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Old, Frail, and Uninsured: Accounting for Puzzles in the U.S. Long-Term Care Insurance Market

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Abstract: Half of U.S. 50-year-olds will experience a nursing home stay before they die, and a sizable fraction will incur out-of-pocket expenses in excess of $200,000. Surprisingly, only about 10 percent of individuals over age 62 have private long-term care insurance (LTCI), and many applicants are denied coverage by insurers because they are frail. This paper proposes an equilibrium optimal contracting framework that features demand-side frictions due to Medicaid and supply-side frictions due to adverse selection, market power and administrative costs of paying claims. We find that low LTCI take-up rates and rejections among poor individuals are due to Medicaid. Supply-side frictions, however, are responsible for rejections among frail affluent individuals, and both types of frictions matter for those in the middle class.

JEL classification: D82, D91, E62, G22, H30, I13

Key words: long-term care insurance, Medicaid, adverse selection, insurance rejections

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1 Introduction

Nursing home expense risk in the U.S. is significant. About half of U.S. 50-year-olds will experience a nursing home (NH) stay before they die, with a sizeable fraction staying three or more years and incurring out-of-pocket expenses in excess of $200,000. Medicaid, the only form of public insurance for long-term NH care, is only available to individuals with low assets and either low income or impoverishing medical expenses. Given the extent of NH risk and the limited coverage of public insurance, one would expect that many individuals would participate in the private long-term care insurance (LTCI) market. However, the U.S. market is very small: only about 10% of individuals over age 62 have private LTCI. It also has a number of other puzzling features. Many applicants for LTCI are rejected by insurers via medical underwriting because they are deemed to be frail and too costly to insure. Representative policies for those offered insurance only provide partial coverage against long-term care (LTC) risk and charge premia well in excess of the actuarially-fair levels. Yet, profits in this market are low.[1]

Results presented here indicate that demand- and supply-side frictions play significant and distinct roles in accounting for these puzzling features of the U.S. LTCI market. Using an equilibrium optimal contracting model we find that Medicaid crowds out the demand for private LTCI among poor individuals because Medicaid is a secondary payer and benefits for those who qualify are free. Supply-side distortions are also important. In our setup the optimal strategy for an insurer who has market power but faces adverse selection and administrative costs is to deny coverage to frail risk groups.[2] These supply-side distortions are particularly important in accounting for low take-up rates among affluent individuals who are unlikely to qualify for Medicaid benefits. Both types of frictions contribute toward producing low LTCI take-up rates among the middle-class.

Our analysis is the first that shows how to derive optimal contracts in the presence of adverse selection that are empirically relevant. In particular, we show how to generate contracts that exhibit denials of coverage to risk groups with high levels of frailty, partial coverage and high premia to risk groups who are offered insurance, and correlations of LTCI ownership and nursing home entry that are consistent with the data. Our results are novel because the previous literature has concluded that the conventional theory of adverse selection described in Rotschild and Stiglitz (1976) is inconsistent with the pattern of LTCI coverage in the U.S. Conventional theory predicts that an insurer's optimal menu to a group of individuals with the same observable characteristics always includes a full insurance contract. The full insurance contract is chosen by those with the highest risk exposure, whereas, those with a lower risk exposure choose partial or no insurance. In contrast, in the LTCI market, all individuals in frail risk groups are denied coverage. A second, related problem is that, if one properly controls for the information set of the insurer, standard theory predicts that LTCI take-up rates should be higher among NH entrants as compared to non-entrants. In the data, however, LTCI take-up rates among those who enter a NH

[1] Sources for these facts and figures can be found in Section 2.1.

[2] We refer to a group of individuals who is identical from the perspective of the insurer as a risk group. In the insurance literature it is common to use the term risk or insurance pool instead. To avoid confusion we only use the term pooling when referring to the properties of optimal contacts.

are either the same as or even lower than LTCI take-up rates among non-entrants. These counterfactual implications of the theory have led previous researchers to abstract from modeling the supply-side of the market, exogenously specifying insurance contracts instead, and/or to posit multiple sources of private information.

How do market power and administrative costs generate rejections (denials of coverage) and partial insurance coverage in the presence of adverse selection? Individuals in our model have private information about whether they face a good (low) or a bad (high) risk of NH entry. When the issuer has market power, proportionate administrative costs of paying claims make both types more costly to insure, resulting in a reduction in their coverage. Moreover, they increase the cost of insuring bad types more than good types and the decline in coverage of bad types is proportionately larger. As Chade and Schlee (2016) illustrate, if these proportional costs are large enough, they may result in optimal contracts that pool bad and good types together. Pooling occurs because even though bad types are more costly to insure, they cannot receive less coverage without violating incentive compatibility. If the costs are large enough, no profitable non-zero pooling contract exists and rejection occurs.

To produce rejections among risk groups with high frailty in our model, we assume that the distribution of private information is more polarized in high frailty groups. This mechanism is consistent with Hendren (2013) who provides empirical evidence that adverse selection is more severe in risk groups that are more likely to be rejected by insurers. It is also consistent with empirical evidence we provide in this paper that shows that the dispersion of self-reported nursing home entry probabilities increases with our index of frailty.

Medicaid can also produce rejections and partial coverage because it lowers the demand for private LTCI. First, by providing individuals with a guaranteed minimum consumption floor in the NH state, Medicaid reduces the extent of NH expense risk faced by individuals. Second, Medicaid is a secondary payer. This means that, for individuals who meet the means test, private insurance benefits reduce Medicaid benefits one-for-one. The presence of Medicaid reduces the set of profitable contracts that poorer individuals, in particular, are willing to take. If no profitable contracts exist then no trade is possible and rejection occurs. Medicaid also reduces the extent of coverage offered by traded contracts when individuals face uncertainty about their resources at the time of NH entry. This effect, which is novel in the literature, arises for individuals who are partially insured against the NH shock in that they will be eligible for Medicaid in some (low-resource) states. Since the LTCI contract is not contingent on the realization of individual resources at NH entry, these individuals prefer only partial private LTCI coverage.

We assess the empirical relevance of these factors in a detailed quantitative model of the market. Our model has a rich cross-sectional structure, in particular, individuals vary by income, wealth and frailty which are correlated with NH entry risk and observable to the insurer. The insurer offers each risk group a menu of contracts that maximize his profits subject to participation and incentive compatibility constraints. The model is calibrated to match cross-sectional variation in frailty, wealth, survival risk, NH entry risk, and LTCI take-up rates using data from the Health and Retirement Survey (HRS). To construct a frailty index for HRS respondents, we adapt a methodology from the Gerontology literature

[3]For papers in this literature see, for example, Dapp et al. (2014), Ng et al. (2014), Rockwood and Mitnitski (2007), Searle et al. (2008) and the references therein.

such that our index summarizes underwriting criteria used in the LTCI industry.[3] Lifetime NH risk of HRS respondents is estimated using an auxiliary model along the lines of Hurd et al. (2013). To assess the baseline calibration, we compare various moments generated from the model that were not calibration targets to their data counterparts. In particular, we demonstrate that the calibrated baseline economy replicates the variation in self-assessed NH entry risk by frailty, the distribution of insurance use across NH entrants, and conditional and unconditional correlations between entry and LTCI take-up rates. Finally, our main results about the distinct roles of demand- and supply-side frictions are derived by comparing the baseline to alternative economies in which one or more frictions are absent.

In addition to our main results, our analysis provides new insights into three strands of the literature. We model the optimal contract design problem of the insurer whereas the common practice in the LTCI literature is to instead posit exogenous insurance contracts.[4] To illustrate why this distinction matters consider Brown and Finkelstein (2008) who find that Medicaid has a large crowding-out effect on private LTCI under the assumption that contacts are exogenous. Specifically, they find that, due to Medicaid, only individuals in the top one-third of the wealth distribution would be willing to purchase LTCI even if the contract was actuarially-fair and provided full coverage against LTC risk. We find that the crowding out effect of Medicaid is much smaller. More than 60% of individuals purchase private LTCI in a version of our economy that features Medicaid but no private information or administrative costs and is thus fairly similar to the setup of Brown and Finkelstein (2008).[5] The reason why LTCI take-up rates are so much higher in our model is because it is optimal for the insurer to offer partial coverage to most individuals when Medicaid is present. These individuals qualify for Medicaid NH benefits in some states of nature and consequently prefer a contract that offers partial coverage.

Our analysis also provides a resolution to what Ameriks et al. (2016) refer to as the "LTCI puzzle." They find that 66% of respondents in the Vanguard Research Initiative survey have a positive demand for an actuarially-fair state-contingent insurance product that pays out when individuals require assistance with activities of daily living (ADLI). However, only 22% of their sample own LTCI. According to our model the reason for the low LTCI take-up rates among this wealthier population is that for many of them the gains from trade are exhausted by administrative costs and private information frictions on the supply-side.

Finally, our analysis provides a bridge between the theoretical optimal contracting literature and the empirical literature on adverse selection. Previous research on optimal contracting has found that adverse selection models with a single source of private information generally exhibit a positive correlation between risk exposure and insurance coverage. Hellwig (2010) derives this result in a principal agent setting, Stiglitz (1977) and Chade and Schlee (2012) derive it in a setting with a single monopolistic insurer, and Lester et al. (2015) show that it obtains in a framework that admits varying degrees of market power. Based on this theoretical finding, Chiappori and Salanie (2000) propose testing for adverse selection in an insurance market by estimating the correlation between insurance coverage and insurance claims controlling for the information set of the insurer. A significantly positive correlation

[4]Some recent examples of research that assumes exogenous contracts include Ameriks et al. (2016), Ko (2016), Lockwood (2016), and Mommaerts (2015).

[5]One difference is that in our model the insurer has market power and insurance premia are not actuarially-fair.

is evidence in favor of adverse selection. The most relevant empirical paper for the LTCI market, Finkelstein and McGarry (2006), provides evidence that individuals have private information about their NH risk. However, when they implement the correlation test, they fail to find a positive correlation between LTCI ownership and NH entry despite their best effort to control for the information set of insurers. Similar findings have been documented in other insurance markets. Chiappori and Salanie (2000), for instance, document a negative correlation between the level of insurance and claims in the French auto insurance market and Fang et al. (2008) find that holders of Medigap insurance spend less on medical care as compared to non-holders. These empirical findings have led researchers to conclude that individuals have multiple sources of private information and to construct new adverse selection models with this feature.[6,7]

To the best of our knowledge, this is the first paper to demonstrate that a quantitative optimal contracting model with a single source of private information can reproduce the empirical finding that the correlation between insurance ownership and loss occurrence is small and possibly even negative. The quantitative model implies that insurance ownership is uncorrelated with loss occurrence in most risk groups because either both private types in the risk group have LTCI or neither do. In fact, only 0.2% of individuals are offered optimal menus that are informative about the presence of adverse selection.[8] Consequently, it is difficult to detect adverse selection in finite samples of data or in other words the correlation test has low power. Moreover, as we demonstrate, if the econometrician does not perfectly control for the insurer's information set, a negative correlation between LTCI ownership and average NH entry across risk groups can dominate the small positive correlation within risk groups.

A number of other papers have investigated the role of demand-side factors in accounting for low LTCI take-up rates. Koijen et al. (2016) estimate the demand for LTC and life insurance in an asset pricing framework and find large discrepancies from efficient risk sharing. Lockwood (2016) argues that bequest motives imply that many individuals would prefer to self-insure against NH risk by saving and as compared to purchasing LTCI. Davidoff (2010) argues that home equity, one common way of saving, may be a poor substitute for LTCI. Barczyk and Kredler (2016) find that the demand for NH care is very elastic due to the availability of informal care options in a model with intra-family bargaining. Mommaerts (2015) and Ko (2016) develop non-cooperative models of formal and informal care. Mommaerts (2015) finds that informal care reduces the demand for LTCI and that this effect is most pronounced among more affluent individuals but, retirees still have a substantial residual demand for LTCI. One potential source of variation in private NH entry probabilities in our model is differences in the availability of informal care. This interpretation is supported by Ko (2016) who finds that private information about access to informal care by family members is an important source of adverse selection in the LTCI market.

[6] For examples of adverse selection models where individuals have multiple sources of private information see Einav et al. (2010) and Guerrieri and Shimer (2015).

[7] However, results in Chiappori et al. (2006) and Fang and Wu (2016) suggest that it is also challenging to produce a negative correlation between insurance coverage and risk exposure in models with multiple sources of private information.

[8] The optimal menus that are informative consist of a positive contract that the bad type accepts and an offer of no insurance that the good type prefers.

The remainder of the paper proceeds as follows. Section 2 provides a set of facts that motivate our analysis. In Section 3 we present a qualitative analysis of our main economic mechanisms using a simplified model. The quantitative model is presented in Section 4. Section 5 describes how we calibrate the quantitative model and assess the baseline calibration. Section 6 contains our results and our concluding remarks are in Section 7.

2 Motivation

Our research is motivated by a number of puzzling features of the U.S. LTCI market. In this section, we describe these features in detail.

2.1 Market Size

The risk of a costly long-term NH stay, which we define as a stay that exceeds 100 days, is large and public health insurance coverage of such stays is limited. Using HRS data and an auxiliary simulation model, we estimate the the lifetime probability of a long-term NH stay is 30%.[9] On average, those who experience a long-term NH stay spend about 3 years in a NH. According to the U.S. Department of Health, NH costs averaged $205 per day in a semi-private room and $229 per day in a private room in 2010. The two main public insurers are Medicare and Medicaid. Medicare, which provides universal coverage of short-term rehabilitative NH stays, partially covers up to 100 days of NH care. Medicaid provides a safety net for those who experience high LTC expenses but it is both income and asset-tested. Consequently, Medicaid is only an option for individuals who either have low wealth and retirement income (categorically needy) or who have already exhausted their personal resources to pay for high medical expenses (medically needy). As a result, many individuals face a significant risk of experiencing large LTC expenses near the end of their life. For example, a NH stay of three years can result in out-of-pocket expenses that exceed $200,000. Indeed, Kopecky and Koreshkova (2014) find that the risk of large OOP NH expenses is the primary driver of wealth accumulation during retirement.

Given the extent of NH risk in the U.S., one would expect that the market for private LTCI would be large. It is consequently surprising that only 10% of individuals 62 and older in our HRS sample have private LTCI. Moreover, private LTCI benefits account for only 4% of aggregate NH expenses while the share of out-of-pocket payments is 37%.[10]

2.2 Rejections

Many applications for LTCI are rejected. Murtaugh et al. (1995) in one of the earliest analyses of LTCI underwriting estimates that 12–23% of 65 year olds, if they applied, would be rejected by insurers because of poor health. Their estimates are based on the National Mortality Followback Survey. Since their analysis, underwriting standards in the LTCI

[9] In comparison, using HRS data and a similar simulation model, Hurd et al. (2013) estimate that the lifetime probability of having any NH stay for a 50 year old ranges between 53% and 59%.

[10] Medicare and Medicaid account for 18% and 37%, respectively. This breakdown is for 2003 and is from the Federal Interagency Forum on Aging-Related Statistics.

Table 1: Percentage of HRS respondents who would answer "Yes" to at least one LTCI prescreening question.

	Age		
	55–56	60–61	65–66
All	40.5	43.7	49.6
Top Half of Wealth Distribution Only	31.1	33.6	39.1

Data source: Authors' calculations using our HRS sample.

market have become more strict. We estimate rejection rates of as high as 36% for 55–65 year olds by applying underwriting guidelines from Genworth and Mutual of Omaha to a sample of HRS individuals.[11]

To understand how we arrive at this figure, it is helpful to explain how LTCI underwriting works. Underwriting occurs in two stages. In the first stage, individuals are queried about their prior LTC events, pre-existing health conditions, current physical and mental capabilities, and lifestyle. Some common questions include: Do you require human assistance to perform any of your activities of daily living? Are you currently receiving home health care or have you recently been in a NH? Have you ever been diagnosed with or consulted a medical professional for the following: a long list of diseases that includes diabetes, memory loss, cancer, mental illness, heart disease? Do you currently use or need any of the following: wheelchair, walker, cane, oxygen? Do you currently receive disability benefits, social security disability benefits, or Medicaid?[12] A positive answer to any one of these questions is sufficient for the insurers to reject applicants before they have even submitted a formal application. Many of these same questions are asked to HRS participants. As Table 1 shows, the fraction of individuals in our HRS sample who would respond affirmatively to at least one question is large even for the youngest age group and for the top half of the wealth distribution. Question 3 received the highest frequency of positive responses. If we are conservative and omit question 3 the prescreening declination rate ranges from 18–24%.

If applicants pass the first stage, they are invited to make a formal application. Medical records and blood and urine samples are collected and the applicants cognitive skills are tested. One in five formal applications are denied coverage.[13] Assuming a 20% rejection rate at each round, the resulting overall rejection rate is roughly 36% for 55–66 years old in our HRS sample.

2.3 Pattern of LTCI take-up rates

It is clear from the prescreening questions above that one of the objectives of LTCI underwriting is to screen out individuals with poor health and low wealth. If this screening is

[11]We subsequently refer to this sample of individuals as our HRS sample and details on our sample section criteria are reported in the Appendix.

[12]Source: 2010 Report on the Actuarial Marketing and Legal Analyses of the Class Program

[13]Source: American Association for Long-Term Care Insurance

Table 2: LTCI take-up rates by wealth and frailty

Frailty Quintile	Wealth Quintile 1–3	4	5
1	0.071	0.147	0.233
2	0.065	0.158	0.205
3	0.049	0.131	0.200
4	0.037	0.113	0.157
5	0.025	0.107	0.104

For frailty (rows), quintile 5 has the highest frailty and, for wealth (columns), quintile 5 has the highest wealth. We only report the average of wealth quintiles 1–3 because take-up rates are very low for these individuals. The wealth quintiles reported here are marginal and not conditional on the frailty quintile, so for example only around 7% of the people in the lowest frailty quintile are in the bottom wealth quintile, while 33% are in the top wealth quintile. Data source: 62–72 year olds in our HRS sample.

successful it will have a depressing effect on both the size and the composition of the market. LTCI take-up rates are likely to be particularly low among those who have low wealth and/or poor health. Medicaid also affects the composition of the market. This program is likely to have the strongest impact on the LTCI take-up rates of the poor because they are most likely to qualify for Medicaid NH benefits. It is consequently interesting to document how LTCI take-up rates vary by wealth and health status in our HRS sample.

To obtain a measure of observable health status for HRS respondents, we summarize all the measures of health collected by LTC insurers and observable in the HRS in a single frailty index. Following the recommendations in the Gerontology literature, all of the measures included in the index are equally weighted. Self-reported health and NH risk are not included since these variables are not observable by insurers.[14]

Table 2 reports LTCI take-up rates by frailty and wealth quintiles for 62–72 year olds in our HRS sample. We focus on the 62–72 age group because it covers the ages when most individuals purchase LTCI and yields a sample size big enough to have a meaningful number of individuals in each combination of wealth and frailty quintile. The average LTCI take-up rate for this group is 9.4%.[15] The table shows that there is substantial variation in LTCI take-up rates across wealth and frailty quintiles. The take-up rate of individuals in the top wealth quintile and lowest frailty quintile is more than 4 times higher than that of individuals in the lowest three wealth quintiles and highest frailty quintile. Notice that within each frailty quintile, take-up rates increase with wealth. Medicaid, since it has a larger effect on the poor, likely plays an important role in accounting for this pattern. Also notice that, within each wealth quintile, take-up rates decline with frailty. Insurance rejections of high risk individuals are likely an important driver of this pattern, especially for those in wealth quintile 5 who are least affected by Medicaid.

[14] Details on the construction of the frailty index, including a list of included variables, is in the Appendix.

[15] Note that this number cannot be arrived at by summing across rows or columns of Table 2 because there are not equal numbers of individuals in each node.

2.4 Pricing and coverage

Those who successfully navigate the LTCI underwriting process face high premia for insurance policies that only provide partial coverage against LTC risk. Brown and Finkelstein (2007) estimate individual loads and comprehensiveness for common LTCI products. They find that individual loads, which are defined as one minus the expected present value of benefits relative to the expected present value of premia paid, range from 0.18 to 0.51 depending on whether or not adjustments are made for lapses. In other words, LTCI policies may be twice as expensive as actuarially fair insurance. These loads are high relative to loads in other insurance markets. For instance, Karaca-Mandic et al. (2011) estimate that loads in the group medical insurance market range from 0.15 for firms with 100 employees to 0.04 for firms with more than 10,000 employees.

About two-thirds of LTCI policies bought in the year 2000 paid a maximum daily benefit that was fixed in nominal terms over the life of the contract. These policies only covered a fraction of the expected lifetime NH costs. Brown and Finkelstein (2007) estimate that a "representative" LTCI policy only covered about 34% of expected lifetime costs. They also consider how loads vary with comprehensiveness and conclude that loads do not rise systematically with the comprehensiveness of the policy.

Brown and Finkelstein (2011) provide more recent estimates of personal loads and comprehensiveness using data from the year 2010. In 2010 average loads were higher: 0.32 without lapses and 0.50 with lapses. However, coverage was better. A representative policy covered about 66% of expected lifetime LTC costs. The main reason for the improvement in coverage is that later policies included a benefit escalation clause. In addition, the maximum daily benefits tended to be higher and the exemption period tended to be lower as compared to policies issued in 2000.

Even though personal loads increased between 2000 and 2010, sales declined, concentration increased and profits fell. New sales of LTCI in 2009 were below 1990 levels and, according to Thau et al. (2014), over 66% of all new policies issued in 2013 were written by the largest three companies. Still, insurers are experiencing losses on their LTCI product lines.[16] This is, in part, because the cost of administering claims is high. Individuals receiving LTCI benefits need to be monitored to verify that their health status has not changed. In addition, insurers are required to hold a substantial amount of reserves because investment returns, lapse rates and morbidity vary significantly over the life of policies.[17]

2.5 Private information

The observations described above are consistent with the hypothesis that individuals have private information about their NH risk exposure, that this information is correlated with frailty and wealth, and that LTC insurers use this information to reject high risk groups. In further support of the hypothesis, Finkelstein and McGarry (2006) find direct evidence of

[16] The top three insurers are Genworth, Northwest and Mutual of Omaha. For information about losses on this business line see, e.g., *The Insurance Journal*, February 15, 2016, `http://www.insurancejournal.com/news /national/2016/02/15/398645.htm` or Pennsylvania Insurance Department MUTA-130415826.

[17] We discuss administrative costs in more detail in Section 5.4.

private information in the LTCI market. Specifically, they find that individuals' self-assessed NH entry risk is positively correlated with both actual NH entry and LTCI ownership even after controlling for characteristics observable by insurers. Hendren (2013) shows that Finkelstein's and McGarry's findings are driven by individuals in high risk groups. Specifically, he finds that self-assessed NH entry risk is only predictive of a NH event for individuals who would likely be rejected by insurers. Hendren's measure of a NH event is independent of the length of stay. Since we focus on stays that are at least 100 days, we repeat the logit analysis of Hendren (2013) using our definition of a NH stay and our HRS sample. We get qualitatively similar results. We find evidence of private information at the 10 year horizon (but not at the 6 year) in a sample of individuals who would likely be rejected by insurers. However, for a sample of individuals who would likely not be rejected we are unable to find evidence of private information.[18]

Interestingly, even though Finkelstein and McGarry (2006) find evidence of private information in the LTCI market, they fail to find evidence that the market is adversely selected using the positive correlation test proposed by Chiappori and Salanie (2000). When they do not control for the insurer's information set, they find that the correlation between LTCI ownership and NH entry is negative and significant. Individuals who purchase LTCI are less likely to enter a NH as compared to those who did not purchase LTCI. When they include controls for the insurer's information set, they also find a negative although no longer statistically significant correlation. Finally, when they use a restricted sample of individuals who are in the highest wealth and income quartile and are unlikely to be rejected by insurers due to poor health they find a statistically significant negative correlation.

3 A Simple Model with Adverse Selection

In this section we establish formal conditions under which one can account for some of the main qualitative features of the LTCI market described above. The arguments in this section are developed using a simplified one-period version of the baseline model that nests the standard adverse selection setup as a special case. This simplified setup allows us to graphically illustrate some of the most important economic mechanisms driving our quantitative results. First, we demonstrate that supply-side frictions, namely, private information, market power and variable administrative costs, can generate less than full insurance, coverage denials, high loads on individuals, and low profits. We then illustrate how demand-side frictions, namely, reduced demand due to the presence of Medicaid, can independently generate these same qualitative features. Finally, we discuss which parameters are important for producing partial insurance and rejections when both supply-side and demand-side frictions are present.

To start, suppose the economy consists of a continuum of individuals and a single monopolistic issuer of private LTCI. The assumption of a single issuer is a parsimonious way to capture the concentration we documented above in this market. Monopoly power also plays a central role in breaking the standard result that individuals with the highest private risk exposure receive full insurance in adversely selected markets.[19] Each individual has a

[18] See the Appendix for more details.

[19] Chade and Schlee (2016) compare and contrast perfect competition with monopoly and explain why

type $i \in \{g, b\}$. They each receive endowment ω but face the risk of entering a NH and incurring costs m. The probability that an individual with type i enters a NH is θ^i. A fraction $\psi \in (0, 1)$ of individuals are good risks who face a low probability $\theta^g \in (0, 1)$ of a NH stay. The remaining $1 - \psi$ individuals are bad risks whose NH entry risk is $\theta^b >> \theta^g$. Let η denote the fraction of individuals who enter a NH then $\eta \equiv \psi\theta^g + (1-\psi)\theta^b$. Each individual observes his true NH risk exposure but the insurer only knows the structure of uncertainty. A menu consists of a pair of contracts (π^i, ι^i), one for each private type $i \in \{g, b\}$. Each contract consists of a premium π^i that the individual pays to the issuer and an indemnity ι^i that the issuer pays to the individual if he incurs NH costs m.

Denote consumption of an individual with risk type i as c^i_{NH} in the NH state and c^i_o otherwise. An individual's utility function is

$$\begin{aligned} U(\theta^i, \pi^i, \iota^i) =& \theta^i u(\omega - \pi^i - m + \iota^i) + (1 - \theta^i)u(\omega - \pi^i), \\ =& \theta^i u(c^i_{NH}) + (1 - \theta^i)u(c^i_o), \end{aligned} \tag{1}$$

and the associated marginal rate of substitution between premium and indemnity is

$$\frac{\partial \pi}{\partial \iota}(\theta^i) = -\frac{U_\iota(\cdot)}{U_\pi(\cdot)} = \frac{\theta^i u'(c^i_{NH})}{\theta^i u'(c^i_{NH}) + (1 - \theta^i)u'(c^i_o)} \equiv MRS(\theta^i, \pi^i, \iota^i). \tag{2}$$

Assume that the utility function has the property that $MRS(\theta^i, \pi^i, \iota^i)$ is strictly increasing in $\theta^i, i \in \{g, b\}$. Under this assumption, which is referred to as the single crossing property, any menu of contracts that satisfies incentive compatibility will be such that if $\theta^{i'} > \theta^i$ then $\pi^{i'} \geq \pi^i$ and $\iota^{i'} \geq \iota^i$.

The optimal menu of contracts for individuals maximizes the insurer's profits subject to participation and incentive compatibility constraints and can be found by solving

$$\max_{\pi^i, \iota^i} \psi[\pi^g - \theta^g(\lambda\iota^g)] + (1 - \psi)[\pi^b - \theta^b(\lambda\iota^b)] \tag{3}$$

subject to

$$(PC_i) \quad U(\theta^i, \pi^i, \iota^i) - U(\theta^i, 0, 0) \geq 0, \quad i \in \{g, b\}, \tag{4}$$

$$(IC_i) \quad U(\theta^i, \pi^i, \iota^i) - U(\theta^i, \pi^j, \iota^j) \geq 0, \quad i, j \in \{g, b\}, \ i \neq j, \tag{5}$$

where $\lambda \geq 1$ reflects variable administrative costs incurred by the issuer when paying claims.

We now review two classic properties of contracts under adverse selection that are standard in the literature.[20] The first property is that the equilibrium contract is always a separating one with a binding participation constraint for the good types and a binding incentive compatibility constraint for the bad types. When Equation (4) binds for the good types and Equation (5) binds for the bad types the optimal menu also satisfies the two

monopoly power is needed to produce partial coverage at the top, pooling and empirically relevant patterns of rejections under adverse selection. An open question that we do not pursue here is how much market power is required. Lester et al. (2015) propose a framework that one could, in principal use, to investigate how the optimal contracts vary with the extent of market power.

[20] See, for example, Rotschild and Stiglitz (1976) and Stiglitz (1977).

first-order conditions

$$\psi MRS(\theta^g,\pi^g,\iota^g)+(1-\psi)\left[\frac{U_\pi(\theta^b,\pi^g,\iota^g)}{U_\pi(\theta^b,\pi^b,\iota^b)}MRS(\theta^g,\pi^g,\iota^g)+\frac{U_\iota(\theta^b,\pi^g,\iota^g)}{U_\pi(\theta^b,\pi^b,\iota^b)}\right]=\lambda\psi\theta^g, \quad (6)$$

$$MRS(\theta^b,\pi^b,\iota^b)=\lambda\theta^b. \quad (7)$$

The equilibrium of our model is always separating and characterized by these conditions when there are no variable administrative costs ($\lambda = 1$) and bad types do not know for sure that they will enter a NH ($\theta^b < 1$). The second standard property in the literature is that bad types are always offered full insurance. When $\lambda = 1$ and $\theta^b < 1$ this property also applies to our model. To see this, note that, if $\iota^b = m$ then consumption in the NH state is the same as consumption in the non-NH state. In this case, $MRS(\theta^b,\pi^b,\iota^b) = \theta^b$, which is the optimality condition (7) when $\lambda = 1$. By the same token, it is never optimal to offer full insurance to good types. Insurance of the good type is always incomplete with $\iota^g < m$.

Figure 1a illustrates a typical optimal menu in this case. The good types get the contract at point G_1 and the bad types get the contract at point B_1.[21] Note that pooling contracts cannot be equilibria in this setting because starting from a pooling contact at point G_1, the insurer can always increase total profits by offering the bad types a more comprehensive contract. Separating equilibria where the good types have a $(0, 0)$ contract can occur though. However, the optimal menu will always consist of at least one nonzero contract that offers full insurance and is preferred by bad types. It follows that the standard setup is inconsistent with our motivating observation that U.S. LTCI policies provide only partial coverage. Moreover, the standard setup is inconsistent with denials of coverage because in the model agents are always offered two contracts and one of them is positive. Thus a zero contract is a choice and not a denial in the standard setup. We now describe how to modify the model to make it consistent with these two properties of the U.S. LTCI market.

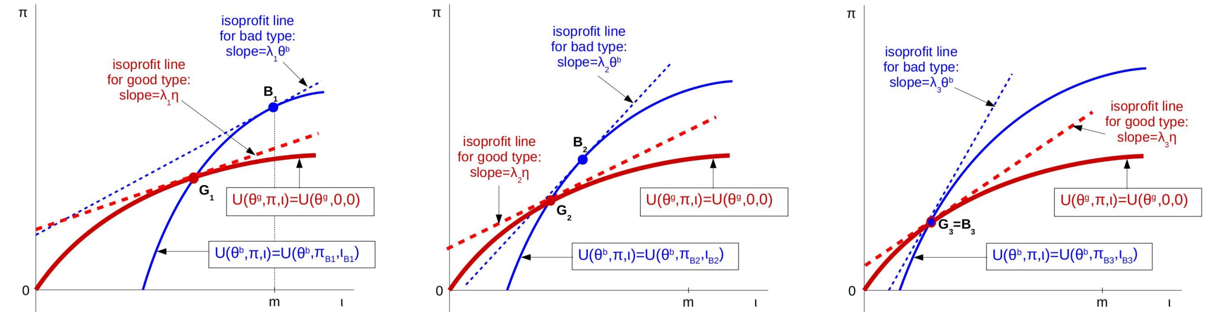
3.1 Optimal Contracts with Variable Administrative Costs

In this section we show that imposing variable administrative costs on the insurer, i.e., setting $\lambda > 1$, can result in equilibria where both types are offered less than full insurance as well as equilibria where neither type is offered a positive contract. These findings and the line of reasoning follows the analysis of Chade and Schlee (2014) and Chade and Schlee (2016) who study the impact of imposing variable costs on a monopolist insurer in the presence of private information and a continuum of types.[22]

To illustrate the impact of variable administrative costs on the optimal menu, consider the impact of slightly increasing λ above 1, i.e., moving from Figure 1a to Figure 1b. Increasing λ increases the slopes of the firm's isoprofit lines. The increased costs of paying out claims are offset by a combination of increased loads on the good types and reduced profits. Indemnities

[21] For simplicity, the good types' contract in the figure is illustrated as the optimal pooling contract, i.e., the contract satisfying $MRS(\theta^g,\pi^g,\iota^g) = \lambda\eta$. Rearranging the first-order conditions, one can show that equation (6) is equivalent to $MRS(\theta^g,\pi^g,\iota^g) = \lambda\left[\frac{\psi\theta^g+(1-\psi)\theta^b A}{\psi+(1-\psi)B}\right]$, where $A \equiv U_\iota(\theta^b,\pi^g,\iota^g)/U_\iota(\theta^b,\pi^b,\iota^b)$ and $B \equiv U_\pi(\theta^b,\pi^g,\iota^g)/U_\pi(\theta^b,\pi^b,\iota^b)$. The figure corresponds to cases where A and B are close to 1.

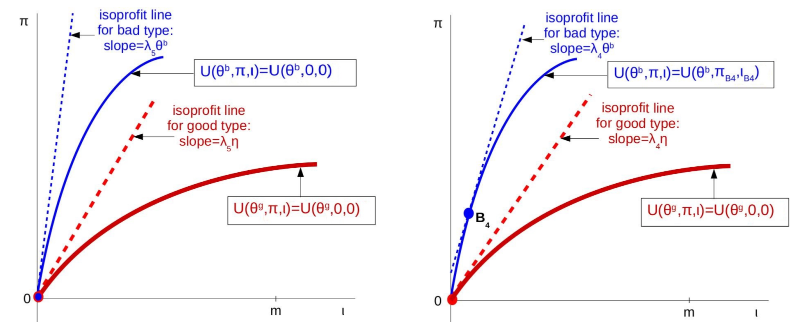
[22] Our findings are also related to previous results by Hendren (2013). In his setting, and in ours, even if $\lambda = 1$, the only contract offered is a $(0, 0)$ pooling contract if ψ is sufficiently small.



(a) Separating equilibrium with $\lambda = 1$

(b) Separating equilibrium with $\lambda > 1$

(c) Pooling equilibrium with $\lambda > 1$



(d) No trade equilibrium with $\lambda > 1$

(e) Only bad types have insurance with $\lambda > 1$

Figure 1: An illustration of the effects of increasing the insurer's proportional overheard costs factor (λ) on the optimal menu. The blue (red) lines are the indifference curves of bad (good) types. The dashed blue lines are isoprofits from contracts for bad types and the red dashed lines are isoprofits from a pooling contract.

and premia of both types fall and the optimal contracts move southwestward along the individuals' indifference curves. Thus if $\lambda > 1$, the property of the standard model that bad types get full insurance no longer holds as both types are now offered contracts where indemnities only partially cover NH costs.

Proposition 1. *If $\lambda > 1$, then the optimal menu features incomplete insurance for both types, i.e., $\iota^i < m$ for $i \in \{b, g\}$.*

Proof. See Appendix. □

Since the marginal costs of paying out claims to the bad type are higher than to the good types, as λ increases, the contracts will also get closer together. Once λ is large enough, the insurer will no longer be able to increase profits by offering a separate contract to the bad types as opposed to offering a single (pooling) contract. Figure 1c depicts such a case where both types get the same nonzero contract. Once a pooling contract occurs, the equilibrium under any larger values of λ will also involve pooling. As λ continues to increase, the pooling contract will move along the good types participation constraint with loads on both types rising and profits gradually falling, until λ is so large that no profitable nonzero pooling

contract exists. Figure 1d illustrates the case where the optimal menu consists of the single pooling contract $(\pi, \iota) = (0, 0)$. We adopt the same terminology as Chade and Schlee (2016) and subsequently refer to this case as either a no-trade equilibrium or a rejection. Proposition 2 provides necessary and sufficient conditions for such equilibria to occur in the presence of positive variable administrative costs.

Proposition 2. *There will be no trade, i.e., the optimal menu will consist of a single (0,0) contract iff*

$$MRS(\theta^b, 0, 0) \leq \lambda\theta^b, \tag{8}$$
$$MRS(\theta^g, 0, 0) \leq \lambda\eta, \tag{9}$$

both hold.

Proof. See Appendix. □

No trade equilibria occur when the amount individuals are willing to pay for even a small positive separating or pooling equilibrium is less than the amount required to provide nonnegative profits to the insurer. Condition (8) rules out profitable separating menus where only bad types have positive insurance, such as the one illustrated in Figure 1e. Condition (9) rules out profitable pooling and separating menus where both types are offered positive insurance.

3.2 Optimal Contracts in the Presence of Medicaid

We have shown that, in the presence of private information, positive variable administrative costs lower profits for the insurer, produce optimal menus in which both types receive less than full insurance, and can generate rejections. We will now show that equilibria with these features can also be generated if there is a means-tested social insurance program in the model that, like the U.S. Medicaid program, guarantees a minimum consumption floor to individuals who incur NH costs. In order to isolate the effects of the Medicaid program on the optimal contracts in this section, we assume that $\theta^b < 1$ and that there are no variable administrative costs ($\lambda = 1$).

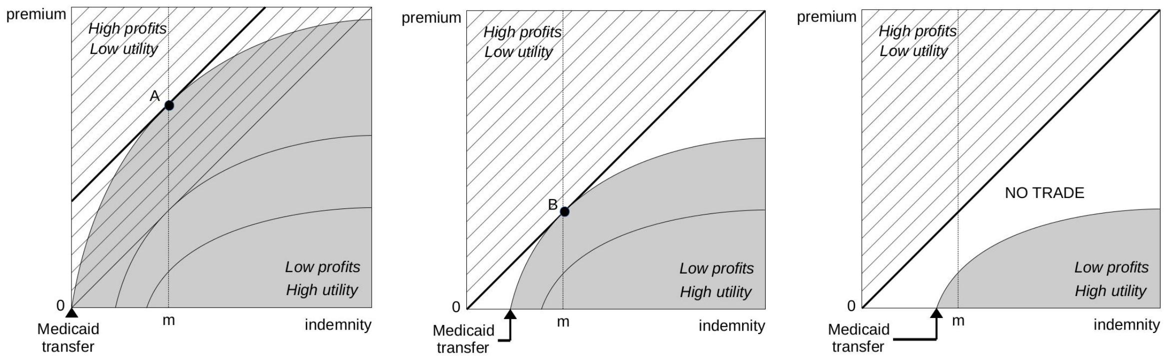
Assume that individuals who experience a NH event receive means-tested Medicaid transfers according to

$$TR(\omega, \pi, \iota) \equiv \max\left\{0, \underline{c}_{NH} - [\omega - \pi - m + \iota]\right\}, \tag{10}$$

where $\underline{c}_{NH}$ is the consumption floor. Then consumption in the NH state is

$$c^i_{NH} = \omega + TR(\omega, \pi^i, \iota^i) - \pi^i - m + \iota^i. \tag{11}$$

By providing NH residents with a guaranteed consumption floor, Medicaid increases utility in the absence of private insurance thus reducing demand for such insurance. Moreover, Medicaid is a secondary payer which means that, when $\underline{c}_{NH} > \omega - \pi - m + \iota$, marginal increases in the amount of the private LTCI indemnity ι are exactly offset by a reduction in Medicaid transfers, so individual utility stays constant at $u(c_{NH}) = u(\underline{c}_{NH})$. Thus, for



(a) Non-binding consumption floor (b) Low consumption floor (c) High consumption floor

Figure 2: An illustration of the effects of Medicaid on the trading space. The straight lines are the insurer's isoprofit lines and the curved lines are the individual's indifference curves.

individuals that meet the means-test, the marginal utility of the insurance indemnity is zero and only private LTCI contracts in which $\iota - \pi$ exceeds $\underline{c}_{NH} + m - \omega$ are potentially attractive.

Suppose that without Medicaid, the optimal contract of one of the types is given by point A in Figure 2a. Figure 2b illustrates the impact of introducing Medicaid with a small value of $\underline{c}_{NH}$. Notice that the optimal indemnity is unchanged. However, the individual's outside option has improved, and to satisfy the participation constraint, the premium is reduced. Because the insurer gives the individual the same coverage at a lower price, his profits decline. As $\underline{c}_{NH}$ increases, an equilibrium, such as the one depicted in Figure 2c, will eventually occur. In this case, $\underline{c}_{NH}$ is so large that the insurer can not give the agent an attractive enough positive contract and still make positive profits. The optimal contract is $(0, 0)$.

Now assume that when individuals are choosing their LTCI contract, they face uncertainty about the size of their endowment. Specifically, assume that ω is distributed with cumulative distribution function $H(\cdot)$ over the bounded interval $\Omega \equiv [\underline{\omega}, \overline{\omega}] \subset \mathbb{R}_+$. An individual's utility function is

$$U(\theta^i, \pi^i, \iota^i) = \int_{\underline{\omega}}^{\overline{\omega}} \Big[\theta^i u(c^i_{NH}(\omega)) + (1 - \theta^i) u(c^i_o(\omega))\Big] dH(\omega), \tag{12}$$

where

$$c^i_o(\omega) = \omega - \pi^i, \tag{13}$$

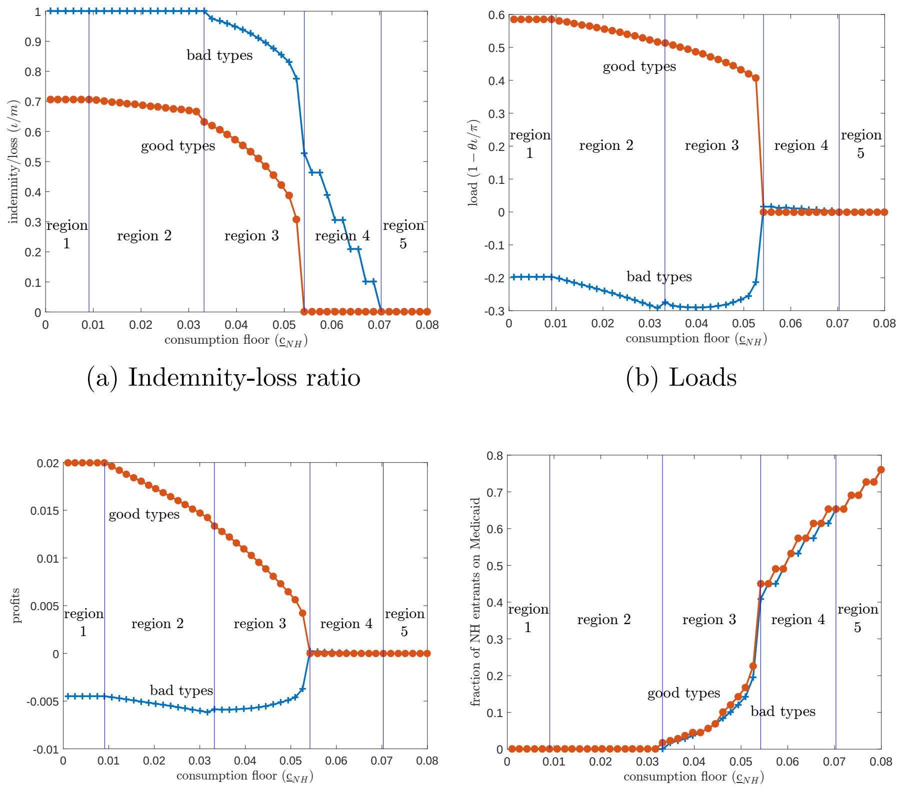
$$c^i_{NH}(\omega) = \omega + TR(\omega, \pi^i, \iota^i) - \pi^i - m + \iota^i, \tag{14}$$

and the Medicaid transfer is defined by (10).

Endowment uncertainty will be used in the baseline model to capture the fact that, in reality, at the time of LTCI purchase, most individuals do not know whether and to what

[23] Another way to capture these facts would be to have more model periods and have individuals face uncertainty about the timing and duration of their NH stay.

extent Medicaid will cover their costs if they have a NH event.[23] We introduce it now because it allows the optimal LTCI contracts to achieve qualitative properties that are consistent with the data. In particular, Medicaid can generate rejections and reduce insurer's profits without this uncertainty, but it cannot produce partial insurance for the bad risk type. When endowment uncertainty is absent, due to Medicaid's secondary payer status, a NH event is either insured by Medicaid or private insurance but never both.[24] Thus, if bad types purchase any private insurance, they will prefer full coverage. With endowment uncertainty, in contrast, an individual may rely on both types of insurance. In the case of a NH event, he may be eligible for Medicaid under only some realizations of the endowment and use private LTCI in the other states. However, he will not want full private LTCI coverage because, due to Medicaid, he is already partially insured against NH risk in expectation.



(a) Indemnity-loss ratio

(b) Loads

(c) Profits

(d) Fraction of NH entrants on Medicaid

Figure 3: Impact of varying the Medicaid consumption floor, $\underline{c}_{NH}$, on the indemnity-loss ratio, loads, profits, and the fraction of NH entrants on Medicaid when the endowment is stochastic.

Figure 3 illustrates how the optimal contracts, profits and Medicaid take-up rates evolve as the Medicaid consumption floor, $\underline{c}_{NH}$, is increased from zero in the setup with endowment uncertainty. The figure is divided into 5 distinct regions. In region 1, the consumption

[24]In equilibrium, if an individual is eligible for Medicaid his LTCI contract must be $(0, 0)$ as any nonzero contract would involve the same consumption in the NH state as a $(0, 0)$ contract but lower consumption in the non-nursing state.

floor is so low that even if an individual has no private LTCI and the smallest realization of the endowment he will not qualify for Medicaid. In this region, Medicaid has no effect on the optimal contracts. In region 2, Medicaid influences the contracts even though, in equilibrium, neither type receives Medicaid transfers. In this region, Medicaid has a similar effect to that illustrated in Figure 2b. For some realizations of the endowment, good types qualify for Medicaid if the contract is $(0, 0)$. This tightens their participation constraint and the contract offered to them has to be improved. A better contract for good types tightens, in turn, the incentive compatibility constraint for bad types. The insurer responds by reducing premiums for both types, and the indemnity of the good types and loads on both types fall. Since Medicaid's presence has resulted in more favorable contracts for individuals, the insurer's profits fall. In region 3, Medicaid has the same effects as in region 2 but now, in addition, both types receive Medicaid benefits in equilibrium for some realizations of ω. As discussed above, the partial insurance of NH shocks via Medicaid results in optimal contracts that feature partial coverage and, in this region, both types have less than full private insurance. Proposition 3 provides a sufficient condition for this to occur.

Proposition 3. *If* $\underline{\omega} < \underline{c}_{NH}$ *then the optimal menu features incomplete insurance for both types, i.e.,* $\iota^i < m$ *for* $i \in \{b, g\}$.

Proof. See Appendix. □

In region 4, the consumption floor is so high that the good types, who's willingness to pay for private LTCI is lower than the bad types, choose to drop out of the private LTCI market. Notice that, even though the average loads are declining as the consumption floor increases, the load on bad types jumps up upon entry into this region. In regions 1–3, the contracts exhibit cross-subsidization with bad types benefiting from negative loads and good types facing positive loads. In region 4, the insurer is able to make a small amount of positive profits by offering a positive contract that is only attractive to the bad types. Finally, in region 5, Medicaid has a similar effect to that depicted in Figure 2c. The consumption floor is so large that there are no terms of trade that generate positive profits from either type. The insurer rejects applicants when the consumption floor is in this region as the optimal menus consist of a single $(0, 0)$ contract.

In the Appendix, we establish that the single-crossing property continues to obtain when Medicaid is present. However, due to the non-convexities Medicaid creates, conditions (8) and (9) in Proposition 2 are no longer sufficient conditions for rejections to occur, and, although still necessary, are not very useful. Proposition 4 provides a stronger set of necessary conditions for rejections in the presence of Medicaid and a stochastic endowment.

Proposition 4. *If the optimal menu is a* $(0, 0)$ *pooling contract then*

$$U(\theta^b, \lambda\theta^b\iota, \iota) < U(\theta^b, 0, 0), \quad \forall \iota \in \mathbb{R}_+, \tag{15}$$

and

$$U(\theta^g, \lambda\eta\iota, \iota) < U(\theta^g, 0, 0), \quad \forall \iota \in \mathbb{R}_+. \tag{16}$$

If condition (15) fails, then one can find a profitable contract that bad types would take,

and, if condition (16) fails, then one can find a profitable pooling contract that good types would take. The conditions are not sufficient because, while they rule out profitable pooling contracts and separating contracts where good types get no insurance, they do not rule out separating contracts where both types get positive insurance. Absent Medicaid, there can never exist a separating contract that increases profits if the optimal pooling contract is $(0, 0)$. However, the non-convexities introduced by Medicaid break this property. As a result, even when the optimal pooling contract generates negative profits, a profitable separating contract might still exist.

Figure 3 highlights some important distinctions between our model, where contracts are optimal choices of an issuer, and previous research by, for instance, Brown and Finkelstein (2008), Mommaerts (2015), and Ko (2016), who model demand-side distortions in the LTCI market but set contracts exogenously. In regions 2 and 3, notice that Medicaid's presence only impacts the pricing and coverage of the optimal private contracts. In these regions, the insurer responds to the reduced demand for private LTCI by adjusting the terms of the contracts but still offers positive insurance. In contrast, in regions 4 and 5, Medicaid's presence also impacts the fraction of individuals who have any private LTCI. Notice that the Medicaid recipiency rates of both types increase as the consumption floor is increased in these regions. This means that, even though good types do not have LTCI in region 4 and no individuals have it in region 5, Medicaid is covering their NH costs only for a subset of the endowment space. For some realizations of ω, they self-insure. Thus, in these regions, Medicaid is crowding-out demand for private LTCI despite providing only incomplete coverage itself. This crowding-out effect is also present in models with exogenous contracts, however, the effects of Medicaid on the terms of positive contracts is not. Thus, allowing the insurer to adjust the contracts in response to the presence of Medicaid is important because, if the terms of the contracts cannot adjust, then the crowding-out effect of Medicaid on the size of the LTCI market will be overstated.

3.3 Varying Rejection Rates across Risk Groups

The analysis so far has considered the problem of an insurer that offers insurance to a single risk group.[25] We now turn to describe how the extent of rejections changes as we vary observable characteristics of individuals. This discussion provides intuition for the results found using the quantitative model which features an environment with a rich structure of public information and thus multiple risk groups. We want the model to account for the observations in Table 2. That table shows, for instance, that LTCI take-up rates are low for those with low wealth. An explanation for this observation is that risk groups with low expected endowments are more likely to be rejected by the insurer due to Medicaid. The following proposition formalizes this claim.

Proposition 5. *When $\overline{\omega} - m \leq \underline{c}_{NH}$, the possibility of rejection in equilibrium increases if the distribution of endowments on $[\underline{\omega}, \overline{\omega}]$ is given by $H_1(\cdot)$ instead of $H(\cdot)$ where $H_1(\cdot)$ is first-order stochastically dominated by $H(\cdot)$.*

Proof. See Appendix. □

[25] Recall that we use the term risk group to refer to a group of individuals that is identical to the insurer.

It immediately follows from Proposition 5 that the possibility of rejections increases if the expected endowment decreases when $\overline{\omega} - m \leq \underline{c}_{NH}$. When $\overline{\omega} - m > \underline{c}_{NH}$, decreasing the expected endowment may also lead to an increased possibility of rejection. However, in this case, it is also possible that the likelihood of rejections goes down since, absent Medicaid, lowering an individual's endowment raises his demand for insurance.

Table 2 also shows that LTCI take-up rates are declining in frailty. Insurers are more likely to reject frail individuals. In the quantitative model, individuals will vary by endowments and frailty, both of which will be observable by the insurer, and the distribution of private information will vary across these observable types. The following proposition shows two ways of varying the distribution of private information with frailty to generate an increasing possibility of rejection.

Proposition 6. *When $\lambda > 1$ and θ^b is sufficiently close to 1, the possibility of rejection in equilibrium increases if:*

1. *θ^b increases;*
2. *θ^b increases and θ^g decreases such that the mean NH entry probability $\eta \equiv \psi\theta^g + (1 - \psi)\theta^b$ does not change.*

Proof. See Appendix. □

Either of the two ways mentioned in the proposition can, in theory, be used to generate the decreasing pattern of LTCI take-up rates with frailty in the table. If, as in way 1, only θ^b increases then both the mean and the dispersion of the NH entry probabilities will increase. However, way 2 states that increasing the dispersion of entry probabilities while holding the mean fixed by varying both θ^b and θ^g can also generate increased rejection rates. In short, to generate an increase in rejection rates with frailty, both ways require an increase in the dispersion in NH entry probabilities with frailty. However, way 1 also requires an increase in the mean. We show in Section 5.3 that the increase in dispersion is consistent with the pattern of NH entry probabilities in the data, while the increase in the mean implied by case 1 is inconsistent. Thus both θ^b and θ^g must vary with frailty to generate patterns of both LTCI take-up rates and NH entry probabilities that are consistent with the data.

Propositions 5 and 6 show that risk groups with low expected endowments and/or high polarization of private information are more likely to be rejected. The propositions also provide a basis for understanding how our model can generate either small positive correlations or even negative correlations between LTCI coverage and NH entry consistent with the findings documented in Finkelstein and McGarry (2006).[26,27] Recall that the measure of LTCI coverage they use to estimate these correlations is whether an individual has or does not have LTCI. Their data does not allow them to ascertain how the size of LTCI policies varies with NH entry. Thus, if information observed by the insurer is perfectly controlled for, the correlation is only identified off risk groups where one private information type purchases

[26] We review their findings in Section 2.5.

[27] Note that when they control for information observable by insurers their estimated negative coefficient is not statistically significant. Thus one cannot rule out the possibility of a zero or even a small positive correlation between LTCI coverage and NH entry.

LTCI and the other type does not. Returning to Figure 3, note that this only occurs in region 4. In all of the other regions, either both types hold LTCI or neither type holds LTCI. If region 4 is small then the correlation between LTCI coverage and observed risk exposure will be nonnegative but small.

The model may also produce a negative correlation if the econometrician only observes a subset the insurer's information set. To see this, suppose that the insurer's information set allows him to assign each individual into one of two risk groups: group 1 and group 2 where the groups are such that $\theta_1^b < \theta_2^b$ but otherwise identical. Clearly, $\eta_1 < \eta_2$, i.e., group 2 has a higher average NH entry rate than group 1. Suppose that the optimal menu for group 1 is nonzero for both types. By Proposition 6, group 2 may be rejected. In this case, all members of group 1 will have LTCI and no members of group 2 will have it, and yet, the average NH entry probability of group 2 is higher than that of group 1. If the econometrician's information set does not allow him to discern individuals in group 1 from individuals in group 2, he will measure a negative correlation between LTCI coverage and NH entry.

In general, finding a negative correlation requires that three things occur. First, the econometrician must have less information than the insurer. Second, LTCI take-up rates need to be declining in η. Corollary 7 states one way this second assumption can occur.[28]

Corollary 7. *Consider two groups of individuals,* 1 *and* 2, *such that NH risk is on average higher in group* 1 *than group* 2 ($\eta_1 > \eta_2$). *If* $\lambda > 1$ *then it is possible that, in equilibrium, group* 1 *is rejected and group* 2 *is not* ($\iota_1^i = 0 < \iota_2^i$, $i \in \{b, g\}$).

Proof. See Appendix. □

Third, the quantitative features of the optimal contracts must be of a certain form. For instance, suppose that risk group 2 is rejected as before but that risk group 1's optimal menu is a separating one with zero insurance for the good types instead. In this scenario, the differential LTCI take-up rates in group 1 acts as a countervailing force and the econometrician may measure a positive relationship between LTCI coverage and NH entry.

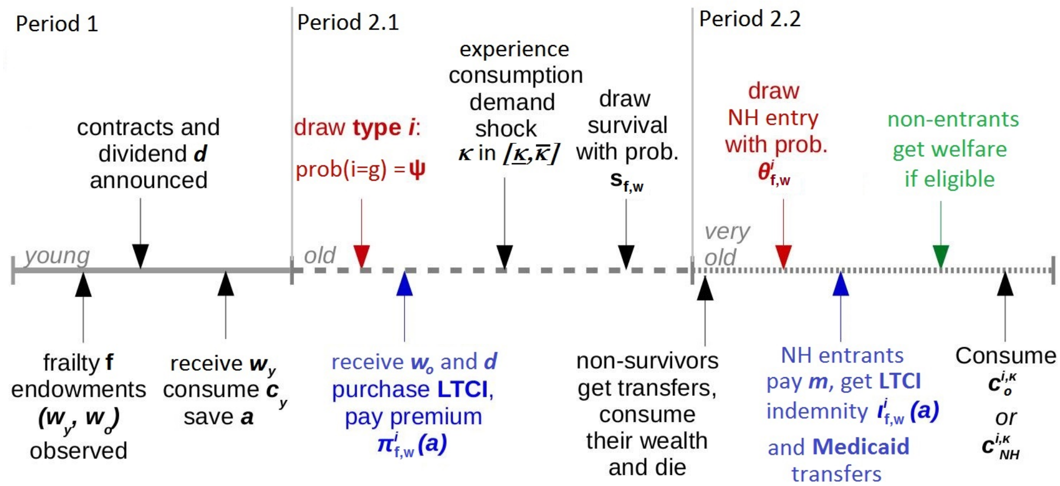
From these examples it is clear that the correlations between LTCI coverage and NH entry in our model have the potential to be either small and positive or even negative. We discuss the properties of the quantitative model in this respect in Section 6.3.

4 Quantitative model

We consider an endowment economy with two periods, where period 2 is divided into two subperiods: period 2.1 and period 2.2. The economy consists of three kinds of actors: a continuum of individuals, a monopolist provider of private LTCI, and a government. We refer to individuals as young in period 1, old in period 2.1, and very old in period 2.2. All individuals become old. In period 1, individuals make a consumption-savings decision. We model this decision because an individual's savings impacts his Medicaid eligibility and offered menu of LTCI contracts. At the beginning of the period 2.1, the insurer issues policies

[28]The corollary uses Proposition 6. In the Appendix we discuss how to get LTCI take-up rates to decline in η using Proposition 5.

Figure 4: Timeline of events in the baseline model.



and pays out profits to old individuals as dividends. Between periods 2.1 and 2.2 a survival shock occurs and some old die. At the beginning of period 2.2, the very old face the risk of experiencing a NH event. The government taxes individuals and uses tax revenue to finance a welfare program for retirees and a Medicaid program for NH residents.

4.1 Individual's problem

Figure 4 lays out the timing of events in the model. At birth an individual draws his frailty status f and lifetime endowment of the consumption good $\mathbf{w} = [w_y, w_o]'$ which are jointly distributed with density $h(f, \mathbf{w})$. Frailty status and endowments are noisy indicators of NH risk. He also observes his probability of surviving from period 2.1 to period 2.2, $s_{f,\mathbf{w}}$, which varies with f and $\mathbf{w}$, his period 2 dividend income from ownership of the firm $d\Pi$ where Π is the insurer's profits, and the menus of LTCI contracts that will be available in period 2.

A young individual receives w_y and then chooses consumption c_y and savings a. At the beginning of period 2, the individual receives w_o and $d\Pi$ and observes his true risk of entering a NH conditional on surviving to period 2.2: $\theta^i_{f,\mathbf{w}}$, $i \in \{g, b\}$ with $\theta^g_{f,\mathbf{w}} < \theta^b_{f,\mathbf{w}}$. The individual's true NH entry risk is private information that is not observable by the government or the insurer and the individual realizes a low (good) NH entry probability, $i = g$ with probability ψ. We assume that NH entry probabilities depend on f and $\mathbf{w}$ but that ψ is independent of them. He then chooses a LTCI contract from the menu offered to him by the private insurer. The insurer conditions the menu of contracts offered to each individual on their frailty status, endowments, and asset. Each menu contains two incentive compatible contracts: one for the good types and one for the bad types. A contract consists of a premium $\pi^i_{f,\mathbf{w}}(a)$ that the individual pays to the insurer and an indemnity $\iota^i_{f,\mathbf{w}}(a)$ that the insurer pays to the individual if the NH event occurs.

After purchasing LTCI, individuals experience a demand shock that induces them to

[29] The demand shock acts to create uncertainty along the lines that we analyzed in the simple model in Section 3.2.

consume a fraction κ of their young endowment where $\kappa \in [\underline{\kappa}, \overline{\kappa}] \subseteq [0,1]$ has density $q(\kappa)$.[29] We use this demand shock to capture the following features of NH events in a parsimonious way. On average, individuals have 18 years of consumption between their date of LTCI purchase and their date of NH entry. However, the timing of a NH event is uncertain and individuals who experience a NH event later in life than others are likely to have consumed a larger fraction of their lifetime endowment beforehand.

Very old individuals may experience a NH event at cost m. To capture the fact that many retirees die without ever entering a NH, we assume that with probability $s_{f,\mathbf{w}}$ the retiree survives to this stage and with probability $1 - s_{f,\mathbf{w}}$ he does not. If he does not survive, we assume that he anticipates his death and consumes all his wealth before dying.[30]

Individuals who experience a NH event may receive benefits from a public means-tested LTCI program (Medicaid). Medicaid is a secondary insurer in that it guarantees a consumption floor of $\underline{c}$ to those who experience a NH shock and have low wealth and low levels of private insurance.

An individual of type $(f, \mathbf{w})$ solves the following maximization problem, where the dependence of choices and contracts on h and w is omitted to conserve notation,

$$U_1(f, \mathbf{w}) = \max_{a \geq 0, c_y, \mathbf{c}_{NH}, \mathbf{c}_o} u(c_y) + \beta U_2(a), \tag{17}$$

with

$$U_2(a) = \left[\psi u_2(a, \theta^g_{f,\mathbf{w}}, \pi^g, \iota^g) + (1-\psi) u_2(a, \theta^b_{f,\mathbf{w}}, \pi^b, \iota^b)\right], \tag{18}$$

and

$$u_2(a, \theta^i, \pi^i, \iota^i) = \int_{\underline{\kappa}}^{\overline{\kappa}} \Big\{ u(\kappa w_y) + \alpha \Big[s_{f,\mathbf{w}} \big(\theta^i u(c^{i,\kappa}_{NH}) + (1-\theta^i) u(c^{i,\kappa}_o)\big) + (1 - s_{f,\mathbf{w}}) u(c^{i,\kappa}_o) \Big] \Big\} q(\kappa) d\kappa, \tag{19}$$

subject to

$$c_y = w_y(1-\tau) - a, \tag{20}$$

$$c^{i,\kappa}_o + \kappa w_y = (1-\tau)(w_o + ra + d\Pi) + a - \pi^i(a), \quad i \in \{g, b\}, \tag{21}$$

$$c^{i,\kappa}_{NH} + \kappa w_y = (1-\tau)(w_o + ra + d\Pi) + a + TR(a, \pi^i(a), \iota^i(a), m, \kappa) - \pi^i(a) - m + \iota^i(a) \tag{22}$$

where α and β are subjective discount factors. The Medicaid transfer is

$$TR(a, \pi, \iota, m, \kappa) = \max\left\{0, \underline{c}_{NH} - \left[(1-\tau)(w_0 + ra + d\Pi) + a - \kappa w_y - \pi - m + \iota\right]\right\}, \tag{23}$$

r denotes the real interest rate and τ is a tax.

[30]There is evidence that individuals anticipate their death. Poterba et al. (2011) have found that most

In the U.S. retirees with low means also receive welfare through programs such as the Supplemental Security Income program. We capture these programs in a simple way. After solving the agent's problem above which assumes that there is only a consumption floor in the NH state, we check whether they would prefer, instead, to save nothing and consume the following consumption floors: $\underline{c}_{NH}$ in the NH state and $\underline{c}_o$ in the non-NH state. If they do, we allow them to do so and assume that they do not purchase LTCI.[31]

4.2 Insurer's problem

The insurer observes each individual's endowments $\mathbf{w}$, frailty status f, and assets a. He does not observe an individual's true NH entry probability, $\theta^i_{f,\mathbf{w}}$, $i \in \{g, b\}$, but knows the distribution of NH risk in the population and the individual's survival risk $s_{f,\mathbf{w}}$. We assume that the insurer does not recognize that asset holdings depend on $\mathbf{w}$ and f via household optimization. We believe that this is realistic because most individuals purchase private LTCI relatively late in life. Note that the demand shock, κ, is realized after LTCI is contracted.

The insurer creates a menu of contracts $\left(\pi^i_{f,\mathbf{w}}(a), \iota^i_{f,\mathbf{w}}(a)\right)$, $i \in \{g, b\}$ for each group of observable types that maximizes expected revenues taking into account that individual's face survival risk after insurance purchase. Following Chade and Schlee (2014) and Chade and Schlee (2016), we assume that the insurer faces two types of administrative costs associated with paying claims: a variable cost λ, that is proportional to the total payout and a fixed cost, k.[32] His maximization problem is

$$\Pi(h, \mathbf{w}, a) = \max_{(\pi^i_{f,\mathbf{w}}(a), \iota^i_{f,\mathbf{w}}(a))_{i\in\{g,b\}}} \psi\Big\{\pi^g_{f,\mathbf{w}}(a) - s_{f,\mathbf{w}}\theta^g_{f,\mathbf{w}}\left[\lambda\iota^g_{f,\mathbf{w}}(a) + kI(\iota^g_{f,\mathbf{w}}(a) > 0)\right]\Big\} \tag{24}$$

$$+ (1-\psi)\Big\{\pi^b_{f,\mathbf{w}}(a) - s_{f,\mathbf{w}}\theta^b_{f,\mathbf{w}}\left[\lambda\iota^b_{f,\mathbf{w}}(a) + kI(\iota^b_{f,\mathbf{w}}(a) > 0)\right]\Big\}$$

subject to

$$(IC_i) \quad u_2(a, \theta^i_{f,\mathbf{w}}, \pi^i_{f,\mathbf{w}}(a), \iota^i_{f,\mathbf{w}}(a)) \geq u_2(a, \theta^i_{f,\mathbf{w}}, \pi^j_{f,\mathbf{w}}(a), \iota^j_{f,\mathbf{w}}(a)), \quad \forall i, j \in \{g, b\}, i \neq j \tag{25}$$

$$(PC_i) \quad u_2(a, \theta^i_{f,\mathbf{w}}, \pi^i_{f,\mathbf{w}}(a), \iota^i_{f,\mathbf{w}}(a)) \geq u_2(a, \theta^i_{f,\mathbf{w}}, 0, 0), \quad \forall i \in \{g, b\} \tag{26}$$

Equation (25) restricts attention to insurance contracts that are incentive compatible and equation (26) requires that insurance contracts deliver at least as much utility to each individual as she can achieve by self-insuring against NH risk. There is no need to impose a non-negativity restriction on profits since, the insurer has the option of offering a contract with a zero indemnity. Let $\tilde{h}(f, \mathbf{w}, a)$ denote the measure of agents with frailty status f,

retirees die with very little wealth and Hendricks (2001) finds that most households receive very small or no inheritances. This assumption eliminates any desire for agents to use LTCI to insure survival risk.

[31] Modelling the Supplemental Security Income program in this way helps us to generate the low levels of savings of individuals in the bottom wealth quintile without introducing additional nonconvexities into the insurer's maximization problem.

[32] We did not analyze fixed administrative costs in Section 3 because they just shift the isoprofit schedules down in a parallel fashion. But, it is clear from the analysis that they can also produce rejections. We discuss the distinction between these two costs in Section 5.4.

endowment $\mathbf{w}$, and asset holdings a. Total profits for the insurer are given by

$$\Pi = \sum_{\mathbf{w}} \sum_{f} \sum_{a} \Pi(f, \mathbf{w}, a)\tilde{h}(f, \mathbf{w}, a). \tag{27}$$

4.3 Government's problem

In period 1, the government collects taxes on individuals' income when young and saves the revenue at rate r. In period 2, it collects taxes on individuals' income when old and finances the two means-tested welfare programs in the economy. Given the two consumption floors, $\{\underline{c}_{NH}, \underline{c}_o\}$, the government sets the tax rate τ to satisfy the government budget constraint

$$REV = \sum_{\mathbf{w}} \sum_{f} TR^{f,\mathbf{w}} h(f, \mathbf{w}), \tag{28}$$

where $TR^{f,\mathbf{w}} h(f, \mathbf{w})$ is aggregate government transfers to individuals of type $(f, \mathbf{w})$ via the two welfare programs and

$$REV = \sum_{\mathbf{w}} \sum_{f} \tau(1+r)\omega_y h(f, \mathbf{w}) + \sum_{\mathbf{w}} \sum_{f} \sum_{a} \tau \left[\omega_o + ra + d\Pi\right] \tilde{h}(f, \mathbf{w}, a),$$

is aggregate government revenue.

4.4 Equilibrium

We solve for a competitive equilibrium under the assumption that the real interest rate is exogenous. The U.S. economy has strong international financial linkages and it is unlikely that changes in LTCI arrangements would have a large effect on U.S. real interest rates. In order to place private and social insurance for longterm care on an equal footing we recognize the costs of financing Medicaid. Medicaid is financed with an income tax and this distorts savings incentives. We thus solve a fixed point problem that insures that the government budget constraint is satisfied, that insurance markets clear, and that total dividend income received by individuals equals total profits generated by the private LTC insurer.

Definition 1. Competitive Equilibrium. Given a distribution of individuals by frailty and endowments $h(f, \mathbf{w})$, a real interest rate r, and consumption floors $\{\underline{c}_{NH}, \underline{c}_o\}$, a competitive equilibrium consists of a set of insurance contracts $\{\pi^i_{f,\mathbf{w}}(a), \iota^i_{f,\mathbf{w}}(a)\}$, $i \in \{g, b\}$; profits Π; a government income tax rate τ; consumption allocations $\{c_y^{f,\mathbf{w}}, c_o^{f,\mathbf{w},i,\kappa}, c_{NH}^{f,\mathbf{w},i,\kappa}\}$, $i \in \{g, b\}$; and savings policy $a^{h,\mathbf{w}}$ such that the consumption allocations and saving policy solve the individuals' problems and the insurance contracts solve the insurer's problem, total dividend income is equal to total profits of the insurer, the distribution of agents by frailty, endowments and assets is such that

$$\tilde{h}(f, \mathbf{w}, a) = \begin{cases} h(f, \mathbf{w}), & \text{if } a = a^{f,\mathbf{w}}, \\ 0, & \text{otherwise,} \end{cases}$$

and the government budget constraint holds.

5 Calibration and Assessment

Solving the model is computationally intensive due to the large number of risk groups and the nonconvexities in individual budget sets created by the means-test. We allow for 101 different income levels and 5 different frailty levels so there are 505 risk groups and thus 505 distinct optimal menus to be computed. When computing the optimal menu for a given risk group we need to check different possible configurations of contracts and it is not unusual to encounter non-convergence due to the setting of the initial conditions.[33] These computational issues dictate that we parameterize the model by informally calibrating it to data targets.[34]

5.1 Preferences and technology

Individuals cover a substantial fraction of NH expenses using their own resources. Given the size of these expenses, it makes sense to assume that households are risk averse and thus willing to pay a premium to avoid this risk. A common choice of the risk aversion coefficient in the macroeconomics incomplete markets literature is $\sigma = 2$. We use this value. The preference discount factor and interest rate in conjunction with σ jointly determine how much people save for retirement. The preference discount factor β is set to reproduce average wealth of 62–72 year olds in our HRS sample relative to average lifetime earnings.[35] The resulting annualized value of β is 0.94.

On average individuals in our dataset enter a NH at age 83 or about 18 years after they retire. The parameter α captures the discounting between the age of retirement and LTCI purchase, and the age when a NH event is likely to occur. We choose α to reproduce the average wealth of NH entrants immediately before entering the NH relative to the average wealth of 62–72 year olds. This ratio is 1.60 in our dataset and 1.56 in our model.[36]

We assume that savings earn a risk-free real return of 2% per annum. However, when computing the overall return on savings between the first and second model period, we recognize that savings are not accumulated uniformly during individuals' working careers. Younger workers have zero or even negative net worth and it is only midway through their working career that they start provisioning for retirement. We capture this in a simple way by assuming that the gross return at retirement of 1 unit of savings for those aged 21–34 is zero. From 35 to 64 we tabulate the total return at retirement of one unit of savings for each age and then average the returns from ages 21–64.[37] This procedure results in an effective annualized return on savings of 0.0062.

[33] See the appendix for more details on the computation.

[34] A table summarizing the calibration is provided in the appendix.

[35] The specific target is 0.222. Our choice of this age group is based on two considerations. First, if we limit attention to those aged 65 we would only have a small number of observations. Second, the average age when individuals purchase LTCI in our sample is 67 and this is the midpoint of the interval we have chosen.

[36] To calculate this number in the data, we average the wealth of NH entrants in the wave that precedes their NH entry wave.

[37] By taking the simple average we are implicitly assuming that individuals are saving the same fraction of their working-age income in each period between 35 and 64.

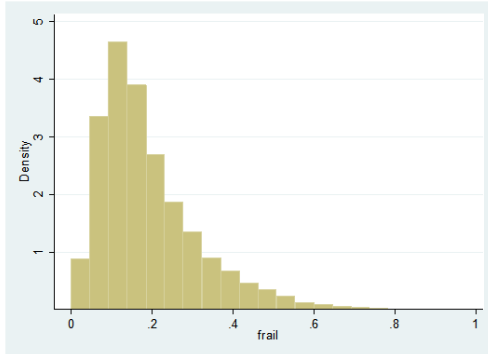


Figure 5: Distribution of frailty for 62–72 year-olds in our HRS sample. Severity of frailty is increasing with the index value and the maximum is normalized to one.

Table 3: Mean frailty by PE quintile in the data and the model.

	PE Quintile				
	1	2	3	4	5
Data	0.23	0.22	0.19	0.16	0.15
Model	0.23	0.20	0.18	0.16	0.13

Data source: Authors' calculations using our HRS sample.

5.2 Frailty and endowment distributions

The distribution of frailty in the model is calibrated to replicate the distribution of frailty of individuals aged 62–72 in our HRS sample. We focus on 62–72 year-old individuals because frailty is observed by the insurer at the time of LTCI purchase. In our HRS sample, the frailty of 62–72 year-old individuals is negatively correlated with their permanent earnings (PE).[38] To capture this feature of the data we assume that the joint distribution of frailty and the endowment stream, $h(f, \mathbf{w})$, is a Gaussian copula. This distribution has two attractive features: the marginal distributions do not need to be Gaussian and the dependence between the two marginal distributions can be summarized by a single parameter $\rho_{f,\mathbf{w}}$. The value of this parameter is set to -0.4 so that the model generates the variation in mean frailty by PE quintile observed in the data. Table 3 shows the targeted values and model counterparts.

Figure 5 shows the empirical frailty distribution. We approximate it using a beta distribution with $a = 1.53$ and $b = 6.70$. The parameters of the distribution are chosen such that mean frailty in the model is 0.18 and the Gini coefficient of the frailty distribution is 0.35, consistent with their counterparts in the data. When computing the model, we discretize frailty into a 5-point grid. We use the mean frailty of each quintile of the distribution as grid values.

The marginal distribution of endowments is assumed to be log-normal. We equate en-

[38]We use annuitized income to proxy for PE and assign individuals the annuitized income of their household head. See the appendix for details.

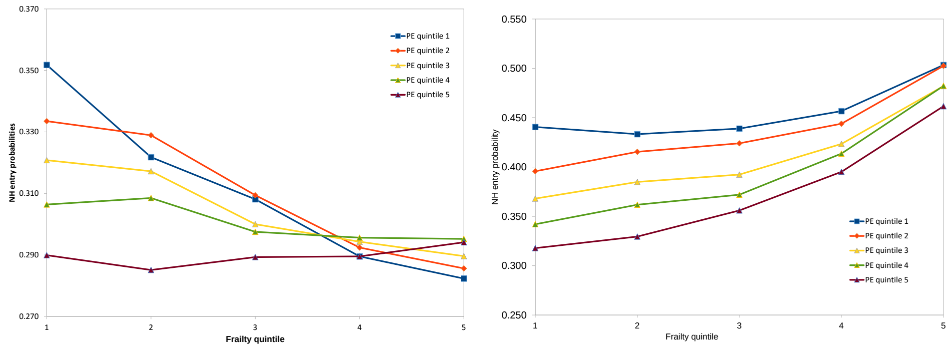


Figure 6: Lifetime NH entry probabilities by frailty and PE quintile: unconditional (left panel) and conditional on surviving to age 80 (right panel). NH entry probabilities are probabilities of a NH stay of at least 100 days. The probabilities are based on our auxiliary simulation model which is estimated using HRS data.

dowments to the young with permanent earnings and normalize the mean young endowment to 1. This is equivalent to a mean permanent earnings of \$1,049,461 in year 2000 dollars which is approximated as average earnings per adult aged 18–64 in year 2000 multiplied by 40 years.[39] The standard deviation of the log of endowments to the young is set to 0.8 because it implies that the Gini coefficient for the young endowment distribution is 0.42. This value is consistent with the Gini coefficient of the permanent earnings distribution for individuals 65 and older in our HRS.

Endowments to the old are a stand in for social security and private pension benefits. Individuals with the lowest income value get a 60% replacement rate. As income increases, the replacement rate falls in a linear way to 40% at the highest income level. The minimum and maximum levels of the replacement rate are chosen so that the model reproduces the average social security replacement rate in the year 2000 and the ratio of wealth at retirement of quintile 5 to quintile 2. The average replacement rate, taken from Biggs and Springstead (2008), is 45% and the ratio of quintile 5's wealth to that of quantile 2's is 18 in our HRS sample.

We want our model to capture the fact that individuals who enter a NH early in their retirement period, on average, have more wealth than individuals who enter later. We thus assume that the distribution of demand shocks κ is such that $1 - \kappa$ is log-normal. We then use the parameters of this distribution to target two data facts. The first data target is that 46% of NH residents are on Medicaid in our HRS sample. In the model, the corresponding value is 45%. The second data target is the level of quintile 5's wealth at the time of NH entry. This number is 0.53 in our dataset and 0.52 in our model. This results in the mean value of κ being 0.55 and the standard deviation of log κ set to 0.245.

[39] To derive average earnings per adult aged 18-64 in year 2000 we divide aggregate wages in 2000 taken from the Social Security Administration by number of adults aged 18-64 in 2000 taken from the U.S. Census.

Table 4: LTCI take-up rates by wealth and frailty: data and model

Frailty	Data			Model		
	Wealth Quintile			Wealth Quintile		
Quintile	1–3	4	5	1–3	4	5
1	0.071	0.147	0.233	0.076	0.127	0.243
2	0.065	0.158	0.205	0.069	0.179	0.218
3	0.049	0.131	0.200	0.062	0.114	0.183
4	0.037	0.113	0.157	0.025	0.111	0.182
5	0.025	0.107	0.104	0.015	0.111	0.085

For frailty (rows) Quintile 5 has the highest frailty and for wealth (columns) Quintile 5 has the highest wealth. We merge wealth quintiles 1–3 because take-up rates are very low for these individuals. Data source: 62–72 year olds in our HRS sample.

5.3 Nursing home entry and survival probabilities

Notice that the moments used to calibrate the joint distribution of frailty and endowments are all exogenous and independent of the survival and NH entry probabilities. Thus, having calibrated this distribution, we can use it to assign individuals in the model to frailty and PE quintiles, and thereby partition the population into 25 groups, one for each frailty/PE quintile combination. To reduce the number of parameters, we assume individuals within the same group have the same survival probability and the same set of true NH entry probabilities.[40]

To obtain survival and lifetime NH entry probabilities by frailty and PE quintile groups from the data, we use an auxiliary simulation model similar to that in Hurd et al. (2013). All NH entry probabilities are probabilities of experiencing a long-term (at least 100 day) NH stay. We focus on long-term NH stays because stays of less than 100 days are heavily subsidized by Medicare. The left panel of Figure 6 shows the probability a 65 year-old will enter a NH before death by frailty and PE quintiles estimated using the simulation model. NH entry risk does not vary much with frailty for PE quintile 5 and is actually decreasing in frailty for PE quintiles 1–4. These patterns occur because frailty is an indicator of both NH entry risk and mortality risk. The right panel of the figure shows that, once we condition on surviving to age 80, the lifetime NH entry probabilities of 65 year-olds increase with frailty. To obtain the survival probabilities, we estimate the probability that a 65 year-old will survive to either age 80 or until a NH event occurs for each of the 25 groups. We use survival until age 80 *or* a NH event because, this way, regardless of which one we target, our calibrated model will be able to match both the unconditional NH entry probabilities and the entry probabilities conditional on survival that are reported in Figure 6. The resulting survival probabilities of each frailty and PE quintile are provided in the appendix. Not surprisingly, the relationship between frailty and survival is negative in all PE quintiles.

[40]We wish to emphasize that these groups are not risk groups because individuals in a given group are not identical to the insurer. The insurer observes 101 distinct levels of permanent earnings and thus will offer different menus to individuals in a given group.

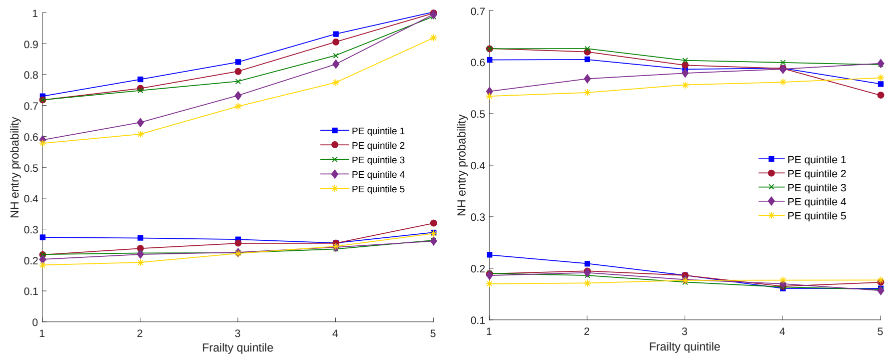


Figure 7: Nursing home entry probabilities in the model unconditional (left panel) and conditional on surviving (right panel) for good and bad types by frailty and PE quintile.

We calibrate the 51 parameters that govern the distribution of NH entry probabilities across private information types in each frailty-income quintile group to reproduce group-specific NH entry probabilities and LTCI take-up rates. We start by normalizing the probability of NH entry of the bad types in quintile 1 of the permanent earnings distribution and quintile 5 of the frailty distribution to 1. This leaves 49 NH type-specific entry probabilities and ψ to be set. The first set of targets we use are the 25 NH entry probabilities reported in the right panel of Figure 6. In our model, the probability of NH entry conditional on survival is $\eta^s_{f,\mathbf{w}} = \psi\theta^g_{f,\mathbf{w}} + (1-\psi)\theta^b_{f,\mathbf{w}}$. This expression is used to back out the NH entry probabilities of good types, $\theta^g_{f,\mathbf{w}}$'s, in each frailty-income quintile group given ψ and that group's $\theta^b_{f,\mathbf{w}}$.

The second set of targets are the 15 LTCI take-up rates of individuals in all combinations of quintiles 1-3, 4, and 5 of the wealth distribution and quintiles 1 through 5 of the frailty distribution reported in the lower panel of Table 4. The resulting system is not identified because the number of free parameters $(50-25=25)$, exceeds the number of data moments (15). This gap is bridged by restricting the shape of the NH entrance probabilities of bad types, $\theta^b_{f,\mathbf{w}}$ in wealth quintiles 1–3. In particular, we assume that the NH entry probabilities of bad types in these wealth quintiles vary with frailty at the same rate as average NH entry rates vary with frailty. This produces 10 restrictions on the $\theta^b_{f,\mathbf{w}}$'s and reduces the number of free parameters to 15 so that the system is exactly identified. Our decision to restrict the parameters in this way is based on two considerations. First, only a very small number of individuals in quintiles 1 and 2 have LTCI in our dataset. Second, in the model, no individuals in these quintiles buy LTCI because they are guaranteed to get Medicaid if they incur a NH event.[41]

Table 4 reports the 15 LTCI take-up rates implied by the model using our calibration scheme and the data targets. The fit of the model is not perfect due to the fact that we discretize the state space to compute the model. Note, however, that the take-up rates generated by the model increase with wealth and decline with frailty for both the rich and poor. Our calibration scheme also does a good job of reproducing the average LTCI take-up

[41] This difference between the model and the data is present for a variety of reasons including measurement error, our parsimonious specification of the Medicaid transfer function, and the fact that we have not modeled all shocks faced by retirees such as spousal death.

Table 5: Standard deviation of self-reported (private) NH entry probabilities by frailty: data and model

	Frailty Quintile				
	1	2	3	4	5
Data	1.00	0.99	1.03	1.27	1.47
Model	1.00	1.09	1.21	1.36	1.52

The standard deviations (SDs) are normalized such that the SD of frailty quintile 1 is 1. Data values are SDs of self-reported probabilities of entering a NH in the next 5 years for individuals aged 65–72 excluding observations where the probability is 50%. The pattern in the data is robust to variations in the way we construct the SDs including how we handle those reporting a probability of 0, 100% or 50%. Data source: Authors' calculations using our HRS sample.

rate. In our HRS sample, 9.4% of retirees aged 62–72 have LTCI and in the model 9.3% of 65 year-olds have a nonzero LTCI contract. The fact that we are able to reproduce the average LTCI take-up rate suggests that the restrictions we have imposed on the $\theta^b_{f,\mathbf{w}}$'s for wealth quintiles 1-3 are broadly consistent with our data.

This calibration scheme yields a value for ψ of 0.7 and type-specific NH entry probabilities that are reported in Figure 7. Notice that the dispersion in private NH entry risk increases with frailty. The increase in dispersion is needed for the model to reproduce the negative relationship between LTCI take-up rates and frailty that is observed within each PE quintile in the data. Recall from Proposition 6 that increasing the dispersion of the NH entry probability distribution increases the possibility of rejection and, thus, puts downward pressure on the take-up rates. To get lower take-up rates via case 1 requires the average NH entry rate to increase with frailty. The counterpart to the average NH entry rate in our baseline model is the unconditional lifetime NH entry rates shown in the left panel of Figure 6.[42] Notice that these rates do not increase with frailty: they are flat for PE quintile 5 and decline with frailty in all the other quintiles. Thus the increase in dispersion is due to a combination of increasing unconditional NH entry probabilities for bad types and decreasing probabilities for good types such that the variation in the average unconditional entry probability with frailty in the model matches its variation in the data.

One way to assess this aspect of our model is to provide independent evidence that dispersion in private NH entry probabilities, and thus the severity of the private information friction, increases with frailty. The first row of Table 5 reports normalized standard deviations of self-reported NH entry probabilities for 65–72 year-old HRS respondents by frailty quintile. These probabilities are not exactly comparable to the private NH entry probabilities in the model for two reasons. First, they are self-reported probabilities of NH entry in the next 5 years whereas the model values are lifetime NH entry probabilities. Second, the self-reported probabilities are very noisy with 1/3 of respondents reporting 0.5 and another third reporting either 0 or 1. The second row of Table 5 reports the distribution of private NH entry probabilities by frailty quintile that emerge from our calibration. Despite the noise,

[42]To see this note that the baseline model is equivalent to a model without survival risk but NH entry probabilities given by $\tilde{\theta}^i_{f,\mathbf{w}} = s_{f,\mathbf{w}}\theta^i_{f,\mathbf{w}}$.

a comparison of the first and second rows of the table show that the dispersion of private information is increasing in frailty in both the data and the model.

5.4 Nursing home and insurance costs

In practice NH care expenses have two components. The first component is nursing and medical care and the second component is room and board. We interpret this second component as being part of consumption and thus a choice and not a medical expense shock. We estimate the medical expense component of the cost of a NH stay in the following way. Stewart et al. (2009) estimate that the average total cost of a NH stay was \$60,000 in the year 2003. Barczyk and Kredler (2016) estimate that the medical expense component accounts for 56.2% of the total cost. Using this factor yields \$33,720 per year for the medical component of a NH stay. Braun et al. (2015) estimate that the average duration of NH stays that exceed 90 days is 3.25 years. Medicare provides NH benefits for up to the first 100 days. To account for this, we subtract 100 days resulting in an average benefit period of 2.976 years. Multiplying the annual medical cost by the average length of a NH stay yields total medical expenses of \$100,351 or a value of $m = 0.0956$ when scaled by average lifetime earnings.

As we pointed out in Section 2.4 private LTCI products are costly to administer. Insurers need to verify that those claiming benefits qualify under the terms of the contract.[43] Regulatory rules for setting rates on LTCI policies have changed during our sample period. Prior to 2000, a common requirement was that insurers fix the minimum percentage of premium revenue earmarked for losses (benefits to the insured) at 60% leaving 40% to cover administrative costs and profits. This scheme resulted in large increases in premia as the group of insured aged and claims increased. In 2000, the rules were changed to set caps on losses over the lifetime of the policy. Following lower than expected investment returns, lower than expected lapse rates and higher than expected morbidity experiences on policies issued in the 1980s and 1990s new policies issued since 2014 are expected to embed a 10% margin in their pricing of premia to reflect the risk of adverse claims experiences over the life of the policy.[44] We interpret these regulations as suggesting that about 30% of premium revenue is for administrative expenses and about 10% is for (risky) profits.

Our model has two parameters that govern administrative costs of paying claims. The parameter λ is a proportional cost of paying claims and k is a fixed cost. We have seen that λ affects the type of equilibrium (separating or pooling) and also plays a central role in producing rejections. The fixed cost k shifts the isoprofit lines down in a parallel fashion and thus can also generate rejections. As k increases, risk groups with less comprehensive contracts are more likely to be rejected because in these less profitable groups the insurer's net revenue will not be large enough to cover the higher fixed costs. We choose λ and k by targeting two moments. The first moment is total costs incurred by the insurer as a

[43]Since health status can improve, insurers also need to monitor those receiving benefits on an ongoing basis. It is not unusual for individuals to have multiple qualifying events. In recent years insurers have also gotten involved in care management. In some cases, the insured or the family of the insured prefer to keep the insured out of a nursing home. Thus there may be opportunities to coordinate with the insured and their family to come up with alternative accommodations that are preferred by the family and reduce claims.

[44]See King (2016) for more details.

Table 6: Distribution of insurance across NH residents: data and model

	LTCI	Medicaid	Both	Neither
Data	8.2	45.6	2.7	43.4
Model	9.2	44.8	0.1	45.9

Percent of NH residents covered by LTCI only, Medicaid only, both, or neither in the data and the model. Data source: Authors' calculations using our HRS sample.

fraction of total premium revenue which is 30%. The second moment is an average load on individuals of 0.40 which is in the middle of the range of average loads documented in the literature.[45]

5.5 Government programs and taxes

We set the consumption floors $\underline{c}_{NH}$ and $\underline{c}_o$ to 0.02 which corresponds to $7,053 per year under the assumption that an average stay lasts 2.976 years. This value was chosen to match estimates of consumption floors from the previous literature.[46] The tax rate that is needed to clear the government budget constraint is 0.011.

5.6 Insurance Distribution

We have not explicitly targeted the distribution of insurance use (private, public, none) across NH residents. It follows that comparing the predictions from the model with data along this dimension is another way to assess our model. Table 6 shows that the model does an excellent job generating the distribution of insurance across NH residents in the HRS data. The biggest difference is that the fraction of individuals who end up with both LTCI and Medicaid is positive in the model but smaller than in the data. In the model, these are individuals who, ex-ante, bought LTCI because they were not covered by Medicaid for all realizations of the demand shock but, ex-post, drew a realization of κ that resulted in Medicaid eligibility.

6 Results

In Section 2, we motivated our analysis by describing some of the main features of the U.S. private LTCI market. We now turn to discuss how well our model accounts for these observations and why. The results from Section 3 demonstrated that, at a qualitative level, the main features of the market could be accounted for, independently, by either supply-side frictions due to administrative costs and adverse selection or demand-side frictions due to Medicaid. One of our main objectives here is to understand the role of each of these

[45] See Section 2.4 for details.

[46] See Kopecky and Koreshkova (2014) for more details.

Table 7: Rejection rates in the Baseline, the No Administrative Costs, the No Medicaid, and the Full Information economies

Scenario Description	Baseline	No Admin. Costs $\lambda = 1,\ k = 0$	No Medicaid $\underline{c}_{nh} = 0.001$	Full Information $\theta^i_{f,\mathbf{w}}$ public
Average	90.5	34.9	15.0	50.5
By PE Quintile				
1	100	100	20.0	100
2	100	74.3	0.0	100
3	87.5	0.0	0.0	38.1
4	87.1	0.0	0.0	0.0
5	78.0	0.0	54.9	14.5
By receiving Medicaid NH benefits conditional on surviving				
Would	44.5	34.2	4.0	41.1
Would not	46.0	0.7	11.0	9.4

Rejection rates are percentage of individuals who are only offered a single contract of $(0, 0)$ by the insurer. Note that for PE quintiles that the figures are expressed as a percentage of individuals in that quintile. However, the bottom two rows of the table are a decomposition of the average rejection rate for that economy.

mechanisms. To help distinguish between them, we compare the baseline economy with three other scenarios. In each scenario, the endowments and pre-tax interest rate are the same, and the proportional income tax rate is adjusted to balance the government budget constraint. In the No Administrative Costs economy we remove the insurer's variable and fixed costs by setting $\lambda = 1$ and $k = 0$. In the No Medicaid economy, the NH consumption floor $\underline{c}_{NH}$ is reduced to 0.001.[47] The Full Information economy, which is designed to understand the effects of private information, assumes that the insurer can directly observe each individual's true NH risk exposure, $\theta^i_{f,\mathbf{w}}$.

6.1 LTCI take-up rates and rejections

We explained in Section 2.3 that LTCI take-up rates decline with frailty for both poor and rich individuals in our HRS sample and we have calibrated the baseline model to reproduce these observations (see Table 4). Our model has two mechanisms for generating low LTCI take-up rates. Some risk groups are rejected by the insurer because there is no basis for trade. The menu of contracts offered to these risk groups consists of a single $(0, 0)$ contract. As we explained in Section 3, for other risk groups the menu consists of two contracts, a non-zero one and a $(0, 0)$ one, and the good type chooses the $(0, 0)$ one.[48] It turns out that

[47] We do not reduce $\underline{c}_{NH}$ to zero because then some individuals would experience negative consumption. Also note that the non-NH consumption floor, $\underline{c}_o$, does not vary across economies.

[48] This result follows from Lemma 1 (see the Appendix) which establishes that the single crossing condition obtains in our setup with Medicaid and stochastic endowments.

Table 8: LTCI take-up rates by wealth and frailty: Baseline and Full Information economies

	Baseline			Full Information		
Frailty	Wealth Quintiles			Wealth Quintiles		
Quintile	3	4	5	3	4	5
1	0.145	0.127	0.243	0.624	1.000	0.811
2	0.163	0.179	0.218	0.745	1.000	0.875
3	0.161	0.114	0.183	0.668	1.000	0.890
4	0.083	0.111	0.182	0.537	1.000	0.911
5	0.081	0.111	0.085	0.545	1.000	0.918

The LTCI take-up rates in wealth quintiles 1 and 2 are 0 in both economies.

these latter menus are very rare in the Baseline economy. Only 0.2% of individuals choose not to purchase private LTCI.[49] Thus, the low LTCI take-up rates produced by our model are almost exclusively due to rejections. Consistently, as the first column of Table 7 shows, the rejection rate in our Baseline economy is 90.5%. It is 100% for individuals in PE quintiles 1 and 2 and declines with permanent earnings in quintiles 3–5.

Recall from Section 2.2, that we estimated that roughly 36% of 55–66 year old HRS respondents would be rejected based on medical underwriting if they applied for LTCI. Some care must be exercised in relating rejections or denials based on this data to our model concept of rejections. In our model, a rejection is a no-trade result. For there to be a basis for trade, the insurer must be able to offer a profitable contract that is attractive to the purchaser. Our data measure of rejections primarily captures no trade that arises because from the insurer's perspective insuring a particular risk group is not profitable. Unfortunately, our HRS dataset does not include questions that would allow us to ascertain how many individuals do not have private LTCI because they perceive that it is too expensive. However, survey results in Ameriks et al. (2016) suggest that pricing is also an important reason for low take-up rates. It follows that our estimate of a rejection rate of 36% using HRS data is best thought of as providing a lower bound on situations where no trade might occur.

We now turn to analyze the roles of administrative costs on the insurer and Medicaid in producing rejections and thus low LTCI take-up rates. To get at their individual roles, compare columns 2 and 3 of Table 7, which report rejection rates in the No Administrative Costs economy and the No Medicaid economy, to column 1. Removing either administrative costs or Medicaid has a big effect on the average rejection rate. However, removing administrative costs has a larger impact on the rejection rates of the rich, whereas removing Medicaid has a larger impact on the rejection rates of the poor. In particular, notice that removing administrative costs eliminates rejections of individuals in quintiles 3–5. When

[49]The small fraction of individuals who choose not to purchase LTCI is partly due to frictions from Medicaid and administrative costs and partly due to private information. When we run a counterfactual simulation that removes Medicaid and administrative costs the fraction of individuals who choose not to purchase LTCI increases to 2.4%.

administrative costs are removed profits from offering insurance increase. This has a bigger effect on contracts for higher income risk groups whose demand is not as greatly reduced by the presence of Medicaid. In contrast, when Medicaid is removed, rejections are eliminated in quintiles 2–4 and substantially reduced in quintile 1. They do not fall to zero in quintile 1 because some individuals in quintile 1 are so poor that they cannot afford NH care and must rely on Medicaid even though the Medicaid consumption floor is extremely low. Interestingly, while removing Medicaid is needed to substantially reduce rejections of the poor (those in quintile 1 and 2) and removing insurer costs is needed to substantially reduce rejections of those in quintile 5, rejections in quintiles 3–4 can be eliminated by removing either one.

Individuals who are rejected by the insurer can be divided into two groups: those who, if they survive to the very old age and enter a NH, will receive Medicaid NH benefits and those who will not. In the baseline economy 46.0% of individuals are rejected by the insurer but ultimately are too affluent to satisfy the Medicaid means test. This latter group pays all of their NH expenses out of pocket. Removing either administrative costs or Medicaid reduces the fraction of individuals in this group. When administrative costs are removed, only 0.7% of individuals are rejected and end up paying out of pocket for NH care. These are individuals in PE quintile 2 whose demand for LTCI was low because their likelihood of ending up on Medicaid if NH entry occurred was high, but who ultimately ended up with too much wealth to meet the Medicaid means test. When Medicaid is removed, 11.0% of individuals are rejected and end up paying out of pocket for NH care. These are individuals in PE quintile 5 who were rejected by the insurer because of poor health.

Finally, to understand the role that private information plays, the final column of Table 7 shows the rejection rates in the Full Information economy. Absent private information, rejection rates fall from 90.5% to 50.5%. This decline is due to a decline in rejections of individuals in PE quintiles 3–5. Removing private information increases profitability and, like removing administrative costs, this has a larger effect on the rejection rates of higher income individuals. Table 8 reports LTCI take-up rates for the Baseline economy and the Full Information economy. Notice that LTCI take-up rates in the Full Information economy are not only too large but also have the wrong pattern in wealth quintiles 4–5. It is clear from these results that private information plays an essential role in generating the decline in LTCI take-up rates with frailty within the upper wealth quintiles.

Previous research by Braun et al. (2015) and De Nardi et al. (2013) has found that Medicaid and other means-tested social insurance for retirees is highly valued. It insures against a range of risks faced by retirees including lifetime earnings risk, NH risk, spousal death risk and longevity risk. It is thus interesting to consider how effective this form of social insurance is in providing for those who are rejected due to the supply-side distortions. A comparison of the Baseline with the No Administrative Costs and the Full Information economies using Table 7 suggests that Medicaid NH benefits are not very effective in insuring against supply-side frictions. When going from the No Administrative Costs economy to the Baseline economy, the fraction of individuals who are rejected but receive Medicaid NH benefits increases from 34.2% to 44.5%, while the fraction of individuals who are rejected but pay out-of-pocket for NH care goes from 0.7% to 46%. The pattern of changes is similar when moving from the Full Information economy to the Baseline economy. In other words, most of the individuals who are rejected due to administrative costs or private information

end up paying for NH care out of pocket.

6.2 Coverage, loads and profits

6.2.1 Coverage and loads in the baseline economy

A second feature of the U.S. market for LTCI that we discussed in Section 2 is that individuals who purchase LTCI only receive partial coverage. The most common policies cover between one-third and two-thirds of lifetime NH expenses and policies that offer comprehensive lifetime coverage have essentially disappeared from the market. Insurance contracts in our Baseline economy capture this feature of the market. Indemnities cover 58% of NH medical costs on average. Table 9 shows how the average fraction of NH costs covered varies by private information type, wealth and frailty. Notice that, conditional on having a positive amount of coverage and on private type, comprehensiveness of the policies does not vary much across observable characteristics. Comprehensiveness does, however, vary significantly across private types. The indemnity covers about 75% of NH costs for bad types and about 50% of NH costs for good types. Combining these results with the previous results on rejections indicates that the insurer is reacting to adverse selection in two ways. He is screening out risk groups that are not profitable by offering $(0, 0)$ contracts, and he is incentivizing individuals in profitable risk groups to reveal their private type by offering a menu featuring a less and a more comprehensive contract.

How does our model generate partial coverage and, in particular, partial coverage for bad risks? In Section 3, we described three different ways to break the classic adverse selection result that bad risks receive full coverage. One way to break it is by generating pooling contracts with positive levels of insurance. However, it can also be broken with separating contracts if the insurer faces administrative costs, or if Medicaid is present and endowments are stochastic. It turns out that positive pooling contracts do not arise in our Baseline Economy. Thus, partial coverage of bad types is due to the second and third factors.

The pricing of LTCI in our Baseline economy is also broadly consistent with pricing in the U.S. LTCI market. Recall that Brown and Finkelstein (2007) and Brown and Finkelstein (2011) find that the average load in the LTCI market is in the range 0.18 and 0.5, depending on whether or not the loads are adjusted for policy lapses and the sample period. They also find that the relationship between loads and comprehensiveness is non-monotonic and that for some individuals loads are negative. The average load was a calibration target (0.40). However, loads by wealth and frailty level were not calibration targets. Table 9, shows that there is considerable cross-subsidization. In all wealth and frailty quintiles, bad types have negative loads as they are subsidized by good types. The loads are also nonlinear in wealth. For good types, wealth quintile 4 receives the most coverage with the lowest load. Coverage is lower and loads are higher in both wealth quintiles 3 and 5. For bad risk types, in contrast, loads and coverage are both increasing in wealth. Still, most individuals in the model are good types and the average pattern of loads by wealth exhibits the same shape as that of the good types.

In the model economy, the insurer creates a separate menu for each risk group and thus, in equilibrium, offers hundreds of menus. However, in reality, insurers usually pool individuals into two or three risk groups. One remedy to this discrepancy between the model and the

Table 9: Comprehensiveness and individual loads by private type and frailty and wealth quintiles in the Baseline economy.

	Wealth Quintile				
	1	2	3	4	5
Good risks (θ^g)					
Fraction of NH costs covered	NA	NA	0.489	0.538	0.497
Load	NA	NA	0.621	0.591	0.599
Bad risks (θ^b)					
Fraction of NH costs covered	NA	NA	0.701	0.766	0.764
Load	NA	NA	-0.089	-0.082	-0.018
	Frailty Quintile				
	1	2	3	4	5
Good risks (θ^g)					
Fraction of NH costs covered	0.507	0.517	0.503	0.497	0.489
Average load	0.598	0.598	0.607	0.609	0.617
Bad risks (θ^b)					
Fraction of NH costs covered	0.744	0.753	0.749	0.748	0.741
Load	-0.061	-0.042	-0.057	-0.057	-0.061

The fraction of NH costs covered is the average indemnity divided by the medical expense cost of a nursing-home stay or (ι/m) for individuals with a positive amount of insurance. NA denotes cases where the denominator is zero.

data would be to assume that the insurer faces a fixed cost of creating each menu. A fixed menu cost gives the insurer an incentive to reduce the number of risk groups by combining individuals with different observable characteristics into the same group. We do not model this fixed cost because it significantly complicates the problem of finding the optimal set of menus.[50] However, as Table 9 shows, the extent of coverage and loads on each private type do not vary much across observable characteristics in our Baseline economy. This fact suggests that a small fixed cost of creating menus could substantially reduce the number of menus offered by the insurer.

6.2.2 Coverage and loads in other economies

We next analyze the individual roles of private information, Medicaid, and administrative costs in producing partial coverage, high loads and low take-up rates. The first three columns of Table 10 show the average LTCI take-up rates, fractions of NH costs covered, and loads on good and bad risk types in the Baseline economy, the No Administrative Costs economy, and the No Medicaid economy. Notice that removing either administrative costs or Medicaid not only increases take-up rates but also the comprehensiveness of contracts offered with the removal of Medicaid having the larger effect. Loads, in contrast, may increase or decline depending on which friction is removed. Without administrative costs, loads on individuals

[50] Finding the profit maximizing optimal set of menus with fixed menu costs is a challenging combinatorics problem in our setting because there are a very large number of contracts that have to be considered.

Table 10: LTCI take-up rates, comprehensiveness and individual loads by private type in the Baseline, the No Administrative Costs, the No Medicaid, and the Full Information economies

Scenario	Baseline	No Admin. Costs	No Medicaid	Full Information
Description		$\lambda = 1$, $k = 0$	$\underline{c}_{nh} = 0.001$	$\theta^i_{f,\mathbf{w}}$ public
Good risks (θ^g)				
LTCI take-up rate	0.092	0.612	0.847	0.495
Fraction of NH costs covered	0.503	0.543	0.620	0.858
Load	0.603	0.551	0.709	0.492
Bad risks (θ^b)				
LTCI take-up rate	0.095	0.651	0.850	0.495
Fraction of NH costs covered	0.747	0.859	0.807	0.588
Load	-0.054	-0.156	0.118	0.263

The fraction of NH costs covered is the average indemnity divided by the medical expense cost of a nursing-home stay or (ι/m) for individuals with a positive amount of insurance.

are lower than in the Baseline economy. Without Medicaid, demand for LTCI increases and the insurer can increase loads and trade still occurs. The final column of the table presents the same statistics for the Full Information economy. Under full information, not only are take-up rates higher, but contracts are, on average, more comprehensive. However, the impact of going from the Baseline economy to full information differs for good and bad risk types. Good risks experience an increase in coverage and a decline in loads. Bad risks, on the other hand, experience exactly the opposite. The intuition for this finding dates back to Arrow (1963) who demonstrates that the amount of insurance available to those with high risk exposures falls if insurance markets open after their risk exposure is observed.

Brown and Finkelstein (2008) and Ameriks et al. (2016) use a different strategy to assess the roles of high loads, incomplete coverage and Medicaid in accounting for low LTCI take-up rates. Both of these papers specify contracts exogenously and consider counterfactuals in which individuals are offered full insurance against NH risk at an actuarially-fair price. Brown and Finkelstein (2008) find that only the top one-third of individuals, when ranked by wealth, purchase a full-coverage actuarially-fair LTCI policy when Medicaid is present. In our setting with endogenous contracts Medicaid affects not only the fraction of individuals who are offered non-zero contracts but also the comprehensiveness and pricing of the contracts. These results demonstrate that counterfactuals that do not allow the insurer to adjust the size of contracts offered, likely overstate the crowding-out effect of Medicaid. Indeed, the crowding-out effect of Medicaid is much weaker in our optimal contracting model as compared to Brown and Finkelstein (2008). To see this, consider a version of our Baseline economy in which two supply-side frictions — private information and administrative costs — are removed. Medicaid is present with the consumption floor set at the baseline level. Insurance is not actuarially fair in this scenario, the average load is 0.35, due to the fact that the insurer is a monopolist. Nevertheless, 64% of individuals purchase LTCI. Table 11 reports

Table 11: LTCI take-up rates, comprehensiveness, and individual loads in the economy with no private information and no administrative costs.

	Wealth Quintile				
	1	2	3	4	5
LTCI take-up rates	0	0.20	1	1	1
Fraction of loss covered	NA	0.29	0.73	0.93	1.0
Average load	NA	0.073	0.34	0.41	0.35
	Frailty Quintile				
	1	2	3	4	5
LTCI take-up rates	0.81	0.74	0.65	0.57	0.42
Fraction of loss covered	0.95	0.93	0.88	0.89	0.84
Average load	0.36	0.35	0.34	0.35	0.33

NA denotes cases where the denominator is zero.

LTCI take-up rates, comprehensiveness of coverage and average loads by wealth and frailty quintiles in this alternative economy. LTCI take-up rates are 100% in wealth quintiles 3–5. Medicaid crowds out most private insurance in wealth quintile 2 and all private insurance in quintile 1. Wealth quintile 2 is particularly interesting because the load on insurance for this group is only 0.073 and thus close to the actuarially-fair benchmark. These individuals are not interested in a full-coverage private LTCI product because for some values of the demand shock they qualify for Medicaid NH benefits.[51] Indeed, 80% of individuals in wealth quintile 2 prefer to rely exclusively on Medicaid while 20% purchase a LTCI policy that covers 29% of NH costs.

In contrast to individuals in the lower wealth quintiles, those in quintile 4 receive extensive coverage (93%) and those in quintile 5 receive full coverage. For the latter group, the chance of receiving Medicaid NH benefits is particularly low and full coverage is attractive. This final property of the model is related to Ameriks et al. (2016). They find that 66% of individuals in a sample of affluent individuals with median wealth of $543,000 have demand for an ideal state-contingent LTCI product that is priced in an actuarially-fair manner. However, only 22% of their respondents hold LTCI and they refer to this as a "LTCI puzzle." For purposes of comparison, in our model, average wealth in wealth quintile 5 is $821,000 and average wealth in quintile 4 is $364,000.[52] Recall that in our baseline economy individuals in wealth quintiles 4–5 have LTCI take-up rates of 13% and 21%, respectively. Thus, we find that the LTCI puzzle that Ameriks et al. (2016) document for wealthy individuals can be attributed to supply-side distortions induced by private information and administrative costs.

We have focused on these two examples because they are the most relevant to our analysis. However, it is common practice in the literature to abstract from the contract design problem of the issuer when modeling the LTCI market. Some recent examples include Lockwood (2016) who analyzes optimal saving and bequests in a setting with exogenously specified LTCI and Mommaerts (2015) and Ko (2016) who analyze the informal care market under

[51] Recall that the optimal contracts have this same property in the simple model. See Figure 3.

[52] Both figures are expressed in terms of year 2000 dollars.

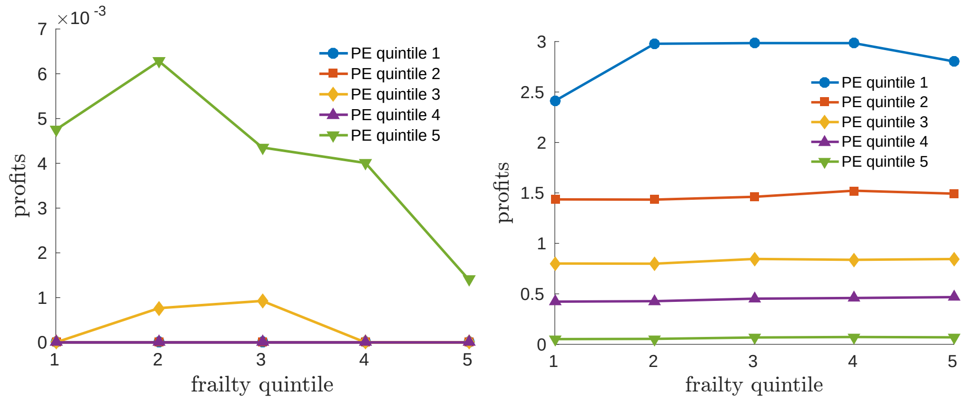


Figure 8: Profits Baseline and No Medicaid Specifications.
The left (right) panel reports profits as a percent of average lifetime earnings by frailty and PE quintiles in the Baseline economy (No Medicaid) economy. Note the difference in scale of the two plots.

the assumption that an alternative option is an exogenously specified LTCI contract. It is conceivable that modeling the supply-side of the LTCI market would also provide new insights into savings decisions of the old in the presence of bequest motives and their demand for informal care.

6.2.3 Profits

In Section 2.4 we documented that profits in the U.S. private LTCI market are low. Profits are also low in our Baseline economy. They are less than 0.001% of average lifetime earnings. The left panel of Figure 8, which reports profits by frailty and PE quintiles, reveals that most profits come from insuring healthy rich individuals as most of the other risk groups are rejected and profits are thus zero. Medicaid, administrative costs and private information all work to reduce profits. Medicaid, however, has the largest impact. When it is removed profits rise to 1.1% of average lifetime earnings.[53] The right panel of Figure 8 shows profits by frailty and PE quintiles in the No Medicaid economy. In this economy, in contrast to the Baseline, the insurer generates most of his profits from the poor. Profits fall monotonically with permanent earnings and do not vary much by frailty. Notice that Medicaid has a large effect on profits for two reasons. First, Medicaid's presence dramatically reduces the fraction of profitable risk groups and when Medicaid is removed the fraction of zero-profit rejected pools declines. Second, Medicaid substantially lowers the profits the insurer receives from risk groups that are getting a positive amount of insurance. For example, when Medicaid is removed, the profits on individuals in PE quintile 5 increase by a factor of 20.

6.3 Insurance ownership and NH entry

We now illustrate that our baseline economy generates, at least at a qualitative level, a broad range of correlations between self-assessed NH entry risk, NH entry and LTCI ownership

[53]Profits are 0.38% of average lifetime earnings in the No Administrative Costs economy and 0.25% in the Full Information economy.

documented in Finkelstein and McGarry (2006).[54] These results are interesting because we have posited a single source of private information. We wish to emphasize that none of the statistics that we are about to report are calibration targets and that the results from the model are population moments. First, they find a positive correlation between self-assessed NH entry risk and NH entry and interpret this as evidence of private information, Figure 7 shows that, in the baseline economy, individuals' private NH entry probabilities are positively correlated with actual NH entry. This is true both on average and within risk groups. Second, they find that individuals act on their private information by documenting a positive correlation between self-assessed NH entry risk and LTCI ownership. The baseline economy has this property too. The LTCI take-up rate of bad types is 9.5% while the take-up rate of good types is 9.2%. Moreover, bad types have a higher LTCI take-up rate than good types no matter whether or how we control for the information set of the insurer, or whether or not we condition on survival. Third, the correlations between NH entry and LTCI take-up rates in the baseline economy are consistent with those they report. Table 12 reports NH entry rates of LTCI holders and non-holders. Only 36.4% of LTCI holders in the baseline economy enter a NH whereas 40.5% of non-holders enter consistent with the negative correlation they find when they do not control for the insurer's information set.

Chiappori and Salanie (2000) ascertain that to properly test for the presence of private information one must fully control for the information set of the insurer. Finkelstein and McGarry (2006) control for the information set of the insurer by computing the correlation between NH entry and LTCI ownership with two different sets of controls. The first set only controls for observable variation in health. The second set controls for both observable health variables and individuals' wealth and income quartiles. In both cases, they find a small negative but not statistically significant correlation. Only when they consider a special sample of individuals who are in the fourth quartile of the wealth and income distributions and have no health issues that would likely lead them to be rejected by insurers do they find a statistically significant negative correlation. Consistently, as Table 12 shows, if we only control for frailty, we continue to find a negative correlation but the size of the differential between the entry rates of LTCI non-holders and holders is now smaller in each group.[55] If, in addition to frailty, we also control for wealth and income quartile the differences in the take-up rates between non-holders and holders becomes even smaller.[56] In addition, the correlation is negative for precisely half of the groups and positive for the other half, and the average differential is essentially zero.[57] However, like Finkelstein and McGarry (2006), if we focus on individuals in the top wealth and income quartile and the lowest frailty quintile we find a negative correlation between LTCI take-up rates and NH entry. The NH entry rate of LTCI holders in this group is 32.1% while the entry rate of non-holders is 32.4%. Taken together these results demonstrate that our model with a single source of private information encompasses the main empirical results that have led the previous literature to conclude that multiple sources of private information are essential for understanding this insurance market.

[54]See Section 2.5 for an overview of their findings.

[55]We check for conditional independence of NH entry and LTCI ownership because Chiappori and Salanie (2000) point out that this strategy, which is the basis of their χ^2 statistic, is more robust to nonlinearities.

[56]These results are not reported in the table because the number of groups is so large.

[57]Using equal weights for each group the average LTCI ownership rate of entrants is 0.03 percentage points higher than the ownership rate of non-entrants.

Table 12: NH entry rates of LTCI holders and non-holders in the Baseline economy

		Frailty Quintile				
	Average	1	2	3	4	5
LTCI holders	36.4	33.2	35.5	37.5	40.9	47.7
Non-holders	40.5	35.5	37.3	39.8	43.1	49.3

Numbers are percent of survivors to the very old stage of life who enter a NH.

The reason that the correlations between LTCI take-up rates and NH entry are as small as in Finkelstein and McGarry (2006) is because the differential in LTCI take-up rates between good and bad types only occurs in a tiny fraction of risk groups. It follows that tests based on LTCI take rates will have low power in detecting adverse selection in samples of data generated by our model. Moreover, as we illustrated in Section 3.3, our model generates a pattern of rejections that is increasing in observed risk exposure. Thus, when risk groups are bunched together by the econometrician, the negative correlation between average NH entry and LTCI take-up rates across risk groups within a bunch dominates the small positive correlation between NH entry and LTCI take-up rates that arises within some risk groups.

It is worth noting that these results partly reflect a data limitation. HRS data only reports LTCI ownership rates and not the size of coverage. If data was available on the size of LTCI policies, it would be easier to detect adverse selection in correctly measured risk-groups. However, the bunching effect would still be operative as long as there is a difference between the information set of the econometrician and the insurer.

Finally, there are important differences between the moments generated by our model and the statistics reported in Finkelstein and McGarry (2006). They compute correlations between LTCI ownership and NH entry risk over a short horizon. Specifically, they look at NH entry risk within 5 years of observing LTCI ownership. NH entry risk in our model is the lifetime risk.[58] Given the short time horizons that their empirical findings are based on, we believe that the most comparable objects from the model are the NH entry rates conditional on survival that we report in Table 12. That said, with no controls, our model still generates a negative correlation even if we do not condition on survival. However, the size of the differential is smaller and the differentials in the upper frailty quintiles flip sign. The reason survival impacts the size of the differentials is because survival impacts the correlation between average NH entry and LTCI take-up rates across risk groups since, as Figure 6 shows, survival changes the way that NH entry varies with both frailty and wealth.

Medicaid, private information and administrative costs all play important roles in producing these results. Abstracting from them increases the fraction of risk groups where the optimal contract for the good type is $(0, 0)$ but positive for the bad type. However, we have seen above that abstracting from any one of these mechanisms also changes the size and pattern of rejections. With smaller rejection rates, the size of the bias associated with

[58]Alternatively, one could construct an empirical measure of lifetime NH entry risk and compare this to the results from our quantitative model. However, this is not straightforward because lifetime NH risk of HRS respondents is not directly observable and would have to be estimated using an auxiliary model. This creates an additional source of noise and specification error. In our view it is best to compare our model results with the empirical findings of Finkelstein and McGarry (2006).

bunching risk groups together declines.

7 Conclusion

In this paper we have developed a structural model of the U.S. LTCI market that posits a single source of private information and yet is remarkably successful in accounting for a range of puzzling observations about this market. In particular, we find that low take-up rates of private LTCI and contracts that provide only partial coverage of NH costs are to be expected when administrative costs and the crowding out effects of Medicaid are recognized. Insurers optimally respond to these frictions by rejecting individuals who have poor health or low income via medical underwriting, and by both increasing the price and reducing the comprehensiveness of insurance offered. Our analysis also suggests that tests for adverse selection that are based on correlating insurance ownership and insurance claims may give misleading results. In our model, NH entry by owners of LTCI is about the same as NH entry by those who do not own LTCI, even though the insurer contends with private information by offering optimal menus of contracts that honor participation and incentive compatibility constraints.

Appendix

See online appendix.

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1 Proofs of propositions

For the proofs it is useful to denote $h(\cdot)$ as the density function associated with the distribution $H(\cdot)$ and define $\hat{\omega}(\pi, \iota) \equiv \underline{c}_{NH} + m - \iota + \pi$. Note that by Equation (10) in the paper, Medicaid transfers are zero for all $\omega \geq \hat{\omega}$ and positive for all $\omega < \hat{\omega}$.

It is also useful to note that the optimal pooling contract of the insurer's maximization problem defined by Equations (3)–(5) in the paper must satisfy

$$MRS(\theta^g, \pi, \iota) = \lambda\eta, \tag{1}$$

and the participation constraint of the good type, Equation (4) in the paper, with equality.

Proposition 1. *If $\lambda > 1$ then the optimal menu features incomplete insurance for both types, i.e., $\iota^i < m$ for $i \in \{b, g\}$.*

Proof. First, note that the slope of the indifference curve at the full insurance level of indemnity always equals θ^i or

$$MRS(\theta^i, \pi^i, m) = \frac{\theta^i u'(\omega - \pi^i - m + \iota^i)}{\theta^i u'(\omega - \pi^i - m + \iota^i) + (1 - \theta^i)u'(\omega - \pi^i)}\Big|_{\iota^i = m} = \theta^i, \tag{2}$$

for all π^i. Second, note that the slope of the indifference curve declines with the level of indemnity or

$$\frac{\partial MRS(\theta^i, \pi^i, \iota^i)}{\partial \iota^i} = \frac{\theta^i(1 - \theta^i)u''(c_{NH})u'(c_o)}{[\theta^i u'(c_{NH}) + (1 - \theta^i)u'(c_o)]^2} < 0. \tag{3}$$

The good type is always under-insured, regardless of whether the optimal contract is pooling or separating. To see this for the optimal pooling contract (π^p, ι^p), combine Equation (1) with Equation (2) to obtain the following inequality

$$MRS(\theta^g, \pi^p, \iota^p) = \lambda\eta > \theta^g = MRS(\theta^g, \pi^p, m),$$

which holds when $\lambda > 1$ since $\eta = \psi\theta^g + (1-\psi)\theta^b \geq \theta^g$. Then it follows from Equation (3) that $\iota^p < m$. If instead the equilibrium is separating then combine the expression for Equation (6) provided in Footnote 21 in the paper and Equation (2) to obtain

$$MRS(\theta^g, \pi^g, \iota^g) = \lambda\left[\frac{\psi\theta^g + (1-\psi)\theta^b A}{\psi + (1-\psi)B}\right] > \theta^g = MRS(\theta^g, \pi^g, m),$$

where $A \equiv U_\iota(\theta^b, \pi^g, \iota^g)/U_\iota(\theta^b, \pi^b, \iota^b)$ and $B \equiv U_\pi(\theta^b, \pi^g, \iota^g)/U_\pi(\theta^b, \pi^b, \iota^b)$. The inequality holds for $\lambda \geq 1$ since under the single-crossing property any incentive compatible separating contract must be such that $\pi^g < \pi^b$ which implies that

$$A = \frac{\theta^b u'(c^g_{NH})}{\theta^b u'(c^b_{NH})} > \frac{\theta^b u'(c^g_{NH}) + (1-\theta^b)u'(c^g_o)}{\theta^b u'(c^b_{NH}) + (1-\theta^b)u'(c^b_o)} = B,$$

where $c^i_{NH} = \omega - m + \iota^i - \pi^i$ and $c^i_o = \omega - \pi^i$. Thus, from Equation (3), $\iota^g < m$. Finally, combine Equation (7) in the paper and Equation (2) to obtain

$$MRS(\theta^b, \pi^b, \iota^b) = \lambda\theta^b > \theta^b = MRS(\theta^b, \pi^b, m),$$

which holds when $\lambda > 1$. Thus, from Equation (3), $\iota^b < m$. $\square$

Proposition 2. *There will be no trade, i.e., the optimal menu will consist of a single (0,0) contract iff*

$$MRS(\theta^b, 0, 0) \leq \lambda\theta^b, \tag{4}$$
$$MRS(\theta^g, 0, 0) \leq \lambda\eta, \tag{5}$$

both hold.

Proof. The proof follows directly from the proof of Proposition 2 in Chade and Schlee (2014) modified for the 2-type case. $\square$

Lemma 1. ***Single-crossing Property*** *The single-crossing property holds when the endowment is stochastic and Medicaid is present with $\underline{c}_{NH} > 0$.*

Proof. The proof shows that $\frac{\partial MRS(\theta,\pi,\iota)}{\partial\theta} > 0$ for all $\pi, \iota \in \mathbb{R}^+$. Recall that

$$MRS(\theta, \pi, \iota) = -\frac{U_\iota(\theta, \pi, \iota)}{U_\pi(\theta, \pi, \iota)},$$

where

$$U_\iota(\theta, \pi, \iota) = \theta\int_{\hat{\omega}}^{\bar{\omega}} u'(c_{NH})dH(\omega) > 0, \tag{6}$$
$$U_\pi(\theta, \pi, \iota) = -U_\iota(\theta, \pi, \iota) - (1-\theta)B < 0, \tag{7}$$

with $B \equiv \int_{\underline{\omega}}^{\overline{\omega}} u'(c_o)dH(\omega) > 0$. Differentiating the MRS with respect to θ yields

$$\frac{\partial MRS}{\partial \theta} = -\frac{U_{\iota\theta}U_{\pi} - \theta U_{\iota}U_{\pi\theta}}{U_{\pi}^2}, \tag{8}$$

where

$$U_{\iota\theta} = \theta^{-1}U_{\iota} > 0, \tag{9}$$
$$U_{\pi\theta} = -U_{\iota\theta} + B = -\theta^{-1}U_{\iota} + B, \tag{10}$$

and the arguments are omitted to save space. Using Equations (6)–(10) it is easy to show that

$$\frac{\partial MRS}{\partial \theta} = \frac{BU_{\iota\theta}}{\theta U_{\pi}^2} > 0.$$

□

Proposition 3. *If $\underline{\omega} < \underline{c}_{NH}$ then the optimal menu features incomplete insurance for both types, i.e., $\iota^i < m$ for $i \in \{b, g\}$.*

Proof. We start by showing that if $\underline{\omega} < \underline{c}_{NH}$ then the optimal contract for any type $i \in \{b, g\}$, (π^i, ι^i), is such that $\hat{\omega}(\pi^i, \iota^i) > \underline{\omega}$. Suppose instead that $\hat{\omega}(\pi^i, \iota^i) \leq \underline{\omega}$. In this case, no one of type i is on Medicaid in equilibrium. The utility function, Equation (12) in the paper, can be stated as

$$U(\theta^i, \pi^i, \iota^i) = \int_{\underline{\omega}}^{\overline{\omega}} \Big[\theta^i u(c_{NH}) + (1-\theta^i)u(c_o)\Big] dH(\omega),$$

where $c_{NH} = \omega - m + \iota^i - \pi^i$ and $c_o = \omega - \pi^i$, and the marginal rate of substitution is

$$MRS(\theta^i, \pi^i, \iota^i) = \frac{\theta^i \int_{\underline{\omega}}^{\overline{\omega}} u'(c_{NH})dH(\omega)}{\theta^i \int_{\underline{\omega}}^{\overline{\omega}} u'(c_{NH})dH(\omega) + (1-\theta^i)\int_{\underline{\omega}}^{\overline{\omega}} u'(c_o)dH(\omega)}.$$

Following the same proof strategy as that of Proposition 1 it is easy to show that if $\lambda \geq 1$ then $\iota^i \leq m$ for $i \in \{g, b\}$. However, since $\underline{\omega} < \underline{c}_{NH}$ we have

$$\underline{c}_{NH} + m - \iota^i + \pi^i \equiv \hat{\omega}(\pi^i, \iota^i) \leq \underline{\omega} < \underline{c}_{NH},$$

which implies that $\iota^i - \pi^i > m$ and since $\pi^i > 0$ it must be that $\iota^i > m$, a contradiction. We have established that the equilibrium contract for each type $i \in \{g, b\}$ must be such that $\hat{\omega}(\pi^i, \iota^i) > \underline{\omega}$.

If $\hat{\omega}(\pi^i, \iota^i) \geq \overline{\omega}$ then everyone of type i is on Medicaid in equilibrium and the utility function, Equation (12) in the paper, can be stated as

$$U(\theta^i, \pi^i, \iota^i) = \int_{\underline{\omega}}^{\overline{\omega}} \Big[\theta^i u(\underline{c}_{NH}) + (1-\theta^i)u(c_o)\Big] dH(\omega),$$

where $c_o = \omega - \pi^i$. In this case, $MRS(\theta^i, \pi^i, \iota^i) = 0$ for all (π^i, ι^i) and the optimal contract is $(0, 0)$.

We now establish that for $i \in \{b, g\}$, $\iota^i < m$ holds when $\hat{\omega}(\pi^i, \iota^i) \in (\underline{\omega}, \overline{\omega})$ by showing that $\iota^i \geq m$ leads to a contradiction. The utility function, Equation (12) in the paper, can be stated as

$$
\begin{aligned}
U(\theta^i, \pi^i, \iota^i) = &\int_{\underline{\omega}}^{\hat{\omega}(\pi^i,\iota^i)} \left[\theta^i u(\underline{c}_{NH}) + (1-\theta^i)u(c_o)\right] dH(\omega) \\
&+ \int_{\hat{\omega}(\pi^i,\iota^i)}^{\overline{\omega}} \left[\theta^i u(c_{NH}) + (1-\theta^i)u(c_o)\right] dH(\omega),
\end{aligned}
$$

where $c_{NH} = \omega - m + \iota^i - \pi^i$ and $c_o = \omega - \pi^i$, and the marginal rate of substitution is

$$
\begin{aligned}
MRS(\theta^i, \pi^i, \iota^i) &= \frac{\theta^i \int_{\hat{\omega}}^{\bar{\omega}} u'(c_{NH}) dH(\omega)}{\theta^i \int_{\hat{\omega}}^{\bar{\omega}} u'(c_{NH}) dH(\omega) + (1-\theta^i) \int_{\underline{\omega}}^{\overline{\omega}} u'(c_o) dH(\omega)}, \\
&= \left[1 + \frac{(1-\theta^i)}{\theta^i} \frac{\int_{\underline{\omega}}^{\overline{\omega}} u'(c_o) dH(\omega)}{\int_{\hat{\omega}}^{\bar{\omega}} u'(c_{NH}) dH(\omega)}\right]^{-1}.
\end{aligned}
$$

If $\iota^i \geq m$ then $MRS(\theta^i, \pi^i, \iota^i) < \theta^i$. To see this suppose that $MRS(\theta^i, \pi^i, \iota^i) \geq \theta^i$ which implies that

$$
\begin{aligned}
1 + \frac{(1-\theta^i)}{\theta^i} \frac{\int_{\underline{\omega}}^{\overline{\omega}} u'(c_o) dH(\omega)}{\int_{\hat{\omega}}^{\bar{\omega}} u'(c_{NH}) dH(\omega)} \leq \frac{1}{\theta^i} \Leftrightarrow \\
\int_{\underline{\omega}}^{\hat{\omega}} u'(c_o) dH(\omega) \leq \int_{\hat{\omega}}^{\bar{\omega}} \left[u'(c_{NH}) - u'(c_o)\right] dH(\omega). \qquad (11)
\end{aligned}
$$

Since $\iota^i \geq m$, we have $c_{NH} = \omega - \pi^i + \iota^i - m \geq c_o = \omega - \pi^i$, which implies $u'(c_{NH}) - u'(c_o) \leq 0$. Equation (11) becomes

$$
\int_{\underline{\omega}}^{\hat{\omega}} u'(c_o) dH(\omega) \leq \int_{\hat{\omega}}^{\bar{\omega}} \left[u'(c_{NH}) - u'(c_o)\right] dH(\omega) \leq 0, \qquad (12)
$$

which is a contradiction since $u'(c_0) > 0$ and $\underline{\omega} < \hat{\omega} < \overline{\omega}$.

Having established that $\iota^i \geq m$ implies $MRS(\theta^i, \pi^i, \iota^i) < \theta^i$ for $i \in \{b, g\}$ the final step is to show that this condition violates the necessary conditions for an optimal contact. First consider an optimal pooling contract (π^p, ι^p). Note that $\theta^g < \lambda\eta$ since $\lambda \geq 1$ and $\eta = \psi\theta^g + (1-\psi)\theta^b > \theta^g$. So $MRS(\theta^g, \pi^p, \iota^p) < \lambda\eta$ when $\iota^p \geq m$. This is a contradiction because the optimal pooling contract must satisfy Equation (1). It follows that $\iota^p < m$.

Now consider an optimal separating contract. First, consider good types. Under the optimal contract it must be that $\theta^g < \lambda(\psi\theta^g + (1-\psi)\theta^b A)/(\psi + (1-\psi)B)$, since $\lambda \geq 1$ and due to single-crossing (established in Lemma 1) and incentive compatibility $\pi^g < \pi^b$ so

$$
A = \frac{\theta^b \int_{\hat{\omega}(\pi^g,\iota^g))}^{\bar{\omega}} u'(c_{NH}^g) dH(\omega)}{\theta^b \int_{\hat{\omega}(\pi^b,\iota^b)}^{\bar{\omega}} u'(c_{NH}^b) dH(\omega)} > \frac{\theta^b \int_{\hat{\omega}(\pi^g,\iota^g))}^{\bar{\omega}} u'(c_{NH}^g) dH(\omega) + (1-\theta^b) \int_{\underline{\omega}}^{\bar{\omega}} u'(c_o^g) dH(\omega)}{\theta^b \int_{\hat{\omega}(\pi^b,\iota^b)}^{\bar{\omega}} u'(c_{NH}^b) dH(\omega) + (1-\theta^b) \int_{\underline{\omega}}^{\bar{\omega}} u'(c_o^b) dH(\omega)} = B,
$$

where $c^i_{NH} = \omega - m + \iota^i - \pi^i$ and $c^i_o = \omega - \pi^i$. Hence $MRS(\theta^g, \pi^g, \iota^g) < \lambda(\psi\theta^g + (1-\psi)\theta^b A)/(\psi + (1-\psi)B)$ when $\iota^g \geq m$. This is a contradiction because the equilibrium contract for good types must satisfy Equation (6) in the paper. It follows that $\iota^g < m$.

Second, consider bad types. We have established that if $\iota^b \geq m$ then $MRS(\theta^b, \pi^b, \iota^b) < \theta^b \leq \lambda\theta^b$ since $\lambda \geq 1$. This is a contradiction because the equilibrium contract for bad types must satisfy Equation (7) in the paper. It follows that $\iota^b < m$.

□

Proposition 4. *If the optimal menu is a* $(0, 0)$ *pooling contract then*

$$U(\theta^b, \lambda\theta^b\iota, \iota) < U(\theta^b, 0, 0), \quad \forall \iota \in \mathbb{R}_+, \tag{13}$$

and

$$U(\theta^g, \lambda\eta\iota, \iota) < U(\theta^g, 0, 0), \quad \forall \iota \in \mathbb{R}_+. \tag{14}$$

Proof. The proposition is proved by showing that if the conditions don't hold, one can find a menu with at least one nonzero contract that satisfies all the constraints and delivers non-negative profits.

First, assume that condition (13) does not hold but that condition (14) does. If (13) does not obtain, there exists $\iota \in \mathbb{R}_+$ such that

$$U(\theta^b, \lambda\theta^b\iota, \iota) \geq U(\theta^b, 0, 0). \tag{15}$$

Give bad types $(\lambda\theta^b\iota, \iota)$ and good types $(0, 0)$. Under this menu the insurer's profits are

$$\Pi = (1-\psi)\lambda\theta^b\iota + \psi 0 - (1-\psi)\lambda\theta^b\iota - \psi 0 = 0;$$

the participation constraint for the bad types, which is also their incentive compatibility constraint, holds by condition (15); the participation constraint of the good types is trivially satisfied; and the incentive compatibility constraint for the good types is satisfied since

$$U(\theta^g, \lambda\theta^b\iota, \iota) < U(\theta^g, \lambda\eta\iota, \iota) < U(\theta^g, 0, 0),$$

where the first inequality follows from the fact that $\eta < \theta^b$ and the second from condition (14).

Second, assume that condition (14) does not hold which means there exists $\iota \in \mathbb{R}^+$ such that

$$U(\theta^g, \lambda\eta\iota, \iota) \geq U(\theta^g, 0, 0). \tag{16}$$

Give both types $(\lambda\eta\iota, \iota)$. Under this pooling contract the insurer's profits are

$$\Pi = \lambda\eta\iota - \lambda\eta\iota = 0,$$

the participation constraint of the good types holds by condition (16), and the participation constraint for the bad types holds since condition (16) holds and U satisfies the single-crossing property established in Lemma (1). Note that the incentive compatibility constraints are trivially satisfied since both types get the same contract.

□

Proposition 5. *When $\overline{\omega} - m \leq \underline{c}_{NH}$, the possibility of rejection in equilibrium increases if the distribution of endowments on $[\underline{\omega}, \overline{\omega}]$ is given by $H_1(\cdot)$ instead of $H(\cdot)$ where $H_1(\cdot)$ is first-order stochastically dominated by $H(\cdot)$.*

Proof. It is useful to express Equations (13)–(14) as

$$U(\theta^i, \pi^i, \iota) - U(\theta^i, 0, 0) < 0, \quad \forall \iota \in \mathbb{R}^+, i \in \{g, b\}, \tag{17}$$

where

$$U(\theta^i, \pi^i, \iota) = \int_{\underline{\omega}}^{\overline{\omega}} \Big[\theta^i u(\max(\underline{c}_{NH}, \omega - \pi^i - m + \iota)) + (1 - \theta^i) u(\omega - \pi^i)\Big] dH(\omega),$$

with

$$\pi^i = \begin{cases} \lambda \theta^b \iota, & \text{if } i = b, \\ \lambda \eta \iota, & \text{if } i = g. \end{cases}$$

Without loss of generality, assume that $m \geq \iota \geq \pi > 0$.

Let $\Delta U(H)$ and $\Delta U(H_1)$ represent $U(\theta^i, \pi^i, \iota) - U(\theta^i, 0, 0)$ when the endowment distribution is given by $H(\cdot)$ and $H_1(\cdot)$, respectively. Then

$$\Delta U(H) = \int_{\underline{\omega}}^{\overline{\omega}} \tilde{u}(\omega) dH(\omega),$$

and

$$\Delta U(H_1) = \int_{\underline{\omega}}^{\overline{\omega}} \tilde{u}(\omega) dH_1(\omega),$$

where

$$\tilde{u}(\omega) = \Big[\theta^i u(\max(\underline{c}_{NH}, \omega - \pi^i - m + \iota)) + (1 - \theta^i) u(\omega - \pi^i)\Big] - \\ \Big[\theta^i u(\max(\underline{c}_{NH}, \omega - m)) + (1 - \theta^i) u(\omega)\Big].$$

If $\tilde{u}(\omega)$ is non-decreasing then $\Delta U(H) \geq \Delta U(H_1)$ and rejections are weakly more likely under H_1 than H. When $\overline{\omega} - m \leq \underline{c}_{NH}$ we have

$$\tilde{u}(\omega) = \Big[\theta^i u(\max(\underline{c}_{NH}, \omega - \pi^i - m + \iota)) + (1 - \theta^i) u(\omega - \pi^i)\Big] - \Big[\theta^i u(\underline{c}_{NH}) + (1 - \theta^i) u(\omega)\Big],$$

and

$$\frac{d\tilde{u}(\omega)}{d\omega} = \begin{cases} \theta^i u'(\omega - \pi^i - m + \iota) + (1 - \theta^i)[u'(\omega - \pi) - u'(\omega)], & \omega - \pi^i + \iota > \underline{c}_{NH}, \\ (1 - \theta^i)[u'(\omega - \pi^i) - u'(\omega)], & \omega - \pi^i + \iota \leq \underline{c}_{NH}. \end{cases}$$

It is easy to see that $\frac{d\tilde{u}(\omega)}{d\omega} > 0$.

□

Note that the assumption that $\overline{\omega} - m \leq \underline{c}_{NH}$ is sufficient but not necessary. Thus it may be possible to relax this assumption.

Proposition 6. *When $\lambda > 1$ and θ^b is sufficiently close to 1, the possibility of rejection in equilibrium increases if:*

1. *θ^b increases;*

2. *θ^b increases and θ^g decreases such that the mean NH entry probability $\eta \equiv \psi\theta^g + (1 - \psi)\theta^b$ does not change.*

Proof. Note that the proof is for the no Medicaid case, although, as the quantitative results illustrate, the proposition holds even when Medicaid is present. Without Medicaid, rejection will occur in equilibrium iff

$$f^b(\theta^b) \equiv \lambda\theta^b - MRS(\theta^b, 0, 0) \geq 0, \tag{18}$$

and

$$f^g(\theta^g, \theta^b) \equiv \lambda\eta - MRS(\theta^g, 0, 0) \geq 0, \tag{19}$$

where $\eta \equiv \psi\theta^g + (1 - \psi)\theta^b$.

1. Differentiating f^b with respect to θ^b yields

$$\frac{f^b(\theta^b)}{d\theta^b} = \lambda - \frac{\int_{\underline{\omega}}^{\overline{\omega}} u'(\omega - m)dH(\omega) \int_{\underline{\omega}}^{\overline{\omega}} u(\omega)dH(\omega)}{[\theta^b \int_{\underline{\omega}}^{\overline{\omega}} u'(\omega - m)dH(\omega) + (1 - \theta^b) \int_{\underline{\omega}}^{\overline{\omega}} u'(\omega)dH(\omega)]^2}. \tag{20}$$

   When $\theta^b = 1$, Equation (20) is positive since

$$\frac{f^b(\theta^b)}{d\theta^b}|_{\theta^b=1} = \lambda - \frac{\int_{\underline{\omega}}^{\overline{\omega}} u'(\omega)dH(\omega)}{\int_{\underline{\omega}}^{\overline{\omega}} u'(\omega - m)dH(\omega)},$$

   $\lambda \geq 1$ and $u'(\omega) < u'(\omega - m)$ for $m > 0$. It is easy to see that Equation (20) is increasing in θ^b. Thus if θ^b is sufficiently close to 1, increasing θ^b will increase f^b. Differentiating f^g with respect to θ^b yields

$$\frac{f^g(\theta^g, \theta^b)}{d\theta^b} = \lambda(1 - \psi) > 0.$$

   Thus increasing θ^b increases f^g.

2. The proof that f^b is increases is the same as in 1 since f^b does not depend on θ^g. The first term of f^g does not change. The second term only depends θ^g and differentiating it with respect to θ^g yields

$$\frac{dMRS(\theta^g, 0, 0)}{d\theta^g} = \frac{\int_{\underline{\omega}}^{\overline{\omega}} u'(\omega - m)dH(\omega) \int_{\underline{\omega}}^{\overline{\omega}} u(\omega)dH(\omega)}{[\theta^g \int_{\underline{\omega}}^{\overline{\omega}} u'(\omega - m)dH(\omega) + (1 - \theta^g) \int_{\underline{\omega}}^{\overline{\omega}} u'(\omega)dH(\omega)]^2} > 0.$$

   So f^g increases when θ^g declines.

□

Corollary 7. *Consider two groups of individuals, 1 and 2, such that NH risk is on average higher in group 1 than group 2 ($\eta_1 > \eta_2$). If $\lambda > 1$ then it is possible that, in equilibrium, group 1 is rejected and group 2 is not ($\iota_1^i = 0 < \iota_2^i$, $i \in \{b, g\}$).*

Proof. Suppose group 2 is not rejected and group 1 has a higher θ^b then group 2 but is otherwise identical. Assume also that θ^b of group 2 is sufficiently close to 1 so that Proposition 6 holds. Then by part 1. of Proposition 6 it is possible that group 1 is rejected.

□

Another way to get LTCI take-up rates to be declining in η is to assume that $\lambda = 1$ and instead suppose that $\underline{c}_{NH} > 0$ and that expected endowments are negatively correlated with NH entry rates. In particular, if one considers two risk groups $\eta_1 > \eta_2$ the same result can also obtain if the expected endowment of risk group 1 is lower than the expected endowment of risk group 2.

2 Details of the data work

Our HRS sample is constructed from the 1992 to 2012 waves of HRS and AHEAD. The sample is essentially the same as Braun et al. (2015) and Kopecky and Koreshkova (2014). Beyond adding additional data from 1992, 1994, and 2012, there are a few other changes. There is no censoring at -500 and 500 for asset values near 0. We assign an individual's 1998 weight (or post-1998 mean weight, if their 1998 weight is 0) to pre-1998 waves where their weight is 0. The main definitional novelties/changes are now provided. An individual is retired if his labor earnings are less than \$1500 (in 2000 dollars). An individual is considered to have ever had long-term care insurance if they report having been covered in half or more of their observed waves.

Nursing home event A nursing home event occurs when an individual spends 100 days or more in a nursing home within the approximately two year span between HRS interviews or within the period between their last interview and death. If the individual dies less than 100 days after their last interview, but at the time of their death had been in a nursing home for over 100 days, this also counts as a nursing home event. In the HRS, there are several (sometimes inconsistent) variables that provide information about the number of days spent in a nursing home. From the RAND dataset, we use the total nursing nights over all stays during the wave, as well as the number of days one has been in a nursing home (conditional on being in a nursing home at the time of the interview). This information is also pulled from the exit data, as well as the date of entry to a nursing home, provided the individual died there. Interview and death dates are used when a respondent reports having been continuously in a nursing home since the previous wave. Since the information is sometimes conflicting, and one piece often missing when another observed, a nursing home event is assigned if any of the variables suggest a person met the criteria described above.

Permanent Income To calculate permanent income, first sum the household heads social security and pension income and average this over all waves in which the household head is retired. The cumulative distribution of this average is defined as the permanent income, which ranges from 0 to 1. For singles, the household head is the respondent, and for couples it is the male.

Wealth We use the wealth variable ATOTA which is the sum of the value of owned real estate (including primary residence), vehicles, businesses, IRS/Keogh accounts, stocks, bonds, checking/savings accounts, CDs, treasury bills and "other savings and assets" less any debt reported.

2.1 Construction of the Frailty Index

Table 1 lists the variables we used to construct the frailty index for HRS respondents. The choice of these variables is based on Genworth and Mutual of Omaha LTCI underwriting guidelines. To construct the frailty index, first sum the variables listed in the first column of Table 1, assigning each a value according to the second column. Then divide this sum by the total number of variables observed for the individual in the year, as long as the total includes 30 or more variables. The construction of this frailty index mostly follows the guidelines laid out in Searle et al. (2008), and uses a set of HRS variables similar to the index created in Yang and Lee (2009). There are a couple of differences however. Primarily, a few variables that do not necessarily increase with age (e.g. drinking > 15 drinks per week and smoking) were included. Also, cognitive tests are broken into parts which each count as separate variables, essentially increasing their weight in the index relative to Searle et al. (2008), which uses only a single variable for cognitive impairment. Nevertheless, our frailty distribution still closely resembles those of frailty indices used in other papers.

2.2 Evidence of private information

Hendren (2013) finds that self-assessed NH entry risk is only predictive of a NH event for individuals who would likely be rejected by insurers. Hendren's measure of a NH event is independent of the length of stay. Since we focus on stays that are at least 100 days, we repeat the logit analysis of Hendren (2013) using our definition of a NH stay and our HRS sample. We get qualitatively similar results. We restrict the sample to individuals ages 65–80. We find evidence of private information at the 10 year horizon (but not at the 6 year) in a subsample of this sample consisting of individuals who would likely be rejected by insurers. This sample includes individuals who have any ADL/IADL restriction, past stroke, or past nursing or home care. The p-value for a Wald test which restricts the coefficients on subjective probabilities to zero is 0.003 at the 10 year horizon and 0.169 at the 6 year. If all individuals above age 80 are included in the reject sample as well the p-values at both horizons are less than 0.000. For a sample of individuals who would likely not be rejected we are unable to find evidence of private information. The p-value for a Wald test which restricts the coefficients on subjective probabilities to zero is 0.210 at the 10 year horizon and 0.172 at the 6 year.

Table 1: Health Variables for Frailty Index Construction

Variable	Value
Some difficulty with ADL/IADLs:	
Eating	Yes=1, No=0
Dressing	Yes=1, No=0
Getting in/out of bed	Yes=1, No=0
Using the toilet	Yes=1, No=0
Bathing/shower	Yes=1, No=0
Walking across room	Yes=1, No=0
Walking several blocks	Yes=1, No=0
Using the telephone	Yes=1, No=0
Managing money	Yes=1, No=0
Shopping for groceries	Yes=1, No=0
Preparing meals	Yes=1, No=0
Getting up from chair	Yes=1, No=0
Stooping/kneeling/crouching	Yes=1, No=0
Lift/carry 10 lbs	Yes=1, No=0
Using a map	Yes=1, No=0
Taking medications	Yes=1, No=0
Climbing 1 flight of stairs	Yes=1, No=0
Picking up a dime	Yes=1, No=0
Reaching/ extending arms up	Yes=1, No=0
Pushing/pulling large objects	Yes=1, No=0
Cognitive Impairment:	
Immediate Word Recall	+.1 for each word not recalled (10 total)*
Delayed Word Recall	+.1 for each word not recalled (10 total)*
Serial 7 Test	+.2 for each incorrect substraction (5 total)
Backwards Counting	Failed test=1, 2nd attempt = .5, 1st attempt = 0
Identifying obejcts & Pres/VP	.25 for each incorrect answer (4 total)
Identifying date	.25 for each incorrect answer (4 total)
Ever had one of following conditions:	
High Blood Pressure	Yes=1, No=0
Diabetes	Yes=1, No=0
Cancer	Yes=1, No=0
Lung disease	Yes=1, No=0
Heart disease	Yes=1, No=0
Stroke	Yes=1, No=0
Psychological problems	Yes=1, No=0
Arthritis	Yes=1, No=0
BMI ≥ 30	Yes=1, No=0
Drinks 15+ alcoholic drinks per week	Yes=1, No=0
Smokes Now	Yes=1, No=0
Has smoked ever	Yes=1, No=0

*For the 1994 HRS cohort, 40 questions were asked (instead of 20) for word recall. In this year, each missed question receives weight 0.05.

2.3 Description of the auxiliary simulation model

To obtain survival and lifetime NH entry probabilities by frailty and PE quintile groups, we use an auxiliary simulation model similar to that in Hurd et al. (2013). First, using a multinomial logit, we estimate transition probabilities between four states that we observe in the HRS: alive and dead, each with and without a nursing home event in the last two years. These probabilities depend on age, PE, NH event status, and frailty, including polynomials and interactions of these variables. Specifically, age is modeled as a cubic function, frailty as a quadratic, and the others are both linear. Interactions include age with each of the other first-order terms, as well as frailty with PE. We also simulate lifetime frailty paths because we need them since, in contrast to Hurd et al. (2013), we include frailty in the multinomial logit. This is done using estimates from a fixed effects regression of frailty on lagged frailty, age, and age squared.

Simulations begin at age 67. To get the initial distribution of explanatory variables, we first average frailty and population weights across all observations at which an individual is between ages 62-72. PE and the estimated fixed effect are constant within individuals. The initial distribution then draws 500,000 times from this person-level weighted distribution. The model simulates two-year transitions, following the structure of the HRS data, and assigns age of death by randomly choosing an age between their last living wave and the death wave.[1]

3 Computation

Computing an equilibrium in our model is subtle because Medicaid NH benefits are means-tested and Medicaid is a secondary payer of NH benefits. Individual saving policies exhibit jumps and the demand for private insurance interacts in subtle ways with $q(\kappa)$, the distribution of consumption demand shocks.

We start by discretizing the endowment and frailty distributions. The number of grid points for endowments $\mathbf{w}$ is $ny = 101$ and frailty takes on $nf = 5$ grid points. The consumption demand shock κ is also discretized: $nk = 50$.

The specific algorithm for computing an equilibrium proceeds as follows. First, we guess values for profits (which gives us dividends) and taxes and then we iterate over profits and taxes until profits converge and taxes satisfy the government budget constraint. In each iteration, we have to solve for allocations, contracts, and profits for each combination of endowments and frailty in the discretized state space. For each point in the discertized state space $(\mathbf{w}_s, f_j)$, $s = 1, nw$ and $j = 1, nf$, we guess a level of savings: $\hat{a}_{f_j,\mathbf{w}_s}$. Given $\hat{a}_{f_j,\mathbf{w}_s}$, we then solve for the optimal contracts as follows. The optimal contract for a risk group depends on which individuals of observable type $(\mathbf{w}_s, f_j)$ qualify for Medicaid if they incur the NH shock. Thus, it depends on the specific combinations of the κ shock and the private type $i \in \{g, b\}$ that imply that an individual qualifies for Medicaid. Because

[1]Note that a half year is added to death age to account for the fact that reported ages are the floor of a respondent's continuous age. Nursing home entry ages are similarly assigned, but we add only 0.2 years due to the 100 day requirement of a nursing home event. They are also upwardly bound by death age when both occur in the same wave.

Table 2: Model Parameters

Description	Parameter	Value
Risk aversion coefficient	σ	2
Preference discount factor	β	0.94
Retirement preference discount factor	α	0.21
Interest rate (annualized)	r	0.0062
Frailty distribution	h	BETA(1.53,6.70)
Endowment distribution	$[w_y, w_o]'$	LN(-0.32,0.80)
Copula parameter	ρ_{f,w_y}	-0.4
Demand shock distribution	κ	$1-\kappa \sim$ LN(-1.08,0.245)
Fraction of good types	ψ	0.7
Nursing home cost	m	0.0956
Insurer's variable cost of paying claims	λ	1.195
Insurer's fixed cost of paying claims	k	0.016
Medicaid consumption floor	$\underline{c}_{NH}$	0.02
Welfare consumption floor	$\underline{c}_o$	0.02
Tax rate	τ	0.011

of the non-convexities introduced by Medicaid the Kuhn-Tucker conditions of the insurer's problem are not sufficient. However, if one first assumes a distribution of individuals across Medicaid, then a contract satisfying the Kuhn-Tucker conditions is sufficient. So we solve for the optimal contract for all feasible combinations of individuals with different κ's and i's receiving Medicaid. The number of cases that has to be considered is large but it can reduced by the noting that for a given value of κ if a bad type is on Medicaid the good type is also on Medicaid by the single-crossing property and that if a type i qualifies for Medicaid for a value of κ he will also qualify for Medicaid for all larger values of κ.

To solve for the optimal contracts for each Medicaid distribution, first we solve for the optimal pooling contract. Second, we check to see if an optimal separating menu exists. The contract of type g under the optimal separating menu is the same as the optimal pooling contract. So we fix the good type's contract at the optimal pooling one and solve for the optimal separating contract of the bad type (if it exists).[2] The optimal contract for observable type $(\mathbf{w}_s, f_j)$ under the current guess for savings, $\hat{a}_{f_j,\mathbf{w}_s}$ is then the one that maximizes the insurer's profits. Finally, we iterate over savings until we find the value of savings that maximizes expected lifetime utility.

4 Additional Calibration Details

Table 2 lists many of the model's parameters and their calibrated values. The survival probabilities of each frailty and PE quintile in the quantitative model are shown in Figure 1.

[2]If a separating menu doesn't exists it means one of the inequality constraints will be violated.

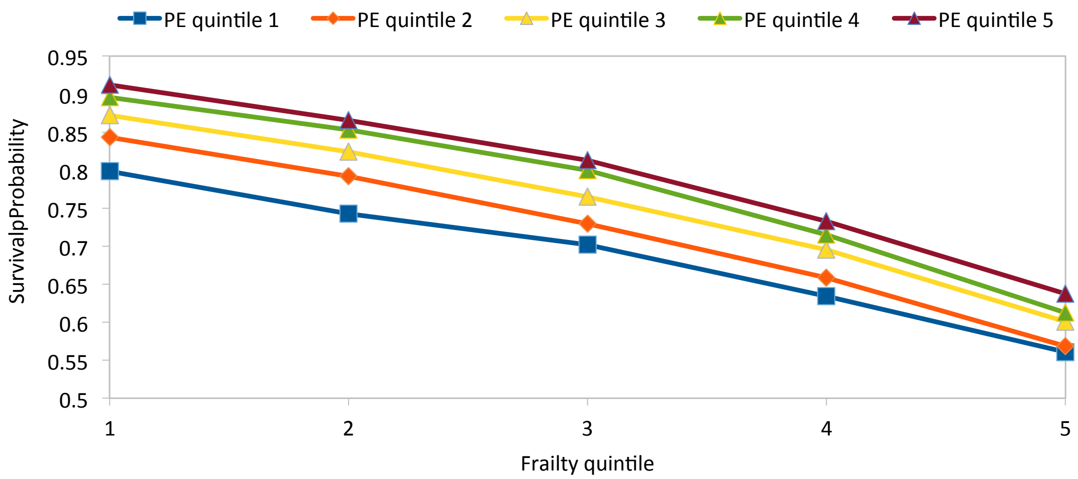

Figure 1: The probability of surviving to age 80 or until experiencing a NH stay by frailty and PE quintile. The probabilities are based on our auxiliary simulation model which is estimated using HRS data.

5 Additional Results

Figures 2 –3 show how Medicaid and adverse selection influence pricing and coverage at alternative frailty and wealth levels. In the Baseline economy, coverage is roughly flat across frailty quintiles for the bad risk types. Removing private information reverses the pattern of coverage and reduces the variation in loads. Premia fall and coverage increases for good types across all frailty quintiles. Bad risks, in contrast, face higher loads and lower levels of coverage in the Full Information economy and see their coverage fall with frailty.[3]

Reducing the scale of Medicaid, in contrast, increases the level of coverage for both private information types at each frailty quintile. However, the loads are also higher. This effect is most pronounced at frailty Q5. In the baseline LTCI insurance covers 49% of the costs for the good risk type and 74% for the bad risk type. When Medicaid is scaled back, coverage increases to 68% for good risks and to 85% for bad risks. Notice also that coverage increases monotonically in frailty in the No Medicaid economy.

Figure 3 reports how coverage and loads vary by wealth quintile for the same three scenarios. In the Baseline and Full Information economies, as wealth increases, bad risk types experience lower coverage and higher loads. Good risk types experience somewhat higher coverage and lower loads as wealth is increased but the overall pattern is hump-shaped. Reducing the Medicaid NH benefit floor has a very big impact on the poor. Their demand for LTCI is inelastic and, as a result, they now face the highest loads but also receive the most coverage.

[3]This final result dates back to Arrow (1963) who demonstrates that the amount of insurance available to those with high risk exposures falls if insurance markets open after their risk exposure is observed.

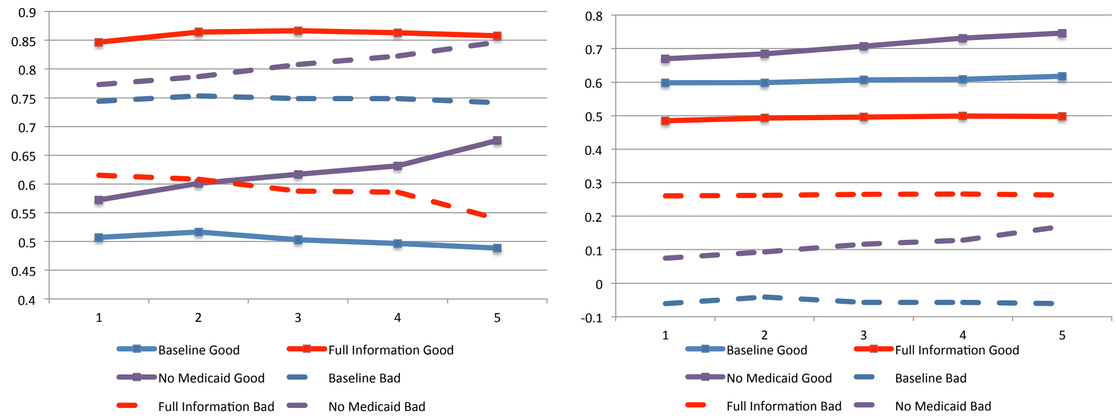

Figure 2: Insurance coverage and loads by frailty quintile.
The left panel reports LTCI indemnities relative to medical costs of a NH stay and the right panel reports loads for the two private information types: good risks and bad risks. Three economies are reported: Baseline, Full Information and No Medicaid.

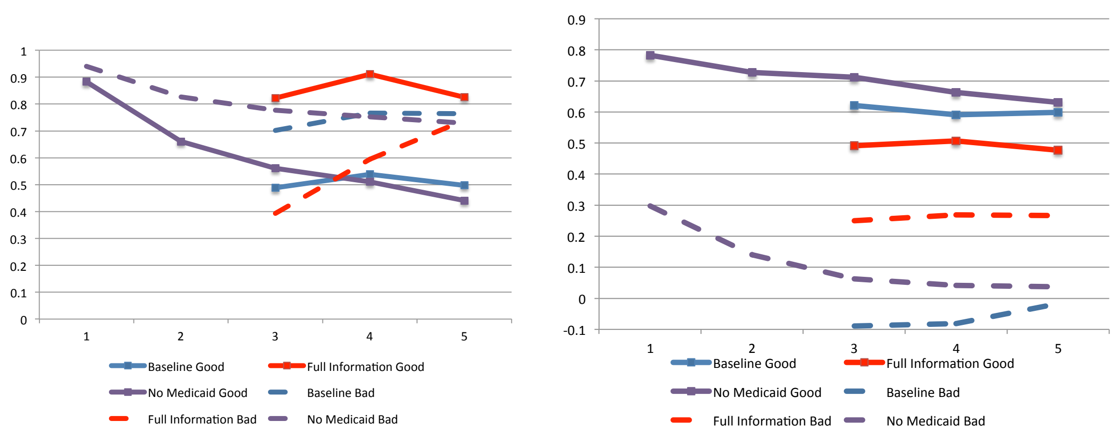

Figure 3: Insurance coverage and loads by wealth quintile.
The left panel reports indemnities relative to the medical cost of a NH stay and the right panel reports loads for the two private information types: good risks and bad risks. Three economies are reported: Baseline, Full Information and No Medicaid.

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