settlement occurs on the books of the central bank. Central bank liabilities have value because a fourth type of agent, *taxpayers*, must exchange their endowment of the numeraire good for central bank liabilities in order to pay taxes to the government.

Under this new arrangement (arrangement 4), period 1 trades no longer consist of exchanges of private promises for period 2 deliveries of the numeraire good, in return for period 1 deliveries of an eponymous good. Instead, transactions are "spot" transactions of eponymous goods in return for a credit entry on the supplier's central bank account.[14] All transactions in central bank claims are final. Since agents of type A, B, and C are not endowed with central bank claims in period 0, they can only obtain these by borrowing them from the central bank in period 1, in the form of "overdrafts" on their account, i.e., credits against promised delivery of the numeraire good in period 2. The central bank may or may not require collateral when it grants these overdrafts. By assumption the central bank cannot monitor.

In this expanded environment, we will wish to consider allocations in which default can occur. If the central bank grants uncollateralized credit to an agent and the agent defaults, the default must be covered by the taxpayers, whose losses are incorporated in the social planner's objective. Taxpayers are assumed to be endowed with sufficient numeraire to cover any default. Central bank

[14]Implicit in our analysis is the reasonable notion that a public can provide DVP (or at least a close approximation to it) more cheaply than a private system could.

claims that left outstanding after the settlement stage (i.e., any net debit positions that are in default) carry a social cost of $\pi > 0$ per claim outstanding, which is borne by the taxpayers.

As a benchmark we consider symmetric pure-strategies equilibria of a trading game, characterized by the following payment system: suppose that all agents always have central bank accounts, as well as access to zero-interest uncollateralized overdraft credit during the trading period. Since there is no doubt concerning the value of central bank claims, trade always occurs in such an environment. The only uncertainty is whether agents will default. The likelihood of default depends on two factors, the first being the extent to which all trades are offsetting, and the second being the value of agents' default shocks D. If all trades are offsetting then the value of the default shocks is irrelevant because there is nothing for an agent to default on: since all payment is in the form of credits and debits on the books of the central bank, offsetting book entries are in effect subject to a form of net settlement. If all transactions do not net out to zero, then an agent will default if and only if he experiences a low default-cost shock (given the structure of demand shocks this can only happen for agent C).

The resulting equilibria can be characterized by the following conditions (in addition to (18)-(21))

$$\ell^{A,B,C} = 0 \quad (97)$$

$$c_\ell^A = c_\ell^{A**} \quad (98)$$

$$c_\ell^B = c_\ell^{B**} \quad (99)$$

$$c_\ell^C = L - \chi\left\{D^C = D_H\right\}(1-F)P \quad (100)$$

$$P \in [\alpha, 1] \quad (101)$$

The desirability of this arrangement vis-a-vis the private arrangement depends critically on the structure of the demand shocks:

Proposition 15 *As $f \uparrow 1$, arrangement 4 is preferred to the private arrangements 1, 2, and 3.*

Proof. Under arrangement 4, the expected social loss (due to potential defaults by C on overdrafts) is given by

$$\pi\left[n\left(1-r^C\right)+(1-n)\right](1-f)\,P \quad (102)$$

since a default by C occurs on a promised payment P when either (1) there is no panic, C is unreliable, and demand is not balanced, as occurs with probability $n(1-r^C)(1-f)$; or (2) if a panic occurs and demand is not balanced, as occurs with probability $(1-n)(1-f)$. These probabilities become arbitrarily small for f sufficiently close to 1. Hence as f approaches 1, (102) is less than the cost of collateral or monitoring necessary to sustain trade under a private arrangement. ■

Corollary 16 *As $f \uparrow 1$ and $\alpha \uparrow 1$, equilibrium allocations of arrangement 4 converge (almost surely) to the full-enforcement allocation.*

Proposition 15 and Corollary 16 demonstrate the appeal of central-bank operated payment systems. That is, settling on the books of a central bank can at least partially overcome problems of limited enforcement, without resorting to costly measures such as collateral or monitoring. Relative to wholly uncoordinated payments (i.e., arrangement 1 above) a central-bank operated payment system (arrangement 4) offers two channels for improvement. The first is the de facto netting of offsetting payments, while the second is the substitution of the government's claims on the taxpayers for private claims. If demand is always balanced, these two channels work in perfect conjunction and the problem of limited enforcement vanishes.

The downside of this arrangement is, of course, that demand may not always be balanced and taxpayer losses may result. For unfavorable parameter values (π too large, n, f, or r^Ctoo small) the expected cost to the taxpayers, i.e., (102), may be large enough to wipe out the social gains of central bank settlement. Clearly, the central bank could always limit its exposure by only granting credit against collateral. Always requiring collateral would eliminate the possibility of transfer from the taxpayers but also wipes out the potential welfare gains of settling on the books of the central bank.

An alternative means of limiting the central bank's exposure is to restrict access to central bank settlement. Consider a hierarchical variation on arrangement 4, in which only A and B enjoy access to central bank settlement with uncollateralized daylight overdraft privileges. Agents of type C do not enjoy access to central bank settlement services but instead must settle through an agent of type A or B. During normal times, equilibrium allocations under this new arrangement (arrangement 5) will be given by Proposition 9. C will be monitored by his settlement agent, and, if found to be unreliable, will be required to post collateral (if found to be reliable, he pays a fee to his settlement agent).

During panics, neither A nor B is required to post collateral. But C's settlement agent will still require C to post collateral before agreeing to settle C's transactions. To guarantee settlement under all demand structures and market prices, the more stringent collateral requirement (68) is necessary. In summary, we have

Proposition 17 *Suppose that $f < 1$. Under arrangement 5, during normal times equilibrium allocations are as in Proposition 9. During panics they are given by (18)-(21), (91)-(93), and*

$$\ell^{A,B} = 0 \tag{103}$$

$$\ell^C = \frac{1}{1-\lambda} \tag{104}$$

Arrangement 5 thus falls in between a private hierarchical arrangement (arrangement 3) and universal access to central bank settlement (arrangement 4). Relative to the private arrangement it offers the advantage of requiring less collateral during panics. Relative to arrangement 4, it reduces central bank exposure, but imposes collateral costs on parties lacking central bank access.

Evidently, arrangement 4 will be socially preferred to arrangement 5 as f approaches unity.

7 Policy implications

Above we have seen that a central-bank operated arrangement may offer both potential advantages and disadvantages as compared to private payment arrangements. If the pattern of payments is predicable enough, or if all potential payment system participants are reliable enough, then settling on the books of a central bank can provide efficiency gains by economizing on the use of costly collateral or monitoring. On the other hand, too widespread access to central bank settlement services can have reduce the efficiency gains from private-sector monitoring of risky counterparties. Most modern large-value payment systems, which typically incorporate both public and private elements, may be seen as attempts to refine this tradeoff. Below we describe some features of these systems, and use the model laid out above to evaluate their potential for efficiency gains.

A feature shared by all large-value payment systems is a certain degree of hierarchy. Particularly for systems that settle on the books of a central bank, membership is typically restricted to banks or bank-like institutions. Proposition 17 spells out a potential rationale for this restriction, namely that such a hierarchy allows monitoring to substitute for collateral and/or taxpayer guarantees. But this rationale becomes less compelling as average payments volume (i.e., the parameter f in our model) rises for non-members (i.e., agents of type C).

Another avenue for limiting central bank credit exposure is for central banks to only grant intraday credit against collateral. This policy has been adopted by a number of prominent large-value systems, including Target (Euro-area) and BOJ-Net (Japan).[15] As mentioned above, the downside of such a policy is the cost of posting collateral. In some cases (particularly for Target), central banks have tried to minimize collateral costs by adopting liberal policies concerning the types of assets of collateral that may be posted, and generous valuations or "haircuts" associated with various types of assets. Fedwire, the large-value payment system operated by the Federal Reserve, has adopted a less conservative approach, allowing some participants access to free intraday credit (up to certain limits) but requiring riskier institutions to post collateral.[16]

Our analysis suggests that such collateral requirements may not always eliminate a central bank's exposure to credit risk. In the event of a widespread shock to participants' net worth (a "panic" in the model), the amount of collateral

[15] See Bank of Japan (1997) for information on collateralized intraday credit over BOJ-Net. The corresponding information for Target is available on the European Central Bank's website, www.ecb.int.

[16] A full description of the Federal Reserve's policies on Payment System Risk is available online at www.federalreserve.gov/paymentsystems/psr/. See also Coleman (2002) for a history of the Federal Reserve's policies in this area.

required to completely eradicate the central bank's exposure could well be excessive. Consequently in such situations there will be a temptation for the central bank to suspend the requirement and to provide many, if not all participants un- or undercollateralized credit, on the likely chance that the resulting exposure will be largely eliminated through netting. Indeed it could be argued that central banks' ability and willingness (at least ex post) to serve as "intraday lender of last resort" provides the most compelling rationale for central banks' provision of settlement services.

The continued existence of privately operated, large-value payment systems might suggest that this last advantage is not decisive. But private systems are to some extent dependent on liquidity provided by a central bank. The clearest case of this can be found in CHAPS, the U.K.'s large-value payment system, which is privately operated but which settles on the books of the Bank of England. In this system, liquidity is provided by intraday repos (essentially, collateralized intraday loans) with the Bank of England.[17] Canada's Large Value Transfer System (LVTS) is another privately operated large value system. It operates as a multilateral net settlement system, but has been provided an explicit guarantee against settlement failures by the Bank of Canada (see Dingle 1998). By contrast, CHIPS (the large-value payment system operated by the New York Clearing House Association), enjoys no explicit guarantee against settlement failures. But the current design of CHIPS allows for payments not settled through normal procedures to be transferred to Fedwire at the end of the day (Intraday Liquidity Management Task Force 2000).

The importance of a central-bank operated "backup" can be illustrated through an elaboration of our model of multilateral net settlement (arrangement 2 above). Suppose that agents in arrangement 2 have, in addition to their access to the multilateral net settlement system, access to a central-bank operated payment system. Agents can settle their period 1 transactions either through netting and delivery of the numeraire good, or alternatively by drawing on their account at the central bank.

If no collateral were required under this new arrangement and *if* the central bank were to operate as arrangement 4 above (with zero-interest, uncollateralized daylight overdrafts), then the new arrangement would have same equilibrium allocations as arrangement 4. Under balanced demand, settlement obligations would completely net out at the end of period 1. If demand were not balanced and all agents were reliable, settlement would be completed in the second stage. Finally, an agent unable to settle in the second stage could draw upon his credit line at the central bank to effect settlement. The advantage of this "backup" settlement stage is that payments finality under this new arrangement would be virtually identical to that under a central-bank operated system; agents trading with a participant in this new arrangement would be guaranteed payment without resorting to costly credit or monitoring. However the disadvantage would be the same as under arrangement 4, which is that credit risk

[17] See Buckle and Campbell (2002) for a model of the CHAPS system that incorporates the Bank of England's intraday liquidity provision.

would be concentrated at the central bank.

In practice, private large-value payment systems do not enjoy the sort of "carte blanche" backup that is provided by the arrangement described above. Membership in these systems is restricted; collateral is required; and access to central bank credit may be subject to the sorts of restrictions described above. Nonetheless we would argue that the presence of backup liquidity through the central bank affords the members of private payment systems assurances that promised payments will be made, without posting the quantity of collateral that would be necessary to provide such guarantees in a purely private context.

8 Summary

The chief purpose of large-value payment systems is to efficiently provide assurances that payment obligations will be discharged. In a world where payment system participants are not always trustworthy (and if they were, there would be little need for elaborate payment systems), a number of devices can be employed to provide such assurance. These include the posting of collateral, monitoring, netting, and the substitution of public-sector obligations for private ones. Above we have investigated the tradeoffs that arise in the application of these various means for ensuring that pledges will be honored.

Our analysis assumes that settlement can always be assured by the posting of sufficient collateral. As posting collateral entails costs, a key issue in payment system design is the extent to which the use of collateral may be avoided. Netting of settlement obligations can economize on collateral but is not a panacea, since the efficacy of netting depends on the extent to which incentives are in place for agents to engage in offsetting trades.

Another means of economizing on collateral is for potentially risky agents to be monitored by more reliable agents. In a decentralized context this can be accomplished if the payment system operates in a hierarchical fashion, whereby some agents are excluded from directly settling their own obligations, but must instead settle through a monitor. Incentive constraints on the monitors imply that monitored agents either end up either paying fees to their monitor, or posting collateral, for the privilege of being able to settle their obligations through the payment system.

We also investigate the extent to which settlement in public-sector obligations, i.e., settlement on the books of a central bank, may enhance the reliability of settlement without the use of costly collateral. Central-bank operated settlement systems offer two noteworthy advantages over private systems. The first of these is that the absolute finality of payment in central bank funds allows agents to engage in DVP or "pure spot" transactions. The second is that settlement in central bank funds is robust, in the sense that agents will always have an incentive to engage in DVP transactions, even in states where the reliability of all private credits may be suspect.

These advantages must be weighed against the associated cost, i.e., increased exposure of the central bank and ultimately the taxpayers to credit risk. This cost may be partly offset by the netting implicit in central bank settlement, or by the judicious application of collateral requirements. Too stringent collateral requirements, however, particularly during times of duress, can undo the potential benefit provided by central bank settlement.

In conclusion, the analysis above has provided an outline of some of the issues involved in the design of large-value payment systems, and of some of the key differences between public and private systems. Ongoing technological change and financial deregulation suggest that payment systems will continue to confront these issues in one form or another. Additional investigation of the topics addressed above may prove useful in delineating the tradeoffs posed by these developments.

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